Higher order fluctuations of net protons in HIC: Parity partners and chiral criticality

K. Redlich, V. Koch, M. Marczenko, C. Sasaki

University of Wrocław

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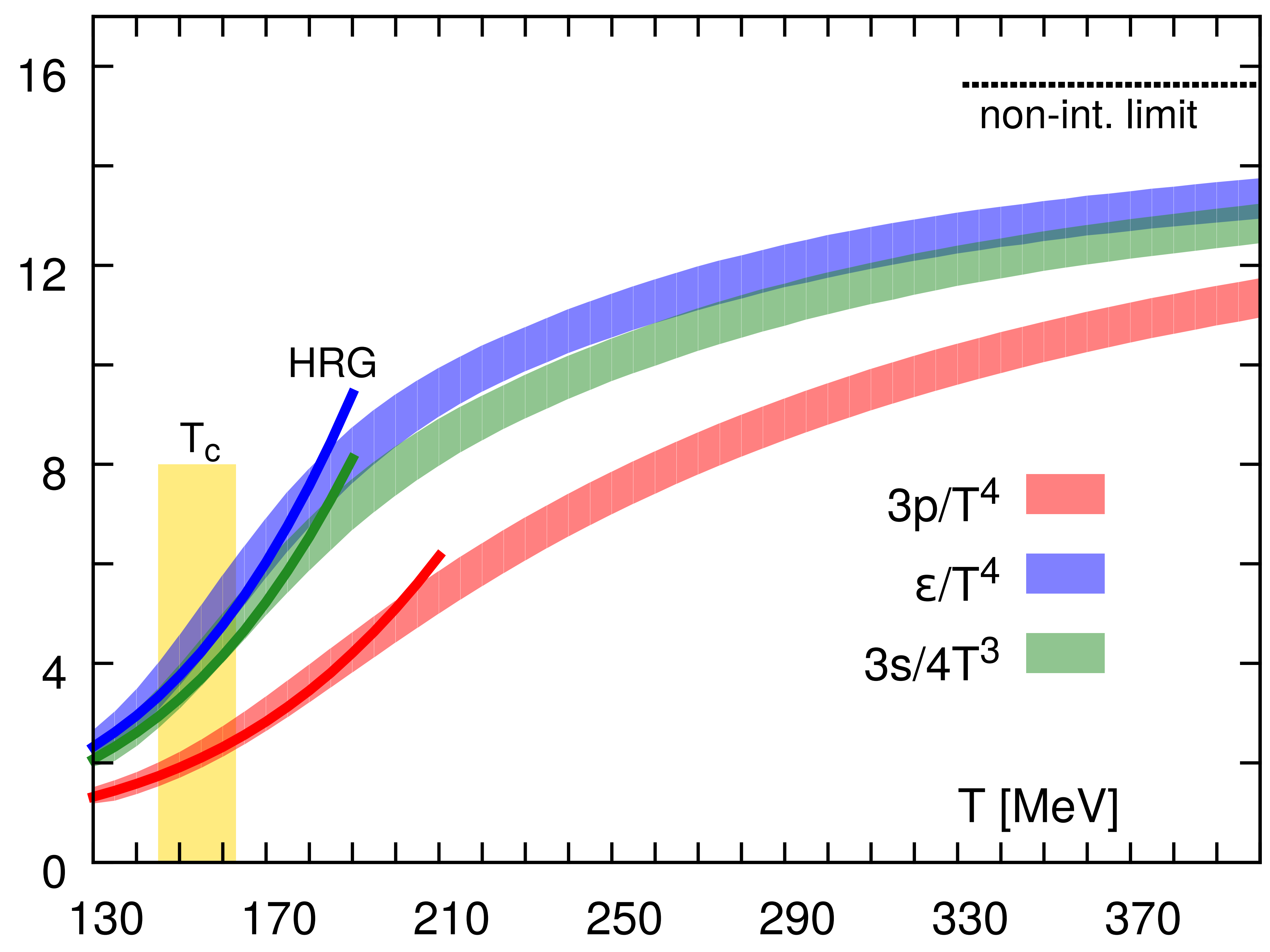
Lattice QCD vs Hadron Resonance Gas

HotQCD, 2014

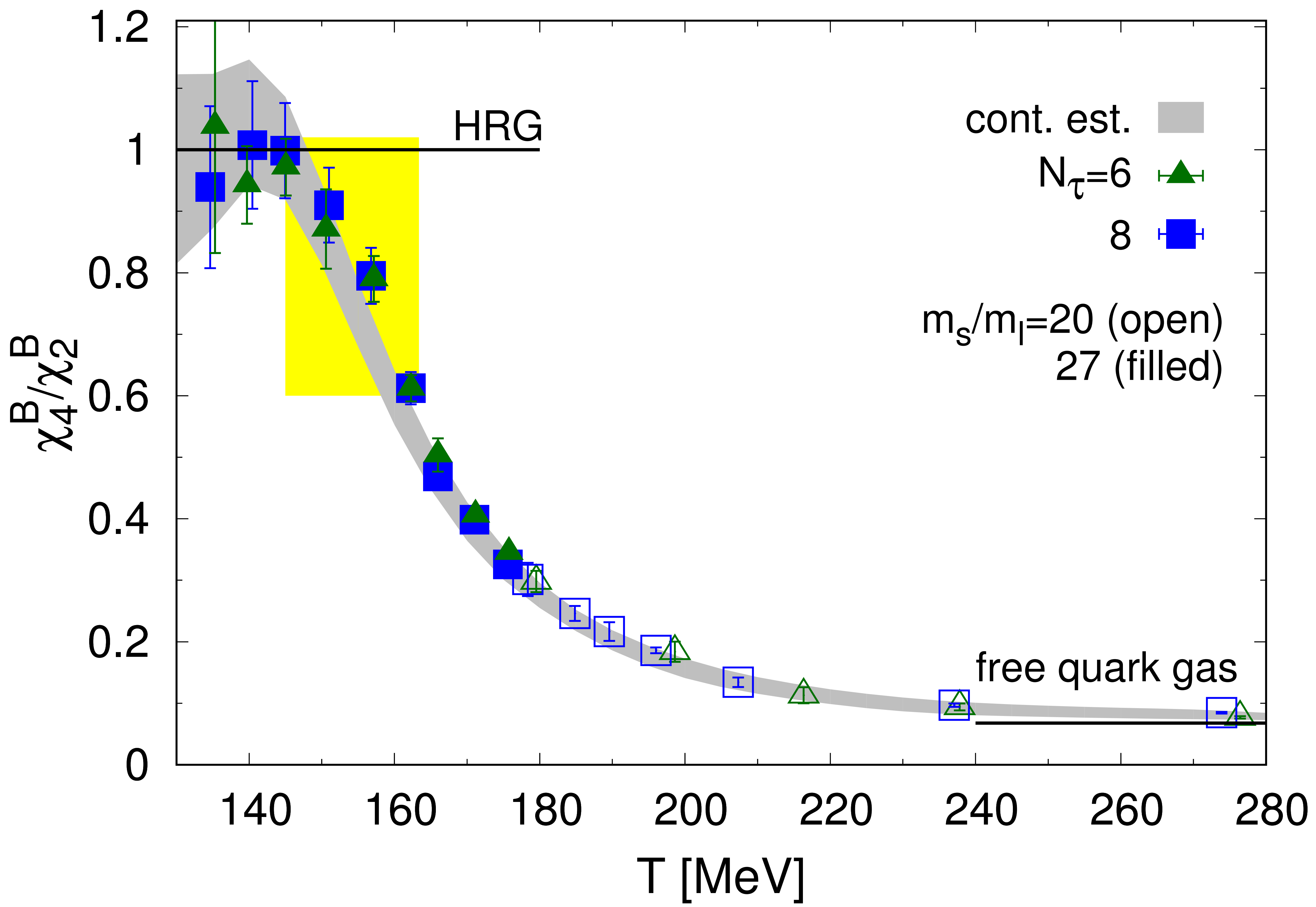


Pressure in the HRG model

$$P^{\text{HRG}} = \sum_{i \in \text{had}} P^{\text{id}}\left(T, \mu_i; m_i\right)$$

Agreement with LQCD EoS up to $\simeq T_c$

HotQCD, 2017

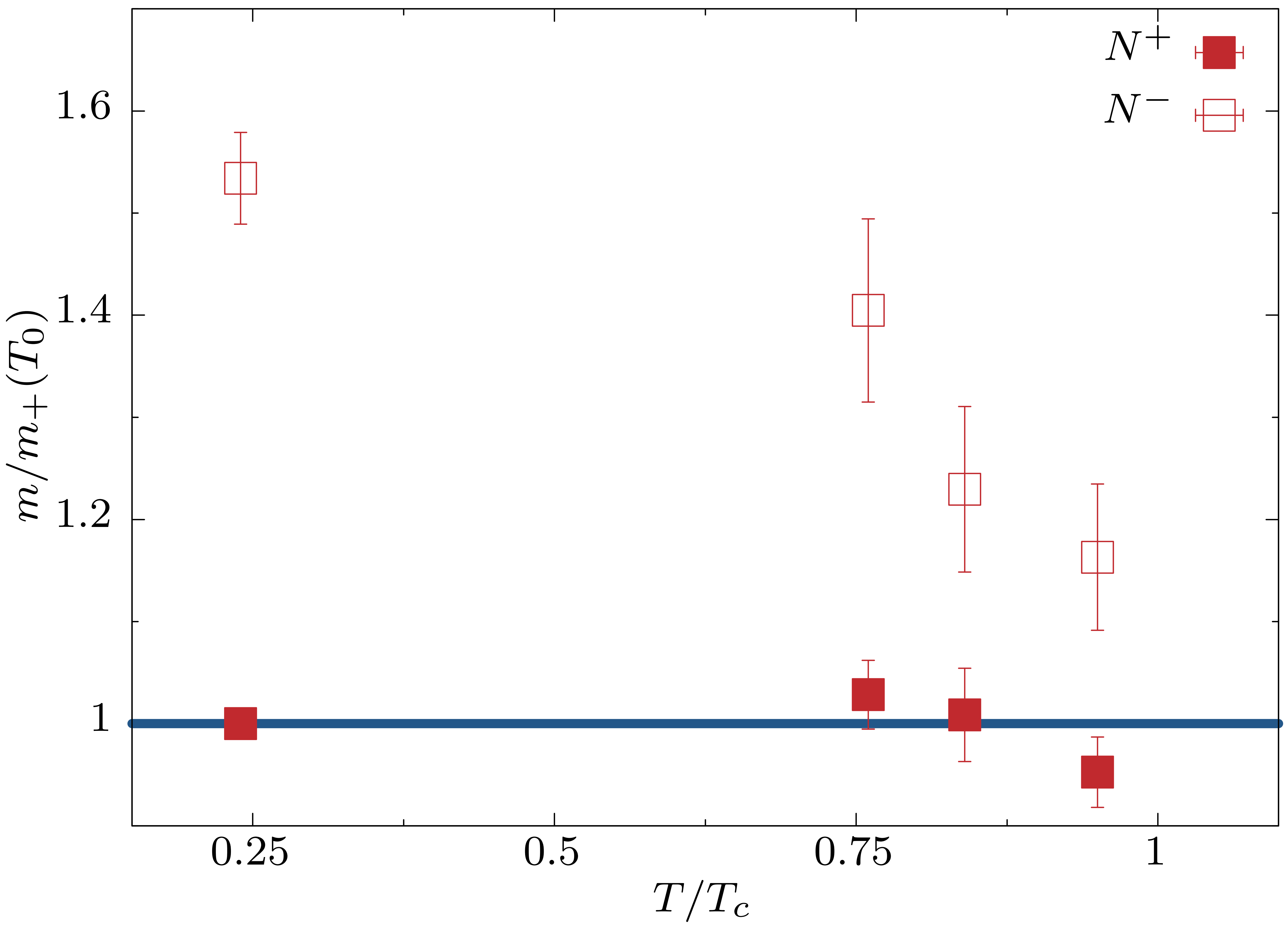


Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T}\right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial\left(\mu_B/T\right)^k}$$

Kurtosis: $\dfrac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Parity Doubling in Lattice QCD
Aarts et al, 2017, 2019



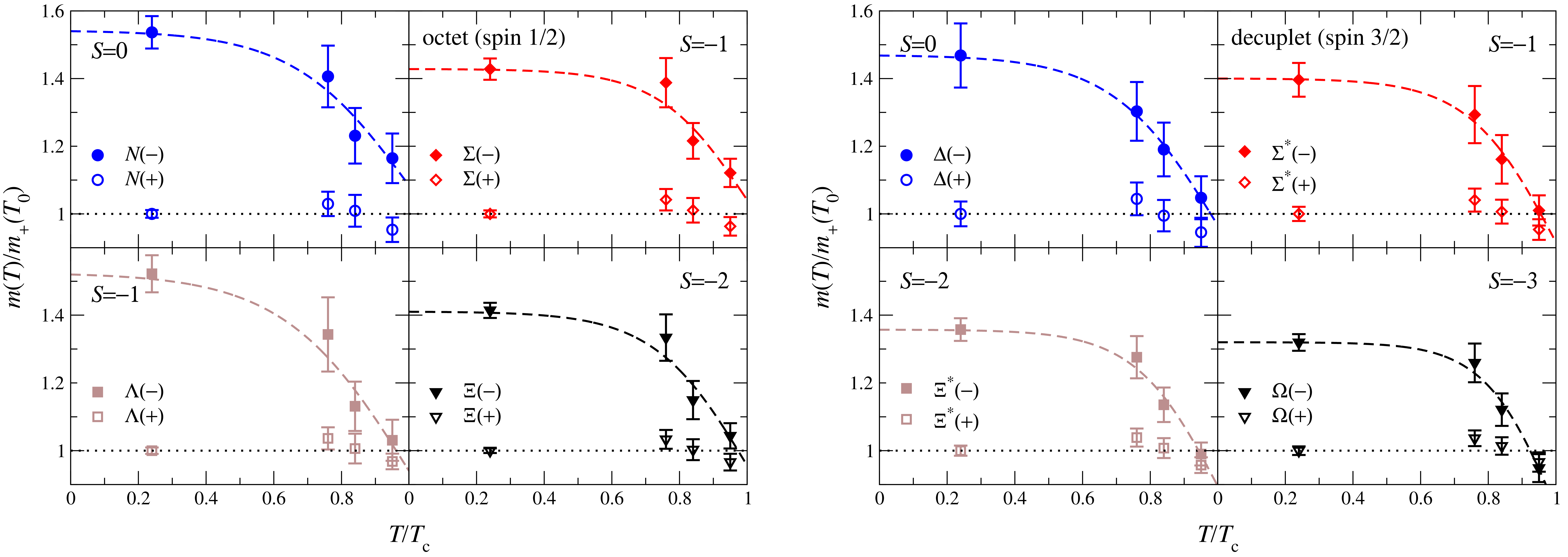
- N^+ nucleon stays nearly unchanged
- N^- chiral partner drops mass towards T_c
- Chiral partners $N^\pm$ degenerate at T_c
- Chiral parents stay massive

Imprint of chiral symmetry restoration in the baryonic sector

LQCD results still obtained with heavy m_π far from continuum limit

Imprint of chiral symmetry restoration in the baryonic sector

Aarts et al, 2019



Clear evidence for partial restoration of chiral symmetry also observed in the strange baryon sector

In-Medium Hadron Resonance Gas

Susceptibilities are sensitive probes of chiral dynamics in different sectors of hadronic quantum numbers

$$\chi_2^B = \frac{\partial^2 P/T^4}{\partial(\mu_B/T)^2} = \frac{1}{VT^3} C_2^B$$

Baryon number fluctuations with chiral in-medium baryon masses

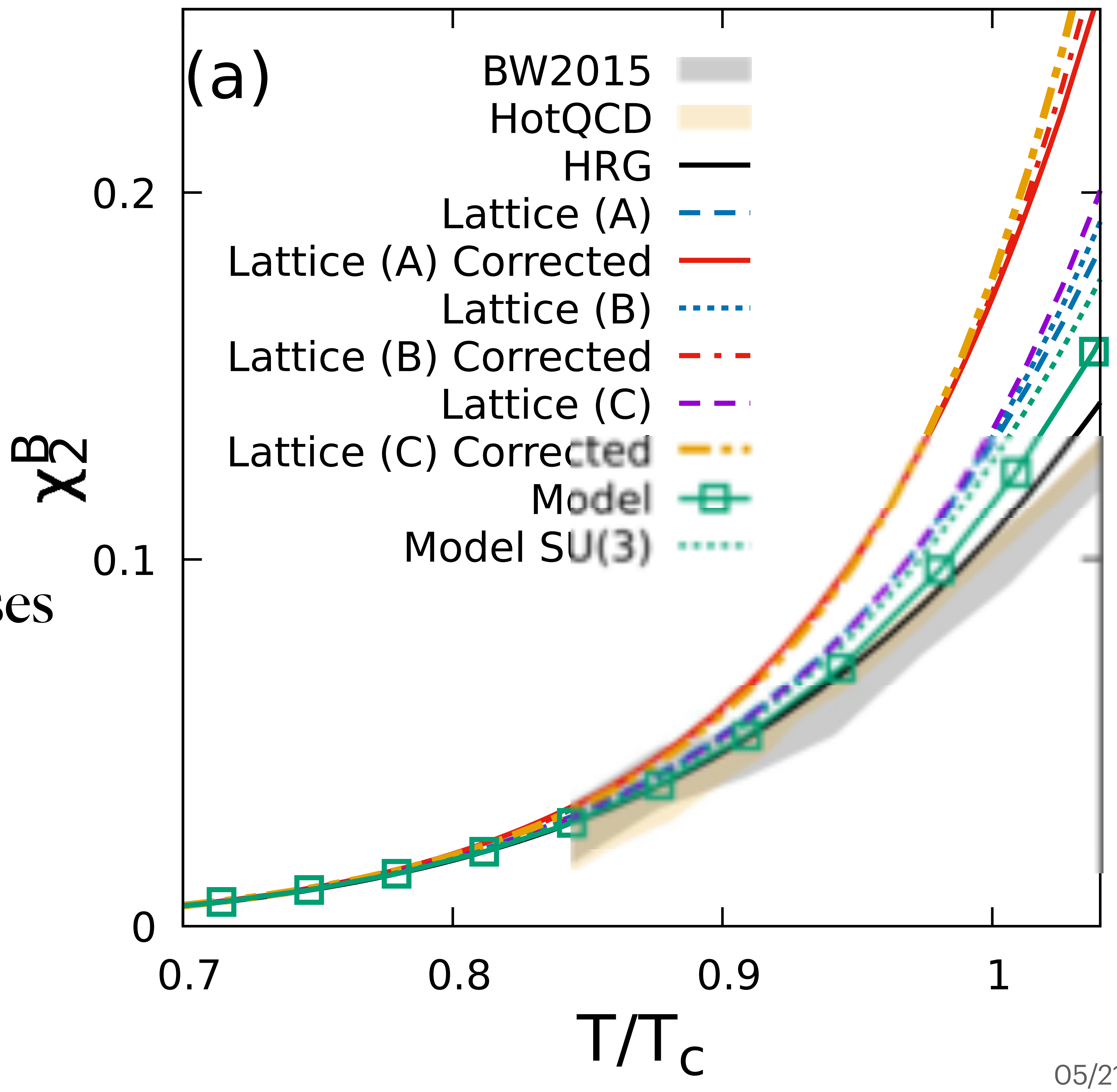
↓

In general too large fluctuations in HRG with in-medium parity partner masses

↓

Missing important contribution of parity partners correlations.

C. Sasaki et al, 2019



For multiplicity $N_B = N_+ + N_-$ Net-baryon number: $\langle N_B \rangle = \langle N_+ \rangle + \langle N_- \rangle$

Second-order fluctuations of the net-baryon number:

$$\langle \delta N_B \delta N_B \rangle = \langle (\delta N_+)^2 \rangle + \langle (\delta N_-)^2 \rangle + 2 \langle \delta N_+ \delta N_- \rangle$$

$$\langle \delta N_\alpha \delta N_\beta \rangle = V T^3 \chi_n^{\alpha\beta} \quad \longleftrightarrow \quad \chi_2^{\alpha\beta} = \frac{d^2 P / T^4}{d(\mu_\alpha / T) d(\mu_\beta / T)}$$

$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

- What are the individual contributions of parity partners N_+ and N_-?
- What is the strength and sign of the correlation χ_2^{+-}?
- Is net-proton a good proxy for net-baryon fluctuations? $\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$ (last two terms crossed out)

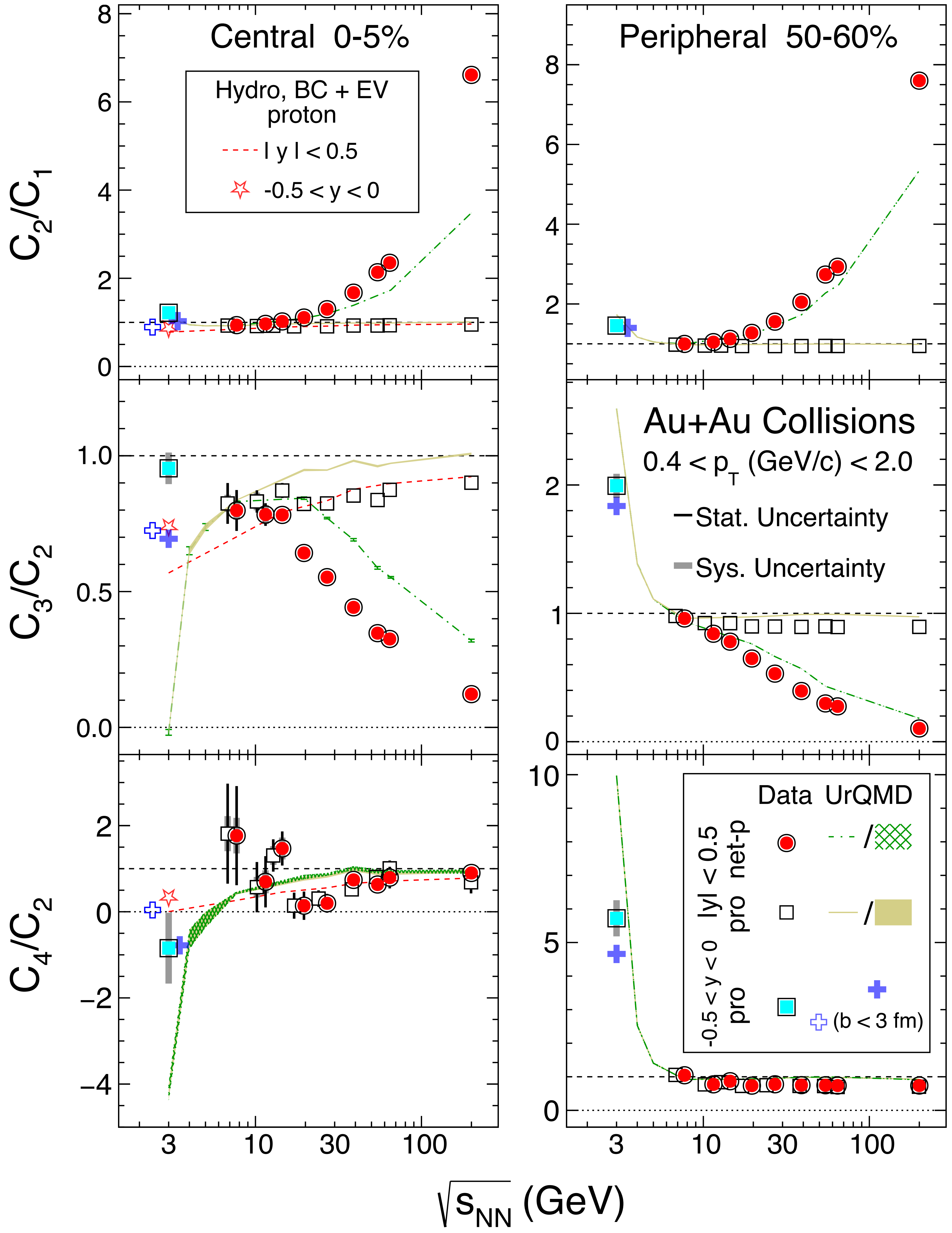
Cumulants vs Susceptibilities

Mean: M	$\langle N_B \rangle$	C_1
Variance: σ^2	$\langle (\delta N_B)^2 \rangle$	C_2
Skewness: S	$\langle (\delta N_B)^3 \rangle / \sigma^3$	$C_3 / C_2^{3/2}$
Kurtosis: K	$\langle (\delta N_B)^4 \rangle / \sigma^3 - 3$	C_4 / C_2^2

$$C_n \equiv VT^3 \frac{d^n P/T^4}{d\left(\mu_B/T\right)^n}\Bigg|_T \quad \longleftrightarrow \quad \chi_n^B \equiv \frac{d^n P/T^4}{d(\mu_B/T)^n}\Bigg|_T$$

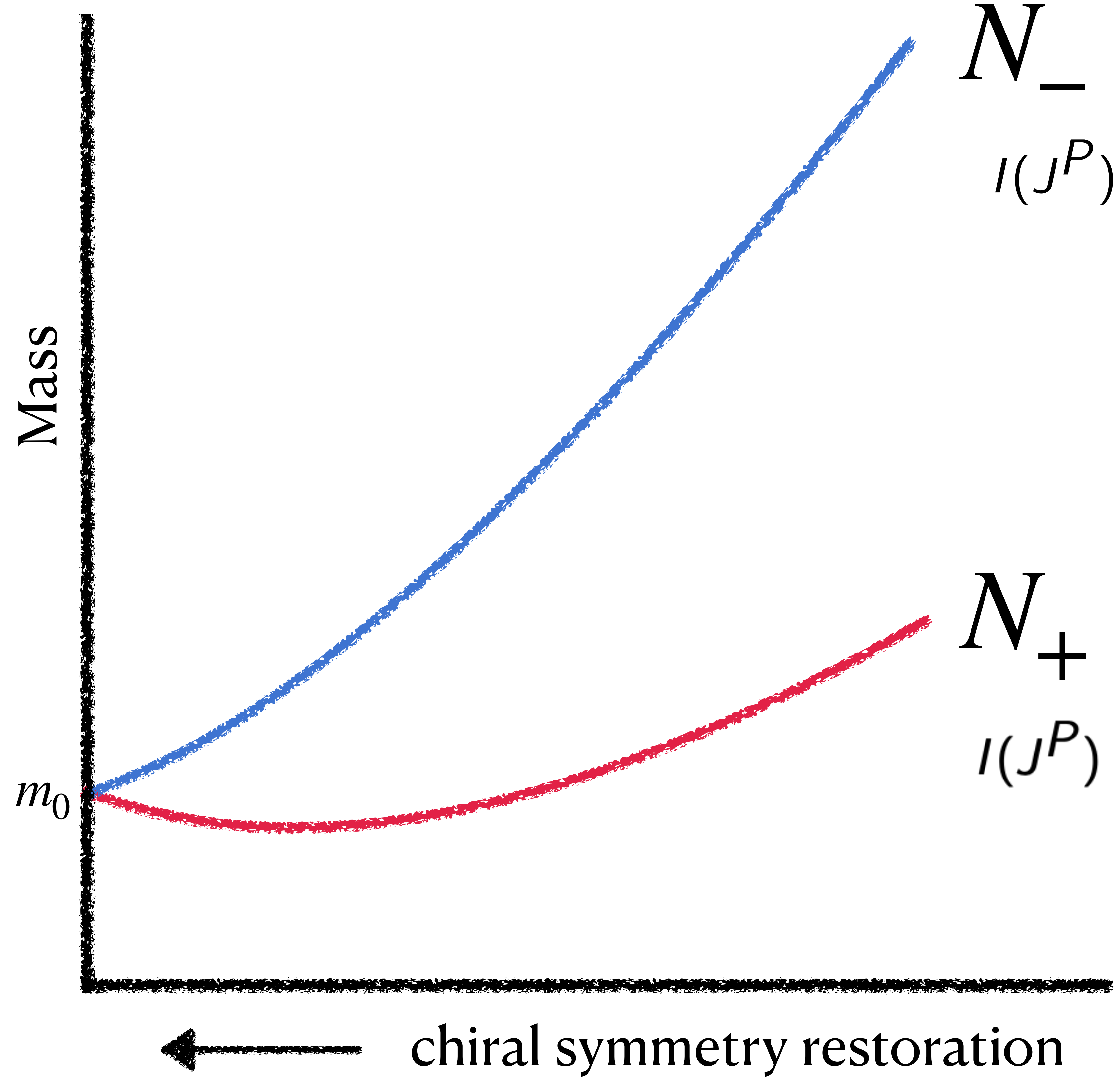
$$C_n = VT^3 \chi_n^B$$

STAR, 2023



Model a'la DeTar, Kunihiro 1989 $\longrightarrow \mathscr{L}_{\text{mass}} \sim m_0 \left(\bar{\psi}_1 \gamma_5 \psi_2 + \bar{\psi}_2 \gamma_5 \psi_1\right)$

$$M_{\pm} = \frac{1}{2}\left(\sqrt{4m_0^2 + a^2\sigma^2} \mp b\sigma\right) \xrightarrow{\sigma \to 0} m_0$$



N_- $N(1535)\ 1/2^-$

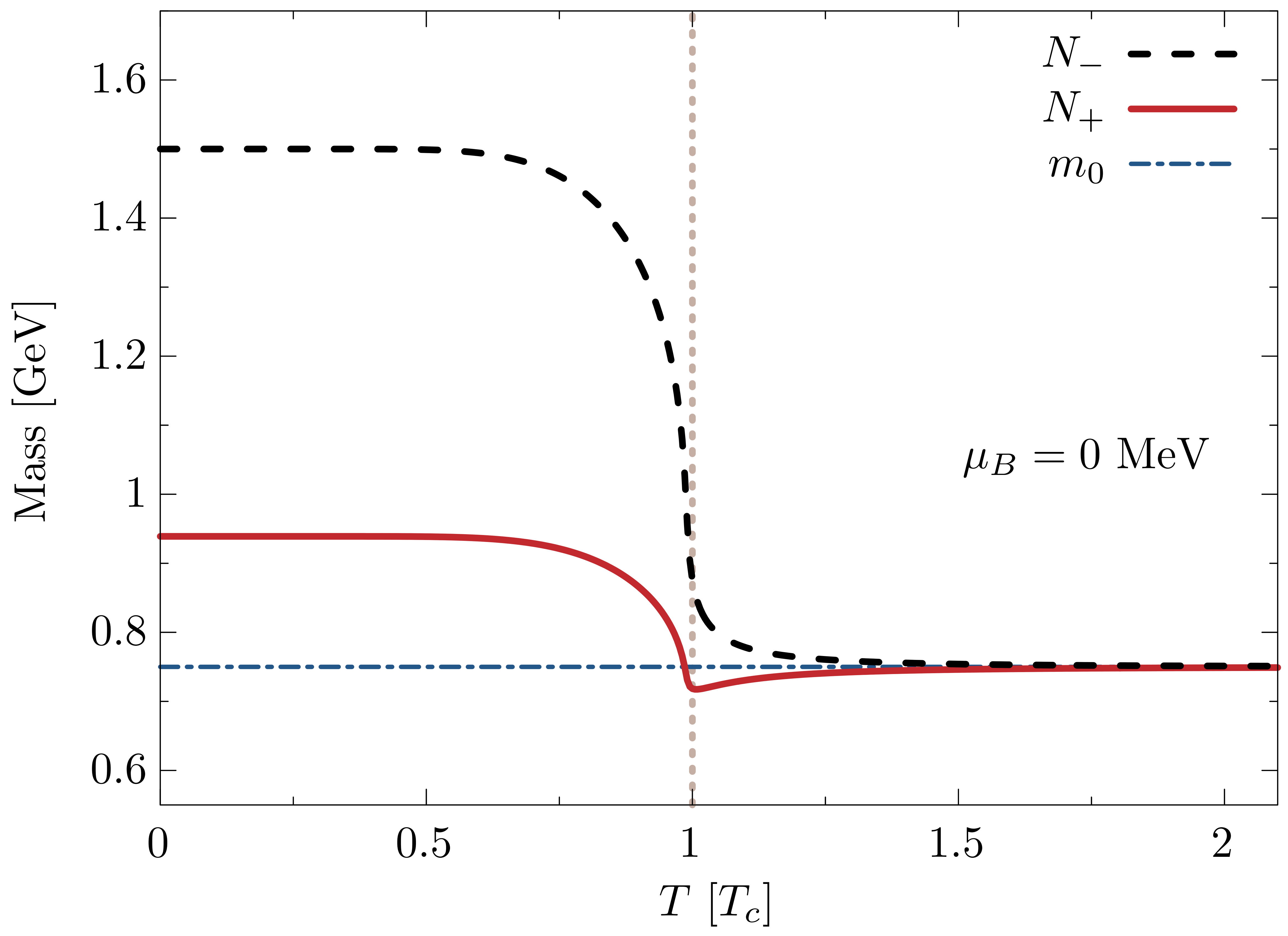
$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ∗∗∗∗

N_+ p n

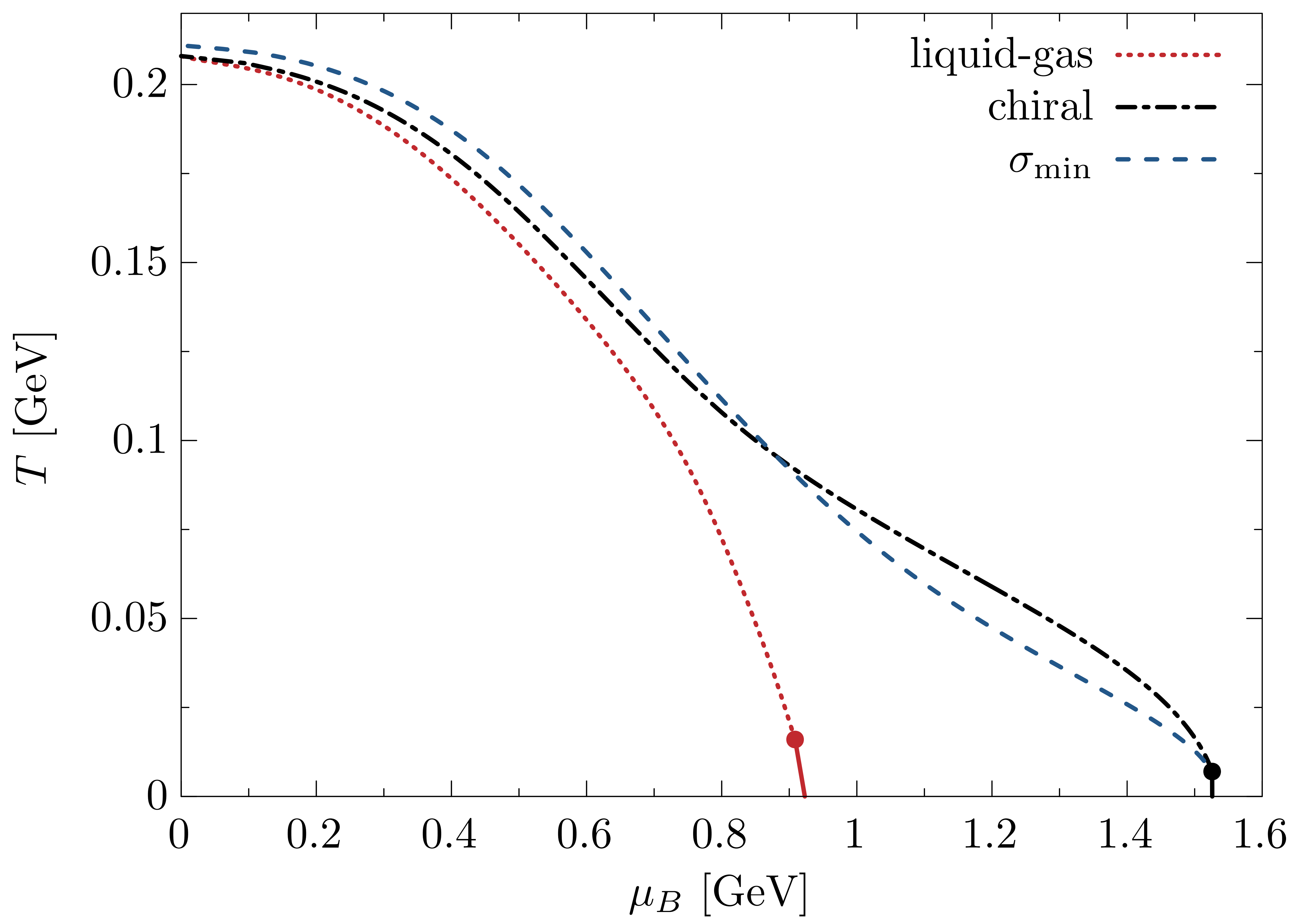
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗∗

Chiral Criticality in Parity Doubling Model

In-medium masses



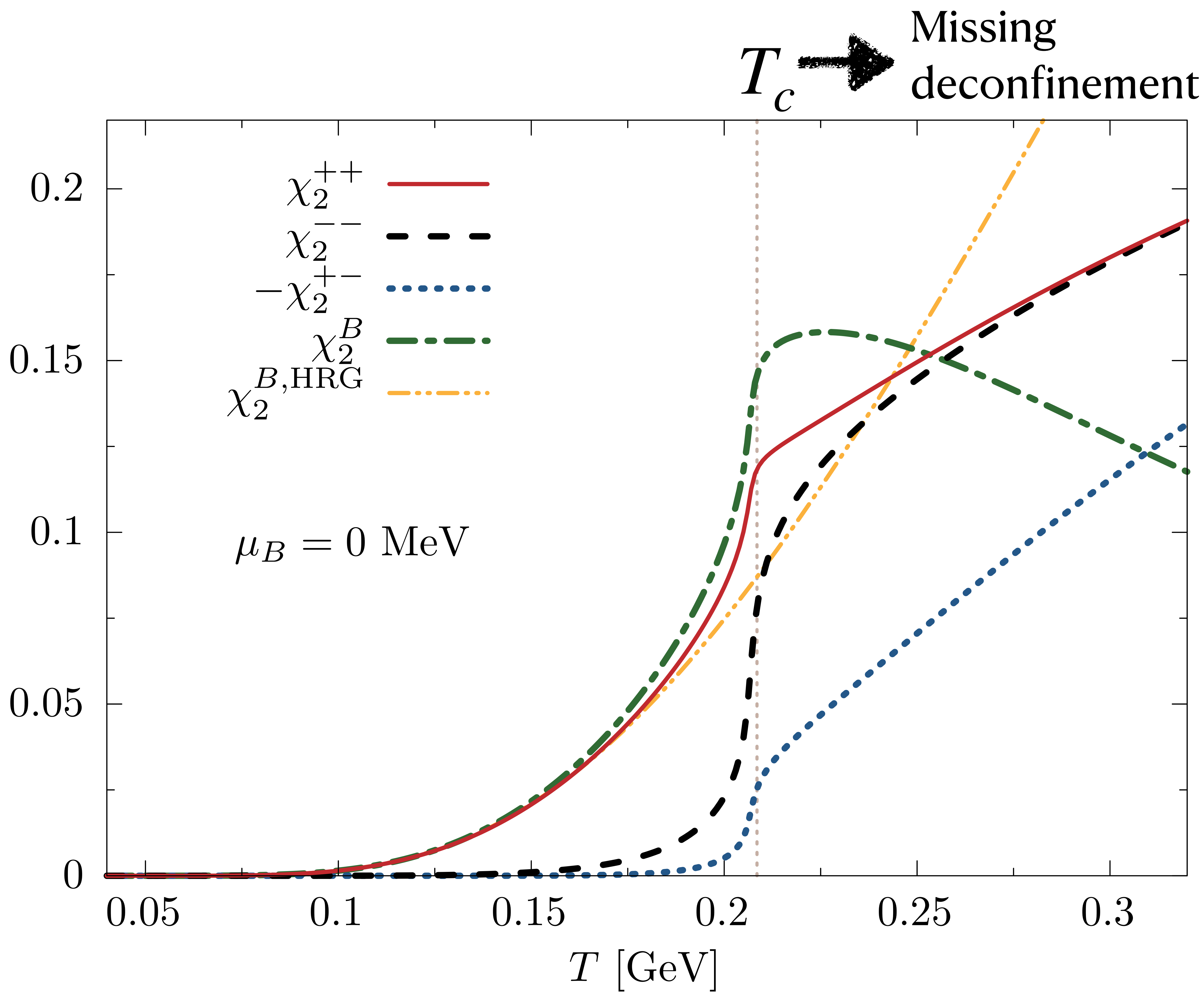
Phase diagram with liquid gas and chiral PTs



- M_- monotonically decreases
- M_+ has a minimum at $\sigma_{\min} = 2\dfrac{b}{a}\dfrac{m_0}{\sqrt{a^2 - b^2}}$

- Position of $\sigma_{\min}$ closely related to the chiral phase transition

Fluctuations of chiral partners near crossover at $\mu_B = 0$

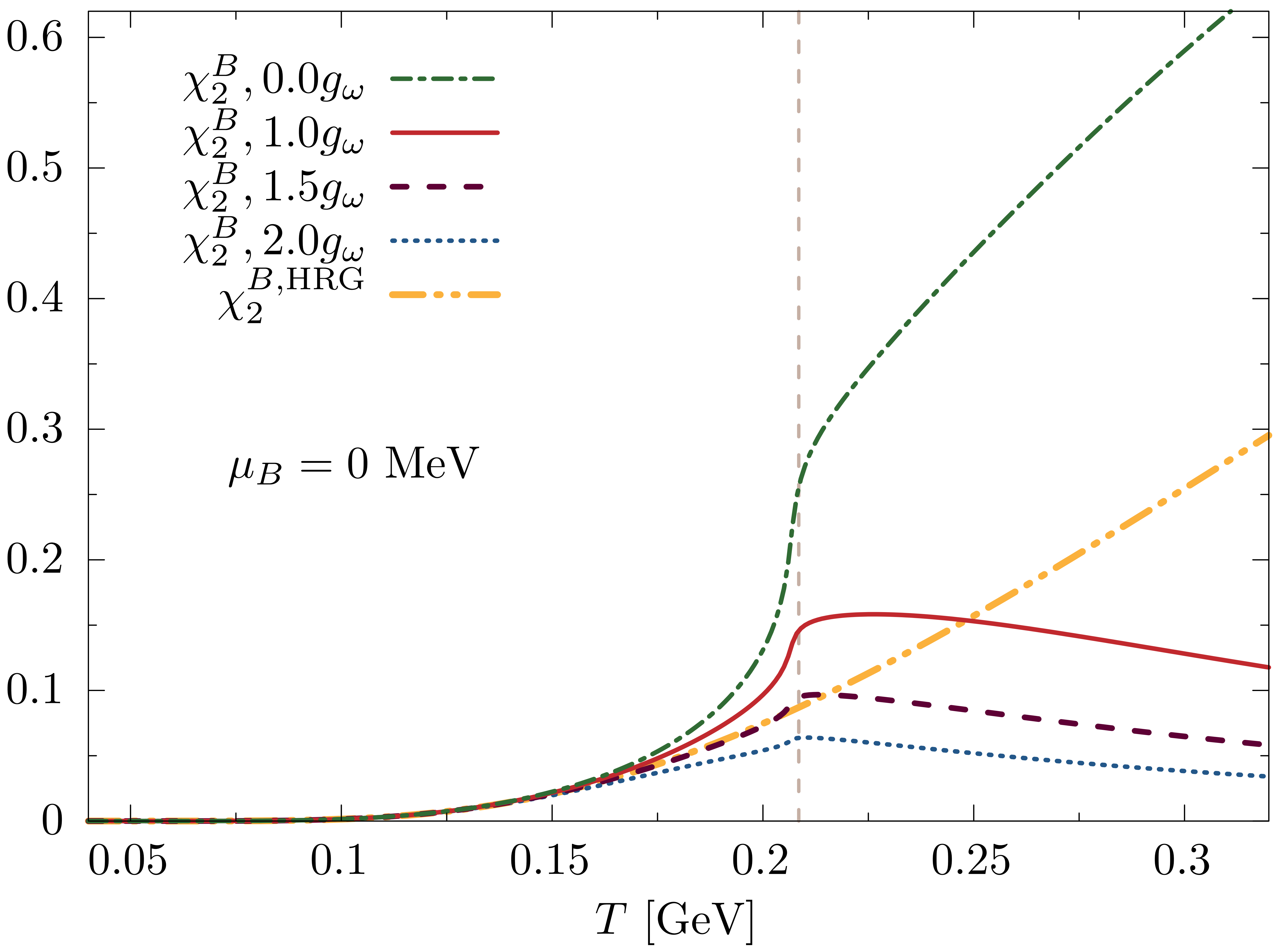


- Overall fluctuations dominated by net-nucleon at $\mu_B = 0$
- Contributions of N_- relevant only in the vicinity of T_c
- Correlations of N_+ and N_- provide negative contribution and set in only near T_c

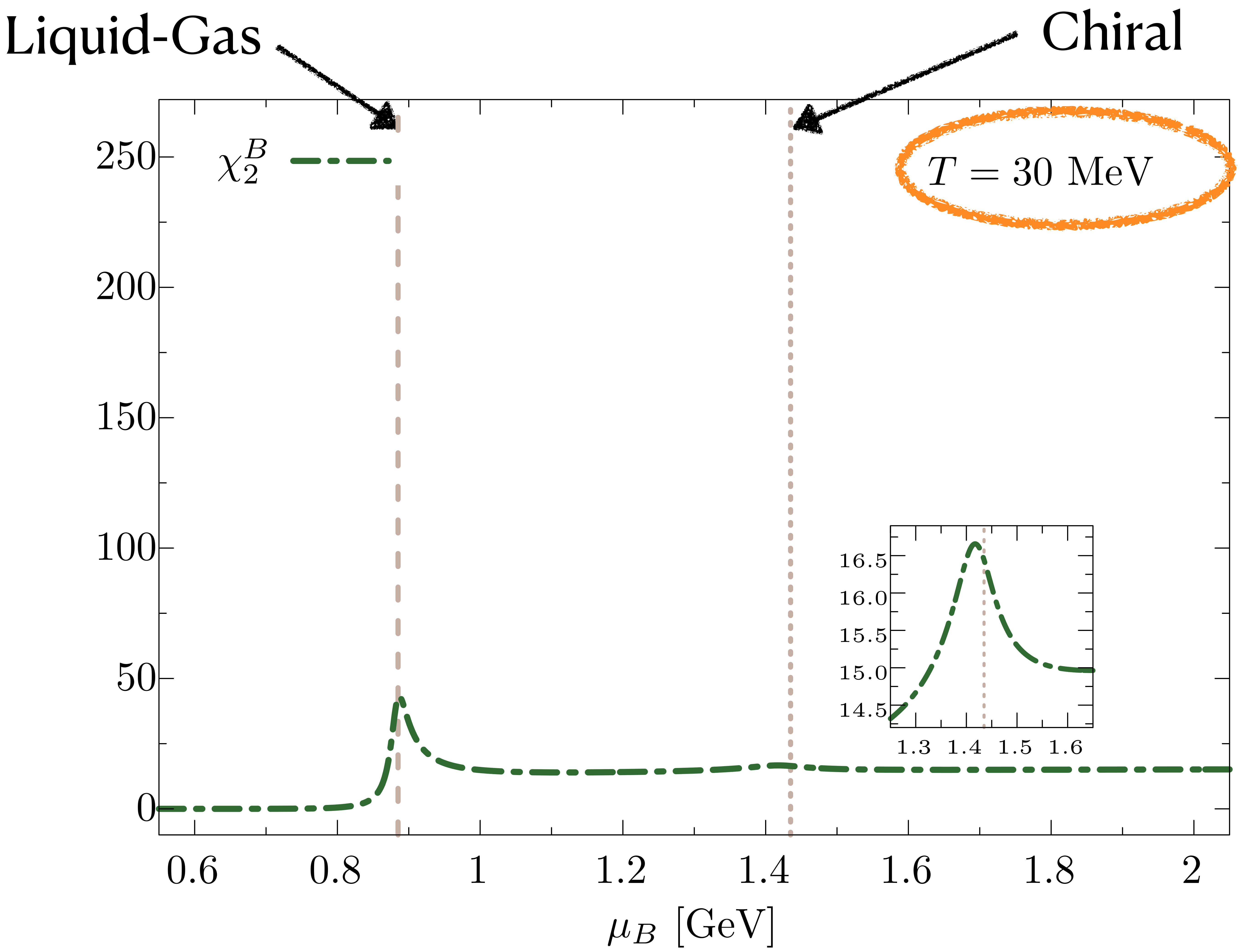
Net-baryon number fluctuations sensitive to an interplay between repulsive interactions and chiral in-medium baryon masses

Influence of the strength of the repulsive interactions

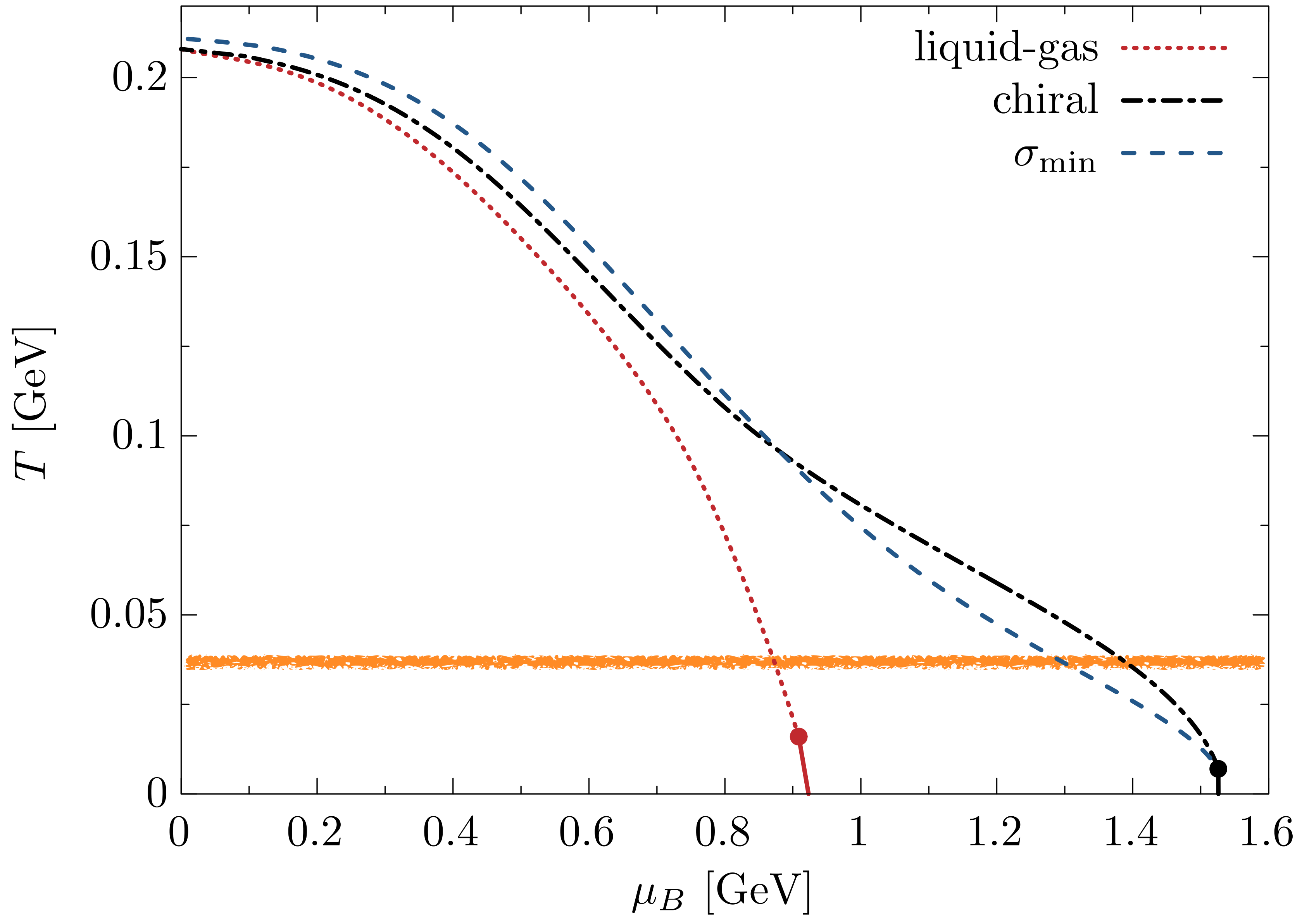
- Clear suppression of fluctuations with an increasing repulsive vector interactions
- Increase of fluctuations due to in-medium chiral masses is reduced via negative correlations
- With particular repulsion strength, fluctuations are pushed down to HRG results with vacuum masses



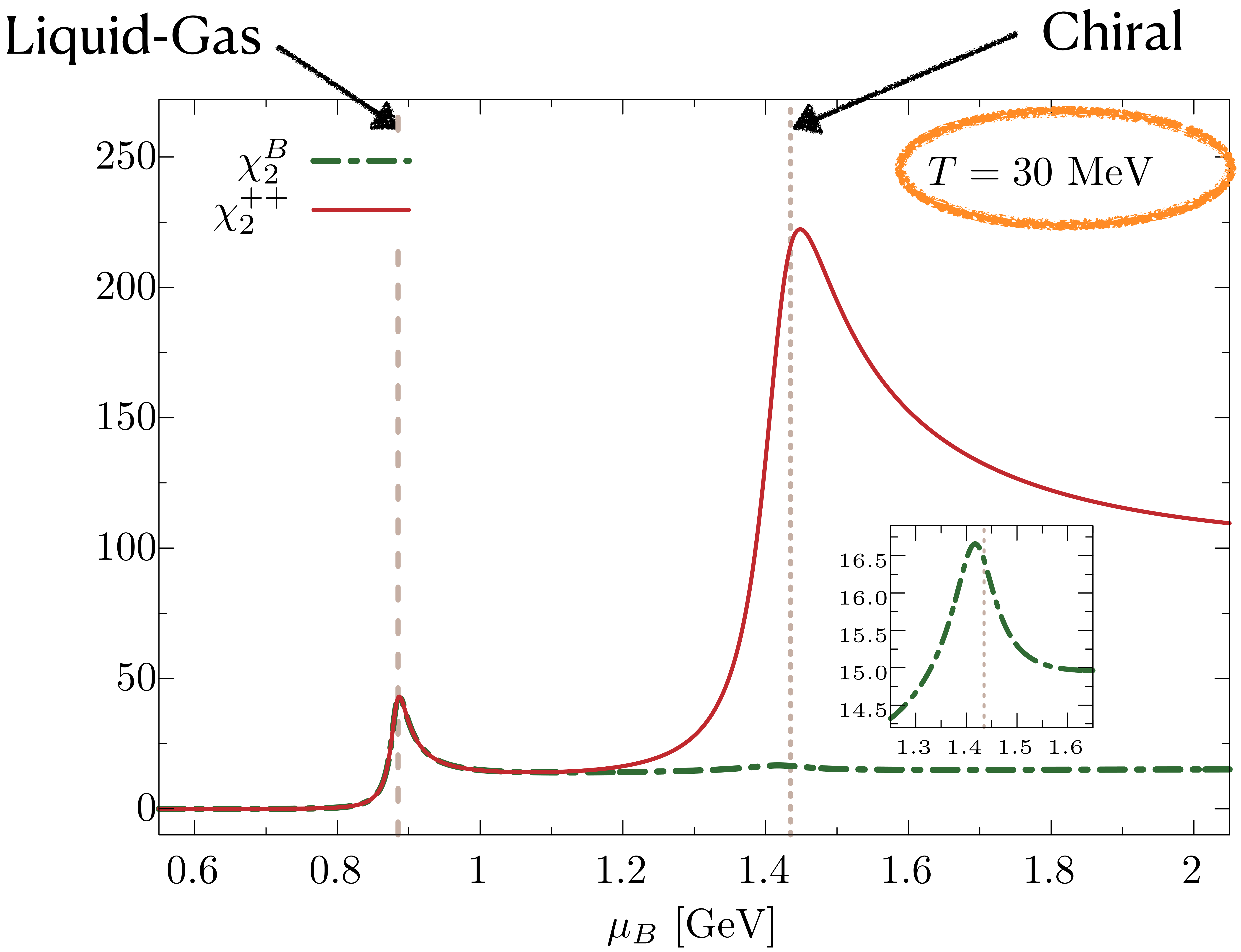
Fluctuations at liquid-gas and chiral transitions

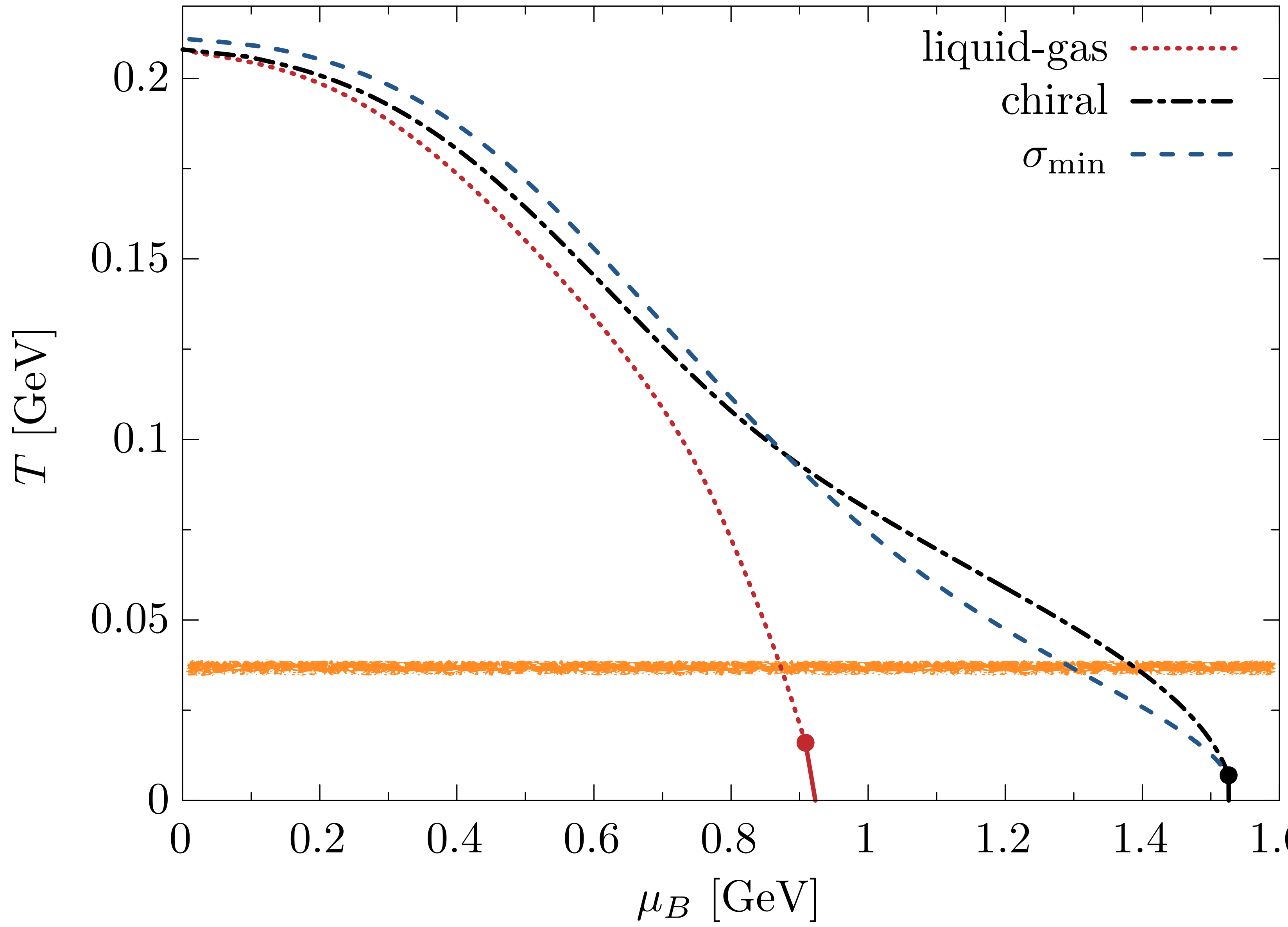


χ_2^B



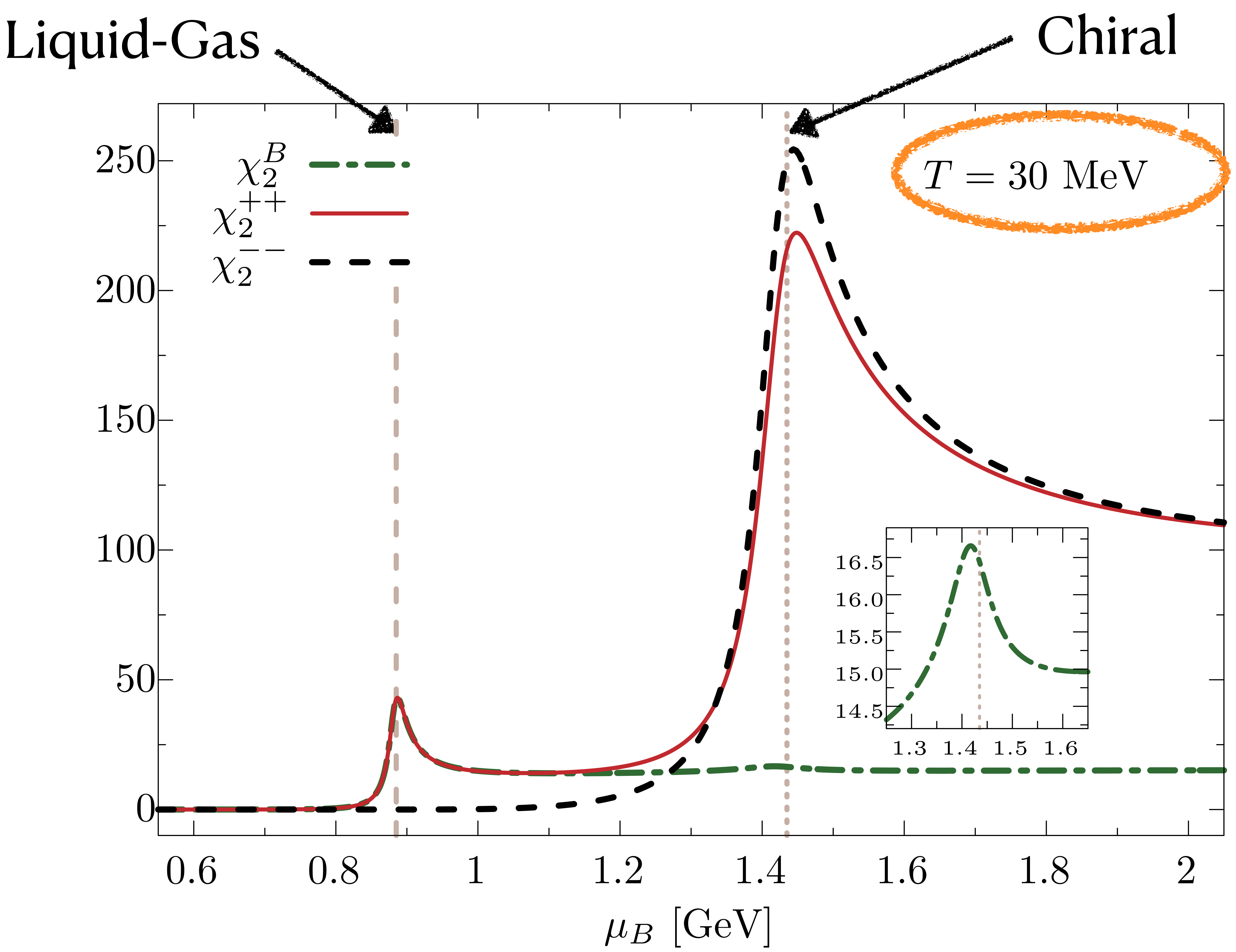
Fluctuations at liquid-gas and chiral transitions

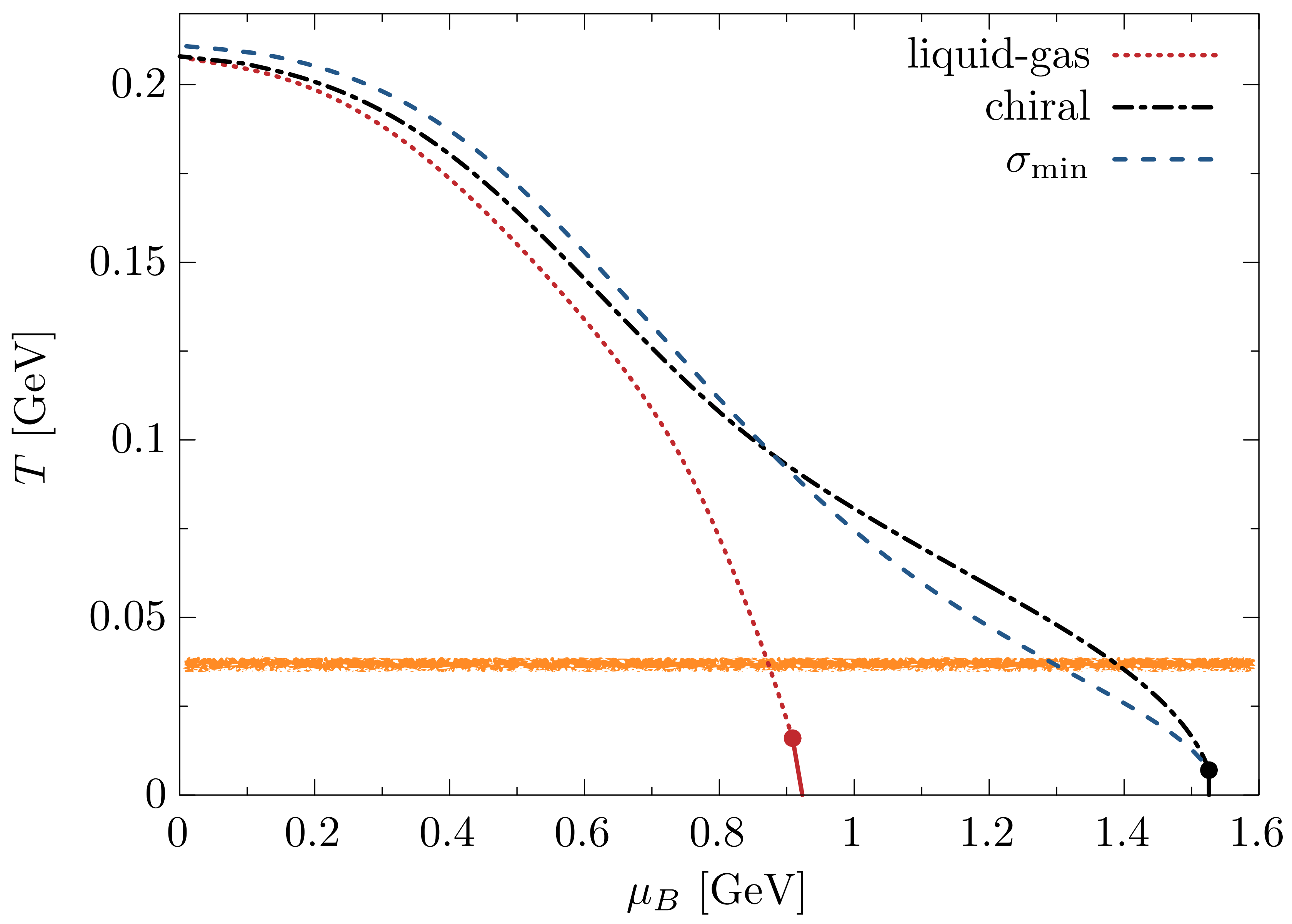




$$\chi_2^B = \chi_2^{++}$$

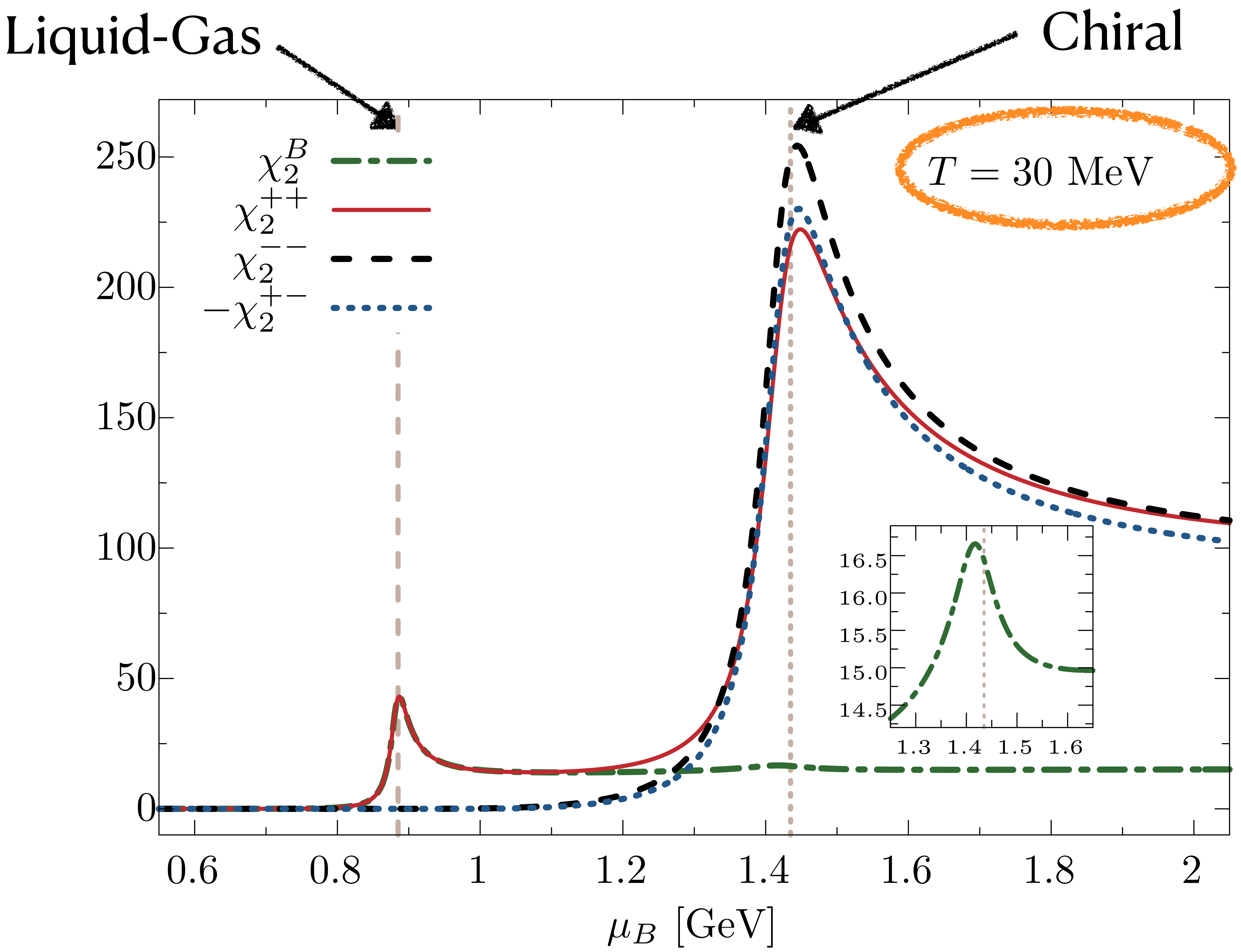
Fluctuations at liquid-gas and chiral transitions

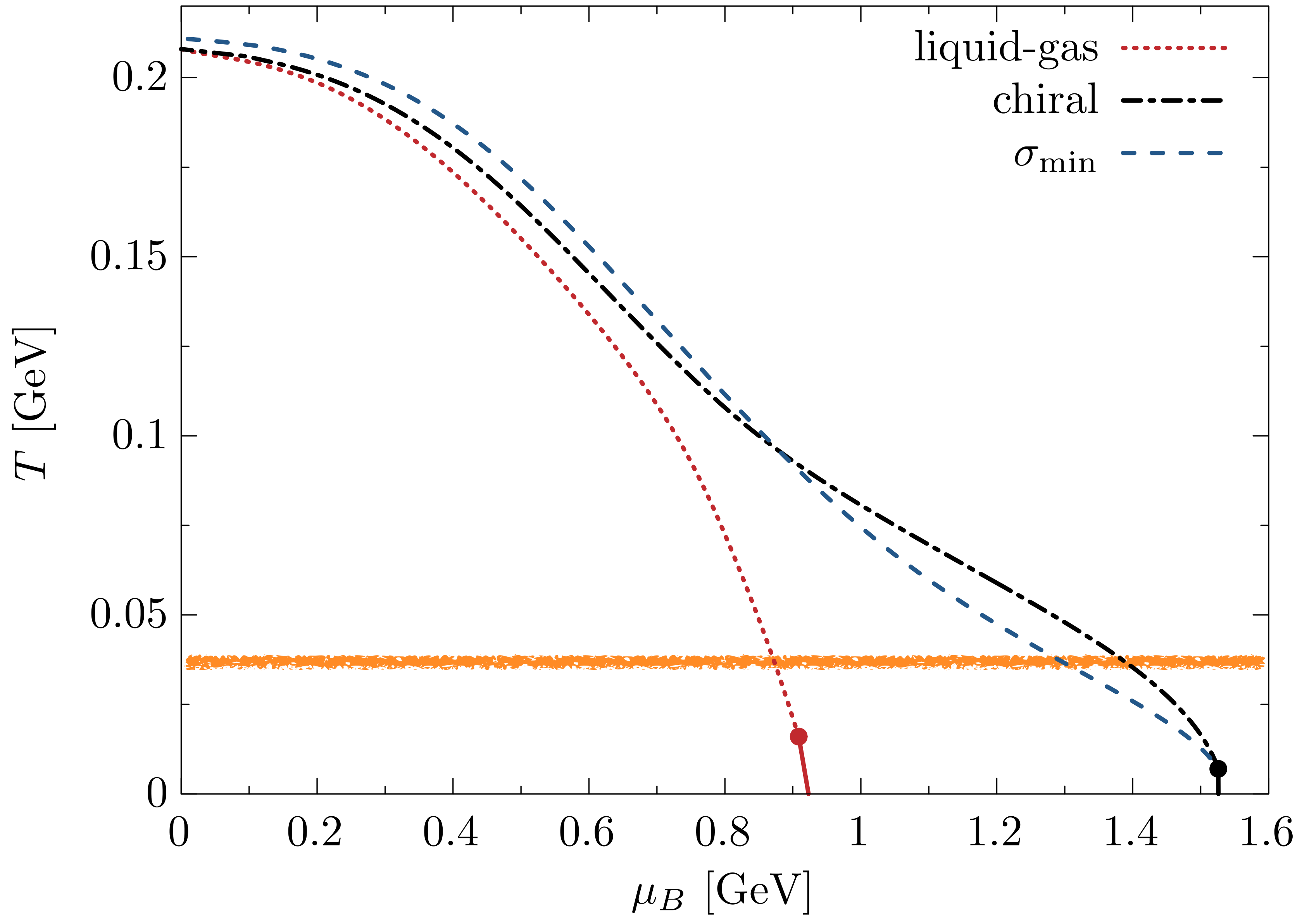




$$\chi_2^B = \chi_2^{++} + \chi_2^{--}$$

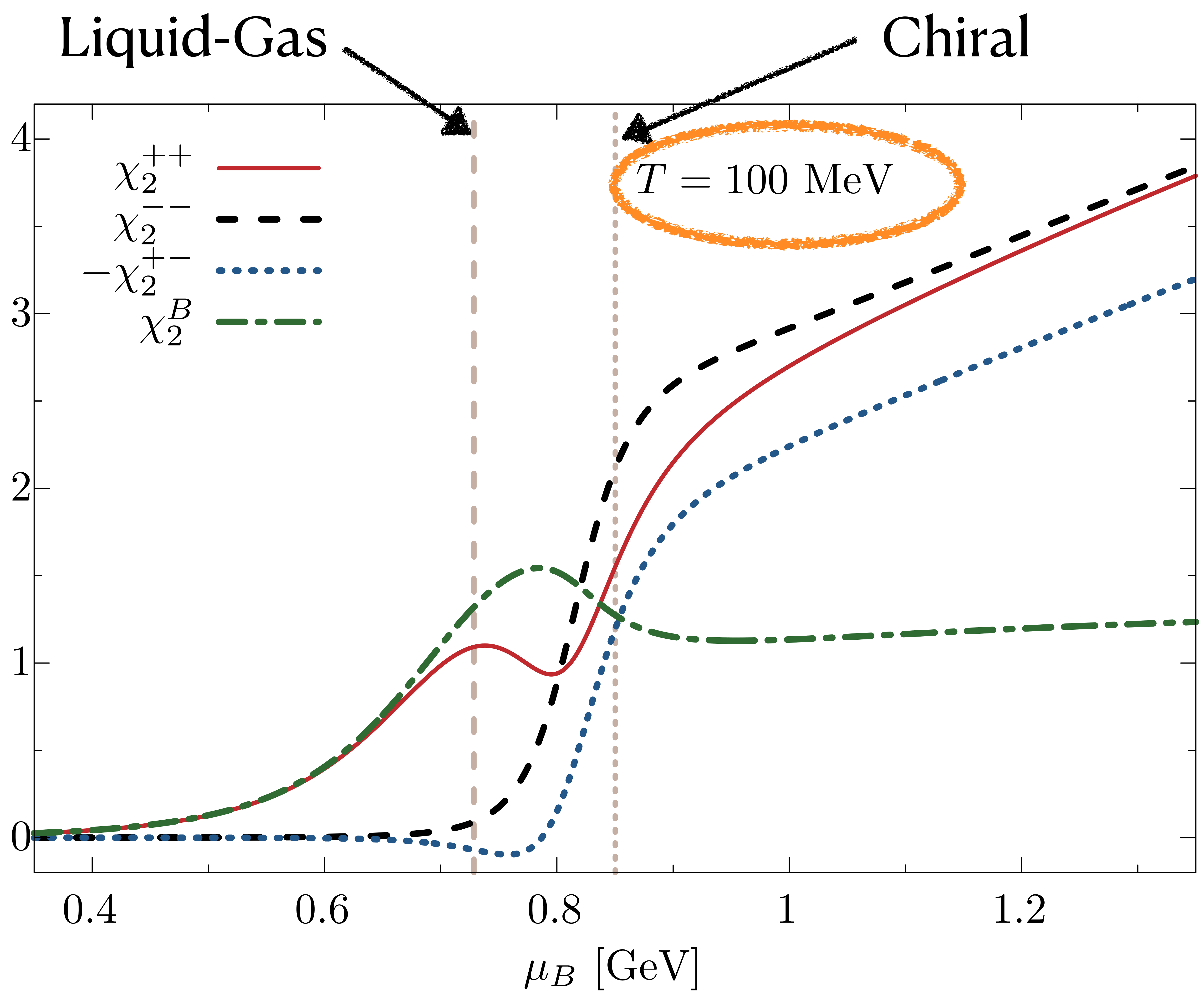
Fluctuations at liquid-gas and chiral transitions

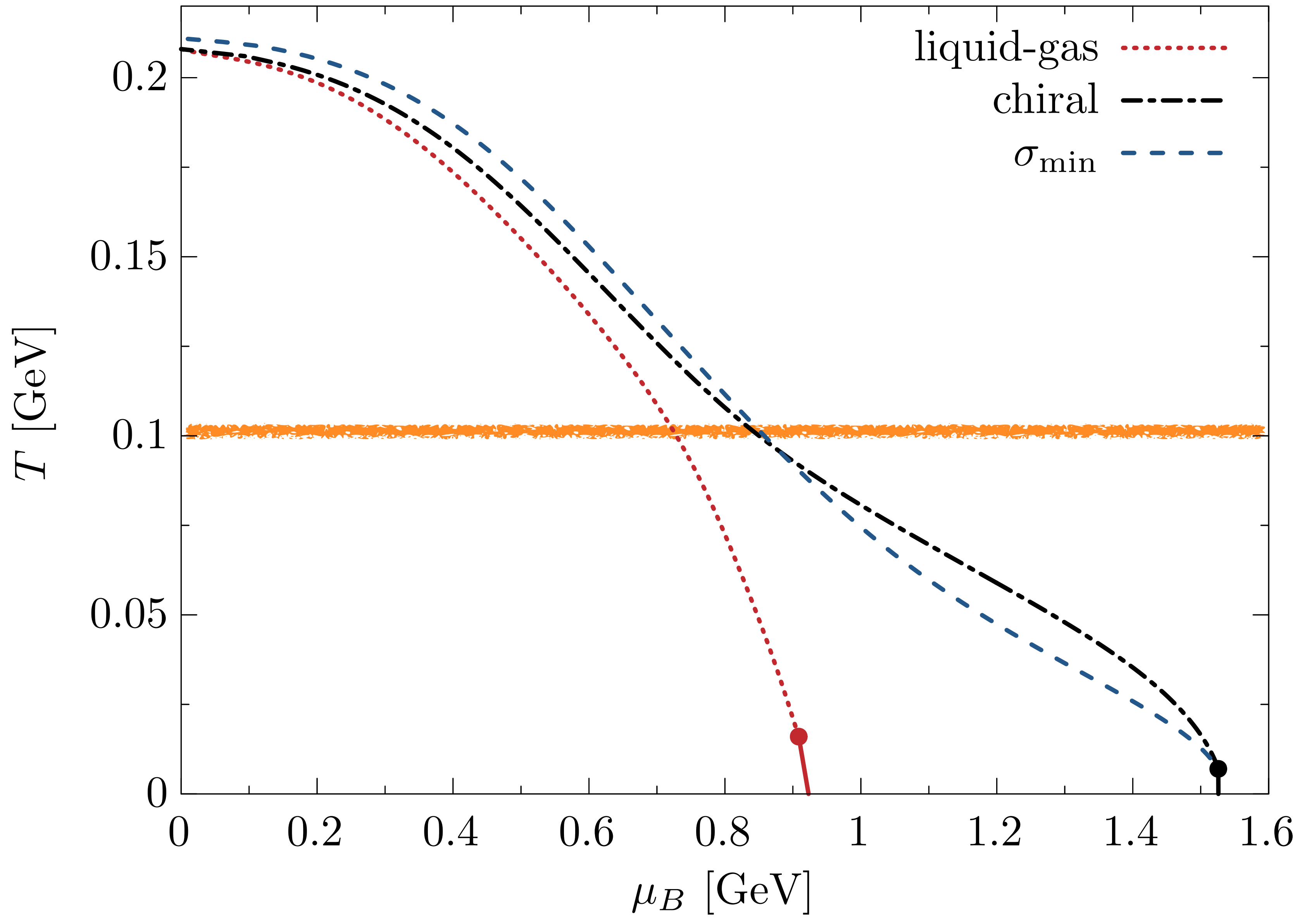




$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

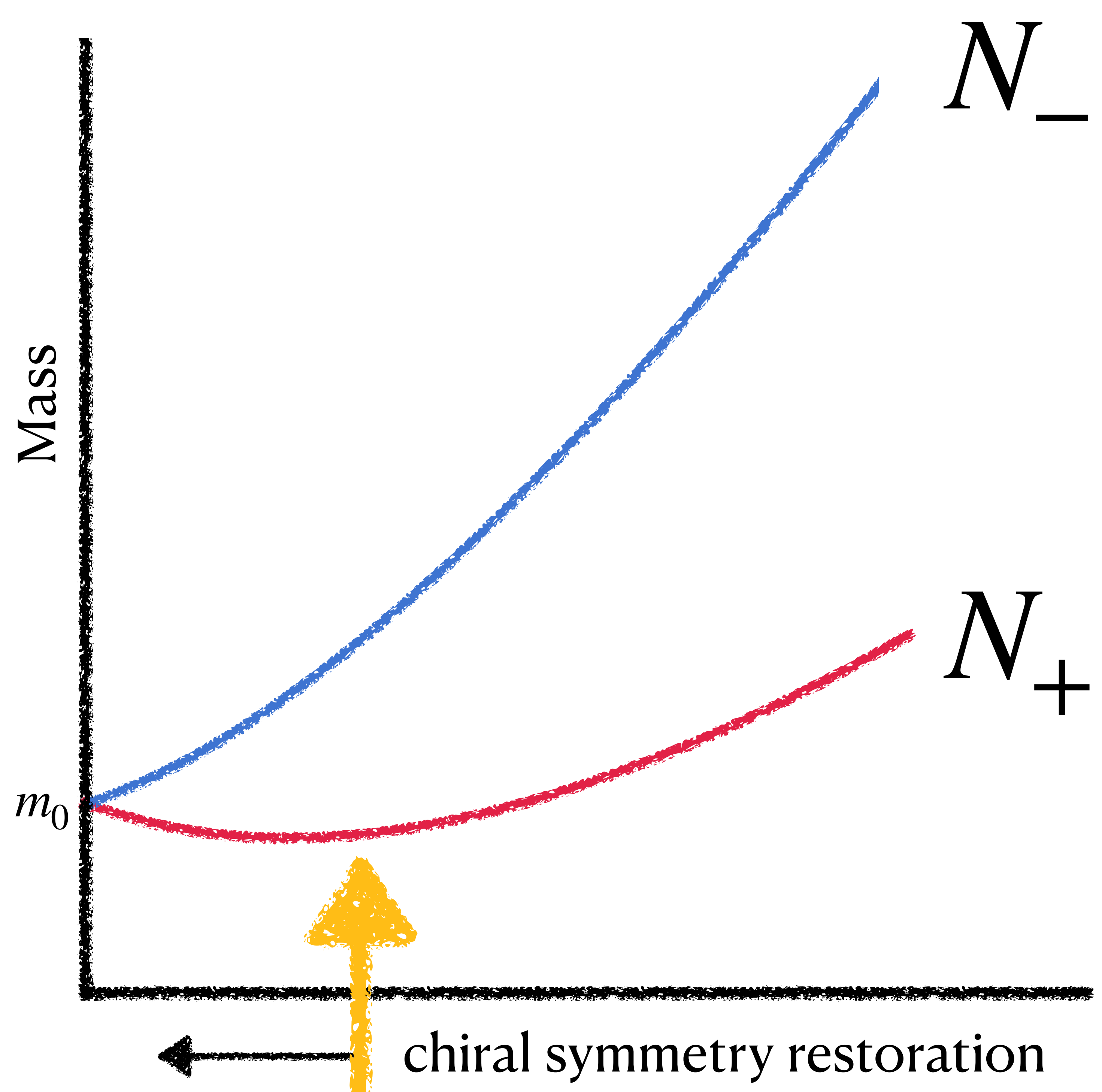
Fluctuations at intermediate temperatures

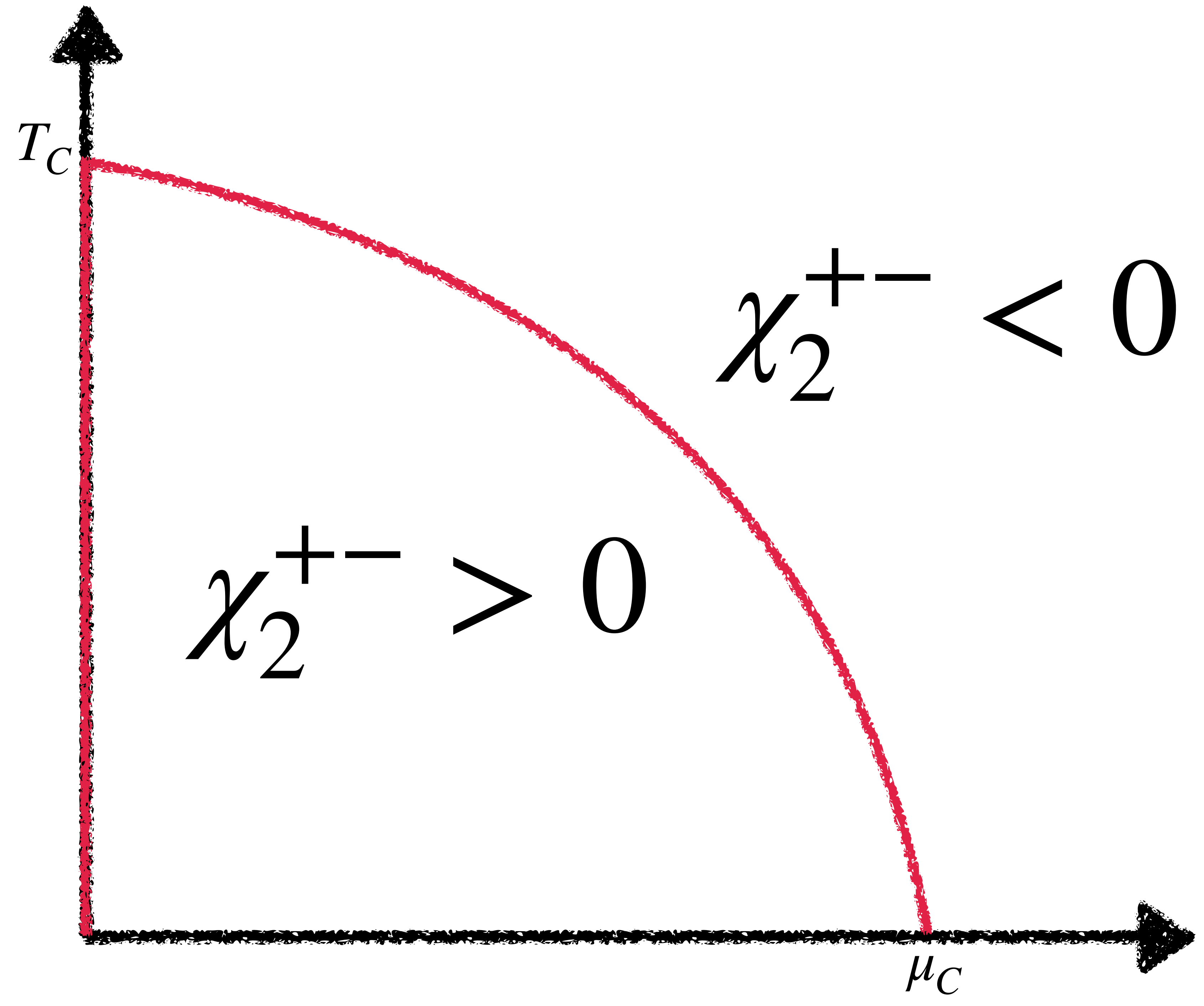




$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

Idealized behavior of the correlator ⟶ no repulsive forces





Min of M_+

$$\chi_2^{+-} \sim \frac{\partial m_+}{\partial \sigma}\frac{\partial m_-}{\partial \sigma}$$

→ but also repulsion

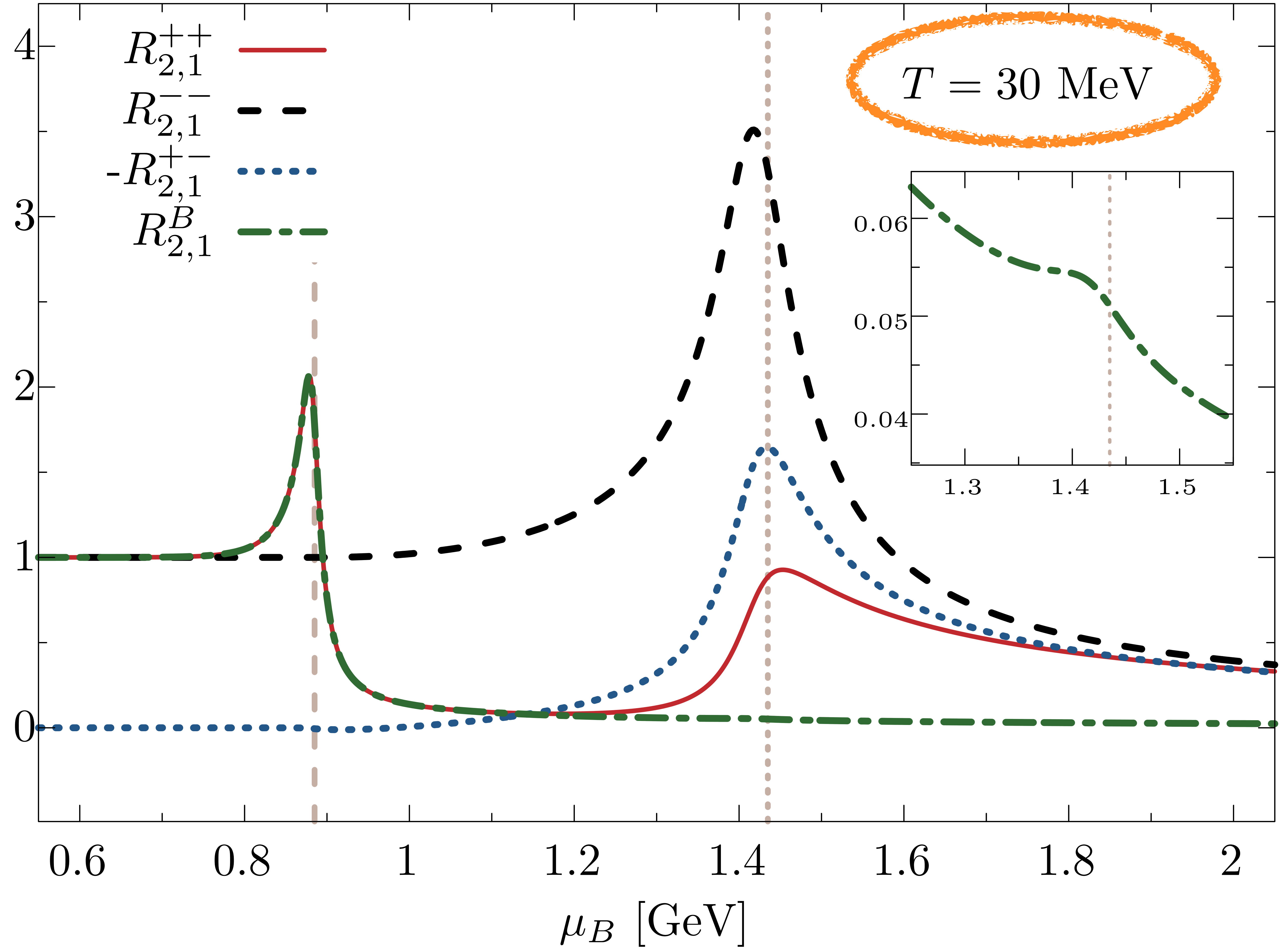
Change of the sign of χ_2^{+-} linked to the chiral phase boundary → interesting quantity to calculate in LQCD

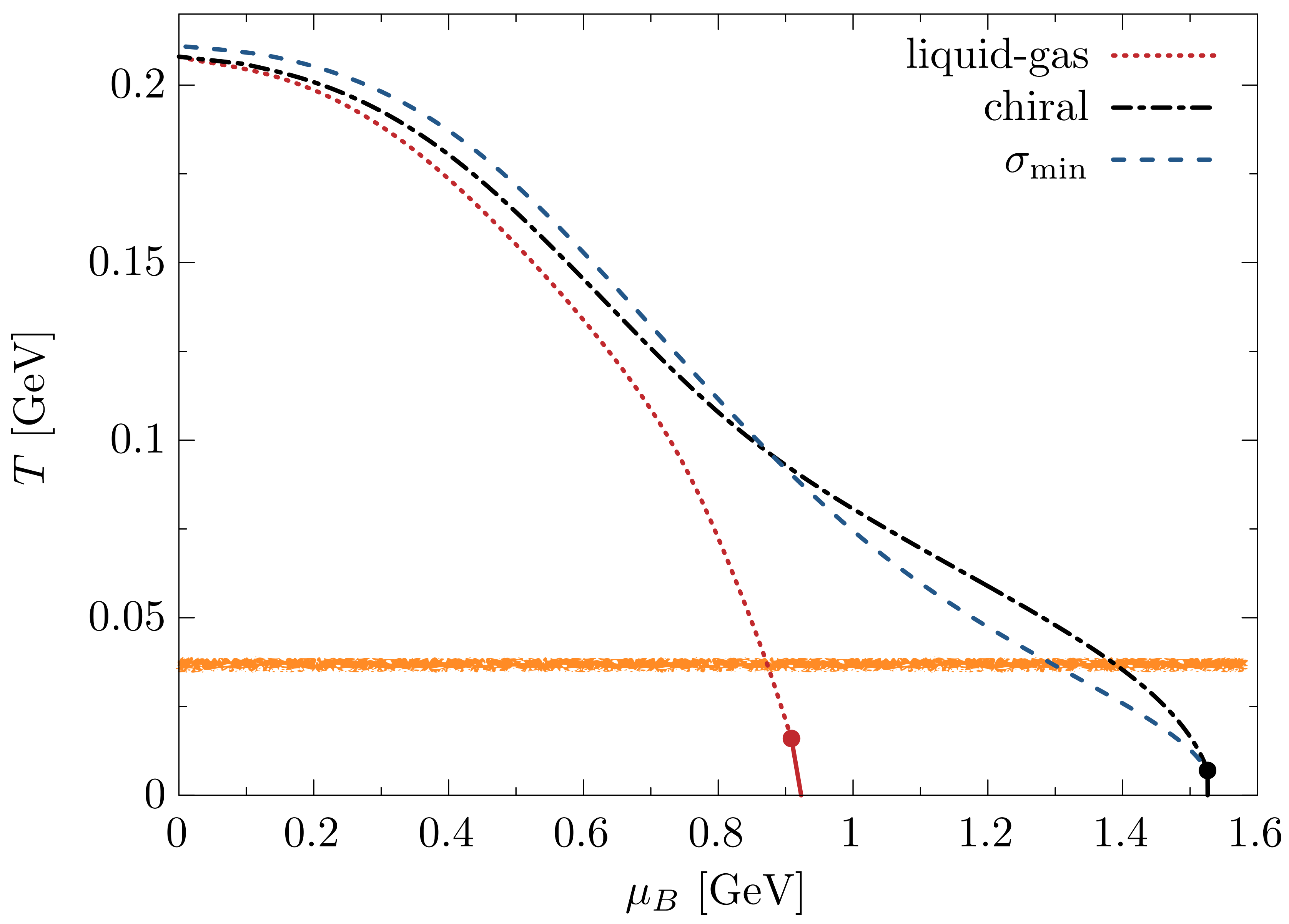
Cumulants $C_n \sim V$

$\downarrow$

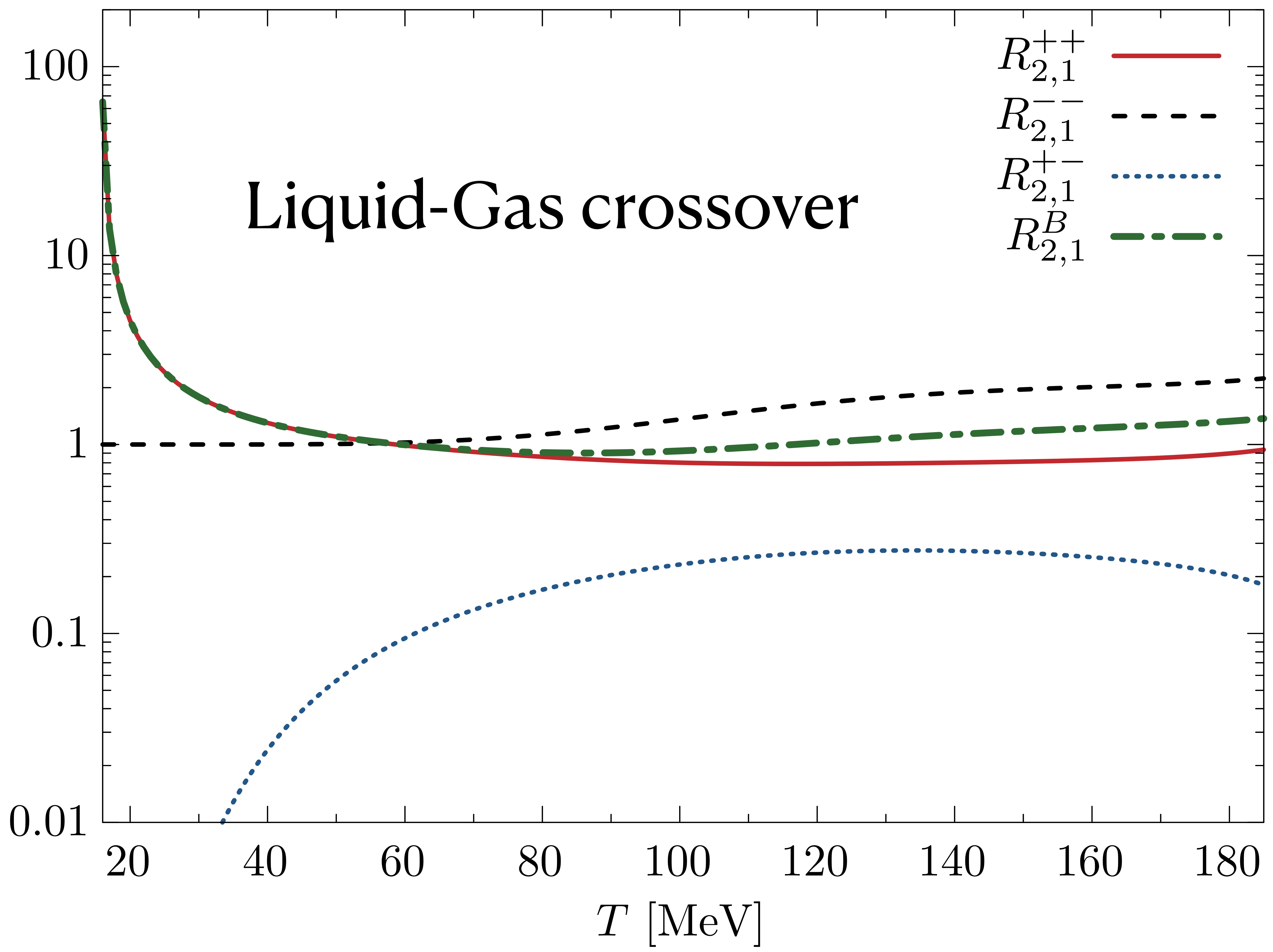
volume cancels in ratios

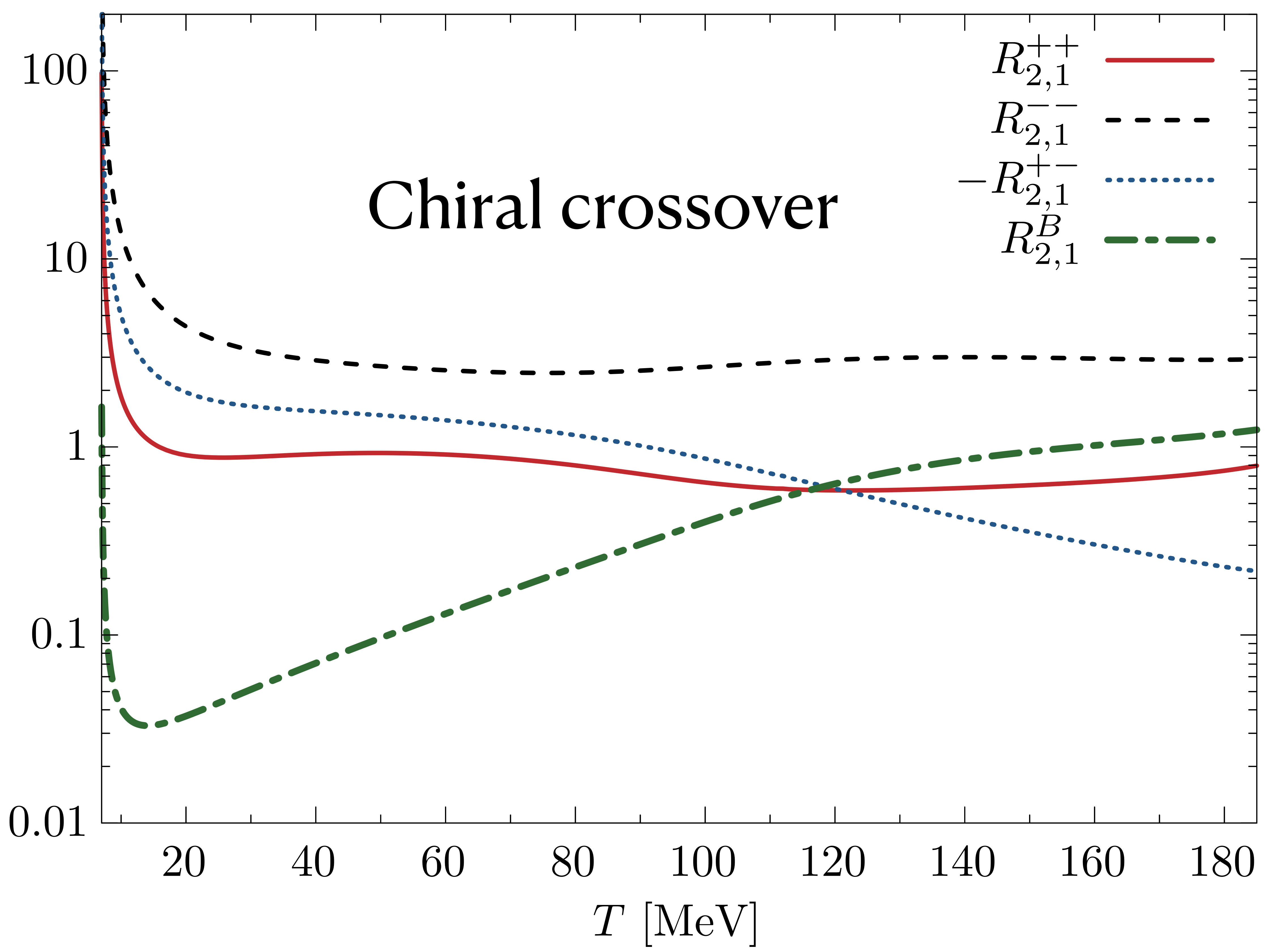
$$R^{\alpha\beta}_{2,1} \equiv \frac{C_2^{\alpha\beta}}{\sqrt{C_1^\alpha C_1^\beta}} = \frac{\chi_2^{\alpha\beta}}{\sqrt{\chi_1^\alpha \chi_1^\beta}} = \frac{\sigma^2}{M}$$





Critical Point → enhanced fluctuations & non-monotonicity





Fluctuations dominated by positive parity

↓

Net-nucleon ~ net-baryon

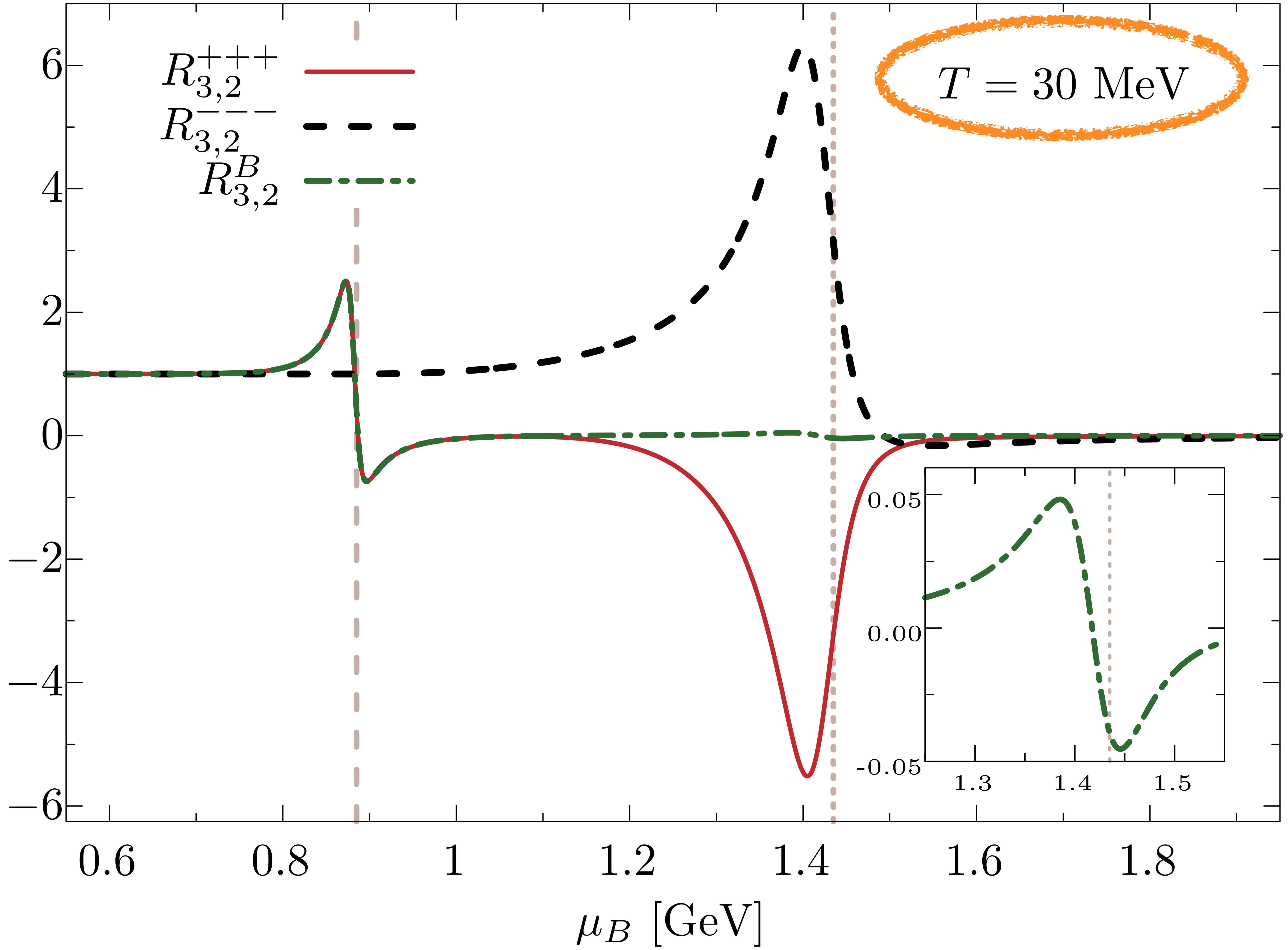
Presence of chiral partners + correlations

↘ ↙

Net-baryon suppressed

Higher-Order Fluctuations of Parity Partners

Marczenko, to appear



Consider $R^{+}_{3,2}$, $R^{-}_{3,2}$, $R^{B}_{3,2}$, where

$$R^{\alpha}_{3,2} \equiv \frac{C^{\alpha\alpha}_{3}}{C^{\alpha\alpha}_{2}} = \frac{\chi^{\alpha\alpha}_{3}}{\chi^{\alpha\alpha}_{2}} = S\sigma$$

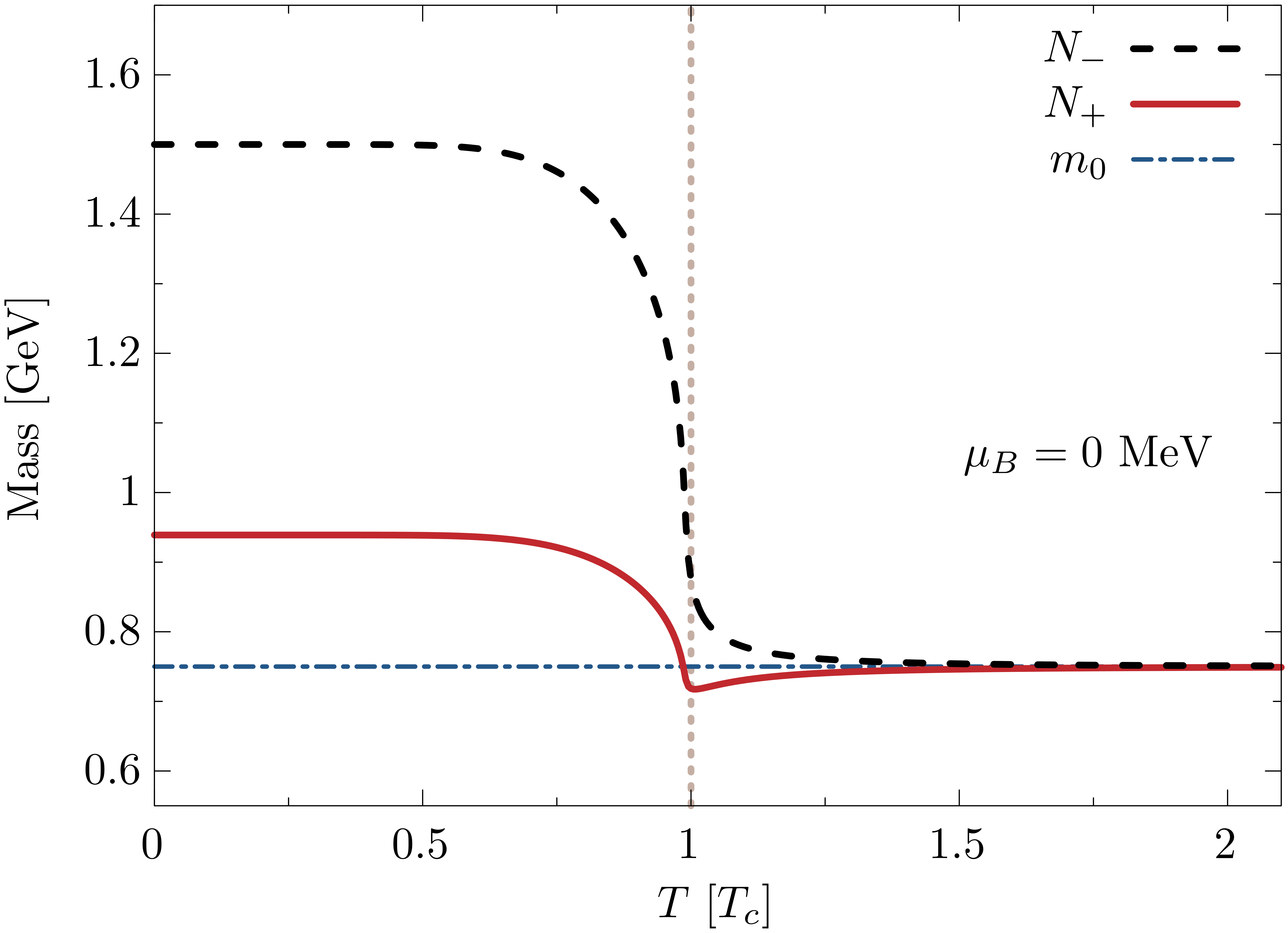
- Very different properties of positive and negative parity partners fluctuation ratios $R^{\alpha}_{3,2}$
- Essentially different from the fluctuations of net baryon number
- Proton number ≠ baryon number fluctuation ratios

Summary

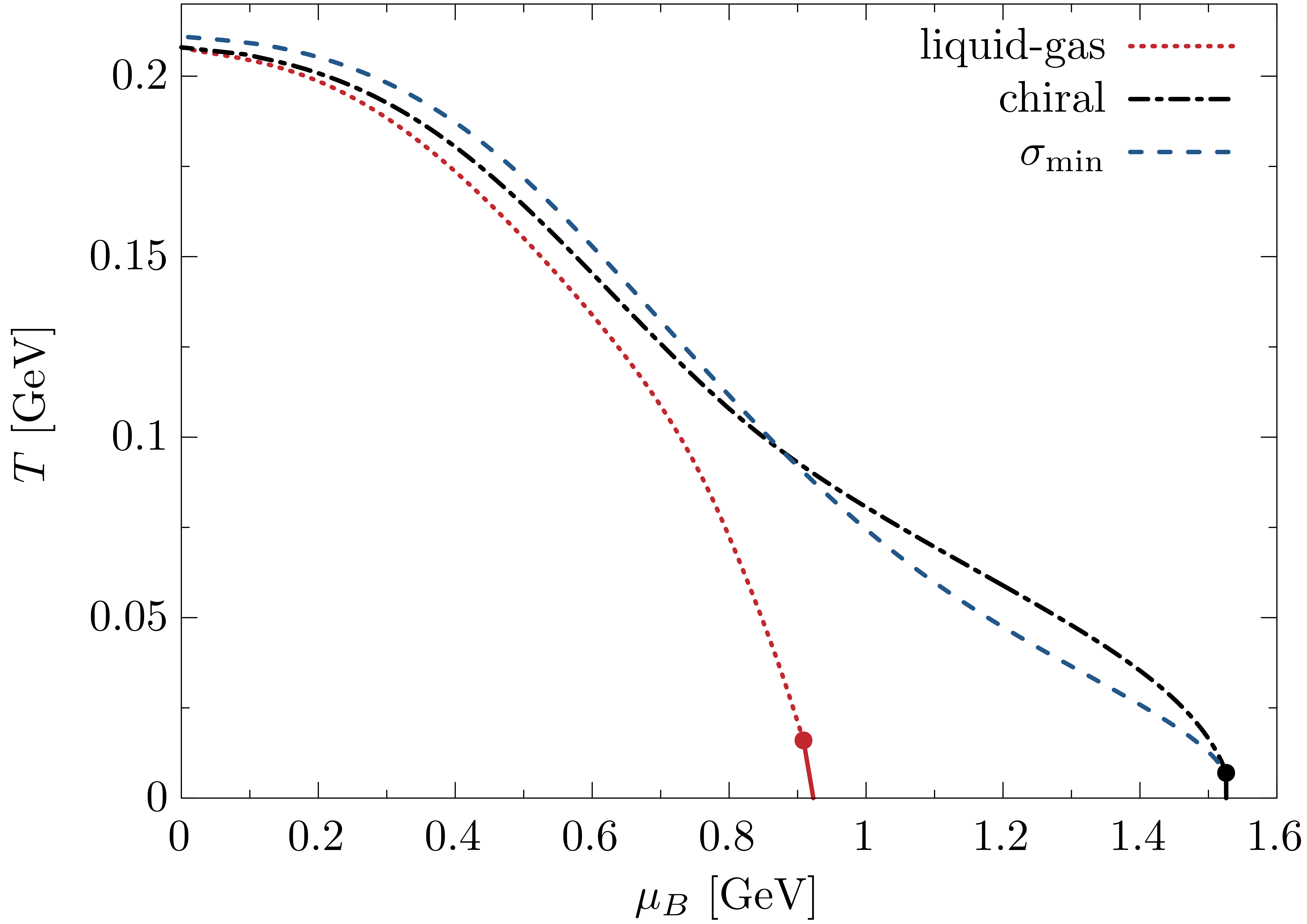
Hadronic parity doublet model for the chiral symmetry

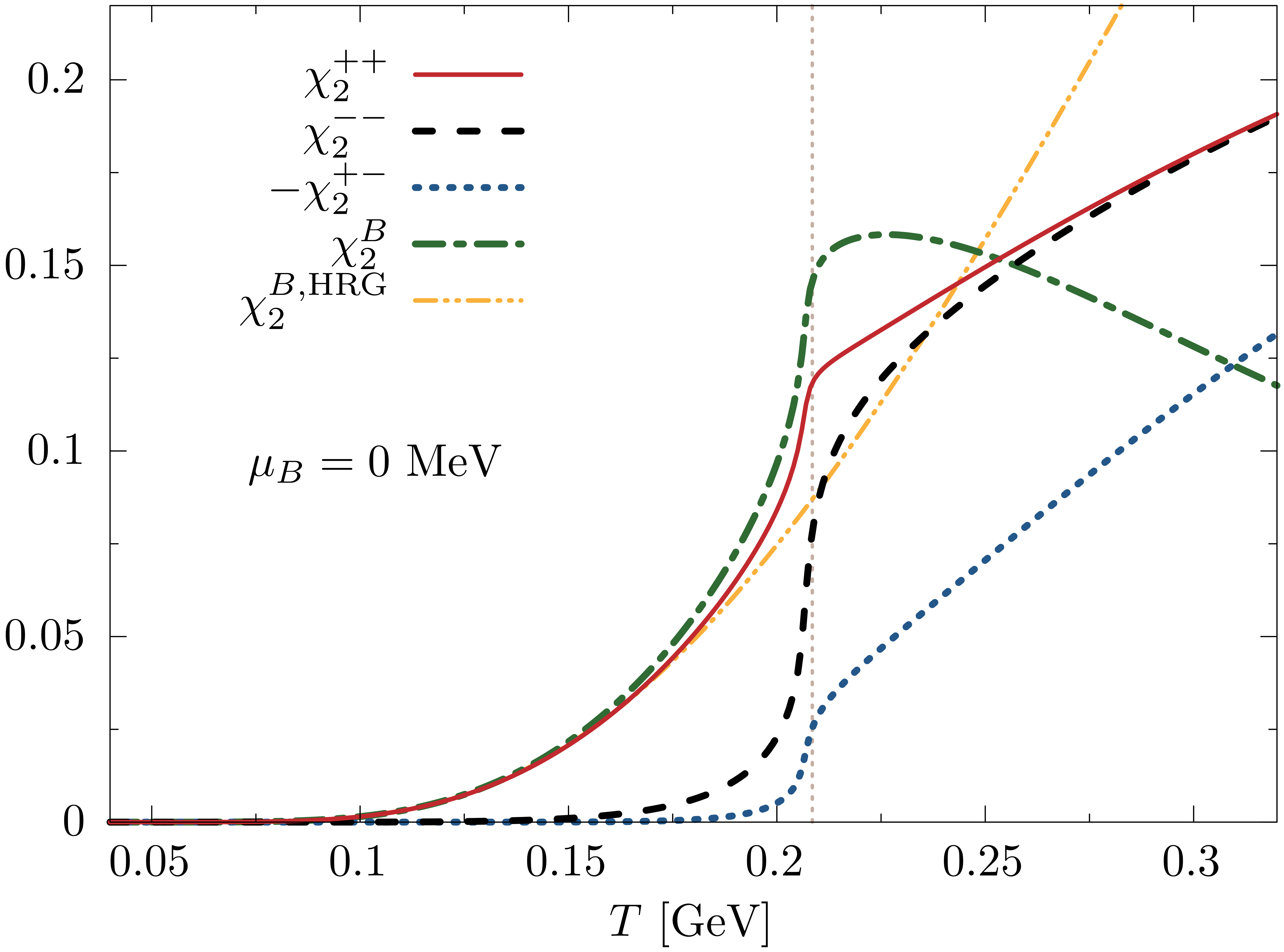
- Exploratory tool to identify contributions of parity partners to fluctuations
- Correlations of parity partners excellent probes for chiral transition
- Net-proton may not reflect the net-baryon fluctuations but measuring net-proton number fluctuations is sufficient to identify chiral CP

Thank You

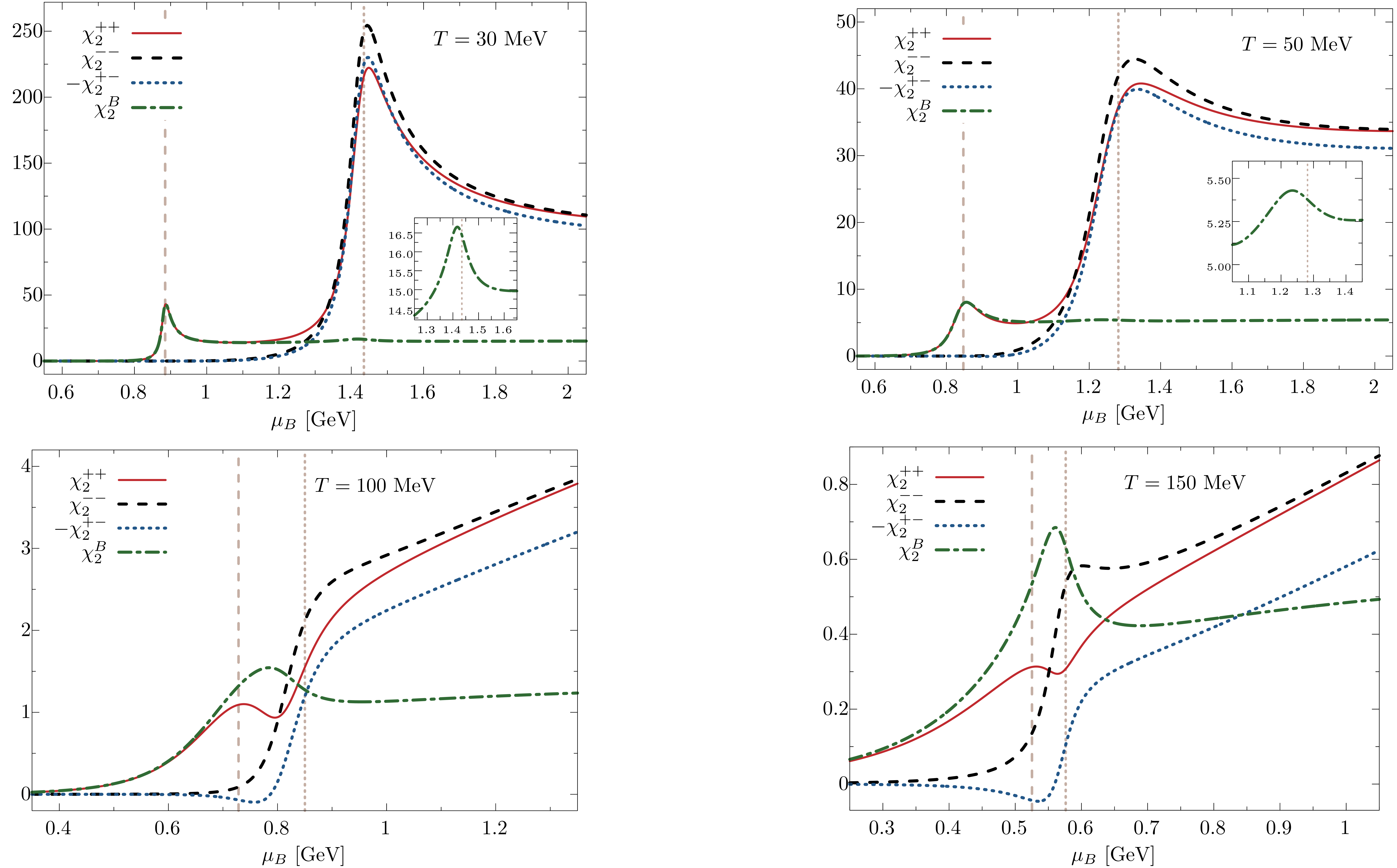


N_-
N_+
m_0
$\mu_B = 0$ MeV
Mass [GeV]
1.6
1.4
1.2
1
0.8
0.6
0
0.5
1
1.5
2
T $[T_c]$

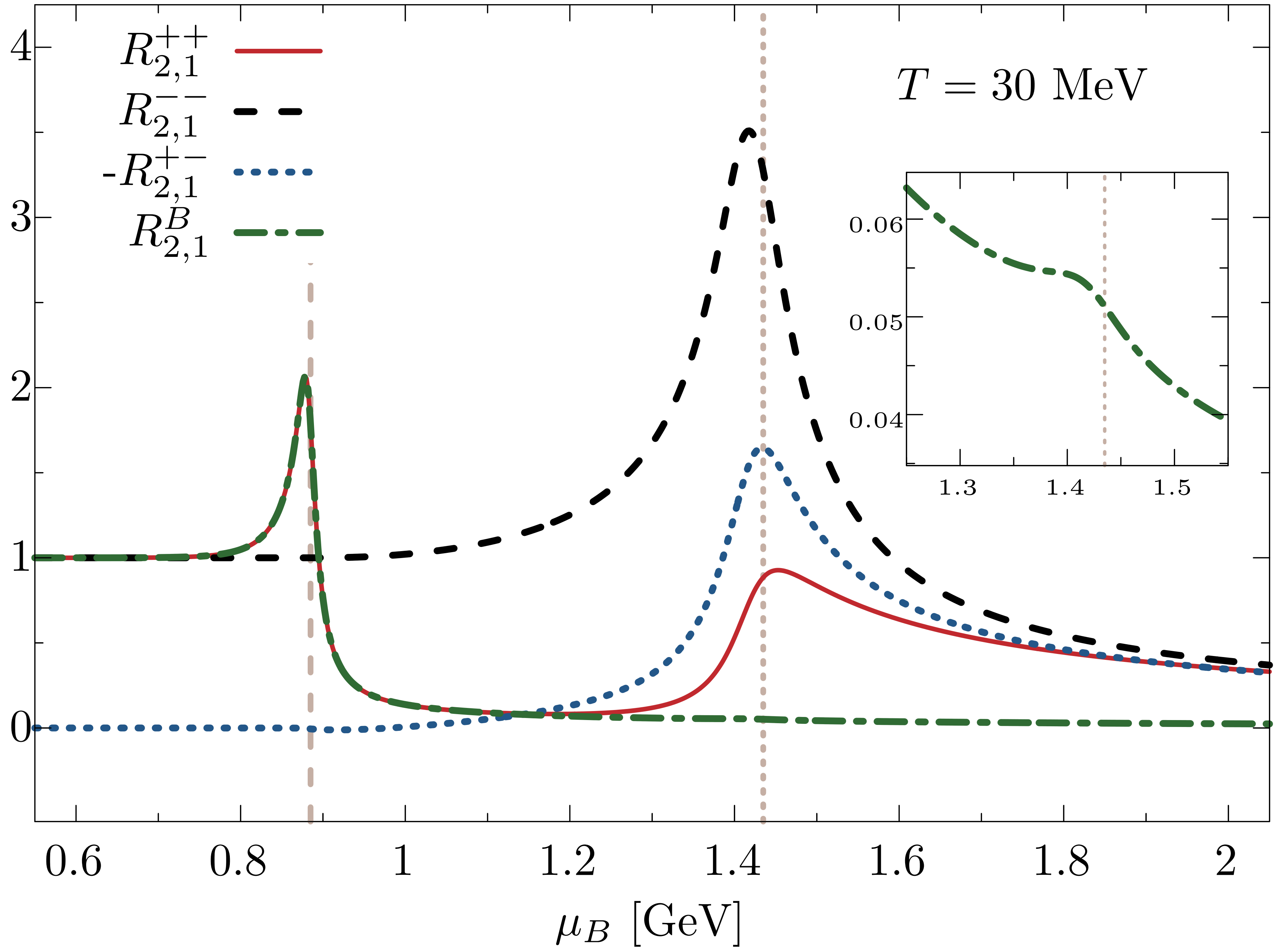

liquid-gas
chiral
$\sigma_{\min}$
T [GeV]
0
0.05
0.1
0.15
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ_B [GeV]

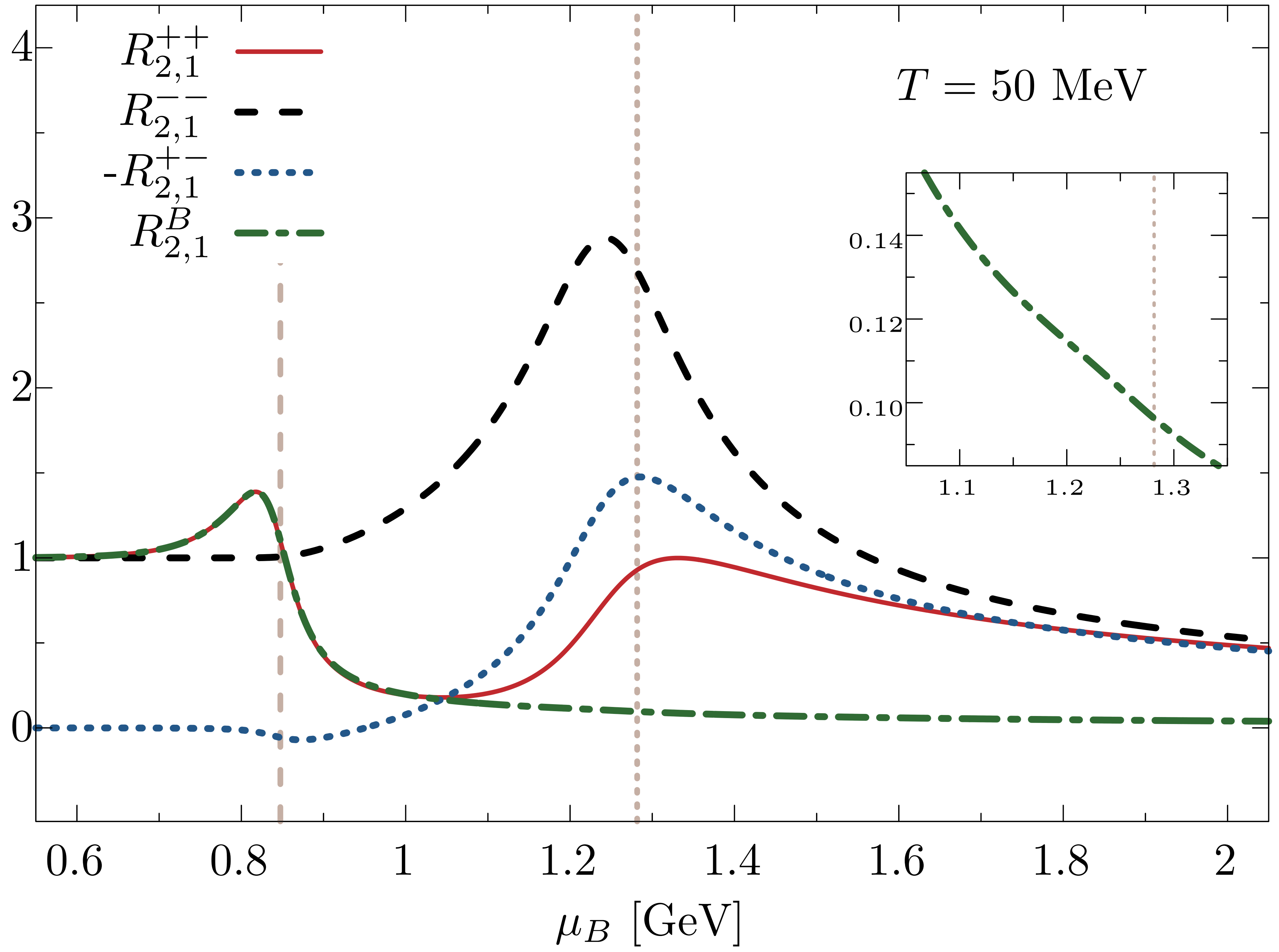


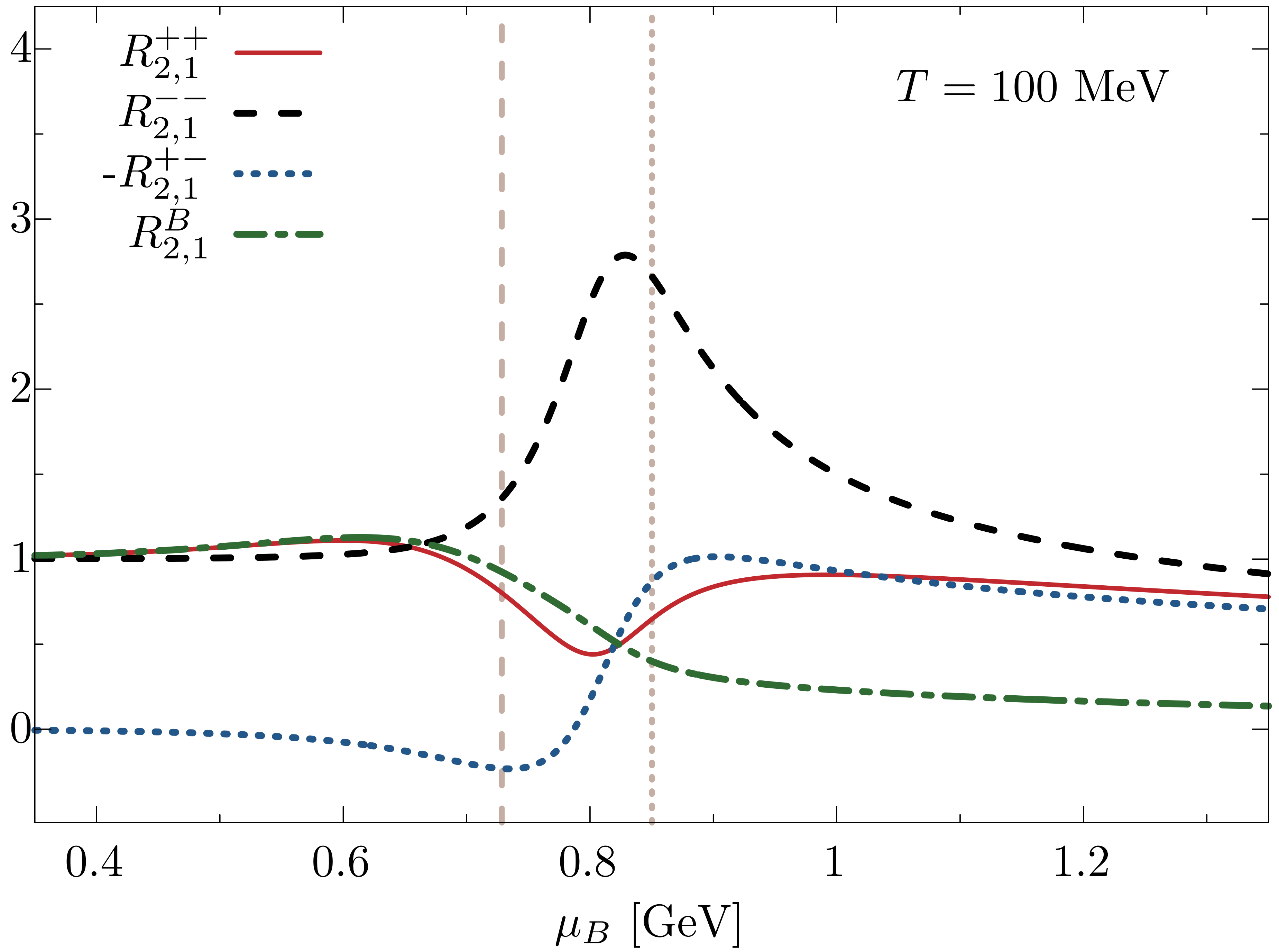
χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^{B}
$\chi_2^{B,\mathrm{HRG}}$
$\mu_B = 0$ MeV
0.2
0.15
0.1
0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
T [GeV]

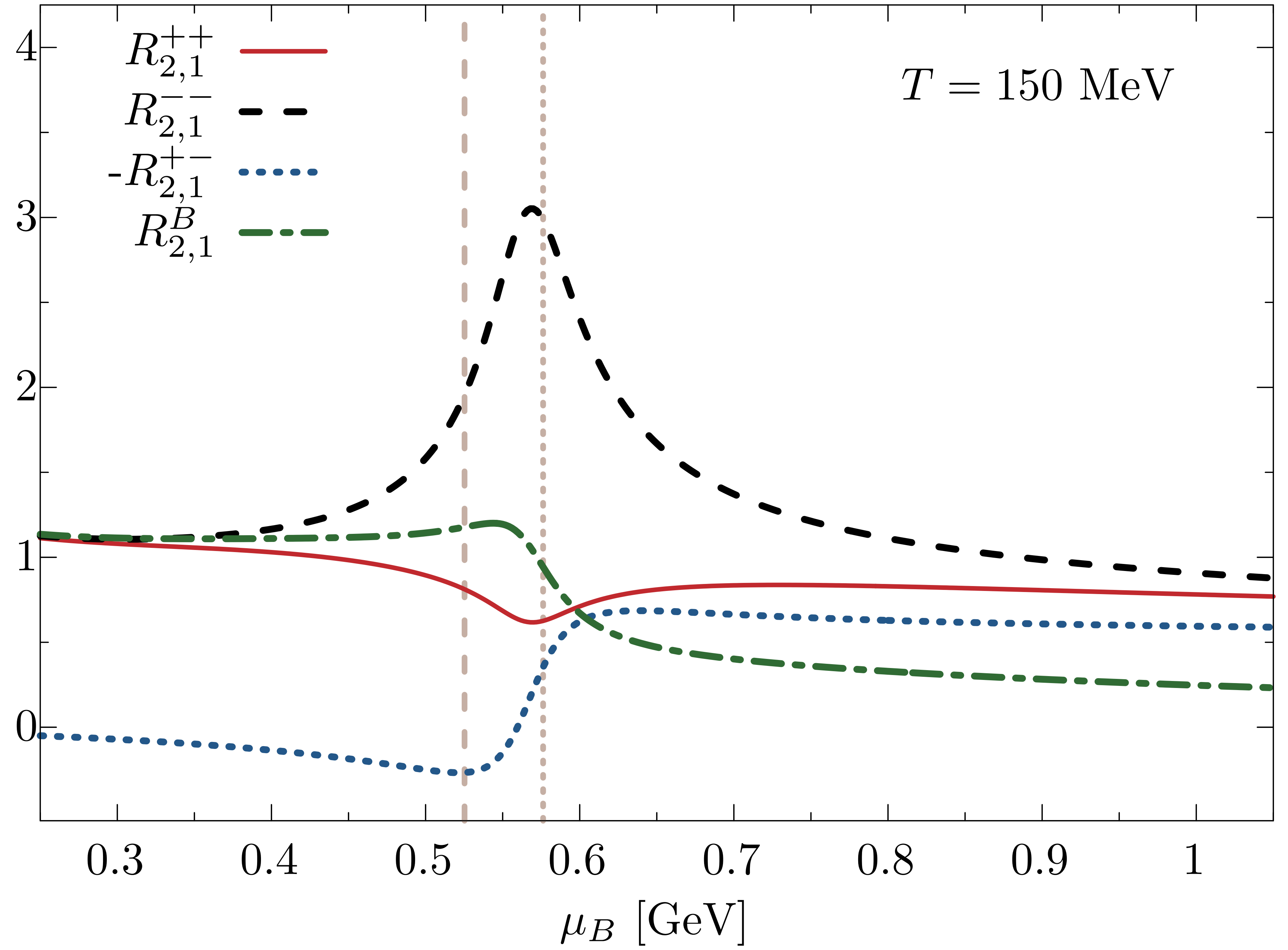


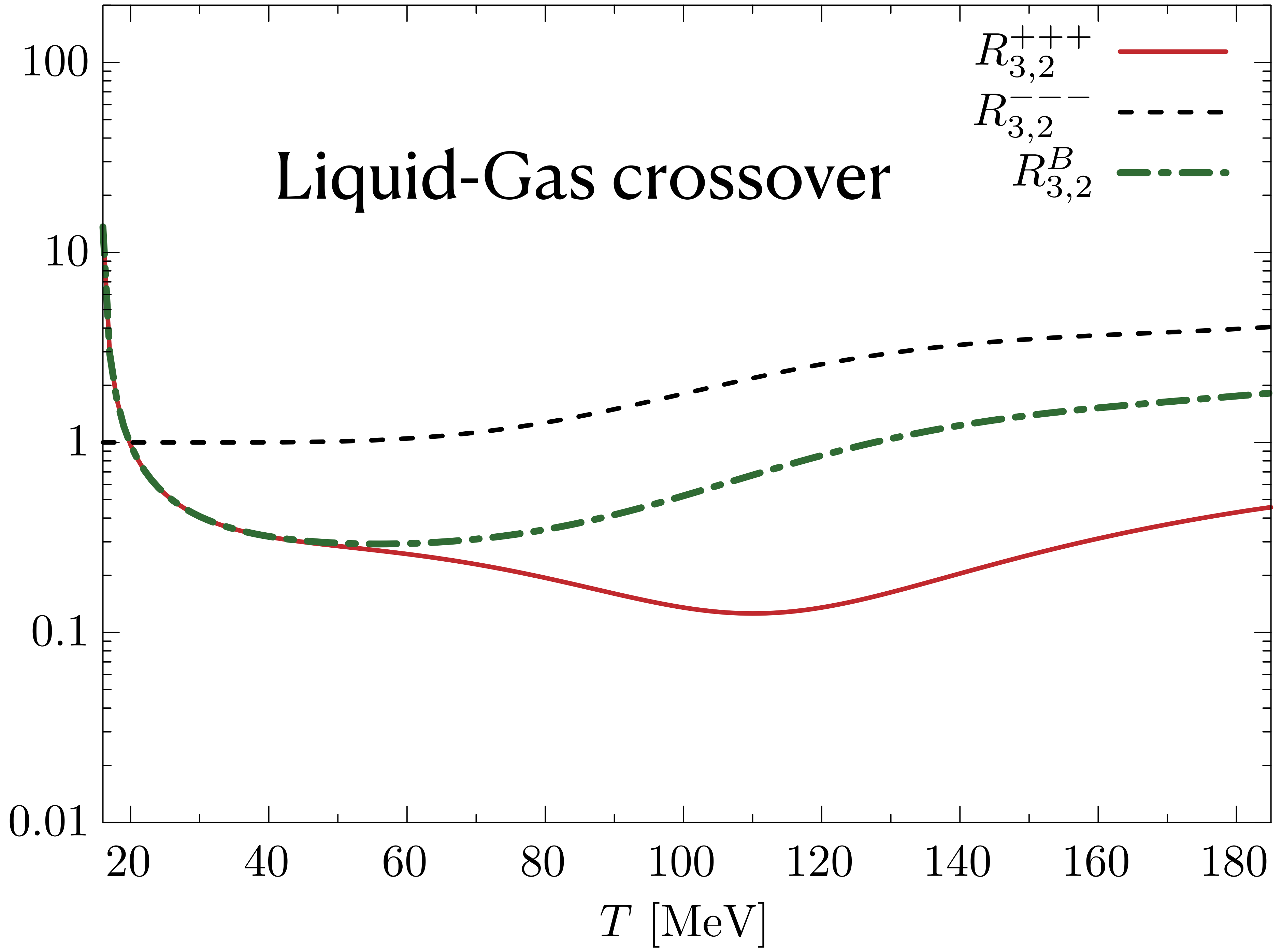
χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^{B}
$T = 30$ MeV
$T = 50$ MeV
$T = 100$ MeV
$T = 150$ MeV
μ_B [GeV]

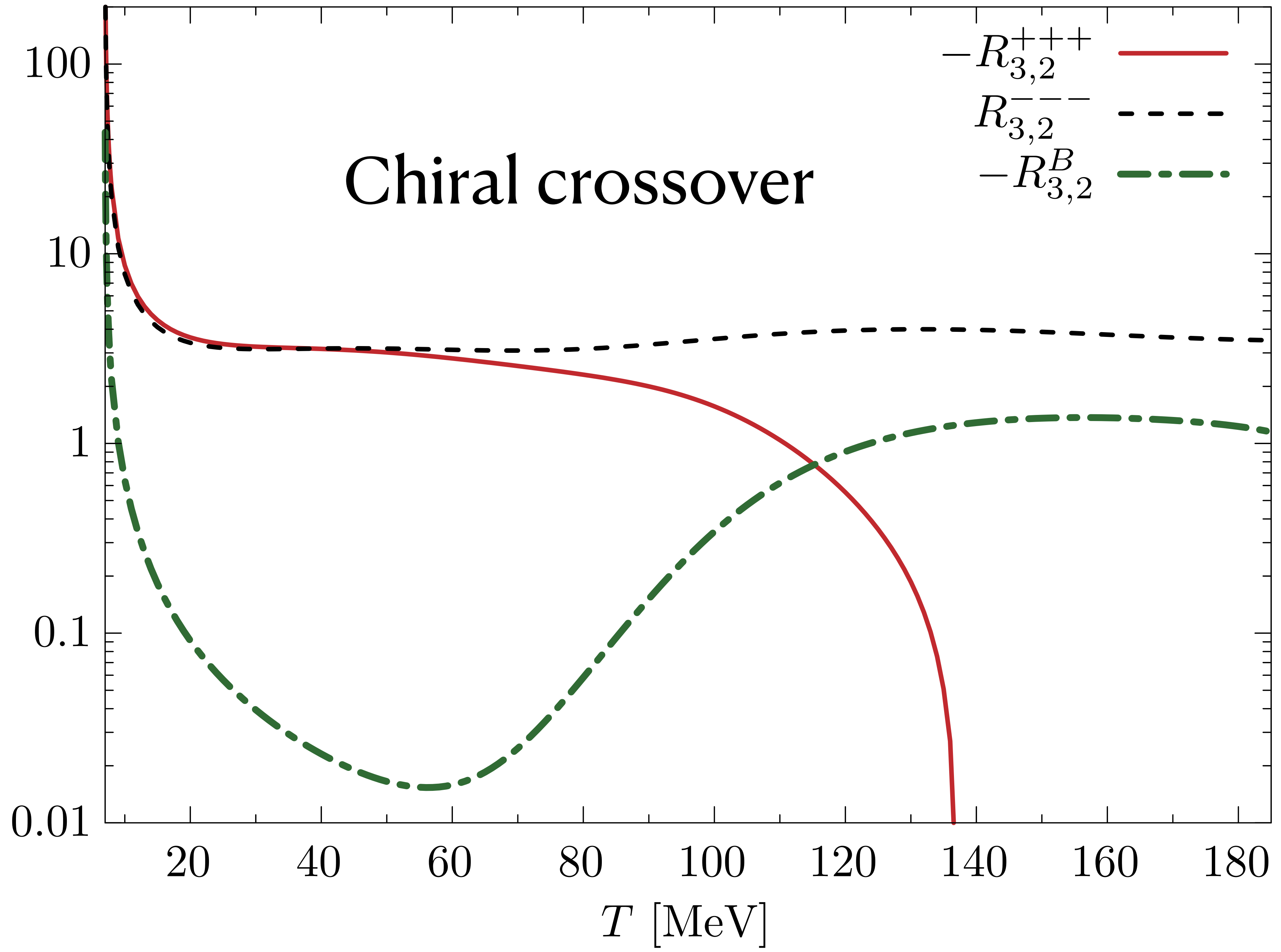

$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 30$ MeV
μ_B [GeV]


$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 50$ MeV
μ_B [GeV]


$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 100$ MeV
μ_B [GeV]


$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 150$ MeV
μ_B [GeV]





$$R_{3,2}^{\alpha\alpha\alpha} \equiv \frac{C_3^{\alpha\alpha}}{C_2^{\alpha\alpha}} = \frac{\chi_3^{\alpha\alpha}}{\chi_2^{\alpha\alpha}} = S\sigma$$

STAR, 2021