TROY: T-MATRIX-BASED ROUTINE FOR HADRON FEMTOSCOPY

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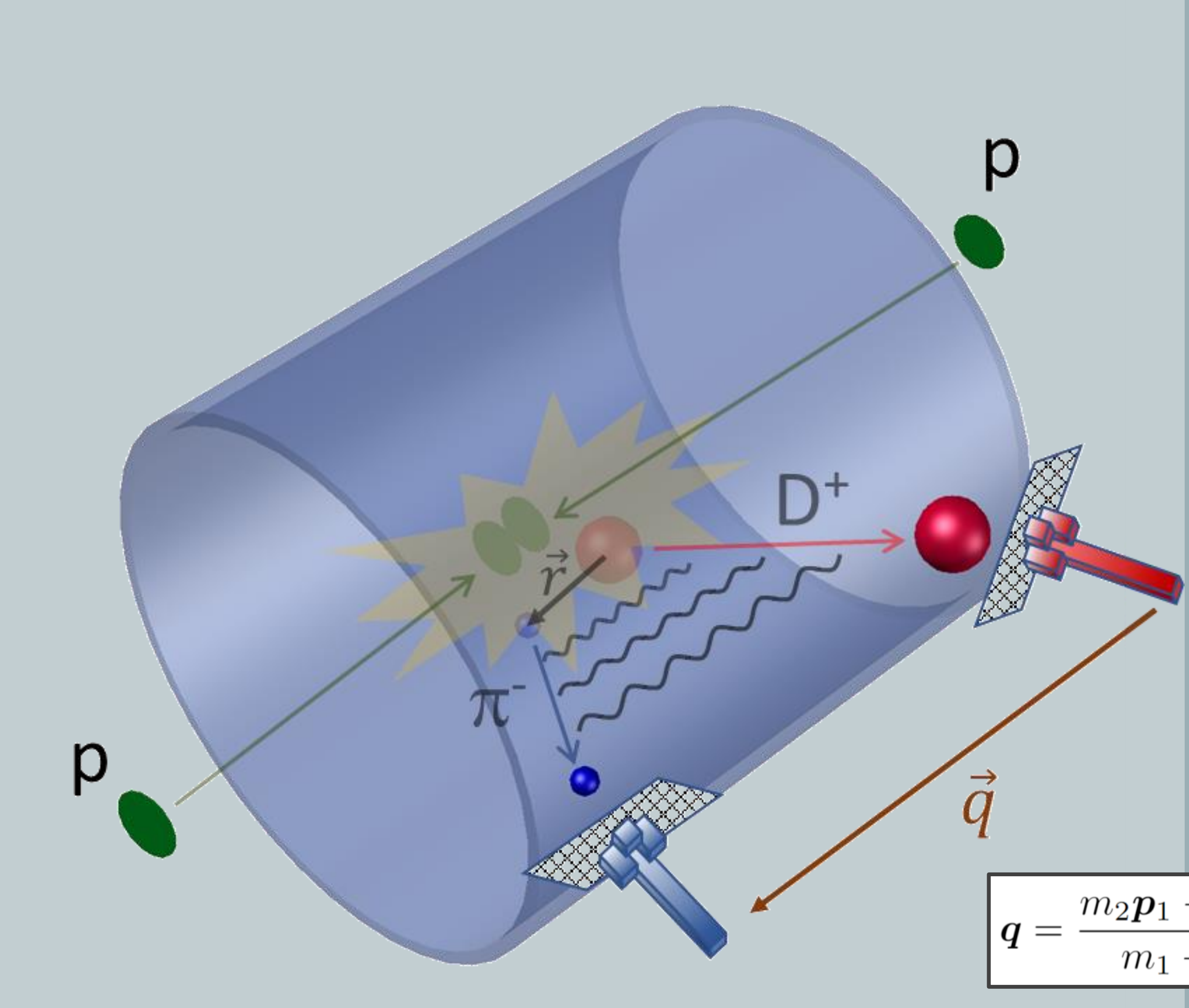
Femtoscopy in RHICs



$$q = \frac{m_2 \boldsymbol{p}_1 - m_1 \boldsymbol{p}_2}{m_1 + m_2}$$

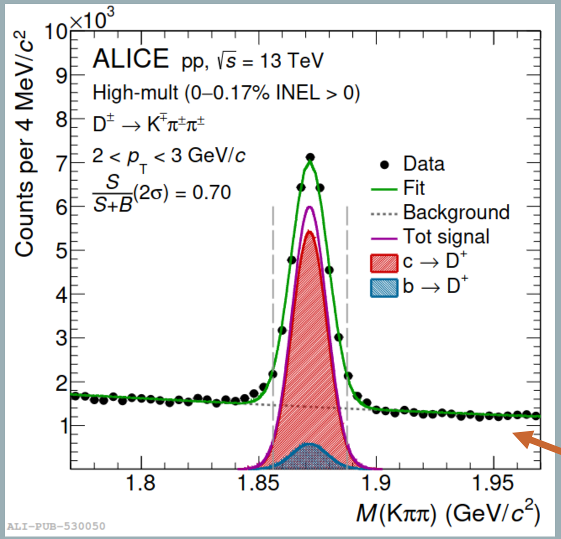
Heinz, Jacak, *Ann.Rev.Nucl.Part.Sci.* 49 (1999) 529-579
Lisa, Pratt, Wiedemann,
Ann.Rev.Nucl.Part.Sci. 55 (2005) 357

Pair Correlation Function

$$C(\boldsymbol{q}) = \mathcal{N}\,\frac{N_{\text{same}}(\boldsymbol{q})}{N_{\text{mixed}}(\boldsymbol{q})}$$

$$C(\boldsymbol{q}) > 1 : \text{correlation}$$
$$C(\boldsymbol{q}) < 1 : \text{anticorrelation}$$

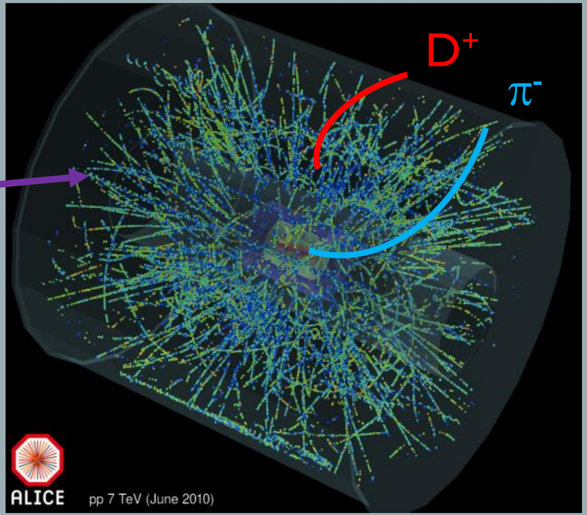
Femtoscopy in RHICs



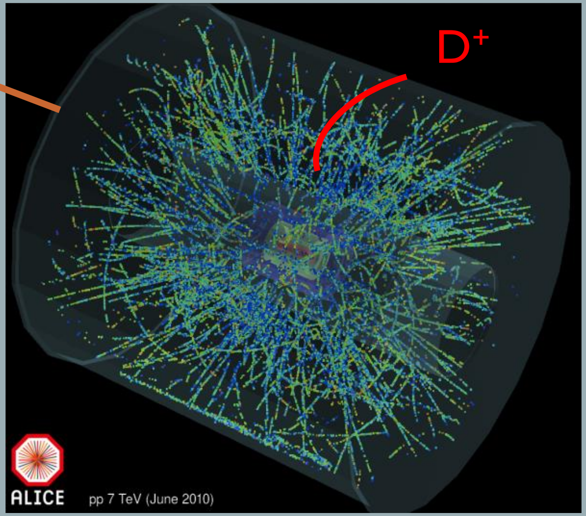
D± reconstruction

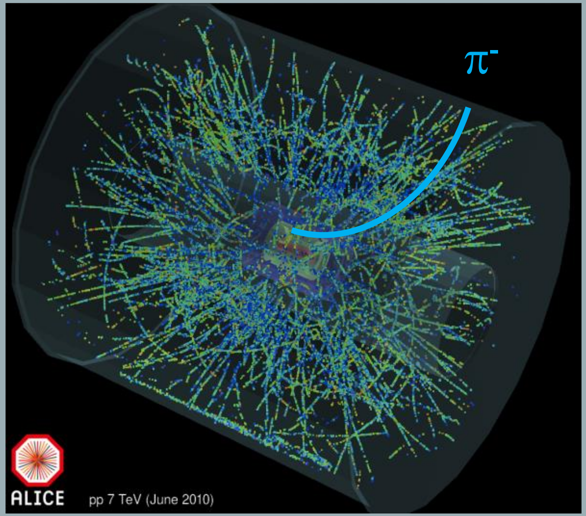
Pair Correlation Function

$$C(\boldsymbol{q}) = \mathcal{N} \frac{N_{\mathrm{same}}(\boldsymbol{q})}{N_{\mathrm{mixed}}(\boldsymbol{q})}$$



'Event mixing' technique





10^6-10^7 events

Koonin-Pratt formula

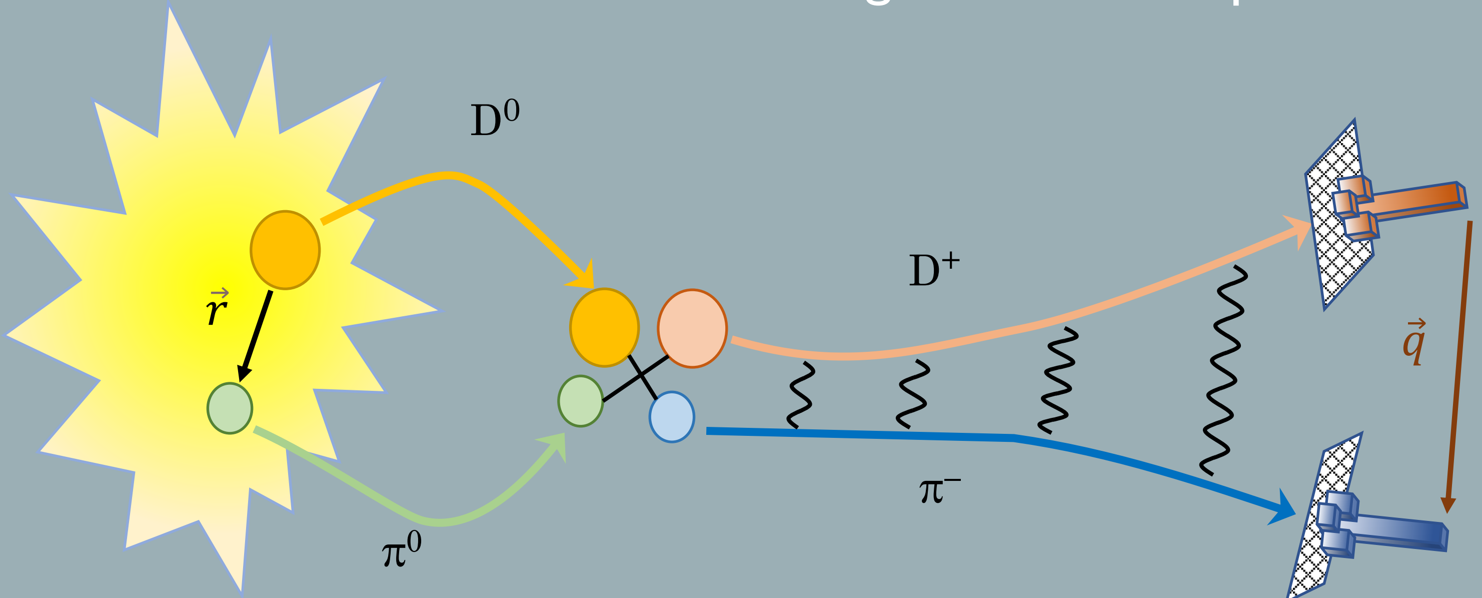
Koonin, *Phys.Lett.B*, 70, 43 (1977)
Pratt, Csorgo, Zimanyi, *Phys.Rev.C*, 42, 2646(1990)

Koonin-Pratt formula

$$C(\boldsymbol{q}) = \int d^3r \sum_i w_i \; S_i(\boldsymbol{r}) \; |\Psi_i(\boldsymbol{q};\boldsymbol{r})|^2$$

Wave function connecting initial channel with observed one

weights related to production mechanism of channels



$C(\boldsymbol{q}) > 1$: attraction
$C(\boldsymbol{q}) < 1$: repulsion

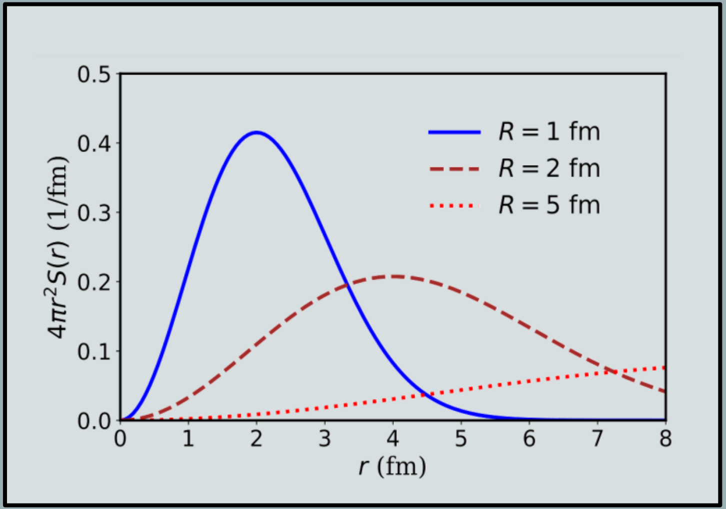
Fabbietti, Mantovani Sarti, Vazquez Doce, *Ann. Rev. Nucl. Part. Sci*, 71, 377 (2021)

Correlation function

TROY FRAMEWORK

Gaussian source function

$$S(r) = \frac{1}{(2\sqrt{\pi}R)^3} \exp\left(-\frac{r^2}{4R^2}\right)$$



Complete Coulomb wave function

$$C(q) = \int d^3r S(r)\, |\Phi_f^{C}(\boldsymbol{q};\boldsymbol{r})|^2 + \int 4\pi r^2\, dr S(r) \left[\sum_i w_i\, |\varphi_i(q;r)|^2 - |\Phi_{0\,f}^{C}(q\,r)|^2\right]$$

s-wave strong + Coulomb wf

s-wave Coulomb wf

Joachain, *Quantum Collision Theory* (1975)

Wave function and scattering T matrix

$$\hat{H}|\Psi\rangle = E|\Psi\rangle$$

$$\hat{H}_0|\Phi\rangle = E|\Phi\rangle$$

$$V|\Psi\rangle = T|\Phi\rangle$$

Interacting wave func.

Free wave func.

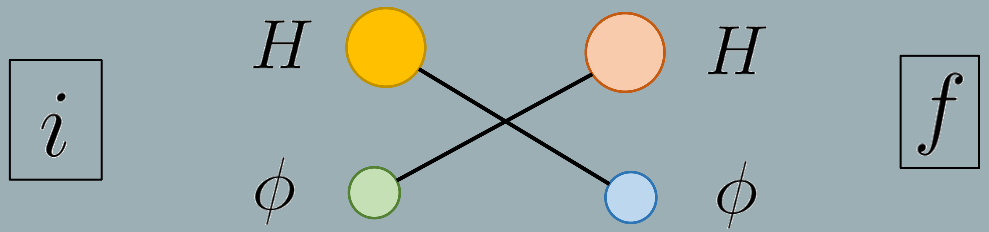
Lippmann-Schwinger equation

$$|\Psi\rangle = |\Phi\rangle + \frac{1}{E - \hat{H}_0 + i\eta} T\,|\Phi\rangle$$

TROY FRAMEWORK

strong + Coulomb

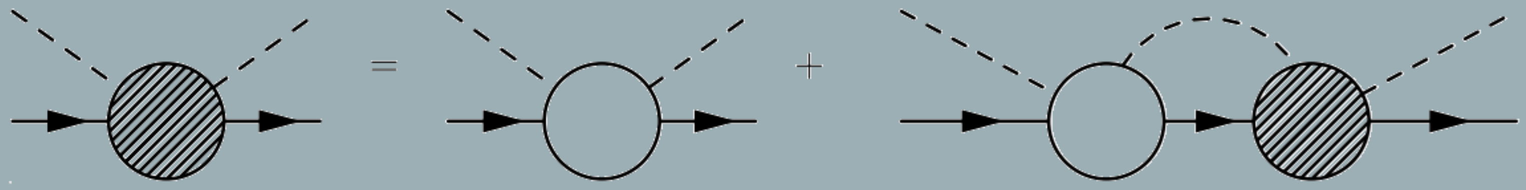
$$\varphi_i(q;r) = j_0(qr)\delta_{if} + \int_0^\infty \frac{4\pi q'^2\, dq'}{(2\pi)^3} \frac{T_{if}(q',q;\sqrt{s})\, j_0(q'r)}{2\omega_{H,i}\, 2\omega_{\phi,i}(\sqrt{s} - \omega_{H,i} - \omega_{\phi,i} + i\eta)}$$



Off-shell T matrix $i \to f$

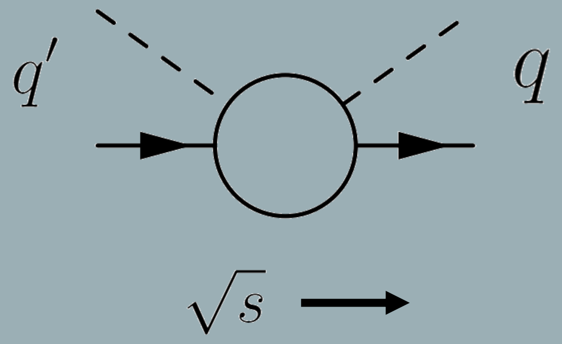
Off-shell T-matrix equation

TROY FRAMEWORK



$$T_{if}(q', q; \sqrt{s}) = V_{if}(q', q; \sqrt{s}) + \sum_l \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} \frac{V_{il}(q', k; \sqrt{s})\, T_{lf}(k, q; \sqrt{s})}{2\omega_{H,l}\, 2\omega_{\phi,l}\, \left(\sqrt{s} - \omega_{H,l} - \omega_{\phi,l} + i\eta\right)}$$

$$V_{if}(q', q; \sqrt{s})$$



q : on-shell
q': off-shell

Regulator: Form Factor

$$f(q, q') = \exp\left(-\frac{q^2 + q'^2}{\Lambda^2}\right)$$

$$\Lambda = 800\ \mathrm{MeV}/c$$

Heavy-meson effective theory

$$\mathcal{L}_{\rm LO} = \mathcal{L}_{\rm LO}^{\rm ChPT} + \langle \nabla^\mu H \nabla_\mu H^\dagger \rangle - m_H^2 \langle H H^\dagger \rangle - \langle \nabla^\mu H^{*\nu} \nabla_\mu H_\nu^{*\dagger} \rangle + m_H^2 \langle H^{*\nu} H_\nu^{*\dagger} \rangle$$
$$+ ig \langle H^{*\mu} u_\mu H^\dagger - H u^\mu H_\mu^{*\dagger} \rangle + \frac{g}{2m_D} \langle V_\mu^* u_\alpha \nabla_\beta H_\nu^{*\dagger} - \nabla_\beta V_\mu^* u_\alpha H_\nu^{*\dagger} \rangle \epsilon^{\mu\nu\alpha\beta} ,$$
$$\mathcal{L}_{\rm NLO} = \mathcal{L}_{\rm NLO}^{\rm ChPT} - h_0 \langle H H^\dagger \rangle \langle \chi_+ \rangle + h_1 \langle H \chi_+ H^\dagger \rangle + h_2 \langle H H^\dagger \rangle \langle u^\mu u_\mu \rangle + h_3 \langle H u^\mu u_\mu H^\dagger \rangle$$
$$+ h_4 \langle \nabla_\mu H \nabla_\nu H^\dagger \rangle \langle u^\mu u^\nu \rangle + h_5 \langle \nabla_\mu H \{u^\mu, u^\nu\} \nabla_\nu H^\dagger \rangle$$
$$+ \tilde{h}_0 \langle H^{*\mu} H_\mu^{*\dagger} \rangle \langle \chi_+ \rangle - \tilde{h}_1 \langle H^{*\mu} \chi_+ H_\mu^{*\dagger} \rangle - \tilde{h}_2 \langle H^{*\mu} H_\mu^{*\dagger} \rangle \langle u^\nu u_\nu \rangle - \tilde{h}_3 \langle H^{*\mu} u^\nu u_\nu H_\mu^{*\dagger} \rangle$$
$$- \tilde{h}_4 \langle \nabla_\mu H^{*\alpha} \nabla_\nu H_\alpha^{*\dagger} \rangle \langle u^\mu u^\nu \rangle - \tilde{h}_5 \langle \nabla_\mu H^{*\alpha} \{u^\mu, u^\nu\} \nabla_\nu H_\alpha^{*\dagger} \rangle ,$$

$$H = \left(D^0 \; D^+ \; D_s^+ \right)$$

$$H_\mu^* = \left(D_\mu^{*0} \; D_\mu^{*+} \; D_{s,\mu}^{*+} \right)$$

$$u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$$

$$u = \exp\left[\frac{i}{\sqrt{2} f_\pi} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \right]$$

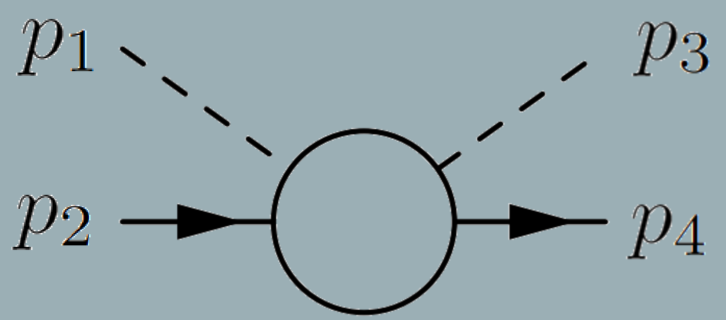
$h_i, \tilde{h}_i$: NLO low-energy constants
Guo et al. *Eur. Phys. J.C79*, 1, 13 (2019)

Kolomeitsev, Lutz *Phys.Lett.* B582 (2004) 39; Hofmann, Lutz *Nucl.Phys. A*733 (2004) 142; Guo, Hanhart. Krewald, Meissner *Phys.Lett.* B666 (2008) 251; Geng, Kaiser, Martin-Camalich, Weise *Phys.Rev.D*82,05422 (2010); Abreu, Cabrera, Llanes-Estrada, JMT-R. *Annals Phys.* 326 (2011) 2737...

Heavy-meson effective theory

$$
\begin{aligned}
V_{ij}(p_1,p_2,p_3,p_4) \;=\; & \frac{1}{f_\pi^2}\Big[\frac{C_{\rm LO}^{ij}}{4}\left[(p_1+p_2)^2-(p_2-p_3)^2\right]-4C_0^{ij}h_0+2C_1^{ij}h_1 \\
& -2C_{24}^{ij}\Big(2h_2(p_2\cdot p_4)+h_4\big((p_1\cdot p_2)(p_3\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\big)\Big) \\
& +2C_{35}^{ij}\Big(h_3(p_2\cdot p_4)+h_5\big((p_1\cdot p_2)(p_3\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\big)\Big)\Big]
\end{aligned}
$$

$$
p_1^\mu = \left(\frac{s+m_1^2-m_2^2}{2\sqrt{s}},\boldsymbol{p}\right)
$$



Montaña, Ramos, Tolos, JMT-R, *Phys.Rev.D*, 102, 096020 (2020)
Montaña, PhD Thesis, U. Barcelona 2022

L=0 partial wave

$$
V_{ij}^{\rm s-wave}(p,p';\sqrt{s}) = \frac{1}{2}\int_{-1}^{1} d\cos\theta_{\boldsymbol{pp'}}\; V_{ij}(p_1,p_2,p_3,p_4)
$$

Off-shell T matrix

We just need real energies, but let us look into complex energy plane to pin down possible resonances

$$z = \sqrt{s} \in \mathbb{C}$$

$$T_{if}(q', q; z) = V_{if}(q', q; z) + \sum_l \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} \frac{V_{il}(q', k; z) T_{lf}(k, q; z)}{2\omega_{H,l} 2\omega_{\phi,l}(z - \omega_{H,l} - \omega_{\phi,l})}$$

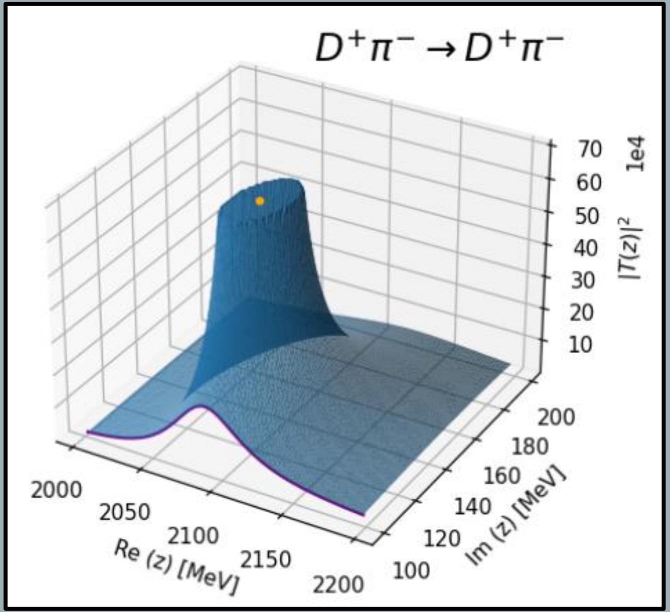
Channel S=0, Q=0 : $D^0\pi^0$ $D^+\pi^-$ $D^0\eta$ $D_s^+K^-$

Channel S=1, Q=+1: $D_s^+\pi^0$ D^0K^+ D^+K^0 $D_s^+\eta$

Off-shell T matrix

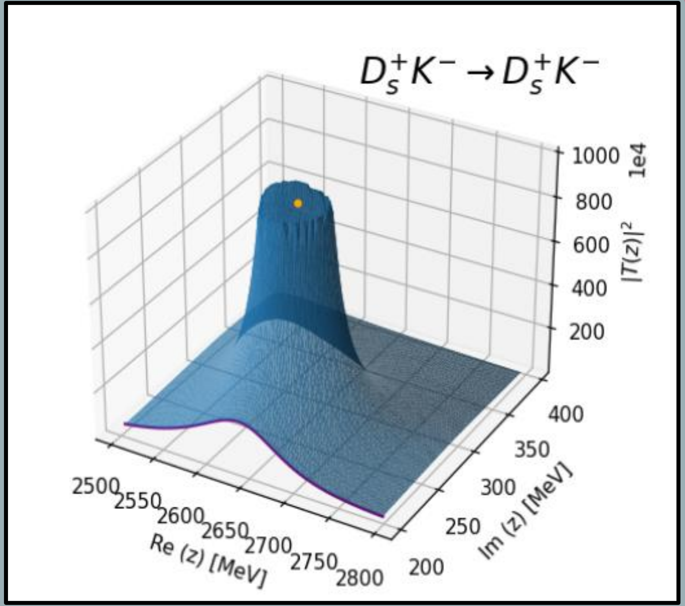
Channel S=0, Q=0

Channel S=1 Q=+1



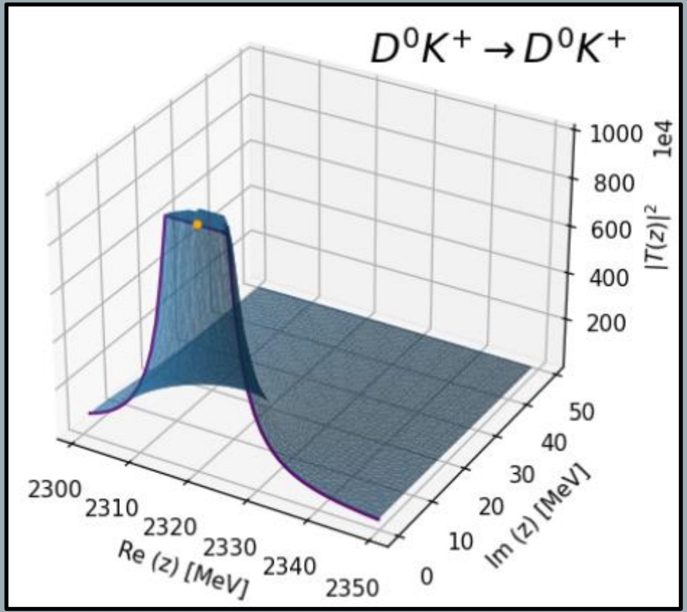
Riemann sheet (-,-,+,+)

z_{pole} = (2092 + i 129) MeV



Riemann sheet (-,-,-,+)

z_{pole} = (2647+ i 265) MeV



Riemann sheet (+,+,+,+)

z_{pole}=(2320 + i 0) MeV

Two-pole structure of $D_0^*(2300)$

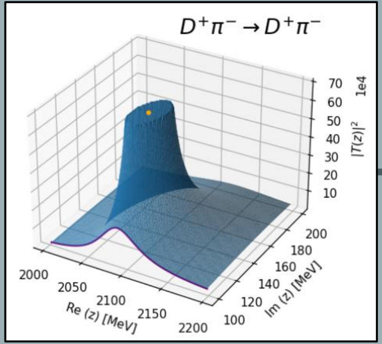
M. Albadalejo *et al. Phys.Lett.B* 767 (2017) 465 ; Guo *et al. Eur.Phys.J.C79* (2019)13; U. Meissner, *Symmetry* 12 (2020) 6, 981; JMT-R, *Symmetry* (2022)13 (2021), 8, 1400

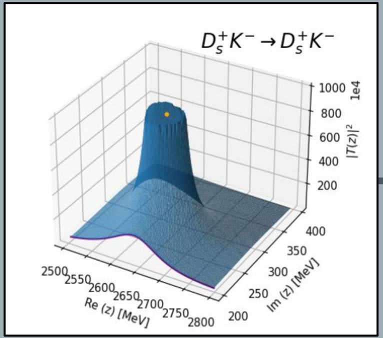
$D_{s0}^*(2317)$ bound state

below D^0K^+ (not coupled to $D_s^+\pi^0$)

Scattering lengths

JMT-R, Ramos, Tolos, 2307.03640 [hep-ph]





(S,Q)	channel	a(fm)	character
(-1,0)	D^+K^-	0.083	Attractive
(0,0)	$D^+\pi^-$	0.253	Attractive
	$D_s^+K^-$	-0.114+i 0.693	Attractive
(0,+2)	$D^+\pi^+$	-0.102	Repulsive
(1,0)	$D_s^+\pi^-$	0.0033	Attractive
(1,+2)	$D_s^+\pi^+$	0.0031	Attractive
	D^+K^+	-0.026+i 0.083	Repulsive
(2,+2)	$D_s^+K^+$	-0.220	Repulsive

$$a_i = -\frac{T_{ii}(m_1+m_2)}{8\pi(m_1+m_2)}$$

- $a > 0$: attractive
- $a < 0$: repulsive/strongly attractive
- $a \in \mathcal{C}$: open channel below

Coulomb interaction

$$\varepsilon\alpha = \pm\frac{1}{137}$$

We add truncated Coulomb potential in T-matrix calculation

$$V^{\mathrm{C}}(|\boldsymbol{p}'-\boldsymbol{p}|;\mathcal{R}_C) = \int_0^{\mathcal{R}_C} d^3r\ \mathrm{e}^{i(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{r}}\ \frac{\varepsilon\alpha}{r} = \frac{4\pi\varepsilon\alpha}{|\boldsymbol{p}'-\boldsymbol{p}|^2}\left[1-\cos(|\boldsymbol{p}'-\boldsymbol{p}|\mathcal{R}_C)\right]$$

$$\mathcal{R}_C = 60\ \mathrm{fm}$$

s-wave projection:

$$V^{\mathrm{C}}_{\mathrm{s-wave}}(p,p';\mathcal{R}_C) = \frac{2\pi\varepsilon\alpha}{pp'}\left\{\mathrm{Ci}\left[|p'-p|\mathcal{R}_C\right] - \mathrm{Ci}\left[(p'+p)\mathcal{R}_C\right] + \ln\left(\frac{p'+p}{|p'-p|}\right)\right\}$$

We have numerically checked against known Coulomb wave funcs when $V_{strong}=0$

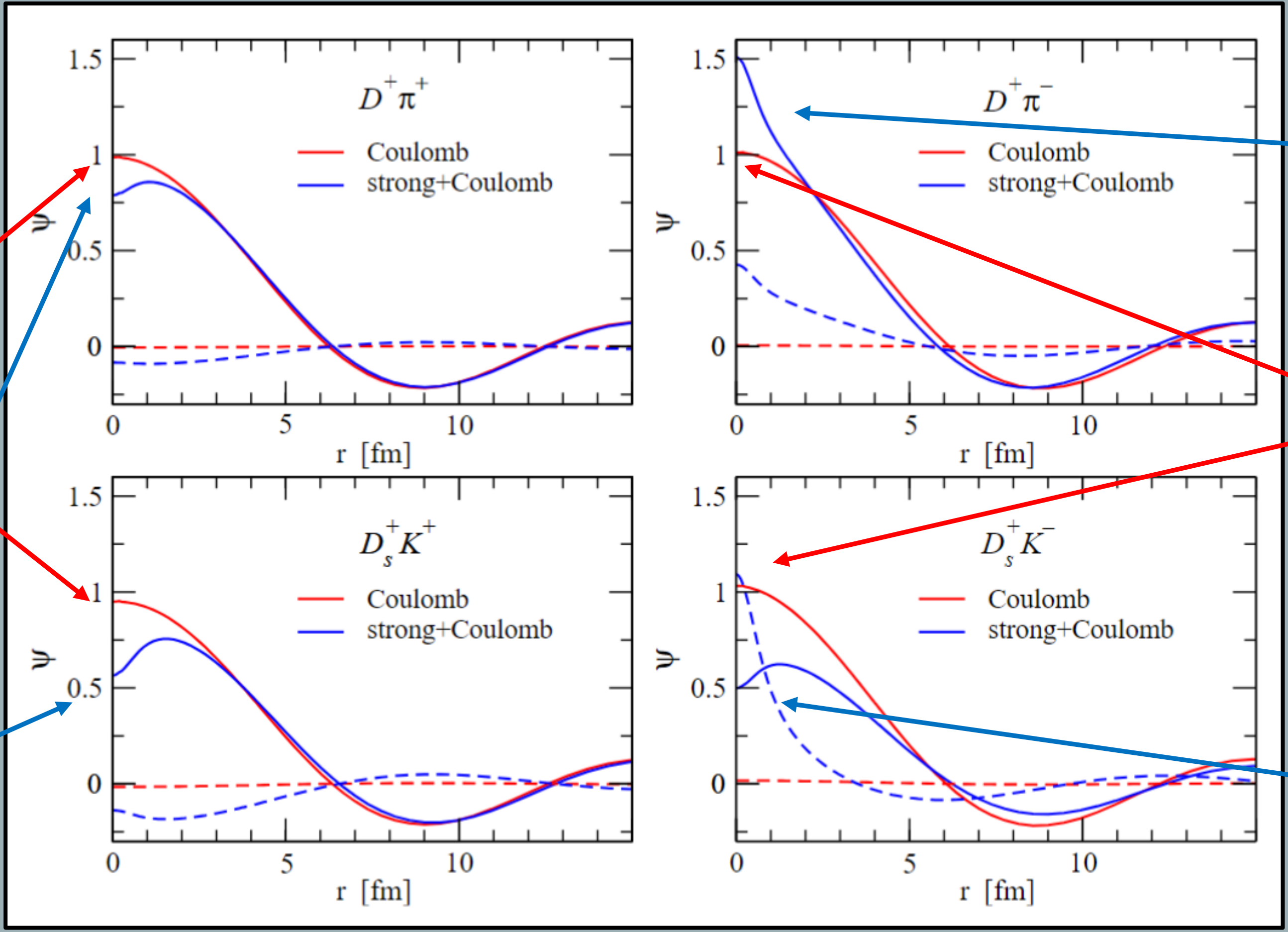
Joachain, Quantum Collision Theory (1975); Holzenkamp, Holinde, Speth, *Nucl.Pys.A*, 500, 485 (1989)

Total wave function

Solid: Re Ψ
Dashed: Im Ψ

Repulsive Coulomb: $\Psi(0) < 1$

Extra repulsion from V_{strong}



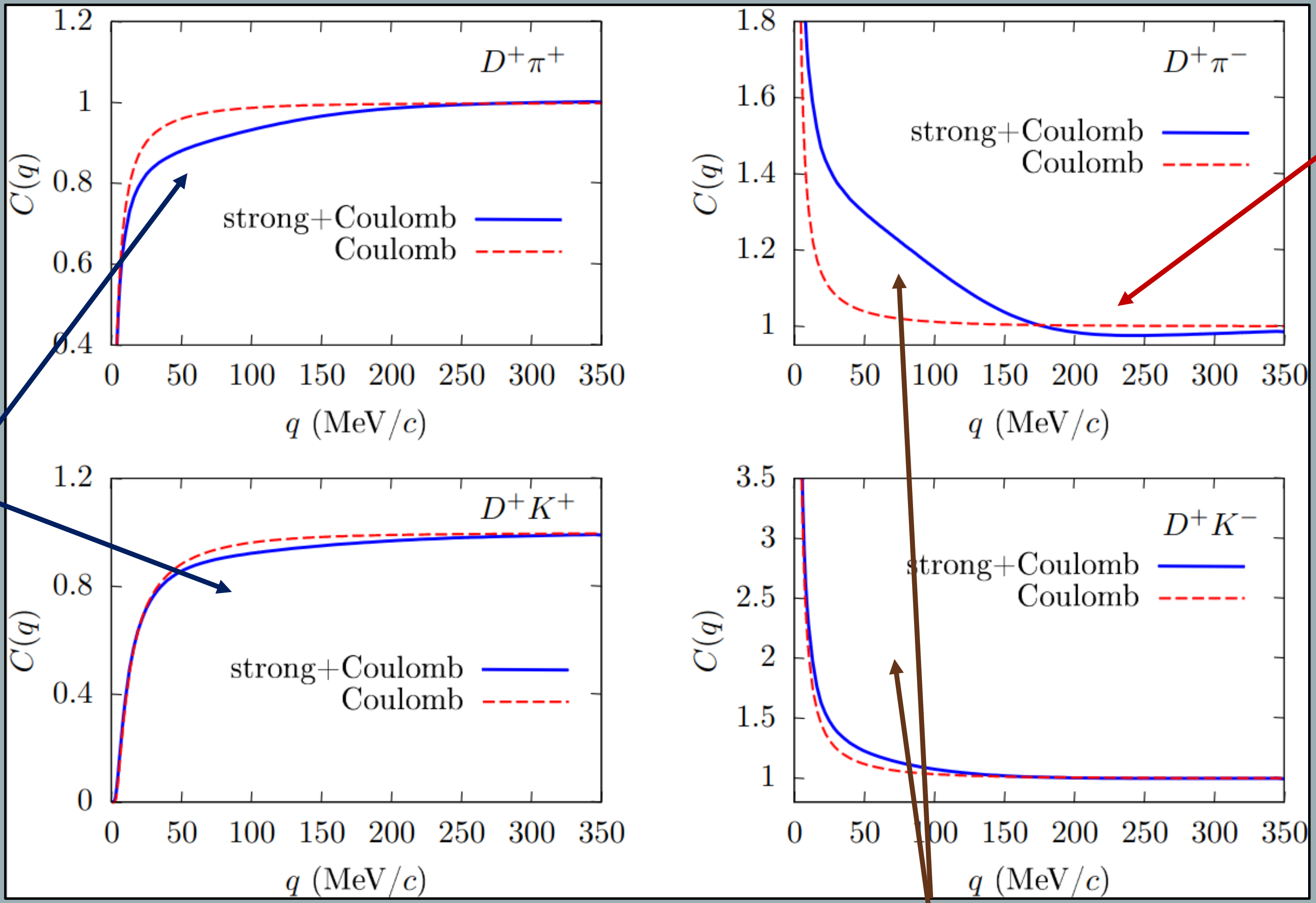
Extra attraction from V_{strong}

Attractive Coulomb: $\Psi(0) > 1$

Effect of resonance and open channel

$q = 100\ \mathrm{MeV}/c$

D-meson correlation functions



Lower pole of $D_0^*(2300)$ makes depletion <1 for q=250 MeV/c

Strong repulsion in like-sign correlations

$R = 1$ fm and $w_i = 1$ for all channels

Extra attraction in unlike-sign correlations

JMT-R, Ramos, Tolos, 2307.03640 [hep-ph]

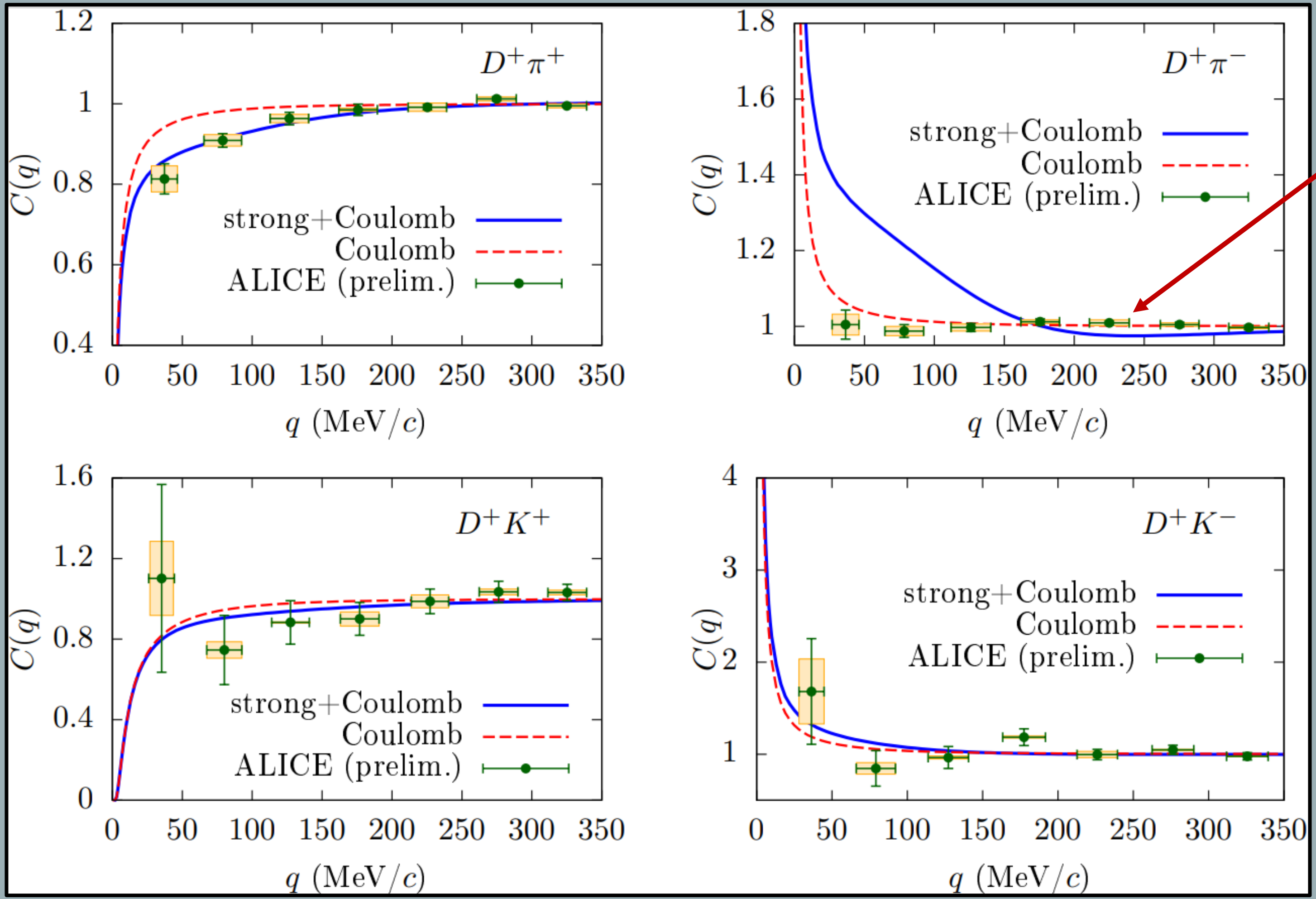
D meson correlation functions

ALICE data still preliminary:

QM2022 Conference

&

LHCP2023 Conference



$R = 1$ fm and $w_i = 1$ for all channels

Lower pole of $D_0^*(2300)$ makes depletion <1 for q=250 MeV/c

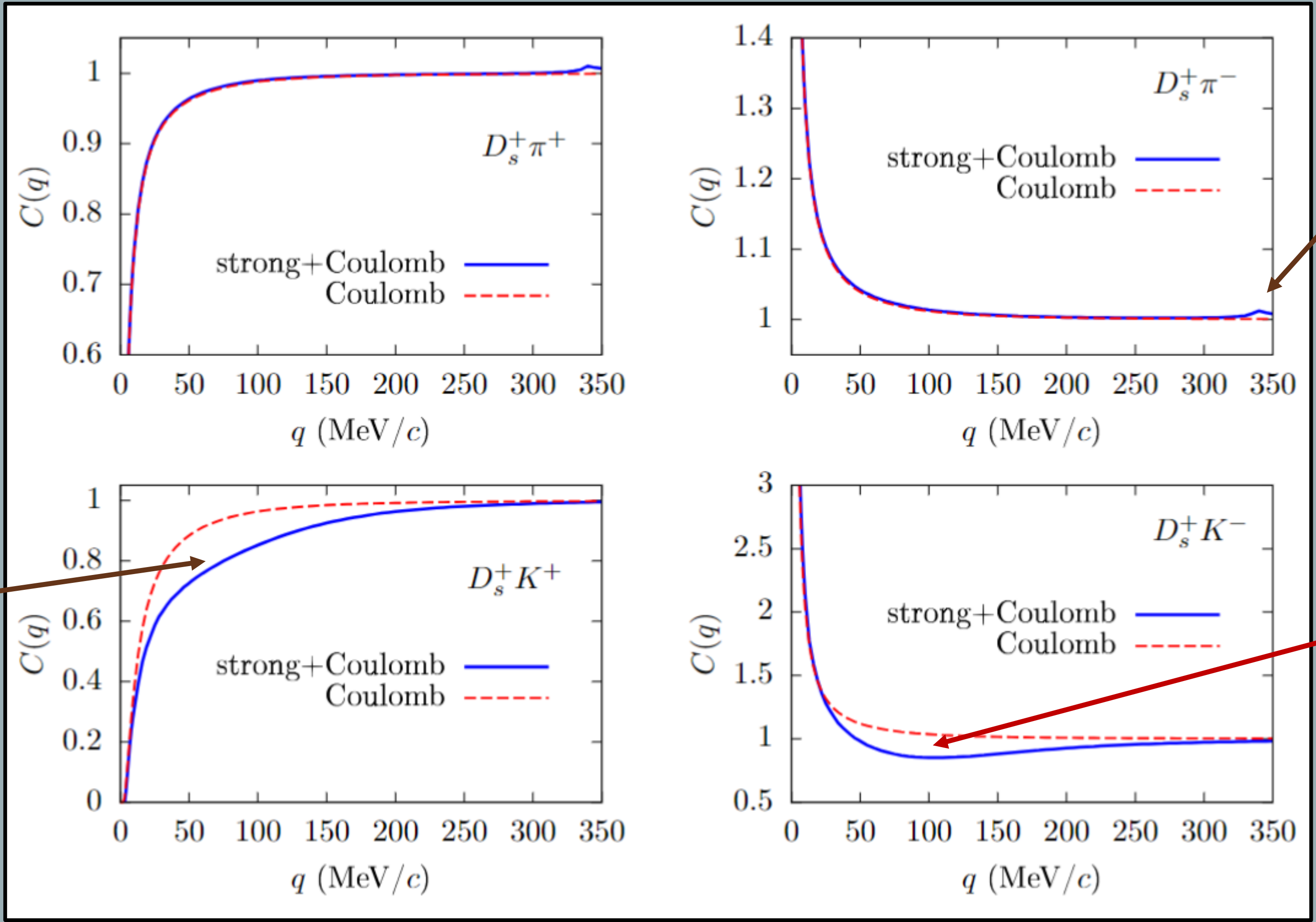
Strong deviations in $D^+\pi^+$ channel!

JMT-R, Ramos, Tolos, 2307.03640 [hep-ph]

D_s meson correlation functions

Small strong effect in pion channels

Strong repulsion predicted in $D_s^+K^+$ correlation



Cusps due to kinematic threshold

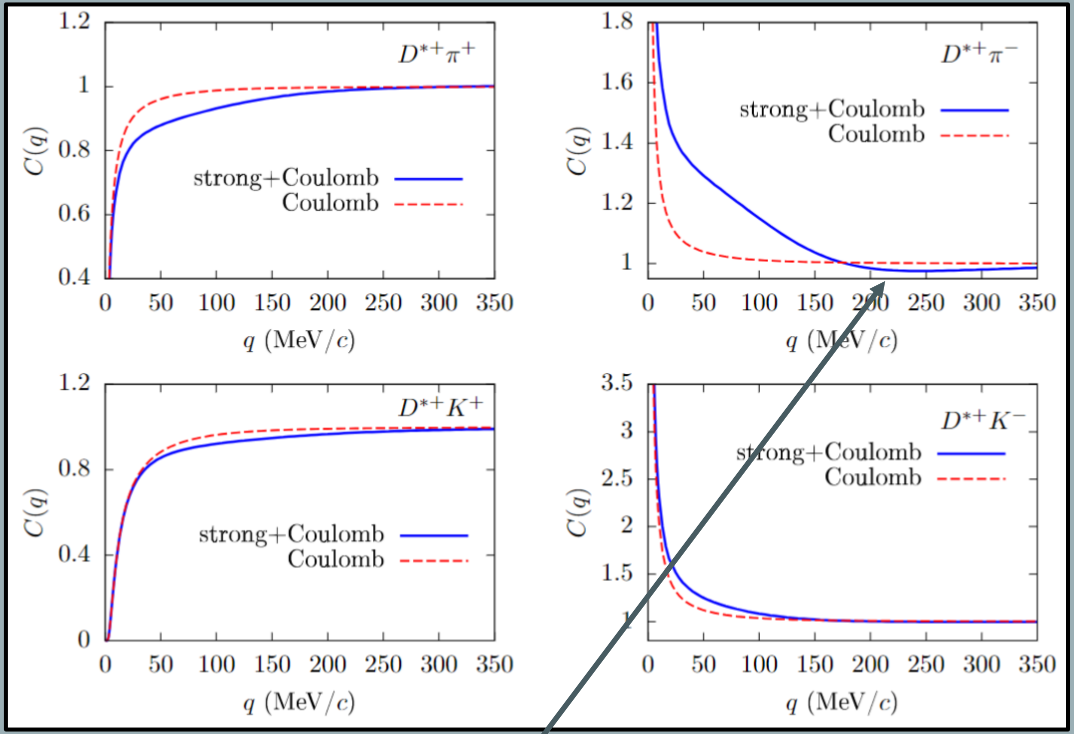
Higher pole of $D_0^*(2300)$ makes depletion<1 for q=100 Mev/c

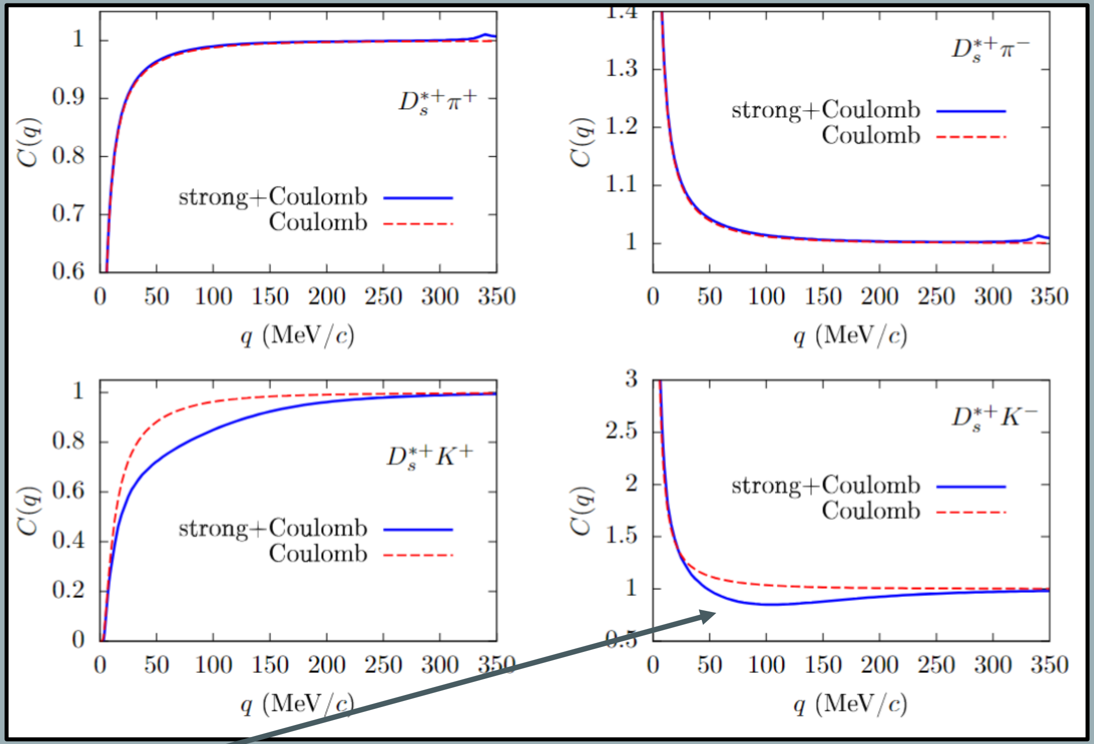
$R = 1$ fm and $w_i = 1$ for all channels

JMT-R, Ramos, Tolos, 2307.03640 [hep-ph]

D* and D_s* correlation functions

Analogous interactions from Heavy-Quark Spin Symmetry





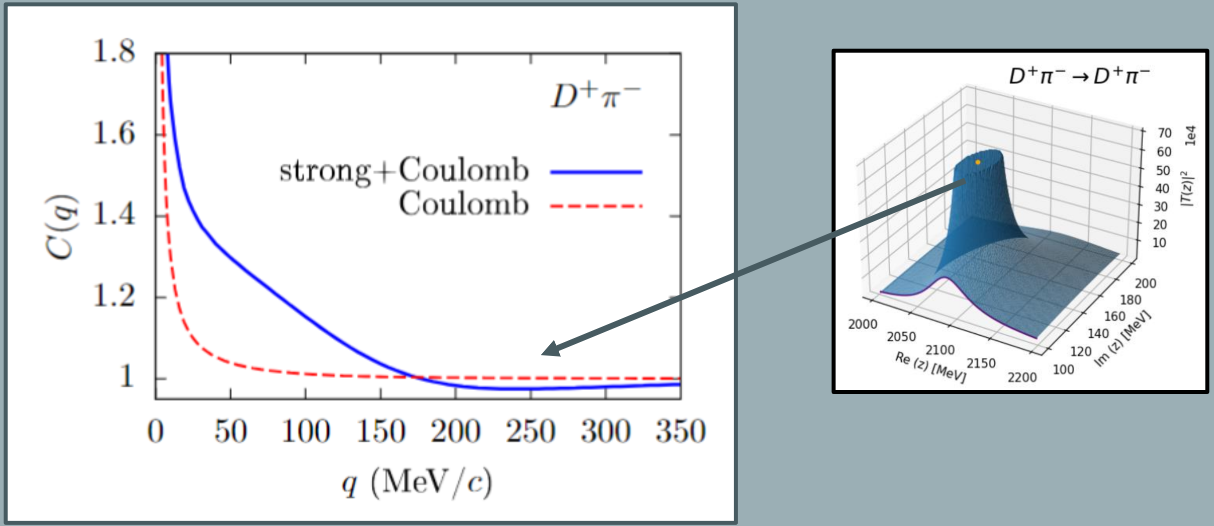
Depletions due to lower pole at q=250 MeV and higher pole at q=100 MeV of $D_1^*(2460)$

also seen in neutral channels: Albaladejo, Nieves, Ruiz-Arriola, 2304.03107 [hep-ph]

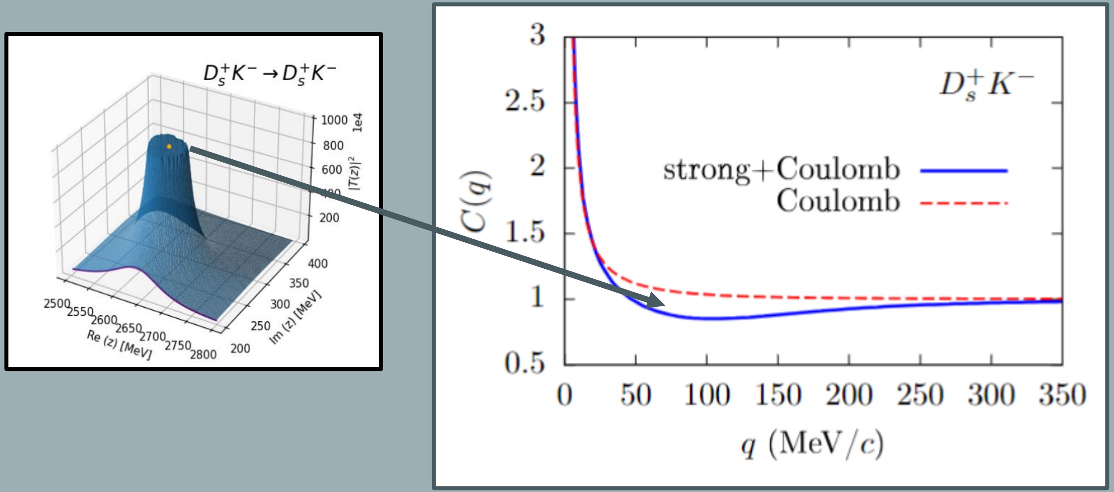
Summary

TROY FRAMEWORK

1. Femtoscopy CFs from T matrix
2. Off-Shell T matrix + Coulomb
3. Good agreement with experimental preliminary data **except $D^+\pi^-$**
4. Depletion due to poles of $D_0^*(2300)$
5. Depletion due to poles of $D_1^*(2460)$
6. Review source and weights



Effects of two-pole state $D_0^*(2300)$ in femtoscopy!



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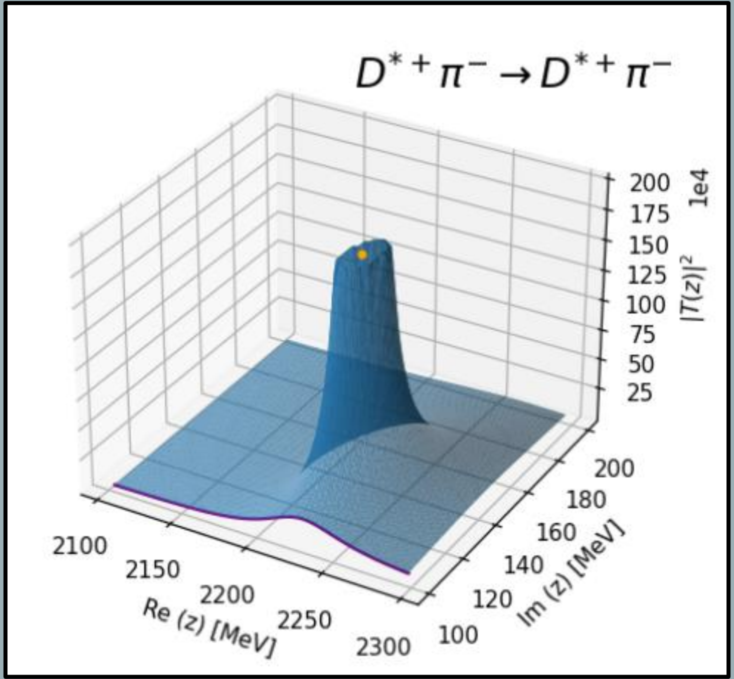
Off-shell T matrix

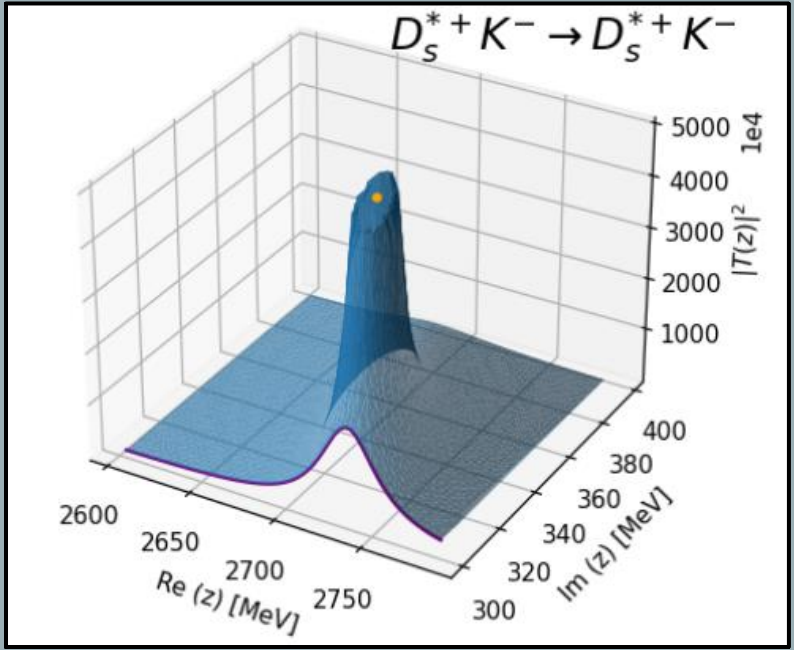
Channel S=0 Q=0: $D^{*0}\pi^0$ $D^{*+}\pi^-$ $D^{+0}\eta$ $D_s^{*+}K^-$

Riemann sheet (-,-,+,+)



J=1

Riemann sheet (-,-,-,+)

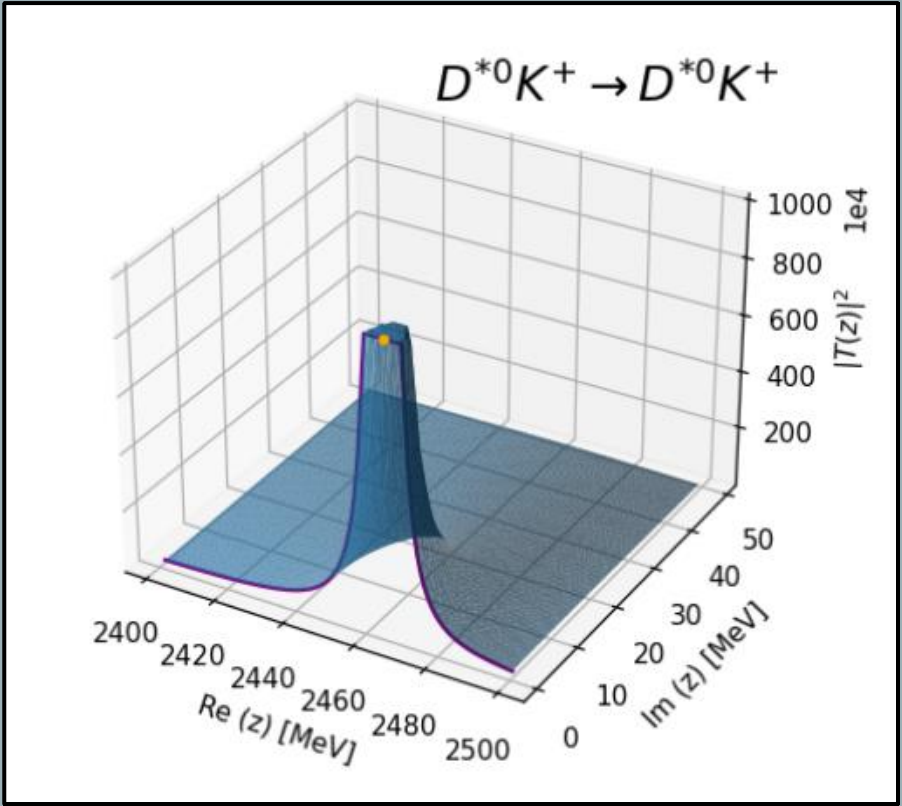


Off-shell T matrix

Channel S=1 Q=+1: $D_s^{*+}\pi^0$ $D^{*0}K^+$ $D^{*+}K^0$ $D_s^{*+}\eta$

J=1

Riemann sheet
(+,+,+,+)



$D_{s1}(2460)$ bound state below $D^{*0}K^+$

On-shell T-matrix

$$T = V(1 - VG)^{-1}$$

Oller, Oset, *Nucl.Phys.*A620 (1997) 438
Oset, Ramos, *Nucl.Phys*A635 (1998) 99

$J = 0$			
Generated state	(S, I)	z (MeV)	On-shell (Montaña *et al.*)
$D_0^*(2300)$ (lower pole)	$(0, \frac{1}{2})$	$2092.4 + \mathrm{i}\,129.5$	$2081.9 + \mathrm{i}\,86.0$
$D_0^*(2300)$ (higher pole)	$(0, \frac{1}{2})$	$2647.2 + \mathrm{i}\,264.8$	$2529.3 + \mathrm{i}\,145.4$
$D_{s0}^*(2317)$	$(1, 0)$	$2320.2 + \mathrm{i}\,0$	$2252.5 + \mathrm{i}\,0$

JMT-R, Ramos, Tolos
2307.03640 [hep-ph]

Montaña, Ramos, Tolos, JMT-R
*Phys.Lett.*B806 (2020) 135464
*Phys.Rev.D*102, 096020, (2020)

(also differences in regularization and isospin vs charge bases)

Off-shell versus On-shell T matrix

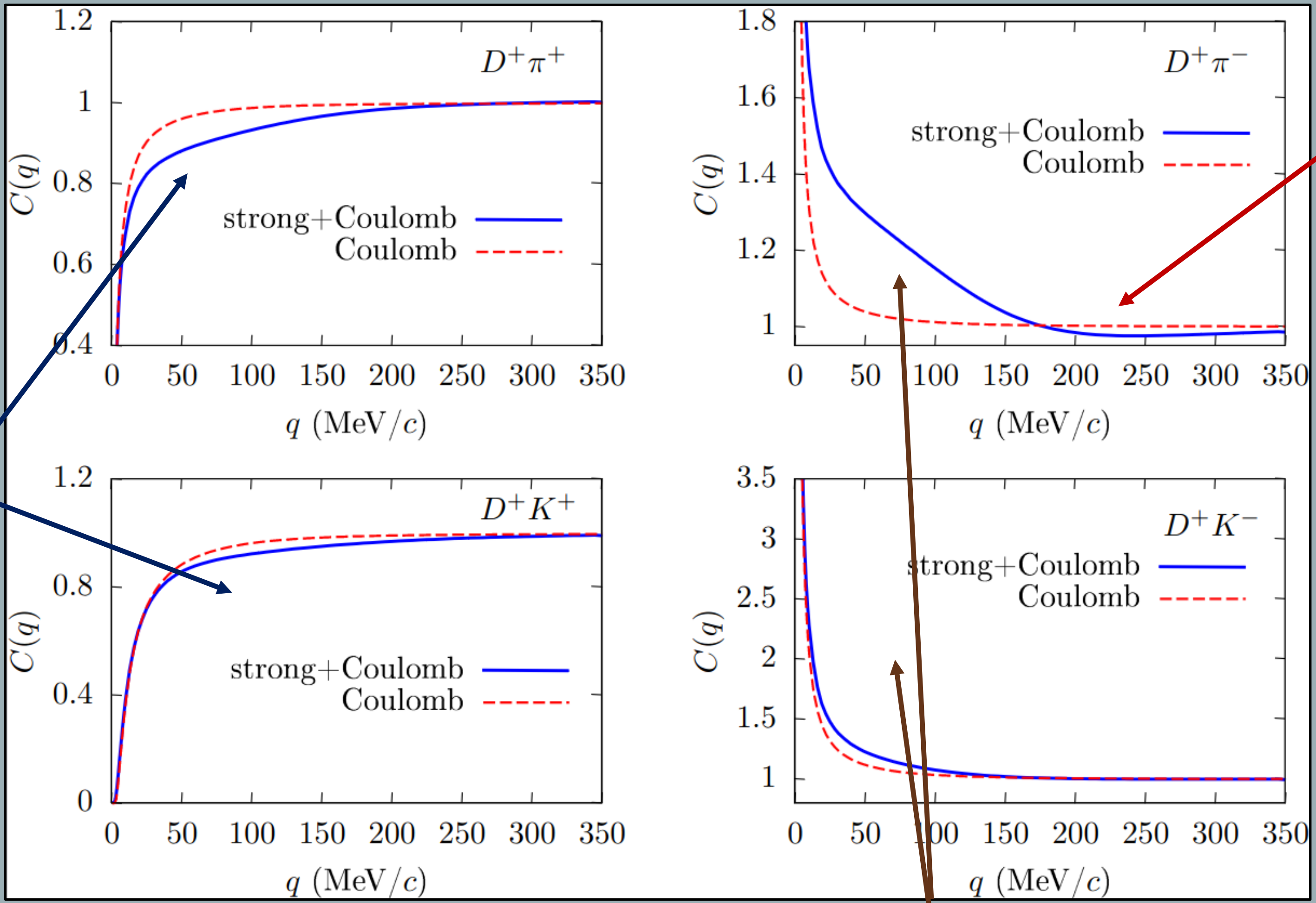
$J=0$			
Generated state	(S, I)	z (MeV)	On-shell (Montaña *et al.*)
$D_0^*(2300)$ (lower pole)	$(0, \frac{1}{2})$	$2092.4 + \mathrm{i}\,129.5$	$2081.9 + \mathrm{i}\,86.0$
$D_0^*(2300)$ (higher pole)	$(0, \frac{1}{2})$	$2647.2 + \mathrm{i}\,264.8$	$2529.3 + \mathrm{i}\,145.4$
$D_{s0}^*(2317)$	$(1, 0)$	$2320.2 + \mathrm{i}\,0$	$2252.5 + \mathrm{i}\,0$

$J=1$			
Generated state	(S, I)	z (MeV)	On-shell (Montaña *et al.*)
$D_1(2430)$ (lower pole)	$(0, \frac{1}{2})$	$2233.6 + \mathrm{i}\,130.8$	$2222.3 + \mathrm{i}\,84.7$
$D_1(2430)$ (higher pole)	$(0, \frac{1}{2})$	$2719.2 + \mathrm{i}\,330.1$	$2654.6 + \mathrm{i}\,117.3$
$D_{s1}(2460)$	$(1, 0)$	$2464.7 + \mathrm{i}\,0$	$2393.3 + \mathrm{i}\,0$

Off-shell
Gaussian form factor
Charge basis

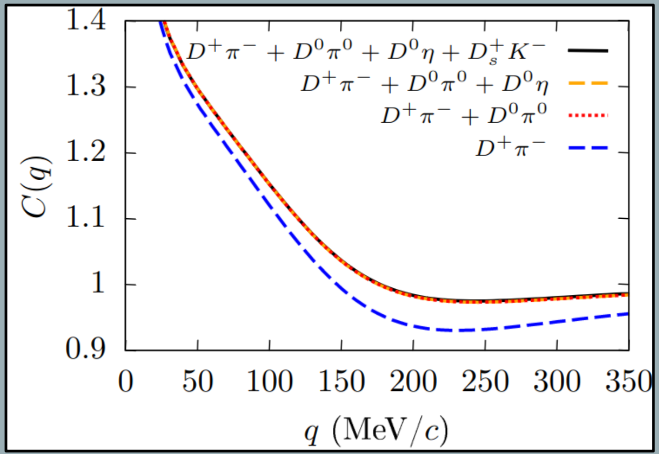
On-shell
Hard cutoff
Isospin basis

D meson correlation functions



Strong repulsion in like-sign correlations

Lower pole of $D_0^*(2300)$ makes depletion <1 for q=250 MeV/c

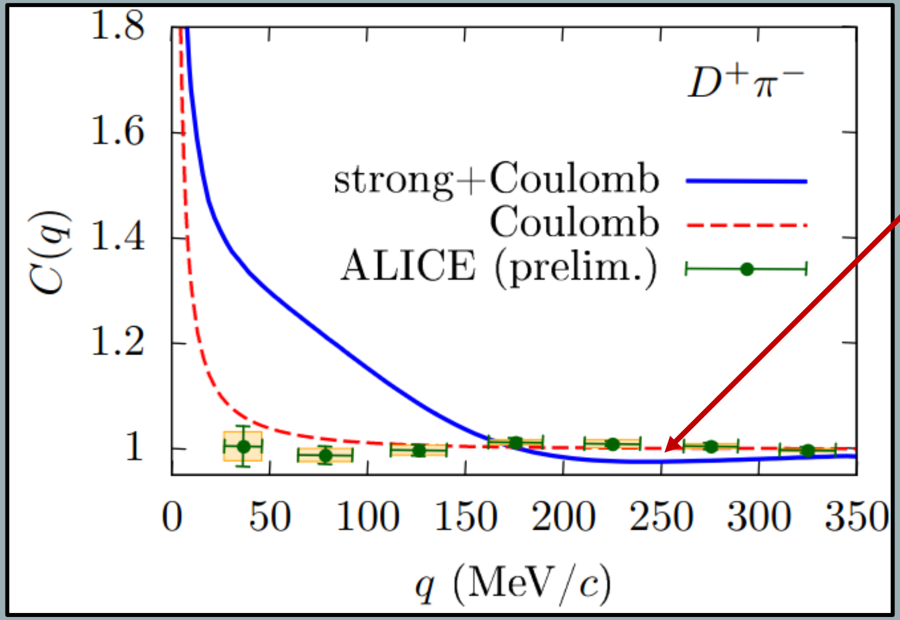


Effect of adding coupled channels

Extra attraction in unlike-sign correlations

JMT-R, Ramos, Tolos, 2307.03640 [hep-ph]

$D^+ \pi^-$ case



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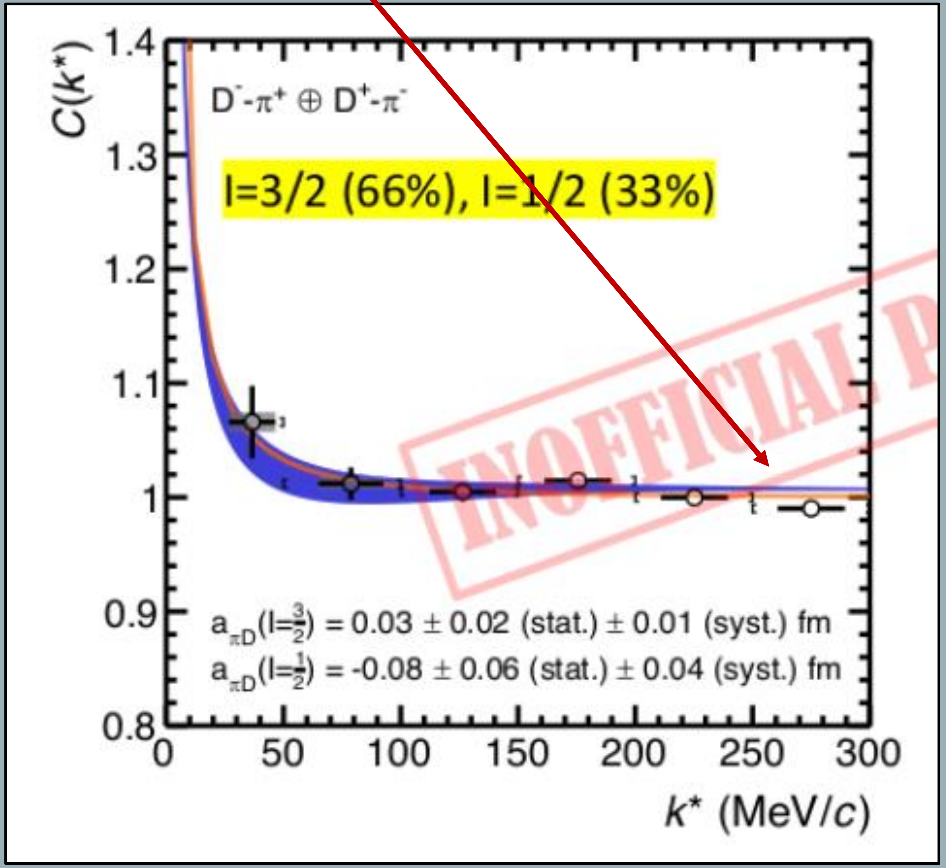
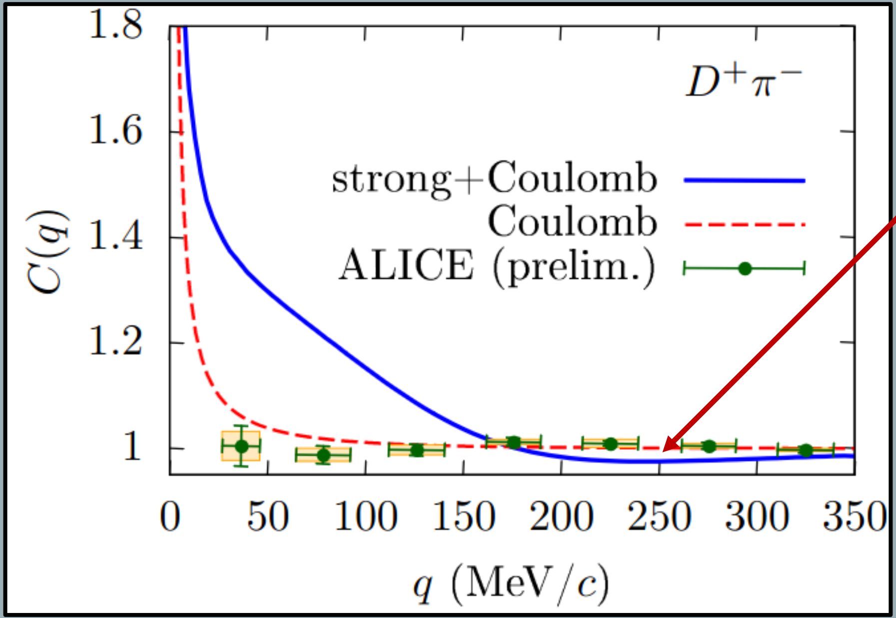
Unofficial ALICE data as of July 2023 shows C(q)<1 for q > 200 MeV : resonance?



Our result with Gaussian source of R=1 fm

$D^+ \pi^-$ case



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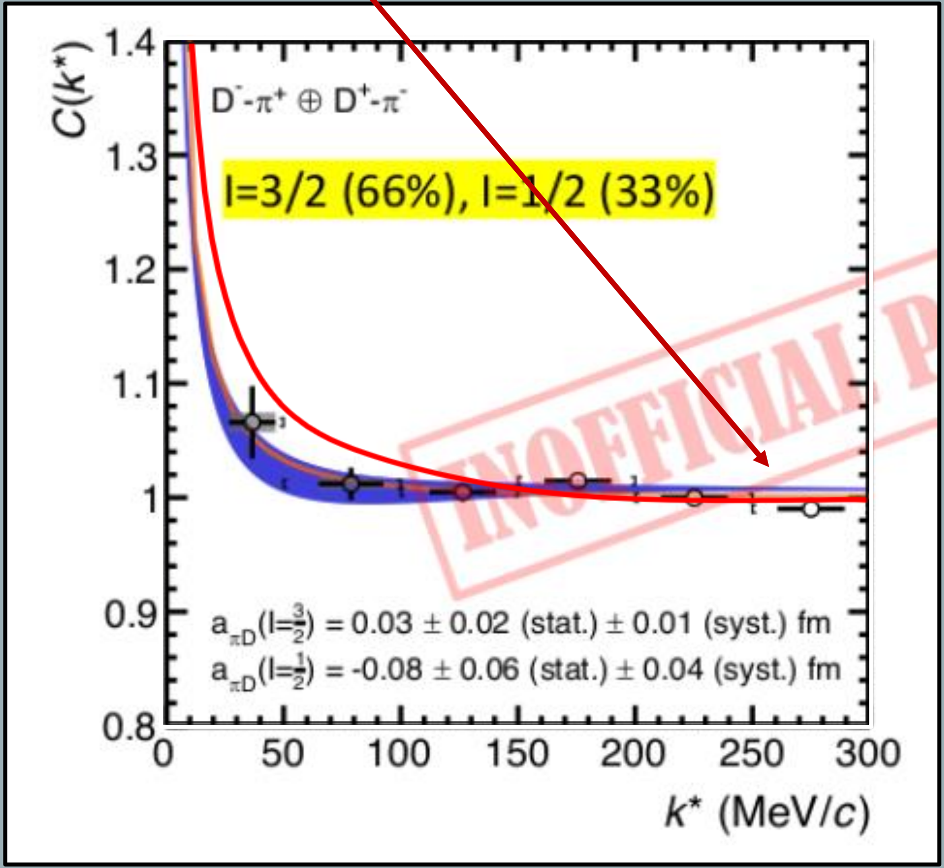

Our result with Gaussian source of R=1 fm

Our result with a secondary source with Gaussian radius of R=3 fm

Relativistic Coulomb

We need to correct for the nonrelativistic nature of Coulomb interaction

$$V^{\mathrm{C,rel}}_{\mathrm{s-wave}}(p,p';\sqrt{s}) = \sqrt{2\omega_1(p)}\sqrt{2\omega_2(p)}\sqrt{\xi(p;s)}\; V^{\mathrm{C}}_{\mathrm{s-wave}}(p,p')\;\sqrt{2\omega_1(p')}\sqrt{2\omega_2(p')}\sqrt{\xi(p';s)}$$

$$\xi(p;s) = 2\mu\frac{\sqrt{s}-\omega_1(p)-\omega_2(p)}{\frac{\lambda(s,m_1,m_2)}{4s}-p^2}$$

Pure kinematic factors

$$\sqrt{2\omega_1(p)}$$

Normalization factors in the Lippmann-Schwinger equation (needed when adding potentials)