Exploring the phase diagram of strong-interaction matter with QCD inspired models

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TU Darmstadt

STRONG-NA7 Workshop &
HFHF Theory Retreat

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HFHF
Helmholtz Forschungsakademie Hessen für FAIR

CRC-TR 211
Strong-interaction matter under extreme conditions

QCD phase diagram

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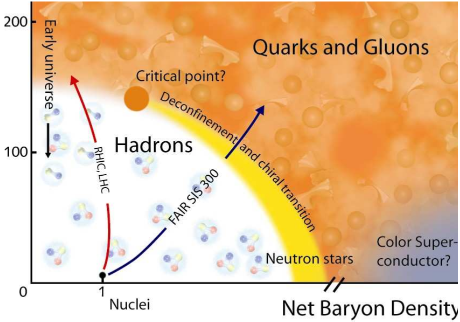
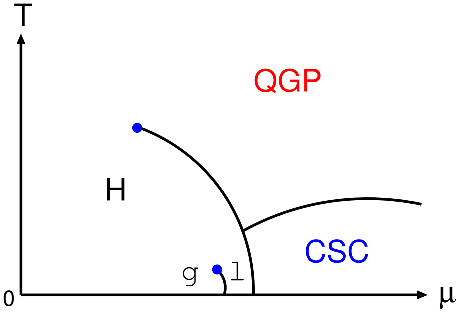
artist's view (CBM @ FAIR poster):



QCD phase diagram

schematic:



- phases depending on T and μ

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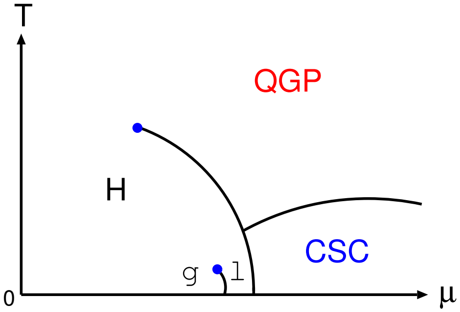

- phases depending on T and μ
- hadronic phase (H)
  - quarks confined in hadrons
  - chiral symmetry broken: $\langle \bar{q}q \rangle \neq 0$
  - nuclear liquid: baryon dominated
  - nuclear gas: meson dominated

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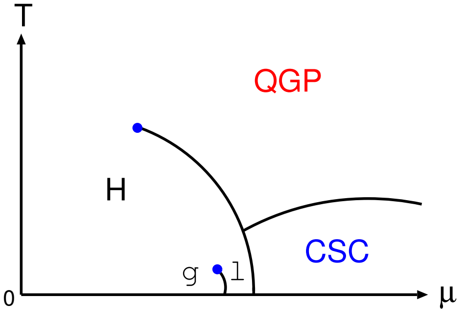

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  - deconfined quarks & gluons
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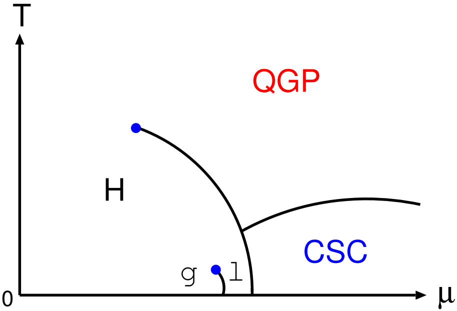

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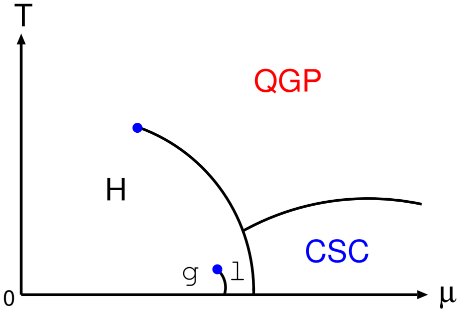

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- color superconductor (CSC)
  - quark pairing: $\langle qq \rangle \neq 0$

QCD phase diagram

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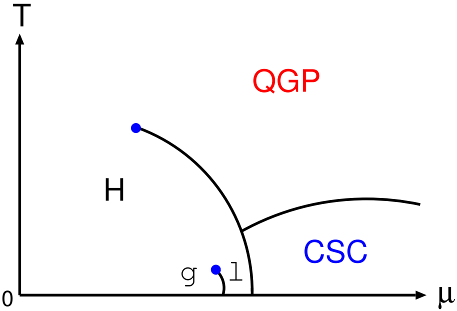

- extensions and variations:
  - non-uniform order parameters ("inhomogeneous phases")
  - additional axes: μ_I, μ_S, magnetic fields, ...

What do we really know?

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- vacuum properties of hadrons

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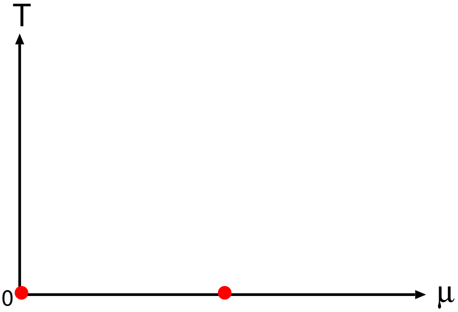

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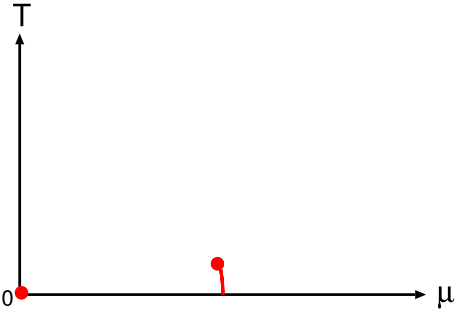

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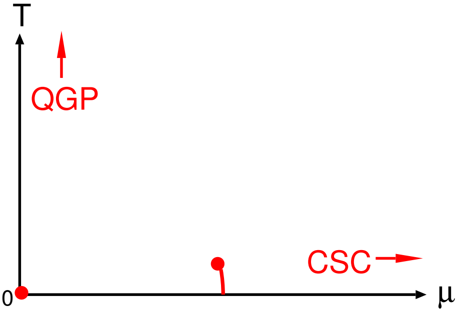

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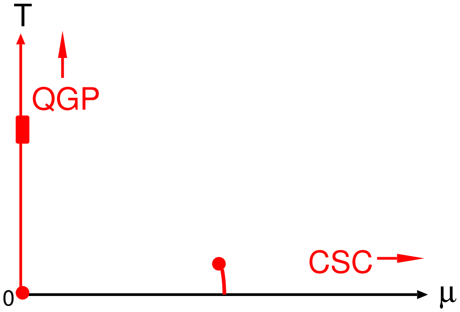

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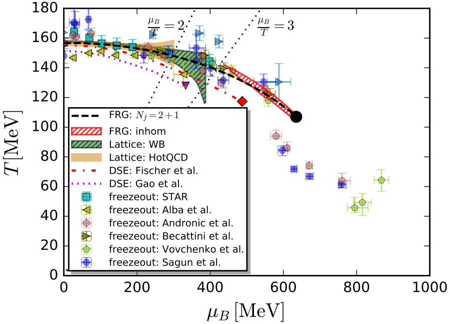

[Fu, Pawlowski, Rennecke, PRD (2020)]

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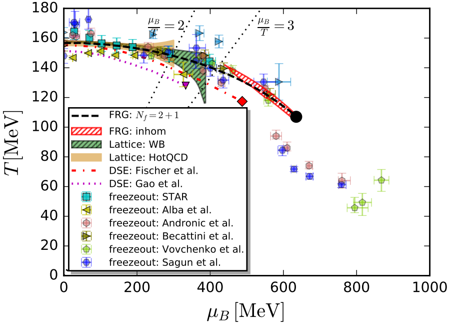

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- HICs: freeze-out points

Why models?

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- typical sentence in papers:

  Unfortunately, present lattice QCD calculation at finite chemical potential is plagued with the so called “sign problem”. Thus, to explore the QCD phase diagram at finite chemical potential, it is necessary to employ some QCD effective models, such as the Nambu–Jona-Lasinio (NJL) model and/or MIT bag model.

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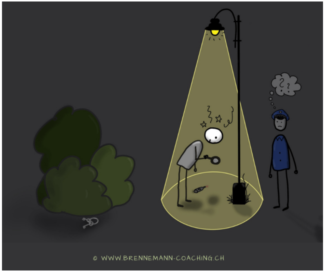
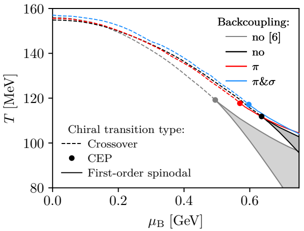
- This reminds me of the man who searches for his key near a street light because it is too dark at the place where he lost it ...



- Even when lattice QCD is not applicable, there are also non-perturbative continuum approaches to QCD ("functional methods"):
  - Dyson-Schwinger Equations (DSEs)
    → Christian Fischer's talk on Thursday
  - Functional Renormalization Group (FRG)

[Gunkel, Fischer, PRD (2021)]



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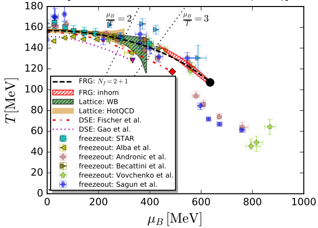

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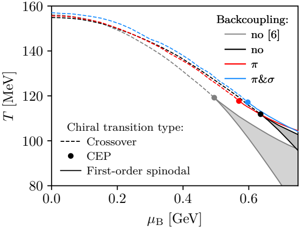

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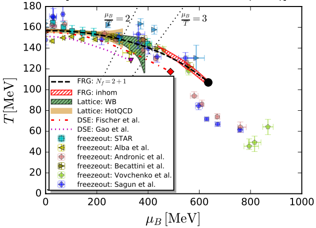

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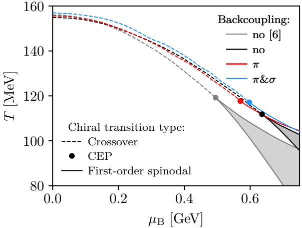

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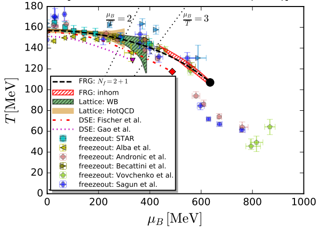

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- but in principle systematically improvable

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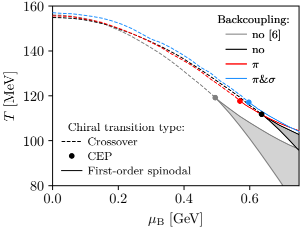

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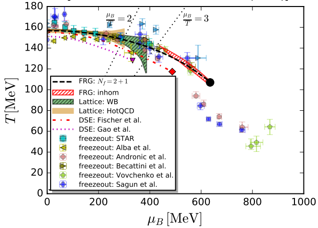

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  - Model-dependent results could be different from QCD.
- Often models have other drawbacks,
  e.g., NJL model:
  - non-renormalizable
    $\rightarrow$ dependence on regularization scheme and cutoff parameters; cutoff artifacts
  - no confinement
  - many possible interaction terms allowed by symmetries $\rightarrow$ many parameters
  - temperature and density dependence of the effective couplings unknown and usually neglected

Why models – some answers ...

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- But we should always keep the limitations in mind and know when to stop ...

Which models?

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Incomplete list of models to explore the phase diagram of strong-interaction matter:
(see also Hubert Hansen's talk on Saturday)

- Hadronic degrees of freedom
  - Hadron Resonance Gas
  - Relativistic Mean Field models (Walecka, Parity Doublet, ...)
- Quark (and gluon) degrees of freedom
  - Bag Models
  - NJL-type models, Quark-Meson model (+ Polyakov-loop extensions)
  - Quark-meson-coupling model
- Combinations and others
  - Hybrid models (e.g., RMF + bag model)
  - Quarkyonic model
  - Holographic models
  - ...

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I will mainly concentrate on NJL and QM models (= my personal expertise).

Outline

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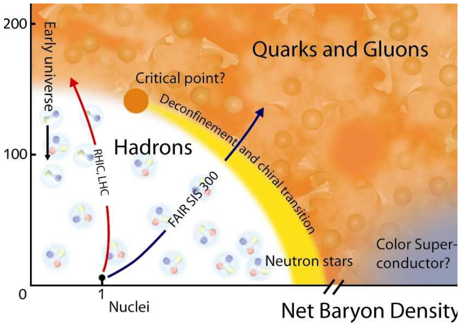
1. Introduction ✔
2. Chiral phase transition and critical endpoint
3. Color superconductivity
4. Inhomogeneous chiral phases



CHIRAL PHASE TRANSITION
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CRC–TR 211

Detour: MIT bag model

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  [Chodos et al., PRD (1974)]
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    (+ perturbative corrections)
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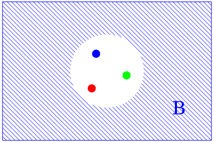

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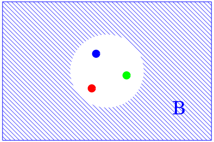
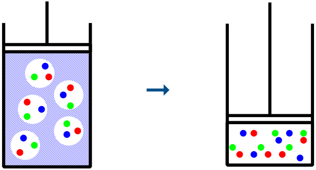


- Deconfinement at large temperature or density:
  - All quarks (and gluons) in one big bag



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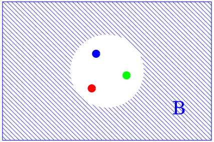

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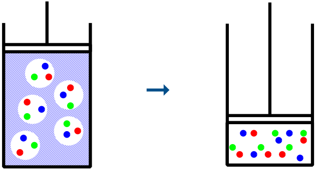

- Thermodynamic limit
  - Pressure relative to the nontrivial vacuum:

$$p_{BM}(T,\mu) = p_q^{ideal}(T,\mu) + p_g^{ideal}(T,\mu) - B \quad (+ \text{ perturbative corrections})$$

Phase diagram

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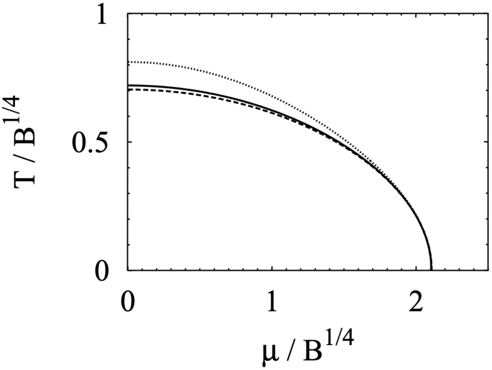
- QGP: $p_{BM} = 37 \cdot \frac{\pi^2}{90} T^4 + \mu^2 T^2 + \frac{\mu^4}{2\pi^2} - B$ (2-flavor bag model)
- Hadronic EoS: $p_\pi = 3 \cdot \frac{\pi^2}{90} T^4$ (ideal massless pion gas)



- drastic change of # d.o.f.
  $\Rightarrow$ 1st order all over
- dominated by B
  (dashed line = no pions)
- $\mu_c \approx 4 T_c$
- including ideal nucleons problematic
  (always favored at large μ)

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- Fits to T_c:

  $p_{BM}(T_c) = p_{pion\ gas}(T_c) \quad \Rightarrow \quad B = (37 - 3)\frac{\pi^2}{90} T_c^4$

  $T_c \approx 155\ \mathrm{MeV} \quad \Rightarrow \quad B \approx 280\mathrm{MeV}/\mathrm{fm}^3 \approx (215\mathrm{MeV})^4$

Chiral symmetry

- Chiral symmetry: $SU(N_f)_L \times SU(N_f)_R = SU(N_f)_V \times$ "$SU(N_f)_A$"
  - $SU(N_f)_V$: $q(x) \to e^{i\theta_a \tau_a} q(x)$
  - "$SU(N_f)_A$": $q(x) \to e^{i\theta_a \tau_a \gamma_5} q(x)$
  - $q(x)$ = quark field operator
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- symmetry of QCD for vanishing quark masses
- explicitly broken by (current) quark masses
  - $m_u = 2.16^{+0.49}_{-0.26}$ MeV, $m_d = 4.67^{+0.48}_{-0.17}$ MeV, $m_s = 93.4^{+8.6}_{-0.3.4}$ MeV (PDG, in $\overline{\text{MS}}$ at 2 GeV scale)

Chiral symmetry

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- Chiral symmetry: $SU(N_f)_L \times SU(N_f)_R = SU(N_f)_V \times$ "$SU(N_f)_A$"
  - $SU(N_f)_V$: $q(x) \rightarrow e^{i\theta_a\tau_a} q(x)$
  - "$SU(N_f)_A$": $q(x) \rightarrow e^{i\theta_a\tau_a\gamma_5} q(x)$
  - $q(x)$ = quark field operator
  - τ_a = generator in flavor space (Pauli or Gell-Mann matrix)
- symmetry of QCD for vanishing quark masses
- explicitly broken by (current) quark masses
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- QCD vacuum: spontaneously broken by $\langle \bar{q}q \rangle \neq 0$ ("chiral condensate")

Spontaneous symmetry breaking

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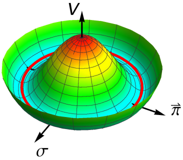
Analogy:

- spontaneous χSB
- spontan. breaking of rotational invariance in a ferromagnet



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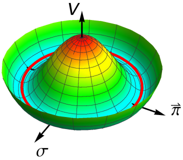

→ lower ground state energy in the broken phase:

$$\Delta\varepsilon = \varepsilon_{broken} - \varepsilon_{symmetric} < 0$$

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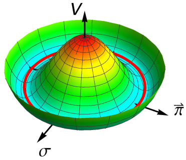

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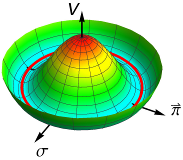

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→ dynamically generated bag constant!

The Nambu–Jona-Lasinio model

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PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

- two papers more than 60 years ago: Phys. Rev. **122**, 345-358; ibid. **124**, 246-254 (1961).
  - no other common paper since then
  - more than 6000 (3000) citations on INSPIRE
- Nambu: Nobel prize in physics 2008 “for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics”
- Nobel lecture presented by Jona-Lasinio: `https://www.nobelprize.org/prizes/physics/2008/nambu/lecture/`

NJL model:
main ideas and results of the original papers

- Lagrangian: $\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$
  - ψ nucleon field
  - 4-point interaction, invariant under chiral transformations
  - chiral symmetry explicitly broken by (small) bare mass m

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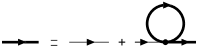

  - dynamical generation of a “constituent mass” $M = m - 2G\langle\bar{\psi}\psi\rangle \gg m$

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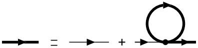

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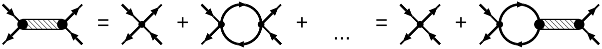
- mesonic excitations:



  - massless pions in the chiral limit ($\rightarrow$ Goldstone theorem, 1961)
  - $m_\pi^2 \propto m$ ($\rightarrow$ Gell-Mann–Oakes–Renner relation, 1968)

Later developments:
brief history of the NJL model

- reinterpretation in the QCD era: schematic model for quarks

  [H. Kleinert, Erice lectures (1976); M.K. Volkov, Annals Phys. (1984); T. Hatsuda, T. Kunihiro, PLB (1984); ...]

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- Polyakov-loop extended NJL model

  [K. Fukushima, PLB (2004); E. Megías, E. Ruiz Arriola, L. L. Salcedo,PRD (2006), C. Ratti, M.A. Thaler, W. Weise, PRD (2006); ...]

  - "statistical realization" of confinement

Thermodynamics of the NJL model: mean-field approximation

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- Lagrangian:

$$\mathscr{L} = \bar{q}(i\partial\!\!\!/ - m)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

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- constant mean fields: $\sigma(x) = \phi = const.$, $\pi_a(x) = 0$

→ mean-field Lagrangian:

$$\mathscr{L}_{MF} = \bar{q}(i\partial\!\!\!/ - m + 2G\phi)q - G\phi^2 \equiv \mathscr{L}_M - \mathcal{V}_M$$

with

$\mathscr{L}_M = \bar{q}(i\partial\!\!\!/ - M)q$ free fermion with mass $M = m - 2G\phi$

$\mathcal{V}_M = G\phi^2 = \frac{(M-m)^2}{4G}$ field independent "potential"

Thermodynamics of the NJL model: thermodynamic potential

- Grand potential per volume ("thermodynamic potential"): $\Omega(T,\mu) = -\frac{T}{V}\ln\mathcal{Z}$

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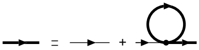

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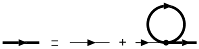
  - $\frac{\partial \Omega_{MF}}{\partial M} = 0$ ➞ gap equation: 
- Thermodynamics: $p = -\Omega, \quad n = -\frac{\partial \Omega}{\partial \mu}, \quad s = -\frac{\partial \Omega}{\partial T}, \quad \varepsilon = -p + Ts + \mu n, \quad \ldots$

NJL bag pressure

NJL bag pressure

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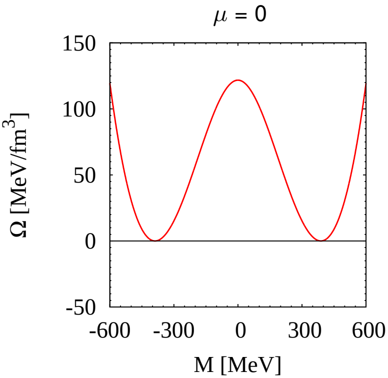
- NJL thermodynamic potential in vacuum (chiral limit):



- dynamically generated bag pressure
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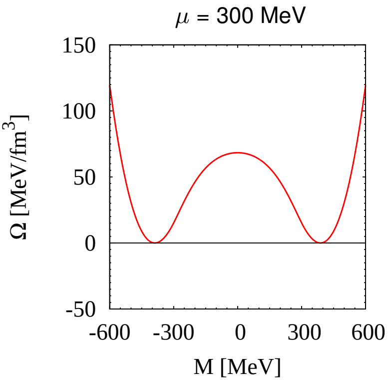
NJL bag pressure

- NJL thermodynamic potential at $T = 0$ (chiral limit):



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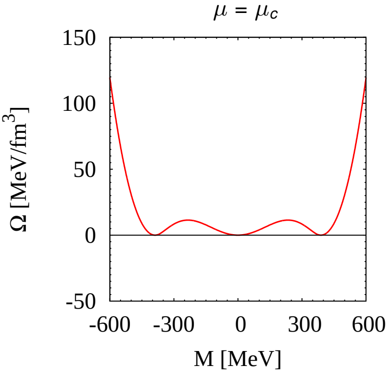

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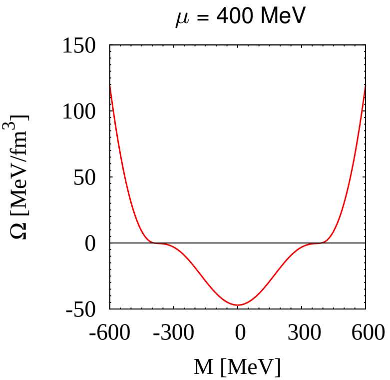

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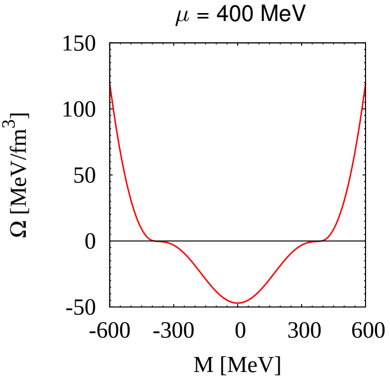

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NJL bag pressure

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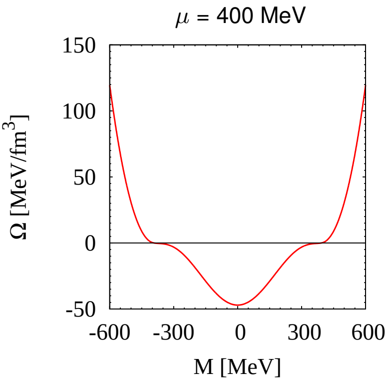

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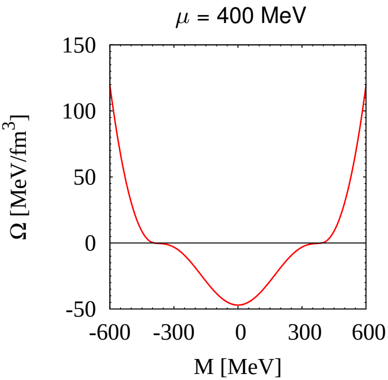

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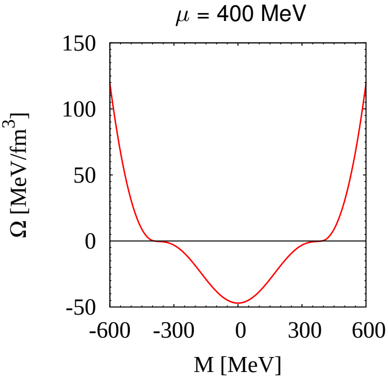

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  and temperature!

Energy per Baryon

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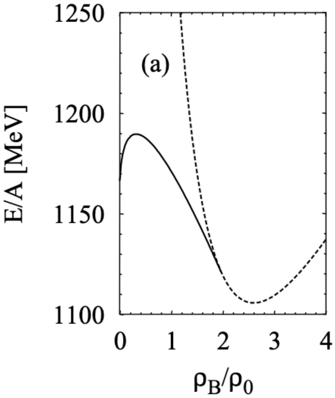


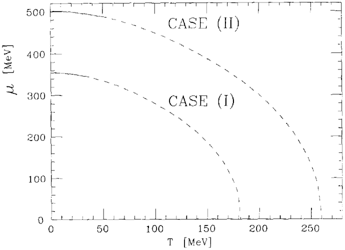
solid: chirally broken solution
dashed: restored solution

- selfbound quark matter in the restored phase
- "schematic nucleon droplets" [MB, NPA (1996)]
- chirally broken solution
  → no confinement

Phase diagram

- first NJL phase diagram:

[M. Asakawa, K. Yazaki, NPA (1989)]



CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

Masayuki ASAKAWA and Koichi YAZAKI

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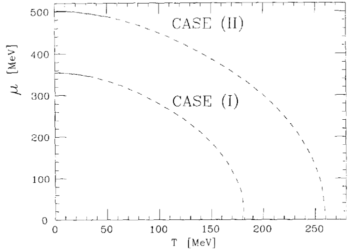

CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

Masayuki ASAKAWA and Koichi YAZAKI

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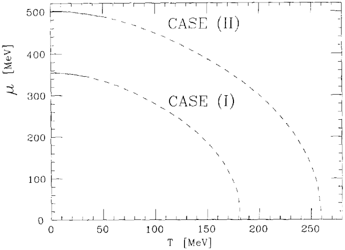

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- first-order phase transition at low T and large μ,
  cross-over at high T and low μ $\longrightarrow$ critical endpoint !
- location depends on parameter choice

Influence of vector interactions

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$\rightarrow$ $\Omega_{MF}(T,\mu;M,\tilde{\mu}) = \Omega_M(T,\tilde{\mu}) + \frac{(M-m)^2}{4G} - \frac{(\mu-\tilde{\mu})^2}{4G_V}, \qquad \tilde{\mu} = \mu - 2G_V\, n$

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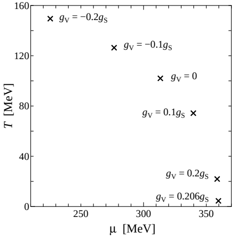
- location of the CEP (PNJL):

[K. Fukushima, PRD (2008)]



- Positive (negative) G_V weaken (strengthen) the first-order phase transition.
- The CEP can be shifted around or removed completely!

Another way to shift the CEP around

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- 't Hooft interaction in the 3-flavor model:

$$\mathcal{L}_D = K \left\{ \det_f \left(\bar{\psi}(1 + \gamma_5)\psi \right) + \det_f \left(\bar{\psi}(1 - \gamma_5)\psi \right) \right\}$$

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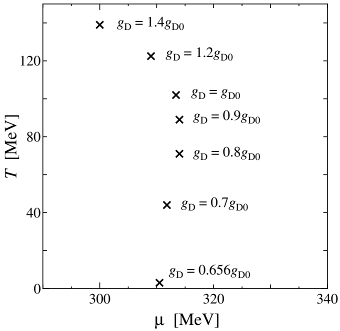

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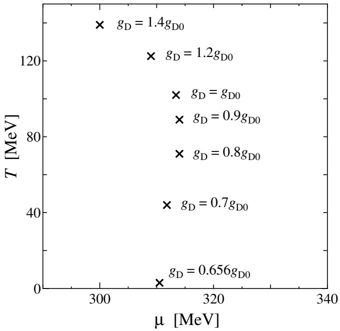

→ The (P)NJL model is not suited for quantitative predictions

Compilation of critical points

[M. Stephanov, PoSLAT (2006)]

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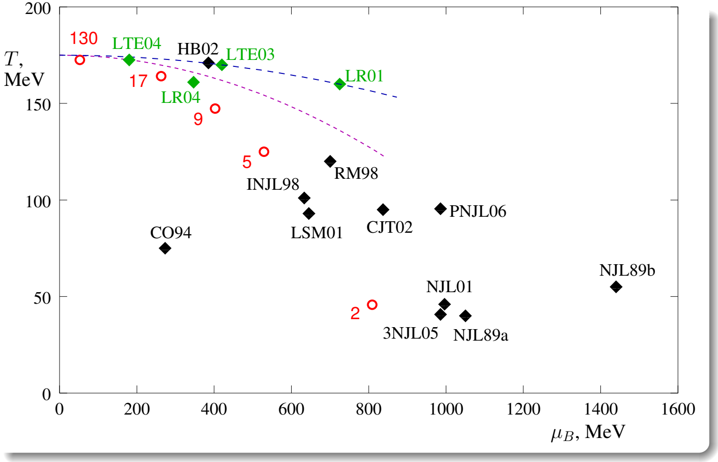

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  And, as we will discuss, they can help to interprete these.

Regularization

- $\Omega_{MF} = -12 \int \frac{d^3p}{(2\pi)^3} \left\{ E_p + (\textit{thermal part}) \right\} + \frac{(M-m)^2}{4G}$,

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  - sharp 3-momentum cutoff: $\int\limits_0^\infty dp\, f(p) \rightarrow \int\limits_0^\Lambda dp\, f(p)$
  - Pauli-Villars: $E_p \rightarrow \sum_{j=0}^{N} c_j\, E_{p,j}, \quad E_p = \sqrt{\vec{p}^{\,2} + M_j^2}$

    e.g., $M_j^2 = M^2 + j\lambda^2, \quad c_0 = 1,\ c_1 = -3,\ c_2 = 3,\ c_3 = -1$

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  Should we better employ renormalizable models to avoid artifacts?

Quark-meson model

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- Lagrangian: $\mathcal{L}_{\text{QM}} = \mathcal{L}_{mes} + \mathcal{L}_q$
  - $\mathcal{L}_{mes} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U(\sigma,\vec{\pi})\,,$

    $U(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma,$ chiral limit: $h = 0$
  - $\mathcal{L}_q = \bar{\psi}\left(i\partial\!\!\!/ - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi$
- Mean-field approximation: σ, $\vec{\pi}$ classical fields
- Mean-field thermodynamic potential quite similar to NJL, but renormalizable
- Typical renormalization conditions:

  determine g, v, λ, h by fitting M, f_π, m_σ, m_π at given Λ, then $\Lambda \to \infty$

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[Carignano, MB, Schaefer, PRD (2014)]

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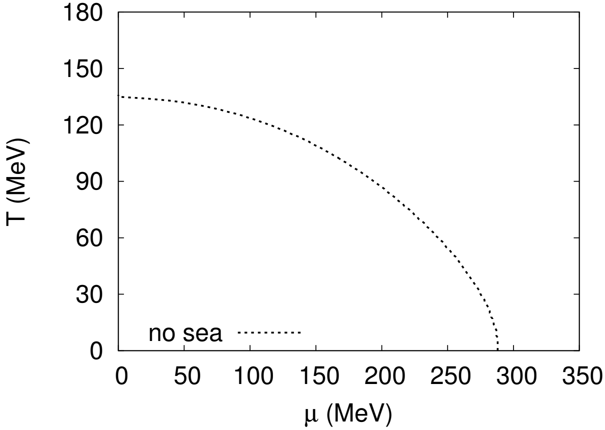

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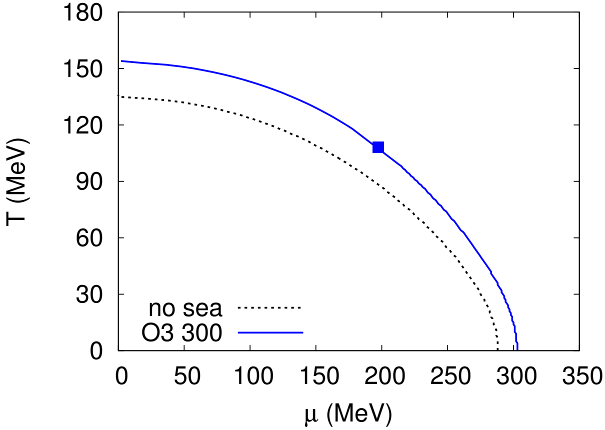

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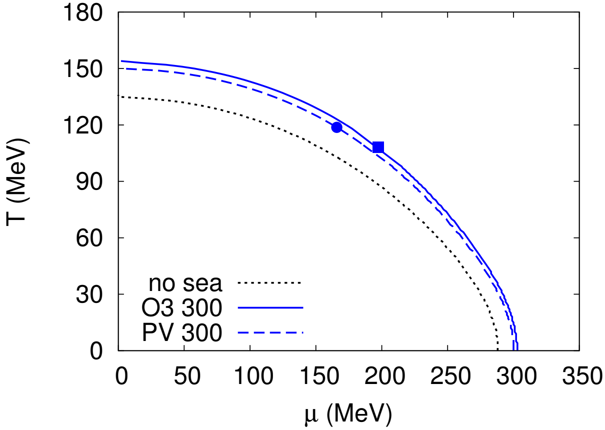

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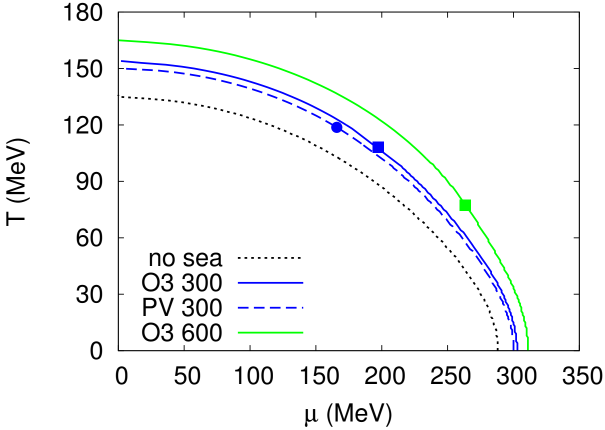

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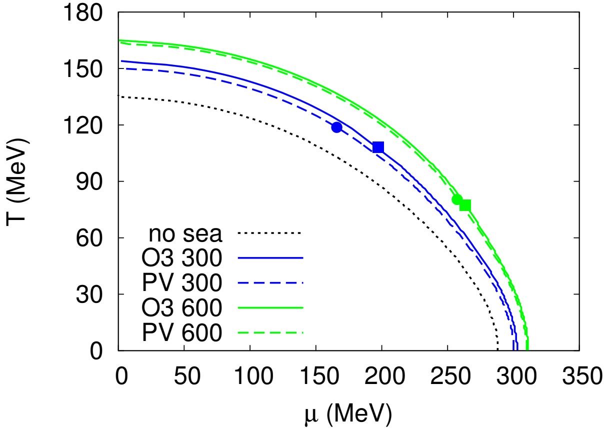

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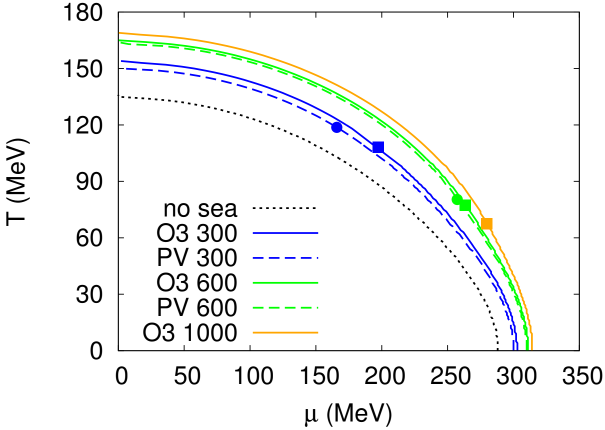

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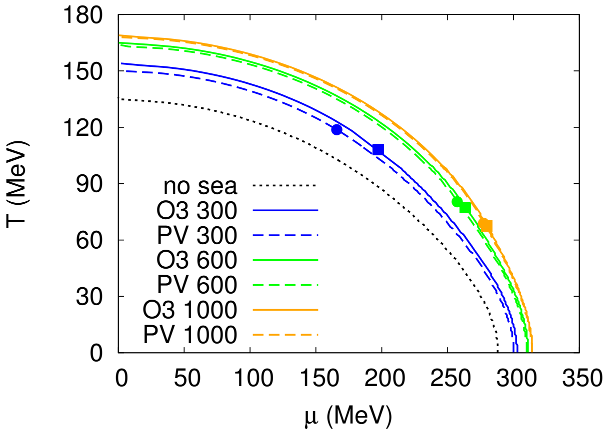

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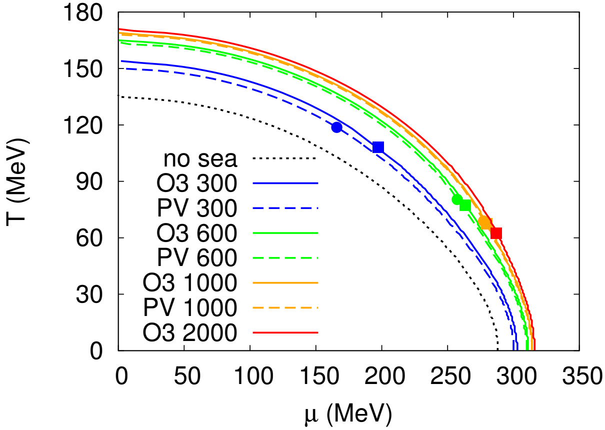

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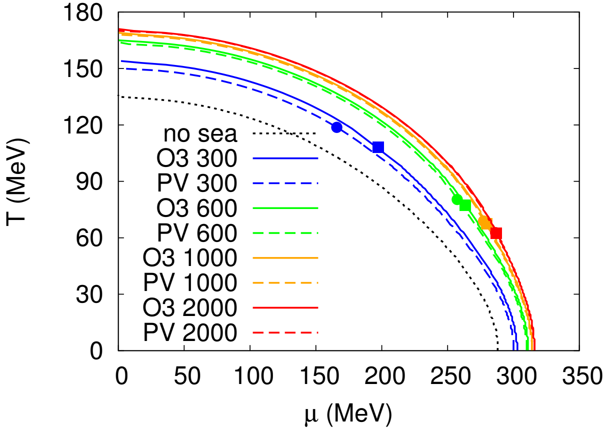

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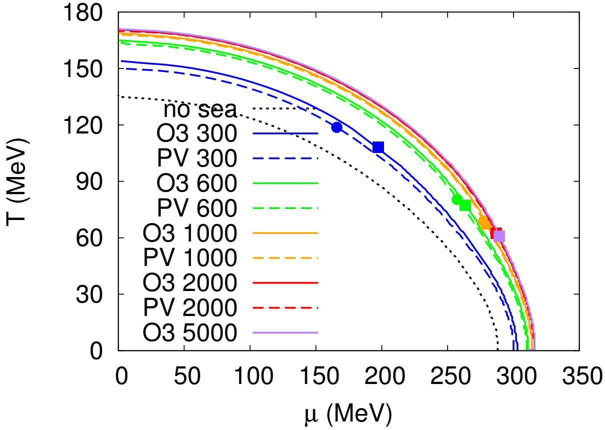

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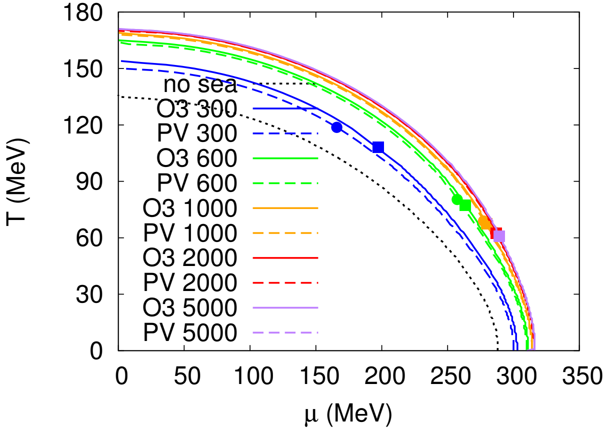

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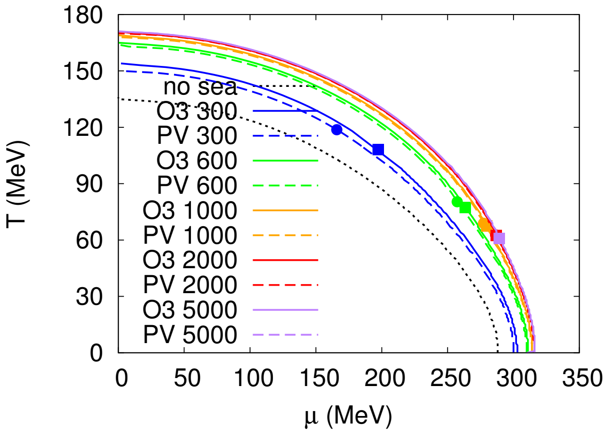

- Convergence reached at $\Lambda \approx 2$ GeV.

Vacuum instabilities

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- Thermodynamic potential for $T = \mu = 0$
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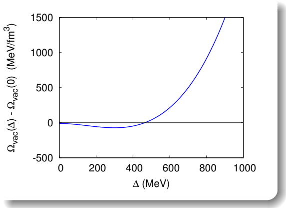
Λ = 600 MeV



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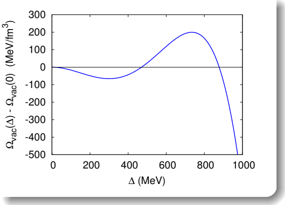

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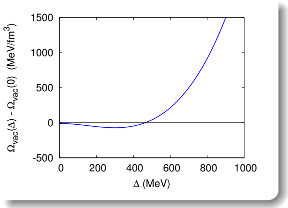

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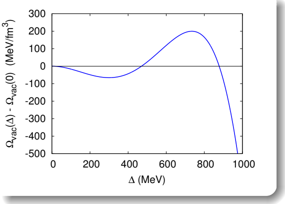

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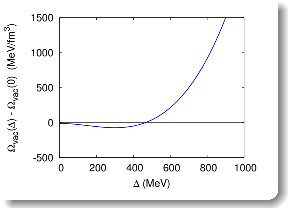

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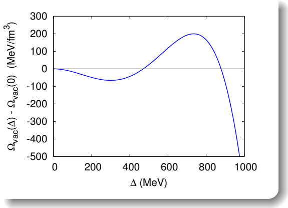

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- Can the problem be cured by including bosonic fluctuations ($\rightarrow$ FRG)?

Model extensions and applications (not shown in the lecture for time reasons)

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PNJL model

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- main shortcoming of the NJL model: no confinement
  - no gluons
  - unphysical $q\bar{q}$ decays of mesons
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- P(olyakov loop extended) NJL model: [K. Fukushima, PLB (2004)]

$$\mathcal{L}_{PNJL} = \bar{q}(i\not{D} - m)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}(\ell,\bar{\ell})$$

  - covariant derivative: $D_\mu = \partial_\mu - iA_\mu, \quad A_\mu = \delta_\mu^0 A_0$ constant background field
  - $\mathcal{U}(\ell,\bar{\ell})$ phenomenological potential ($\leftrightarrow$ pure gluon pressure)

PNJL model: thermodynamics

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- thermodynamic potential (thermal quark part):

$$\Omega_{q,th} = -2N_f T \int \frac{d^3p}{(2\pi)^3} \Big\{ \ln\Big(1 + 3\,\ell\, e^{-(E_p-\mu)/T} + 3\,\bar{\ell}\, e^{-2(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}\Big) + \ln\Big(1 + 3\,\bar{\ell}\, e^{-(E_p+\mu)/T} + 3\,\ell\, e^{-2(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}\Big)\Big\}$$

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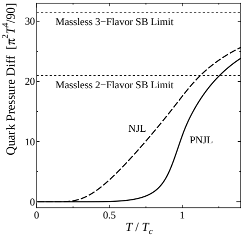

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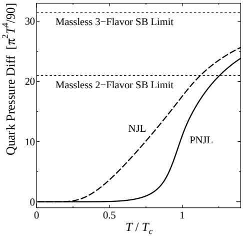

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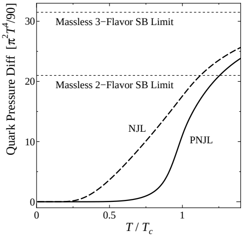
- chiral and deconfinement transitions (partially) synchronized

pressure



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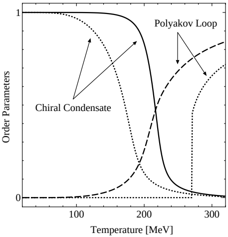
$\langle \bar{q}q \rangle$, $\ell = \bar\ell$



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- How reliable are these methods?

Assessing nonzero μ on the lattice

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- lattice QCD:
  - standard Monte Carlo methods fail at (real) $\mu \neq 0$ ("sign problem")
- suggested work-arounds:
  - Taylor expansion around $\mu = 0$
  - extrapolation from imaginary μ
- How reliable are these methods?

→ Check for models where real $\mu \neq 0$ are accessible!

Taylor expansion

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- Taylor expansion of the pressure: $\frac{p}{T^4}(T,\mu) = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$
- lattice: $n = 2, 4, 6, 8$

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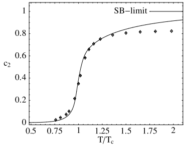
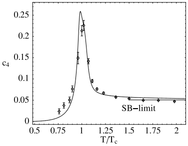
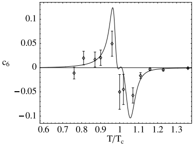
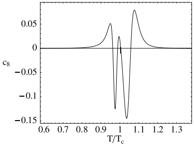
- comparison with PNJL: [S. Rößner, C. Ratti, W. Weise, PRD (2007); lattice: C.R. Allton et al., PRD (2002,2003)]









Taylor expansion: test of concept

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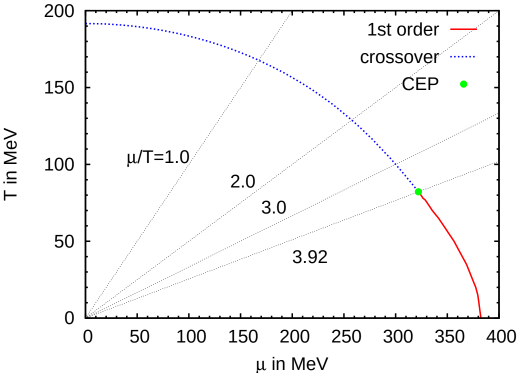

- crossover line:

  maxima of $\frac{\chi_{mm}}{T^2} = -\frac{1}{T^2}\frac{\partial^2\Omega}{\partial m^2}$

  along $\frac{\mu}{T} = const.$

- endpoint:

  $T_c = 82.2$ MeV

  $\mu_c = 322.0$ MeV

  $\frac{\mu_c}{T_c} = 3.92$

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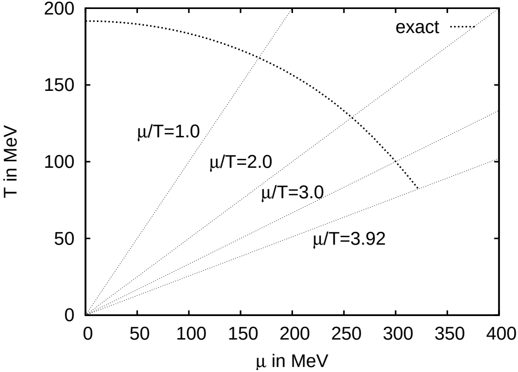

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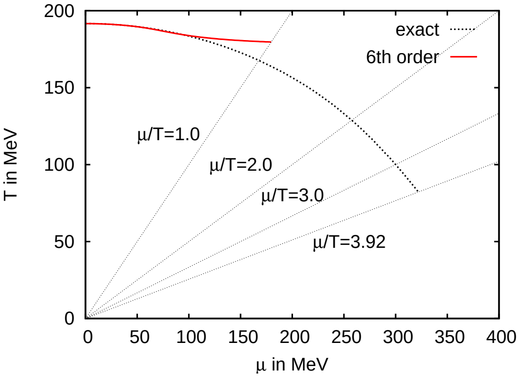

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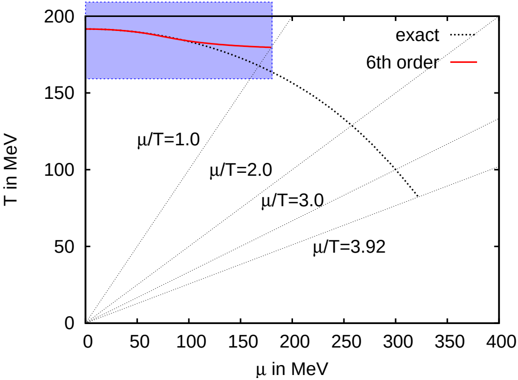

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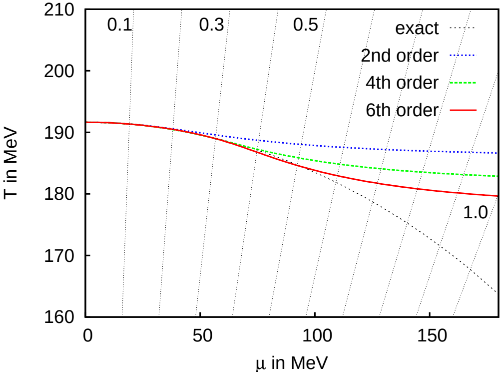

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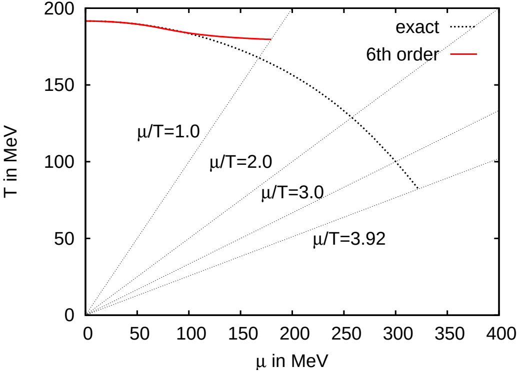

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  I would say: similar conclusion

PNJL beyond mean-field approximation

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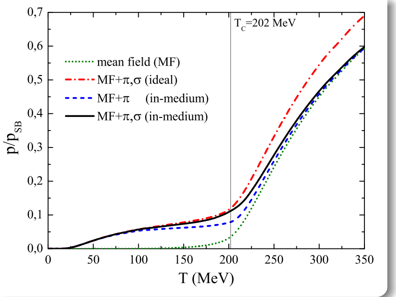

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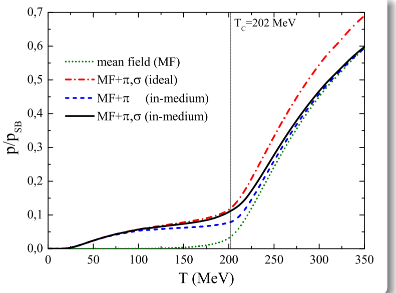

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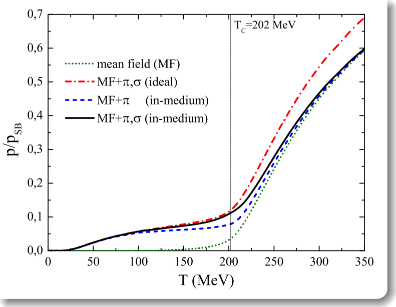

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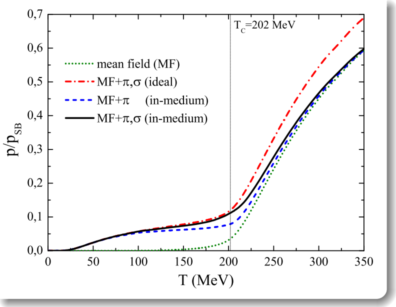

- $T \lesssim T_c$:
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- $T \lesssim 100$ MeV:
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- $T > T_c$:
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Isospin chemical potential

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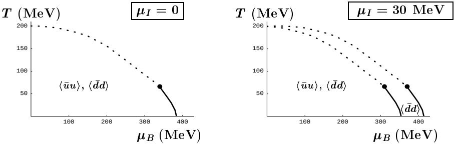

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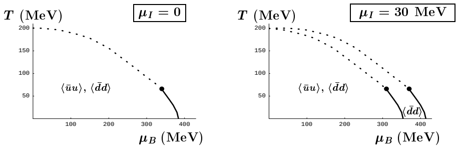

- generalized interaction: $\mathcal{L}_{int} = \mathcal{L}_1 + \mathcal{L}_2$
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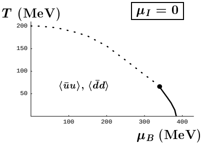

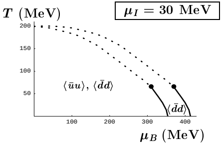

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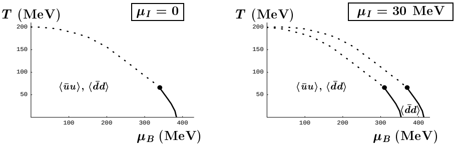

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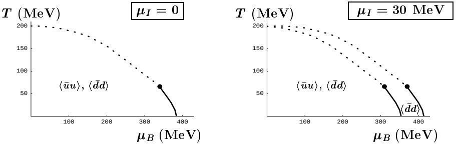

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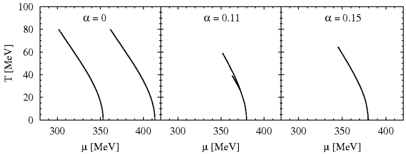

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Discussion: vector interactions

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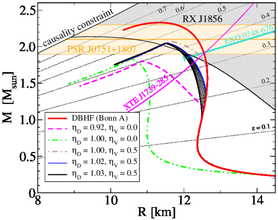

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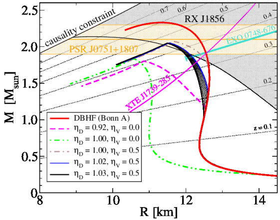

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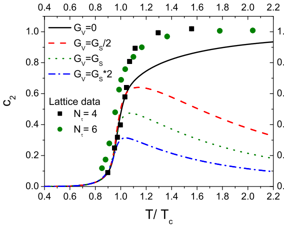

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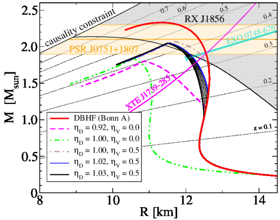

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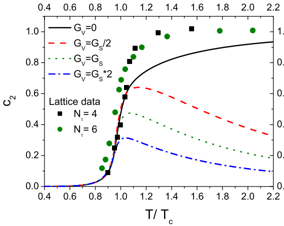

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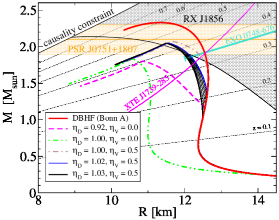

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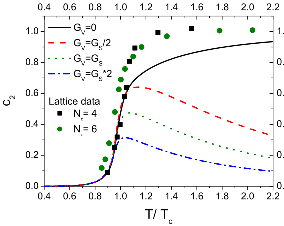

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- possible, but that adds further parameters to the model ...