Heavy flavour hadronization

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Thanks to:
V. Minissale, M.L. Sambataro, S. K. Das, Y. Sun, V. Greco

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28 September – 4 October 2023
Giardini Naxos, Sicily, Italy

Outline

Basic concepts, motivation and model setting

Heavy hadrons in AA collisions:

- **Λc , D spectra and ratio: RHIC and LHC**

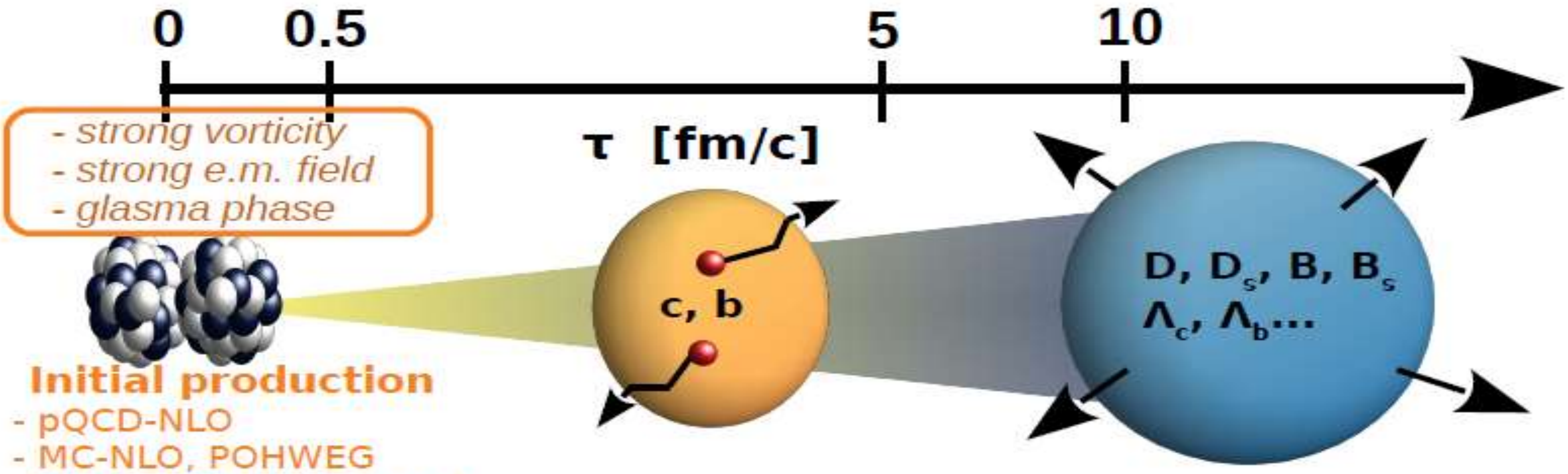
Heavy hadrons in small systems (pp @ 5.02 TeV):

- **Λc/D^0**
- **Ξc/D^0 , Ωc/D0**

Multi-charm production PbPb vs KrKr vs ArAr vs OO:

- comparing evolution with A-A to SHM
- looking at <r> dependence of Ω_{ccc} production

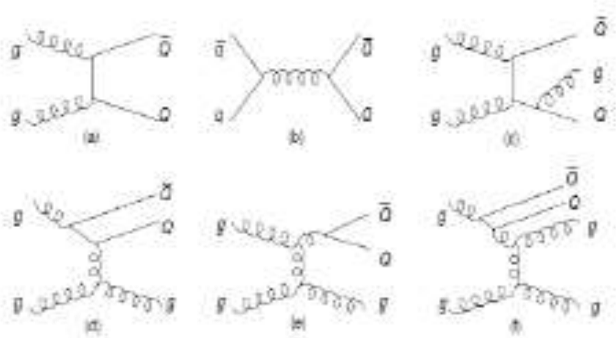
Heavy quarks in uRHIC



Initial production

- pQCD-NLO
- MC-NLO, POHWEG
- CNM effect[pp,pA exp.]

$$\sigma_{pp\to c\bar{c}} = \int_0^1 dx_1 dx_2 \sum_{i,j} f_i(x_1,Q^2) f_j(x_2,Q^2) \sigma_{ij\to c\bar{c}}(x_1,x_2,Q^2),$$



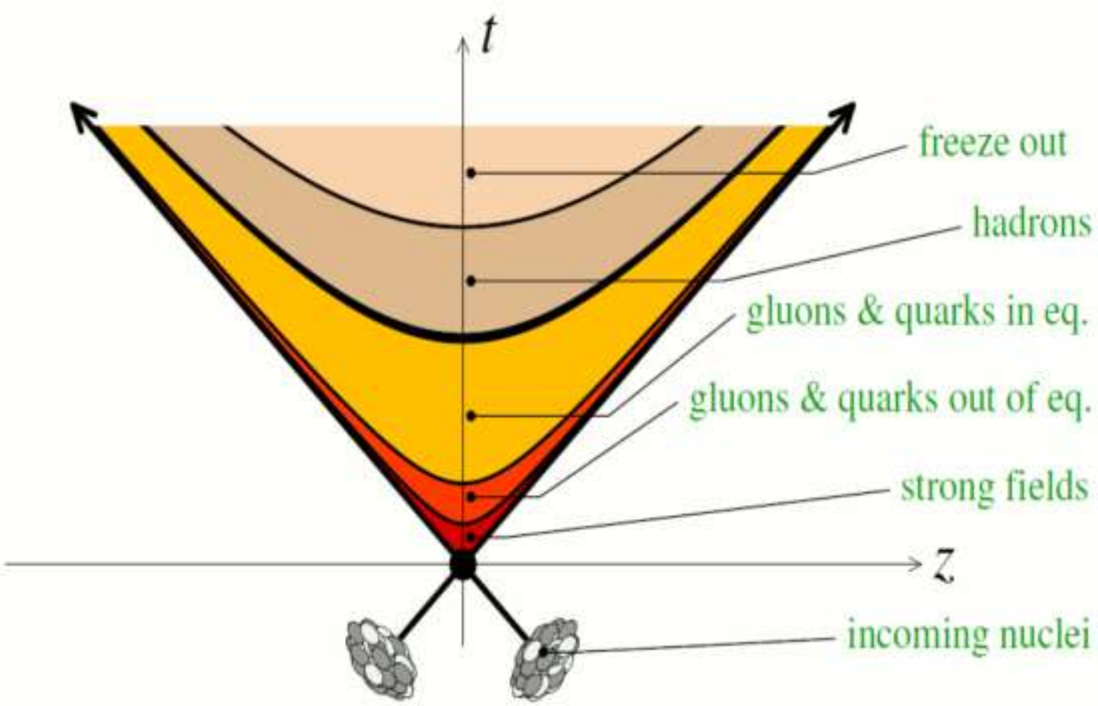
Dynamics in QGP

- Transport approaches: Boltzmann/Fokker-Planck
- Themalization
- Transp. Coeff. of QCD matter $D_s(T)$
- Jet Quenching

Hadronization

- SHM/coalescence and/or fragm. D, D_s, B, B_s, Λ_c, Λ_b, Ξ_c, Ω_c...
- Λ_c/D in pp,pA,AA
- R_{AA}, collective flow harmonics

Hadronisazion in heavy ion collisions



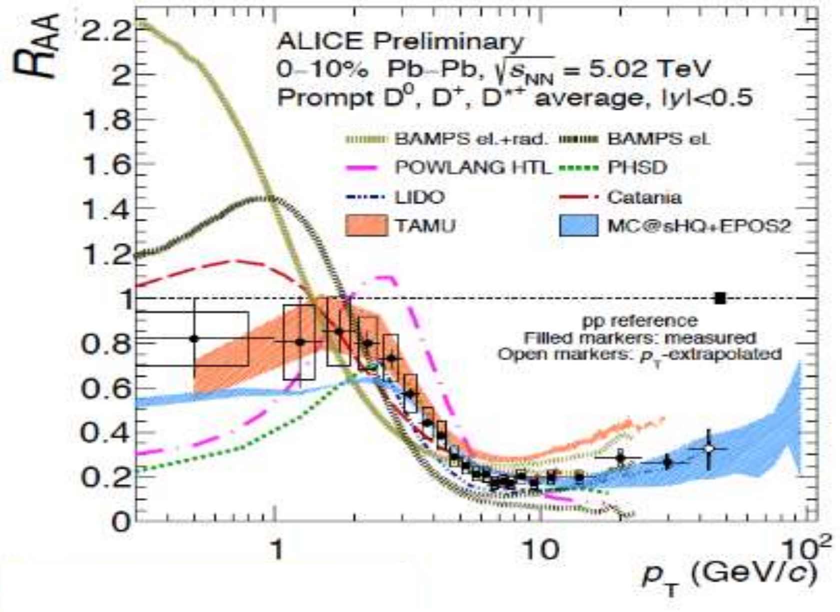
- **Hadronisation:**
  the mechanism by which quarks and gluons produced in hard partonic scattering processes form the hadrons
- **No first-principle description of hadron formation**
  - Non-perturbative problem
  - Necessary to resort to models
- **Hadronisation of the QGP medium at the pseudo-critical temperature**
  Transition from a deconfined medium composed of quarks, antiquarks and gluons to color-neutral hadronic matter

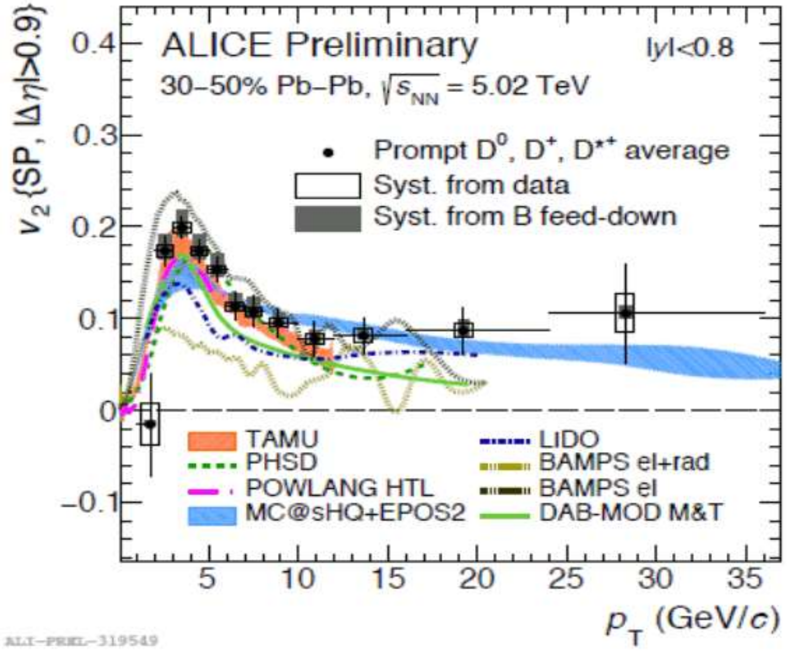
Hadronization (impossible to neglect)

- source of systematic uncertainty in final observable R_{AA} and v_2
  → systematic uncertainty in extracting transport coefficients

- how HF hadronization changes in the presence of a medium

Transport coefficient





Models not really tested at p→0
The new data → determine $D_s(T)$ more properly,
i.e. p→0 where it is defined and computed in lQCD

	Catania	Duke	Frankfurt(PHSD)	LBL	Nantes	TAMU
Initial HQ (p)	FONLL	FONLL	pQCD	pQCD	FONLL	
Initial HQ (x)	binary coll.	binaryy coll.	binary coll.	binary coll.		binary coll.
Initial QGP	Glauber	Trento	Lund		EPOS	
QGP	Boltzm.	Vishnu	Boltzm.	Vishnu	EPOS	2d ideal hydro
partons	mass	m=0	m(T)	m=0	m=0	m=0
formation time QGP	0.3 fm/c	0.6 fm/c	0.6 fm/c (early coll.)	0.6 fm/c	0.3 fm/c	0.4 fm/c
interactions in between	HQ-glasma	no	HQ-preformed plasma	no		no

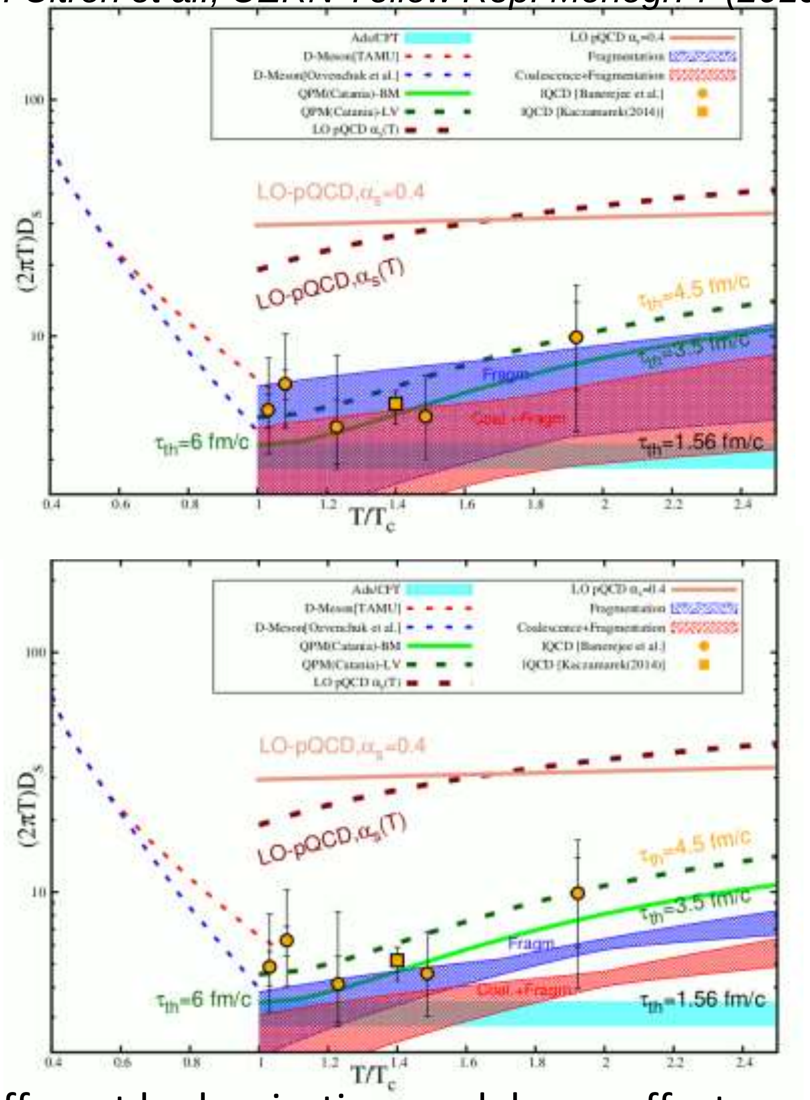
2018-2019

Several Collab. in joint activities:

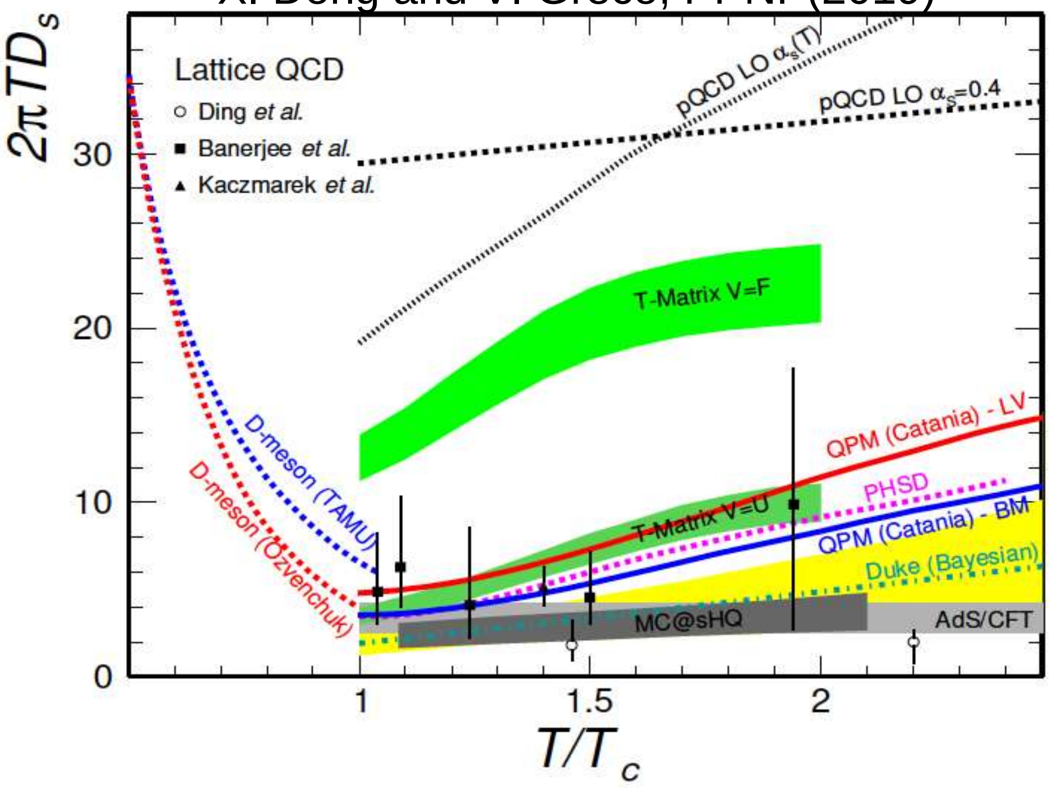
- EMMI-RRTF:
  R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS:
  S. Cao et al.,Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

Transport coefficient

Z. Citron et al., CERN Yellow Rep. Monogr. 7 (2019) 1159



X. Dong and V. Greco, PPNP(2019)



Different hadronization models can affect the extraction of the charm quark diffusion coefficient

New joint activity needed

2018-2019

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- Y. Xu et al., Phys. Rev. C 99 (2019)

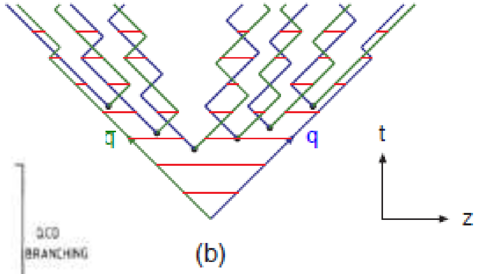
HF Hadronization schemes

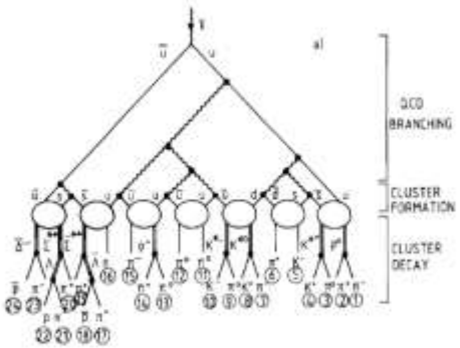
- **Independent fragmentation**

  $q \rightarrow \pi, K, p, \Lambda$ ..

  $c \rightarrow D, D_s, \Lambda_c$, ...

- **String fragmentation** (PYTHIA)





- **In medium hadronization with Cluster decay**

  A. Beraudo et al., arXiv:2202.08732v1 [hep-ph]

- **Coalescence/recombination**

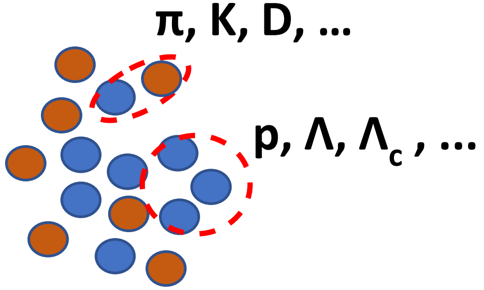
  S. Plumari, V. Minissale et al, Eur. Phys. J. **C78** no. 4, (2018) 348

  S. Cao et al. , Phys. Lett. B 807 (2020) 135561

  Resonance Recombination model

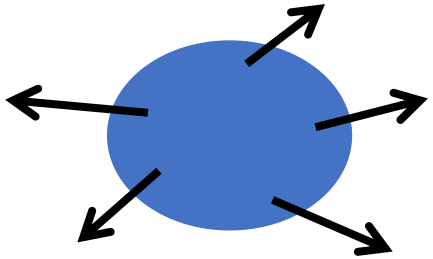
  L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).

  L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).



- **Statistical hadronization model (SHM)**

  A. Andronic et al, JHEP 07 (2021) 035



Relativistic Boltzmann eq. at finite η/s

Bulk evolution

$$p^\mu \partial_\mu f_q(x,p) + m(x)\partial_\mu^x m(x)\partial_p^\mu f_q(x,p) = C[f_q, f_g]$$

$$p^\mu \partial_\mu f_g(x,p) + m(x)\partial_\mu^x m(x)\partial_p^\mu f_g(x,p) = C[f_q, f_g]$$

free-streaming

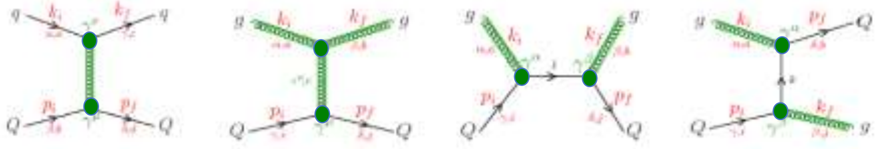
field interaction
ε-3p≠0

collision term
gauged to some η/s≠0

Equivalent to viscous hydro η/s≈0.1

HQ evolution

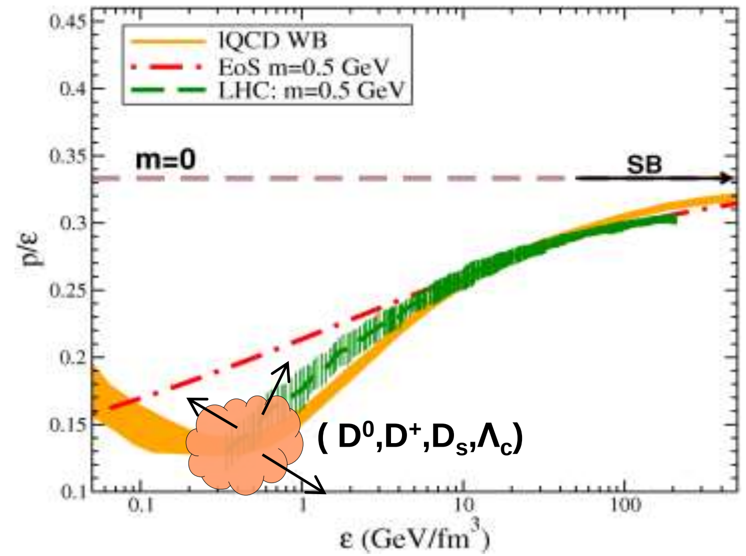
$$p^\mu \partial_\mu f_Q(x,p) = \mathcal{C}[f_q, f_g, f_Q](x,p)$$



$$\mathcal{C}[f_Q] = \frac{1}{2E_1}\int \frac{d^3p_2}{2E_2(2\pi)^3}\int \frac{d^3p_1'}{2E_{1'}(2\pi)^3} \times [f_Q(p_1')f_{q,g}(p_2') - f_Q(p_1)f_{q,g}(p_2)] \times |\mathcal{M}_{(q,g)+Q}(p_1p_2 \to p_1'p_2')|^2 \times (2\pi)^4\delta^4(p_1+p_2-p_1'-p_2') ,$$

M scattering matrix by QPM model fit to lQCD EoS

S. Plumari et al.,J.Phys.Conf.Ser. 981 012017 (2018).



Indipendent fragmentation

Spectrum of heavy quarks produced in pp-collisions can be computed up to NLO in s with available tools

Transition from quark momentum spectrum to hadron momentum, using fragmentation model:

$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f\to h}(z)$$

q → π, K, p, Λ ..

c → D, D_s, Λ_c, ...

Fragmentation function

- **Fragmentation functions** $D_{f\to h}$ are phenomenological functions to parameterize the *non-perturbative parton-to-hadron transition* (z = fraction of the parton momentum taken by the hadron h)

- **Fragmentation functions** assumed **universal** among energy and collision systems and constrained from e^+e^- and ep

- Different models for FFs are currently in use in literature:

- Peterson et al., $D(z) = \frac{C}{z\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^2}$

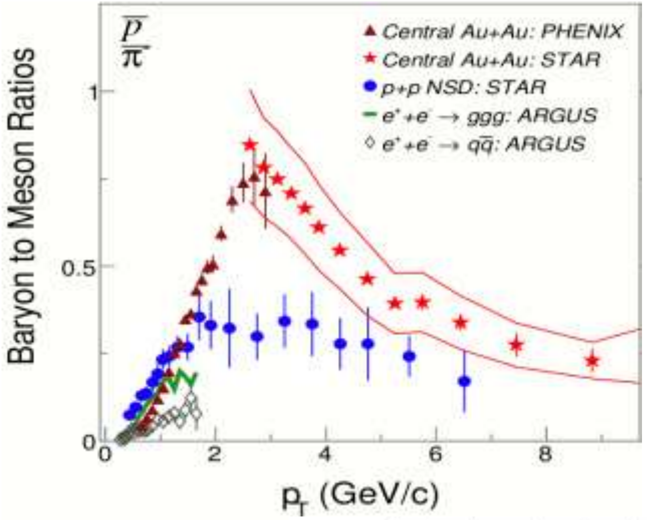
- Kartvelishvili et al., $D(z) = Cz^{\alpha}(1-z)$

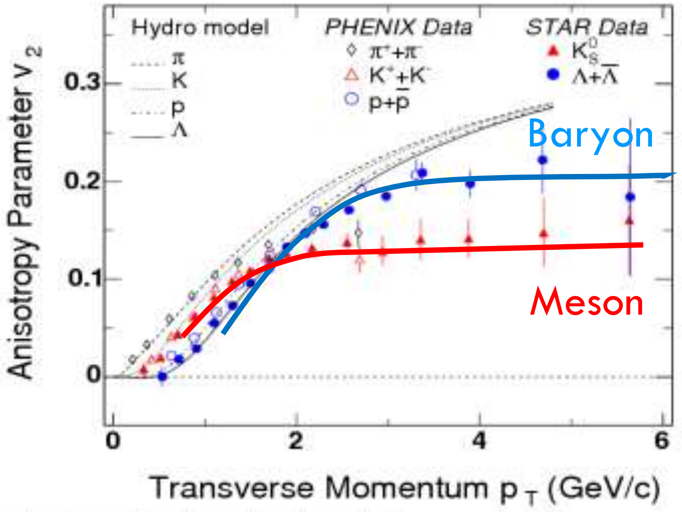
Hadronization: fragmentation and coalescence

R. J. Fries, V. Greco, P. Sorensen Ann.Rev.Nucl.Part.Sci. 58 (2008) 177

Proton to pion ratio Enhancement:

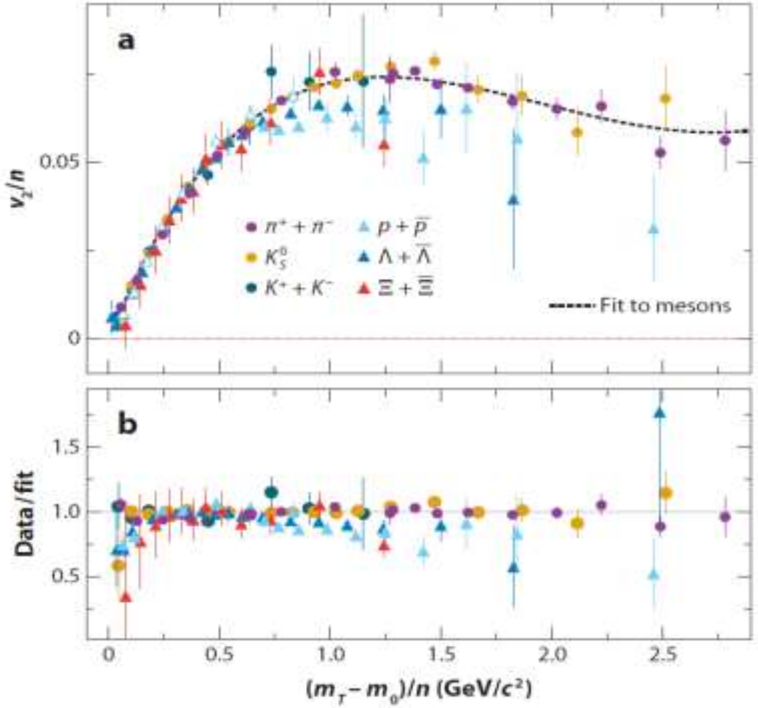
In vacuum from fragmentation functions the ratio is small $\frac{D_{q\to p}(z)}{D_{q\to\pi}(z)} < 0.25$





Elliptic flow splitting:

For p_T>2 GeV Both hydro and fragmentation predicts similar v_2 for pions and protons



Another hadronization mechanism is by coalescence:

Formalism originally developed for light-nuclei production from coalescence of nucleons on a freeze-out hypersurface.

Extended to describe meson and baryon formation in AA collisions from the quarks of QGP through 2→1 and 3→1 processes

V. Greco, C.M. Ko, P. Levai PRL 90, 202302 (2003).
V. Greco, C.M. Ko, P. Levai PRC 68, 034904 (2003).
R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRL 90, 202303 (2003).
R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRC 68,044902 (2003).

Coalescence approach in phase space for HQ

Statistical factor colour-spin-isospin

Parton Distribution function

Hadron Wigner function

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^{n} p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \ldots, x_n; p_1, \ldots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r},\mathbf{q}) = \int d^3r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r}+\frac{\mathbf{r}'}{2}\right)\varphi_M^*\left(\mathbf{r}-\frac{\mathbf{r}'}{2}\right)$$

$\varphi_M(\boldsymbol{r})$ *meson wave function*

Assuming gaussian wave function

$$f_M(x_1, x_2; p_1, p_2) = A_W \exp\left(-\frac{x_{r1}^2}{\sigma_r^2} - p_{r1}^2\sigma_r^2\right)$$

For baryon $N_q=3$

$$f_H(\ldots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2\sigma_{ri}^2\right)$$

Note: only σ_r coming from $\varphi_M(\boldsymbol{r})$ or $\sigma_r*\sigma_p=1$ valid for harmonic oscillator with V(r) $\sigma_r*\sigma_p>1$

Wigner function **width** fixed by root-mean-square charge radius from **quark model**

Meson	$\langle r^2\rangle_{ch}$	σ_{p1}	σ_{p2}
$D^+ = [c\bar{d}]$	0.184	0.282	—
$D_s^+ = [s\bar{c}]$	0.083	0.404	—
Baryon	$\langle r^2\rangle_{ch}$	σ_{p1}	σ_{p2}
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

C.-W. Hwang, EPJ C23, 585 (2002);
C. Albertus et al., NPA 740, 333 (2004)

$$\langle r^2\rangle_{ch} = \frac{3}{2}\frac{m_2^2Q_1 + m_1^2Q_2}{(m_1+m_2)^2}\sigma_{r1}^2 + \frac{3}{2}\frac{m_3^2(Q_1+Q_2)+(m_1+m_2)^2Q_3}{(m_1+m_2+m_3)^2}\sigma_{r2}^2 \quad (8)$$

$\sigma_{ri} = 1/\sqrt{\mu_i\omega}$ Harmonic oscillator relation

$$\mu_1 = \frac{m_1m_2}{m_1+m_2}, \quad \mu_2 = \frac{(m_1+m_2)m_3}{m_1+m_2+m_3}.$$

Normalization $f_H(\ldots)$ *fixed by requiring* P_{coal}(p->0)=1 which fixes A_w , additional assumption wrt standard coalescence which does not have confinement

Coalescence approach in phase space for HQ

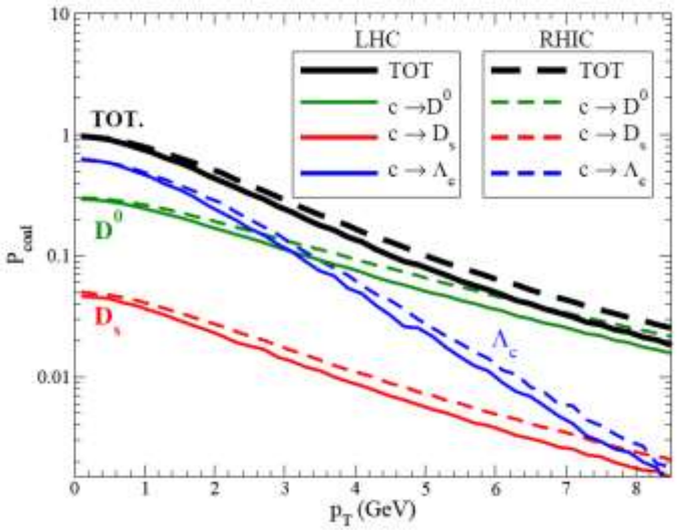
Statistical factor colour-spin-isospin

Parton Distribution function

Hadron Wigner function

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^{n} p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n)\, \delta\left(p_T - \sum_i p_{iT}\right)$$

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2\sigma_{ri}^2\right)$$



S. Plumari, V. Minissale et al., Eur. Phys. J. **C78** no. 4, (2018) 348

- Normalization in $f_W(\dots)$ *fixed by requiring* $P_{coal}(p \to 0)=1$: ….others modify by hand σ_r to enforce confinement for a charm at rest in the medium

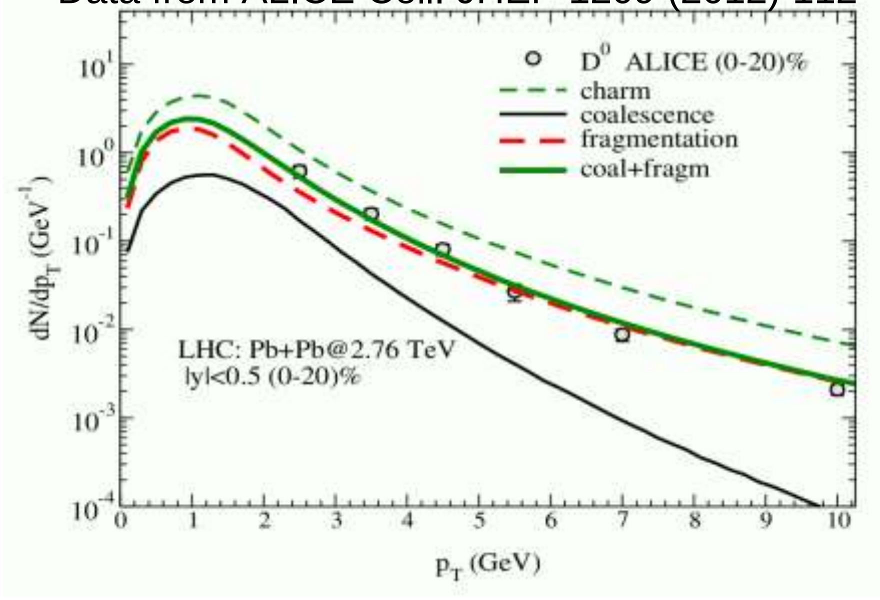
- The charm not “coalescencing” undergo fragmentation:

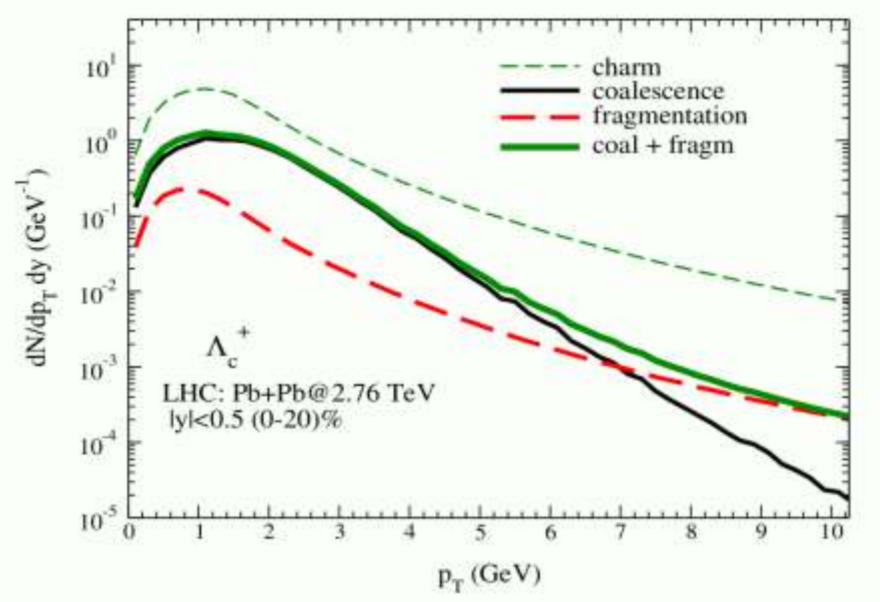
$$\frac{dN_{had}}{d^2p_T\,dy} = \sum \int dz \frac{dN_{fragm}}{d^2p_T\,dy} \frac{D_{had/c}(z, Q^2)}{z^2}$$

charm number conserved at each p_T ,
we have employed e^+e^- FF now PYTHIA

LHC: results

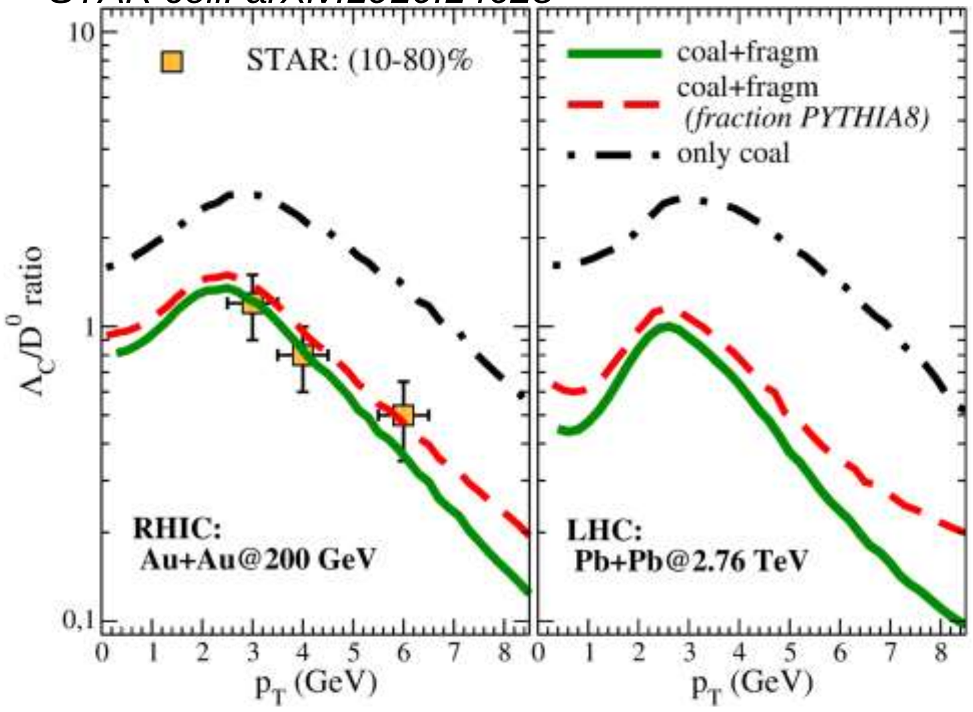
Data from ALICE Coll. JHEP 1209 (2012) 112





wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!

STAR coll. arXiv:1910.14628



The Λ_c/D^0 ratio is smaller at LHC energies: fragmentation play a role at intermediate p_T

S. Plumari, et al., Eur. Phys. J. ***C78*** *no. 4, (2018) 348*

RHIC: Baryon/meson

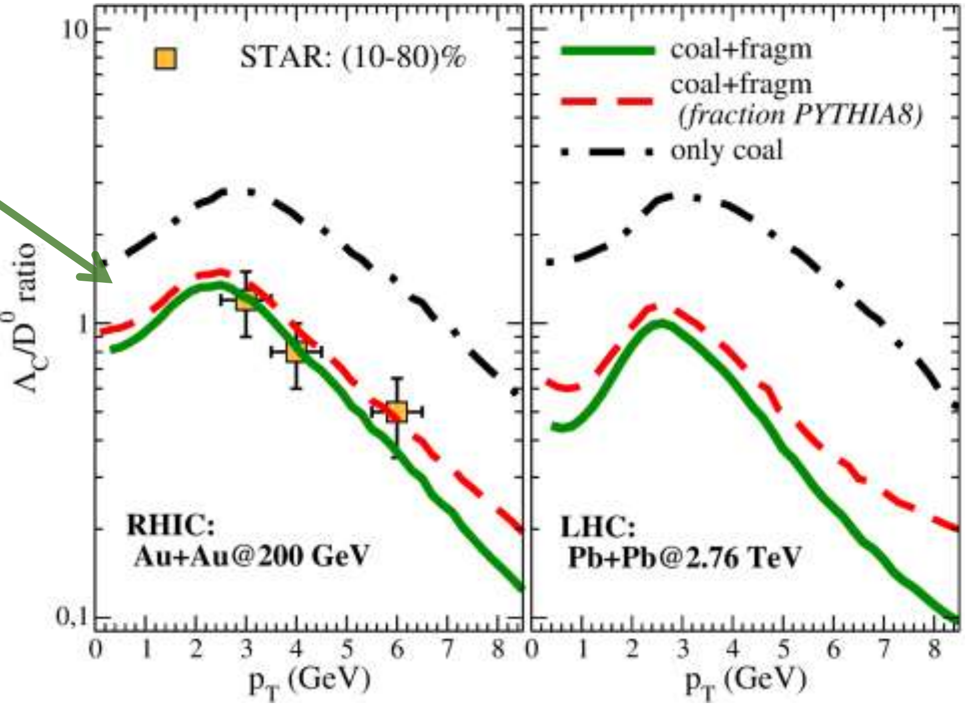
Coalescence

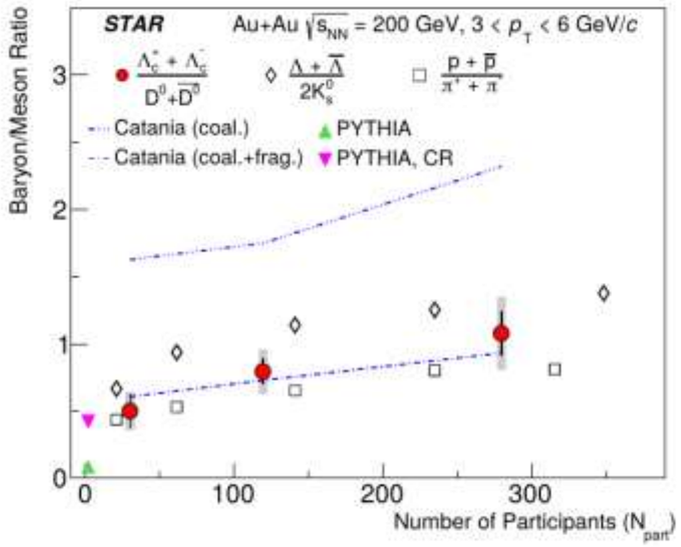
Following: L.W.Chen, C.M. Ko, W. Liu, M. Nielsen, PRC 76, 014906 (2007).
K.-J. Sun, L.-W. Chen, PRC 95, 044905 (2017).
For hypersurface of proper time τ and non relativistic limit:

for $p_T << m$ $$\frac{\Lambda_c^+}{D^0} \propto \frac{g_\Lambda}{g_D}\left(\frac{m_T^\Lambda}{m_T^D}\right) e^{-(m^\Lambda - m^D)/T_C}\, \tau\mu_2$$

$$\mu_2 = \frac{m_3(m_1+m_2)}{m_1+m_2+m_3}$$ Is the reduced mass of the baryon

wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!





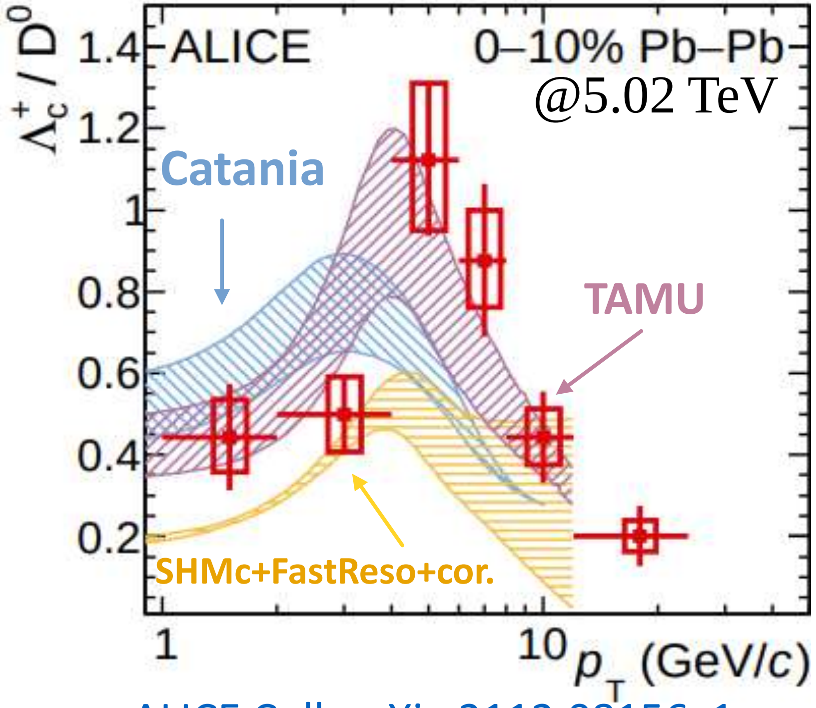
S. Plumari, et al., Eur. Phys. J. ***C78*** *no. 4, (2018) 348*

LHC: results

Results for 0-10% in PbPb @5.02TeV:

Consistent with the trend shown at RHIC and LHC @2.76TeV

Available data at low p_T → differences recombination vs SHM



ALICE Coll. arXiv:2112.08156v1

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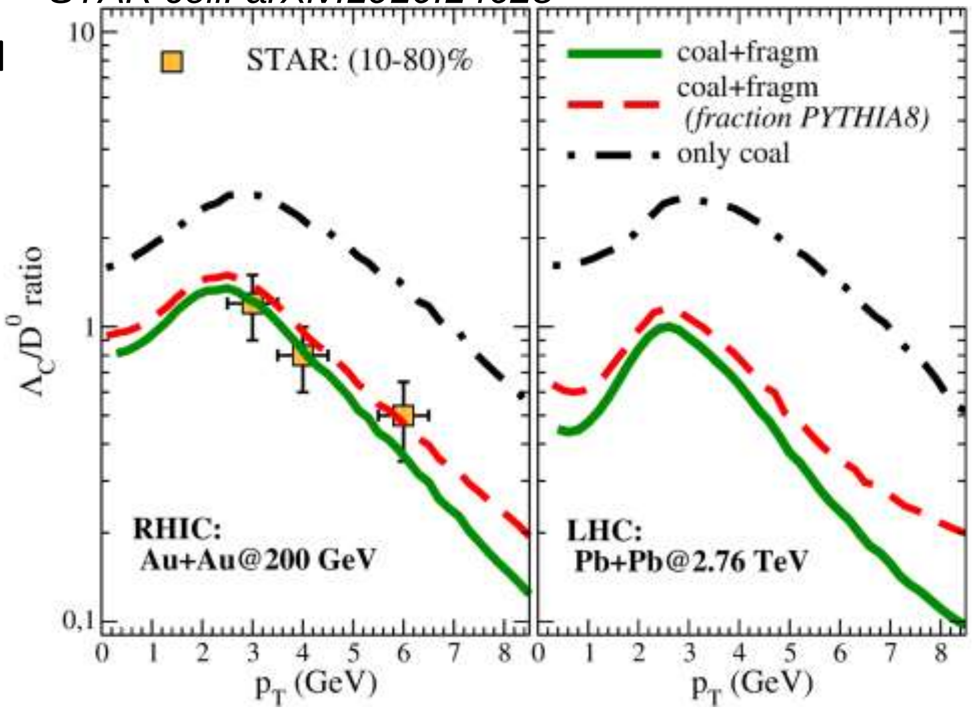

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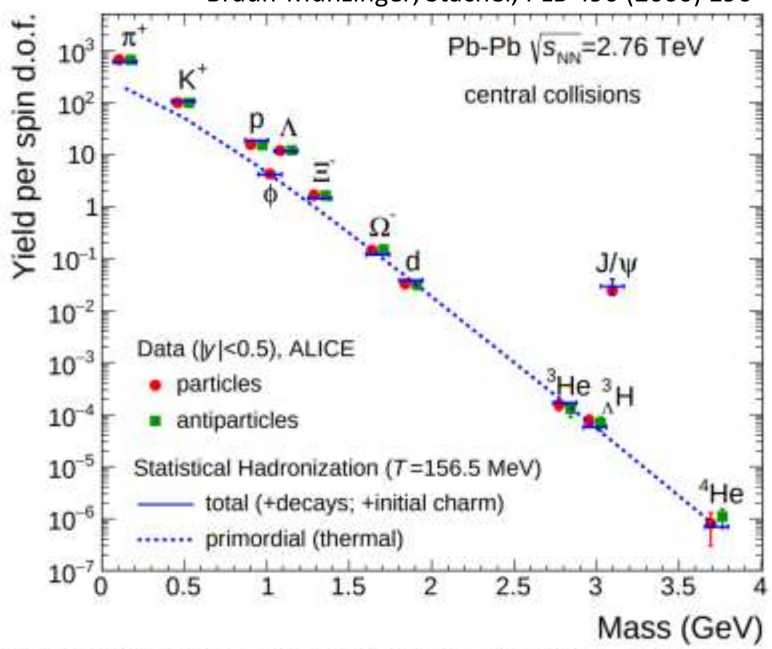
Statistical Thermal Model (SHM) + charm(SHMc)

Braun-Munzinger, Stachel, PLB 490 (2000) 196



grand canonical partition function

$$\ln Z_i = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

chemical potential ↔ conservation quantum numbers (N_B, N_S, N_C)

Equilibrium + hadron-resonance gas + freeze-out temperature.

Production depends on hadron masses and degeneracy, and on system properties.

Charm hadrons *according to thermal weights*

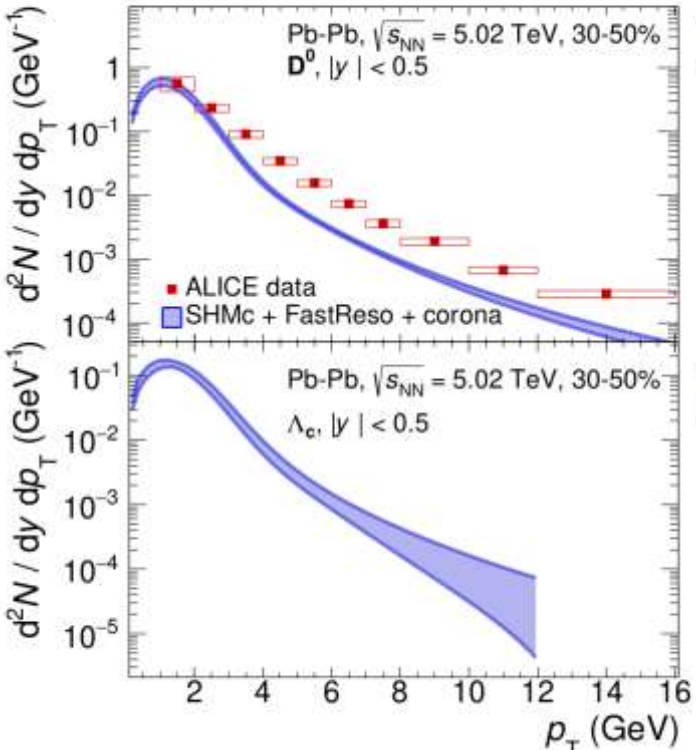
the *total charm content* of the fireball is fixed by the *measured open charm cross section*.

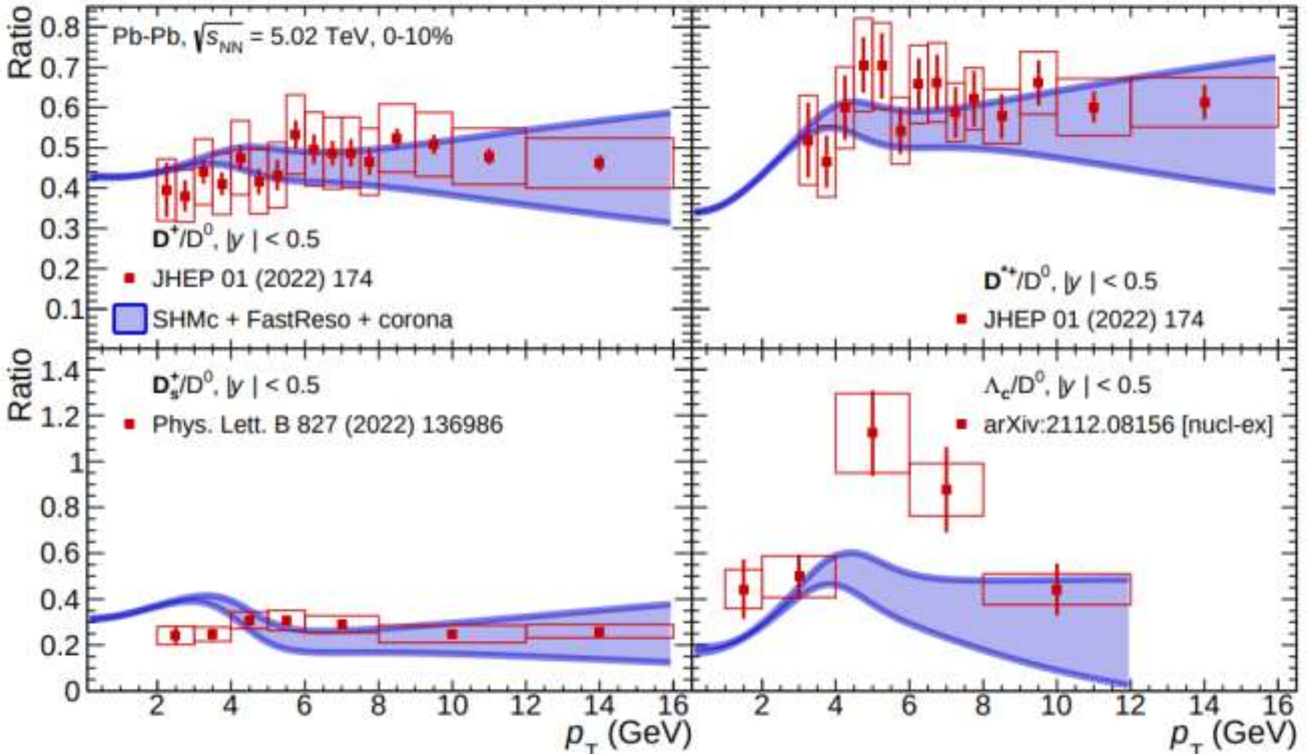
$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c V\left(\sum_i n_{D_i}^{th} + n_{\Lambda_{ci}}^{th}\right) + g_c^2 V\left(\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th}\right)$$

pQCD production $N_{c,\text{anti-c}}$ = 9.6 → g_c = 30.1 (charm fugacity)

Andronic et al., JHEP 07 (2021) 035

SHMc yields+blast wave →p_T spectra





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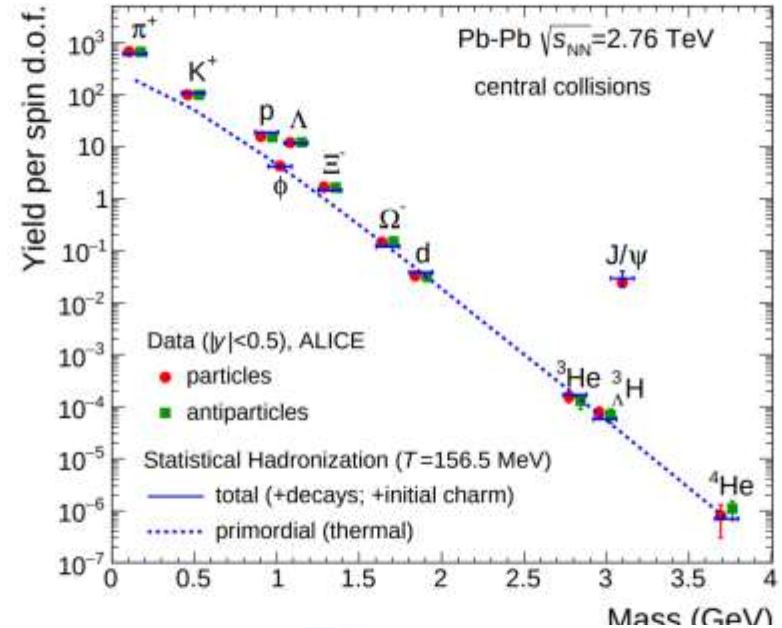

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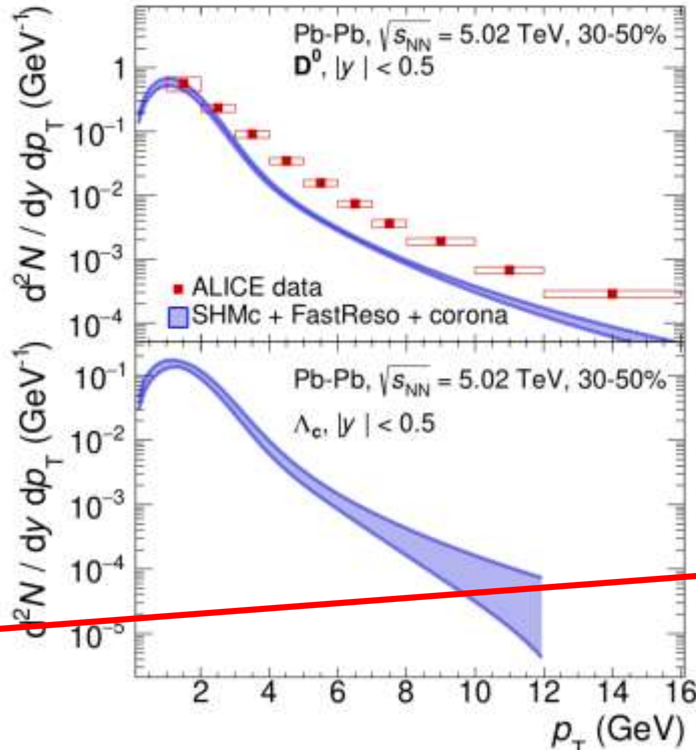
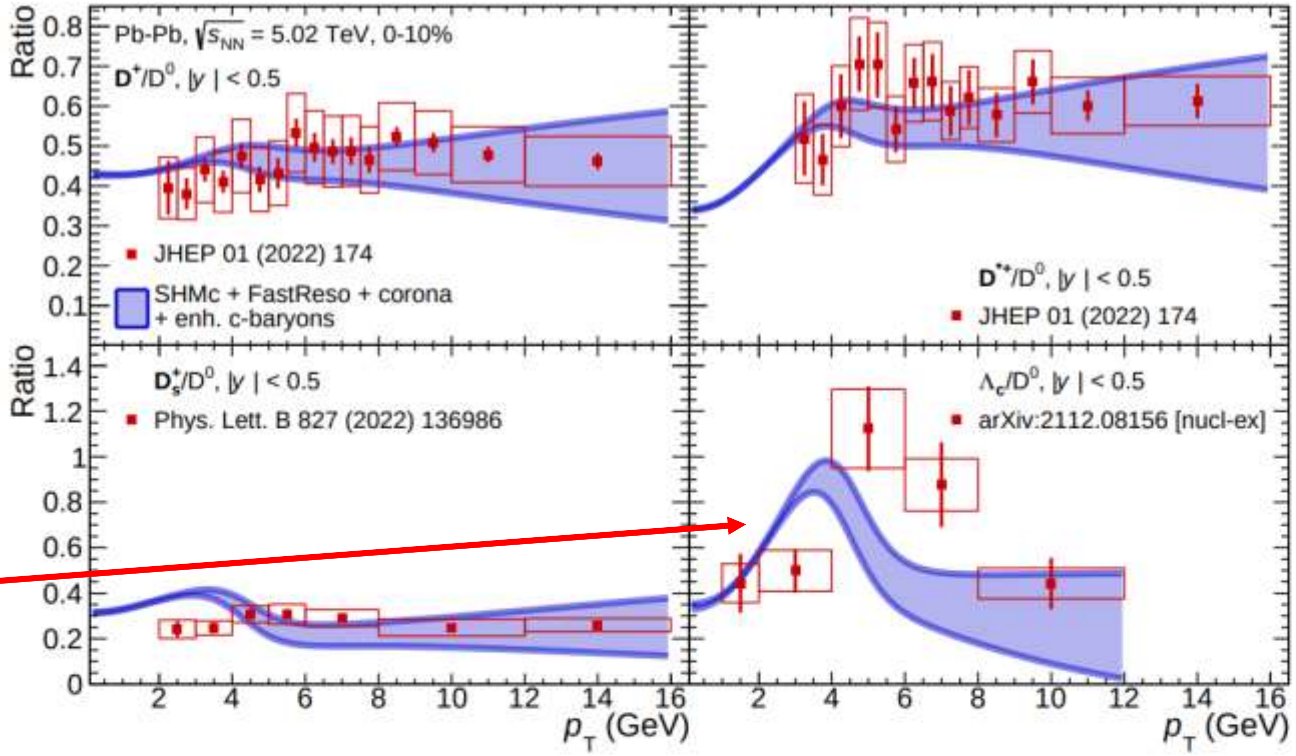
With enhanced set of charmed baryons

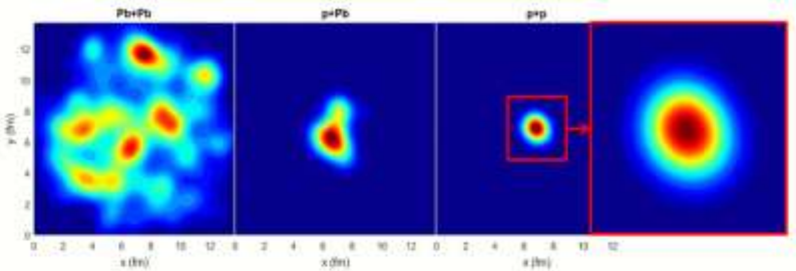


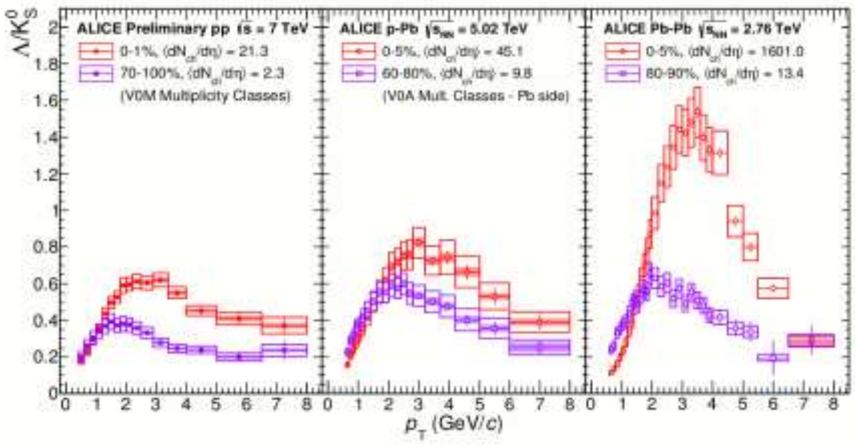


Small systems

Traditional view:

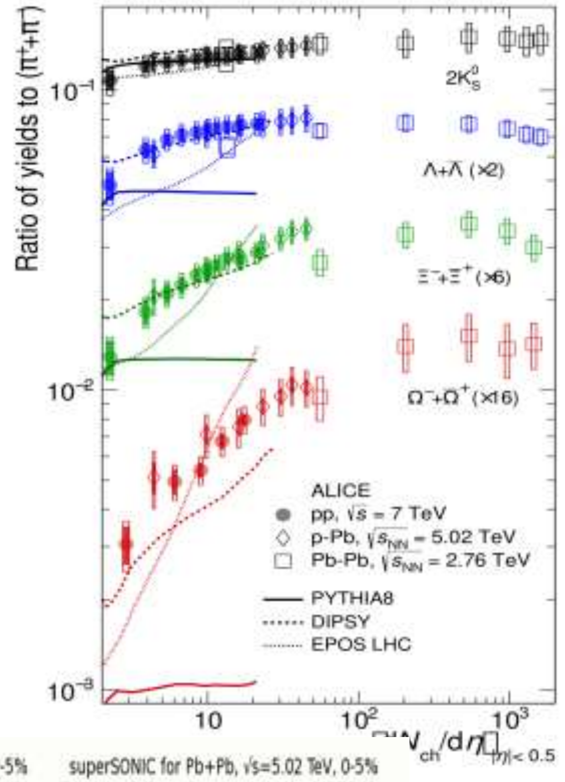
- **QGP in Pb+Pb**
- **no QGP in p+p (“baseline”)**





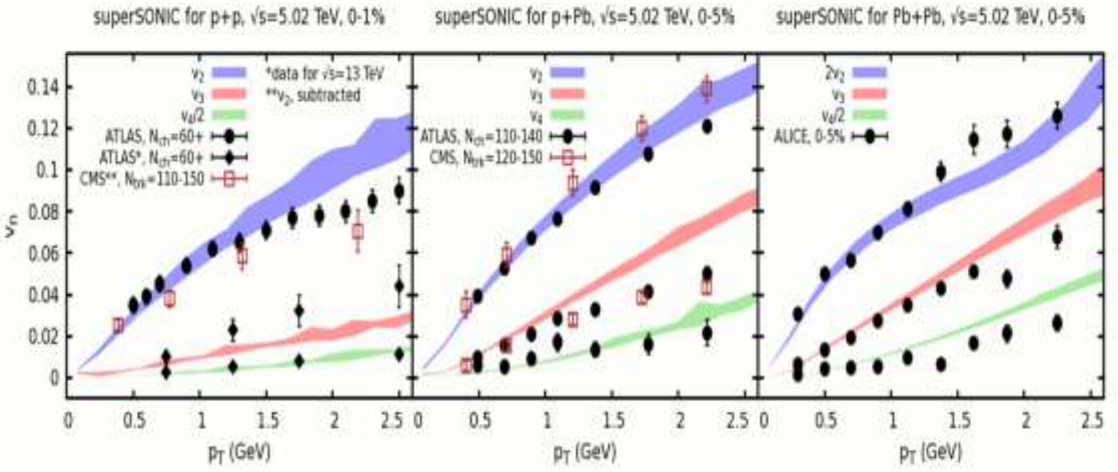
ALICE Coll., PRL 111 (2013) 222301
ALICE Coll., J. Phys.: Conf. Ser. 509 (2014) 012091
ALICE Coll. NPA 956 (2016) 777-780.

ALICE coll. Nature Phys. 13 (2017) 535



Objections to applying hydro in pp

- **Too few particles, cannot be collective**
- **System not in equilibrium**



R. D. Weller, P. Romatschke Phys.Lett. B774 (2017) 351-356

Small systems

Traditional view:

- **QGP in Pb+Pb**
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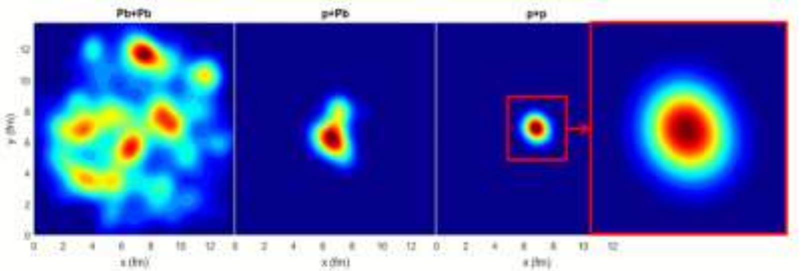


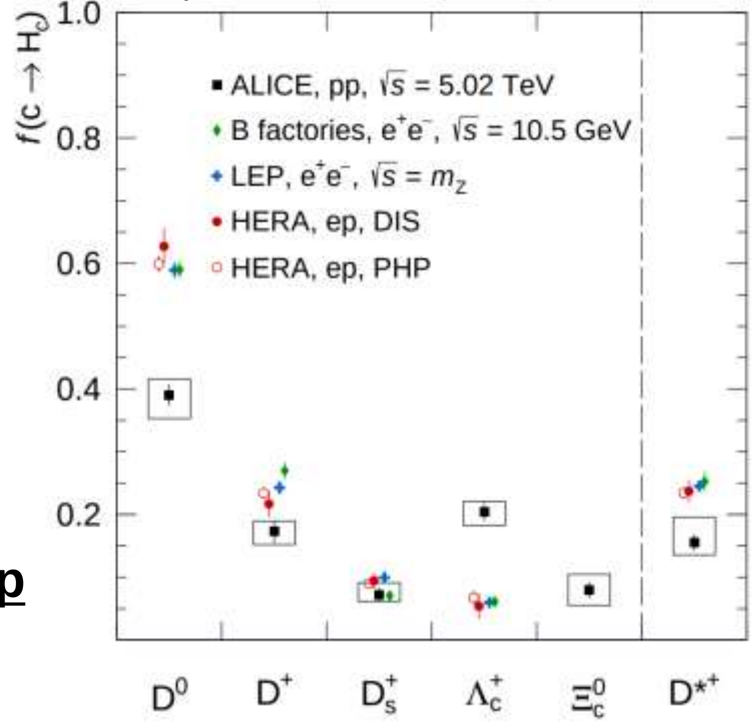
<u>Objections to applying hydro in pp</u>

- **Too few particles, cannot be collective**
- **System not in equilibrium**

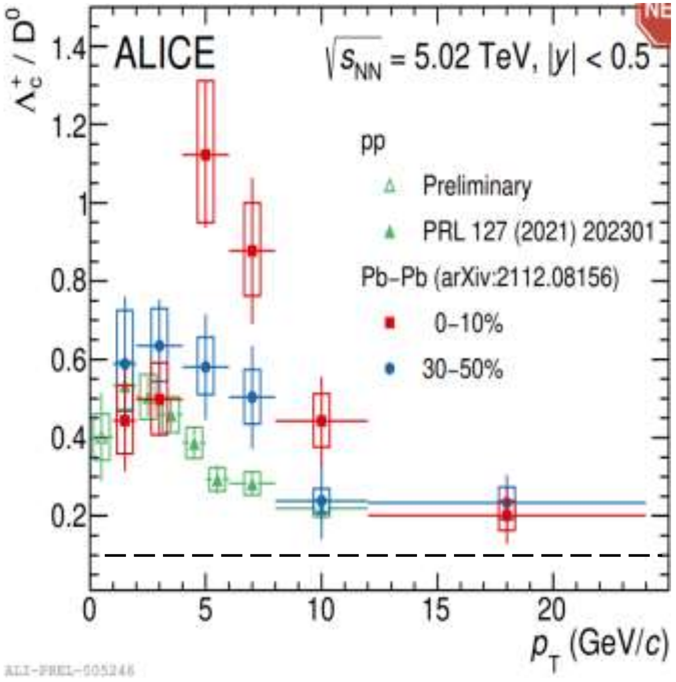
Fragmentation: production from hard-scattering processes (PDF+pQCD).
Fragmentation functions: data parametrization, assumed “universal”

Things get more complicated after experimental evidence with ALICE in pp@5TeV:

ALICE,Phys.Rev.D 105 (2022) 1, L011103



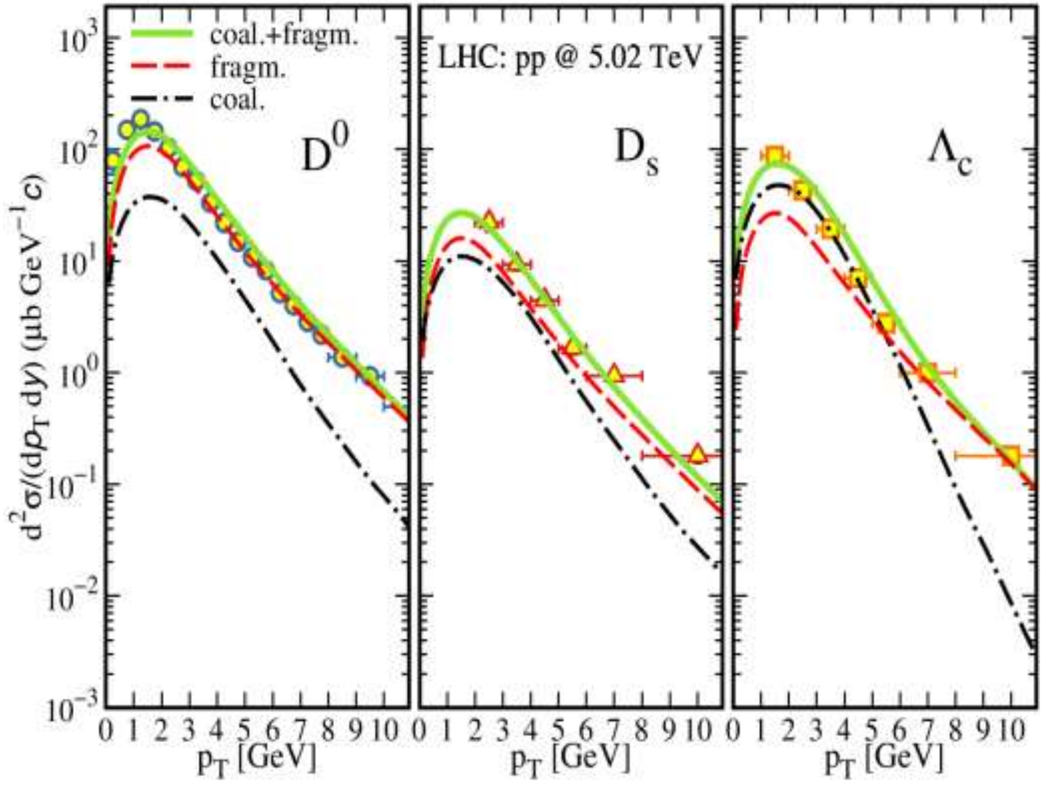
ALICE, PRL 127 202301 (2021)
ALICE, PRC 104 054905 (2021)



- Indication that fragmentation depends on the collision system
- Assumption of their universality not supported by the measured cross sections

Small systems: Coalescence in pp?

Data from: ALICE coll. EPJ C79 (2019) no.5, 388
ALICE coll. Meninno Hard Probes 2018



V. Minissale et al., *Phys.Lett.B* 821 (2021) 136622

- Thermal Distribution (p_T< 2 GeV)

$$\frac{dN_q}{d^2r_T d^2p_T} = \frac{g_g \tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right)$$

- Collective flow $\beta_T = \beta_0 \frac{r}{R}$
- Fireball radius+radial flow constraints dN_{ch}/dy and dE_T/dy
- Minijet Distribution (p_T> 2 GeV)
- NO QUENCHING

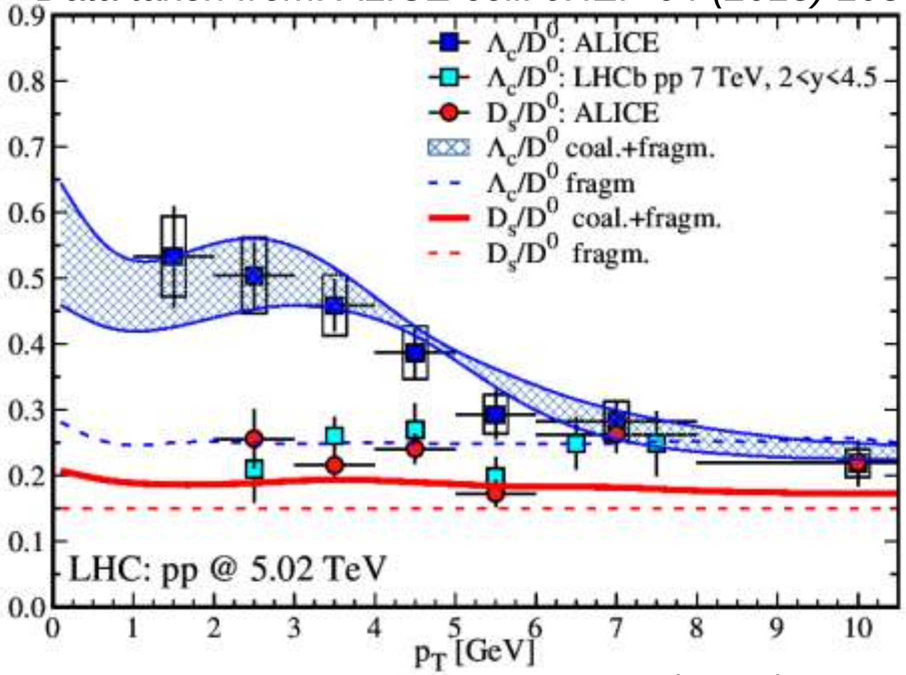
p+p @ 5 TeV

- t_{pp}=1.7 fm/c
- β_0=0.4
- R=2.5 fm
- V~30 fm^3

wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!

Small systems: Coalescence in pp?

Data taken from: ALICE coll. JHEP 04 (2018) 108



V. Minissale et al., *Phys.Lett.B* 821 (2021) 136622

Error band correspond to $<r^2>$ uncertainty in quark model

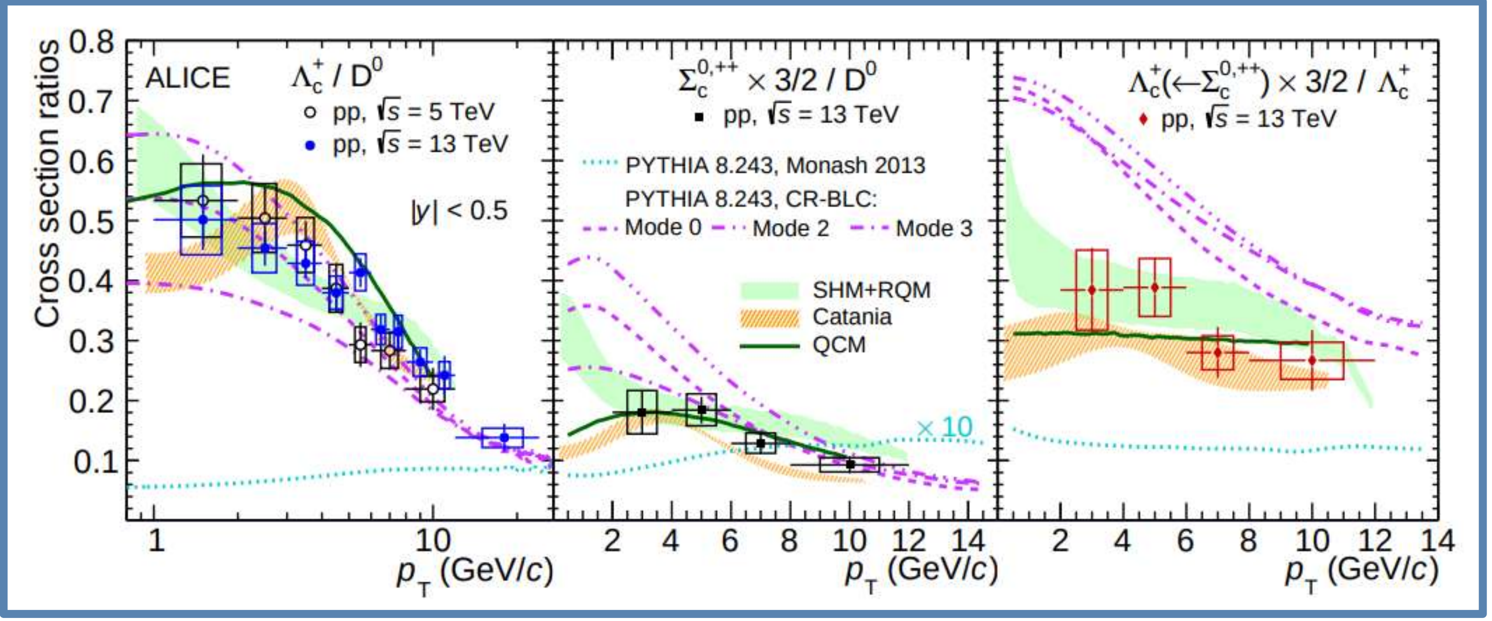
Reduction of rise-and-fall behaviour in Λ_c / D^0 ratio:

-Confronting with AA: Coal. contribution smaller w.r.t. Fragm.

-FONLL distribution flatter w/o evolution trough QGP

-Volume size effect

ALICE Coll., Physical Review Letters 128, 012001 (2022)



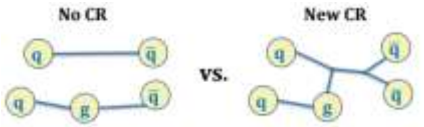
Other models:

He-Rapp, Phys.Lett.B 795 (2019) 117-121:

Increase≈2 to Λ_c production: SHM with resonance not present in PDG

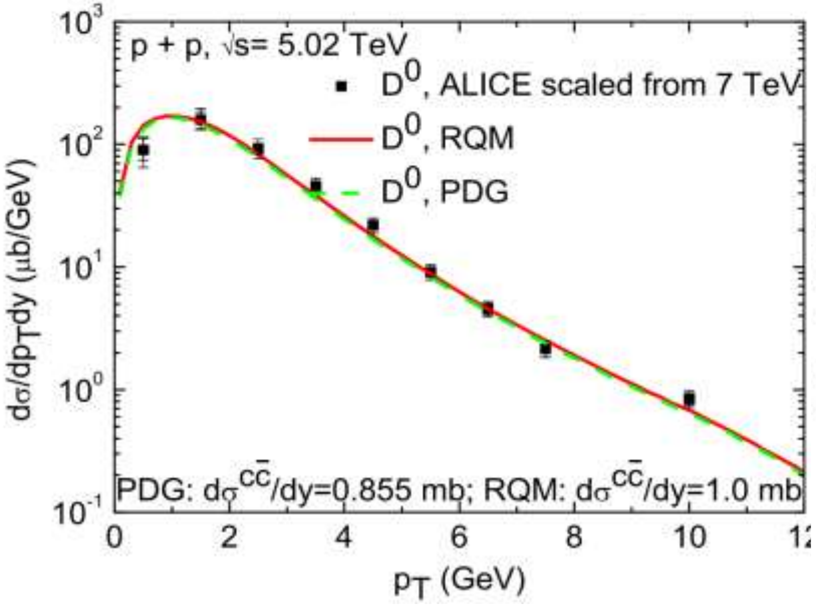
PYTHIA8 + color reconnection

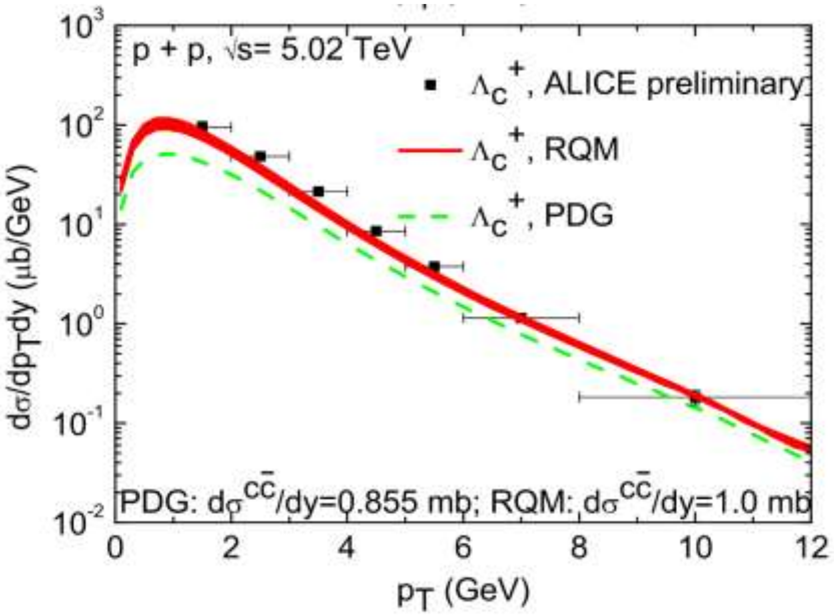
CR with SU(3) weights and string length minimization



Small systems: Coalescence in pp?

He-Rapp, Phys.Lett.B 795 (2019) 117-121





Thermal yields to compute the charmed hadron-chemistry

Transverse-momentum spectra calculated with fragmentation of c-quark spectrum from FONLL

Statistical hadronization for charm hadrons:

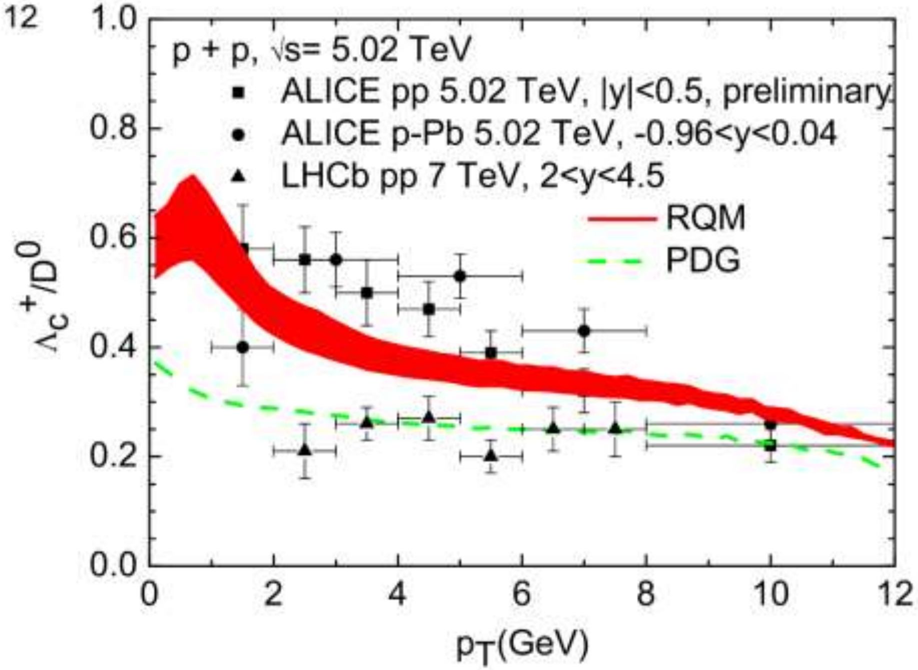
- chemical equilibrium with different charm-hadron species

$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right)$$

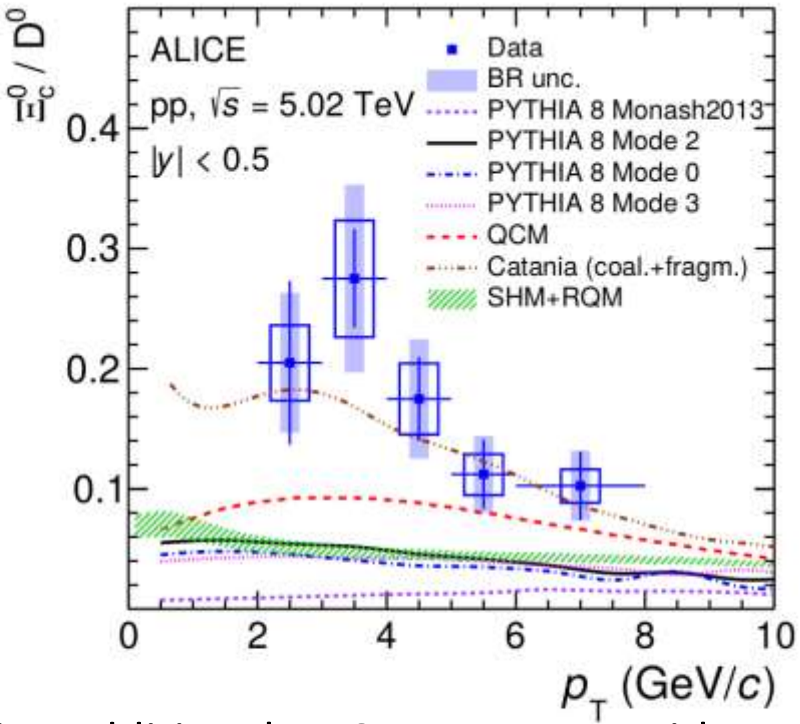
-Increased set of baryons for the Λ_c production:

PDG: $5\Lambda_c$, $3\Sigma_c$, $8\ \Xi_c$, $2\ \Omega_c$

RQM: $18\ \Lambda_c$, $42\ \Sigma_c$, $62\ \Xi_c$, $34\ \Omega_c$



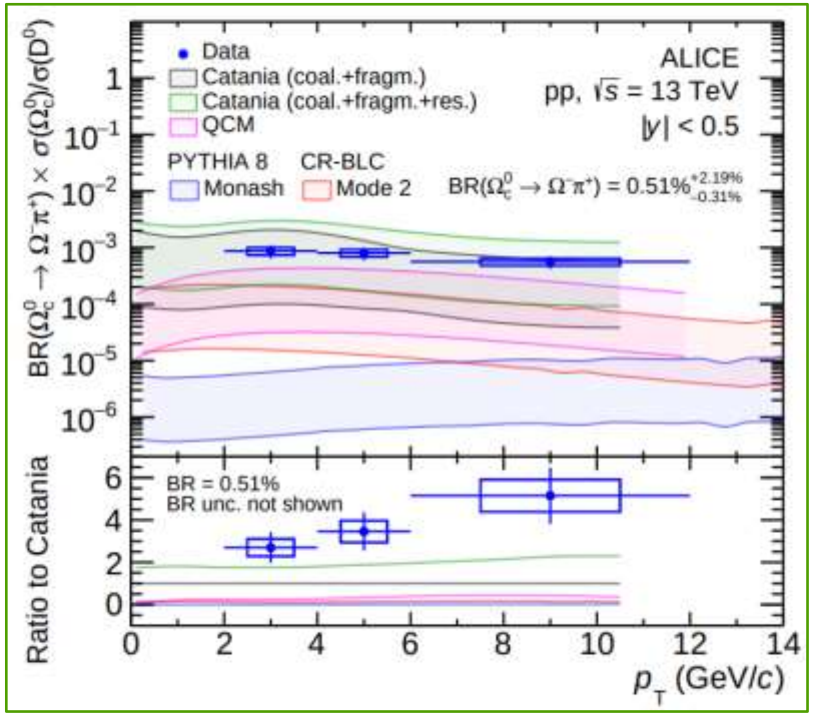
Small systems: Coalescence in pp?



New measurements of heavy hadrons at ALICE:

- Ξ_c/D^0 ratio, same order of Λ_c/D^0: coalescence gives enhancement

- very large Ω_c/D^0 ratio



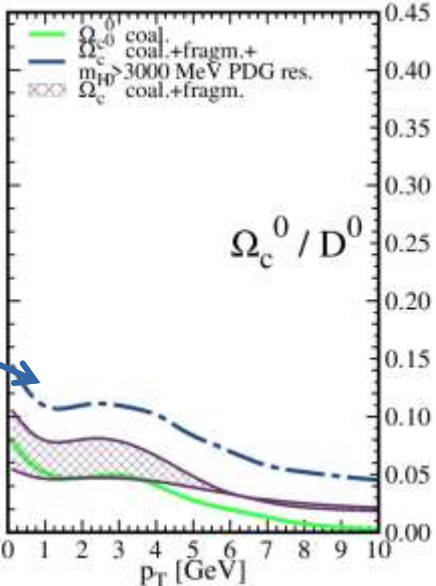
Assuming additional PDG resonances with J=3/2 and decay to Ω_c additional to $\Omega_c^0(2770)$

$\Omega_c^0(3000), \Omega_c^0(3005), \Omega_c^0(3065), \Omega_c^0(3090), \Omega_c^0(3120)$

supply an idea of how these states may affect the ratio

E. Santopinto et. al, EPJC 79 (2019) 12, 1012

Error band correspond to <r^2> uncertainty in quark model



ALICE Coll. JHEP 10 (2021) 159

ALICE Coll. arXiv:2205.13993

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622

Multi-charm in PbPb - KrKr – ArAr -OO

Multi-charm production in PbPb, KrKr, ArAr, OO

Baryon			
$\Xi_{cc}^{+,++} = dcc, ucc$	3621	$\frac{1}{2}\,(\frac{1}{2})$	
$\Omega_{scc}^{+} = scc$	3679	$0\,(\frac{1}{2})$	
$\Omega_{ccc}^{++} = ccc$	4761	$0\,(\frac{3}{2})$	
Resonances			
Ξ_{cc}^{*}	3648	$\frac{1}{2}\,(\frac{3}{2})$	$1.71 \times g.s$
Ω_{scc}^{*}	3765	$0\,(\frac{3}{2})$	$1.23 \times g.s$

like S.Cho and S.H. Lee, PRC101 (2020)
from R.A. Briceno et al., PRD 86(2012)

Strengths of the approach:

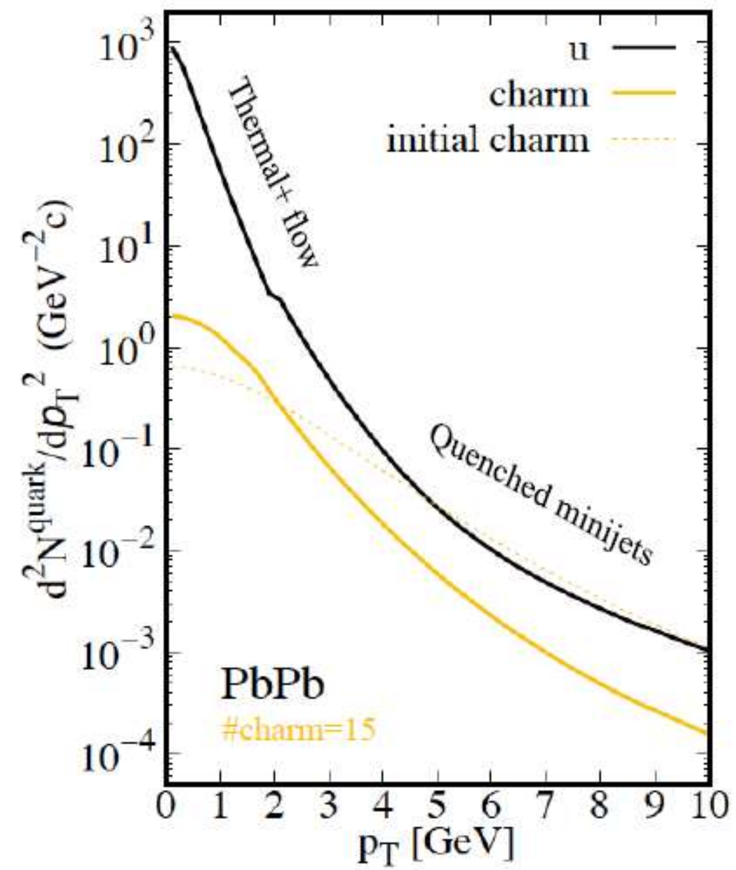
- Does not rely on distribution in equilibrium for charm
  → useful for small AA down to pp collisions and at p_T> 3-4 GeV

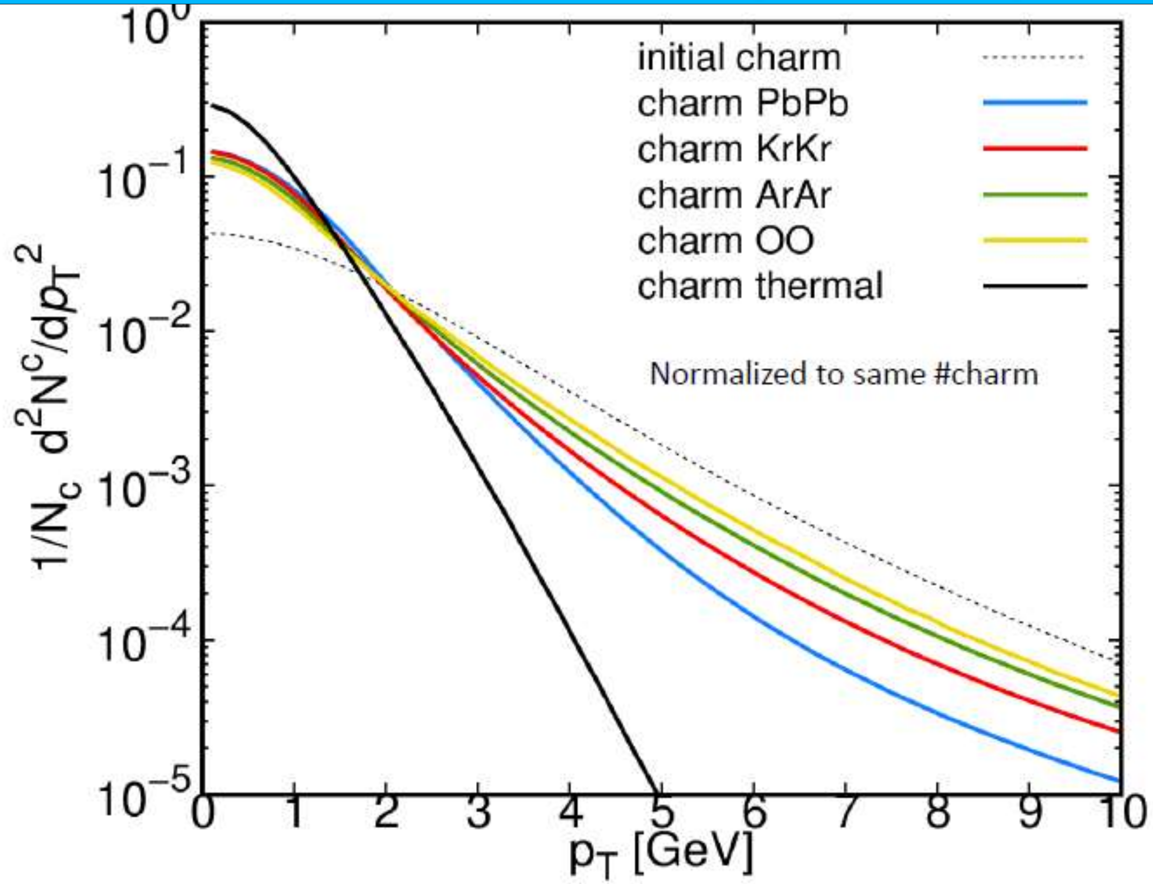
- Provide a p_T dependence of spectra and their ratios vs p_T

Widths from harmonic oscillator rescaling

	Ξ_c	Ω_c	$\Xi_{cc}^{(scal.\omega)}$	$\Omega_{ccc}^{(scal.\omega)}$
$\sigma_{p_1}(GeV)$	0.262	0.345	0.317	0.668
$\sigma_{p_2}(GeV)$	0.438	0.557	0.573	0.771
$\sigma_{r_1}(fm)$	0.751	0.572	0.622	0.295
$\sigma_{r_2}(fm)$	0.450	0.354	0.344	0.256
$\langle r^2\rangle_{ch}(fm^2)$	0.2	-0.12	0.363	0.09
$\langle r^2\rangle(fm^2)$	0.745	0.428	0.545	0.13
ω	$1.03e-2$	$1.5e-2$	$1.03e-2$	$1.5e-2$

Multi-charm production in PbPb, KrKr, ArAr, OO





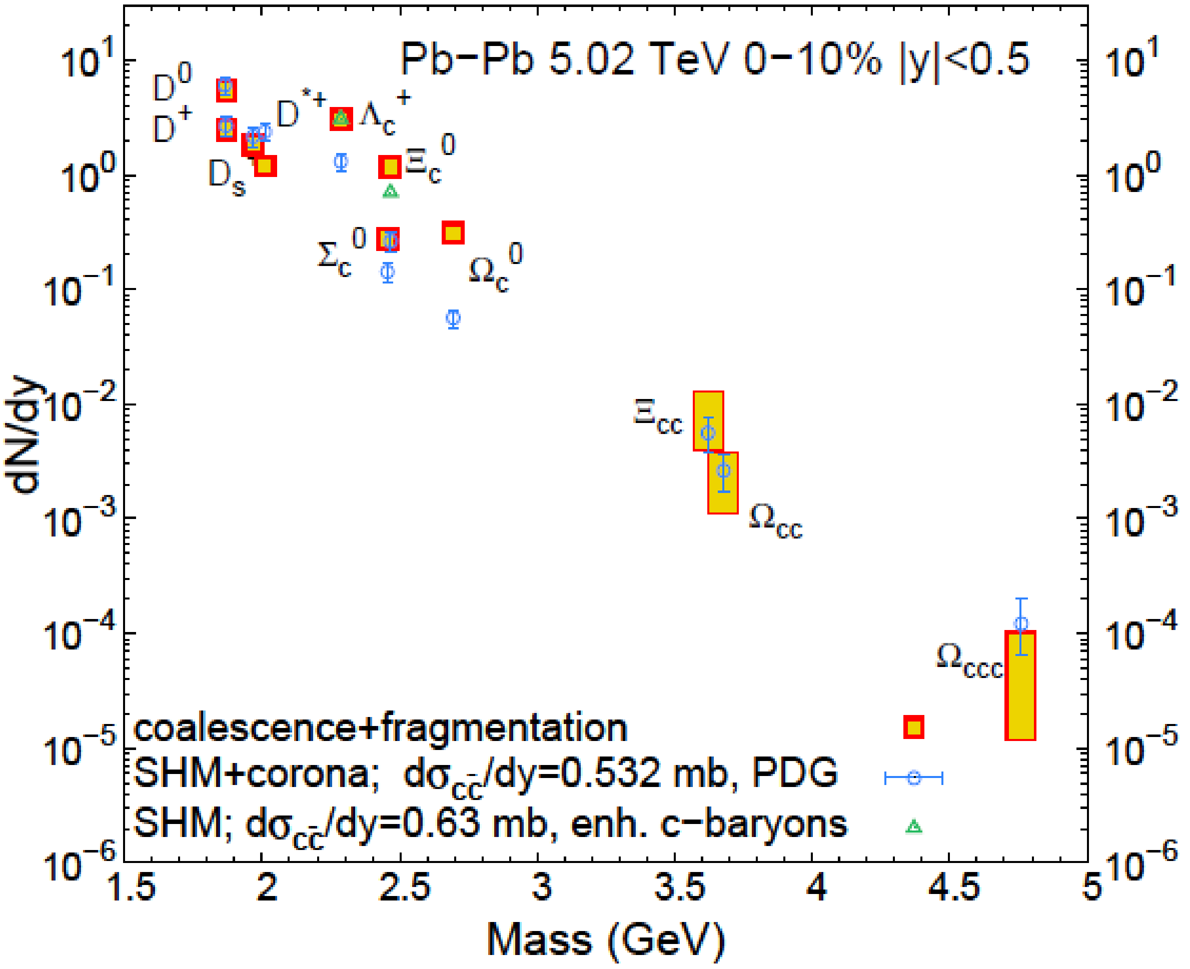
	OO	ArAr	KrKr	PbPb
$R_0(fm)$	2.76	3.75	4.9	6.5
$R_{max}(fm)$	5.2	7.65	10.1	14.1
$\tau(fm)$	4	5	6.2	8
β_{max}	0.55	0.6	0.64	0.7
$V_{\|y\|<0.5}(fm^3)$	345	920	2000	5000

Volume scales with A, now we employ the same value of SHM
A. Andronic et al., JHEP (2021) 035

#charm= 15 (PbPb), 4.35 (KrKr), 1.5(ArAr), 0.4(OO)

Yelds in PbPb: coalescence vs SHM

V. Minissale, S. Plumari, Y. Sun and V. Greco, arXiv:2305.03687.



$\Sigma_c^0, \Xi_c^0, \Omega_c^0$, widths from quark model

Ξ_{cc}, Ω_{cc} widths obtained rescaling with harm. oscillator

$$\sigma_{ri} = \frac{1}{\sqrt{\mu_i \omega}} \qquad \mu_1 = \frac{m_1 m_2}{m_1 + m_2}; \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

→ upper limit: charm thermal distribution

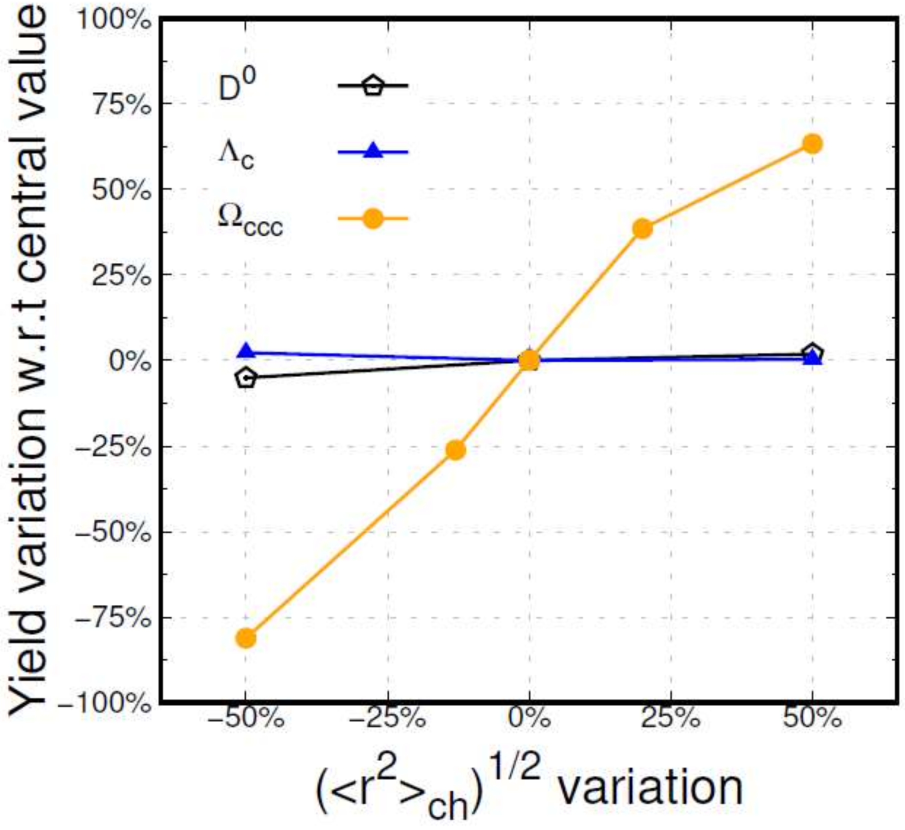
→ lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. (ω from Ω_c^0)

Yelds in PbPb: coalescence

V. Minissale, S. Plumari, Y. Sun and V. Greco, arXiv:2305.03687.

D^0 and Λ_c determine the yield, the radius variation is compensated by the constraint on the charm hadronization

A ± 50% in the radius of Ω_{ccc}
induces **a change in the yield by about 1 order of magnitude**



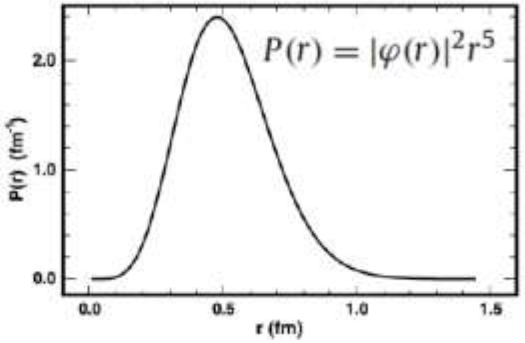
$$V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sum_{i<j}V_{cc}(\mathbf{r}_i,\mathbf{r}_j). \qquad V_{c\bar{c}}(\mathbf{r}_i,\mathbf{r}_j)=-\frac{\alpha}{|\mathbf{r}_{ij}|}+\sigma|\mathbf{r}_{ij}|,$$

Solve the 3-body problem by a 1-body in higher dimensions hyperspherical coordinates method

$$\left[\frac{1}{2m_c}\left(-\frac{d^2}{dr^2}-\frac{5}{r}\frac{d}{dr}\right)+v(r)\right]\varphi(r)=E\varphi(r)$$

$$W(\mathbf{r},\mathbf{p})=\int d^6\mathbf{y}e^{-i\mathbf{p}\cdot\mathbf{y}}\psi\left(\mathbf{r}+\frac{\mathbf{y}}{2}\right)\psi^*\left(\mathbf{r}-\frac{\mathbf{y}}{2}\right)$$

$$W(r,p,\theta)=\frac{1}{\pi^3}\int d^6\mathbf{y}e^{-ipy_1}\varphi(r_y^+)\varphi^*(r_y^-),$$

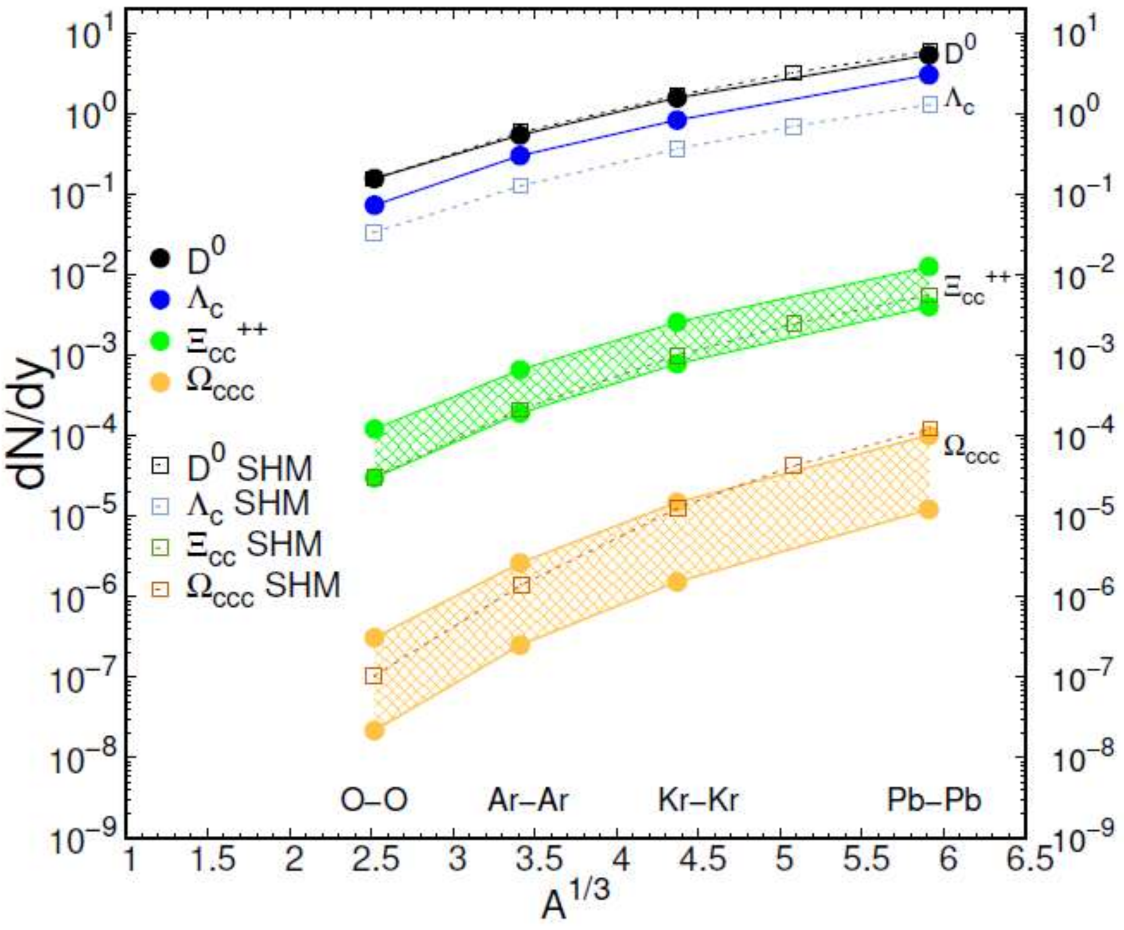


$$\frac{dN}{d^2\mathbf{P}_T d\eta}=C\int_\Sigma\frac{P^\mu d\sigma_\mu(R)}{(2\pi)^3}\int\frac{d^4r_x d^4r_y d^4p_x d^4p_y}{(2\pi)^6}\times F(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3,\tilde{p}_1,\tilde{p}_2,\tilde{p}_3)W(r_x,r_y,p_x,p_y),$$

Ω_{ccc} <r>=0.5 fm & $\sigma_r\cdot\sigma_p\approx 1.5$
similar to Tsinghua PLB746 (2015)

Yelds in PbPb: coalescence vs SHM

V. Minissale, S. Plumari, Y. Sun and V. Greco, arXiv:2305.03687.



$\Sigma_c^0, \Xi_c^0, \Omega_c^0$, widths from quark model

Ξ_{cc}, Ω_{cc} widths obtained rescaling with harm. oscillator

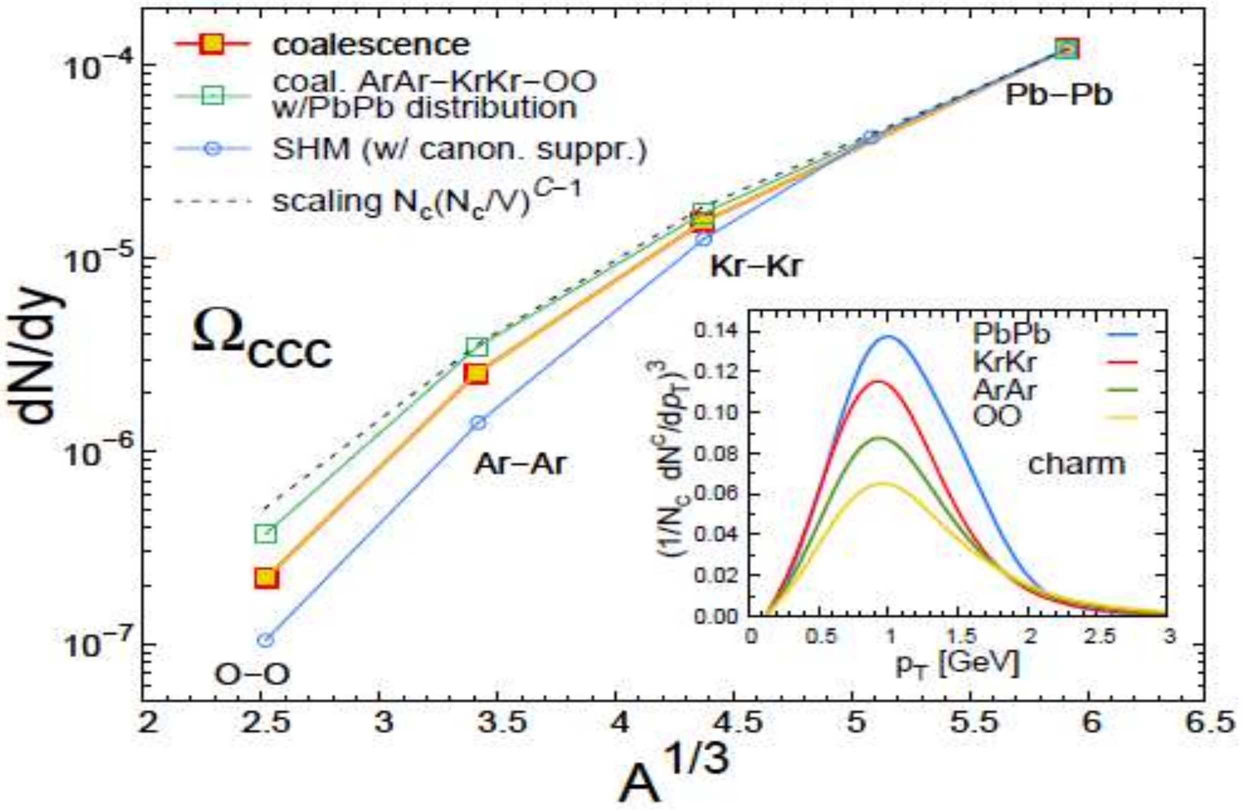
$$\sigma_{ri} = \frac{1}{\sqrt{\mu_i \omega}} \qquad \mu_1 = \frac{m_1 m_2}{m_1 + m_2}; \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

→ upper limit: charm thermal distribution

→ lower limit: PbPb distribution with widths rescaled as standard Harm. Oscill. (ω from Ω_c^0)

	D^0	Λ_c	$\Xi_{cc}^{+,++}$	Ω_{ccc}
OO	0.156	0.0732	$3-12.1\cdot10^{-5}$	$2.2-29.2\cdot10^{-8}$
$ArAr$	0.543	0.301	$1.9-6.6\cdot10^{-4}$	$2.5-26.3\cdot10^{-7}$
$KrKr$	1.564	0.835	$0.78-2.6\cdot10^{-3}$	$1.5-14.9\cdot10^{-6}$
$PbPb$	5.343	3.0123	$4-12.5\cdot10^{-3}$	$0.12-1.01\cdot10^{-4}$

Yelds scaling with A



Scaling of SHM (for A>40)

$$\frac{dN^{AA}}{dy}(h^i) = \frac{dN^{PbPb}}{dy}(h^i)\left(\frac{A}{208}\right)^{(\alpha+3)/3}\frac{f_{can}(\alpha, A)}{f_{can}(\alpha, Pb)}$$

For coalescence, in an homogeneous density background in equilibrium at fixed T, discarding flow and wave functions effects the expected scaling is:

$$V\left(\frac{N_c}{V}\right)^c = N_c\left(\frac{N_c}{V}\right)^{C-1}$$

with $N_c \propto A^{4/3}$ and $V \propto A$

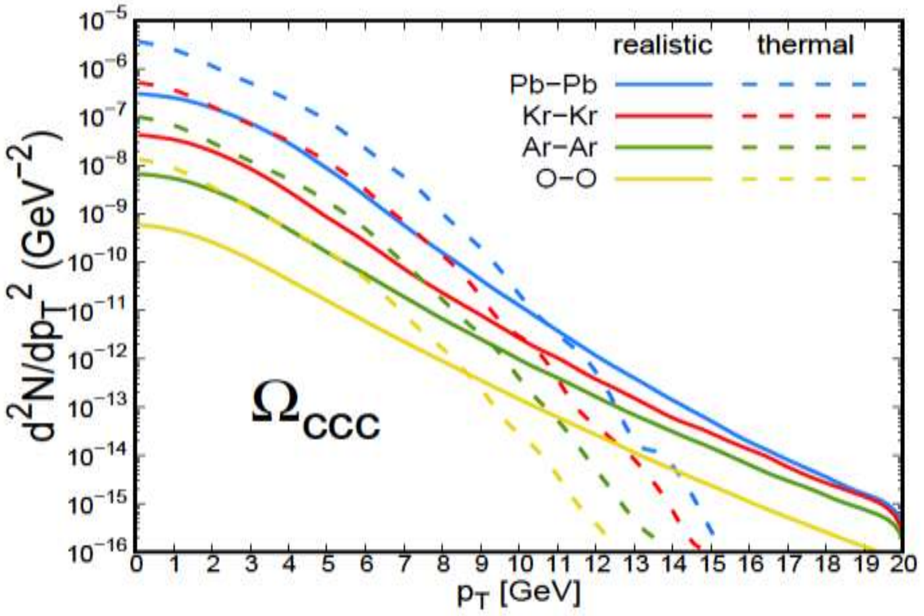
→ the scaling corresponds to $\frac{dN}{dy} \propto A^{\frac{C+3}{3}}$

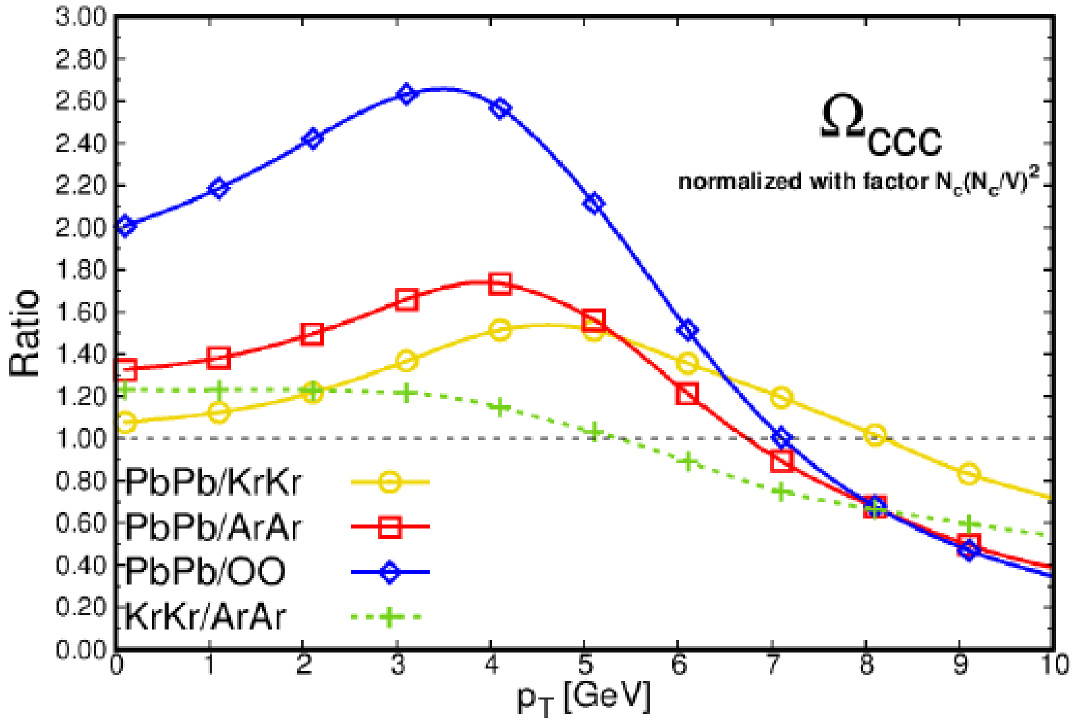
like in SHM w/o canonical suppression

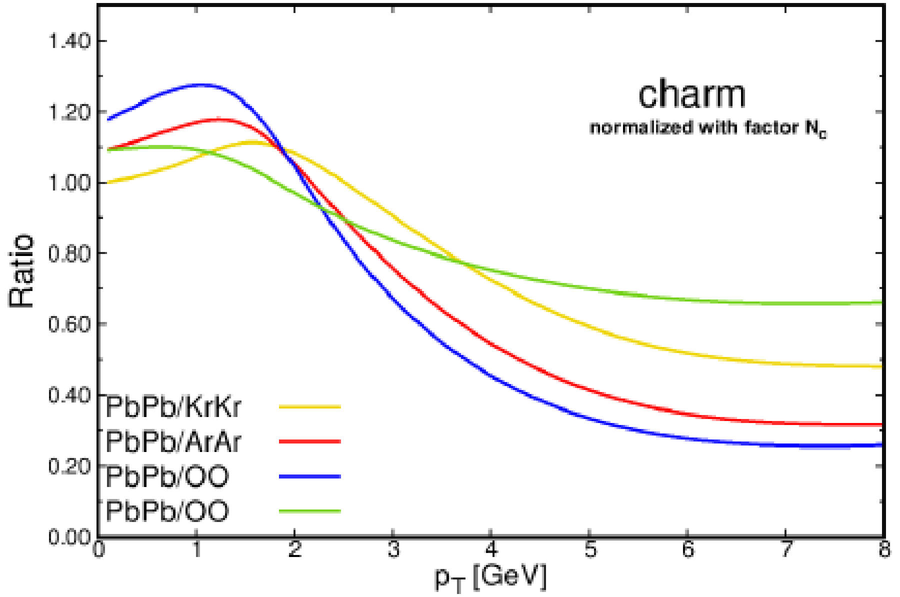
→ If the p_T-distribution does not change we obtain the scaling expected
→ There is an effect due to different charm distributions. In Ar-Ar it reduces Ω_{ccc} by ≈ 1.3 factor, in O-O it is ≈ 1.7
→ the cube of the distribution gives an idea of this difference, but Wigner function mitigate the effect

A larger production of coalescence w.r.t. SHM for small systems:
- Lack of canonical suppression, but e-b-e fluctuations can enhance production? $\langle N^3 \rangle > \langle N \rangle^3$

Ratios of pT distribution Ωccc in PbPb/KrKr/ArAr/OO







- It can be a meter of non-equilibrium. Translation of feature of charm spectra at low p_T into higher momentum region.
- More sensitive for multicharm respect to D mesons and Λ_c. Both effects of light quarks and fragmentation

V. Minissale, S. Plumari, Y. Sun and V. Greco, arXiv:2305.03687.

Conclusion

- **Charm hadronization in AA different than in e^+e^- and ep collisions**
  -Coalescence+fragmentation/Resonance Recombination Model enhancement of Λ_c production at intermediate p_T -> Λ_c/D^0 ~1 for p_T ~ 3 GeV
  -SHM with charm provide information on charm quark thermalization at low p_T

- ***In p+p assuming a medium:***
  - Coal.+fragm. good description of heavy baryon/meson ratio
    (closer to the data for Λ_c/D^0 , Ξ_c/D^0, Ω_c/D^0)
  - SHM+fragmentation able to capture the Λ_c production

- The yield of multi-charm decreases slowly with A in a coalescence approach
  - role of non-equilibrium distribution function

Heavy flavour (charm): Resonance decay

In our calculations we take into account main hadronic channels, including the ground states and the first excited states for D and Λ_c

Statistical factor

$$\frac{[(2J+1)(2I+1)]_{H*}}{[(2J+1)(2I+1)]_{H}}\left(\frac{m_{H*}}{m_H}\right)^{3/2} e^{-(E_{H*}-E_H)/T}$$

MESONS

D^+ *(I=1/2,J=0)*

D^0 *(I=1/2,J=0)*

D_s^+ *(I=0,J=0)*

BARYONS

Λ_c^+ *(I=0, J=1/2)*

Resonances

D^{*+} *(I=1/2,J=1)*	⟶	**$D^0\,\pi^+$**	*B.R. 68%*
		D^+ X	*B.R. 32%*
D^{*0} *(I=1/2,J=1)*	⟶	**$D^0\,\pi^0$**	*B.R. 62%*
		$D^0\,\gamma$	*B.R. 38%*
D_s^{*+} *(I=0,J=1)*	⟶	**D_s^+ X**	*B.R. 100%*
D_{s0}^{*+} *(I=0,J=0)*	⟶	**D_s^+ X**	*B.R. 100%*

Resonances

Λ_c^+(2595) *(I=0,J=1/2)*	⟶	**Λ_c^+**	*B.R. 100%*
Λ_c^+(2625) *(I=0,J=3/2)*	⟶	**Λ_c^+**	*B.R. 100%*
Σ_c^+(2455) *(I=1,J=1/2)*	⟶	**$\Lambda_c^+\pi$**	*B.R. 100%*
Σ_c^+(2520) *(I=1,J=3/2)*	⟶	**$\Lambda_c^+\pi$**	*B.R. 100%*