Angular distributions of hadrons and Using a Bethe Salpeter approach to study hadronization

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Outline

- Introduction
 - hadronization from the QGP state

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- Transport theory
- Coalescence model
- Hadrons in AA collisions
 - p, π, Λ_C, D spectra at RHIC and LHC
- Angular distribution

• Bethe Salpeter approach to hadronization

• Conclusions

- Nuclear matter: Critical energy and temperature in the transition between confined and deconfined phase
- If $T > T_C$ colour charges are deconfined in a Quark Gluon Plasma
- Different value of T and ρ for deconfinement \rightarrow Phase diagram



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<u>Quark Gluon Plasma in</u> Ultrarelativistic Heavy-Ion collisions



 $m_{c,b} >> \Lambda_{QCD}$

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produced by pQCD process (out of equilibrium)

• m_{c,b} >> T₀

negligible thermal production

- $\tau_0 << \tau_{QGP}$
- $\tau_{\text{therm.}} \approx \tau_{\text{QGP}} >> \tau_{g,q}$

HQs experience the full QGP evolution

Carry informations about initial stages, more than light quarks

<u>Quark Gluon Plasma in</u> <u>Ultrarelativistic Heavy-Ion collisions</u>



Relativistic Boltzmann transport for finite $\frac{\eta}{s}$

Bulk Evolution

 $p^{\mu}\partial_{\mu}f_{q,g}(x,p) + M(x)\partial_{\mu}^{x}M(x)\partial_{p}^{\mu}f_{q,g}(x,p) = C_{22}[f_{q,g}]$ Collision $\eta \neq 0$ **Free-streaming Field interaction**

Relativistic Boltzmann transport for finite $\frac{\eta}{s}$

Bulk Evolution

$$p^{\mu}\partial_{\mu}f_{q,g}(x,p) + M(x)\partial_{\mu}^{x}M(x)\partial_{p}^{\mu}f_{q,g}(x,p) = C_{22}[f_{q,g}]$$

Free-streaming Field interaction Collision $\eta \neq 0$

Heavy quark evolution

$$p^{\mu}\partial_{\mu}f_Q(x,p) = C[f_q, f_g, f_Q]$$

- Describes the evolution of the one body distribution function f(x,p)
- It is valid to study the evolution of both bulk and Heavy quarks
- Possible to include f(x,p) out of equilibrium

Coalescence model

Statistical factor colour-
spin-isospinParton distribution
functionHadron Wigner
function $\frac{dN_H}{d^2 P_T} = g_H \int \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i f_{q_i}(x_i, p_i) f_W(x_1...x_n; p_1...p_n) \delta \left(P_T - \sum_{i=1}^n p_{T,i} \right)$

LIGHT

Thermal + flow for u,d,s (
$$p_T < 3 \text{ GeV}$$
)

$$\frac{dN_{q,\bar{q}}}{d^2 p_T} \sim exp\left(-\frac{\gamma - p_T \cdot \beta \pm \mu_q}{T}\right)$$

$$\beta(r) = \frac{r}{R}\beta_{max}$$

$$V = \pi r^2 \tau \cosh(y_Z)$$
+ quenched minijets for u,d,s ($p_T > 3 \text{ GeV}$)

Coalescence model



Wigner function-Wave function

$$\Phi_M^W(\boldsymbol{r},\boldsymbol{q}) = \int d^3r' e^{-i\boldsymbol{q}\cdot\boldsymbol{r}'} \varphi_M(\boldsymbol{r}+\frac{\boldsymbol{r}'}{2}) \varphi_M^*(\boldsymbol{r}-\frac{\boldsymbol{r}'}{2})$$

where $\varphi_M(\mathbf{r})$ is the meson wave function

Assuming gaussian wave function

$$f_H(\dots) = \prod_{i=1}^{N_q - 1} A_W \exp(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2)$$

Wigner function width fixed by root-mean-square charge radius from quark model

<u>C.-W. Hwang, EPJ C23, 585 (2002)</u> <u>C. Albertus et al., NPA 740, 333 (2004)</u>

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2$$

$$\sigma_{ri} = 1/\sqrt{(\mu_i \,\omega)} \qquad \mu_1 = \frac{m_1 m_2}{m_1 + m_2} \qquad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
0.184	0.282	
0.083	0.404	
$\langle r^2 \rangle_{ch}$	$\sigma_{_{p1}}$	σ_{p2}
0.15	0.251	0.424
0.2	0.242	0.406
-0.12	0.337	0.53
	$ \begin{array}{c} \langle r^2 \rangle_{ch} \\ 0.184 \\ 0.083 \\ \langle r^2 \rangle_{ch} \\ 0.15 \\ 0.2 \\ -0.12 \end{array} $	$\begin{array}{c c} \langle r^2 \rangle_{ch} & \sigma_{p1} \\ 0.184 & 0.282 \\ 0.083 & 0.404 \\ \hline \langle r^2 \rangle_{ch} & \sigma_{p1} \\ 0.15 & 0.251 \\ 0.2 & 0.242 \\ -0.12 & 0.337 \\ \end{array}$

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Coalescence model





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$$r^{2}\rangle_{ch} = \frac{3}{2} \frac{m_{2}^{2} Q_{1} + m_{1}^{2} Q_{2}}{(m_{1} + m_{2})^{2}} \sigma_{r1}^{2} + \frac{3}{2} \frac{m_{3}^{2} (Q_{1} + Q_{2}) + (m_{1} + m_{2})^{2} Q_{3}}{(m_{1} + m_{2} + m_{3})^{2}} \sigma_{r2}^{2}$$

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Meson	$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
$D^+ = [c\bar{d}]$	0.184	0.282	
$D_s^+ = [\bar{s}c]$	0.083	0.404	
Baryon	$\langle r^2 \rangle_{ch}$	$\sigma_{_{p1}}$	σ_{p2}
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\tilde{\Omega_c^0} = [ssc]$	-0.12	0.337	0.53

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Numerical implementation of coalescence integral

$$\frac{dN_M}{d^2\boldsymbol{p}_T} = g_M \sum_{i,j} P_q(i) P_{\bar{q}}(j) \delta^{(2)} (\boldsymbol{p}_T - \boldsymbol{p}_{iT} - \boldsymbol{p}_{jT}) f_M(x_i, x_j; p_i, p_j)$$

$$\frac{dN_M}{d^2 \boldsymbol{p}_T} = g_B \sum_{i \neq j \neq k} P_q(i) P_q(j) P_q \delta^{(2)} (\boldsymbol{p}_T - \boldsymbol{p}_{iT} - \boldsymbol{p}_{jT} - \boldsymbol{p}_{kT}) f_M(x_i, x_j, x_k; p_i, p_j, p_k)$$

Baryon

Numerical implementation of coalescence integral

$$\frac{dN_M}{d^2\boldsymbol{p}_T} = g_M \sum_{i,j} P_q(i) P_{\bar{q}}(j) \delta^{(2)} (\boldsymbol{p}_T - \boldsymbol{p}_{iT} - \boldsymbol{p}_{jT}) f_M(x_i, x_j; p_i, p_j)$$

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Baryon

$$p_x^{rel} = \frac{E_2 p_{x1}^{CM} - E_1 p_{x2}^{CM}}{E_1 + E_2}$$
$$p_x^{rel} = \frac{m_2 p_{x1}^{CM} - m_1 p_{x2}^{CM}}{m_1 + m_2}$$

 p_i^{rel} is independent of which weights we use

AA @ RHIC & LHC

Wave function widths σ_p of baryon and mesons are the same at RHIC and LHC



AA @ RHIC & LHC

Wave function widths σ_p of baryon and mesons are the same at RHIC and LHC

Data from: STAR Coll. PRL 113, 142301 (2014), ALICE Coll. JHEP 09 (2012) 112 10 D⁰ ALICE (0-20)% 0 10 D⁰ STAR (0-10)% 0 -- charm Coalescence lower at LHC then at RHIC 10 charm coalescence $(2\pi p_{\rm T})^{-1} {\rm dN/dp_{\rm T}} {\rm dy} ({\rm GeV}^{-2})$ fragmentation coalescence fragmentation coal+fragm coal + fragm dN/dp_T (GeV 10 D^0 10 RHIC: Au+Au@200GeV LHC: Pb+Pb@2.76 TeV Main contribution: Fragmentation |y|<1 (0-10)% lyl<0.5 (0-20)% 10 10 10 10 7 3 5 6 8 0 2 3 4 5 6 7 8 9 10 -1 p_T (GeV) $p_{T}(GeV)$ 10 charm coalescence 10^{1} --- charm Coalescence lower at LHC then at RHIC coalescence fragmentation $(2\pi p_T)^{-1} dN/dp_T dy (GeV^{-2})$ fragmentation 10 coal + fragm coal + fragm $dN/dp_{\rm T} \ dy \ (GeV^{^1})$ 10 Λ_{C} 10^{-2} 10^{-3} Main contribution: Coalescence RHIC: Au+Au@200GeV LHC: Pb+Pb@2.76 TeV 10 |y|<0.5 (0-20)% |y|<1 (0-10)% 10^{-4} 10 1 2 3 4 5 6 7 8 9 10 10^{-5} 10-7 5 6 p_T (GeV) p_T (GeV) 14 RHIC LHC





Coordinate space: initial anisotropy



 $\frac{v_2}{\epsilon}$ mesure the efficiency in converting the eccentricity from coordinate to momentum space



Momentum space: final anisotropy

$$v_{2} = \left\langle \frac{p_{x}^{2} - p_{y}^{2}}{p_{x}^{2} + p_{y}^{2}} \right\rangle = \left\langle \cos(2f_{p}) \right\rangle$$

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T} [1 + 2\nu_2 \cos(2\phi) + \cdots]$$





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V2 bulk ON

V2 charm **OFF**

V2 bulk ON



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The BS is a relativistic eq. which try to describe a bound system via a wave function called BS wave function. The idea come from Feynman treatment of interaction, and it was then formalized by Gell Mann. The idea is the following

$$\boldsymbol{\psi}(\mathbf{x}_2, t_2) = \int K(\mathbf{x}_2, t_2; \mathbf{x}_1, t_1) \boldsymbol{\psi}(\mathbf{x}_1, t_1) d^3 \mathbf{x}_1$$

If we now add a weak potential $U(\mathbf{x}, t)$ we can expand the propagator

$$K^{(1)}(2,1) = -i \int K_0(2,3) U(3) K_0(3,1) d\tau_3$$

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$$K^{(1)}(2,1) = -i \int K_0(2,3) U(3) K_0(3,1) d\tau_3$$

Continuing the perturbation expansion, we can write

$$K_{+}^{(A)}(2,1) = K_{+}(2,1)$$

-i $\int K_{+}(2,3)A(3)K_{+}^{(A)}(3,1)d\tau_{3}$

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If we now add a weak potential $U(\mathbf{x}, t)$ we can expand the propagator

$$K^{(1)}(2, 1) = -i \int K_0(2, 3) U(3) K_0(3, 1) d\tau_3$$

In the case in which we consider two particles in interaction

$$K^{(1)}(3,4;1,2) = -ie^2 \int \int K_{0a}(3,5) K_{0b}(4,6) r_{56}^{-1} \\ \times \delta(t_{56}) K_{0a}(5,1) K_{0b}(6,2) d\tau_5 d\tau_6$$

In the case of two particles, we have to consider all the possible interactions

$$G^{(2A)}(1,2;3,4) = -i\Gamma_{a\sigma}\Gamma_{b\tau}K_{+a}(3,1)K_{+b}(4,2)$$
$$\times\Gamma_{a\tau}\Gamma_{b\sigma}G'(1,4)G'(2,3)$$



In the case of two particles we have to consider all the possible interactions

If we consider only the irreducible graphs

$$K^{(n)}(3,4;1,2) = -i \int \int \int \int d\tau_5 d\tau_6 d\tau_7 d\tau_8$$

$$\times K_{+a}(3,5) K_{+b}(4,6) G^{(n)}(5,6;7,8)$$

$$\times K_{+a}(7,1) K_{+b}(8,2)$$



In the case of two particles, we must consider all the possible interactions

$$K^{(n)}(3,4;1,2) = -i \int \int \int \int d\tau_5 d\tau_6 d\tau_7 d\tau_8$$

$$\times K_{+a}(3,5) K_{+b}(4,6) G^{(n)}(5,6;7,8)$$

$$\times K_{+a}(7,1) K_{+b}(8,2)$$

$$K(3,4;1,2) - K_{+a}(3,1)K_{+b}(4,2)$$

= $i \int \int \int \int d\tau_5 d\tau_6 d\tau_7 d\tau_8 K_{+a}(3,5)K_{+b}(4,6)$
 $\times \bar{G}(5,6;7,8)K(7,8;1,2)$

with $\bar{G} = \{G^{(1)} + G^{(2A)} + G^{(2B)} + G^{(2C)} + G^{(2D)} + G^{(2\alpha)} + G^{(3A)} + \cdots \}$

GRAPH I



If we chose only the G(1) contribution, we are considering **Bound State**



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Now that we have the expansion of the propagator, we can use this in order to derive an integral equation for the wave function

$$K(3,4;1,2) - K_{+a}(3,1)K_{+b}(4,2)$$

$$= i \int \int \int \int d\tau_{5} d\tau_{6} d\tau_{7} d\tau_{8} K_{+a}(3,5)K_{+b}(4,6) \longrightarrow \psi(3,4) = -i \int \int \int \int \int d\tau_{5} d\tau_{6} d\tau_{7} d\tau_{8} K_{+a}(3,5)$$

$$\times \bar{G}(5,6;7,8)K(7,8;1,2) \longrightarrow \psi(3,4) = -i \int \int \int \int \int d\tau_{5} d\tau_{6} d\tau_{7} d\tau_{8} K_{+a}(3,5) \times K_{+b}(4,6) \bar{G}(5,6;7,8)\psi(7,8)$$



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Now that we have the expansion of the propagator, we can use this in order to derive an integral equation for the wave function

$$\psi(3,4) = -i \int \int \int \int d\tau_5 d\tau_6 d\tau_7 d\tau_8 K_{+a}(3,5) \longrightarrow \Im \psi(p_{\mu}) = i \int d^4 p' \bar{G}(p, p'; K) \psi(p_{\mu'}) \times K_{+b}(4,6) \bar{G}(5,6; 7,8) \psi(7,8)$$

Salpeter, E. E., & Bethe, H. A. (1951). A relativistic equation for bound-state problems. *Physical Review*, *84*(6), 1232.

$$\mathfrak{F}\psi(p_{\mu}) = i \int d^4p' \bar{G}(p, p'; K) \psi(p_{\mu}')$$

Since is very difficult to solve such an integral equation, a new equation is often used: **Quasi-Potential equation**

$$\left[4(\mathbf{p}^2+m^2)-E^2\right]\psi(\mathbf{p})-g^2\int\frac{d^3q}{(2\pi)^3}\frac{1}{\sqrt{\mathbf{q}^2+m^2}}\frac{\psi(\mathbf{q})}{(\mathbf{p}-\mathbf{q})^2+\mu^2}=0$$

$$\left[4(\mathbf{p}^2+m^2)-E^2\right]\psi(\mathbf{p})-g^2\int\frac{d^3q}{(2\pi)^3}\frac{1}{\sqrt{\mathbf{q}^2+m^2}}\frac{\psi(\mathbf{q})}{(\mathbf{p}-\mathbf{q})^2+\mu^2}=0$$



$$\langle x | q, \mathbf{p}_1 \mathbf{p}_2 \rangle = V^{-1} e^{i(\mathbf{p}_1 \mathbf{x}_1 + \mathbf{p}_2 \mathbf{x}_2)} \langle x | M, \mathbf{P} \rangle = V^{-1/2} e^{i\mathbf{P} \cdot \mathbf{R}} \varphi_M(\mathbf{y})$$

The idea is to calculate the overlap of the two wave functions, in which the second is the BS Wave function, and insert the squared amplitude in a coalescence integral

$$\langle x | q, \mathbf{p}_1 \mathbf{p}_2 \rangle = V^{-1} e^{i(\mathbf{p}_1 \mathbf{x}_1 + \mathbf{p}_2 \mathbf{x}_2)} \langle x | M, \mathbf{P} \rangle = V^{-1/2} e^{i\mathbf{P} \cdot \mathbf{R}} \varphi_M(\mathbf{y})$$

The idea is to calculate the overlap of the two wave functions, in which the second is the BS Wave function, and insert the squared amplitude in a coalescence integral

Link to coalescence approach

$$N_{M} = C_{M} V^{3} \int \frac{d^{3}P}{(2\pi)^{3}} \frac{d^{3}p_{1}}{(2\pi)^{3}} \frac{d^{3}p_{2}}{(2\pi)^{3}} \times w(\mathbf{p}_{1})w(\mathbf{p}_{2}) \left| \langle q, \mathbf{p}_{1}\mathbf{p}_{1} | M, \mathbf{P} \rangle \right|^{2}$$

Conclusion(beginnings?)

- Study more deeply the motivations behind the fact that the elliptic flow of the light quarks contributes on the v₂ of Λ_C more than on the v₂ of the D mesons in [2,4] slice
- How the v_3 affect angular distributions in meson and baryon and how the v3 of Λ_c is generated
- Why when there is only the v_2 of the bulk turned on we see difference between the v_2 of meson and baryon already at low p_T
- Continue the development of the Bethe-Salpeter approach to coalescence

Meson	Mass(MeV)	l (J)	Decay modes	B.R.
$D^+ = \bar{d}c$	1869	$\frac{1}{2}(0)$		
$D^0 = \bar{u}c$	1865	$\frac{1}{2}(0)$		
$D_s^+ = \bar{s}c$	2011	$\tilde{0}(0)$		
Resonances				
D^{*+}	2010	$\frac{1}{2}(1)$	$D^0\pi^+; D^+X$	68%,32%
D^{*0}	2007	$\frac{1}{2}(1)$	$D^0\pi^0;~D^0\gamma$	62%,38%
D_s^{*+}	2112	Õ(1)	D_s^+X	100%
Baryon				
$\Lambda_c^+ = udc$	2286	$0(\frac{1}{2})$		
$\Xi_c^+ = usc$	2467	$\frac{1}{2}(\frac{1}{2})$		
$\Xi_c^0 = dsc$	2470	$\frac{\tilde{1}}{2}(\frac{\tilde{1}}{2})$		
$\Omega_c^0 = ssc$	2695	$\tilde{0}(\frac{f}{2})$		
Resonances				
Λ_c^+	2595	$0(\frac{1}{2})$	$\Lambda_c^+ \pi^+ \pi^-$	100%
Λ_c^+	2625	$0(\frac{3}{2})$	$\Lambda_c^+ \pi^+ \pi^-$	100%
Σ_c^+	2455	$1(\frac{1}{2})$	$\Lambda_c^+ \pi$	100%
Σ_c^+	2520	$1(\frac{3}{2})$	$\Lambda_c^+ \pi$	100%
$\Xi_{c}^{'+,0}$	2578	$\frac{1}{2}(\frac{1}{2})$	$\Xi_c^{+,0}\gamma$	100%
Ξ_{c}^{+}	2645	$\frac{1}{2}(\frac{3}{2})$	$\Xi_{c}^{+}\pi^{-}$,	100%
Ξ	2790	$\frac{1}{2}\left(\frac{1}{2}\right)$	$\Xi_c^{\prime}\pi$,	100%
Ξ	2815	$\frac{1}{2}\left(\frac{3}{2}\right)$	$\Xi_c^{}\pi$,	100%
Ω_c^0	2770	$\frac{2}{0}(\frac{3}{2})$	$\Omega_c^0 \gamma$,	100%

In our calculations we take into account hadronic channels including the ground states + first excited states

Statistical factor suppression for resonances

$$\frac{[(2J+1)(2I+1)]_{H^*}}{[(2J+1)(2I+1)]_H} \left(\frac{m_{H^*}}{m_H}\right)^{3/2} e^{-(m_{H^*}-m_H)/T}$$

AA @ RHIC & LHC

Wave function widths σ_p of baryon and mesons are the same at RHIC and LHC



S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348