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INVESTIGATION OF THE COLUMBIA PLOT IN THE HIGH MASS REGION

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Strong-HFHF Workshop - Sicily 2023

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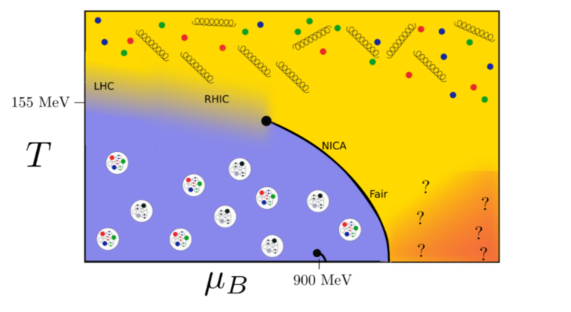
Bergische Universität Wuppertal

QCD Phase-Diagram



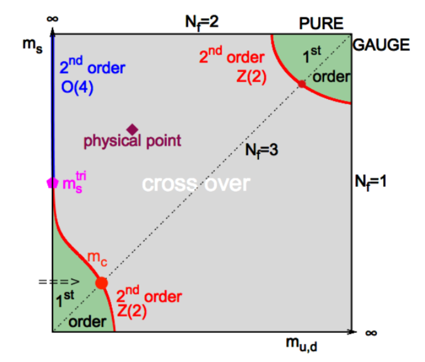
Guenther arxiv:2201.02072

- $\mu = 0 \rightarrow$ Crossover determined by theory and experiment
- $\mu > 0$ Hard to investigate with lattice QCD due to a sign problem

Columbia Plot

Columbia Plot for 0 chemical potential

- Physical masses
  ⇒ <u>Crossover</u>
  (*Fodor et. al. 0611014*)
- Pure gauge theory: Latent heat in cont. limit determined by *Borsanyi et. al. 2202.05234*
  ⇒ <u>1st order</u>



One of three possible Columbia Plots
Forcrand et. al. arxiv:1702.00330

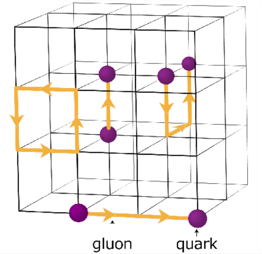
Details on lower left: see talk Julian Bernhardt

Challenges when Investigating Phase Transitions on the Lattice

- 1st order phase transition has super critical slowing down
  $\rightarrow$ Both phases have to be sampled for investiagations of the phase transition
- 2nd order phase transition has critical slowing down
  $\rightarrow$ Large autocorrelation of simulated data.
- Simulating with fermions has high computational cost (scales inverse with the mass of the fermions)

$\rightarrow$ Problem of generating enough statistics or independent samples

Introduction to Lattice QCD



Sketch of a lattice

Lattice QCD

- Discretize the spacetime with spacing a
- Place Spinors ψ at lattice points
- Place Gauge Field as links U_μ in between
- Choose suitable action for Gauge field i.e $S_G[U] = -\frac{\beta}{3}\sum_{n\in\Lambda}\sum_{\mu<\nu}\Re\mathrm{e}\,\mathrm{tr}(\mathbb{1} - P_{\mu\nu}(n))$

Suitable Observables for Investigation

- Polyakov Loop and its susceptibility:

$$P = \frac{1}{N_s^3} \sum_{\vec{x}} P_{\vec{x}} = \frac{1}{N_s^3} \sum_{\vec{x}} \mathrm{tr} \left[\prod_{\tau} U_0(\vec{x}, \tau) \right] \tag{1}$$

$$\chi = N_s^3 (\langle |P|^2 \rangle - \langle |P| \rangle^2) \tag{2}$$

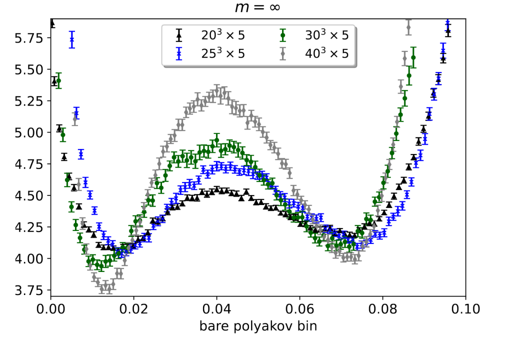
- Third-order Binder cumulant of the Polyakov loop:

$$b_3 = \frac{\langle |P|^3 \rangle - 3\langle |P| \rangle \langle |P|^2 \rangle + 2\langle |P| \rangle^3}{(\langle |P|^2 \rangle - \langle |P|^2 \rangle)^{\frac{3}{2}}} \tag{3}$$

- Latent heat:

$$\Delta(\frac{\epsilon - 3p}{T^4}) = N_t^4 a \frac{\partial \beta}{\partial a} [S_{\mathrm{hot}} - S_{\mathrm{cold}}] \tag{4}$$

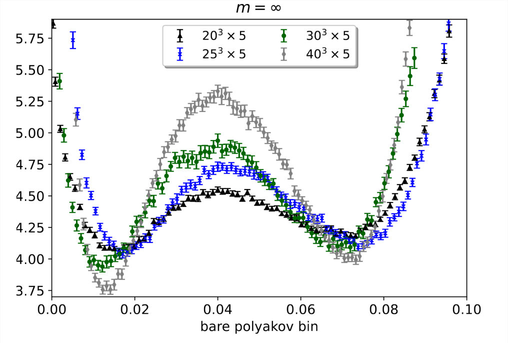
Super critical slowing down – Quenched QCD



Wuppertal-Budapest arxiv: 2212.10155

- Problem: The system is more likely to freeze in one of the two phases

Super critical slowing down – Quenched QCD



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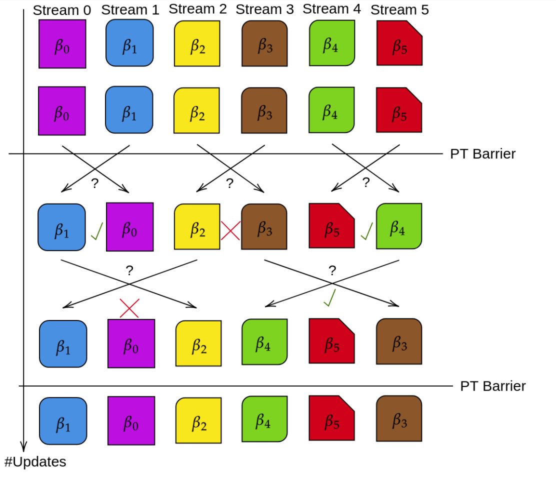
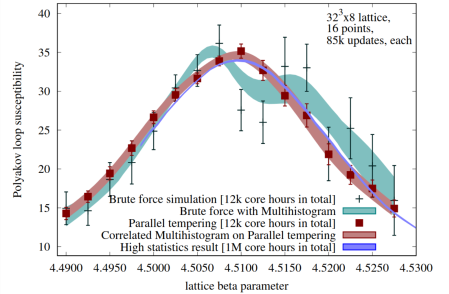
- Problem: The system is more likely to freeze in one of the two phases
- $\rightarrow$ One Ansatz: Parallel Tempering (*arxiv: 9205018 ,9810032*)

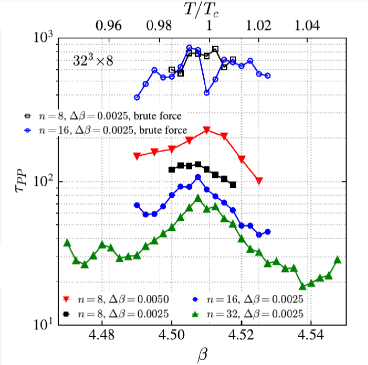
Parallel Tempering - Principle



Principle of the Parallel Tempering Algorithm. β indicates inverse Temperature

Parallel Tempering – Consequences in Quenched QCD

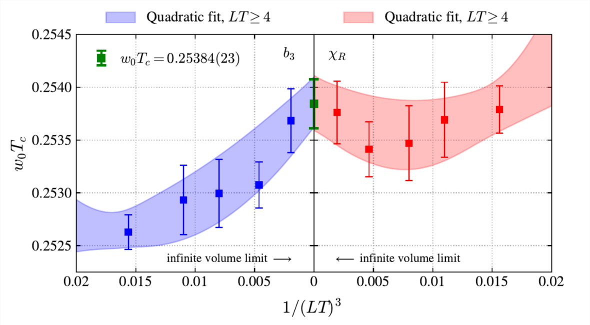




Wupperta-Budapest arxiv: 2202.05234

- Autocorrelation time drops significantly
  $\rightarrow$ More precise simulations with the same effort

Determine Transition Temperature and Latent Heat



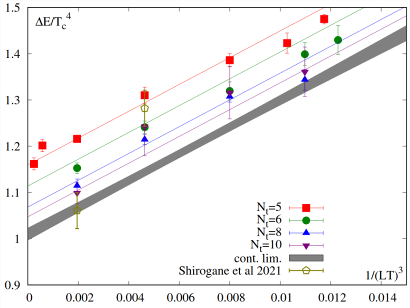
Wuppertal-Budapest arxiv: 2202.05234

Determined transition temperature (in lattice units):

$$w_0 T_c = 0.25384(11)_{\text{stat}}(21)_{\text{sys}}$$

First per-mil accurate result in QCD thermodynamics

Determine Transition Temperature and Latent Heat



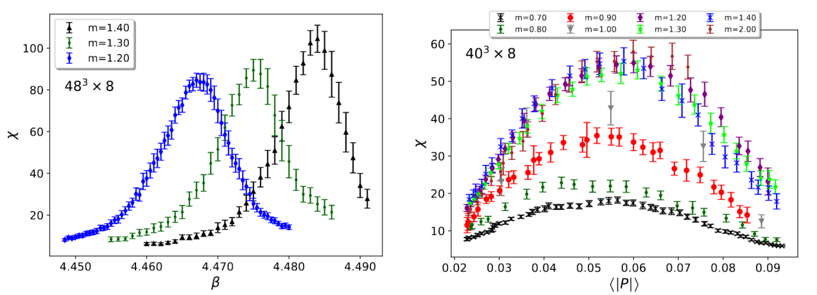
Wuppertal-Budapest arxiv: 2202.05234

- Determined latent heat

$$\Delta(\frac{\epsilon - 3p}{T^4}) = 1.025(21)_{\text{stat}}(27)_{\text{sys}} \quad (5)$$

$\rightarrow$ Confirms first order phase transition

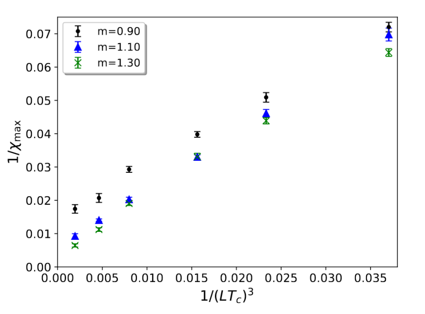
Now include dynamical quarks with large but finite mass and measure peak position again for different masses



Wuppertal-Budapest arxiv: 2212.04192

Determine peak position for each mass and volume simulated

Parallel Tempering applied to Dynamical QCD

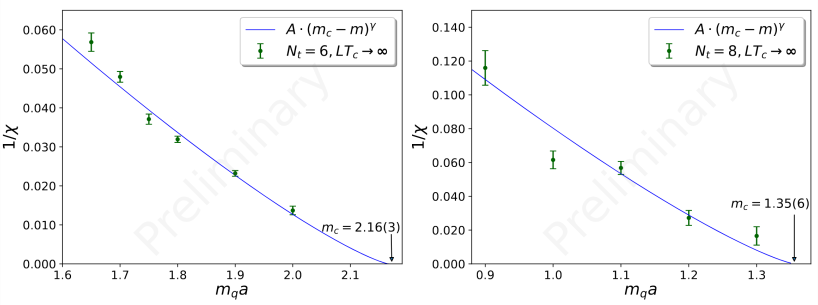


Wuppertal-Budapest 2212.04192

<u>Fit functions</u>

- Linear: $\chi^{-1}(LT_c) = a + b\left(\frac{1}{LT_c^3}\right)$
- With "effective exponent": $\chi^{-1}(LT_c) = a + b\left(\frac{1}{LT_c^3}\right)^c$

Determine m_c



R. Kara Lattice 2022

Note: No continuum limit of the critical mass possible yet, due to critical slowing down and additional computational costs in including fermions into the simulation.

Improvement but not the solution

- Autocorrelation time was decreased significantly
- Transition temperature on per mil accuracy was determined
- Latent Heat > 0 was determined proving the 1st order phase-transition
- **But:** If one reduces the mass of fermions (uses Dynamical Fermions) the problem of adding fermions also add up to the already existing (super) critical slowing down

$\rightarrow$ We could try new kinds of algorithms in the high mass region or in the pure gauge theory.

Procedure of testing new algorithms for Gauge theories

- Work in an easier gauge group SU(2):
  1. System gives far easier environment to test into
  2. Simulations take less time than SU(3) simulations
  3. System is in the same universality class (scaling behavior) as the 3D Ising modell (very simple)
- If algorithms don't work in SU(2) they should not work in SU(3) either
- If algorithms work ins SU(2) they might as well be applicable for SU(3)

Cluster Algorithm for Gauge theorys?

- Idea: build Clusters, that flip the sign of the trace of the Polyakov Loop

Why could something like that work?

- SU(2) and the 3D Ising model are in the same Universality class $\rightarrow$ SU(2) can be mapped into the Ising Model
- For the Ising Model such Algorithms are well known to solve problem of critical slowing down
  (Wolff DOI: 10.1103/PhysRevLett.62.361 ,
  Swendsen-Wang, DOI: 10.1103/PhysRevLett.58.86)

How is SU(2) mapped into the Ising Model?

- Action of SU(2)

$$S = -\frac{\beta}{2} \sum_{P_{\mu,\nu}} \mathrm{Tr} P_{\mu,\nu}$$

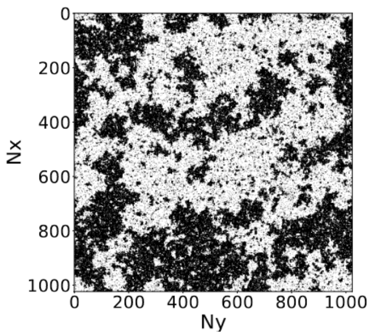
- Hamiltonian of the Ising Model

$$H = -\frac{1}{2} \sum_{\langle ij \rangle} s_i s_j k_{ij}$$

- Idea for connecting them

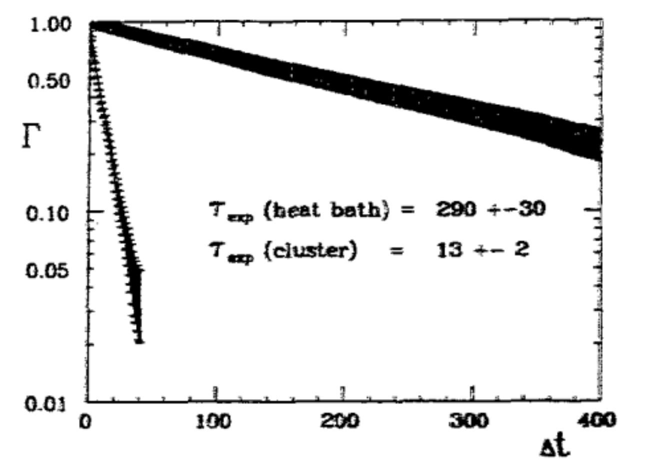
$$s_i = \mathrm{sgn}(\mathrm{Tr}(U_i))$$

$$\Rightarrow k_{ij} = \frac{\mathrm{Tr} P}{\mathrm{sgn} \mathrm{Tr} U_i \mathrm{sgn} \mathrm{Tr} U_j}$$



2D Ising Model near the Critical point

Can this work?



Evertz et. al DOI: 10.1016/0920-5632(91)90886-J

Conclusion

- Algorithmic ideas or mixing of algorithms can help with sampling more efficiently (Parallel Tempering)
- Cluster algorithms which work very good for Spin systems (eg. 3D Ising Model) should be able be implemented in a similar way for gauge theories (SU(2) maybe also SU(3)) due to the fact that they are in the same universality class (not clear for SU(3))
- Mapping of SU(2) into an Ising Model in test stage at the moment

Problems and ideas to solve for upper right corner

Still existing Problems

- (Super) critical slowing down in the 1st and 2nd order (partially improved by using parallel tempering algorithm)
- Computational costs of including dynamical fermions → No continuum limit for m_c in upper right corner due to the upper point

Possible Solutions

- Global heatbath update
- Exploiting of expansions of fermion formulations similar to *arxiv: 1805.03560*

End of the talk

Thank you for your attention!
Any Questions?

Backup - Introduction to lattice QCD SU(3)

In the continuum we have the action for the Gluons:

$$S_G[A] = \frac{1}{4g^2} \sum_{i=1}^{8} \int \mathrm{d}^4 x F_{\mu\nu}^{(i)}(x) F_{\mu\nu}^{(i)}(x) \tag{6}$$

and the fermions:

$$S_F[\psi, \bar{\psi}, A] = \sum_{f=1}^{N_F} \int \mathrm{d}^4 x \, \bar{\psi}^{(f)}(x)(\gamma_\mu(\partial_\mu + iA_\mu) + m^{(f)}) \, \psi^{(f)}(x) \tag{7}$$

Backup - Introduction to lattice QCD SU(3)

Plaquette (smallest simply connected loop on the lattice)

$$P_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu^\dagger(n+\hat{\nu}) U_\nu^\dagger(n). \tag{8}$$

Wilson gauge is one action that approaches the continuum version for a $\to 0$:

$$S_G[U] = -\frac{\beta}{3} \sum_{n\in\Lambda} \sum_{\mu<\nu} \Re\mathrm{e}\,\mathrm{tr}(\mathbb{1} - P_{\mu\nu}(n)) \tag{9}$$

Gauge variables $U_\mu(n)$ introduced to make naive fermion action invariant under local gauge transformation and live on the links between two lattice points

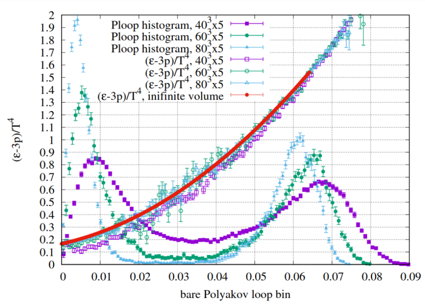
$$U_\mu(n) = \exp\{iaA_\mu(n)\}$$

Backup - Monte-Carlo Sampling

- Use Markov chain Monte-Carlo Algorithms to update the discretized field
- To update the gauge field one uses local update algorithms (Metropolis, Heatbath, (+Overrelexation))
- With the including of fermions has has to use other algorithms (Hybrid Monte-Carlo is used)

$\rightarrow$ After a certain number of updates measure observables on the sampled field (e.g Energy, Magnetization, Plaquette, Polyakov-Loop)

Backup - Determine Transition Temperature and Latent Heat



Budapest-Wuppertal arxiv: 2212.10155

- Latent heat defined via the trace anomaly
- Find minimum of the Polakov Loop histogram

  → The trace anomaly can then be evaluated in both phases

Backup - How does such a Cluster-Algorithm Work

- Let's first look at the Cluster Algorithm in the Ising Model (here Wolff as an example)

+	+	+	+	-	-	-
+	-	-	-	+	-	-
+	-	+	-	-	+	-
-	+	+	-	+	+	-
+	-	+	-	+	+	+

Sketch how the Wolff Algorithm works

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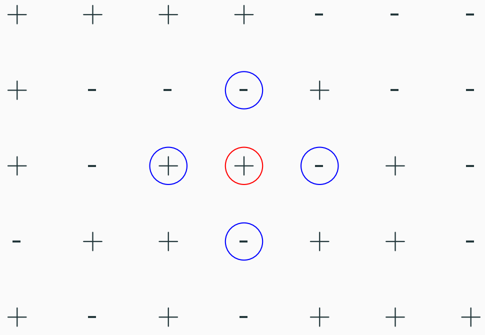

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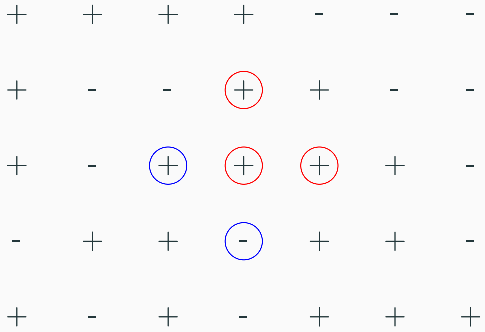

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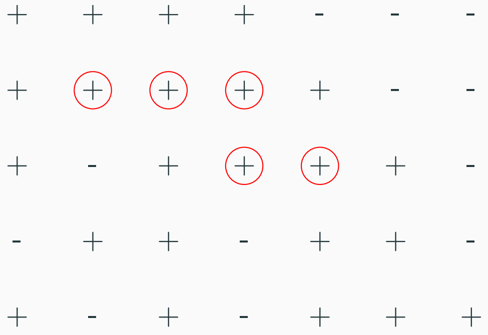

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