Effects of medium expansion on jet transverse momentum broadening

Souvik Adhya[a], Krzysztof Kutak[a], Wiesław Płaczek[b], Martin Rohrmoser[c], Konrad Tywoniuk[d]

[a] IFJ-PAN, Cracow [b] Uniwersytet Jagielloński, Cracow

[c] Cracow University of Technology, Cracow [d]University in Bergen, Bergen

talk based mainly on

[Adhya, Kutak, Płaczek, MR, Tywoniuk, Eur.Phys.J.C 83 (2023) 6, 512, arxiv: 2211.15803]

Medium cases

Static Medium

For $t_0<t<L$: $T=T_0=\text{const.}$ Transverse momentum transfer: $\hat{q}=\hat{q}_0 \propto T_0^{\ 3}$

otherwise: $T=0$ $\hat{q}=0$

Bjorken Medium

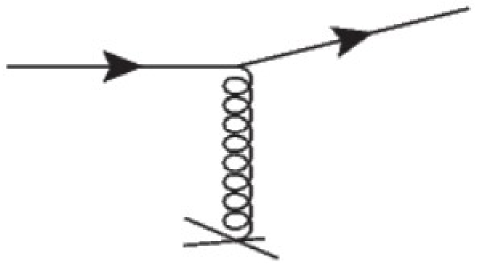
For $t_0<t<L$: $T=T_0\left(\frac{t_0}{t}\right)^{\frac{\alpha}{3}}$ Transverse momentum transfer: $\hat{q}=\hat{q}_0\left(\frac{t_0}{t}\right)^{\alpha}$

otherwise: $T=0$ $\hat{q}=0$

$\alpha=1$ longitudinally expanding medium

Processes in jets in the medium

scattering…



Momentum transfer!

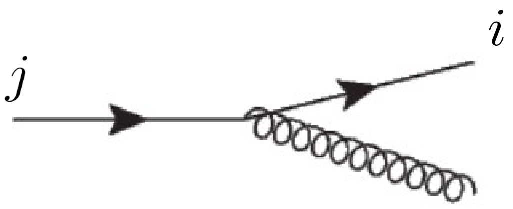
$$p \to p + Q$$

Scattering Kernel:

$$\frac{\partial^3 \mathcal{P}_{\text{scat}}}{\partial t \partial^2 \mathbf{Q}} = \frac{1}{(2\pi)^2} w(\mathbf{Q})$$

Average transfer: $\hat{q}$

…splitting…



Bremsstrahlung as in vacuum.

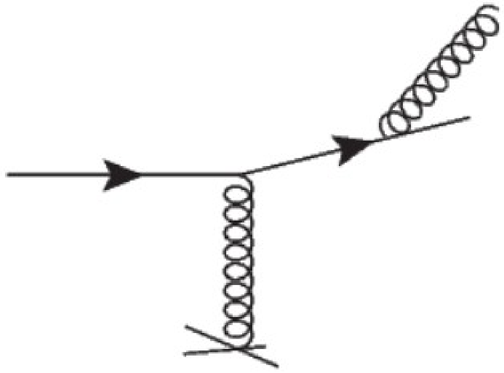
Momentum distribution:

$$p \to zp$$

DGLAP-Kernel:

$$\frac{\partial^2 \mathcal{P}_{\text{split}}}{\partial \hat{t} \partial z} \propto \frac{1}{\hat{t}} P_{ij}(z)$$

…induced radiation



Momentum distribution:

$$p \to zp$$

+Momentum transfer:

$$p \to zp + Q$$

Kernel:

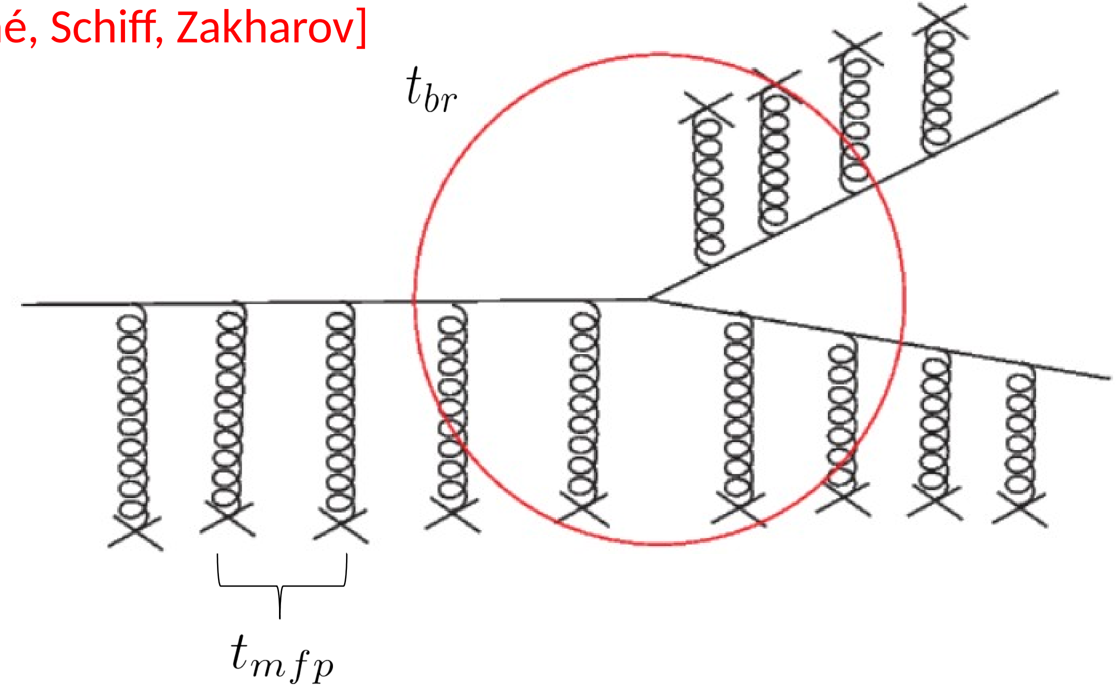
$$\frac{\partial^4 \mathcal{P}_{\text{split}}}{\partial t \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2} \mathcal{K}(\mathbf{Q}, z, p_+)$$

This talk: combination of scattering and induced radiation processes!

Coherent emission

...à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$



$t_{br} \sim t_{mfp}$: one scattering + radiation
...Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, t)$

Splitting Kernels

Static Medium

$$\tilde{\mathcal{K}}^{\rm static}(z,t) = \mathcal{K}(z)\,\frac{\sinh(\kappa(z)\tau) - \sin(\kappa(z)\tau)}{\cosh(\kappa(z)\tau) + \cos(\kappa(z)\tau)}$$

$$\mathcal{K}(z) = \frac{\kappa(z) P_{gg}(z)}{2N_c}$$

$$\tau \equiv t/t_* \qquad t_* \equiv \frac{1}{\bar{\alpha}}\sqrt{\frac{p_0^+}{\hat{q}_0}}$$

$$\bar{\alpha} = \alpha_s N_c/\pi$$

$$\kappa(z) = \sqrt{[1 - z(1-z)]/[z(1-z)]}$$

[Zakharov, JETP Lett. 63, 952 (1996), arxiv:hep-ph/9607440]
[Zakharov, JETP Lett. 65, 615 (1997), arxiv:hep-ph/9704255]
[Adhya, Salgado, Spousta, Tywoniuk, Eur. Phys. J. C 82, 20 (2022), arxiv:2106.02592]
[Adhya, Salgado, Spousta, Tywoniuk, JHEP 07, 150 (2020),arxiv:1911.12193]
[Baier, Dokshitzer, Mueller, Peigne, Schiff, Nucl. Phys. B 483, 291 (1997), arxiv:hep-ph/9607355]
[Baier, Dokshitzer, Mueller, Peigne, Schiff, Nucl. Phys. B 484, 265 (1997), arxiv:hep-ph/9608322]

Bjorken Medium

$$\tilde{\mathcal{K}}^{\rm Bjorken}(z,\tau,\tau_0) = \mathcal{K}(z)\sqrt{\frac{\tau_0}{\tau}}\,\mathrm{Re}\left[(1-i)\frac{J_1(z_L)Y_1(z_0) - J_1(z_0)Y_1(z_L)}{J_1(z_0)Y_0(z_L) - J_0(z_L)Y_1(z_0)}\right]$$

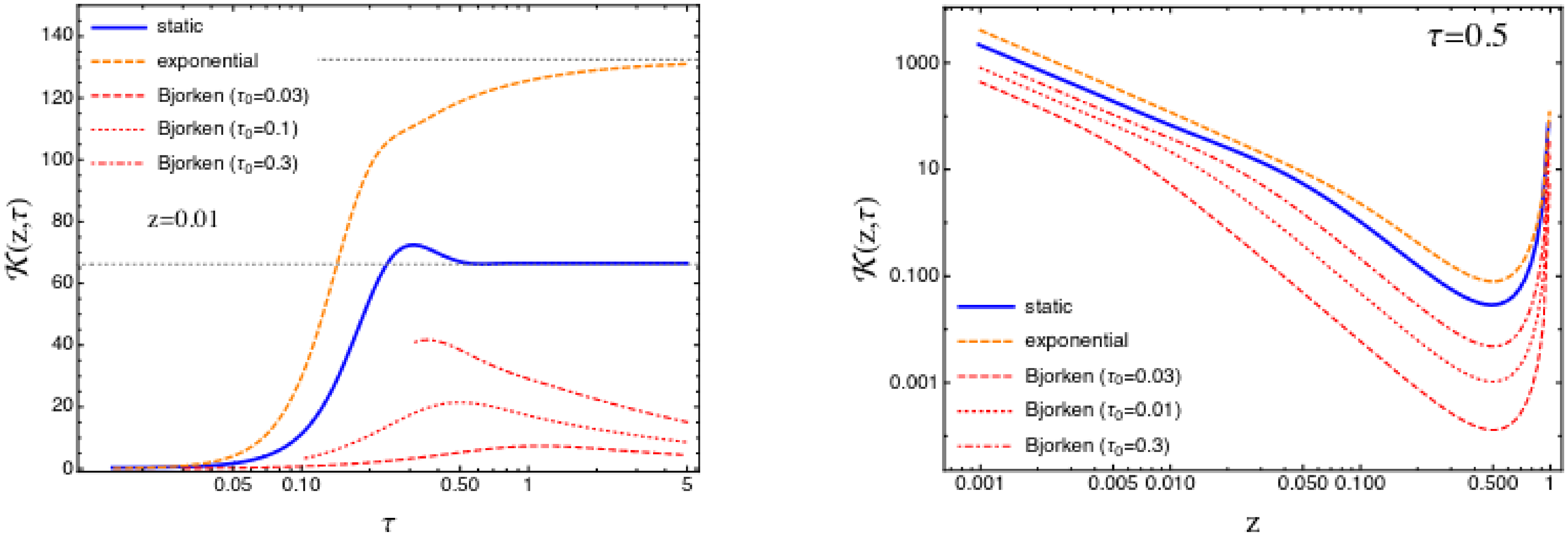
$$z_0 = (1-i)\kappa(z)\tau_0$$

$$z_L = (1-i)\kappa(z)\sqrt{\tau_0\tau}$$

$$\tau_0 \equiv t_0/t_*$$

$$T(t) = \begin{cases} 0 & \text{for} \quad t < t_0, \\ T_0\left(\frac{t_0}{t}\right)^{\frac{1}{3}} & \text{for} \quad t_0 \le t \le L + t_0, \\ 0 & \text{for} \quad t > L + t_0, \end{cases}$$

[Salgado, Wiedemann, Phys. Rev. D 68, 014008 (2003)., arxiv:hep-ph/0302184]
[Baier, Dokshitzer, Mueller, Schiff, Phys. Rev. C 58, 1706 (1998), arXiv:hep-ph/9803473]
[Arnold, Phys. Rev. D 79, 065025 (2009), arxiv:0808.2767]
[Adhya, Salgado, Spousta, Tywoniuk, Eur. Phys. J. C 82, 20 (2022), arxiv:2106.02592]
[Adhya, Salgado, Spousta, Tywoniuk, JHEP 07, 150 (2020), arxiv:1911.12193]



[Adhya, Salgado, Spousta, Tywoniuk, JHEP 07 (2020) 150, arxiv: 1911.12193 [hep-ph]]

Scattering Kernels

Used right now:

$$w(\boldsymbol{l}, t) = \frac{N_c g^2 m_D^2 T}{\boldsymbol{l}^2(\boldsymbol{l}^2 + m_D^2)} = \frac{4\pi \, \hat{q}}{\boldsymbol{l}^2(\boldsymbol{l}^2 + m_D^2)}$$

$$\hat{q} = \alpha_s N_c m_D^2 T \qquad m_D^2 = g^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) = \frac{3}{2} g^2 T^2$$

Evolution equations

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \frac{1}{t_*} \int_0^1 \mathrm{d}z \, \tilde{\mathcal{K}}(z, t) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} \, D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t \right) \theta(z - x) - \frac{z}{\sqrt{x}} \, D(x, \boldsymbol{k}, t) \right]$$

$$+ \int \frac{\mathrm{d}^2 \boldsymbol{l}}{(2\pi)^2} \, C(\boldsymbol{l}, t) \, D(x, \boldsymbol{k} - \boldsymbol{l}, t)$$

$$D(x, \boldsymbol{k}, t) \equiv x \frac{\mathrm{d}N}{\mathrm{d}x \, \mathrm{d}^2 \boldsymbol{k}}$$

$$\int \mathrm{d}^2 \boldsymbol{k}$$

$$C(\boldsymbol{l}, t) = w(\boldsymbol{l}, t) - \delta^{(2)}(\boldsymbol{l}) \int \mathrm{d}^2 \boldsymbol{l}' \, w(\boldsymbol{l}', t)$$

$$\frac{\partial D(x, t)}{\partial t} = \frac{1}{t_*} \int_0^1 \mathrm{d}z \, \tilde{\mathcal{K}}(z, t) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t \right) \Theta(z - x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

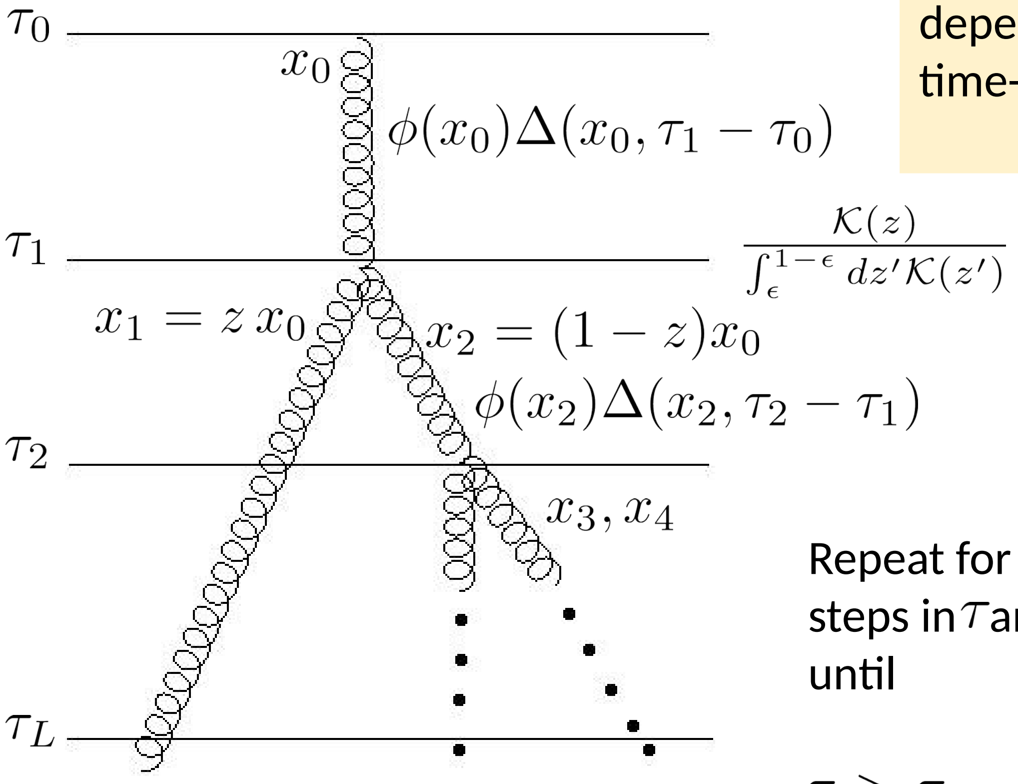
$$D(x, t) = \int \mathrm{d}^2 \boldsymbol{k} \, D(x, \boldsymbol{k}, t) = x \, \mathrm{d}N / \mathrm{d}x$$

Monte-Carlo algorithms for jets

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …

Analogous for the k_T dependent equation and for time-dependent Kernels



$$\phi(x) = \int_\epsilon^{(1-\epsilon)} dz \sqrt{\frac{1}{x}} K(z)$$

$$\Delta(x, \tau_2 - \tau_1) = e^{-\phi(x)(\tau_2 - \tau_1)}$$

Repeat for all steps in τ and x until

$\tau > \tau_L$

Monte-Carlo algorithm that solves for fragmentation functions:

MINCAS

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Monte-Carlo algorithm that solves for multiplicity distributions (jets):

TMDICE

[MR, Comput.Phys.Commun. 276 (2022) 108343]

Source code:

https://github.com/Rohrmoser/TMDICE

Effective length scale

$$L_{\text{eff}} = \int_0^\infty \mathrm{d}t' \sqrt{\frac{\hat{q}(t)}{\hat{q}_0}}.$$

Bjorken Model:

$$L_{\text{eff}} = 2\sqrt{t_0}\left(\sqrt{L+t_0} - \sqrt{t_0}\right)$$

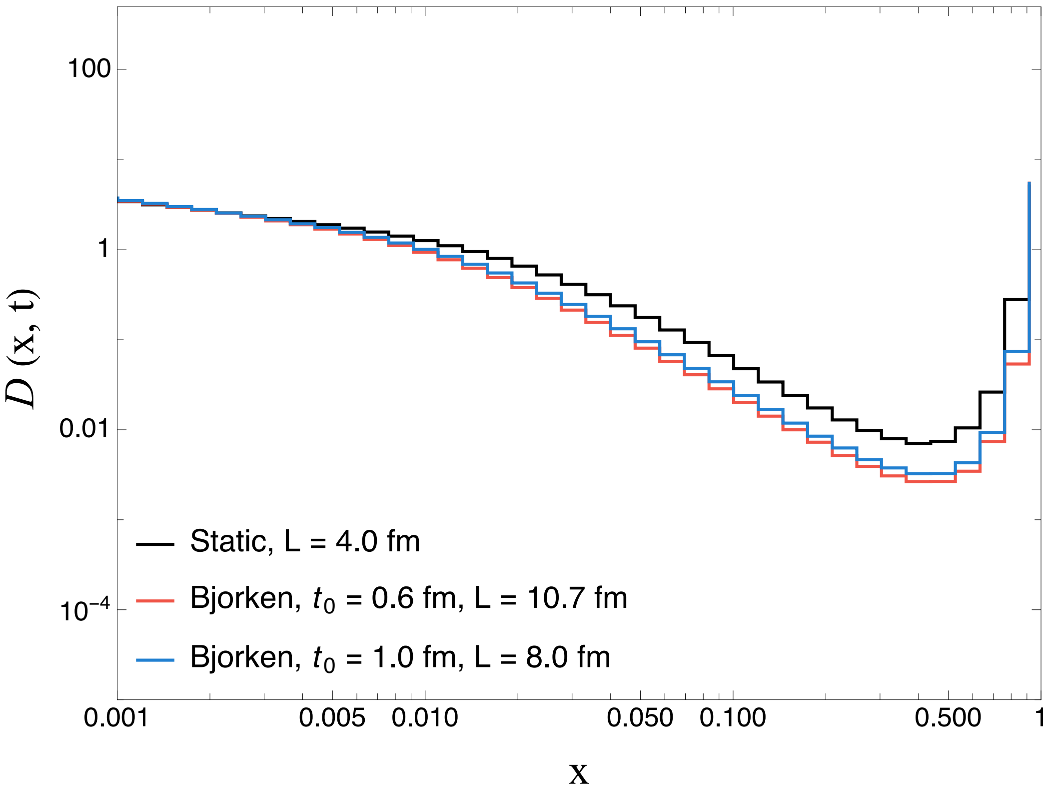
Explored models:

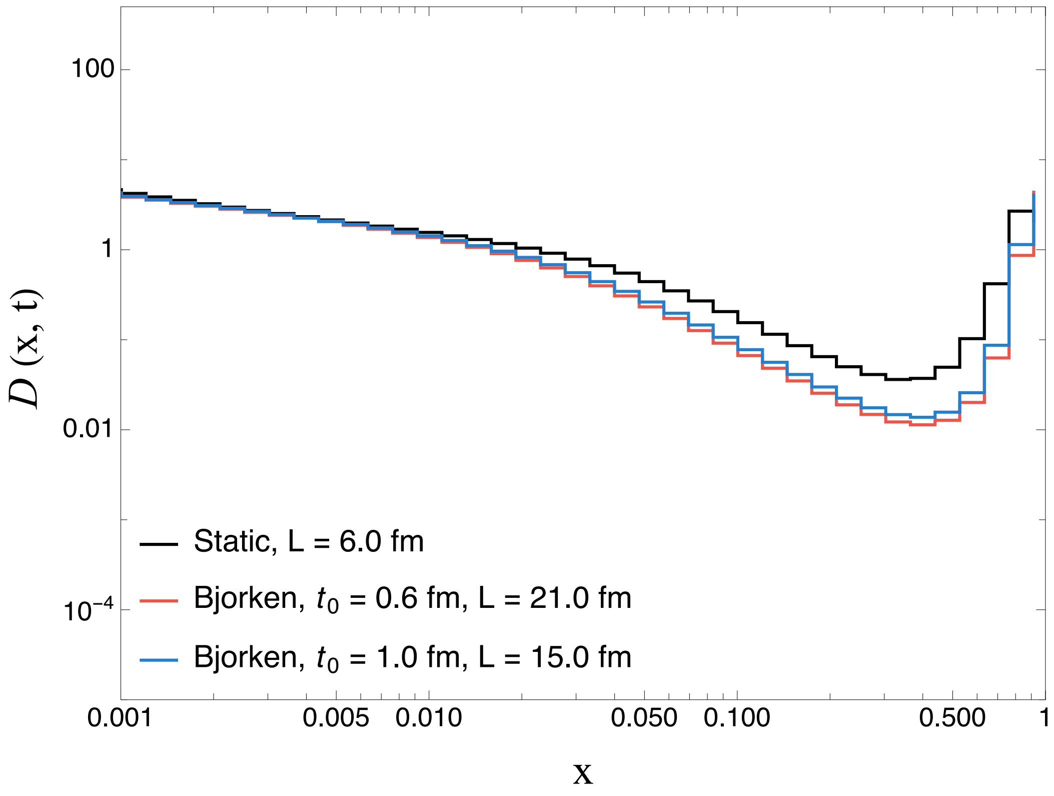
Medium	t_0 [fm]	L_{eff} [fm]	L [fm]
Static	0.0	4.0; 6.0	4.0; 6.0
Bjorken (early)	0.6	4.0; 6.0	10.7; 21.0
Bjorken (late)	1.0	4.0; 6.0	8.0; 15.0

Fragmentation functions

$$\tilde{D}(x, k_T, t) = \int_0^{2\pi} \mathrm{d}\phi_{\boldsymbol{k}}\, k_T\, D(x, \boldsymbol{k}, t)$$

$$D(x, t) = \int_0^{\infty} \mathrm{d}k_T\, \tilde{D}(x, k_T, t)$$



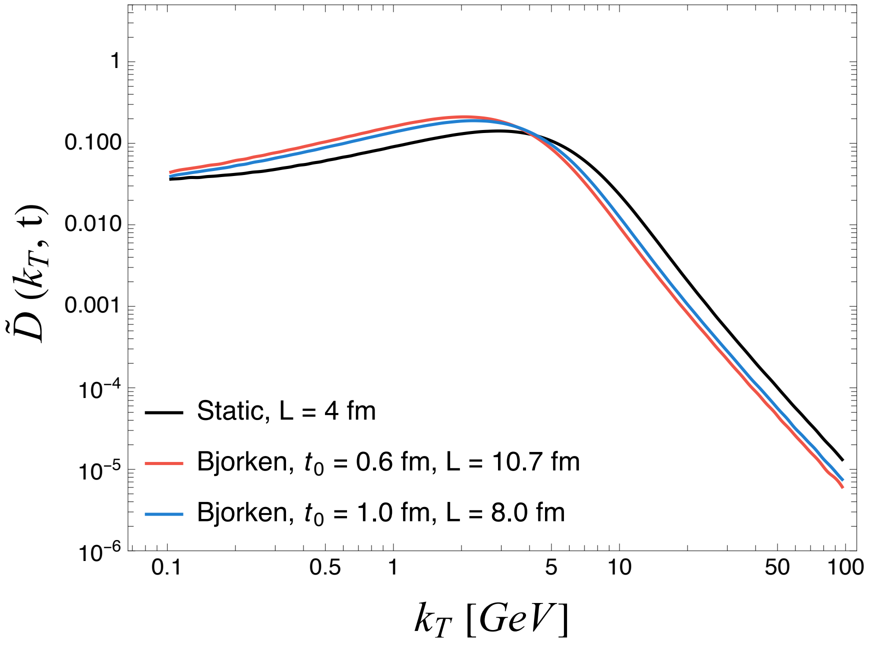


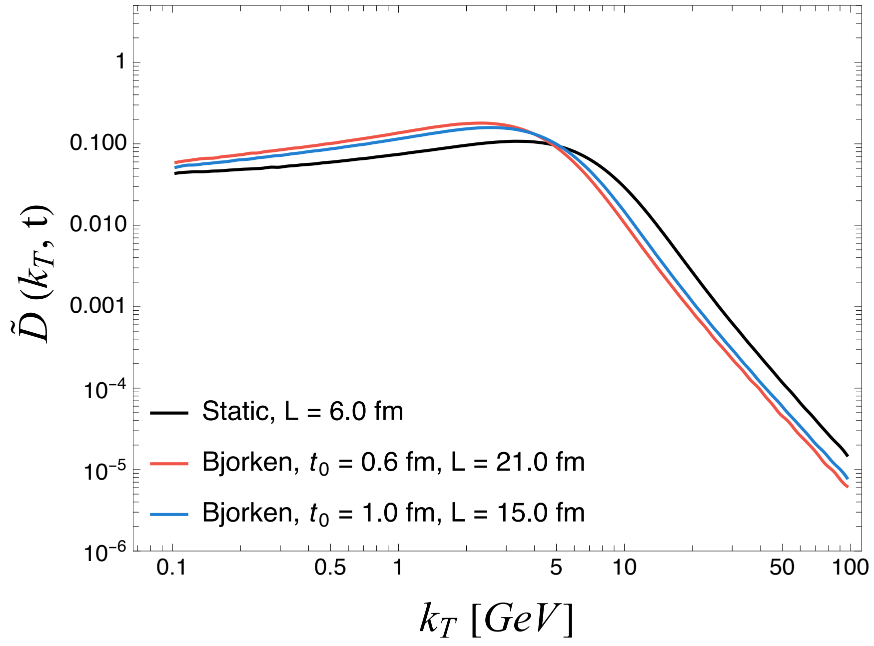
[Adhya, Kutak, Płaczek, MR, Tywoniuk, Eur.Phys.J.C 83 (2023) 6, 512, arxiv: 2211.15803]

Distributions in k_T

$$\tilde{D}(x, k_T, t) = \int_0^{2\pi} d\phi_k \, k_T \, D(x, \boldsymbol{k}, t)$$

$$\tilde{D}(k_T, t) = \int_0^1 dx \, \tilde{D}(x, k_T, t)$$





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angular distributions

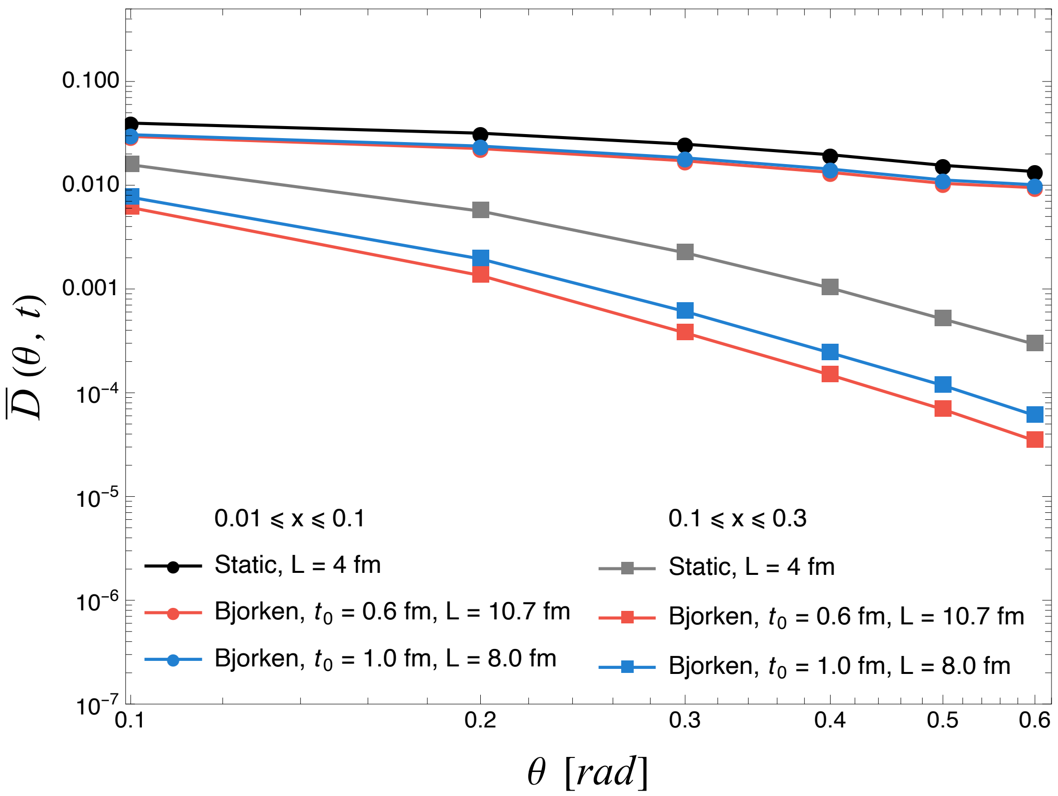
Small angle approximation: $k_T = x\, p_0^{+}\, \theta$

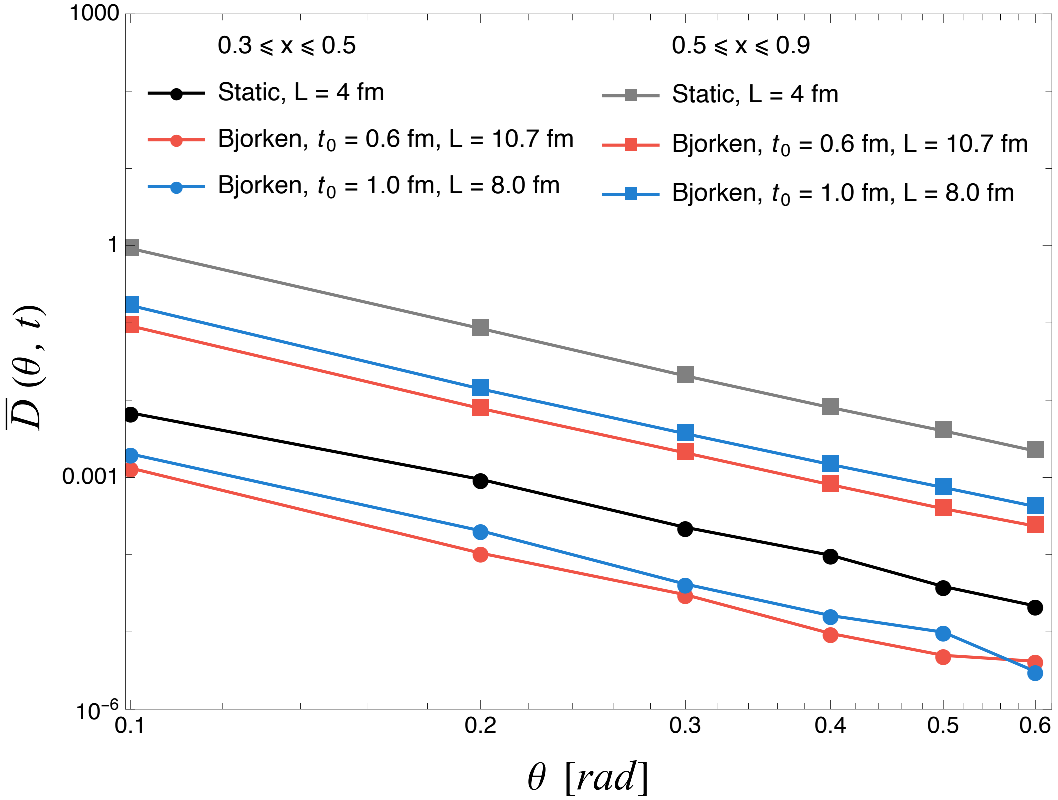
Definition of angular distributions:

$$\bar{D}(x, \theta, t) = x p_0^{+}\, \tilde{D}(x, x p_0^{+} \theta, t)$$

$$D(x, t; \{\theta_{\max}\}) = \int_0^{\theta_{\max}} \mathrm{d}\theta\, \bar{D}(x, \theta, t)$$

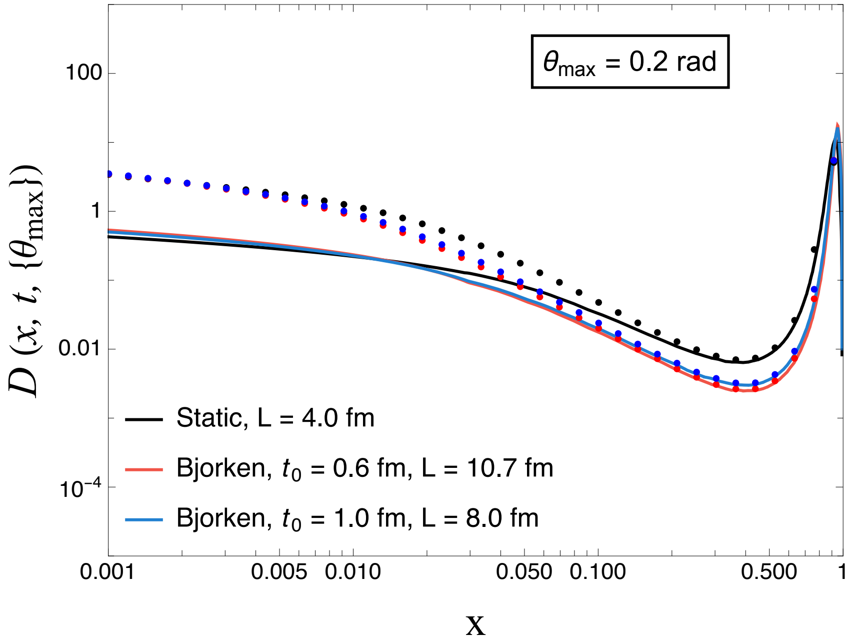
Angular distributions

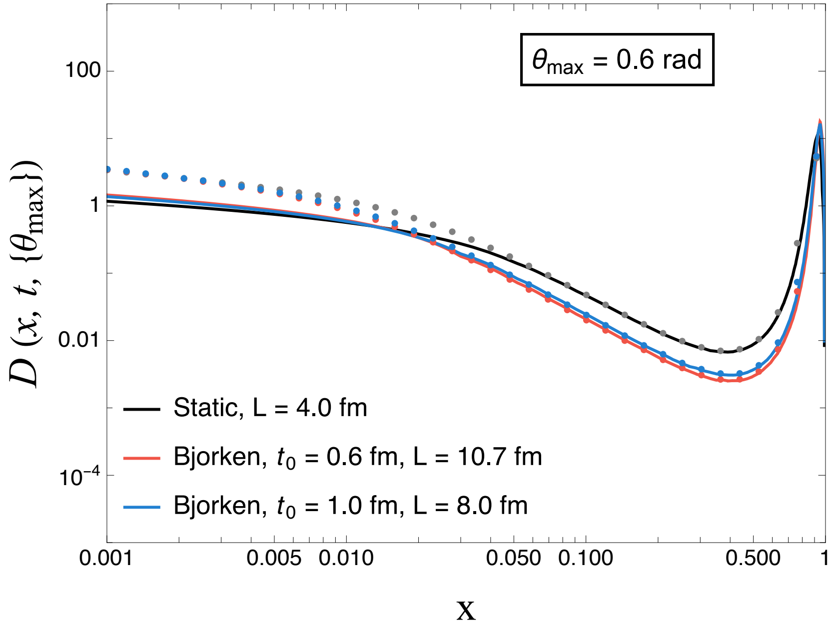


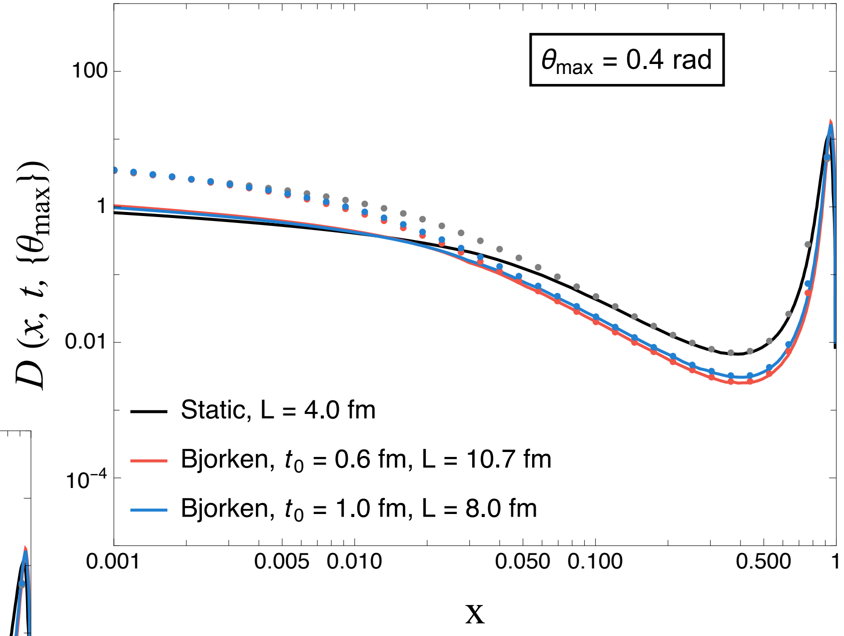


[Adhya, Kutak, Płaczek, MR, Tywoniuk, Eur.Phys.J.C 83 (2023) 6, 512, arxiv: 2211.15803]

Energy dependence of broadening







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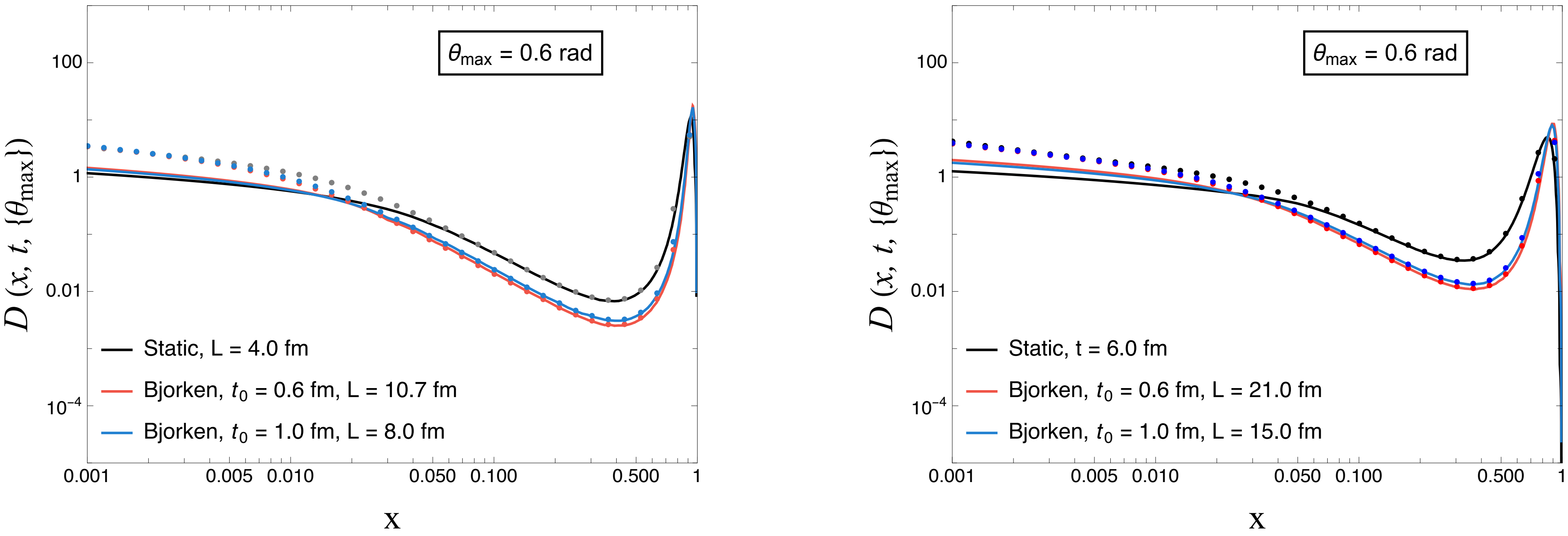
Energy dependence of broadening



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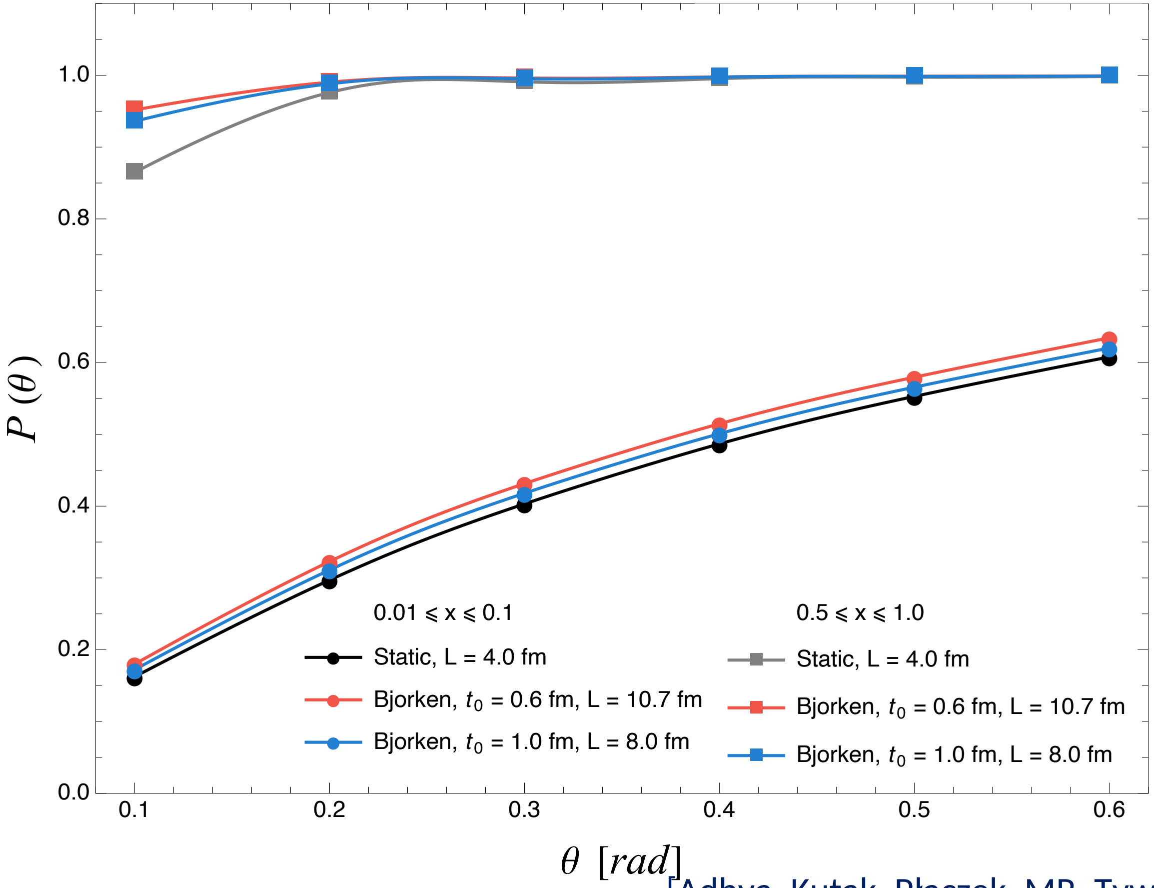
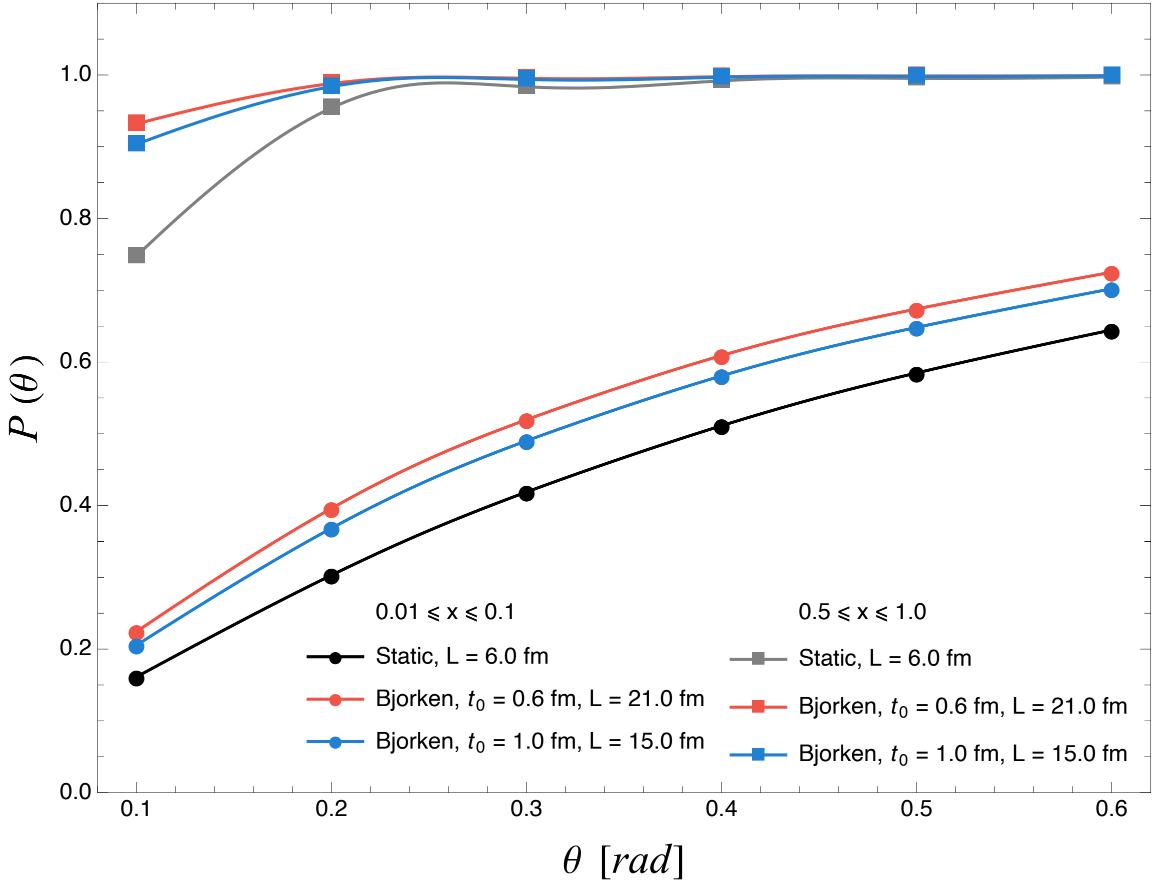
In cone energy

$$P(\theta, t; \{x_{\min}, x_{\max}\}) = \frac{\int_{x_{\min}}^{x_{\max}} \mathrm{d}x \int_0^{\theta} \mathrm{d}\theta' \, \bar{D}(x, \theta', t)}{\int_{x_{\min}}^{x_{\max}} \mathrm{d}x \int_0^{\pi} \mathrm{d}\theta' \, \bar{D}(x, \theta', t)}$$





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Summary & Outlook

- 3 different medium models (static medium, early Bjorken, late Bjorken)
- ‘universal’ distribution at small x (soft emissions)
- Soft particles contribute mostly to angular broadening
- Nearly universal angular broadening at soft emissions
- Hard particles remain collinear; broadening due to consecutive emissions.
- Jets in static media are less collimated than in expanding media.
- Future: Generalization to quarks and gluons.

Thank you for your attention!