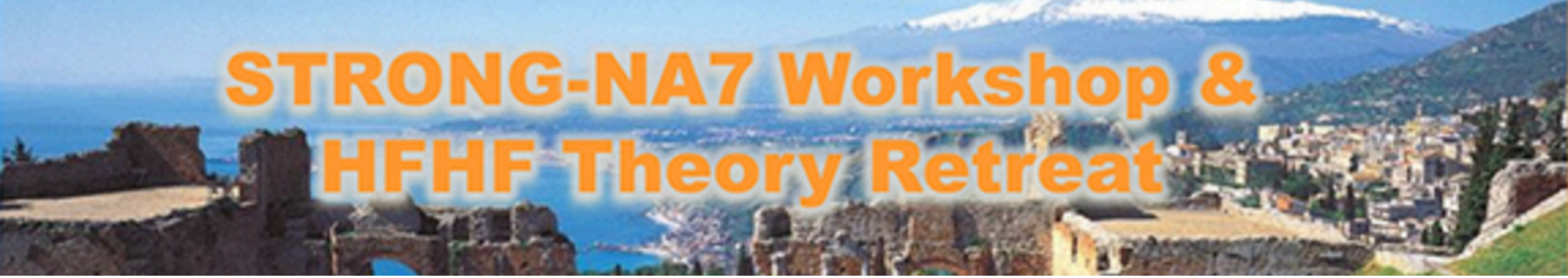


STRONG-NA7 Workshop &
HFHF Theory Retreat

Electromagnetic fields and heavy flavor in relativistic heavy ion collisions

Jiaxing Zhao (SUBATECH)

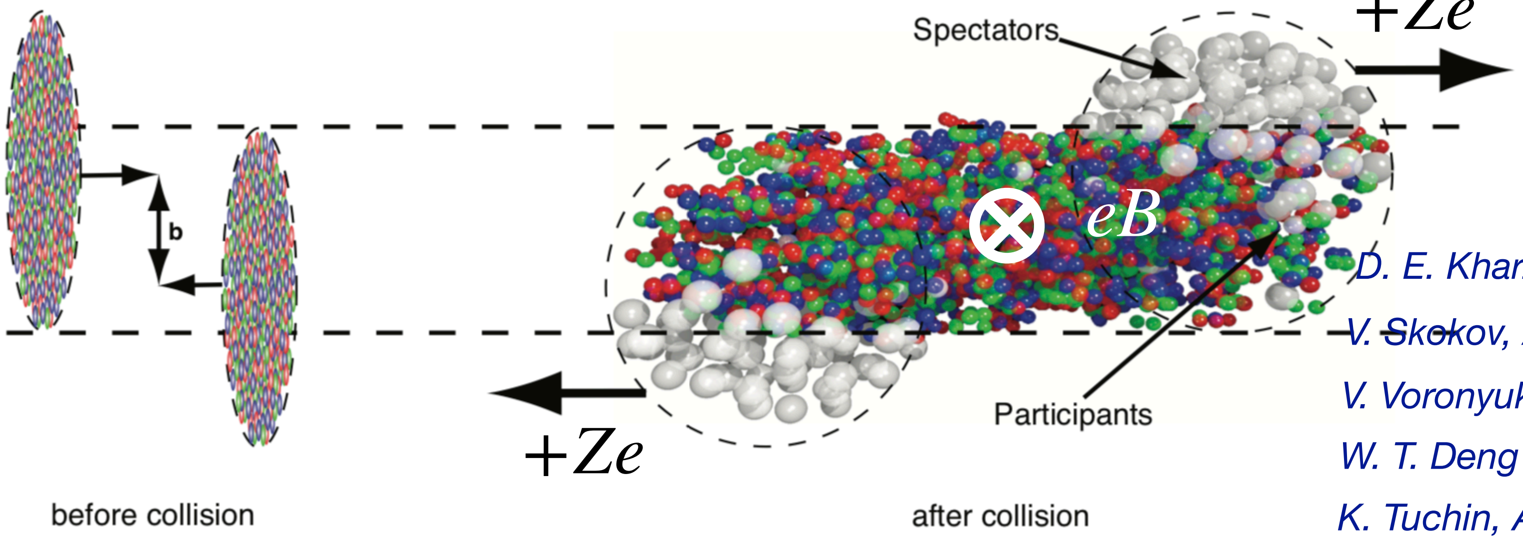
jzhao@subatech.in2p3.fr

29/09/2023

Outline

- *Brief introduction about the electromagnetic fields created in HICs and related HF studies*
- *Electromagnetic fields evolution in the early stage of HIC and Incomplete electromagnetic response of hot QCD matter*
- *Heavy Flavor production in a strong magnetic field.*
- *Summary*

Electromagnetic in HIC



$$eB_y \sim \gamma\beta \frac{Z\alpha}{R_A^2} \sim m_\pi^2$$

D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, NPA 803 (2008) 227-253

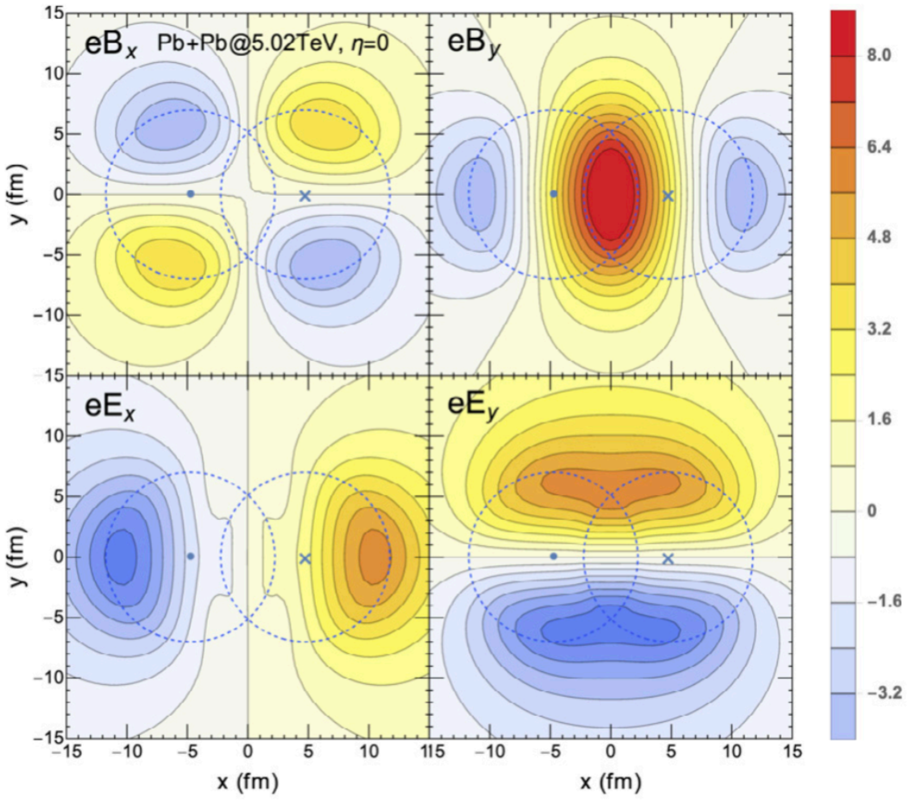
V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925-5932 (2009

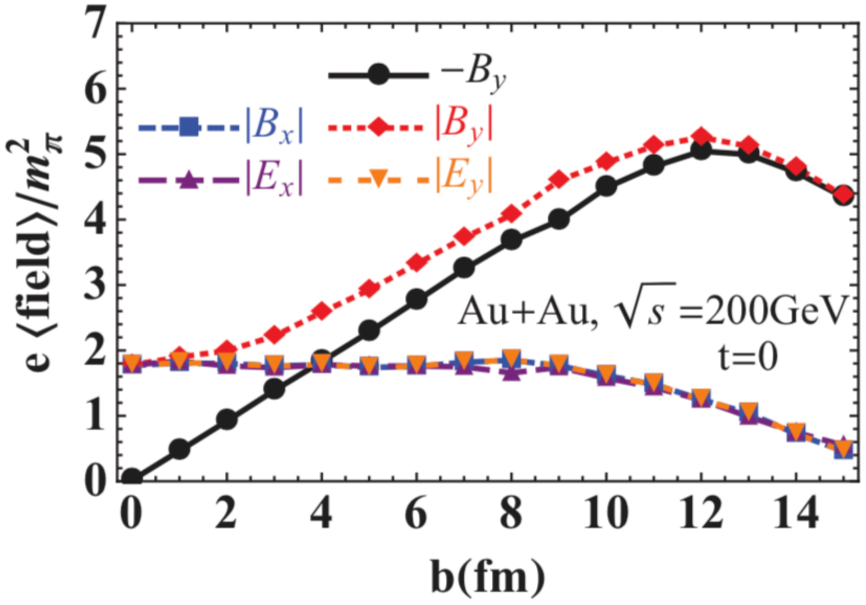
V. Voronyuk, and S. A. Voloshin. et al ,PRC 83 (2011) 054911 .

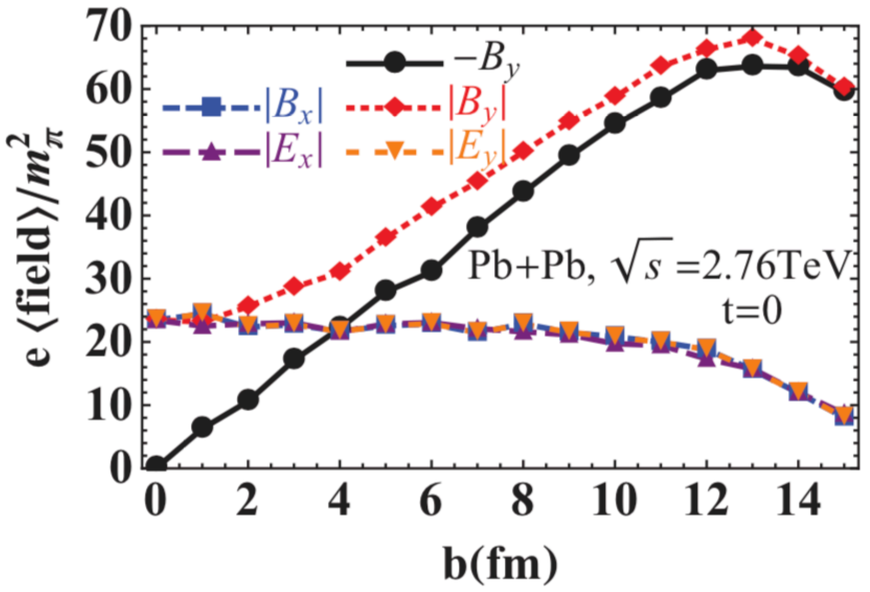
W. T. Deng and X. G. Huang, PRC 85, 044907 (2012).

K. Tuchin, Adv. High Energy Phys. 2013, 490495 (2013).

V. Voronyuk, D. Toneev, W. Cassing, E. Bratkovskaya et al, PRC,83,054911(2011)…



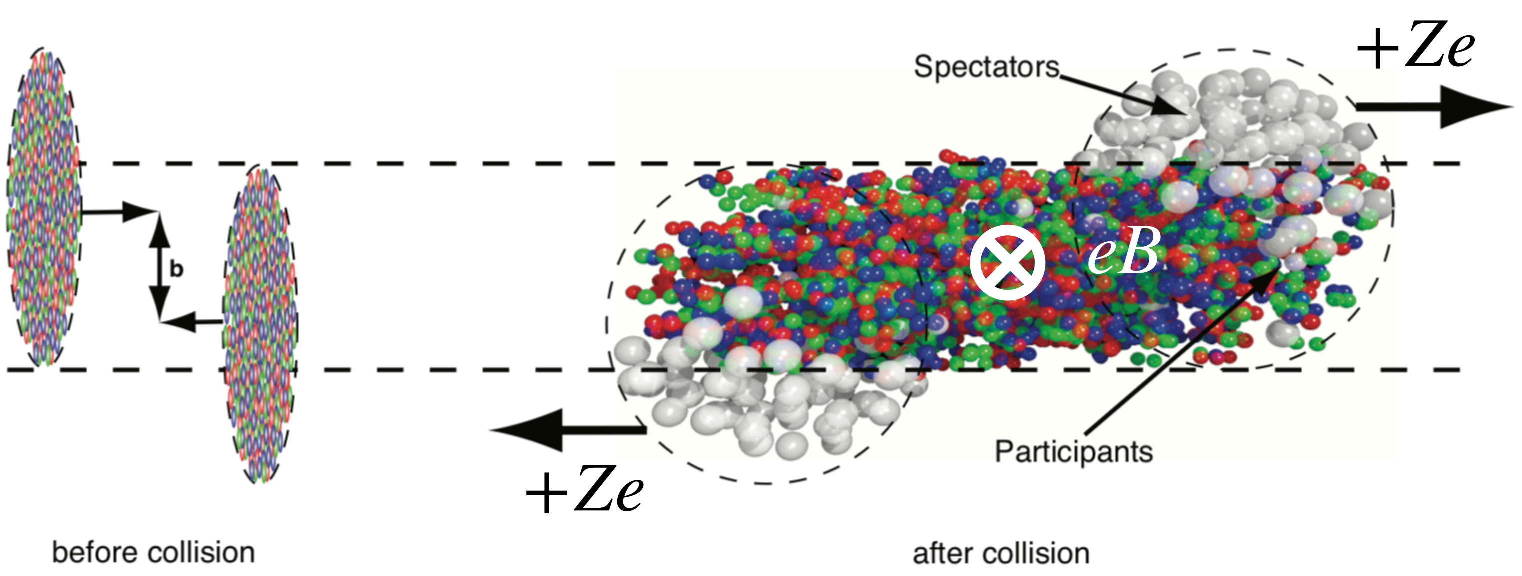


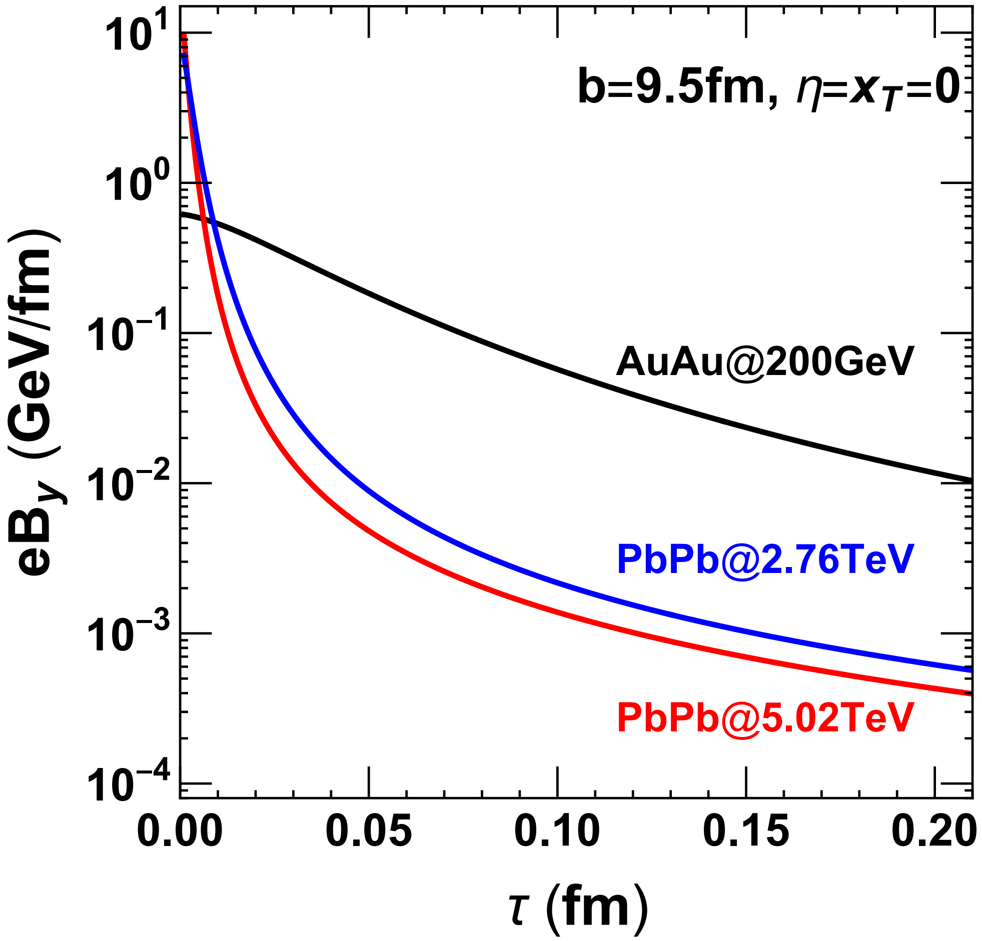


W. Deng and X. Huang, PRC 85, 044907 (2012).

Strongest B field and E field generated in HIC.
(~70m_π^2 at LHC and ~5m_π^2 at RHIC; ~10^{20} Gauss).

Heavy Flavor: a sensitive probe to EM fields





- ✦ $\tau_B \approx R/\gamma \sim 0.1\mathrm{fm/c}$, *lifetime of the strong electromagnetic fields in vacuum.*
- ✦ $\tau_c \sim 1/m_c \approx 0.07\mathrm{fm/c}, \tau_b \sim 1/m_b \approx 0.02\mathrm{fm/c}$, *produced at very stage.*

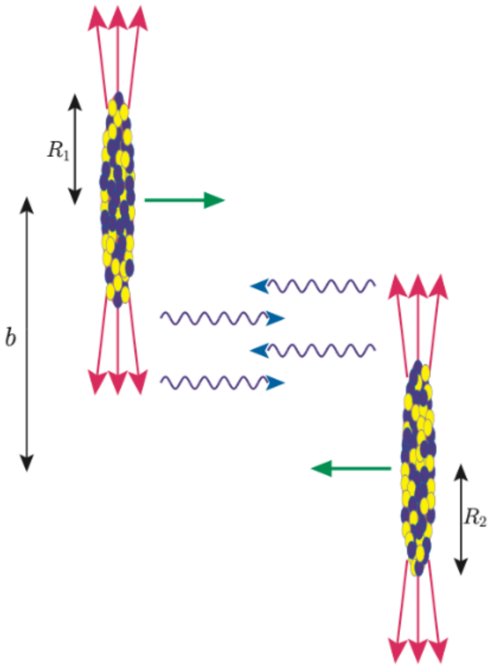
$m_c \sim 1.5\mathrm{GeV} \quad m_b \sim 4.7\mathrm{GeV}$

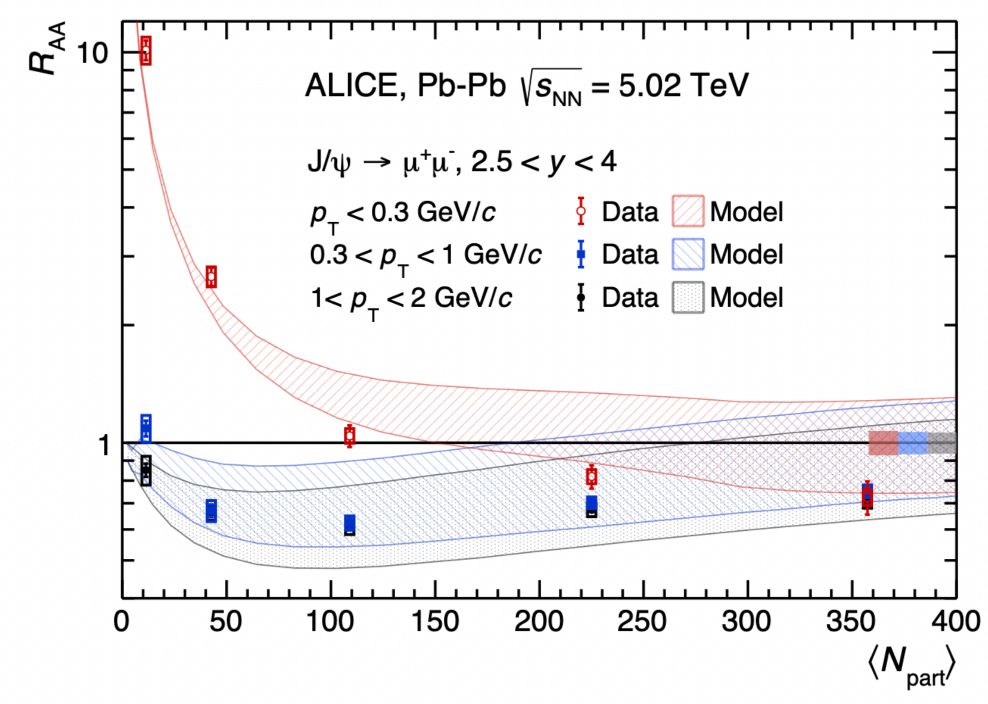
$$\tau_c, \tau_b \lesssim \tau_B$$

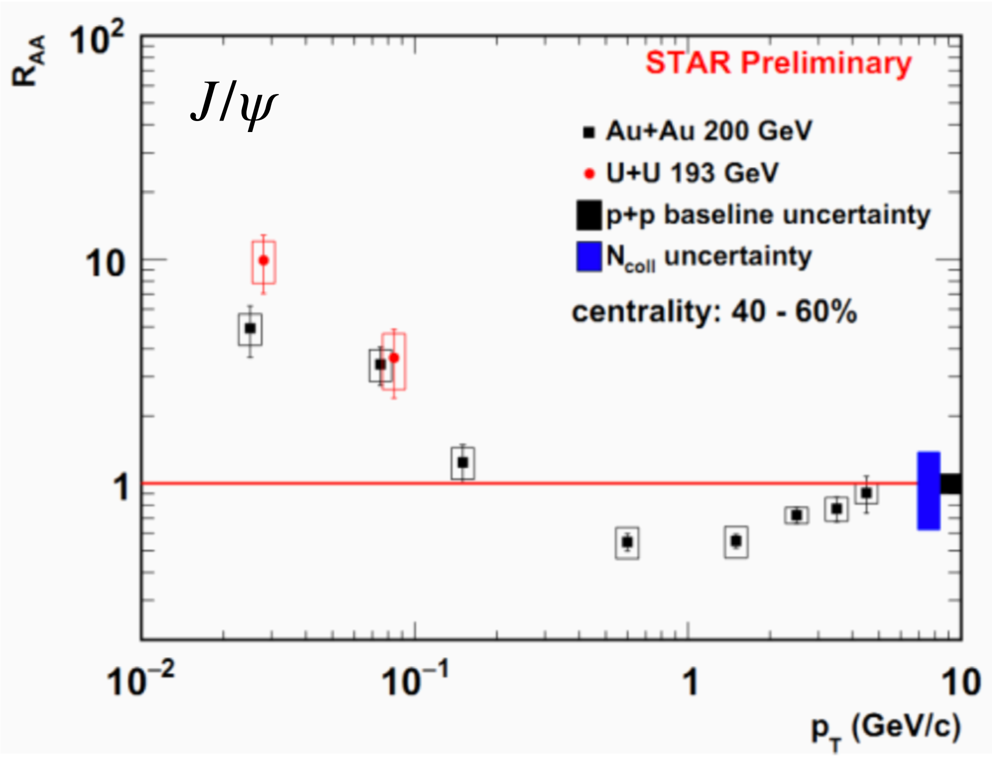
Heavy flavor can be a nice probe!

EM fields related studies(…)

- *Quarkonium photoproduction*





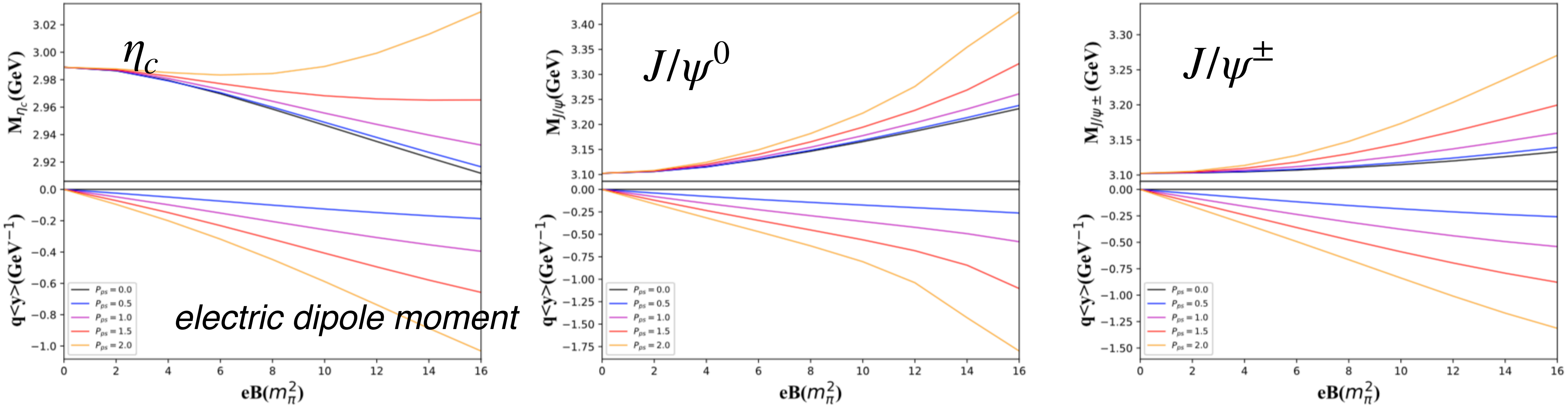


Exceed at very low pT!

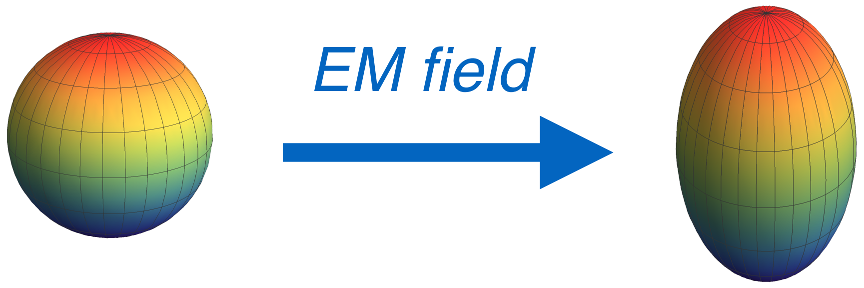
See also:
W. Zha. Z. Tang, Y. Zhang. L. Ruan. Z. Xu. et al.
NPPP. 289-290, PLB 789 (2019) 238-242 ;
B. Chen. et al (PLB. 777(2018)). …

EM fields related studies(…)

- *Quarkonium photoproduction*
- *Change the static properties of heavy flavor quarkonium.*



$$\left[\frac{(\boldsymbol{p}_a - q_a\boldsymbol{A}_a)^2}{2m_q} + \frac{(\boldsymbol{p}_b - q_b\boldsymbol{A}_b)^2}{2m_q} + V(r) - \boldsymbol{\mu}\cdot\boldsymbol{B}\right]\Psi(\boldsymbol{X},\boldsymbol{x}) = (E - 2m_q)\Psi(\boldsymbol{X},\boldsymbol{x})$$



See also:

K. Marasinghe and K. Tuchin, PRC 84 (2011) 044908.

J.Alford and M.Strickland, PRD88, 105017(2013).

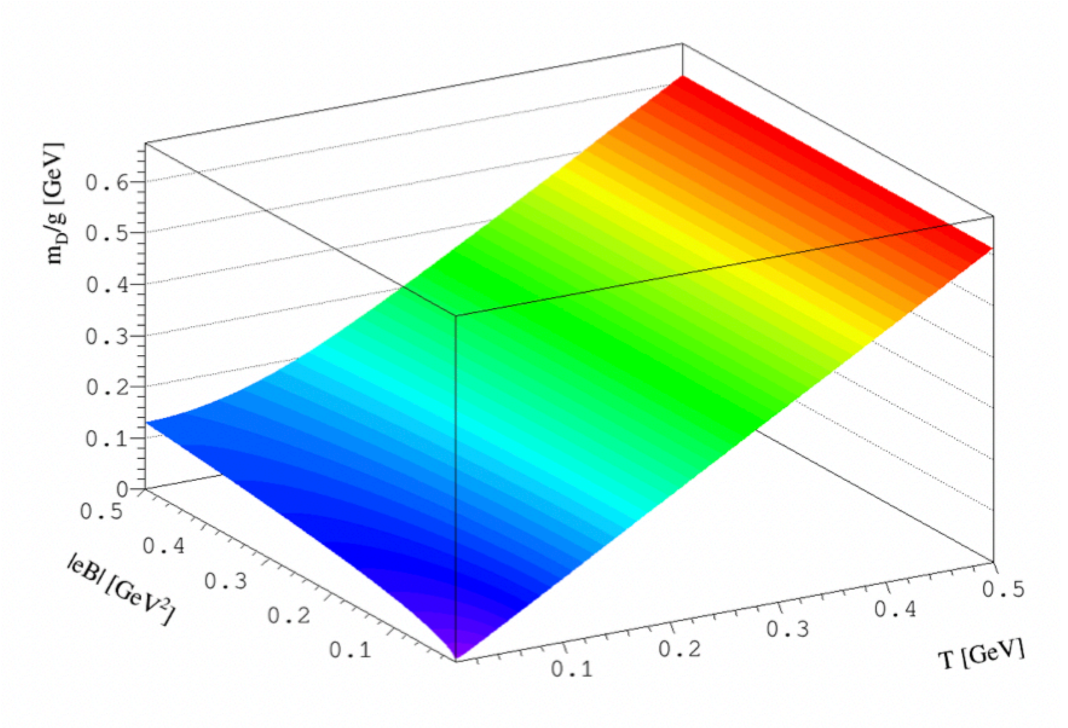
S.Cho, K.Hattori, S.Lee, K.Morita and S.Ozaki, PRL. 113, 172301(2014).
T.Yoshida and K.Suzuki, PRD94, 074043(2016).

S. Iwasaki, M. Oka, K. Suzuki, Eur.Phys.J.A 57 (2021) 7, 222.

*S. Chen, **JZhao**, P. Zhuang, Phys.Rev.C 103 (2021) 3, L031902 …*

EM fields related studies(…)

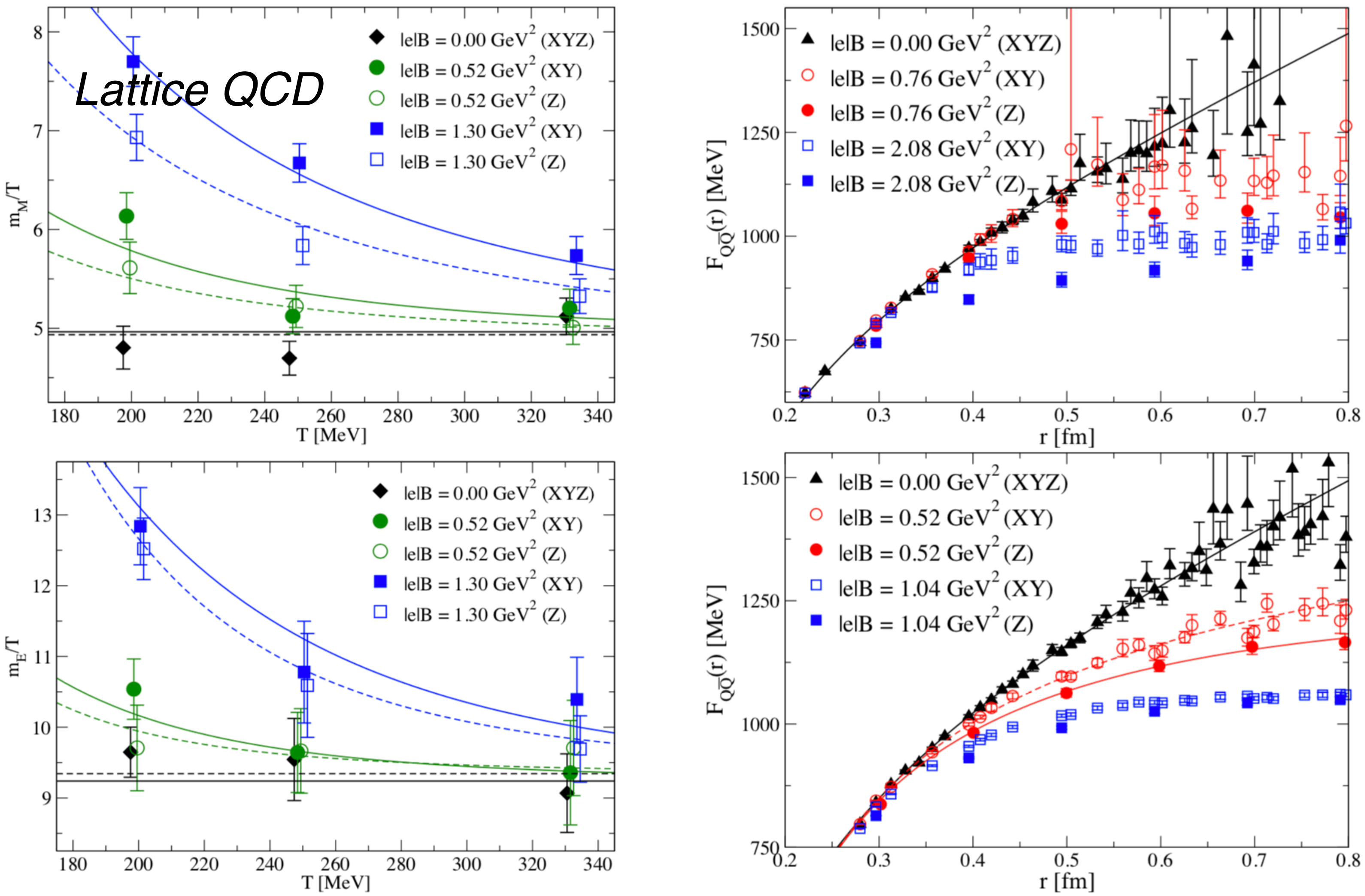
- *Quarkonium photoproduction*
- *Change the static properties of heavy flavor quarkonium.*
- *Change the Debye screening mass and heavy quark potentail.*



$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$
$$m_Q^2(T,B) = -\Pi^{||}_{00}(T,B),$$
$$m_G^2(T) = -\overline{\Pi}^{||}_{00}(T).$$
$$m_Q^2(T,B) = -g^2 T|qB| \sum_{np_zn_1} \Big[(2-\delta_{n_1,0})\,\frac{m^2-\omega_n^2+p_z^2+2n_1|qB|}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2}$$
$$m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

G. Huang, **JZhao**, and P. Zhuang. PRD 107 (2023) 11, 114035

G. Huang, **JZhao**, and P. Zhuang. arXiv:2307.02608.



C. Bonati, M, D’Elia, M, Mariti, et al . PRD 94 (2016) 9, 094007; PRD, 2017, 95(7):074515.

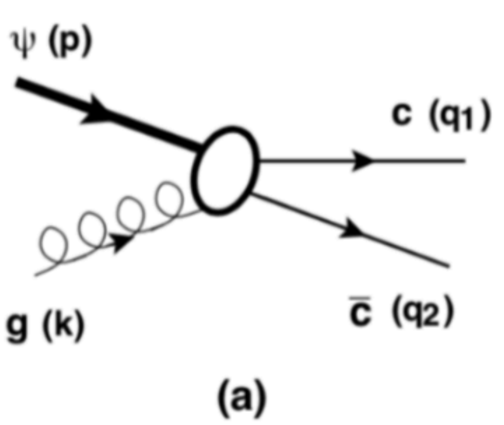
See also:

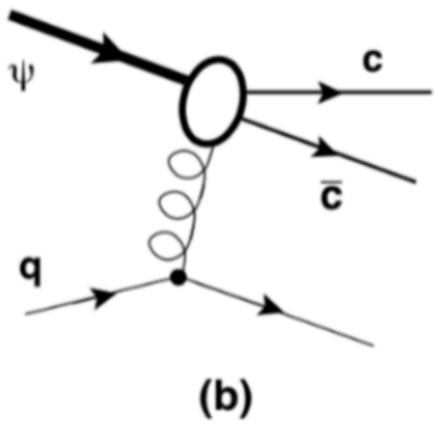
B. Singh, L. Thakur, H. Mishra. Phys. Rev. D, 2018, 97(9):096011.

M. Hasan, K. Patra . Phys. Rev. D, 2020, 102(3):036020.

EM fields related studies(…)

- *Quarkonium photoproduction*
- *Change the static properties of heavy flavor quarkonium.*
- *Change the Debye screening mass and heavy quark potentail.*
- *Quarkonium dynamical dissociation in hot and magnetized QCD medium*

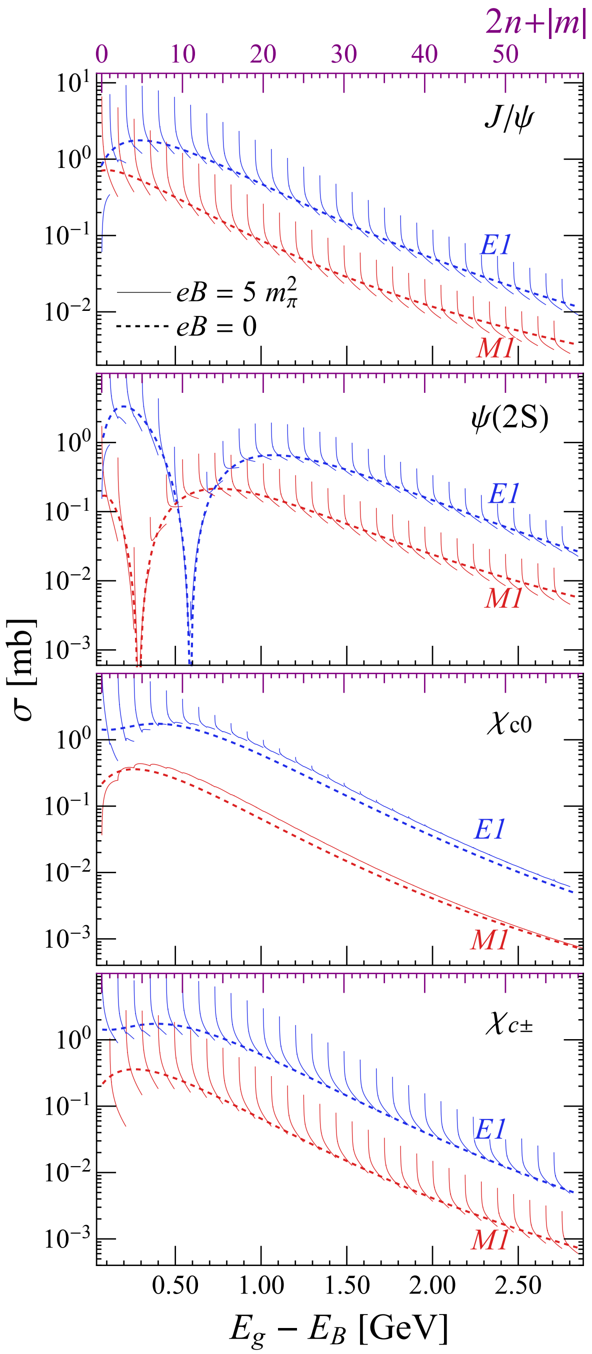


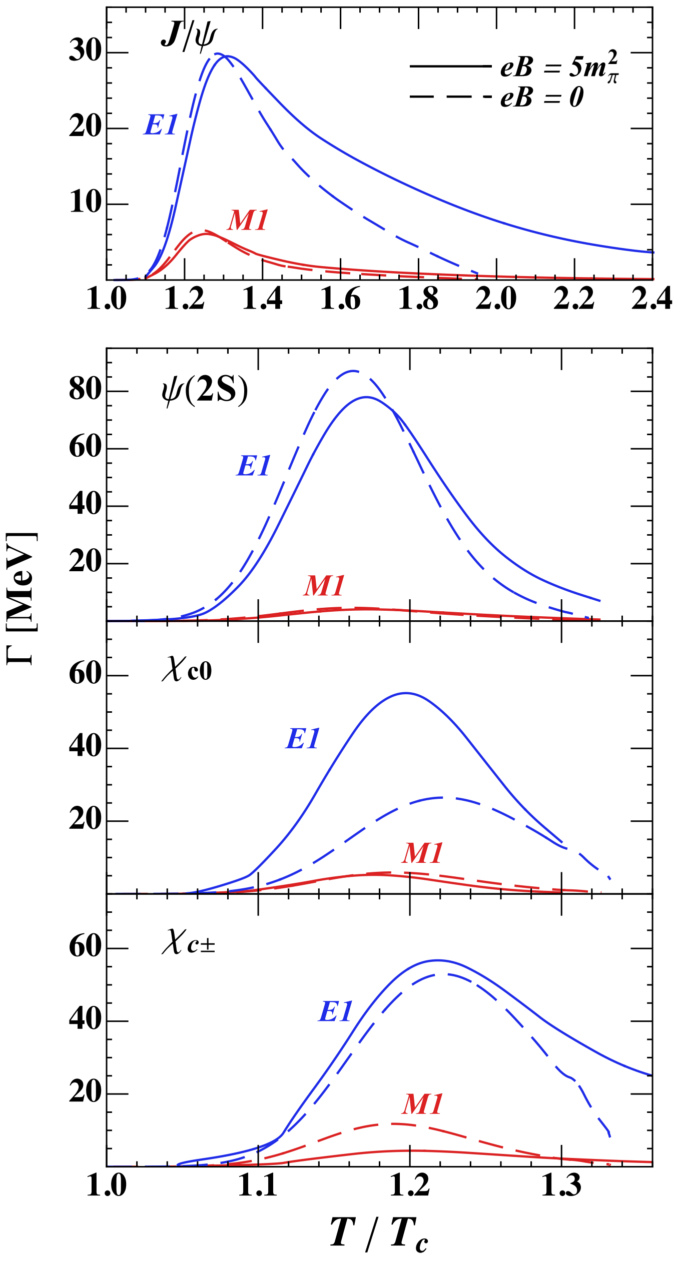


See also:

B. Singh, L. Thakur, H. Mishra. Phys. Rev. D, 2018, 97(9):096011.

M. Hasan, K. Patra . Phys. Rev. D, 2020, 102(3):036020.

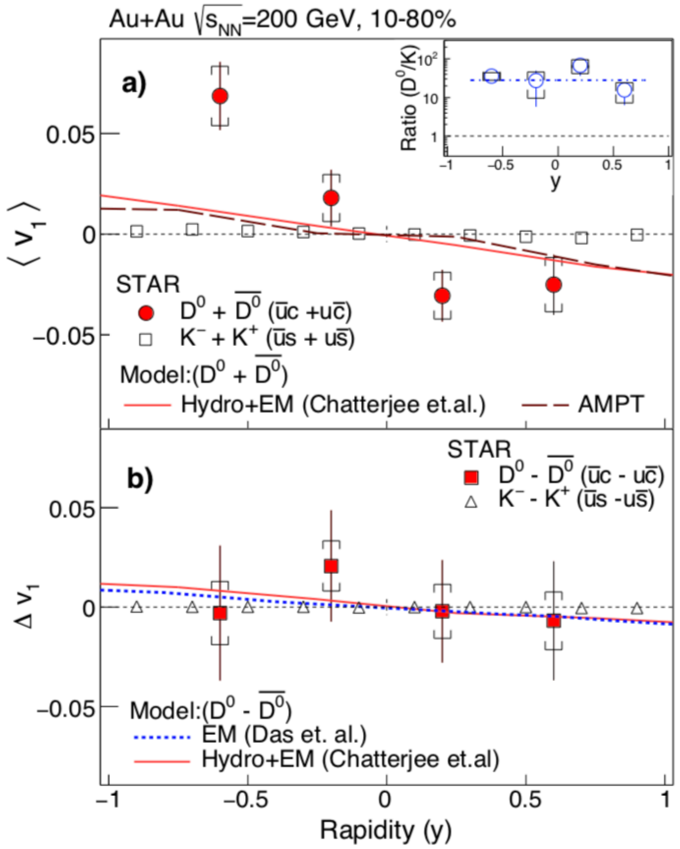


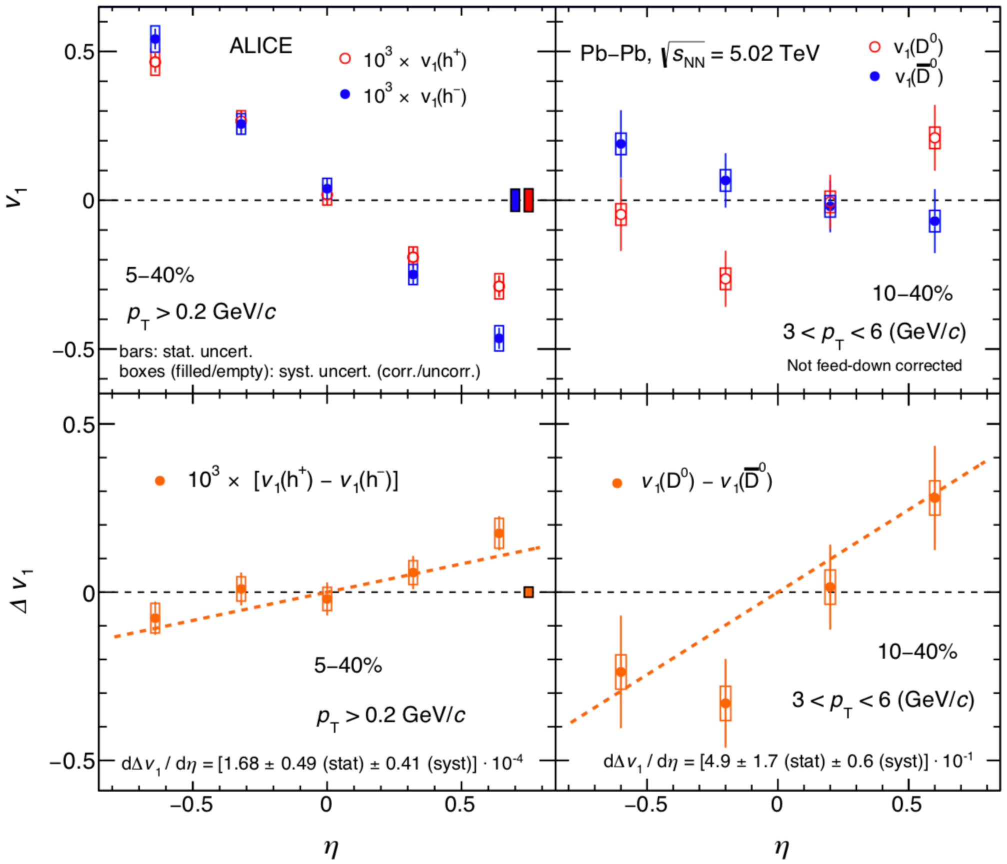


*J. Hu, S. Shi, Z. Xu, **JZhao***, and P. Zhuang, Phys.Rev.D 105 (2022) 9, 094013*

EM fields related studies(…)

- *Quarkonium photoproduction*
- *Change the static properties of heavy flavor quarkonium.*
- *Change the Debye screening mass and heavy quark potentail.*
- *Quarkonium dynamical dissociation in hot and magnetized QCD medium*
- *Probably: splitting of the directed flow of* D *and* $\bar{D}$*, …*



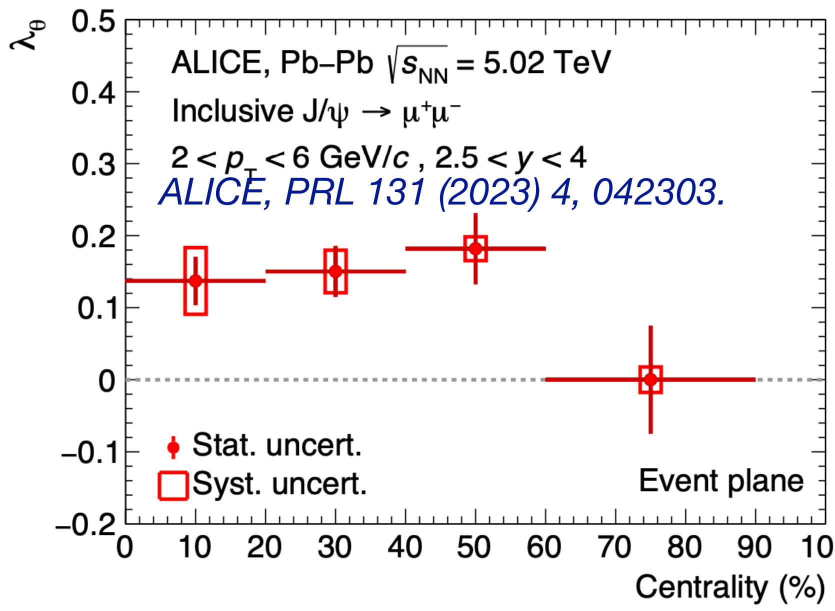


$$\Delta v_1 = v_1(D^0) - v_1(\bar{D}^0)$$

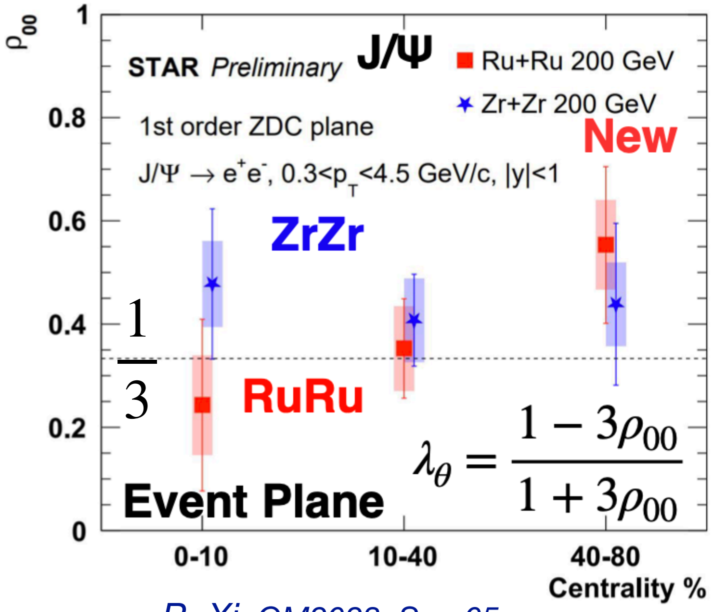
See also:
S. Chatterjee and P. Bozek. PRL120(2018)192301;
Y. Sun, S. Das, S. Plumari, V. Greco. et al. PLB768(2017) 260-264. PLB 816 (2021) 136271.

EM fields related studies(…)

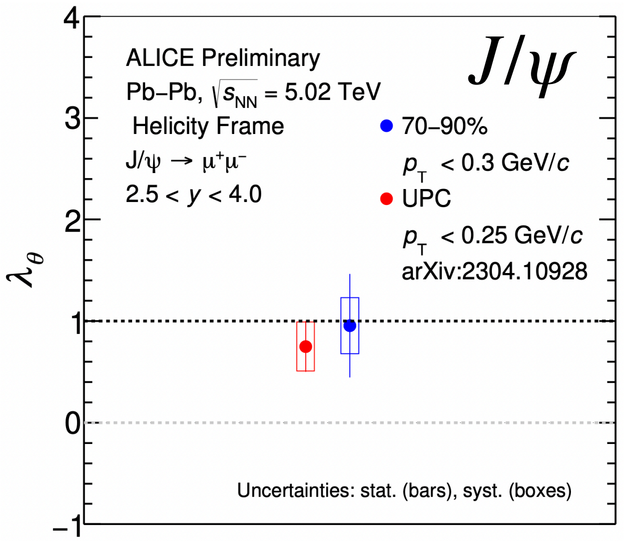
- *Quarkonium photoproduction*
- *Change the static properties of heavy flavor quarkonium.*
- *Change the Debye screening mass and heavy quark potentail.*
- *Quarkonium dynamical dissociation in hot and magnetized QCD medium*
- *Probably: splitting of the directed flow of* D *and* $\bar{D}$*, …*
- *Probably:* $J/\psi, D$ *meson polarization*



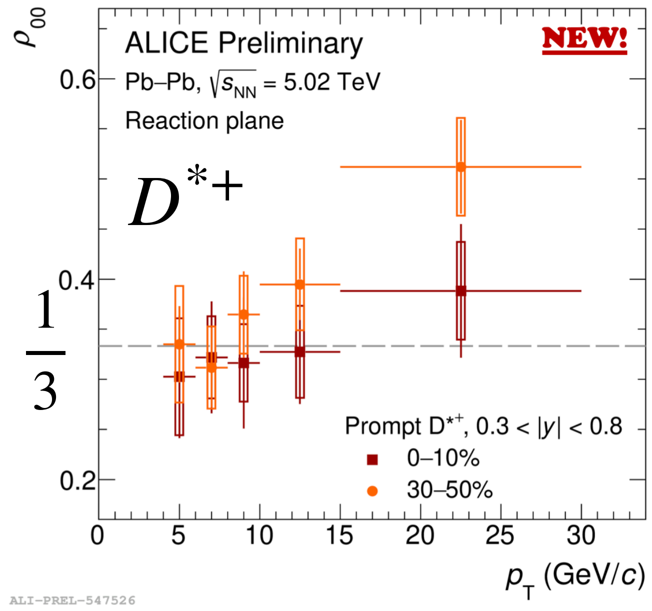
ALICE, PRL 131 (2023) 4, 042303.



B. Xi. QM2023, Sep 05.



J. Brandenburg. QM2023, Sep 08.



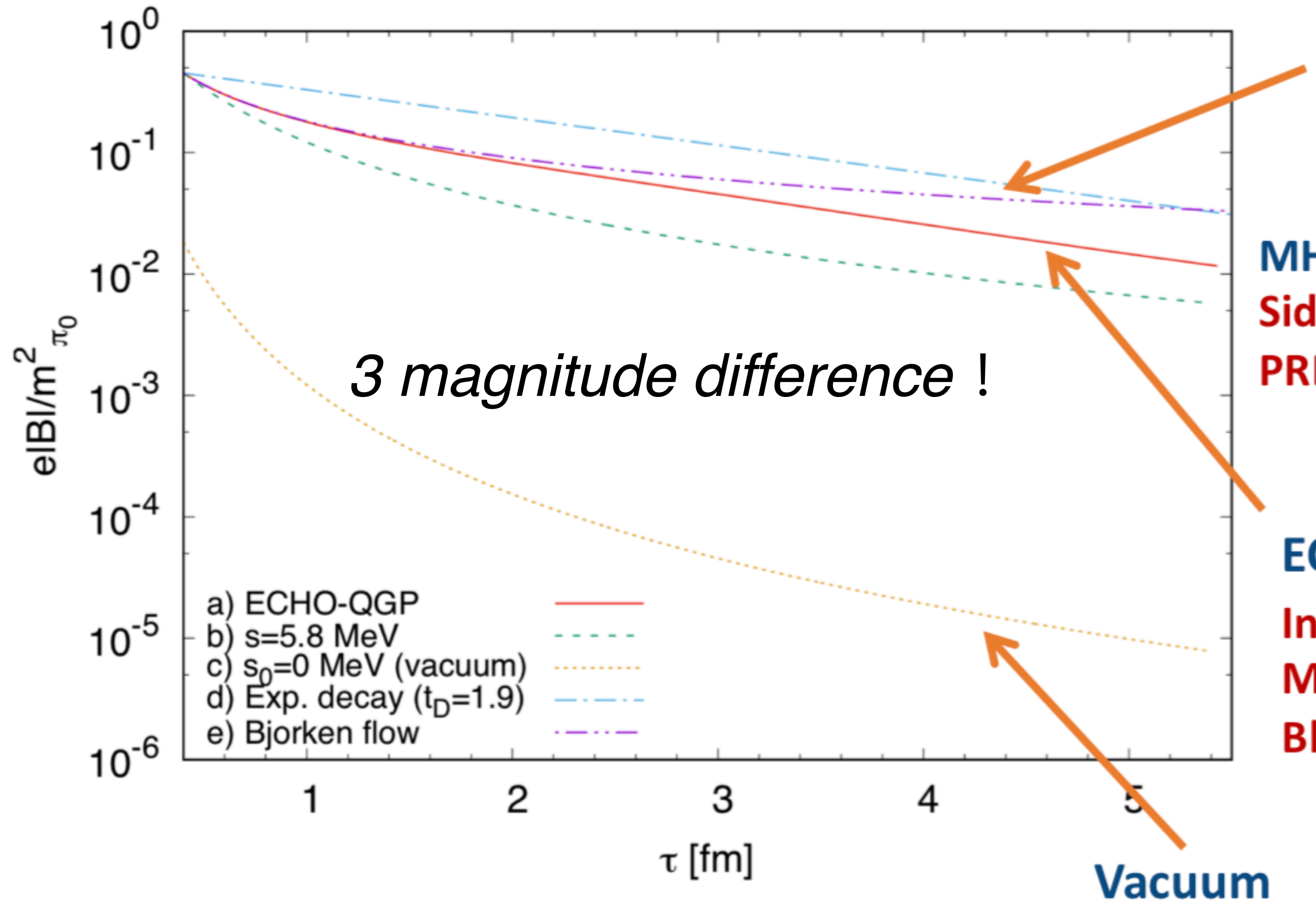
L. Micheletti. QM2023, Sep 05.

Outline

- *Brief introduction about the electromagnetic fields created in HICs and related HF studies*
- *Electromagnetic fields evolution in the early stage of HIC and incomplete electromagnetic response of hot QCD matter*
- *Heavy Flavor production in a strong magnetic field.*
- *Summary*

EM fields evolution and lifetime

Above EM-fields related effects rely on both the strength and lifetime of EM-fields



3 magnitude difference !

Ideal Bjorken MHD

SP, V. Roy, L. Rezzolla,
D.H. Rischke, PRD(2015)

MHD + CME + Chiral anomaly
Siddique, Wang, SP, Wang,
PRD (2019)

ECHO-QGP

Inghirami, Zanna,
Moghaddam, Becattini,
Bleicher, EPJC(2016)

Vacuum
Kharzeev, McLerran,
Warringa, NPA(2008)

G. Inghirami, L. Zanna, A. Beraudo, M. Moghaddam, F. Becattini, and M. Bleicher. Eur.Phys.J.C 76 (2016) 12, 659

Uncertainty : QGP electrical conductivity / QGP expanding / pre-equilibrium stage….

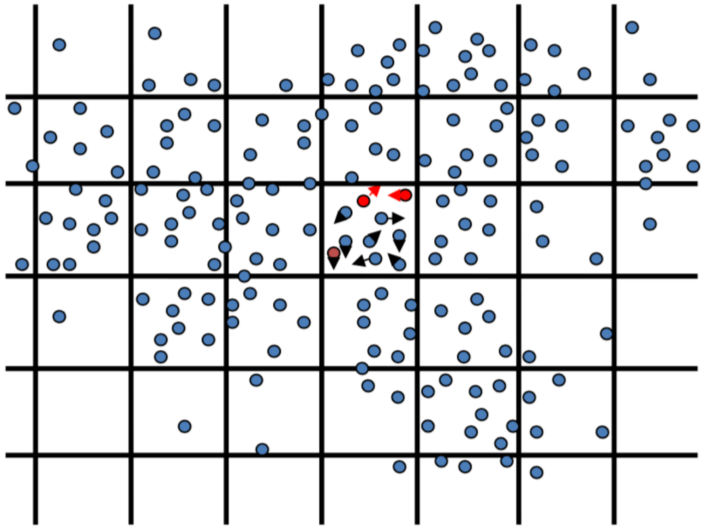
EM fields evolution

- ***EM fields evolution in the pre-equilibrium stage with dynamic generation of quarks.***
- ***Incomplete electromagnetic response of QCD matter***

Boltzmann Approach of MultiParton Scatterings (BAMPS) *Z. Xu, C. Greiner, Phys. Rev. C71, 064901(2005)…*

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}_1}{E_1}\frac{\partial}{\partial \mathbf{r}}+\mathbf{F}_1\frac{\partial}{\partial \mathbf{p}}\right) f_1 = C_{22}+\cancel{C_{23}}+\ldots,$$

$$\partial_\mu F^{\mu\nu} = j^\nu_{ext} + j^\nu_{ind}$$



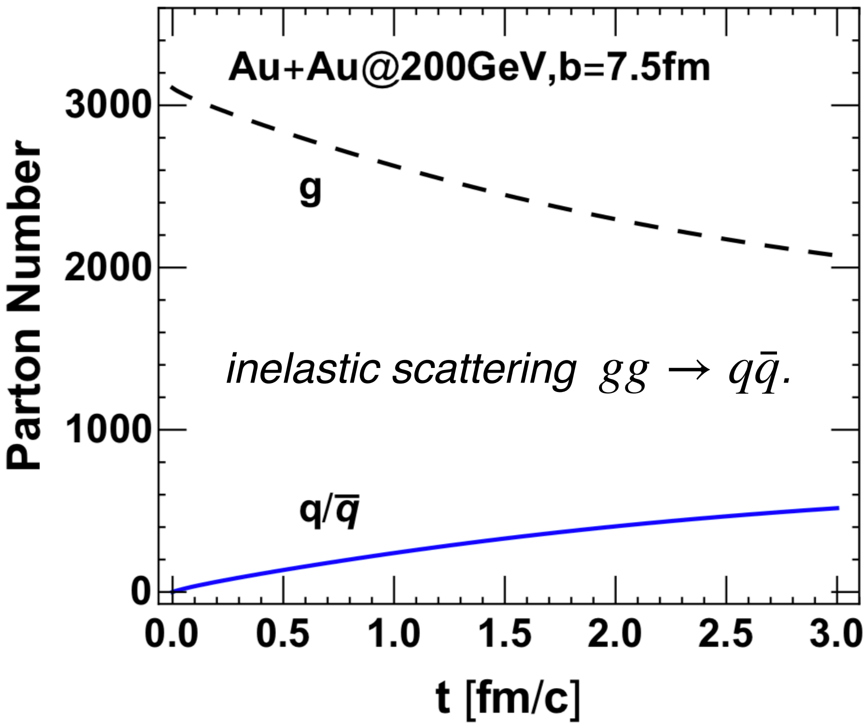
$$\mathbf{F}_1 = q(\mathbf{p}_1/E_1 \times \mathbf{B} + \mathbf{E})$$ *Lorentz force*

The induced em fields are calculated via the Lienard-Wiechert potential:

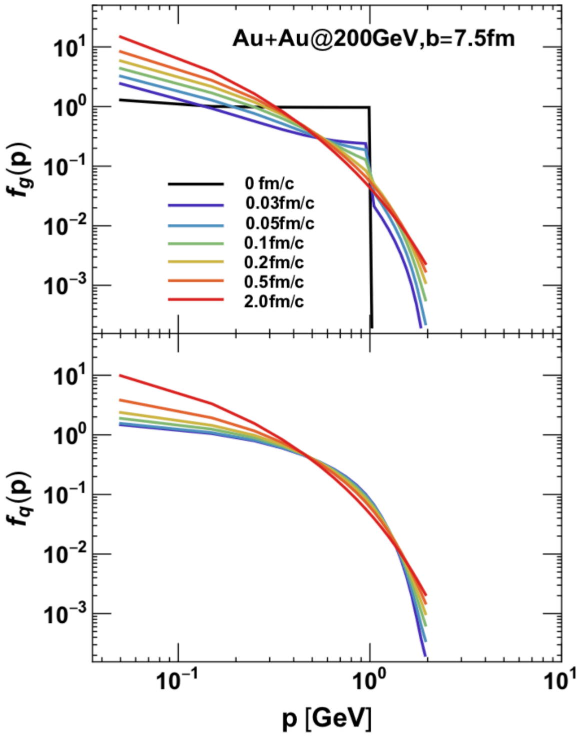
$$e\boldsymbol{E}(\boldsymbol{r},t) = \alpha_{em}\left(\frac{\boldsymbol{n}_s-\boldsymbol{\beta}_s}{\gamma^2(1-\boldsymbol{\beta}_s\cdot\boldsymbol{n}_s)^3|\boldsymbol{r}-\boldsymbol{r}_s|^2}+\frac{\boldsymbol{n}_s\times((\boldsymbol{n}_s-\boldsymbol{\beta}_s)\times\dot{\boldsymbol{\beta}}_s)}{(1-\boldsymbol{\beta}_s\cdot\boldsymbol{n}_s)^3|\boldsymbol{r}-\boldsymbol{r}_s|}\right)_{tr},$$

$$e\boldsymbol{B}(\boldsymbol{r},t) = \alpha_{em}\left(\frac{\boldsymbol{\beta}_s\times\boldsymbol{n}_s}{\gamma^2(1-\boldsymbol{\beta}_s\cdot\boldsymbol{n}_s)^3|\boldsymbol{r}-\boldsymbol{r}_s|^2}+\frac{\boldsymbol{n}_s\times\left(\boldsymbol{n}_s\times((\boldsymbol{n}_s-\boldsymbol{\beta}_s)\times\dot{\boldsymbol{\beta}}_s)\right)}{(1-\boldsymbol{\beta}_s\cdot\boldsymbol{n}_s)^3|\boldsymbol{r}-\boldsymbol{r}_s|}\right)_{tr},$$

EM fields evolution in pre-equilibrium stage



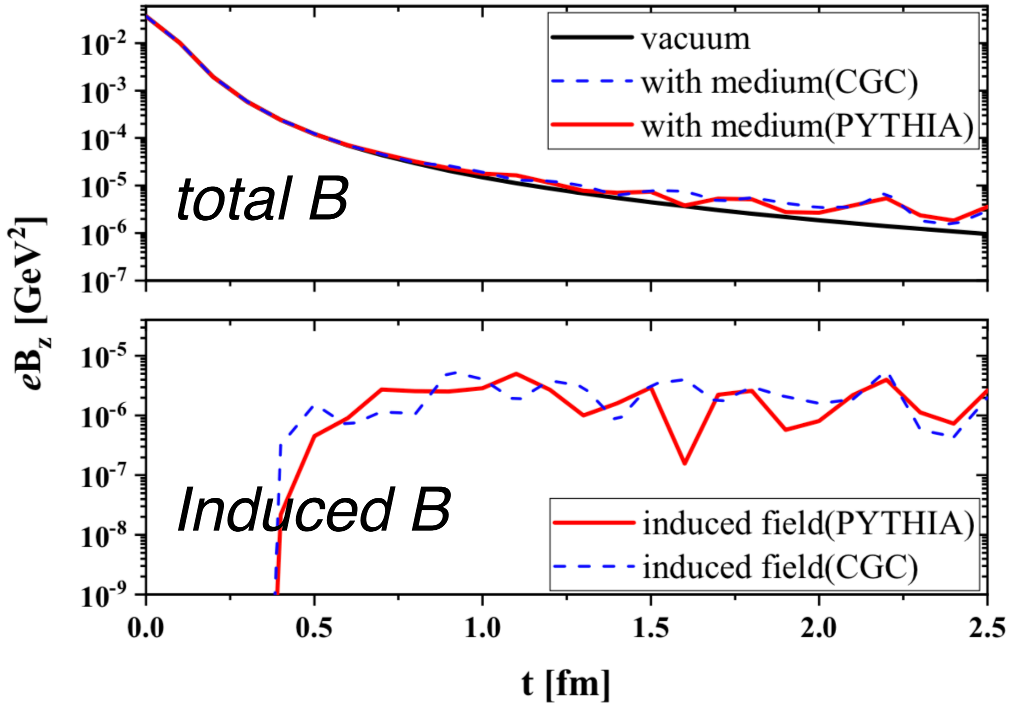
inelastic scattering $gg \to q\bar{q}$.

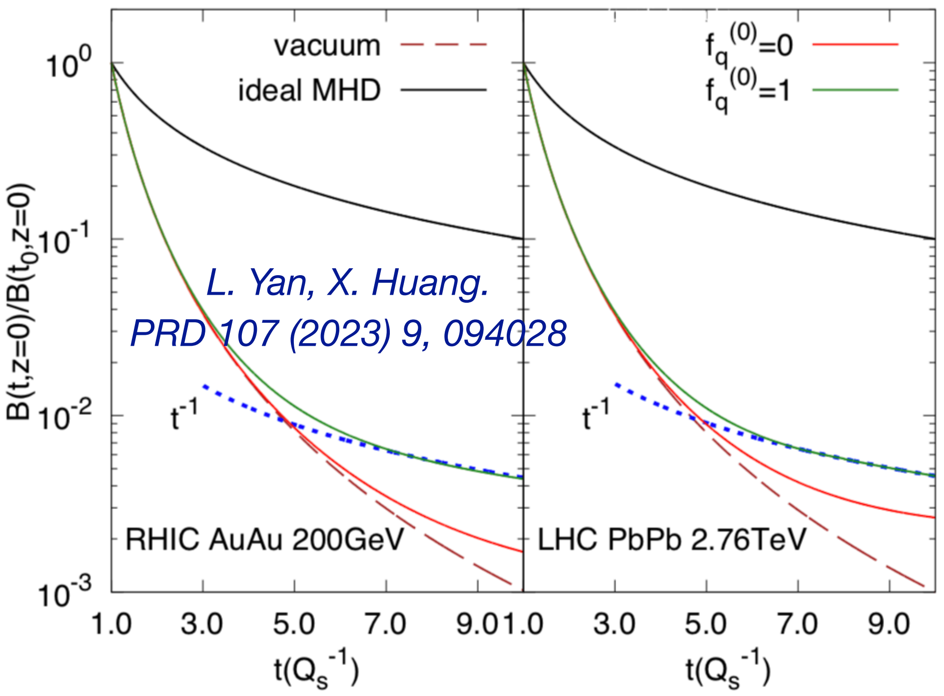


Initial distribution: CGC

$$f_g(\mathbf{p}) = f_0\theta(Q_s - p_t),$$

Channels:
$(1) gg \leftrightarrow gg, (2) gg \leftrightarrow q\bar{q},$
$(3) gq(\bar{q}) \leftrightarrow gq(\bar{q}), (4) qq(\bar{q}\bar{q}) \leftrightarrow qq(\bar{q}\bar{q}),$
$(5) q\bar{q} \leftrightarrow q\bar{q}(q'\bar{q}'), (6) qq' \leftrightarrow qq', (7) q\bar{q}' \leftrightarrow q\bar{q}'$

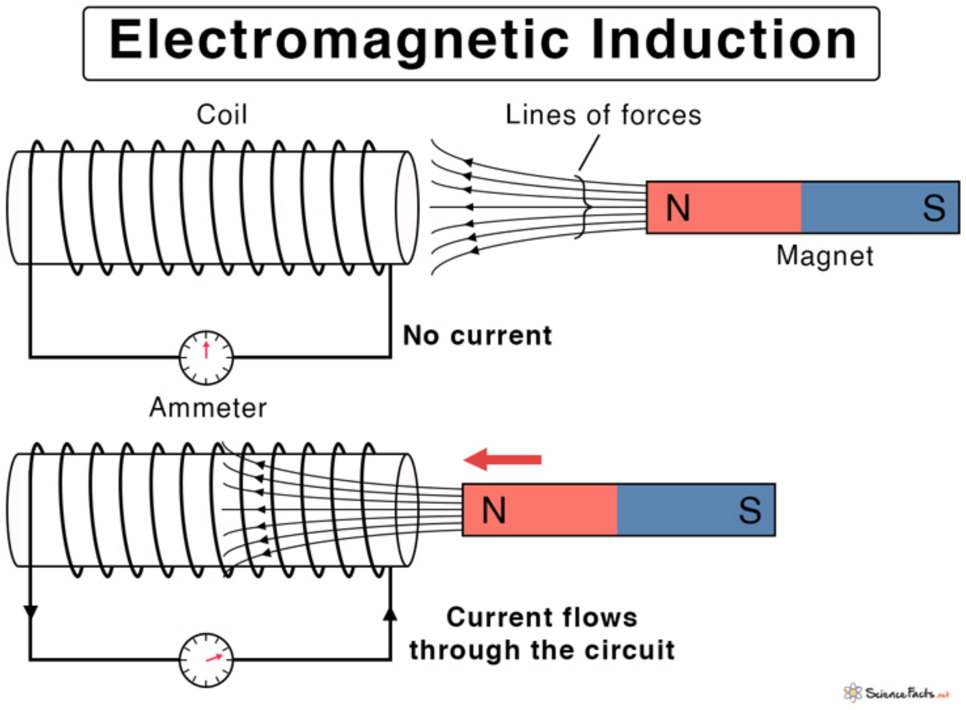




L. Yan, X. Huang.
PRD 107 (2023) 9, 094028

The evolution of the total magnetic field at pre-equilibrium stage is almost same as the vacuum case!

Incomplete electromagnetic response of hot QCD matter



The E field induced due to a changing B field is:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

The E field will generate a Faraday current $\boxed{j^{Ohm} = \sigma_{el} E}$ *This current will create a B field which opposes changes in the external magnetic field (Lenz's law).*

In general case, use the Maxwell equation:

$$\begin{array}{rcl} \nabla \times \mathbf{B} & = & \epsilon\mu\frac{\partial \mathbf{E}}{\partial t} + \boxed{\mu\sigma_{el}(\mathbf{E} + \tilde{\mathbf{v}} \times \mathbf{B})} + \mu\mathbf{j}, \\ \nabla \cdot \mathbf{B} & = & 0, \\ \nabla \times \mathbf{E} & = & -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{E} & = & \rho, \end{array}$$

Ohm's law

Incomplete electromagnetic response of hot QCD matter

We challenge the Ohm's law in hot QCD medium.

*Z. Wang, **JZhao**, C. Greiner, Z. Xu, and P. Zhuang. Phys.Rev.C 105 (2022) 4, L041901*

$$\mathbf{j}^{Ohm} = \sigma_{el}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad ?$$

It needs some time for the current to reach $\mathbf{j}^{Ohm}$.

The lifetime of B-field: τ_B ***The relaxation of induced current:*** $\tau_{Ohm} \sim \sigma_{el}/T^2$

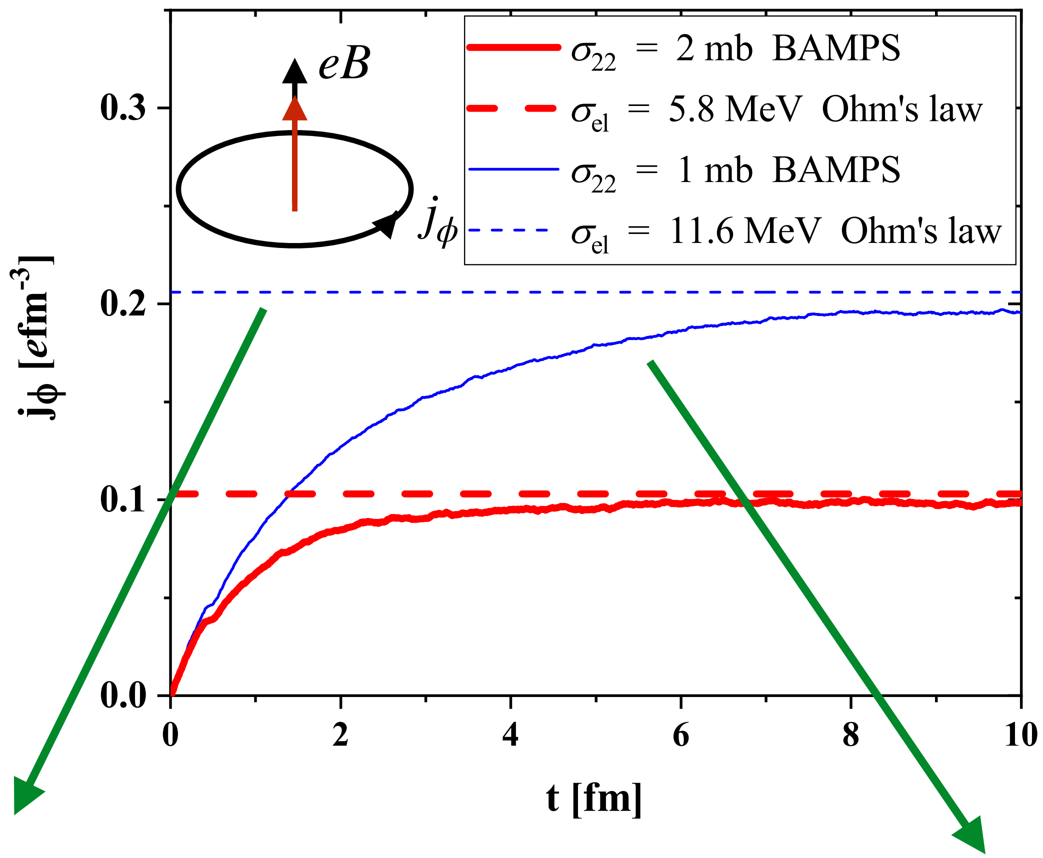
- *Normal case: conductor, QED-plasma…*

  $\tau_{Ohm} \ll \tau_B$

- *QCD medium: quark-gluon plasma*

  $\tau_B \sim R/\gamma \approx 0.06 fm/c$

  $\tau_{Ohm} \sim 1 fm/c \gg \tau_B \quad (\sigma_{el}/T \approx 0.03,\ T = 225 MeV)$



Ohm's law:

$$j_\phi^{Ohm} = \sigma_{el} E_\phi$$

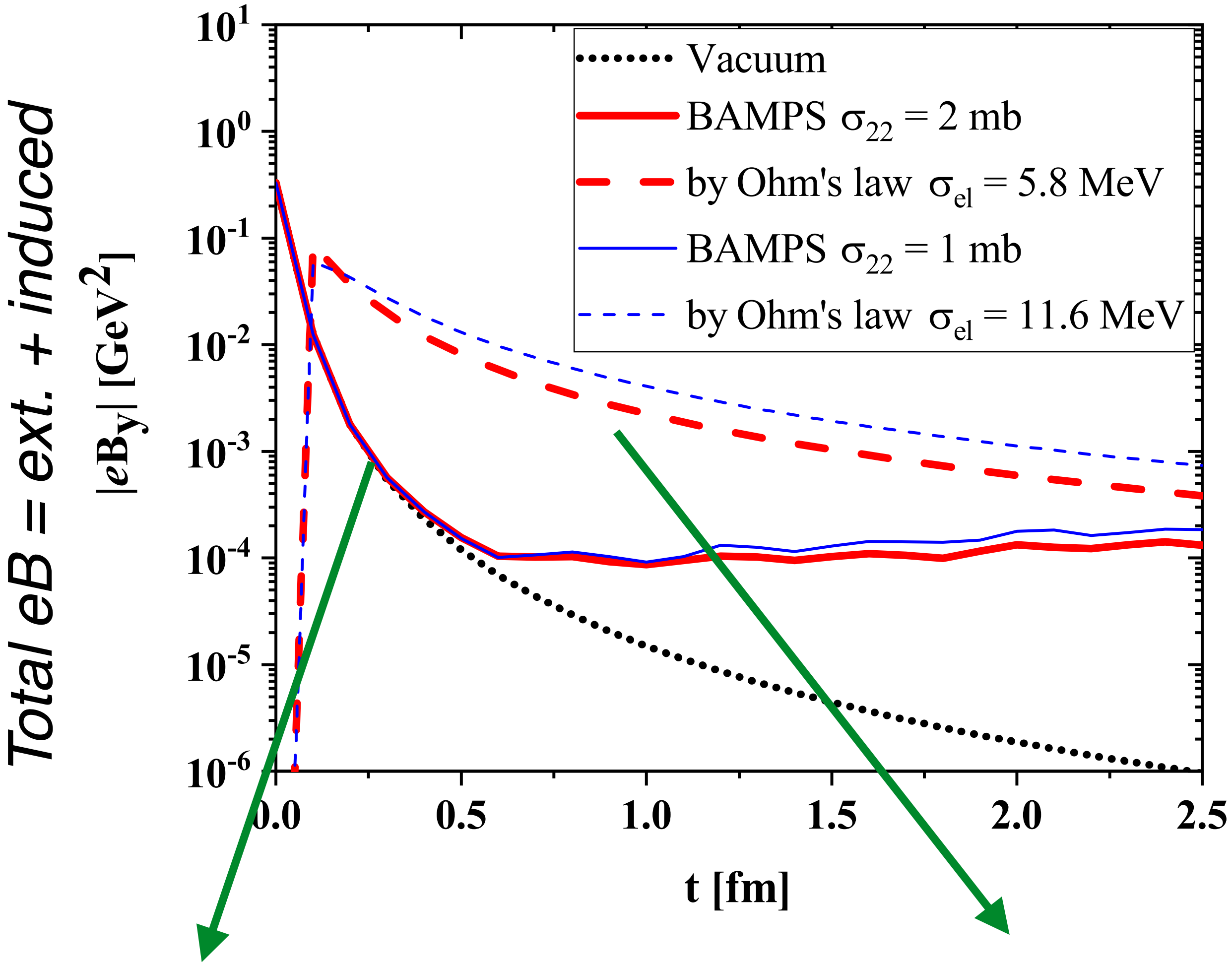
BAMPS:

$$j_\phi(t) = \frac{1}{V}\sum_i q_i \frac{p_\phi^i(t)}{p_0^i(t)},$$

Incomplete electromagnetic response of hot QCD matter

In real heavy ion collisions:

*Z. Wang, **JZhao**, C. Greiner, Z. Xu, and P. Zhuang.*
Phys.Rev.C 105 (2022) 4, L041901



Total eB = ext. + induced

BAMPS

Using Ohm's law
(given by K. Tuchin, also by U. Gursoy, D. Kharzeev)

$$eB_y = \frac{\alpha_{em} b \sigma_{el}}{2(t-z)^2} \exp\left(\frac{-b^2 \sigma_{el}}{4(t-z)}\right).$$

Considering the incomplete electromagnetic response, The induced magnetic field is tiny at early times even the QGP appears at the early stage!

Consider both two effects (pre-equilibrium + incomplete em response), the EM fields may be suppressed largely. This may undermine experimental efforts to measure magnetic-field-related effects in HIC!

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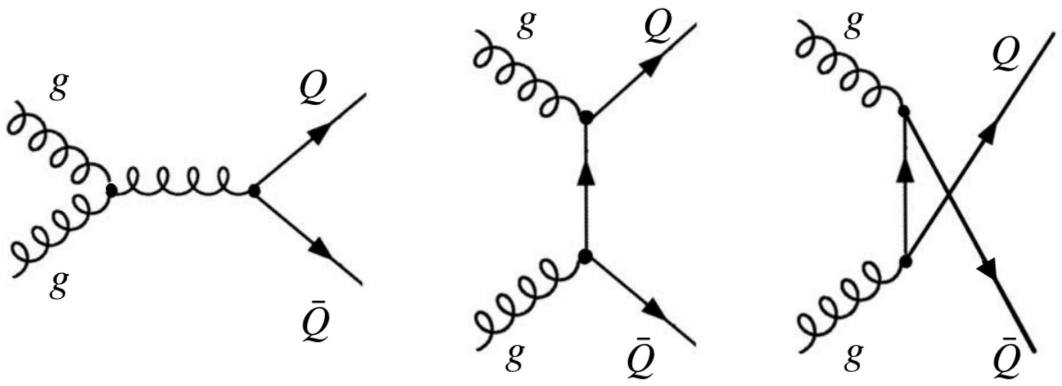
Heavy quark production in a magnetic field

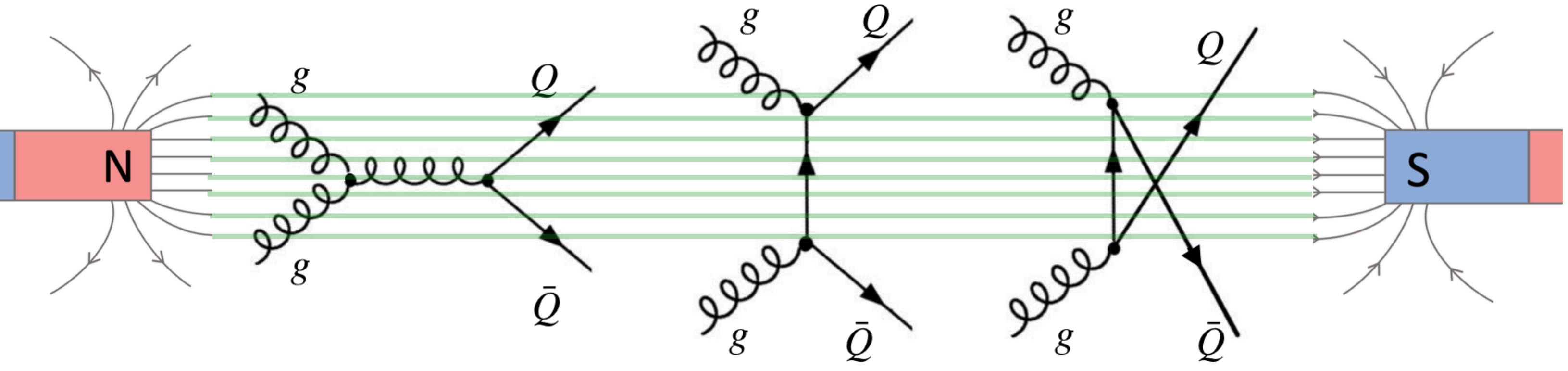


$$\sigma(s) = \frac{\pi\alpha_s^2}{3s}\left[\left(1 + \frac{4m^2}{s} + \frac{m^4}{s^2}\right)\log\left(\frac{1+\chi}{1-\chi}\right) - \left(\frac{7}{4} + \frac{31m^2}{4s}\right)\chi\right]$$

$$\chi = \sqrt{1 - 4m^2/s}$$

B. L. Combridge, Nucl. Phys. B 151, 429 (1979).

Heavy quark production in a magnetic field



Never did before. But in QED sector, there are many studies to characterize the matter-radiation scattering on the magnetar surface.

B. H. Herold, Phys. Rev. D 19, 2868 (1979).
D. B. Melrose and A. J. Parle, Austral. J. Phys. 36, 755 (1983).
C. Thompson, Astrophys. J. 688, 1258 (2008)
A. Kostenko and C. Thompson, Astrophys. J. 869, 44 (2018)

We first investigate the QCD processes in a strong magnetic field created in HIC

1. Dirac equation in external magnetic field

$$[i\gamma^{\mu}(\partial_{\mu} + iqA_{\mu}) - m]\,\psi = 0$$

$$\begin{aligned} \epsilon^2 &= p_z^2 + \epsilon_n^2, \\ \epsilon_n^2 &= m^2 + p_n^2 = m^2 + 2n|q|B \end{aligned}$$

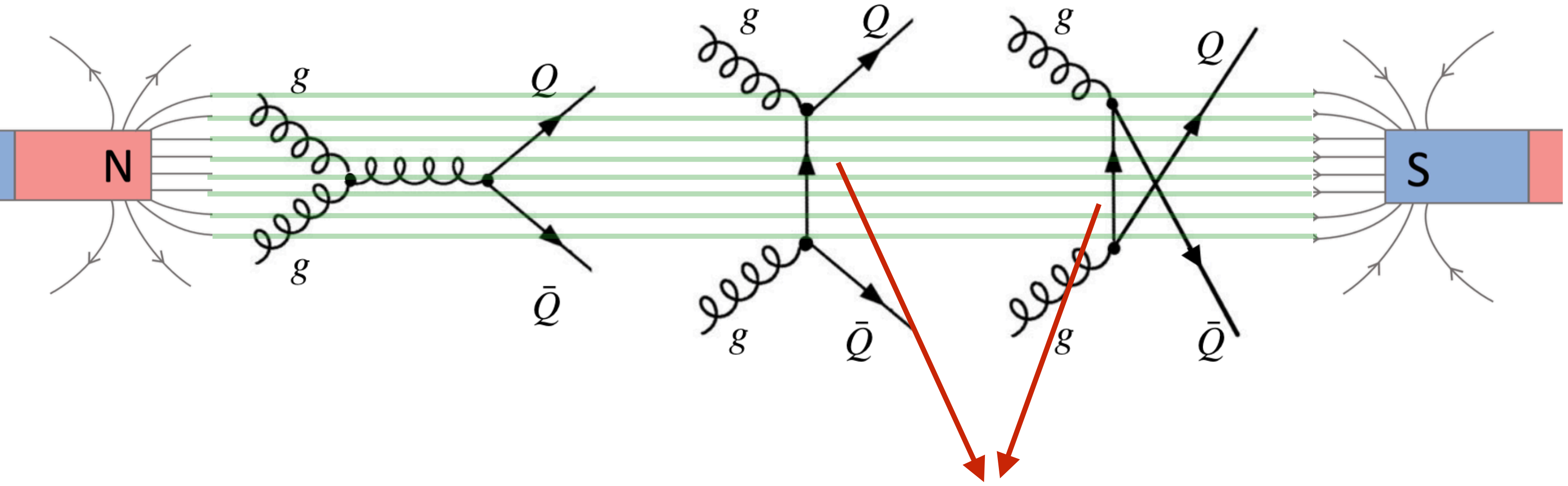
Landau level

$$\psi_{\mp}^{\sigma}(x,p) = \begin{cases} e^{-ip\cdot x} u_{\sigma}(\boldsymbol{x},p) \\ e^{ip\cdot x} v_{\sigma}(\boldsymbol{x},p) \end{cases}$$

$$u_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z p_n \phi_{n-1} \\ (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \\ -ip_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -p_z(\epsilon_n+m)\phi_n \end{bmatrix}, \qquad v_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -p_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -ip_z(\epsilon_n+m)\phi_n \\ -p_z p_n \phi_{n-1} \\ i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \end{bmatrix},$$

$$u_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ -ip_z p_n \phi_n \\ p_z(\epsilon_n+m)\phi_{n-1} \\ ip_n(\epsilon+\epsilon_n)\phi_n \end{bmatrix} \qquad v_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z(\epsilon_n+m)\phi_{n-1} \\ -p_n(\epsilon+\epsilon_n)\phi_n \\ -i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ p_z p_n \phi_n \end{bmatrix}$$

Heavy quark production in a magnetic field



2. Construct the quark propagator in the magnetic field:

$$G(x'-x) = -i\left(\frac{L}{2\pi\lambda}\right)^2 \int dp_z da \sum_{\sigma,n} \left[\theta(t'-t)u_\sigma(\boldsymbol{x}',p)\bar{u}_\sigma(\boldsymbol{x},p)e^{-ip\cdot(x'-x)} - \theta(t-t')v_\sigma(\boldsymbol{x}',p)\bar{v}_\sigma(\boldsymbol{x},p)e^{ip\cdot(x'-x)}\right]$$

$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{\left[m+(\gamma\cdot p)_{||}\right]\left[1-i\mathrm{sgn}(q)\gamma_1\gamma_2\tanh v\right] - \frac{(\gamma\cdot p)_\perp}{\cosh^2 v}\right\} e^{-\frac{v}{|qB|}\left[m^2-p_{||}^2+\frac{\tanh v}{v}p_\perp^2\right]}.$$

Schwinger propagator *J. S. Schwinger, Phys. Rev. 82, 664 (1951).*

3. Change the gluon-quark-antiquark vertex and energy conservation

$$[2\pi\delta(k_z+k_z'-p_z-p_z')]^2 \rightarrow 2\pi L\delta(k_z+k_z'-p_z-p_z'),$$
$$[2\pi\delta(k_y+k_y'-\frac{a'-a}{\lambda^2})]^2 \rightarrow 2\pi L\delta(k_y+k_y'-\frac{a'-a}{\lambda^2}),$$
$$[2\pi\delta(\omega+\omega'-\epsilon-\epsilon')]^2 \rightarrow 2\pi T\delta(\omega+\omega'-\epsilon-\epsilon'),$$

Landau gauge

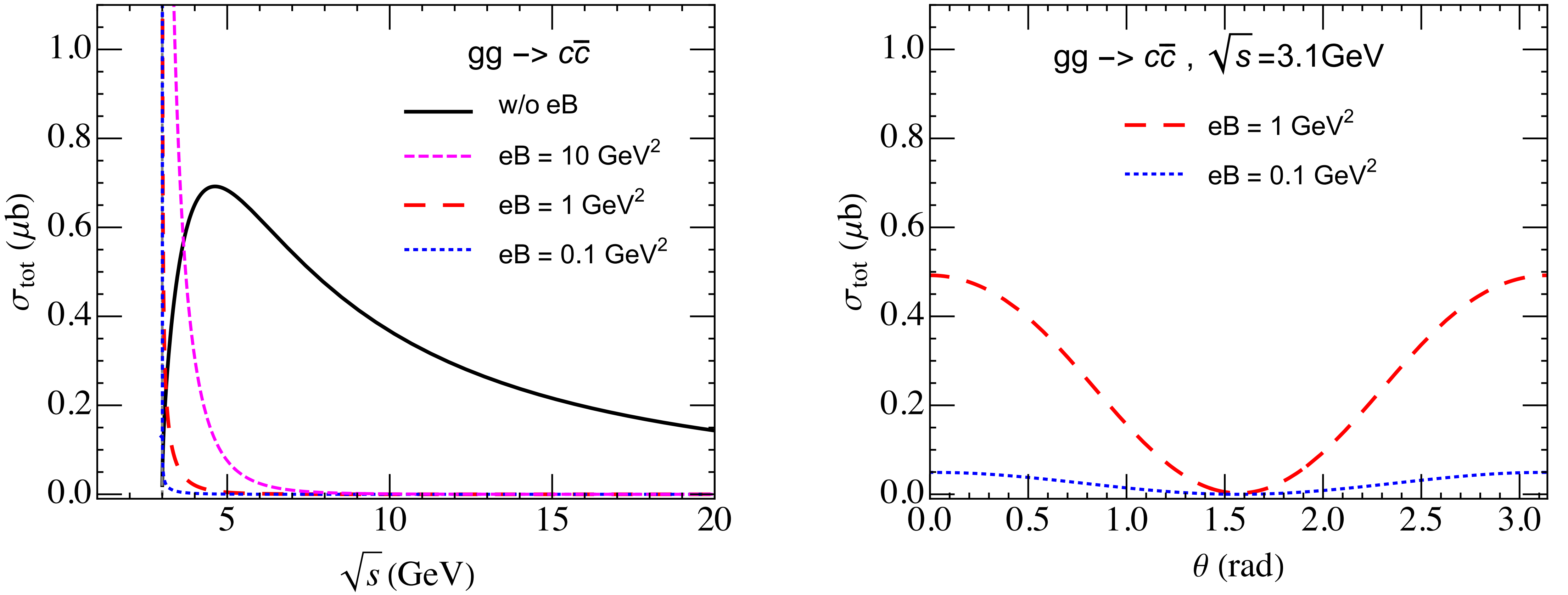
No momentum conservation in the x–direction

Heavy quark production in a magnetic field

With Lowest Landau Level:

*S. Chen, **JZhao**, P. Zhuang. arXiv: 2310….*

$$
\begin{aligned}
\sigma(s,B,\theta) \;=\; & \frac{\pi m^2\alpha_s^2|qB|}{s^3\chi}\left\{\frac{3}{2}\cos^2\theta\left[1-\frac{\sin^2\theta}{1+\sqrt{4m^2/s}}\,\frac{1+\cos^2\theta-4\chi^2}{\sin^4\theta+16m^2/s\cos^2\theta}\,e^{-\frac{s\sin^2\theta}{8|qB|}}\right]\right. \\
& \left. +\;\frac{2}{3}\sin^4\theta\left[\left(\frac{\cos\theta+2\chi}{(\chi+\cos\theta)^2+4m^2/s}\right)^2+\left(\frac{-\cos\theta+2\chi}{(\chi-\cos\theta)^2+4m^2/s}\right)^2-\frac{1}{4}\,\frac{4\chi^2-\cos^2\theta}{\sin^4\theta+16m^2/s\cos^2\theta}\right]e^{-\frac{s\sin^2\theta}{4|qB|}}\right\},
\end{aligned}
$$



Magnetic field and also the gluon polarization angle dependent

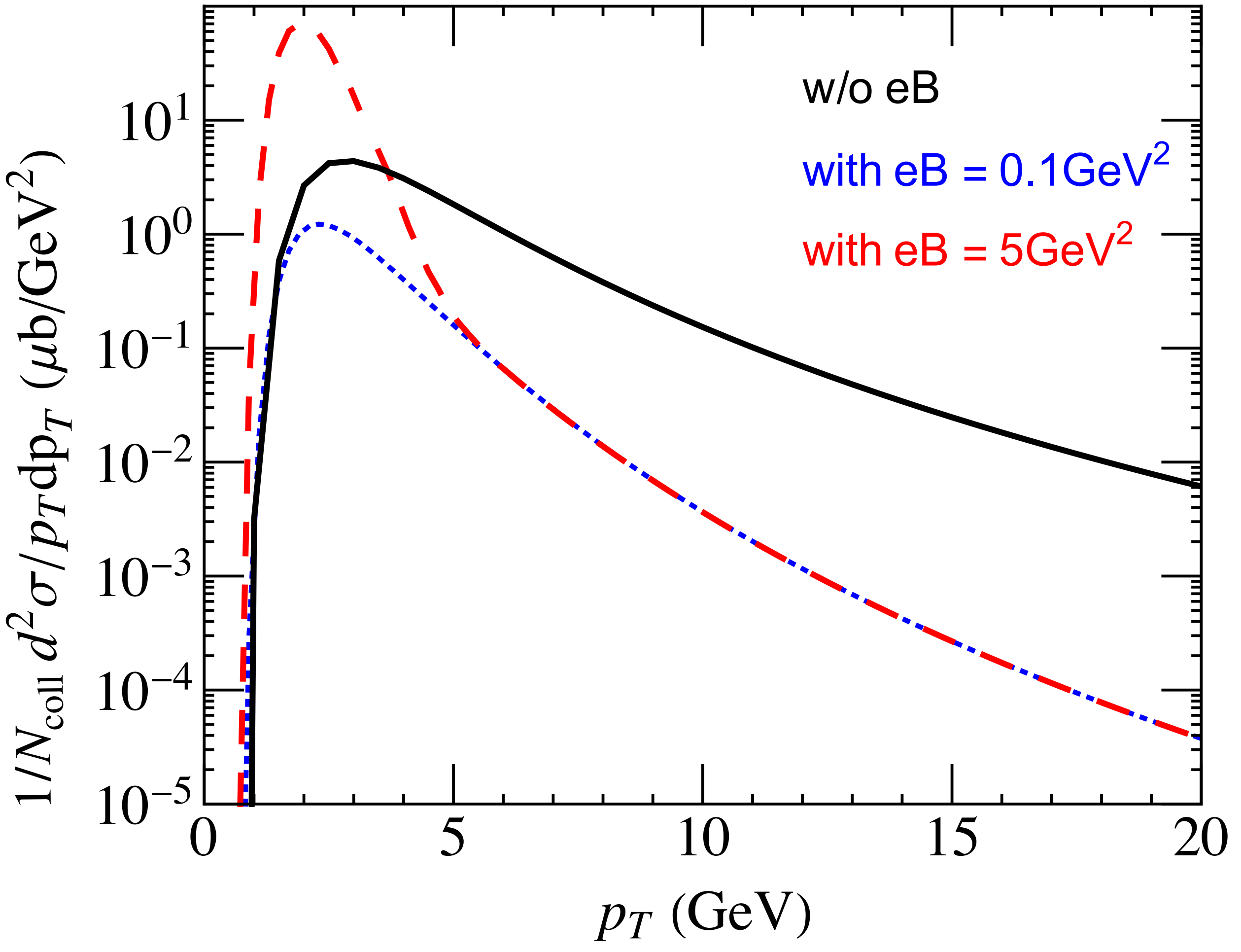
1. *divergence of the cross section at the production threshold —dimension reduction*
2. $\theta = 0,\pi$ *only s-channel contributes;* $\theta = \pi/2$ *only t and u channels.*

Heavy quark production in a magnetic field

*S. Chen, **JZhao**, P. Zhuang. arXiv: 2310….*

$$
\begin{aligned}
\frac{|\mathcal{M}_{gg\to c\bar{c}}|^2}{\pi^2\alpha_s^2} &= 24M^2\cos^2\theta\left(\frac{1}{s}+e^{-\frac{s\sin^2\theta}{8|q|B}}\frac{\sin^2\theta(\frac{s}{4}\sin^2\theta+\frac{t+u}{2}+\frac{(t-u)^2}{s\cos^2\theta})}{(\frac{s}{2}\sin^2\theta+t+u)^2-(\frac{t^2-u^2}{s\cos\theta})^2}\right)\\
&+\left(\frac{(-\frac{\sqrt{s}\cos\theta}{2}+\frac{t-u}{\sqrt{s}\cos\theta})^2}{(\frac{s}{2}\sin^2\theta+t+u-\frac{t^2-u^2}{s\cos\theta})^2}+\frac{(\frac{\sqrt{s}\cos\theta}{2}+\frac{t-u}{\sqrt{s}\cos\theta})^2}{(\frac{s}{2}\sin^2\theta+t+u+\frac{t^2-u^2}{s\cos\theta})^2}-\frac{1}{4}\frac{\frac{(t-u)^2}{s\cos^2\theta}-\frac{s}{4}\cos^2\theta}{(\frac{s}{2}\sin^2\theta+t+u)^2-(\frac{t^2-u^2}{s\cos\theta})^2}\right)\\
&\times\ \frac{64}{3}M^2\sin^4\theta e^{-\frac{s\sin^2\theta}{4|q|B}}.
\end{aligned}
$$

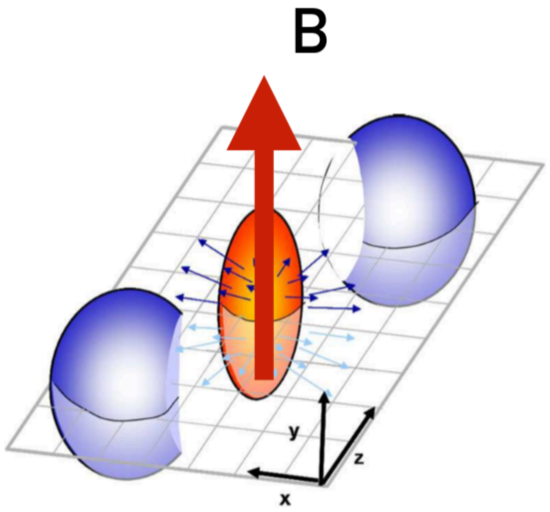
$$
\frac{d^3\sigma^{pp}_{gg\to c\bar{c}}}{dp_T^2dy_cdy_{\bar{c}}} = x_1x_2f_g(x_1,p_T^2)f_g(x_2,p_T^2)\frac{d\sigma_{gg\to c\bar{c}}}{d\hat{t}}(\hat{s},\hat{t},\hat{u}), \qquad \frac{d\sigma_{gg\to c\bar{c}}}{d\hat{t}} = \frac{|\bar{\mathcal{M}}_{gg\to c\bar{c}}|^2}{16\pi s^2}
$$

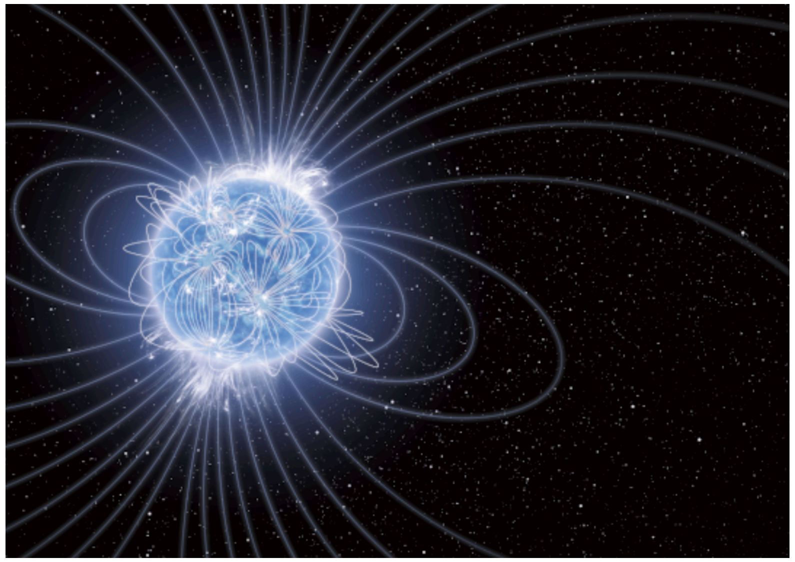


Low momentum enhancement and high momentum suppression!

Summary

- *Strongest EM fields created in the non-central heavy-ion collisions*
- *Considering the dynamic generation of quarks in the pre-equilibrium stage and the incomplete electromagnetic response of QCD matter, the EM fields generated in heavy ion collisions decays rapidly and the magnitude are greatly reduced compared with the initial time.*
- *The magnetic field effect changes strongly the* $Q\bar{Q}$ *spectrum in HIC (with LLL).*

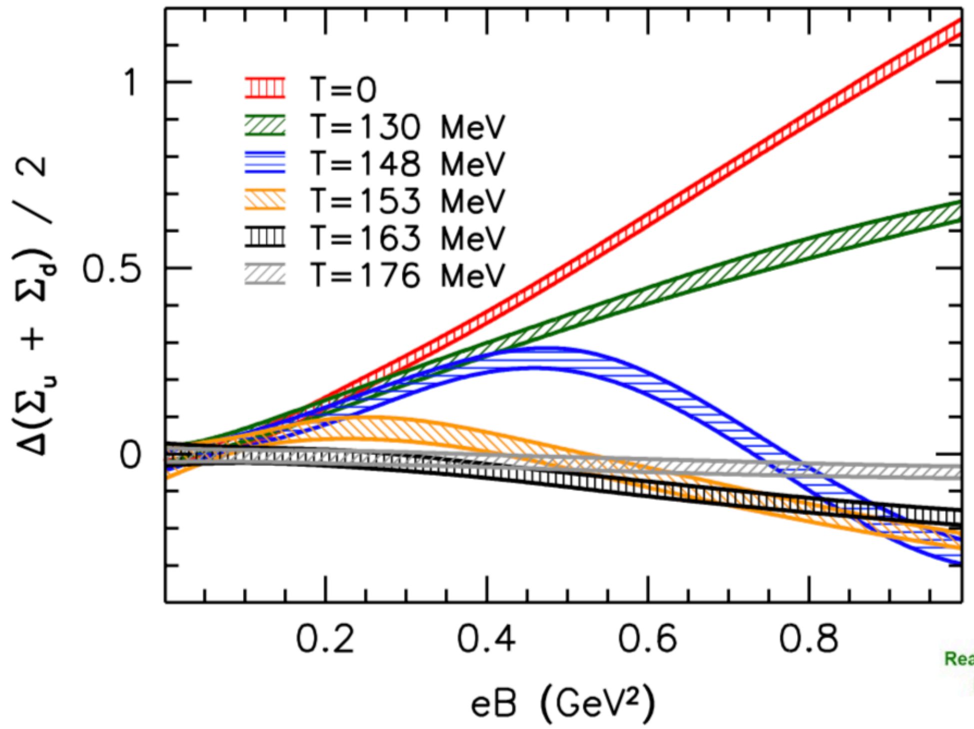




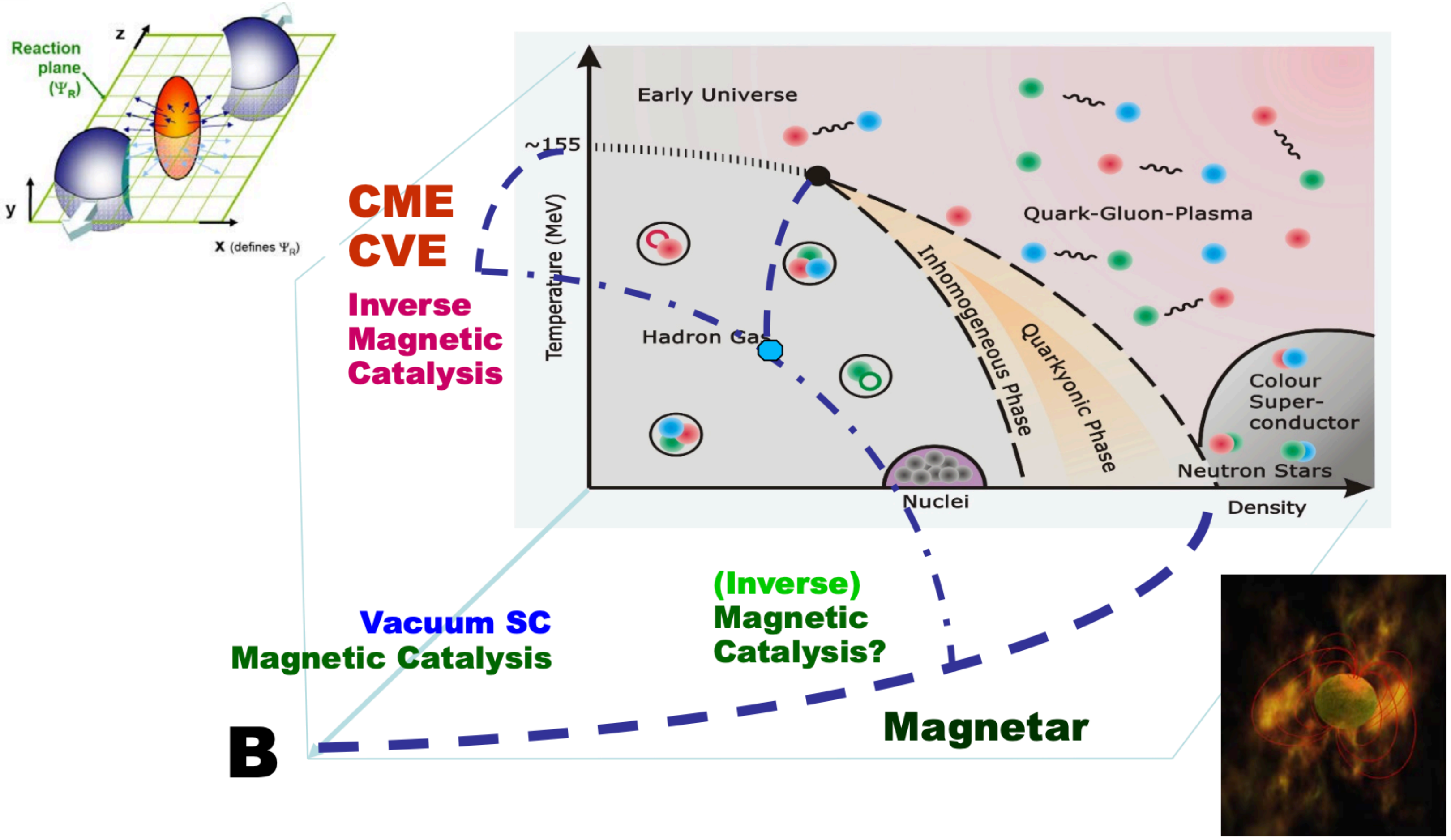
Thanks for your attention!

EM fields related studies(…)

- *magnetic / inverse magnetic catalysis; QCD phase structure*

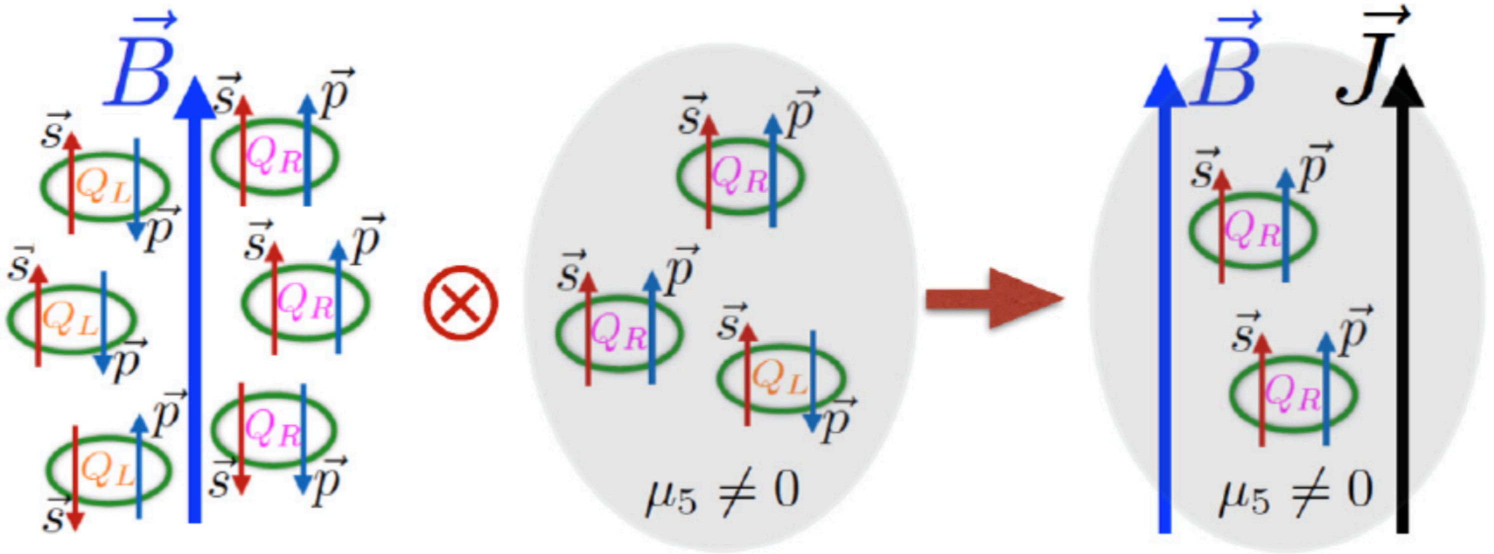


See also:
S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, and A. Schafer
Phys.Rev.D 86 (2012) 071502.
A.J. Mizher, M. N. Chernodub and E.S.Fraga,
Phys.Rev.D 82 (2010) 105016.
E. S. Fraga and A. J. Mizher.
Phys.Rev.D 78 (2008) 025016.…



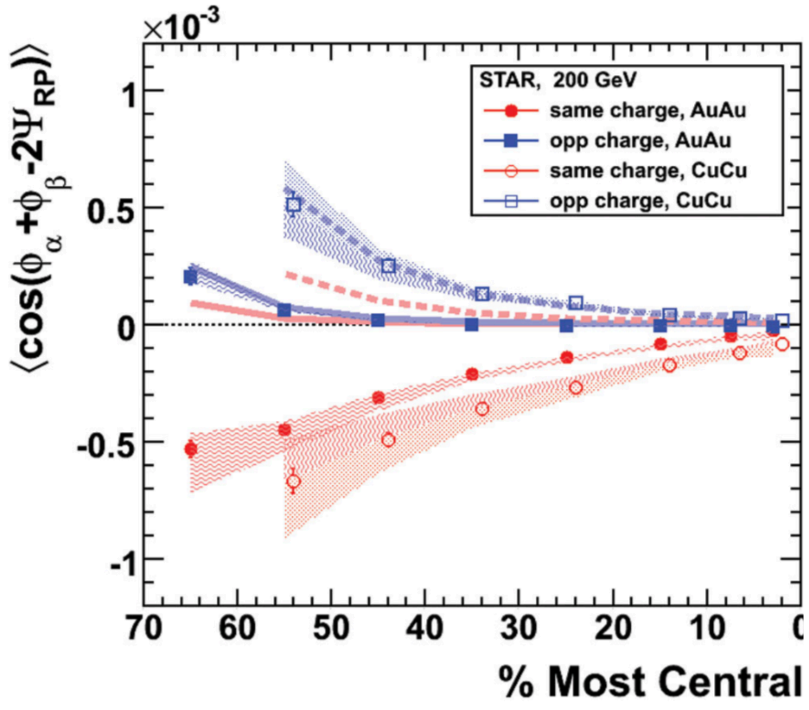
EM fields related studies(…)

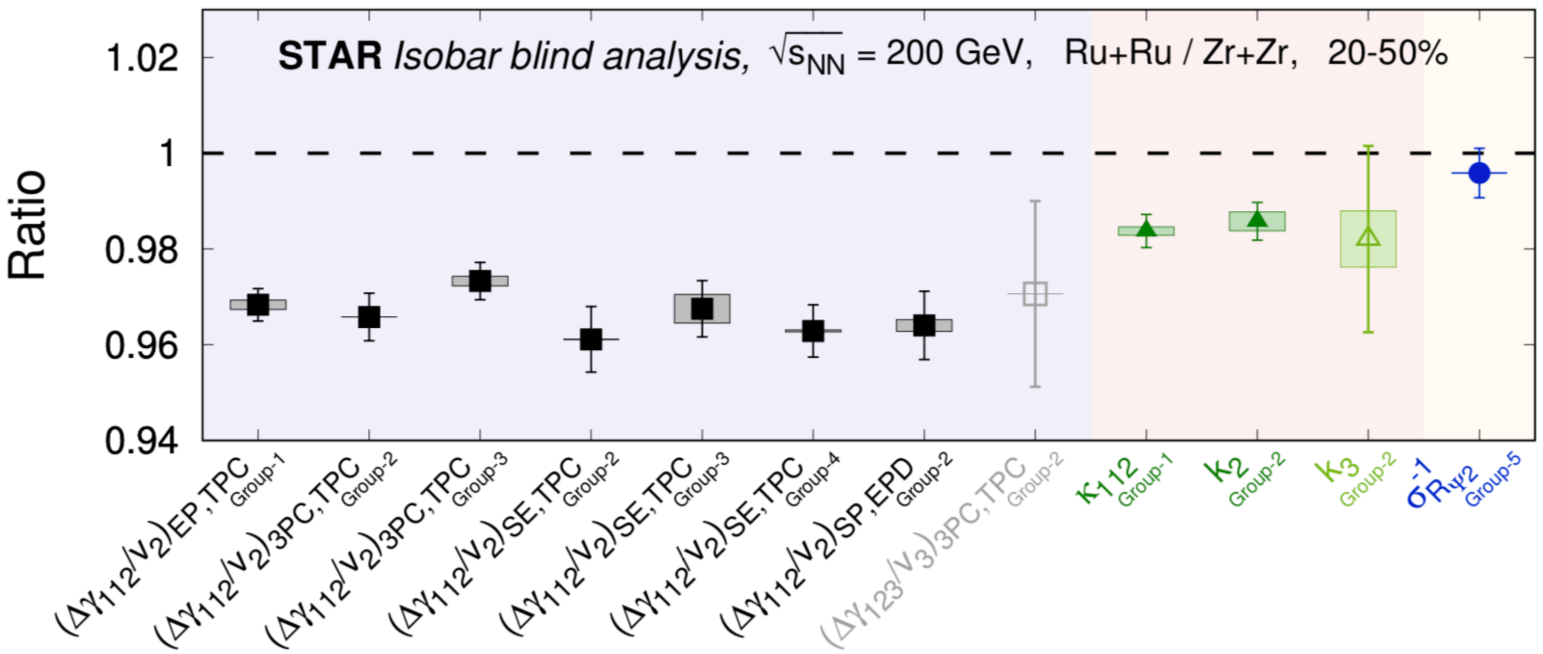
- *magnetic / inverse magnetic catalysis; QCD phase structure*
- *Induce chiral magnetic effect (CME) in the chiral unbalance system*



$$\vec{J} = \frac{Q^2}{2\pi^2}\mu_5 \vec{B}$$

Prog. Part. Nucl. Phys. 88, 1 (2016) …





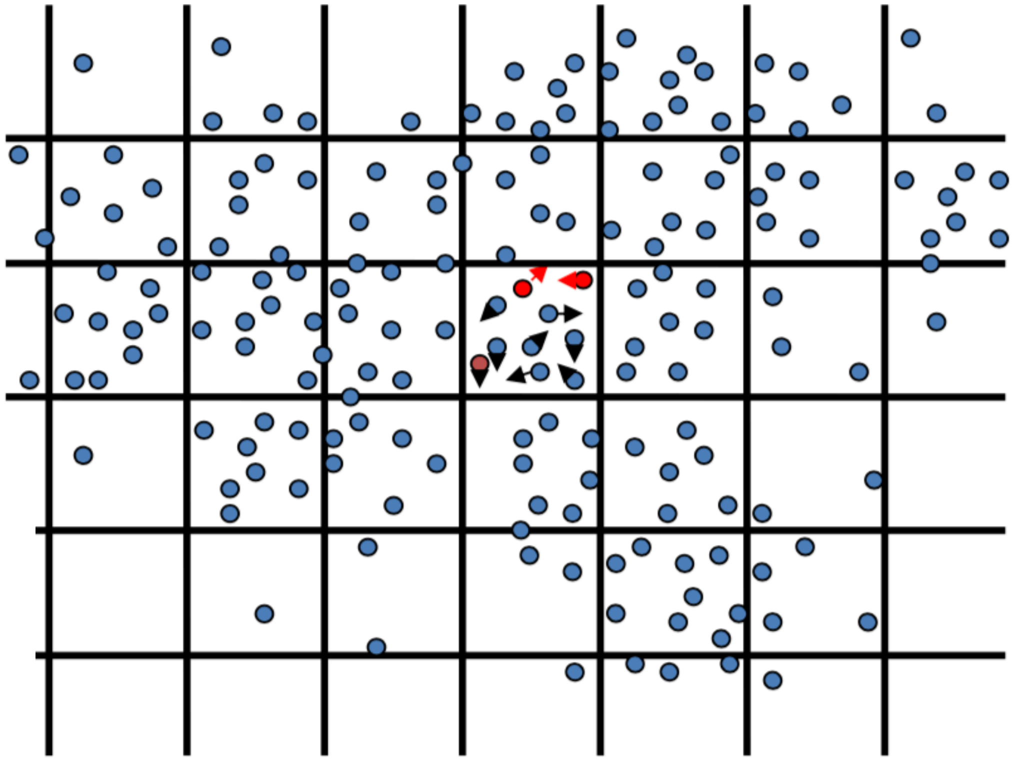
STAR Collaboration, Phys. Rev. Lett. 103 (2009) 251601; Phys. Rev. C 105 (2022) 1, 014901

EM fields evolution in BAMPS

Boltzmann Approach of MultiParton Scatterings (BAMPS)

Z. Xu, C. Greiner, Phys. Rev. C71, 064901(2005)…

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}_1}{E_1}\frac{\partial}{\partial \mathbf{r}}+\mathbf{F}_1\frac{\partial}{\partial \mathbf{p}}\right) f_1 = C_{22}+C_{23}+\dots,$$



$$\mathbf{F}_1 = q(\mathbf{p}_1/E_1\times\mathbf{B}+\mathbf{E})$$ *Lorentz force*

$$\begin{aligned} C_{22} &= \frac{1}{2E_1}\int d\Gamma_2\frac{1}{2}\int d\Gamma_3 d\Gamma_4|\mathcal{M}_{34\to 12}|^2 \\ &\times \left[f_3f_4-f_1f_2\right](2\pi)^4\delta^{(4)}(p_3+p_4-p_1-p_2), \end{aligned}$$

Monte Carlo method: collision probabilities of two particles in spatial cell ΔV of and within a time step Δt :

$$P_{22} = v_{\rm rel}\frac{\sigma_{22}}{N_{\rm test}}\frac{\Delta t}{\Delta V},$$

N_{test} *the number of test particles per real particle to reduce the fluctuation.*

In the limit Δt → 0 and ΔV → 0, the numerical solutions will converge to the exact solutions of the Boltzmann equation

EM fields evolution in BAMPS

Channels: $(1)gg \leftrightarrow gg, (2)gg \leftrightarrow q\bar{q}, (3)gq(\bar{q}) \leftrightarrow gq(\bar{q}), (4)qq(\bar{q}\bar{q}) \leftrightarrow qq(\bar{q}\bar{q}),$
$(5)q\bar{q} \leftrightarrow q\bar{q}(q'\bar{q}'), (6)qq' \leftrightarrow qq', (7)q\bar{q}' \leftrightarrow q\bar{q}'$

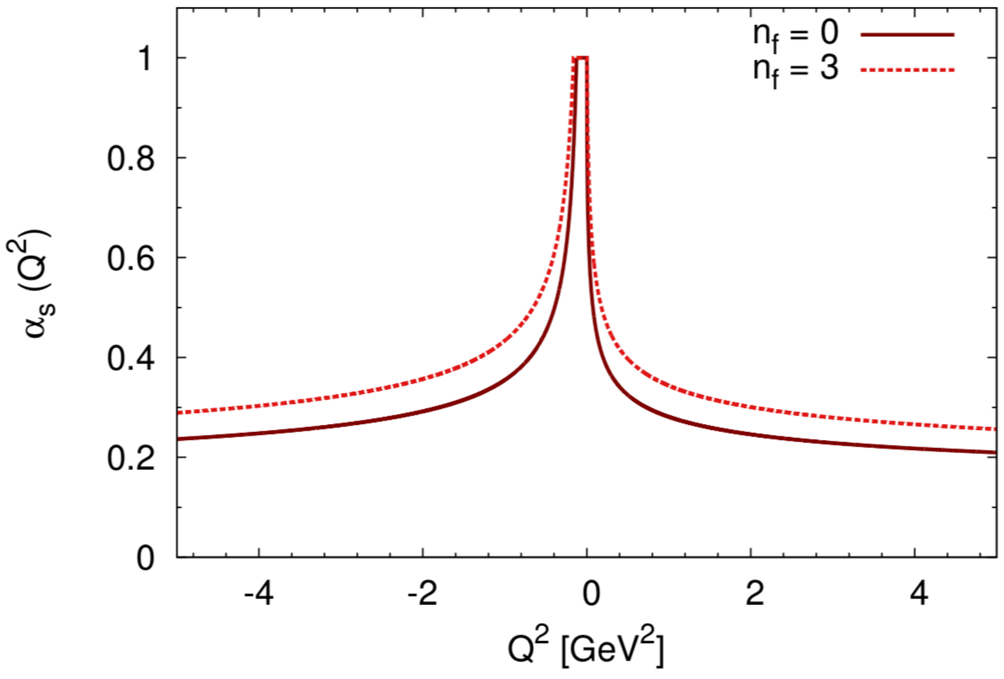
The cross-sections are given by pQCD. The infrared divergences can be fixed by the Debye screening mass of gluons m_D^2 and quarks m_q^2 in the hot medium.

B.L. Combridge, J. Kripfganz, J. Ranft, Phys. Lett. B 70 (1977) 234.
J. Uphoff, F. Senzel, O. Fochler, C. Wesp, Z. Xu and C. Greiner,
Phys. Rev. Lett. 114, 112301 (2015).

$$
\begin{aligned}
|M_{gg\to q\bar{q}}|^2 =& 6\pi^2 \left[\frac{4}{9}\left(\alpha_s^2(t)\frac{tu}{[t-m_q^2(\alpha_s(t))]^2} + \alpha_s^2(u)\frac{tu}{[u-m_q^2(\alpha_s(u))]^2}\right)\right.\\
&+ 2\alpha_s^2(s)\frac{tu}{[s+m_D^2(\alpha_s(s))]^2} + \alpha_s(s)\alpha_s(u)\frac{tu}{[s+m_D^2(\alpha_s(s))][u-m_q^2(\alpha_s(u))]}\\
&\left.+ \alpha_s(s)\alpha_s(t)\frac{tu}{[s+m_D^2(\alpha_s(s))][t-m_q^2(\alpha_s(t))]}\right].
\end{aligned}
$$

$$
\begin{aligned}
|M_{gg\to gg}|^2 =& 72\pi^2\left[3\alpha_s^2(s) - \alpha_s^2(s)\frac{tu}{[s+m_D^2(\alpha_s(s))]^2}\right.\\
&\left.- \alpha_s^2(t)\frac{su}{[t-m_D^2(\alpha_s(t))]^2} - \alpha_s^2(u)\frac{st}{[u-m_D^2(\alpha_s(u))]^2}\right].
\end{aligned}
$$

$$
\begin{aligned}
|M_{qg\to qg}|^2 =& 16\pi^2\left[-\frac{4}{9}\left(\alpha_s^2(s)\frac{su}{[s+m_q^2(\alpha_s(s))]^2} + \alpha_s^2(u)\frac{su}{[u-m_q^2(\alpha_s(u))]^2}\right)\right.\\
&- 2\alpha_s^2(t)\frac{su}{[t-m_D^2(\alpha_s(t))]^2} + \alpha_s(s)\alpha_s(t)\frac{su}{[s+m_q^2(\alpha_s(s))][t-m_D^2(\alpha_s(t))]}\\
&\left.- \alpha_s(t)\alpha_s(u)\frac{su}{[t-m_D^2(\alpha_s(t))][u-m_q^2(\alpha_s(u))]}\right].
\end{aligned}
$$



…

Analytical EM fields with the Ohm’s law

Two nuclei are replaced by two point particles with the charge $q = Ze$ *and mass* $m = Am_N$ *moving in the z direction at impact parameter b*

Using Ohm’s law, the electromagnetic fields in the quark-gluon system are solved by Maxwell's equations:

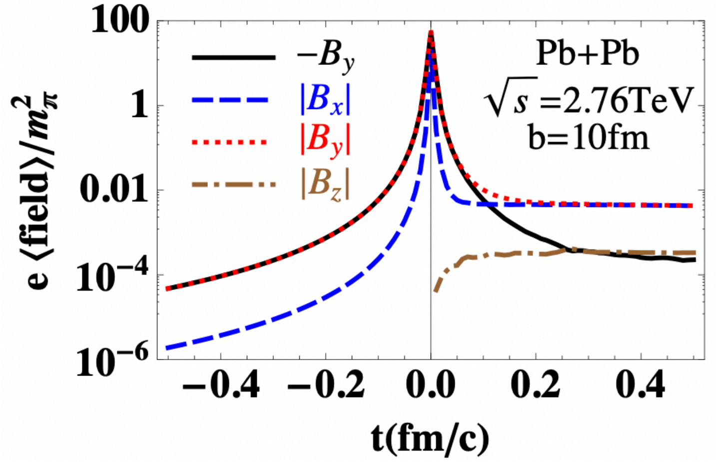
$$\begin{aligned}
&\nabla \cdot \boldsymbol{B} = 0, \\
&\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \\
&\nabla \cdot \boldsymbol{E} = q\delta(x - b/2)\delta(y)\delta(z - vt) + q\delta(x + b/2)\delta(y)\delta(z + vt), \\
&\nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \boxed{\sigma_{el}\boldsymbol{E}} + qv\hat{\mathbf{z}}\delta(x - b/2)\delta(y)\delta(z - vt) - qv\hat{\mathbf{z}}\delta(x + b/2)\delta(y)\delta(z + vt).
\end{aligned}$$

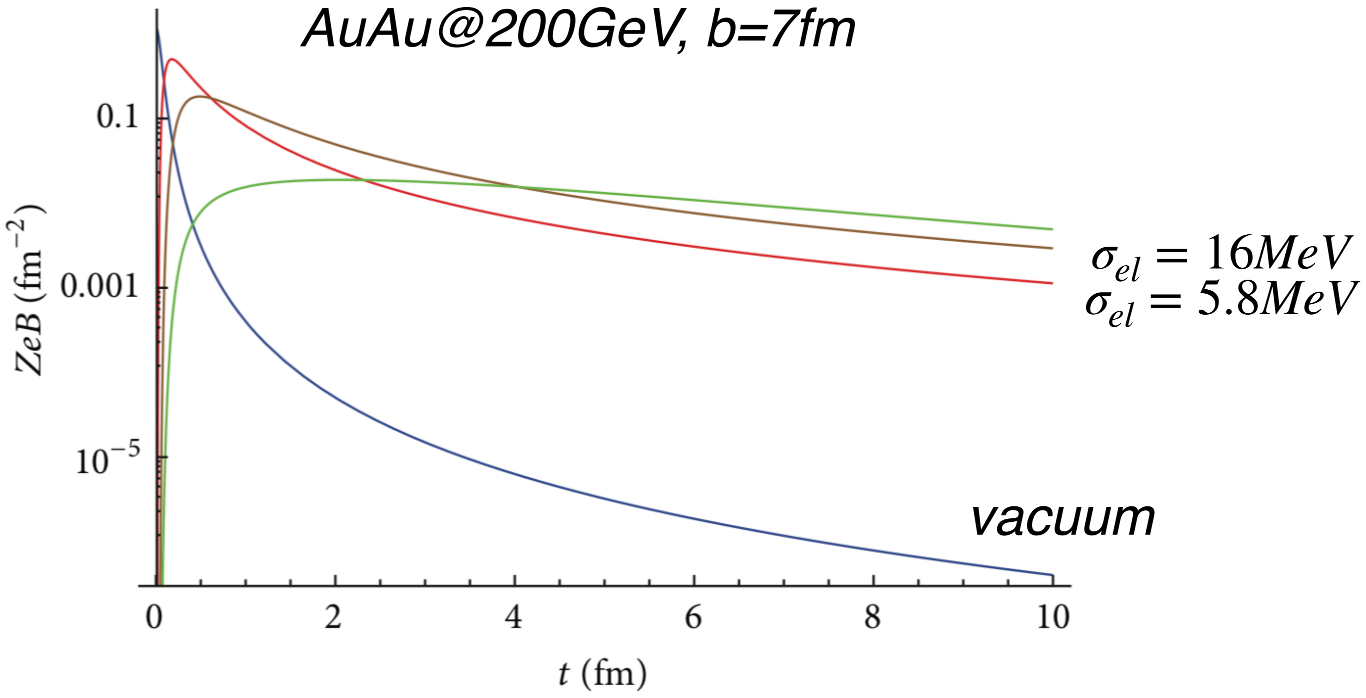
Analytical solution to B and E-field:

K. Tuchin. Adv.High Energy Phys. 2013 (2013) 490495

$$eB_y = \frac{\alpha_{em} b \sigma_{el}}{2(t-z)^2} \exp\left(\frac{-b^2\sigma_{el}}{4(t-z)}\right).$$

$$eE_x = eB_y \coth(Y_0)$$





$t = b^2\sigma_{el}/8$