Strangeness fluctuations in heavy-ion collisions

Marcus Bluhm

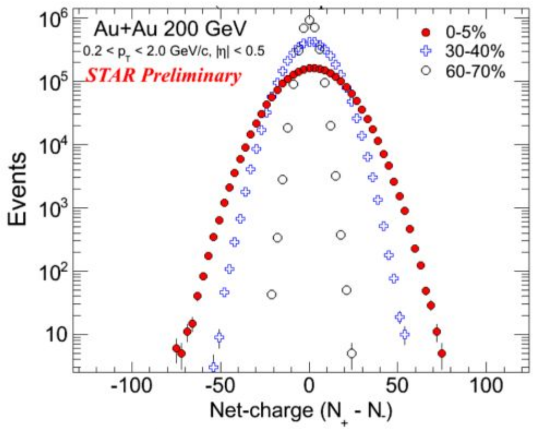
with Marlene Nahrgang, Grégoire Pihan, Taklit Sami and Masakiyo Kitazawa

see also 2111.13977 and 1806.04499

2nd Workshop of the Network NA7-HF-QGP, Giardini Naxos, Italy, 2 October 2023

Significance of fluctuation observables

$\Rightarrow$ experimentally multiplicity fluctuations determined from event-by-event distributions



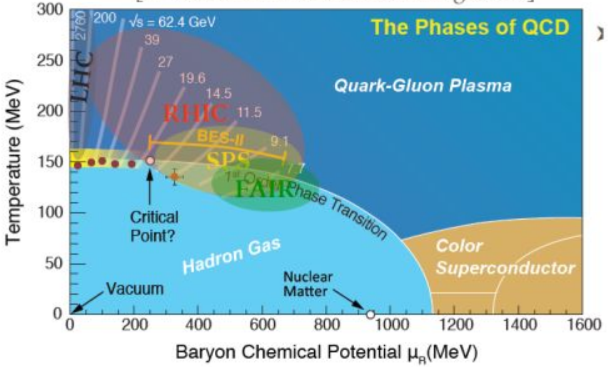
net-electric charge distributions from 1402.1558

- cumulant analysis:
  - mean: $M = \langle N \rangle = \chi_1 = C_1$
  - variance: $\sigma^2 = \langle (\Delta N)^2 \rangle = \chi_2 = C_2$
  - skewness S, kurtosis κ, ...
- net-distributions $N_+ - N_-$ often studied!
- fluctuations in conserved charges of QCD (B, S, Q) sensitive to matter composition
- requires limited phase-space acceptance!

Significance of fluctuation observables

$\Rightarrow$ theoretically fluctuations related to susceptibilities

[B. Müller: BEST Col. Meeting 2016]



from 1906.00936

- allows us to study the phase-structure of QCD matter!
- for a given pressure P susceptibilities follow from

$$\chi_n = VT^3 \frac{\partial^n (P/T^4)}{\partial(\mu/T)^n}$$

- volume cancels in ratios

Connecting theory with experimental data

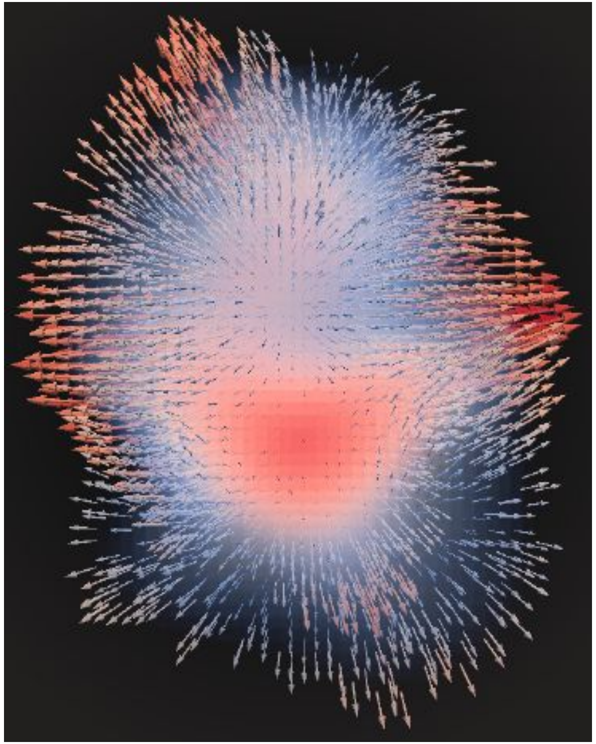
Medium created in a heavy-ion collision is

- extremely short-lived
- spatially extremely small
- inhomogeneous
- highly dynamical

yet still the success of statistical models and fluid dynamical simulations points toward equilibrated matter (at least locally)!

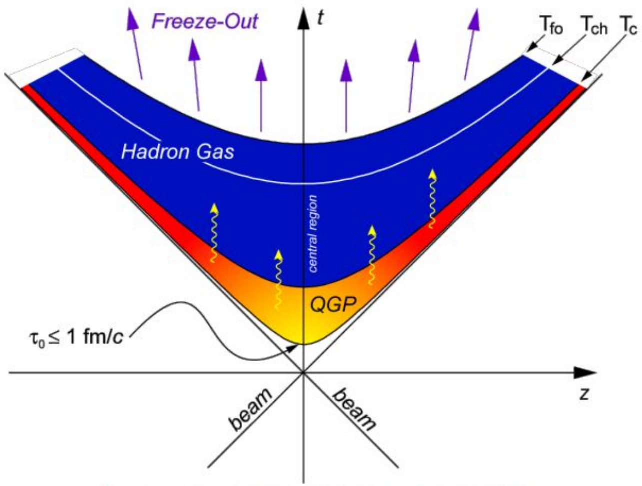
→ Originate fluctuations from an equilibrated source?

2 approaches: statistical and dynamical modeling...



madai.us

Significance of the chemical freeze-out



temporal evolution of a heavy-ion collision

simplified picture:

- after QGP phase and confinement transition, phase with hadronic interactions
- chemical freeze-out when inelastic scatterings stop $\rightarrow$ hadro-chemistry fixed!
- kinetic freeze-out when elastic scatterings cease $\rightarrow$ spectra fixed!

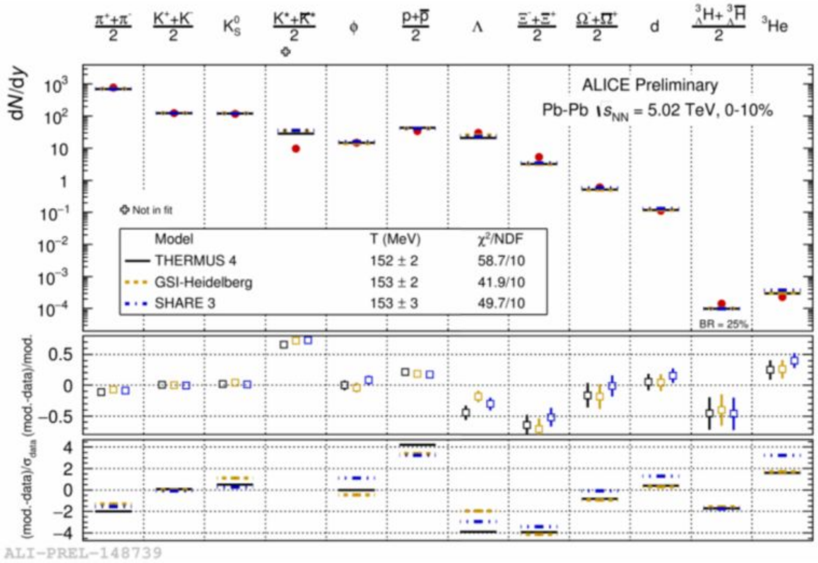
$\Rightarrow$ methods to determine chemical freeze-out conditions:

- traditionally via yields (multiplicities) and yield ratios
- alternatively via event-by-event multiplicity fluctuations

Chemical freeze-out conditions via yields

$\Rightarrow$ determine thermal conditions (T, μ_X) from yield ratios, volume V additionally needed for yields, via statistical hadronization model (SHM) fits

example from ALICE:



from 1808.05823

Chemical freeze-out conditions via yields

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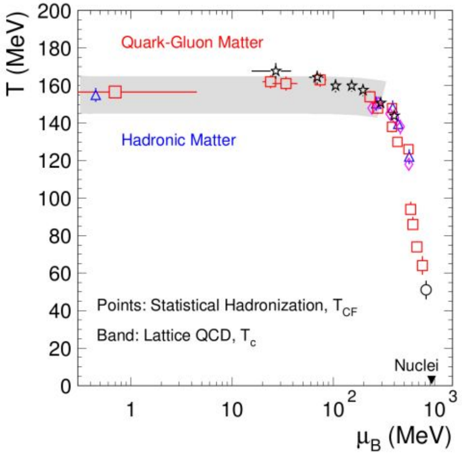
world-data analysis:



from 1710.09425

- analysis at various beam energies $\sqrt{s_{NN}}$ allows us to draw a chemical freeze-out curve

- trend: decreasing $\sqrt{s_{NN}} \rightarrow$ increasing μ_B and decreasing T

Theoretical framework

⇒ employ a Hadron Resonance Gas (HRG) model in grand-canonical ensemble formulation

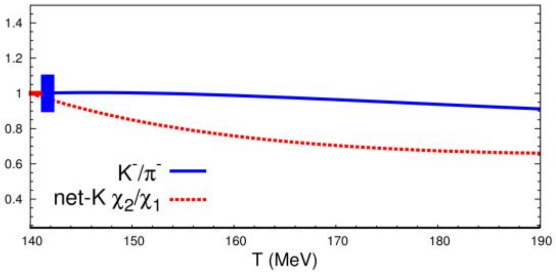
$$P = \sum_i (-1)^{B_i+1} \frac{d_i T}{(2\pi)^3} \int \mathrm{d}^3k \ln\left[1 + (-1)^{B_i+1} z_i e^{-\epsilon_i/T}\right]$$

- particle energy $\epsilon_i = \sqrt{k^2 + m_i^2}$, mass m_i, degeneracy d_i
- fugacity $z_i = e^{\mu_i/T}$ with particle chemical potential $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ and $X_i = B_i, S_i, Q_i$ quantum numbers
- can impose physical conditions met in experiments:
  - net-strangeness neutrality $\langle n_S \rangle = 0$ and initial isospin distribution $\langle n_Q \rangle = a\langle n_B \rangle$ (Au+Au and Pb+Pb at mid-rapidity $a \simeq 0.4$) with $n_X = \sum_i X_i(\partial P/\partial\mu_i)_T$
  - phase-space acceptance limitations in k_T, y and ϕ via $\int \mathrm{d}^3k$ and $\epsilon_i = \cosh(y)\sqrt{k_T^2 + m_i^2}$
- sum over all included PDG particles 319 confirmed (2012) or 738 species (2016)

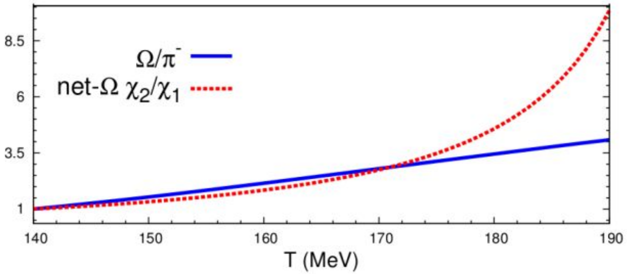
⇒ analyzed experimental data from STAR at RHIC!

Sensitivity - fluctuations vs. yields

⇒ higher-order cumulants of particle multiplicity distributions more sensitive to thermal conditions than particle yield ratios for certain final state hadrons 1504.03262



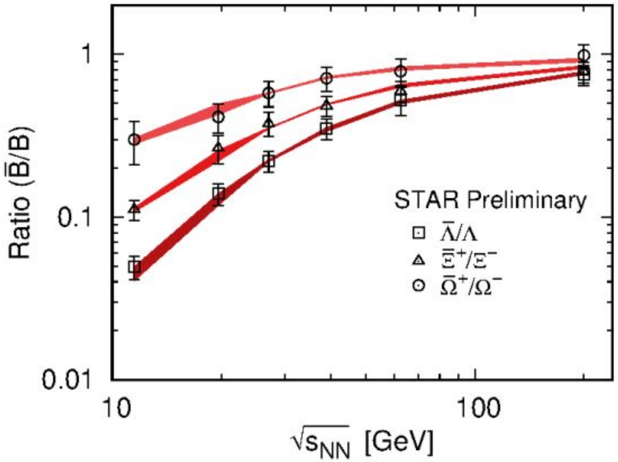
for $\mu_B = 0$



for $\mu_B = 0$

- net-kaon σ_K^2 / M_K sensitivity resolves experimentally achievable accuracy! $\rightarrow$ reliable determination of freeze-out conditions!
- motivates the use of net-kaon fluctuations!
- (anti-)hyperon sensitivity indifferent!

Data analysis - yield ratios

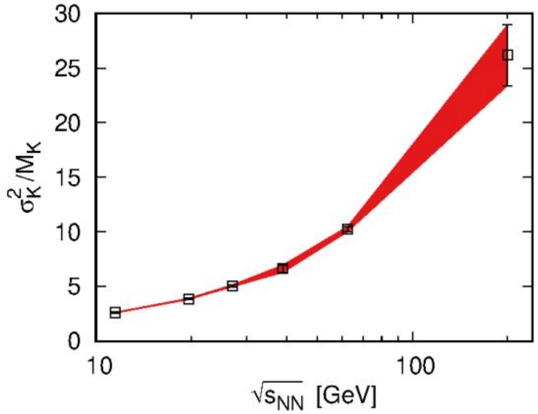


data from nucl-ex/0606014; 1010.0142; and preliminary data from CPOD 2013, 036 (2013)

- anti-hyperon–hyperon ratio $\bar{B}/B$ for Λ, Ξ^- and Ω^- from ϕ- and k_T-integrated yields for given Δy-window
- significant contributions from strong resonance decays
- for Λ electromagnetic decay contributions from Σ^0
- preliminary data on $\bar{B}/B$ confirmed recently 1906.03732

$\Rightarrow$ heavier (anti-)hyperons add some sensitivity to the determination of T, lighter (anti-)hyperons influence stronger μ_B

Data analysis - net-Kaon fluctuations



data from 1709.00773

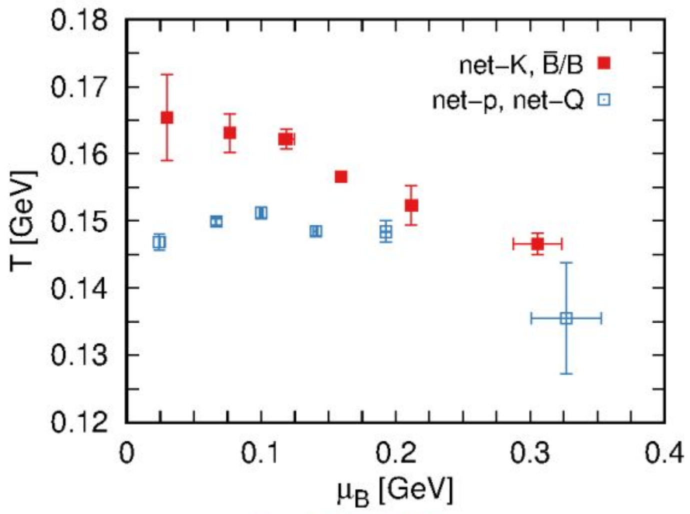
- lowest-order net-kaon fluctuations σ_K^2 / M_K
- data corrected for detector efficiency and centrality bin width
- systematic errors taken into account in the analysis
- acceptance limitations: $0.2 \leq k_T / (\mathrm{GeV/c}) \leq 1.6$ and $|y| \leq 0.5$ with full azimuthal coverage

⇒ significant correlations between K^+ and K^- arise from strong resonance decays

⇒ determination of chemical freeze-out temperature sensitively influenced by net-kaon fluctuation data

Chemical freeze-out conditions

$\Rightarrow$ conditions for T and μ_X determined from a combined analysis (optimal fits) of $\bar{B}/B$-ratios and net-kaon fluctuations σ_K^2/M_K



from 1806.04499

- T and μ_B from strangeness observables

  $\sqrt{s_{NN}}$/GeV= 200, 62.4, 39, 27, 19.6, 11.5 left to right
- error bars from bands in fits
- T and μ_B from net-p and net-Q fluctuations
- significant difference in T for large $\sqrt{s_{NN}}$, approaching each other with decreasing $\sqrt{s_{NN}}$

Chemical freeze-out conditions

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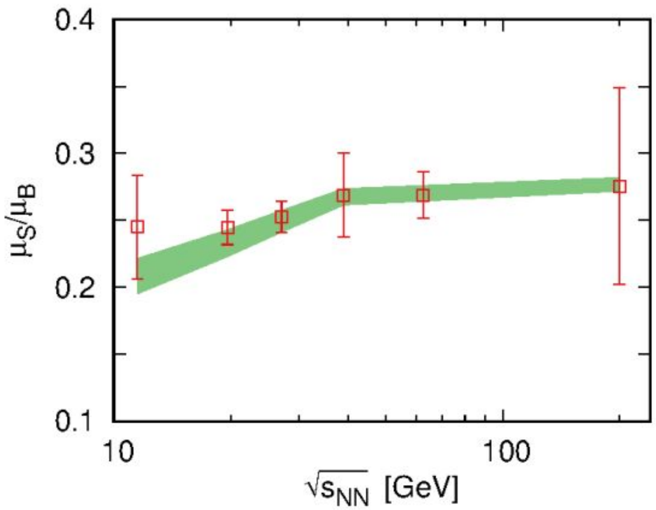


from 1806.04499

- $\mu_Q/\mu_B \lesssim 0$
- strangeness neutrality requires sizeable μ_S/μ_B
- μ_S/μ_B from strangeness observables
- μ_S/μ_B from lattice QCD (Taylor expansion)
- agreement shows impact of unconfirmed strange sector particles in HRG model

Limitations of the approach

- shown error bars base entirely on systematic errors in the analyzed data!
- theoretical uncertainties may stem from:

$\Rightarrow$ regeneration and subsequent decay of K^* resonances not taken into account - isospin randomization effect on net-kaon fluctuations

requires large pion density and long hadronic stage!

$\Rightarrow$ exact global charge conservation on an event-by-event basis not taken into account

makes canonical ensemble formulation necessary!

$\Rightarrow$ fluctuations in the number of participants for a given centrality class not taken into account

stronger for higher-order fluctuations and smaller beam energies!

$\Rightarrow$ resonance decays and elastic scatterings in and out of acceptance window not taken into account

earlier estimates indicate rather small impact!

→ Dynamical evolution of the system completely ignored!

Dynamical evolution of fluctuations

Temporal evolution of conserved charges happens via diffusion:

$$\partial_\tau n_X = \frac{\kappa}{\tau}\partial_y^2\left(\frac{\mu_X}{T}\right)$$

(in Milne coordinates)

strangeness does not evolve independently from other conserved charges X

➡ conserved charges are coupled!

for fluctuation studies include stochastic forcing with amplitude $\sim \kappa$

coupled stochastic diffusion equations:

$$\partial_\tau n_X(\tau, y) = \frac{\kappa_{XY}}{\tau}\partial_y^2\left(\frac{\mu_Y}{T}\right) - \partial_y \xi_X$$

(sum over repeated index Y)

noise-noise correlation function:

$$\langle \xi_X(\tau, y)\xi_Y(\tau', y')\rangle = 2\frac{\kappa_{XY}}{\tau}\delta(\tau-\tau')\delta(y-y')$$

➡ diffusion matrix κ_{XY}

Diffusion matrix and coupled evolution

M. Greif et al. 1711.08680 (see also J.A. Fotakis et al. 2102.08140)
considered a hadronic gas composed of the 19 lightest hadron species
➜ microscopic calculation of κ_{XY} as function of T and μ_B

➜ self-contained study: EoS of hadron gas with 19 lightest species

$$n_X = \frac{1}{2\pi^2} \sum_{\text{hadronic species } j} X_j d_j \int_0^\infty \frac{p^2 \mathrm{d}p}{(-1)^{B_j+1} + e^{(\sqrt{p^2+m_j^2} - B_j\mu_B - Q_j\mu_Q - S_j\mu_S)/T}}$$

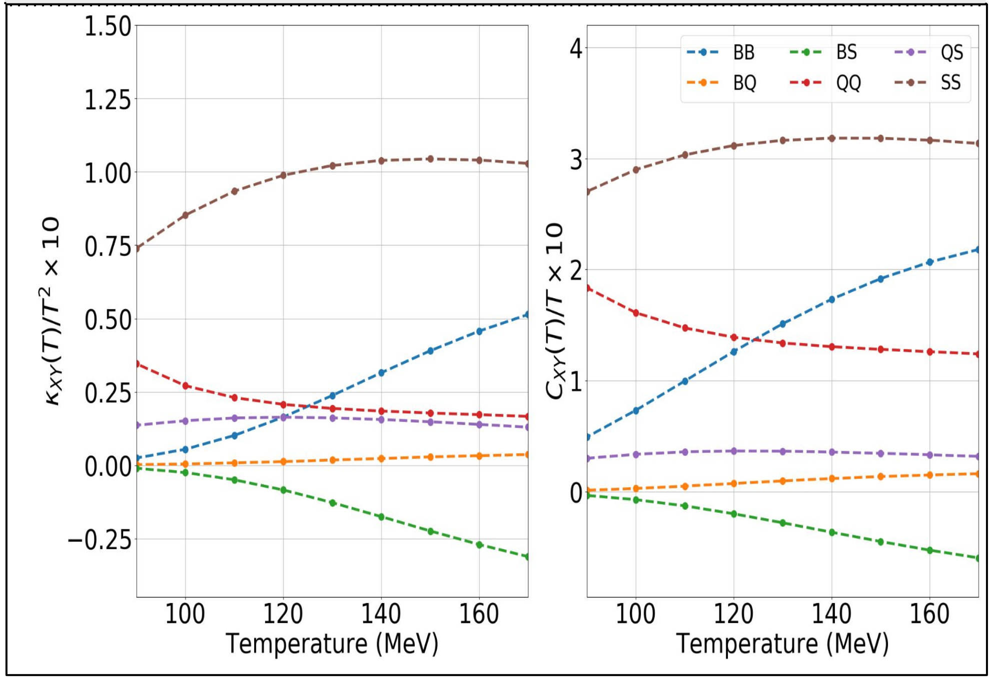
$\pi^{+,-,0}$	$K^{+,-,0}$	$\bar{K}^0$
$p\,\bar{p}$	$n\,\bar{n}$	$\Lambda_0\bar{\Lambda}_0$
$\Sigma_0\;\bar{\Sigma}_0$	$\Sigma^{+,-}$	$\bar{\Sigma}^{+,-}$

temporal evolution of net-strangeness number coupled to net-baryon number and electric charge non-linearly via:

- deterministic currents
- fluctuation dissipation balance
- EoS

Diffusion matrix and Cholesky decomposition

For LHC conditions: $\mu_B = \mu_Q = \mu_S = 0 \text{ MeV}$



noise-noise correlator $\sim \kappa$
➜ stochastic noise components

$$\xi_X \propto C_{XY} Z_Y$$

Cholesky decomposition

C is such that $CC^T = \kappa$ for semi-positive diffusion matrix

expectations: net-strangeness diffusion can best keep up with the expansion, net-strangeness and net-baryon number are anti-correlated, net-electric charge diffusion more effective at small T

Linear approximation to the coupled evolution

expansion of the net-densities to leading order in the chemical potentials:

$$n_X = \chi_{XY}\mu_Y + \mathcal{O}(\mu^2)$$

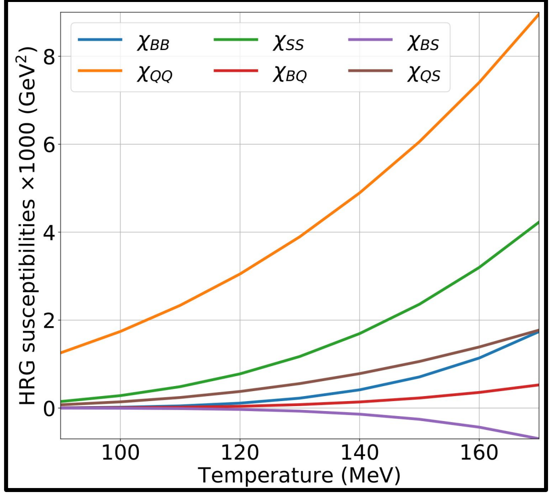
➡ results in coupled diffusion equations that are linear in the net-charge densities

$$\partial_\tau n_X = \frac{D_{XY}(T,\mu_B,\mu_Q,\mu_S)}{\tau}\partial_y^2 n_Y - \sqrt{\frac{2}{\tau}}C_{XY}(T,\mu_B,\mu_Q,\mu_S)\partial_y Z_Y$$

$$D_{XY} = \frac{1}{T}\kappa_{XZ}(\chi^{-1})_{ZY}$$

(huge computational advantage)

HRG susceptibilities for LHC conditions



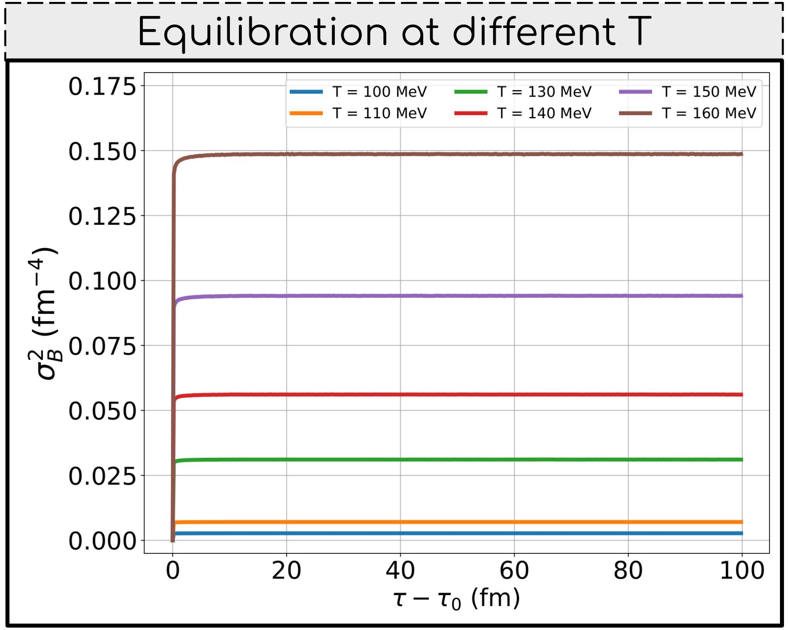
expectations: fluctuations decrease in equilibrium as a function of T, net-electric charge and net-strangeness fluctuations will be large

Decoupled n_B - dynamics: Equilibration

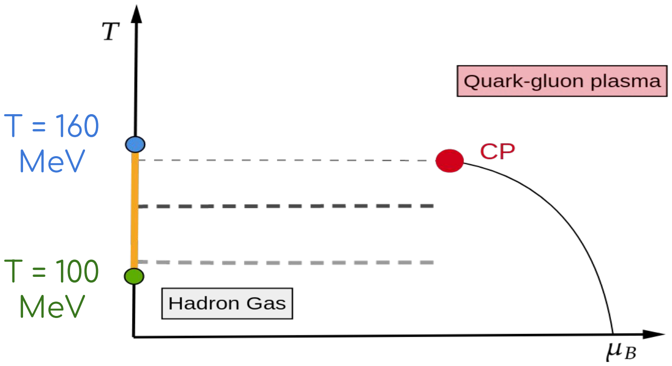
starting from arbitrary initial conditions in fixed size box at fixed T:

$$\kappa = \begin{pmatrix} \kappa_{BB} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- variance of the net-baryon density equilibrates rather quickly
- equilibrium values decrease with T



Decoupled n_B - dynamics: Expanding medium

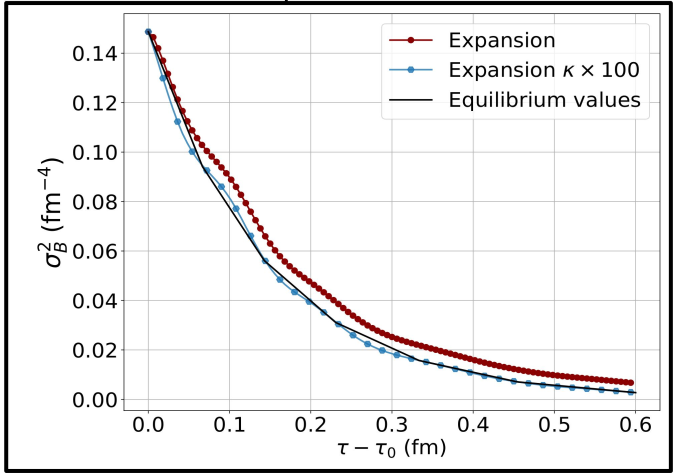


start from the equilibrated system at T=160 MeV and follow trajectory for LHC conditions:

➡ Bjorken expansion

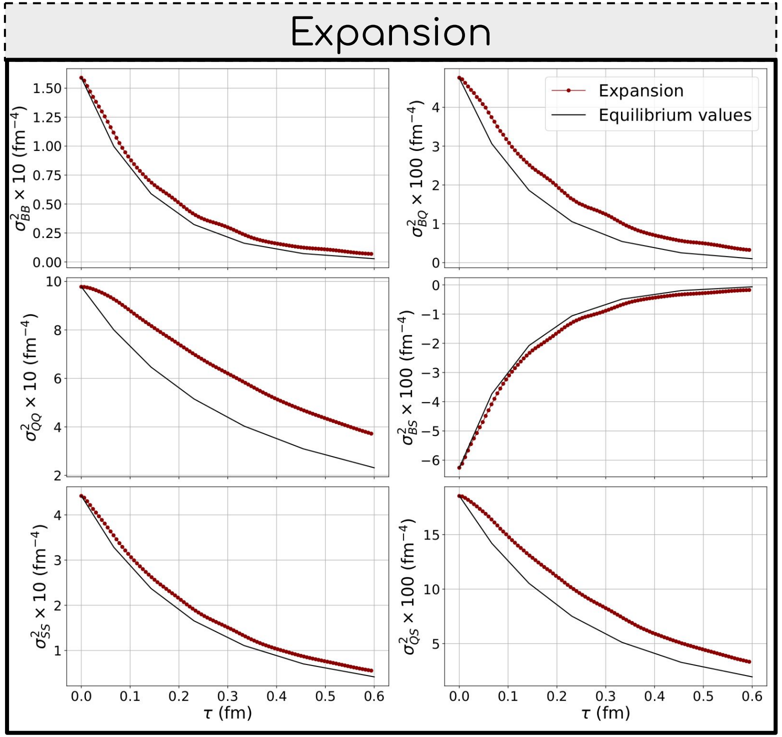
➡ Hubble-like temperature cooling

Expansion



- variance of the net-baryon density goes slightly out of equilibrium rather quickly and remains there
- 100 times stronger diffusion needed to maintain equilibrium

Coupled charge dynamics: Expanding medium



as before but now including charge coupling:

- coupling slightly increases equilibrium values for the variance
- net-baryon number and net-strangeness are anti-correlated (negative co-variance)
- net-electric charge and net-strangeness variances are largest (longest “lifetimes”)
- net-electric charge sector driven out of equilibrium quickly and strongly (maybe approaches equilibrium at later times)
- net-baryon number and net-strangeness remain closer to equilibrium

Attention: only local (co-)variances studied here ➔ electric charge related observables survive longer

Conclusions

- determination of chemical freeze-out conditions from strangeness observables at different beam energies within the HRG model
- in general fluctuations more sensitive than yields and/or their ratios
- freeze-out conditions obtained from strangeness fluctuations compatible with SHM fits to hadron yields
- coupled charge dynamics already impacts density profiles

  see J.A. Fotakis et al. 1912.09103, NPA 1005 (2021)

  but even more so the fluctuations
- with realistic diffusion coefficients difficult to maintain equilibrium even locally on the level of the (co-)variances
- but if source originally in equilibrium ➔ electric charge related sectors preferable as signals decay slower

Role of resonance decays

- net-kaon or net-proton numbers not conserved charges
- resonance decays can significantly modify final particle multiplicity distributions:
  $N_j = N_j^* + \sum_R N_R^* \langle n_j \rangle_R$ with N_j^* directly produced hadrons, $\langle n_j \rangle_R$ associated with branching ratios b_r^R
- event-by-event fluctuations arise from:
  - thermal fluctuations in N_j^* and N_R^*
  - probabilistic character of the decay process (b_r^R only means!)

explicit example for net-kaon number $M_K = M_{K^+} - M_{K^-}$:

$$\text{mean: } M_j = \langle N_j^* \rangle_T + \sum_R \langle N_R^* \rangle_T \langle n_j \rangle_R$$

$$\begin{aligned}\text{variance: } \sigma_K^2 = & \langle (\Delta N_{K^+}^*)^2 \rangle_T + \langle (\Delta N_{K^-}^*)^2 \rangle_T + \sum_R \langle (\Delta N_R^*)^2 \rangle_T \left(\langle n_{K^+} \rangle_R^2 + \langle n_{K^-} \rangle_R^2 \right) \\ & - 2 \sum_R \langle (\Delta N_R^*)^2 \rangle_T \langle n_{K^+} \rangle_R \langle n_{K^-} \rangle_R - 2 \sum_R \langle N_R^* \rangle_T \langle \Delta n_{K^+} \Delta n_{K^-} \rangle_R \\ & + \sum_R \langle N_R^* \rangle_T \left(\langle (\Delta n_{K^+})^2 \rangle_R + \langle (\Delta n_{K^-})^2 \rangle_R \right)\end{aligned}$$

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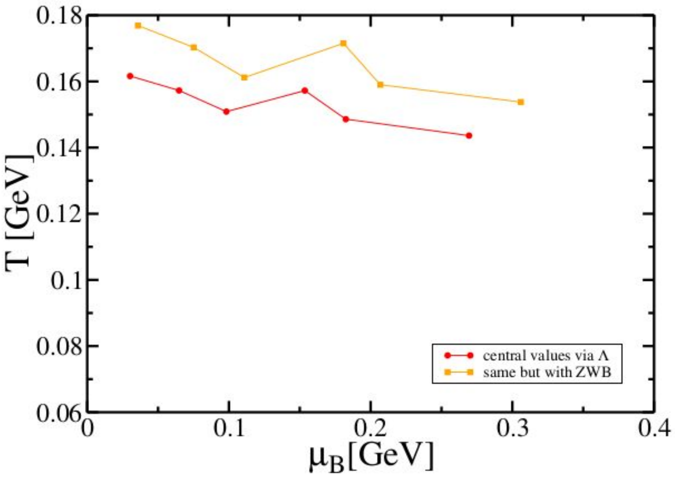
explicit example for net-kaon number $M_K = M_{K^+} - M_{K^-}$:

- resonance decays cause correlation terms
- approach allows for a proper inclusion of $N(1650)$ or $\Xi(1690)^-$
  (not just derivatives w.r.t. μ_S!)
- only thermal averages $\langle \cdot \rangle_T$ can be obtained from HRG model via derivatives, i.e. via χ_n

Improper inclusion of resonance decays

- probabilistic character of resonance decays (ZWB) has non-negligible effect on the results!



exemplary study with and without ZWB

- means unaffected, only σ_K^2 influenced
- combined analysis of $\bar{B}/B$ and σ_K^2/M_K data yields 5% reduction in T and up to 18% reduction in μ_B when ignoring probabilistic contributions!
- consequence of missing correlations between K^+ and K^-

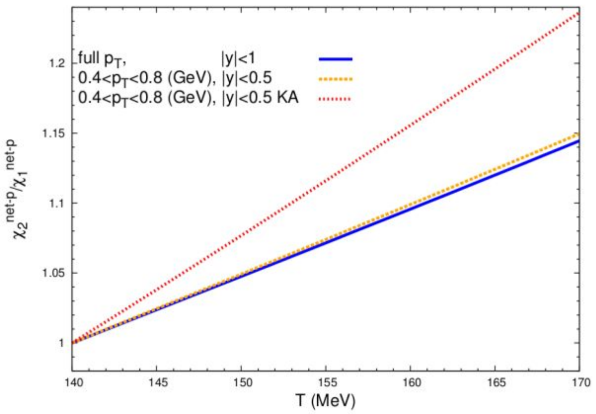
Impact of isospin randomization processes

reactions of the form

$$p(n) + \pi^0(\pi^+) \rightarrow \Delta^+ \rightarrow n(p) + \pi^+(\pi^0)$$

modify primordial protons into undetected neutrons

$\Rightarrow$ isospin of nucleons randomized after 2 cycles; depends on pion density and duration of hadronic phase compared to time for resonance regeneration plus decay



for $\mu_B = 0$ from 1504.03262

- KA-effect can be taken into account 1107.2755, 1205.3292
- can cause up to $5 - 10\%$ deviations in lowest-order fluctuations
- was essential for the determination of freeze-out conditions from net-p and net-Q fluctuations!

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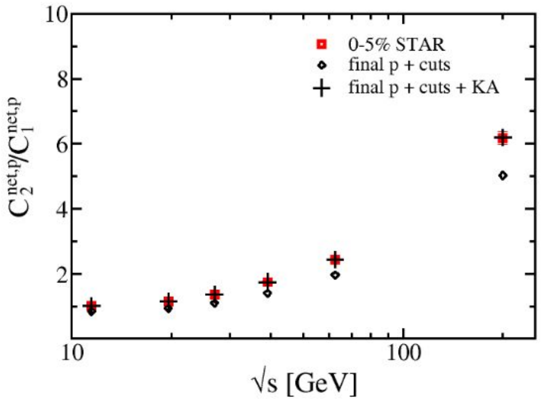

from 1403.4903

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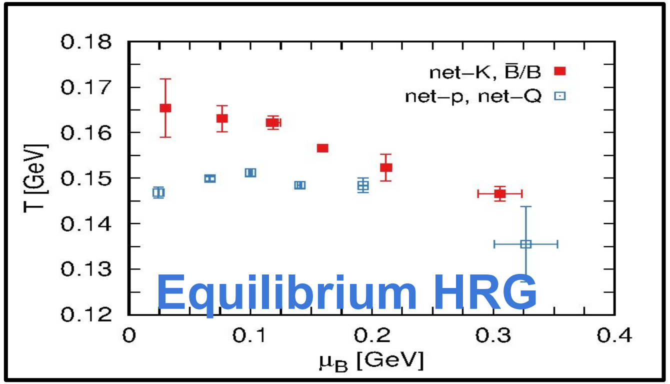
- can cause up to 5 – 10% deviations in lowest-order fluctuations
- was essential for the determination of freeze-out conditions from net-p and net-Q fluctuations!

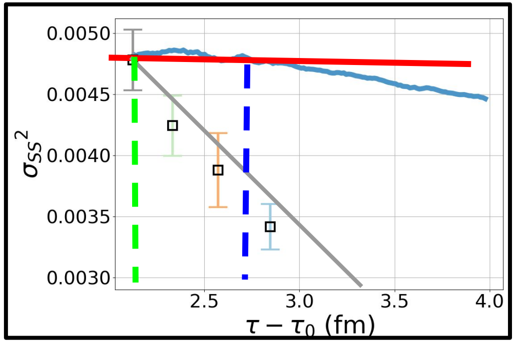
Impact on heavy-ion collision phenomenology

- Can the freeze-out conditions be reliably determined from fluctuation observables?
  - Study of (net-p, net-Q) fluctuations and of (net-K fluctuations, Bbar/B yield ratios)
    - ➔ High sensitivity of fluctuations vs. yields on FO conditions.
  - Separation of strange FO and light FO at highest beam energies.



Bluhm, Nahrgang, Eur.Phys.J.C.79 (2019)
see also: Bellwied et al., Phys.Rev.C.99 (2019)

- Coupled dynamics of fluctuations shows:
  - If the prior QGP evolution/hadronization leads to equilibrium at T≈160MeV: Final fluctuations portray equilibrium at chemical freeze-out in the Q channels.
  - In the S channel: FO temperatures obtained from the comparison of equilibrium HRG vs. experiment are **over-estimated** compared to dynamically expanding systems.



stochastic HRG diffusion