J/Ψ formation within microscopic Langevin simulations

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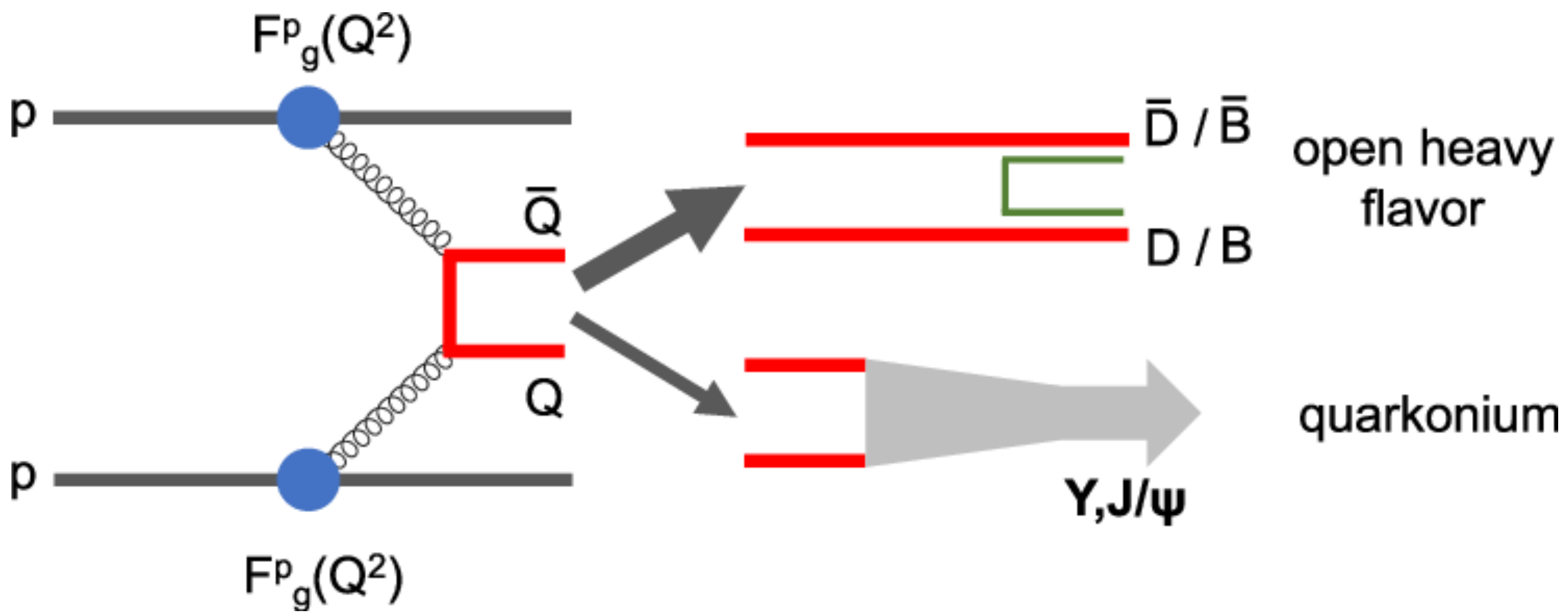
Motivation

Heavy Quarkonia as Hard Probes

- Production in primordial hard collisions

- Measurement of quarkonium states: conclusions about medium properties

➡ Sensitivity to initial conditions, QGP properties, medium interactions



Rothkopf (2020). Heavy quarkonium in extreme conditions. Physics Reports. 858. 10.1016

Motivation

Heavy Quarkonia as Hard Probes

- J/ψ suppression: signal for deconfinement
- Dissociation in QGP due to color screening and elastic scatterings with medium
- Possibility of regeneration processes at higher energies
- Theoretical models: non-relativistic description of heavy quarks

Fokker-Planck equation

- Boltzmann equation for the phase-space distribution of the heavy quarks:

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{E}\frac{\partial}{\partial x} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_Q(t, p, x) = C[f_Q]$$

- Assumption: no mean-field effects, uniform medium

$$\longrightarrow \quad f_Q(\boldsymbol{p}, t) = \frac{1}{V}\int d^3\boldsymbol{x} f_Q(\boldsymbol{x}, \boldsymbol{p}, t)$$

- Collision integral:

$$C[f_Q] = \int d^3\boldsymbol{k}[\omega(\boldsymbol{p}+\boldsymbol{k}, \boldsymbol{k}) f_Q(\boldsymbol{p}+\boldsymbol{k}) - \omega(\boldsymbol{p}, \boldsymbol{k}) f_Q(\boldsymbol{p})]$$

Fokker-Planck equation

➡ Reduction to Fokker-Planck equation:

$$\frac{\partial}{\partial t} f_Q(\boldsymbol{p}, t) = \frac{\partial}{\partial p_i}\left\{ A_i(\boldsymbol{p}) f_Q(\boldsymbol{p}, t) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(\boldsymbol{p}, t)] \right\}$$

- With drag coefficients $A_i(\boldsymbol{p})$ and diffusion coefficients $B_{ij}(\boldsymbol{p})$
- Approximation: constant transport coefficients:

$$A(\boldsymbol{p}) \equiv \gamma = const.\,, \quad B_0(\boldsymbol{p}) = B_1(\boldsymbol{p}) \equiv D = const.$$

Langevin simulations

- Fokker-Planck equation realized with Langevin simulations
- Langevin equation:

$$m\ddot{x} = -m\gamma\dot{x} + \xi$$

- Corresponding update steps for coordinate and momentum in time interval dt:

$$dx_j = \frac{p_j}{m}dt$$

$$dp_j = -\gamma p_j dt + \sqrt{2\gamma m T dt}\,\rho_j$$

- ρ: Gaussian-distributed white noise
- γ: drag coefficient from Abelian plasma model
- Momentum update computed in rest frame of medium

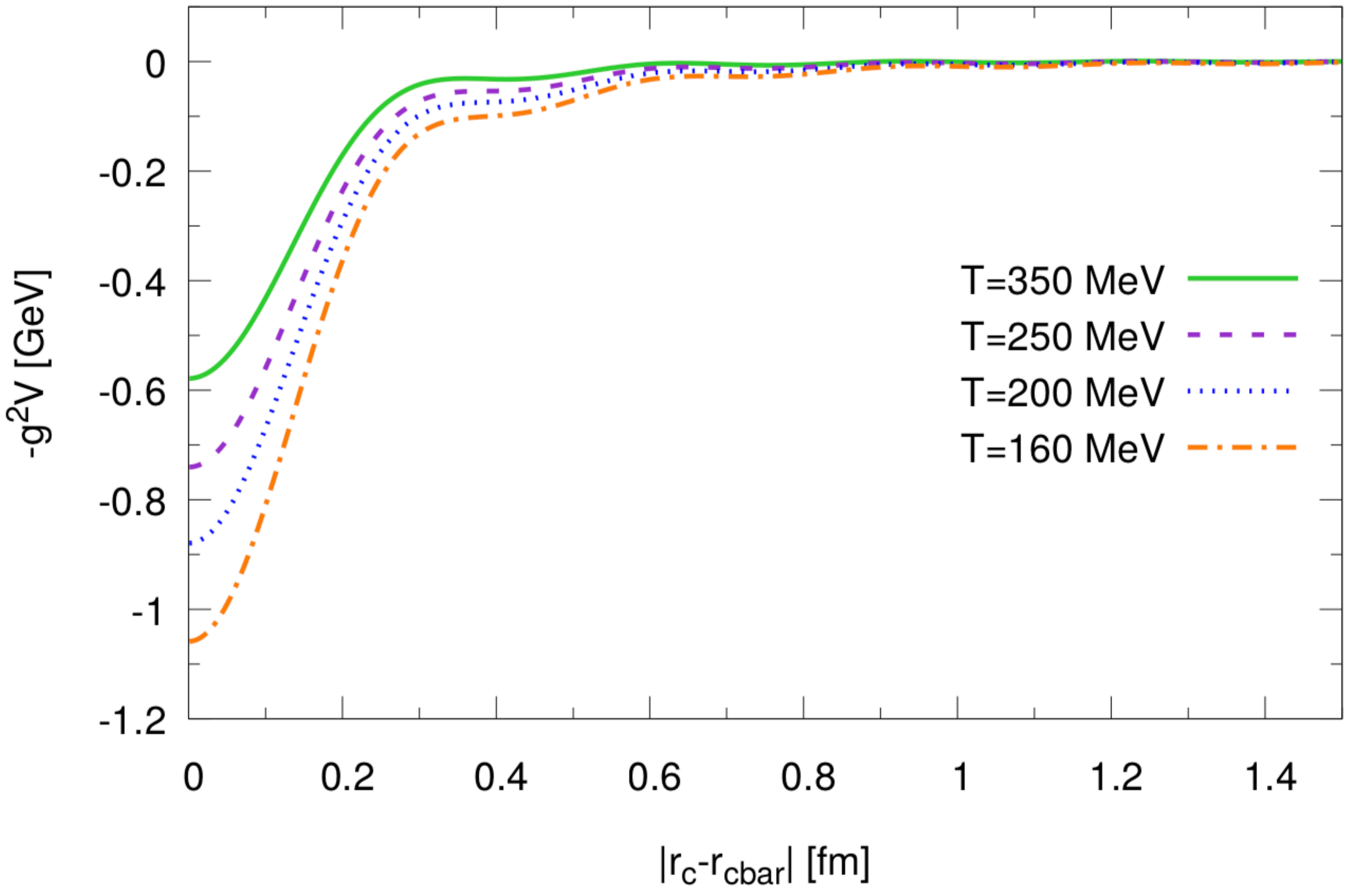
Potential of the Heavy Quarks

- Formalism to describe heavy quarks in Abelian plasma by Blaizot et al.
- Idea: effective theory non-relativistic HQs in plasma of relativistic particles
- Influence functional in infinite-mass limit and large time limit: interpretation as complex potential:

$$\mathscr{V}(\boldsymbol{s}) = -g^2\left[V(\boldsymbol{r}) - V_{ren}(0)\right] - ig^2\left[W(\boldsymbol{r}) - W(0)\right]$$

$$= -\frac{g^2}{4\pi}m_D - \frac{g^2}{4\pi}\frac{\exp(-m_D s)}{s} - i\frac{g^2 T}{4\pi}\phi(m_D s)$$

Potential of the Heavy Quarks



N. Krenz, H. van Hees, C. Greiner, J. Phys.: Conf. Ser. 1070, 012008 (2018)

- Screened Coulomb potential with Cut-Off $\Lambda = 4\ GeV$ and running coupling:

$$g^2 = 4\pi\alpha_s = \frac{4\pi\alpha_s(T_c)}{1 + C\ \ln(T/T_c)},$$

$$m_c = 1.8\ GeV/c^2,$$

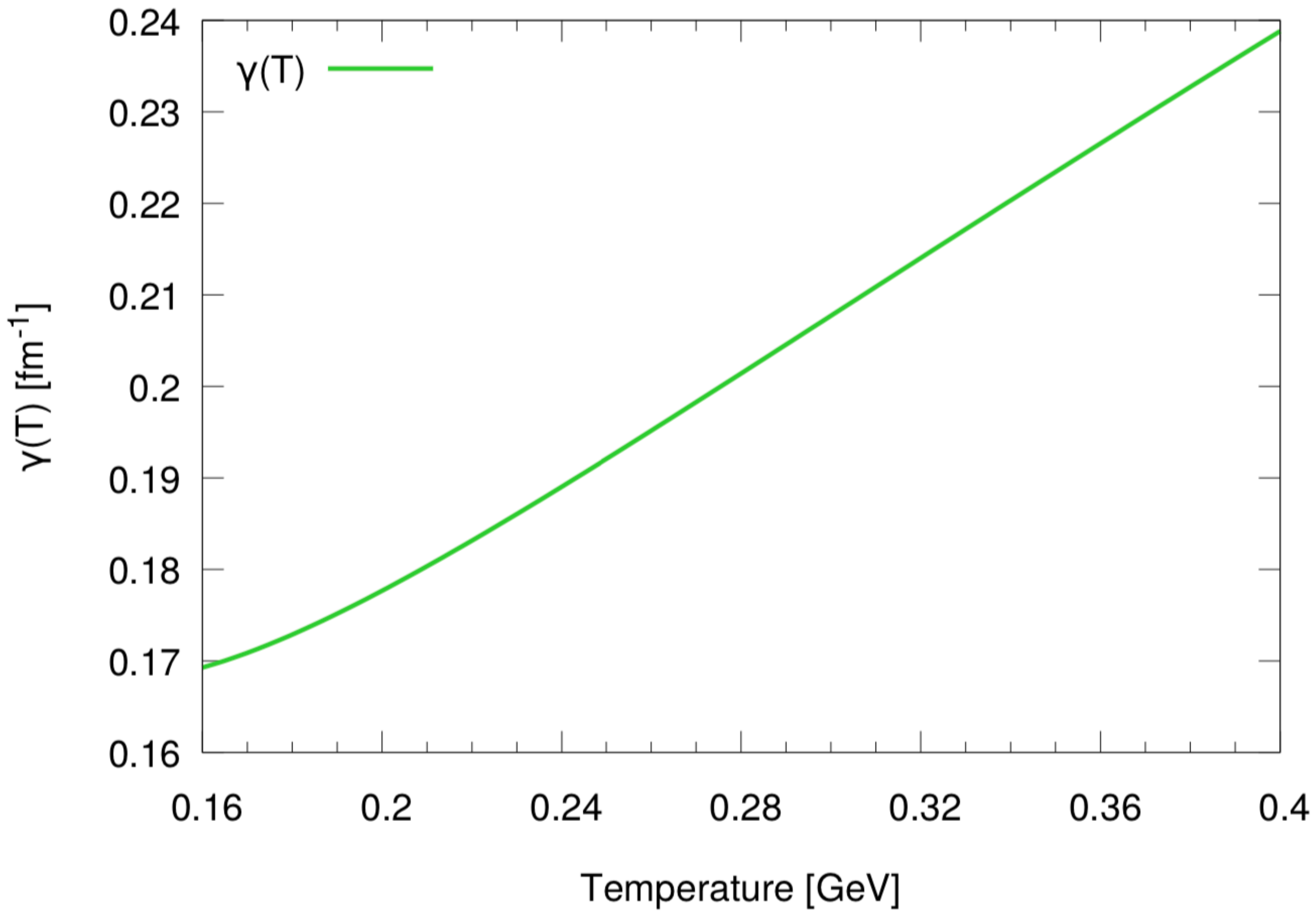
$$T_c = 160\ MeV,$$

$$\alpha_s(T_c) = 0.5,$$

$$C = 0.76$$

Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

Drag Coefficient



N. Krenz, H. van Hees, C. Greiner, J. Phys.: Conf. Ser. 1070, 012008 (2018)

- Drag coefficient:

Langevin:

$$m\ddot{x} = -m\gamma\dot{x} + \xi$$

$$dp_j = -\gamma p_j dt + \sqrt{2\gamma m T dt}\,\rho_j$$

$$\gamma = \frac{m_D^2}{24\pi M}\left[\ln\left(1 + \frac{\Lambda^2}{m_D^2}\right) - \frac{\Lambda^2/m_D^2}{1 + \lambda^2/m_D^2}\right]$$

$$m_D = \sqrt{\frac{4}{3}g^2T^2}$$

Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

Elliptic Fireball

- Elliptic parametrisation of transverse direction:

H. van Hees, M. He, and R. Rapp, Nuclear Physics A, (2015) Vol. 933

$$\frac{x^2}{b^2(\tau)} + \frac{y^2}{a^2(\tau)} \leq 1$$

- Volume of medium in Fireball:

$$V(\tau) = \pi \cdot a(\tau) b(\tau) (z_0 + c\tau) \;, \quad z_0 = c\tau_0$$

with long and short semi-axes $a(\tau)$, $b(\tau)$

- semi-axes:

$$a(\tau) = a_0 + \frac{1}{a_a}\left(\sqrt{1 + a_a^2\tau^2} - 1\right), \; b(\tau) = b_0 + \frac{1}{a_b}\left(\sqrt{1 + a_b^2\tau^2} - 1\right)$$

- a_a, a_b : accelerations chosen to fit to p_T-spectra and elliptic flow of light hadrons

Elliptic Fireball

- Construction of transverse velocity field at midrapidity by expressing $\boldsymbol{r}_\perp(x, y)$ using confocal elliptical coordinates u, v:

$$\boldsymbol{v}_\perp = \frac{r}{r_B}\begin{pmatrix} v_b(\tau)\cos(v) \\ v_a(\tau)\sin(v) \end{pmatrix}$$

v_a, v_b: velocities of the boundary, r_B: distance from center to boundary

- confocal elliptical coordinates u, v given by inversion of

$x = r_0 \sinh(u)\cos(v)$, $y = r_0 \cosh(u)\sin(v)$, $r_0 = a_0\epsilon$

Elliptic Fireball

- Extension to 3D-flow field with longitudinal component and finite rapidity:

➡ Superimpose of model using boost-invariant Bjorken flow

- Resulting 3D-flow field:

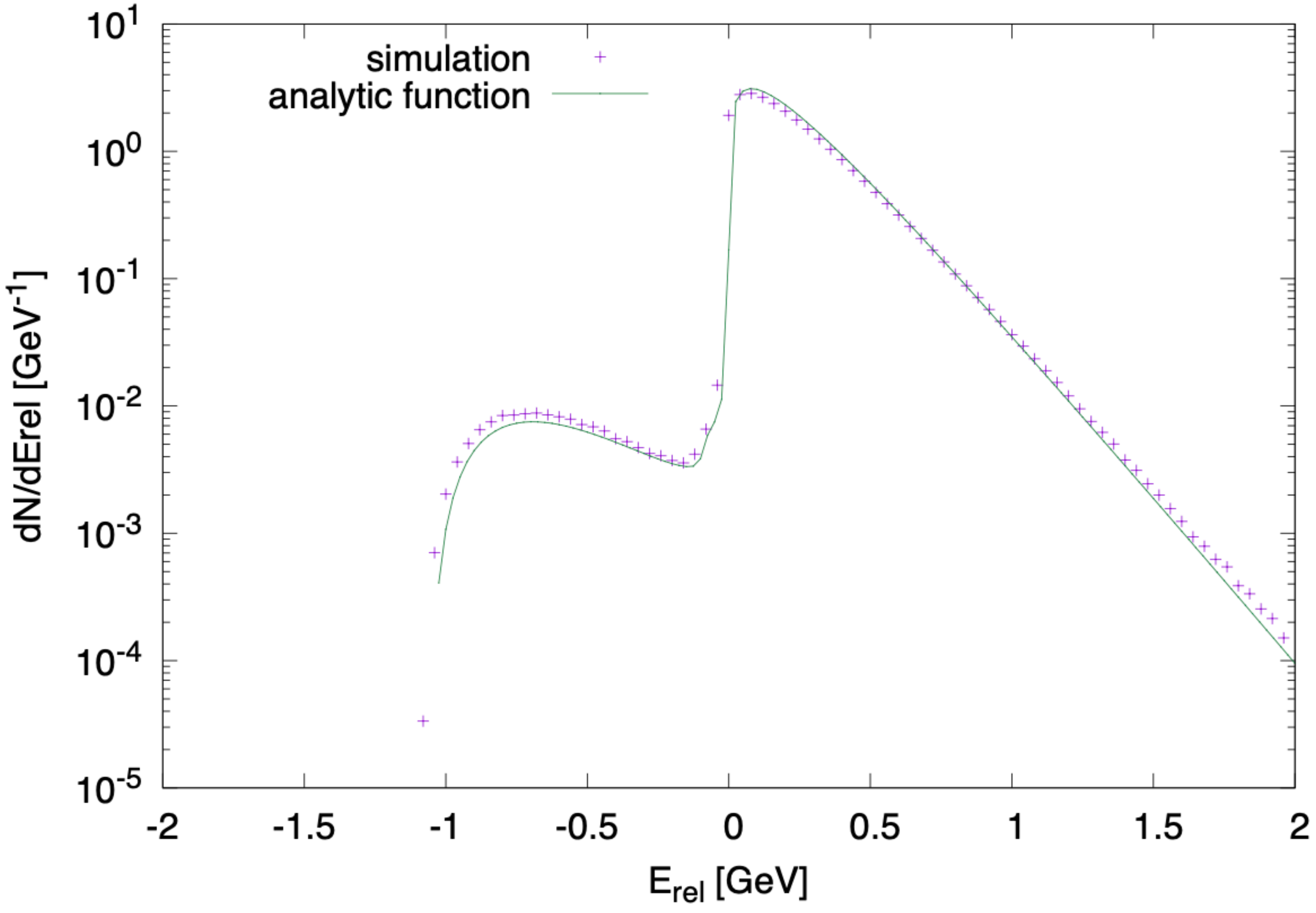
$$v_x = \frac{\tau}{t} v_b(\tau)\cos(\nu)\frac{r}{r_B}\,, \quad v_y = \frac{\tau}{t} v_a(\tau)\sin(\nu)\frac{r}{r_B}, \quad v_z = \tanh(\eta)$$

- Initial momentum distribution of heavy quarks in the fireball given by parametrization fitting charm-quark spectra from PYTHIA

- Initial spatial distribution according to Glauber model

Box Simulations

Energy distribution in equilibrium



- Formation of bound states when energy of charm-anticharm pair is < 0

$$\frac{dN}{dE_{rel}} = (4\pi)^2(2\mu)^{\frac{3}{2}}C\int_0^R drr^2\sqrt{E_{rel} - V(r)}\exp\left(-\frac{E_{rel}}{T}\right)$$

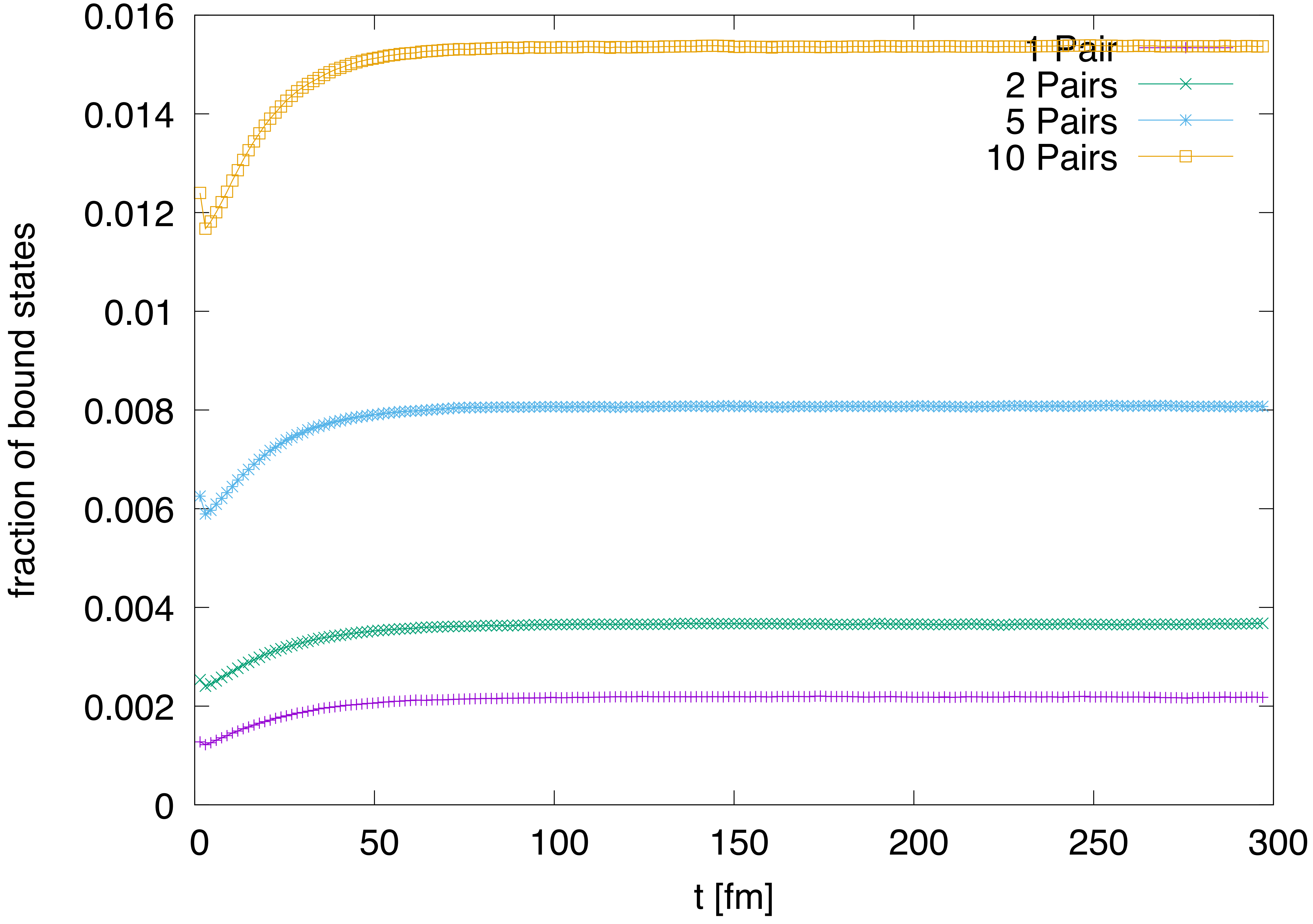
- Box simulation with 1 $c\bar{c}$-pair at $T = 160\ MeV$

➡ leads to right equilibrium density of states

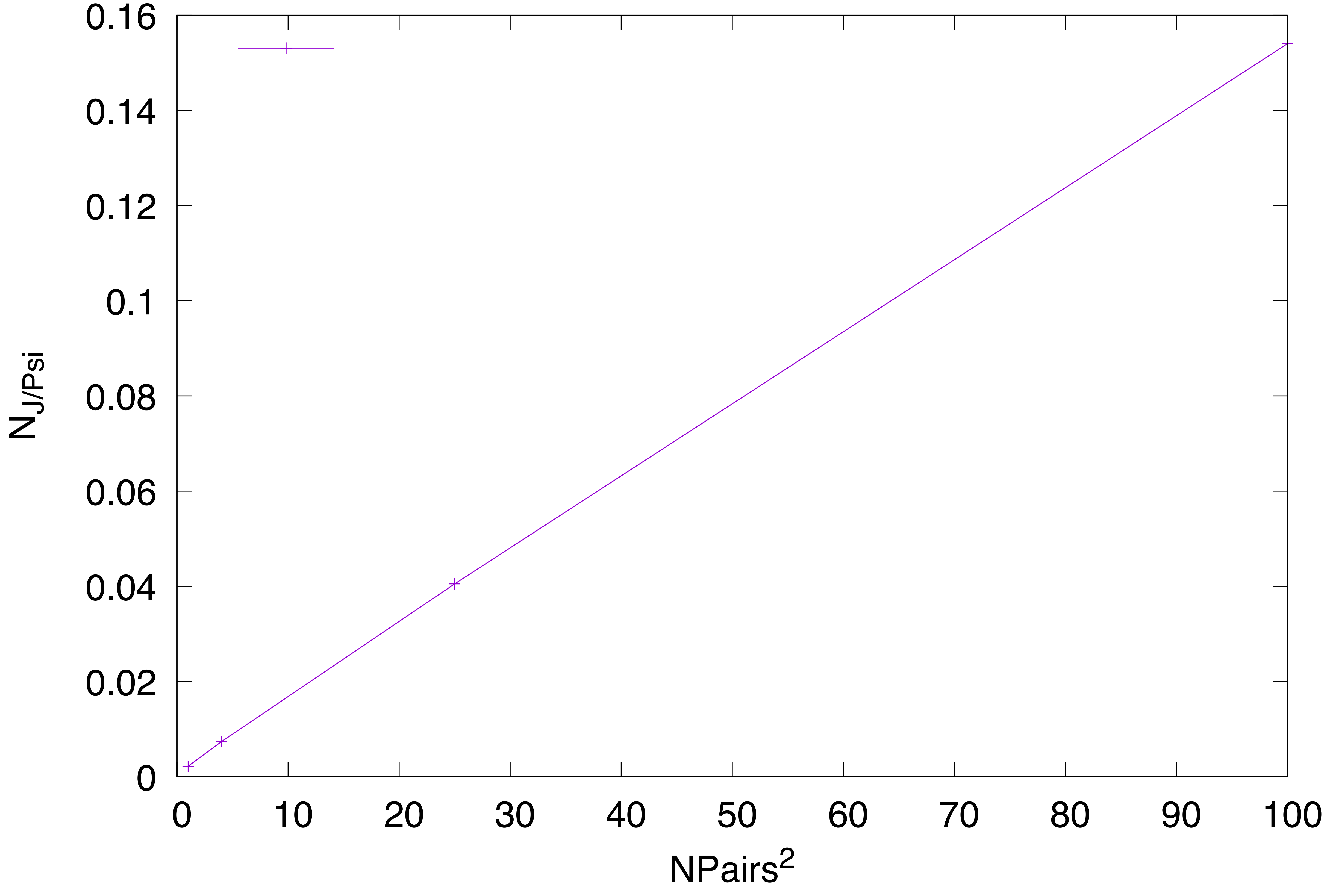
Box Simulations

Bound State Formation in Box Simulation ($T = 180\ MeV$)

Time evolution of fraction of bound states

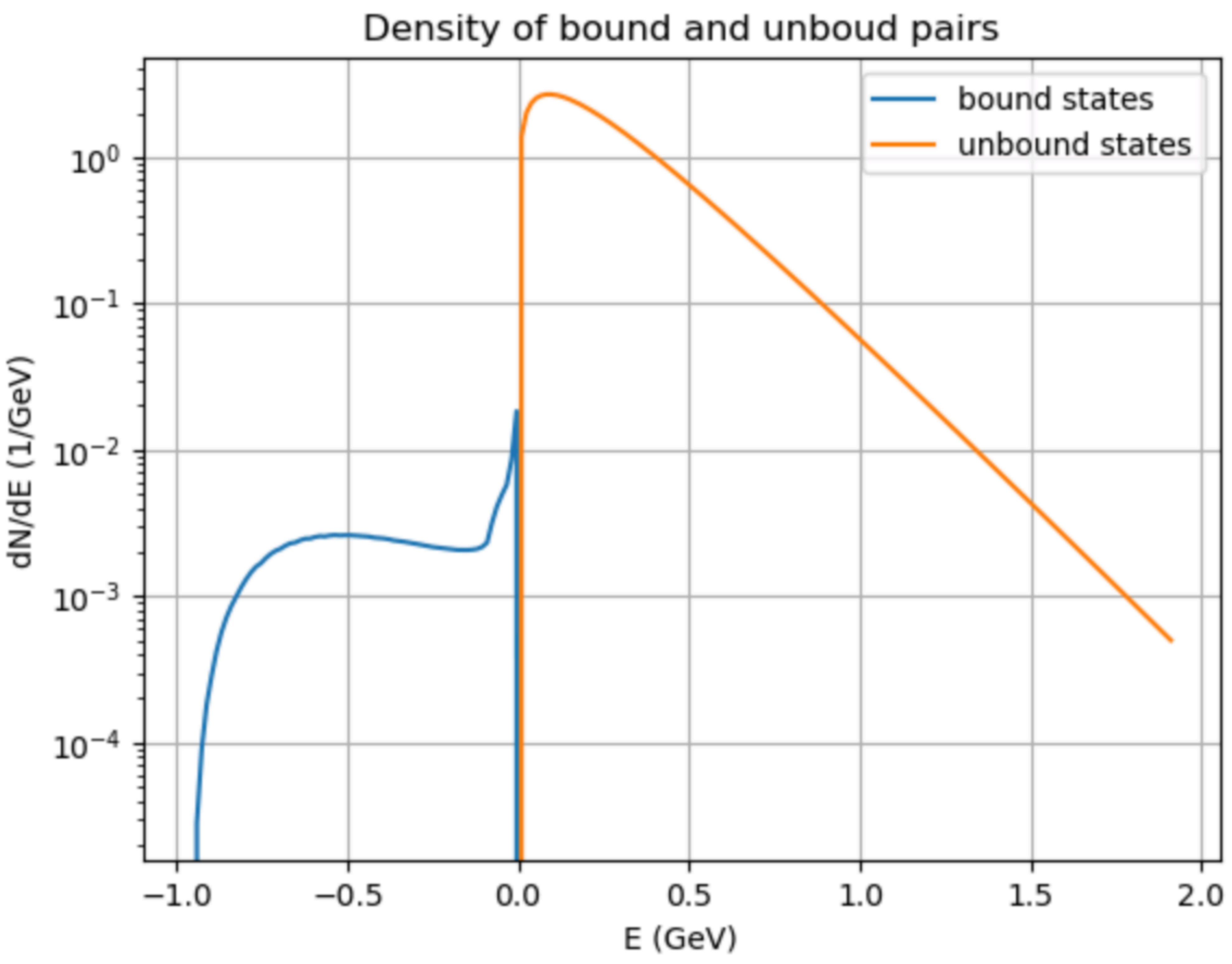


Scaling of J/ψ - yield with number of pairs in the system



Box Simulations

Comparison to theoretically estimated density of bound states

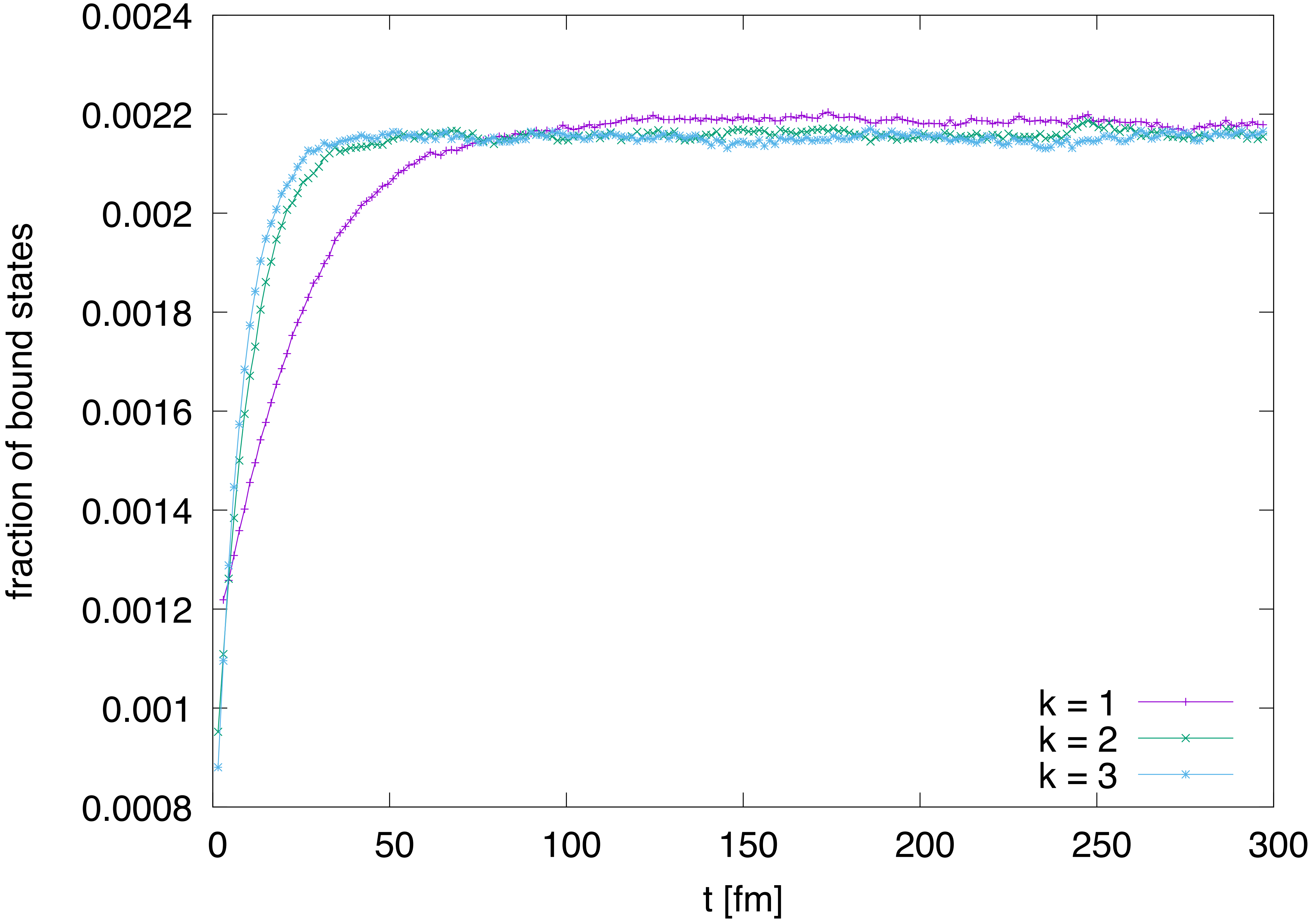


INTEGRATION

Number of pairs	$N_{J/\psi}$ Simulation	$N_{J/\psi}$ Theoretical Estimation
1	0. 0022	0. 00235
2	0. 00735	0. 0094
5	0. 0405	0. 0588
10	0. 154	0. 2353

Box Simulations

Equilibration time for different scalings of drag coefficient γ



- Equilibration time from Langevin equation: $\tau_{eq} = 1/\gamma$
- Faster equilibration for stronger drag force
- Exponential fit:
  - ➡ $k = 2{,}3$: agreement with $\tau_{eq} = 1/\gamma$
  - ➡ Larger values: τ_{eq} too small

Elliptic Flow v_2

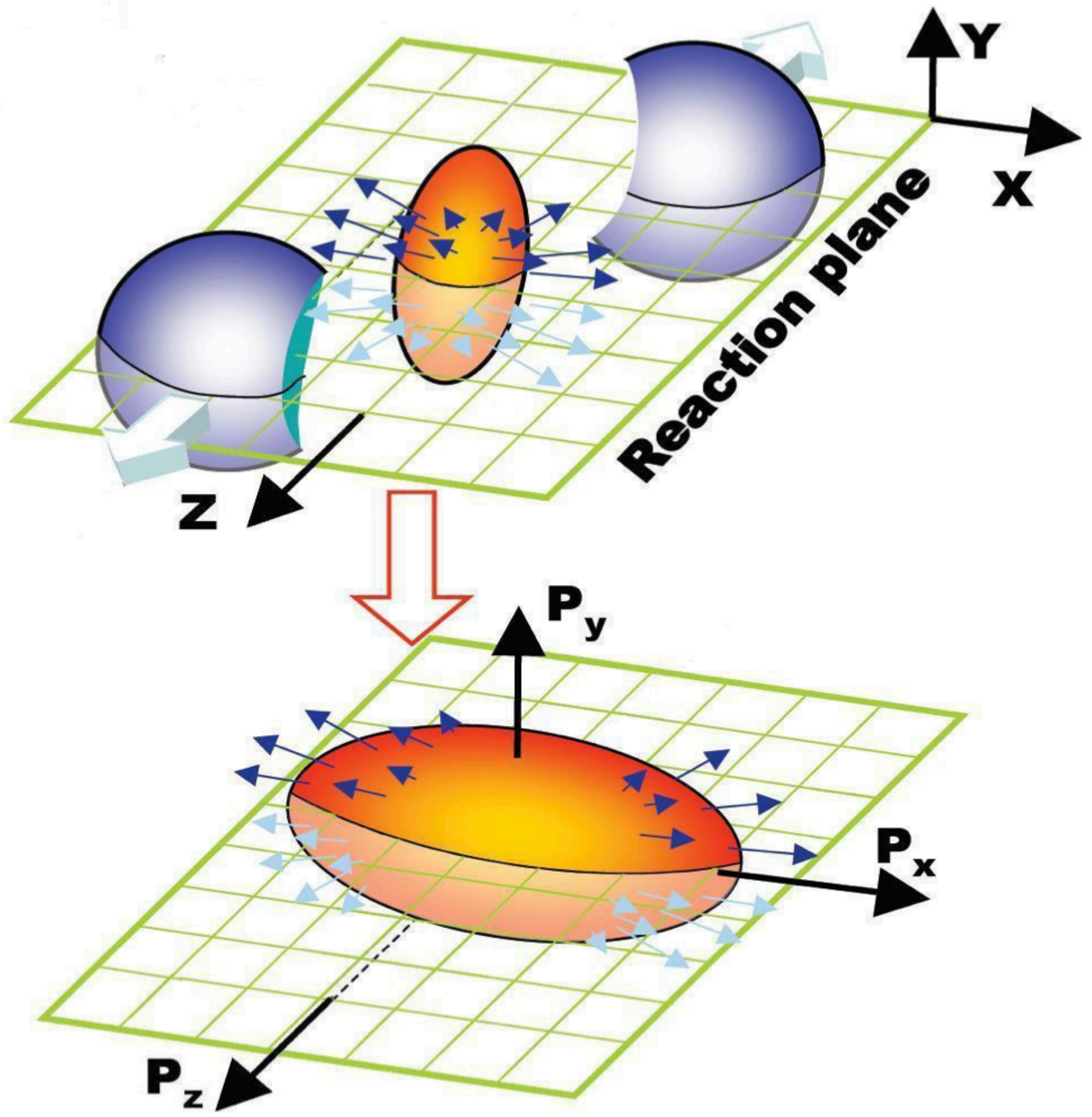
Azimuthal particle distibution:

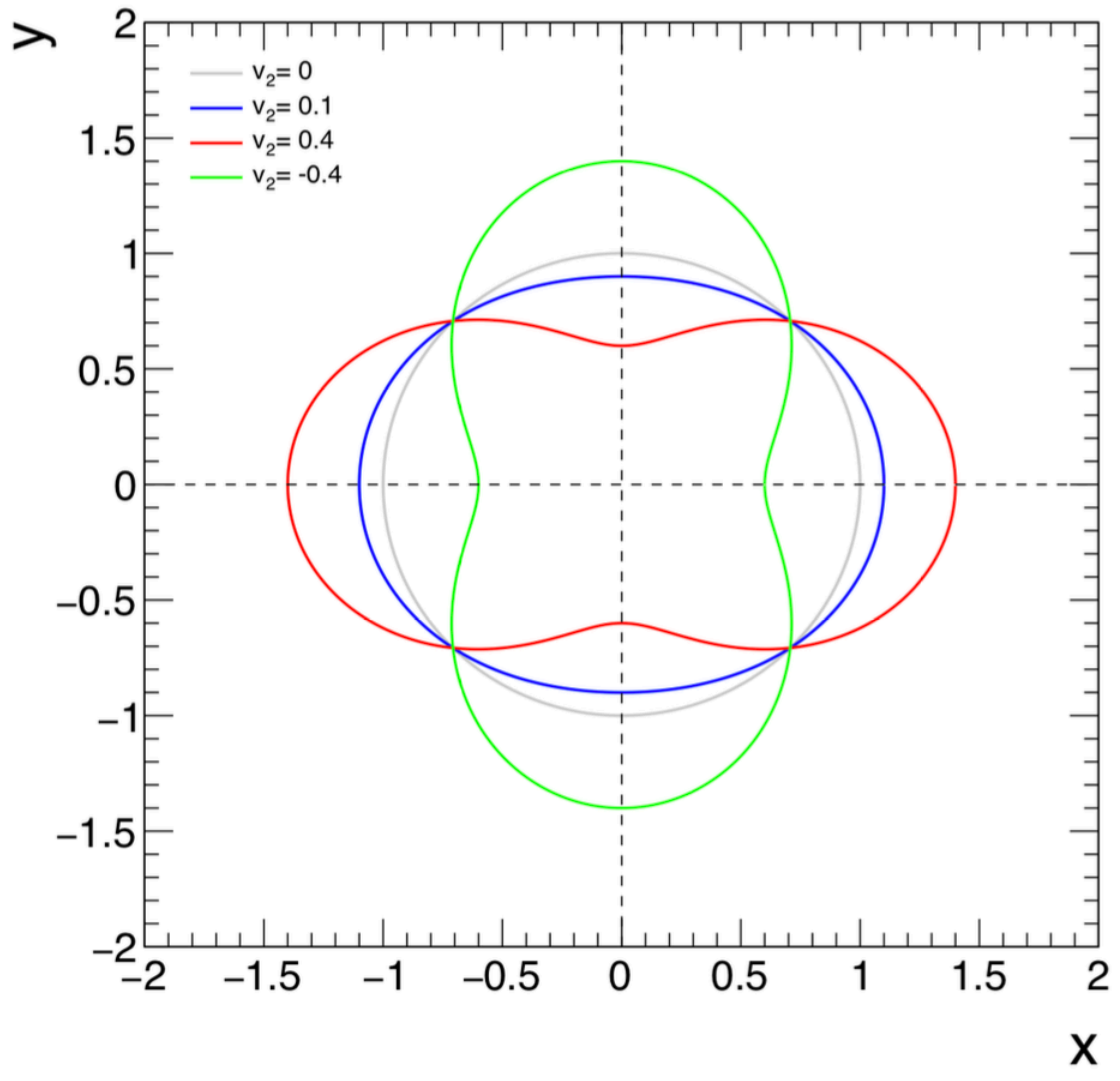
$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^3N}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos(n(\Phi - \Psi_R))\right) \longrightarrow$$

Flow coefficients: $v_n(p_T, y) = \langle \cos[n(\Phi - \Psi_R)] \rangle$

Elliptic Flow:

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

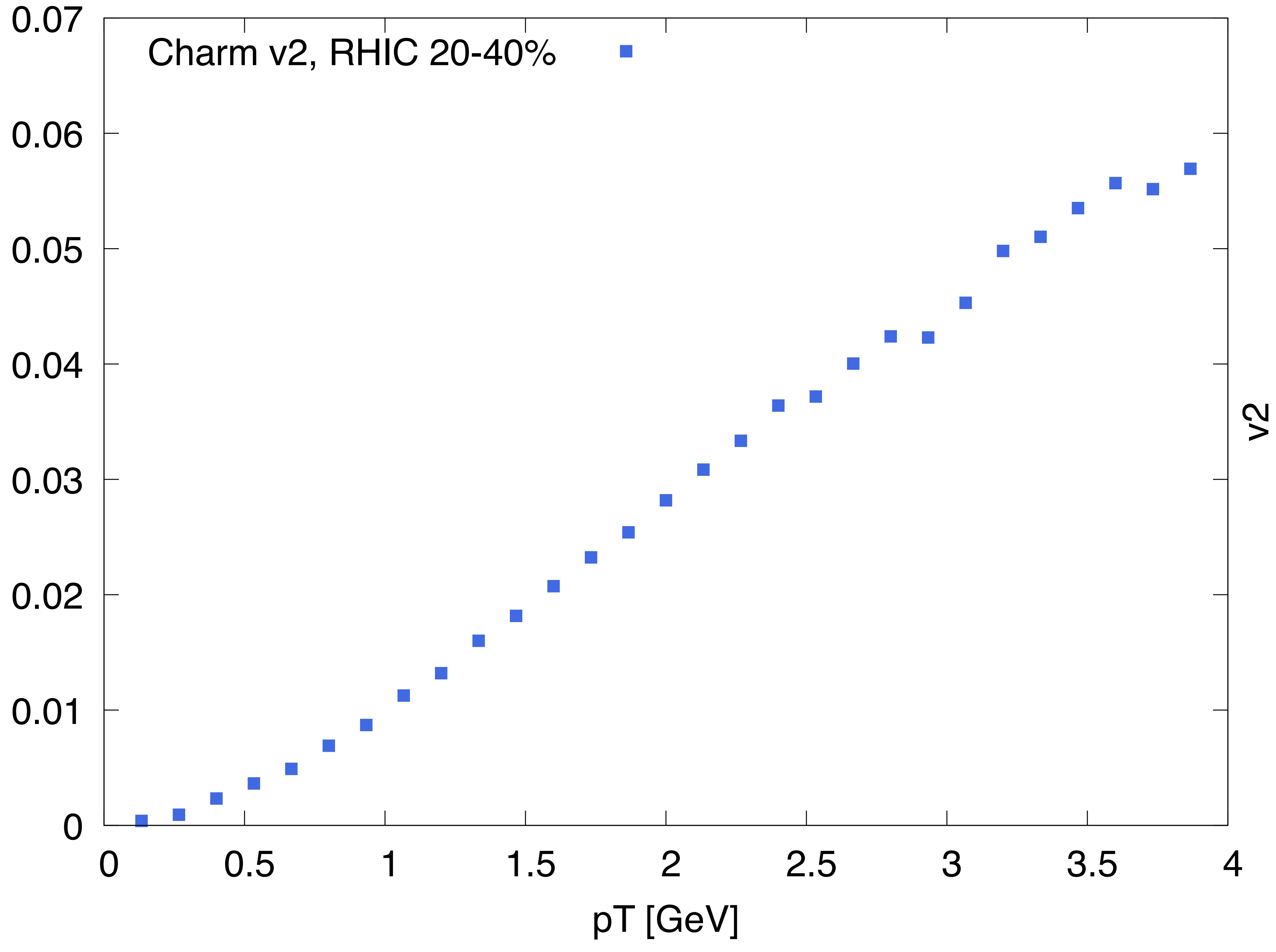




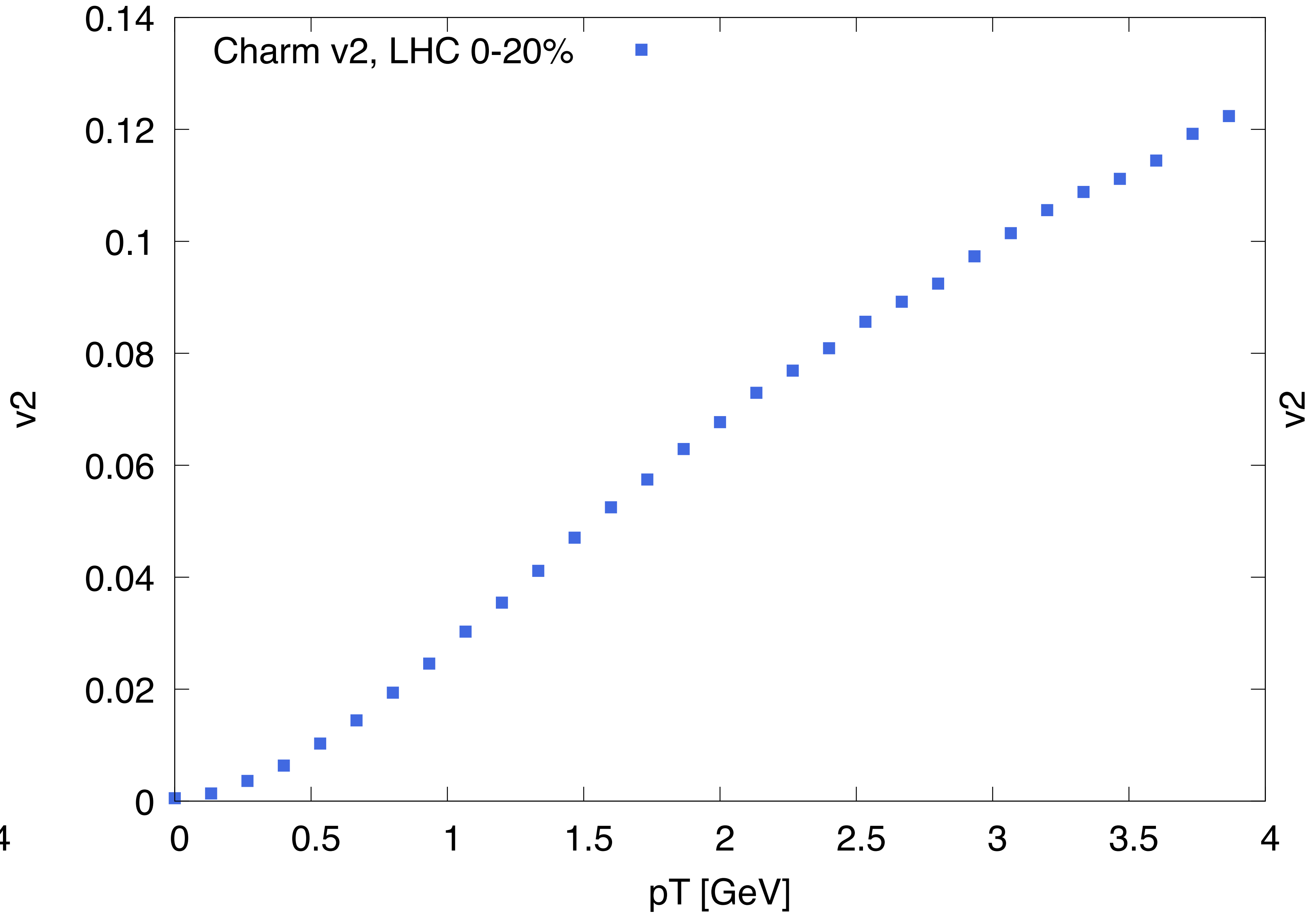
Elliptic Flow v_2

Charm Quarks: 5 Pairs

RHIC, 20-40% Centrality



LHC, 0-20% Centrality



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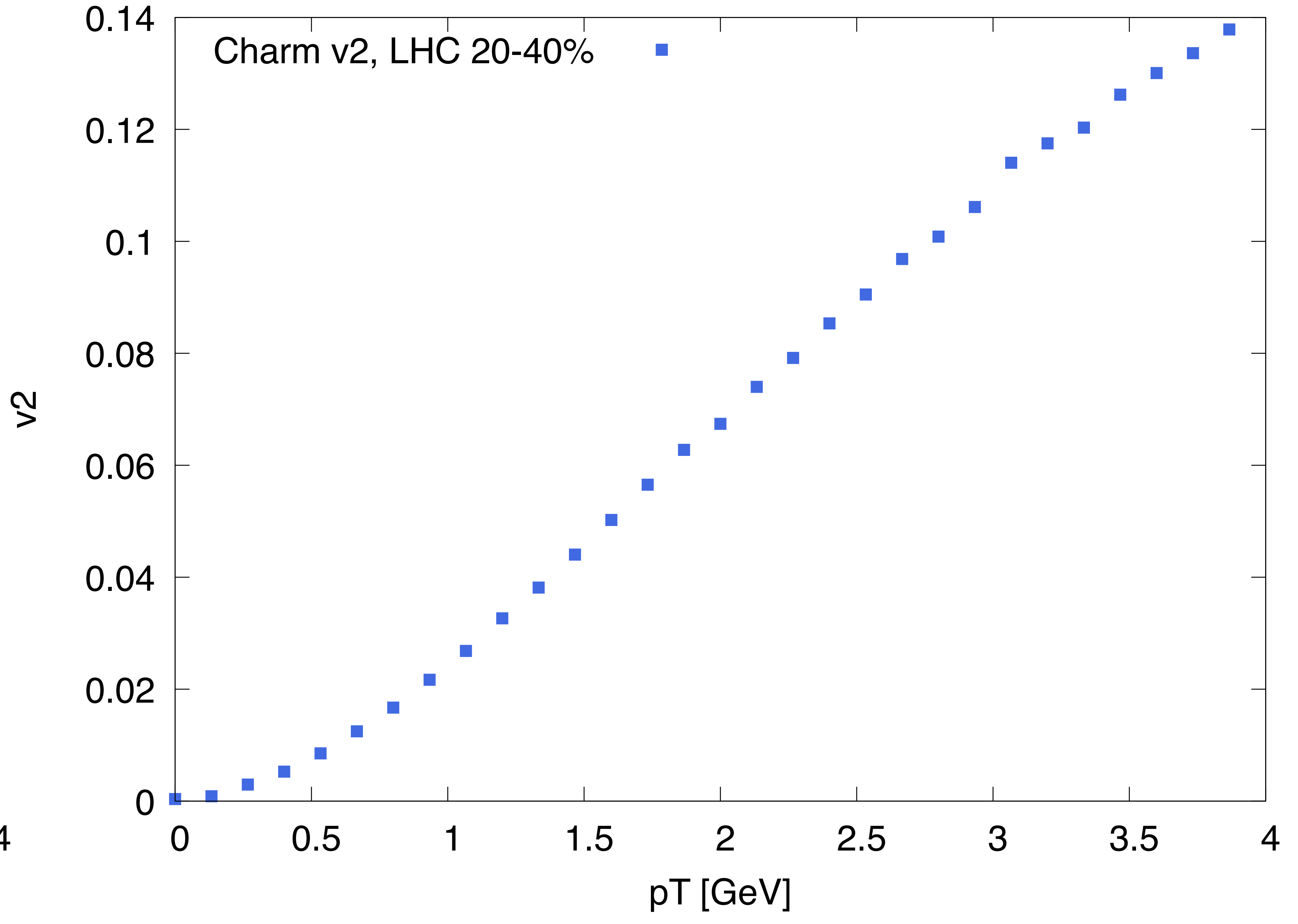

Elliptic Flow v_2

J/ψ: 5 Pairs

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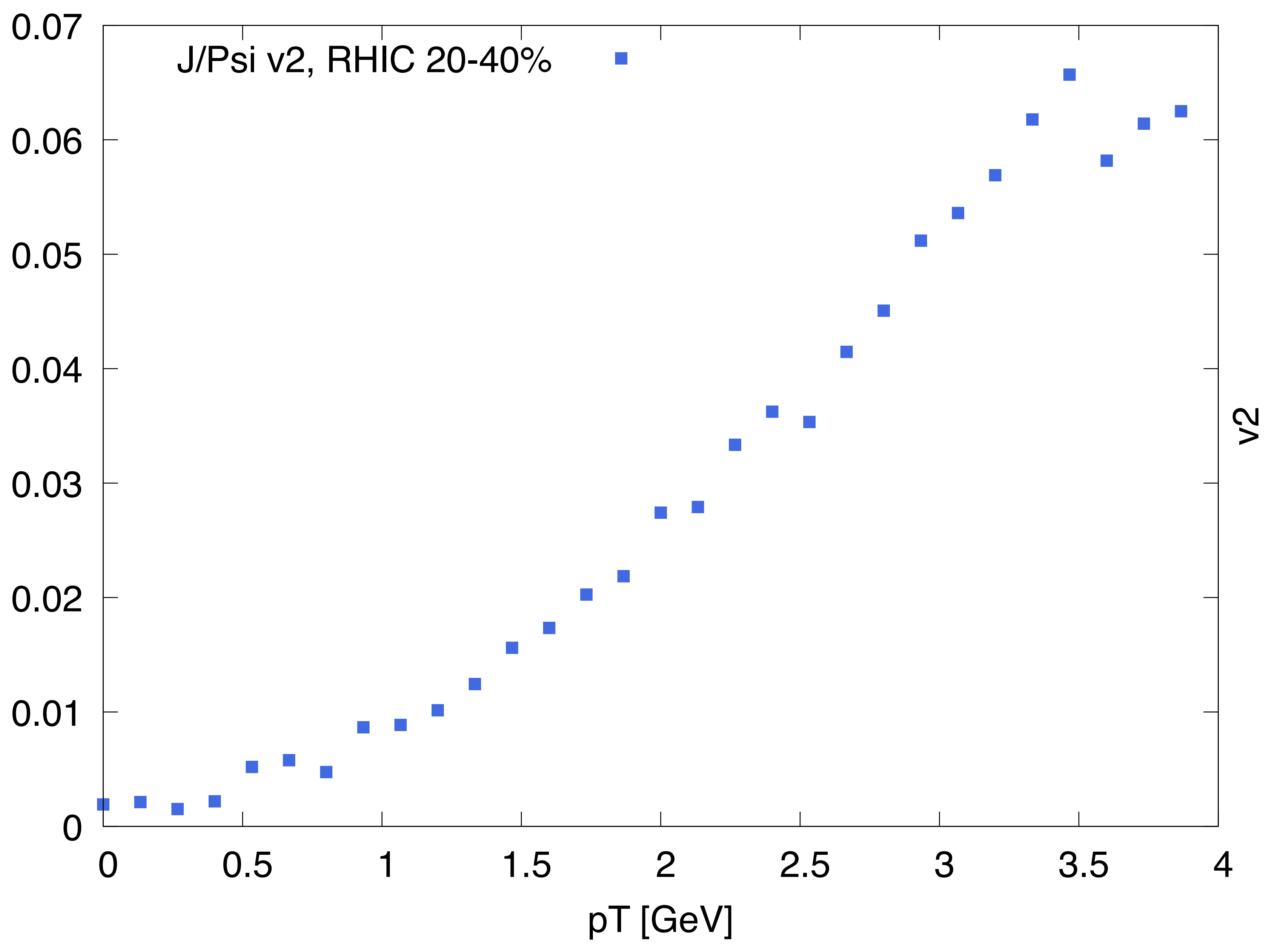

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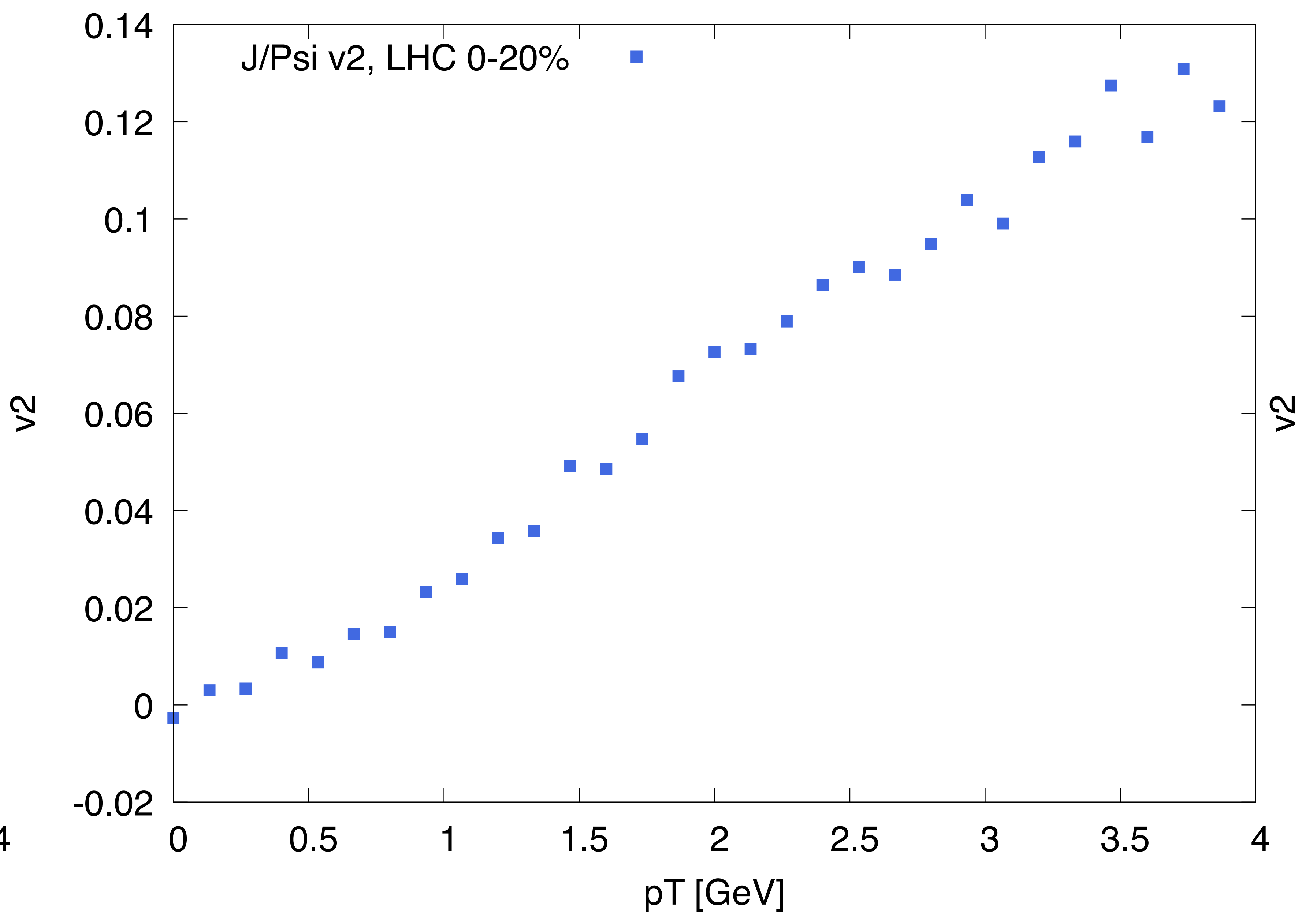

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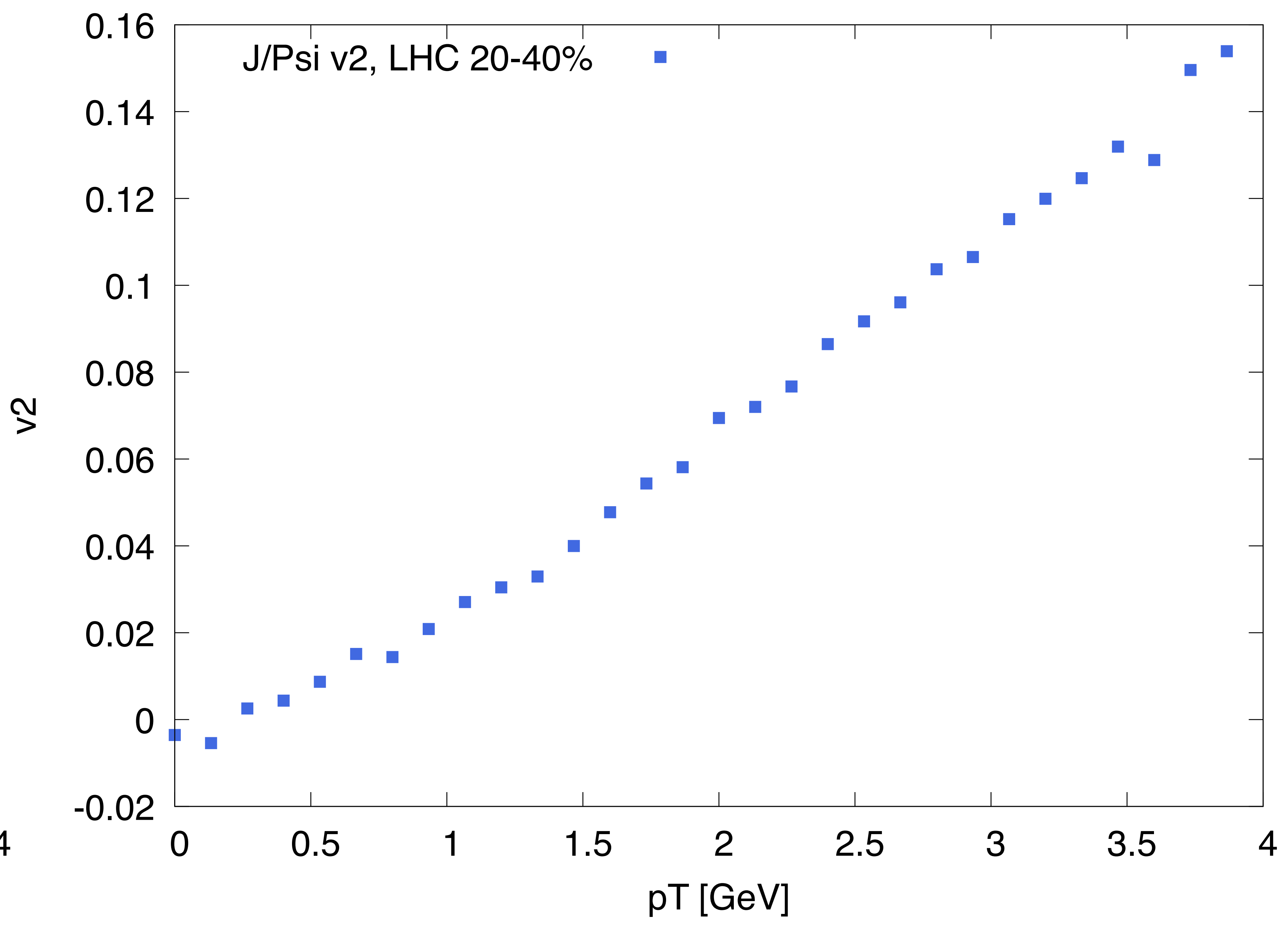


Pb+Pb @ 2.76 TeV (ALICE): $v_2 \simeq 10\,\%$ (central)

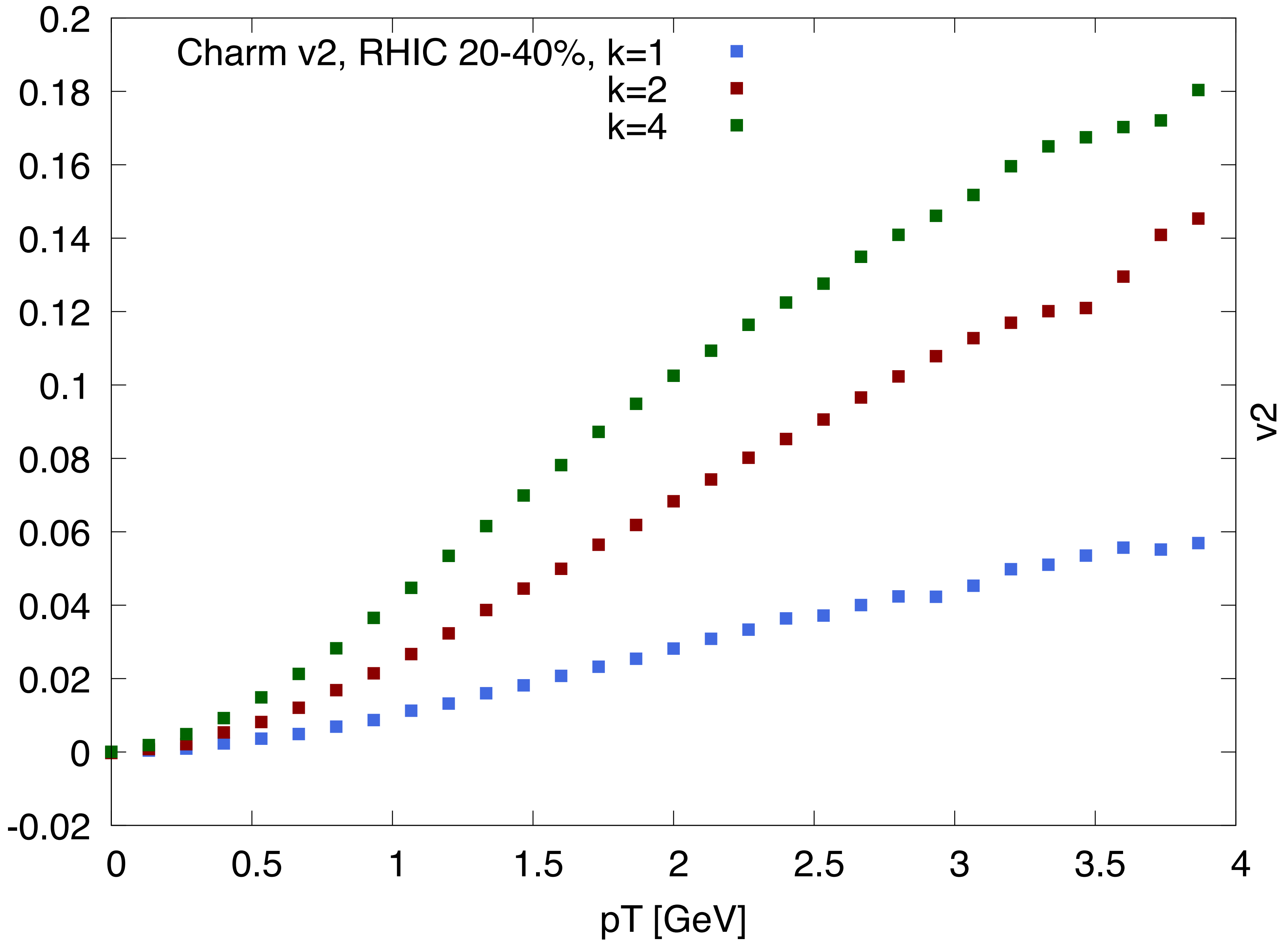
E. Abbas et al. (ALICE Collaboration) $v_2 \simeq 11.6\,\%$ (semicentral)

Phys. Rev. Lett. 111, 162301

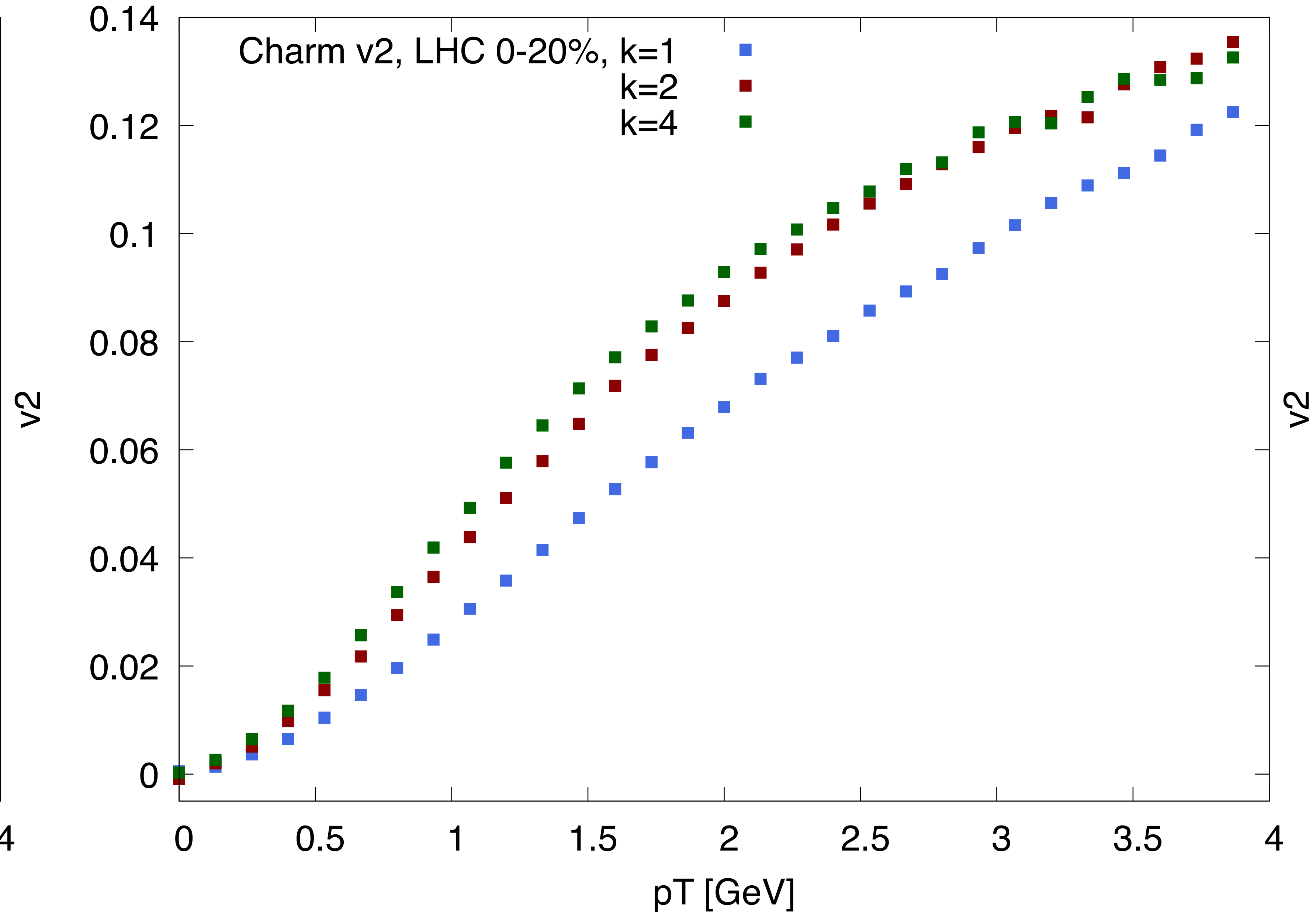
Elliptic Flow v_2 for different scalings of the drag coefficient

Charm Quarks, RHIC & LHC

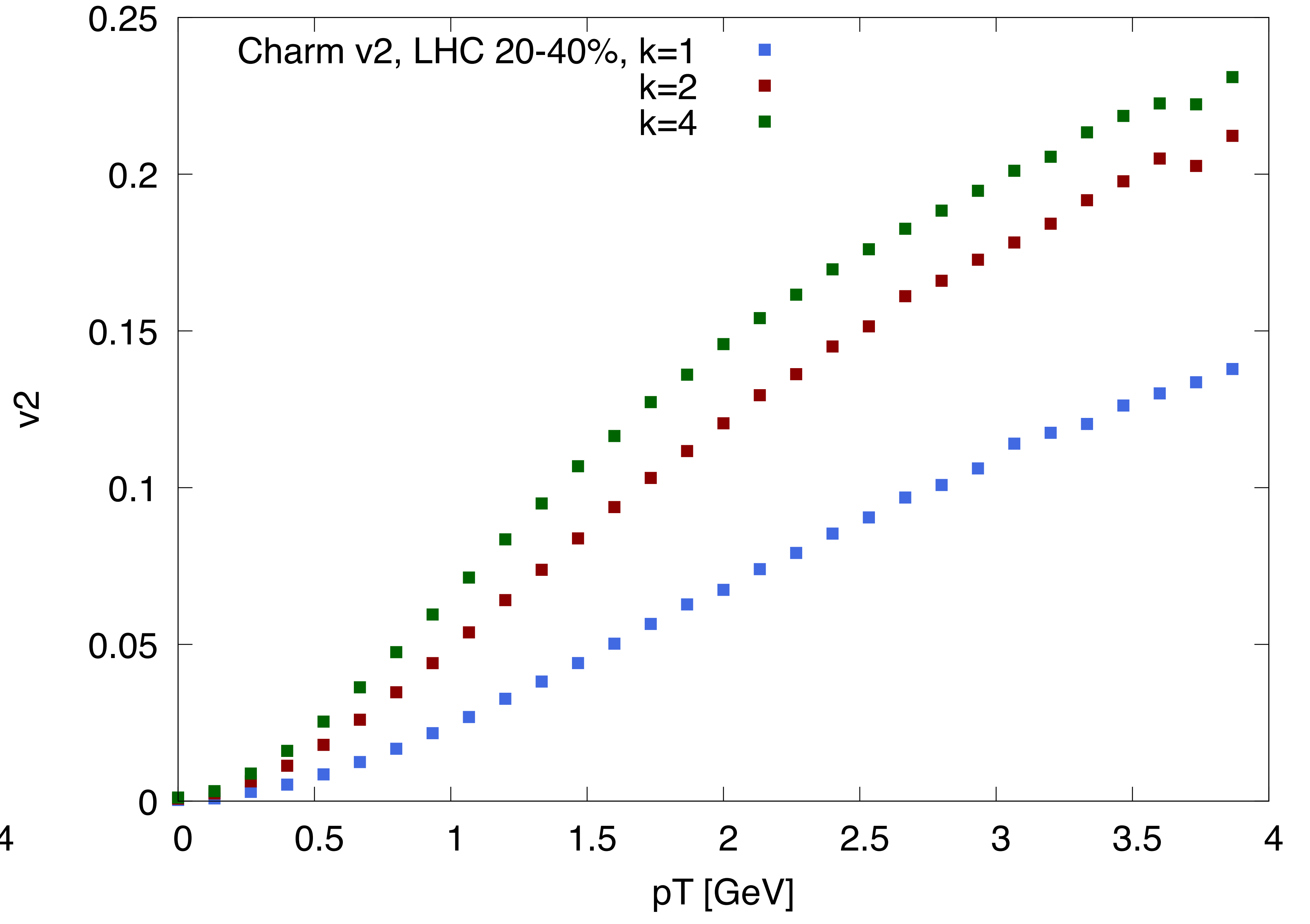
RHIC, 20-40% Centrality



LHC, 0-20% Centrality



LHC, 20-40% Centrality



Elliptic Flow v_2 for different scalings of the drag coefficient

J/ψ, RHIC & LHC

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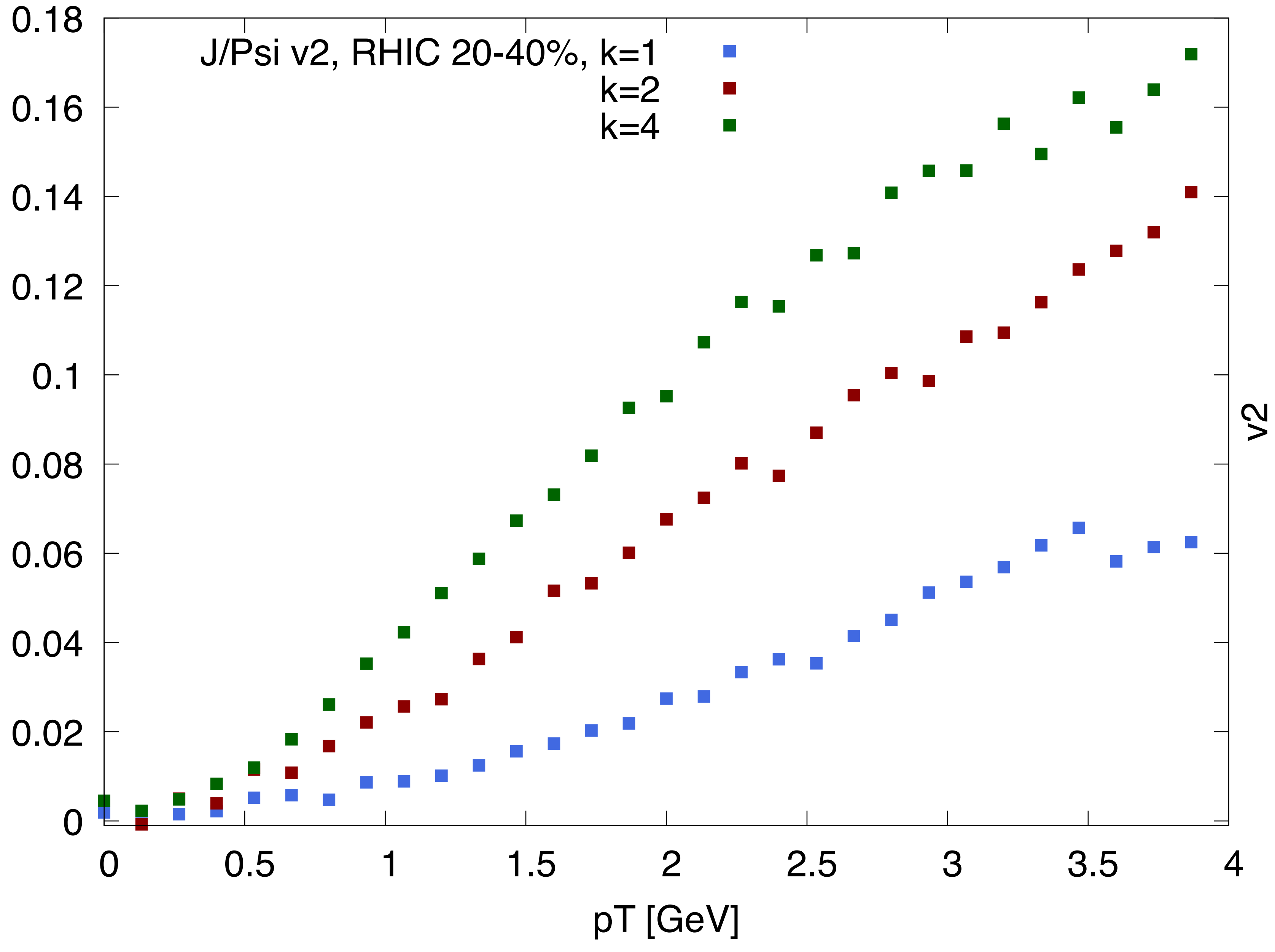
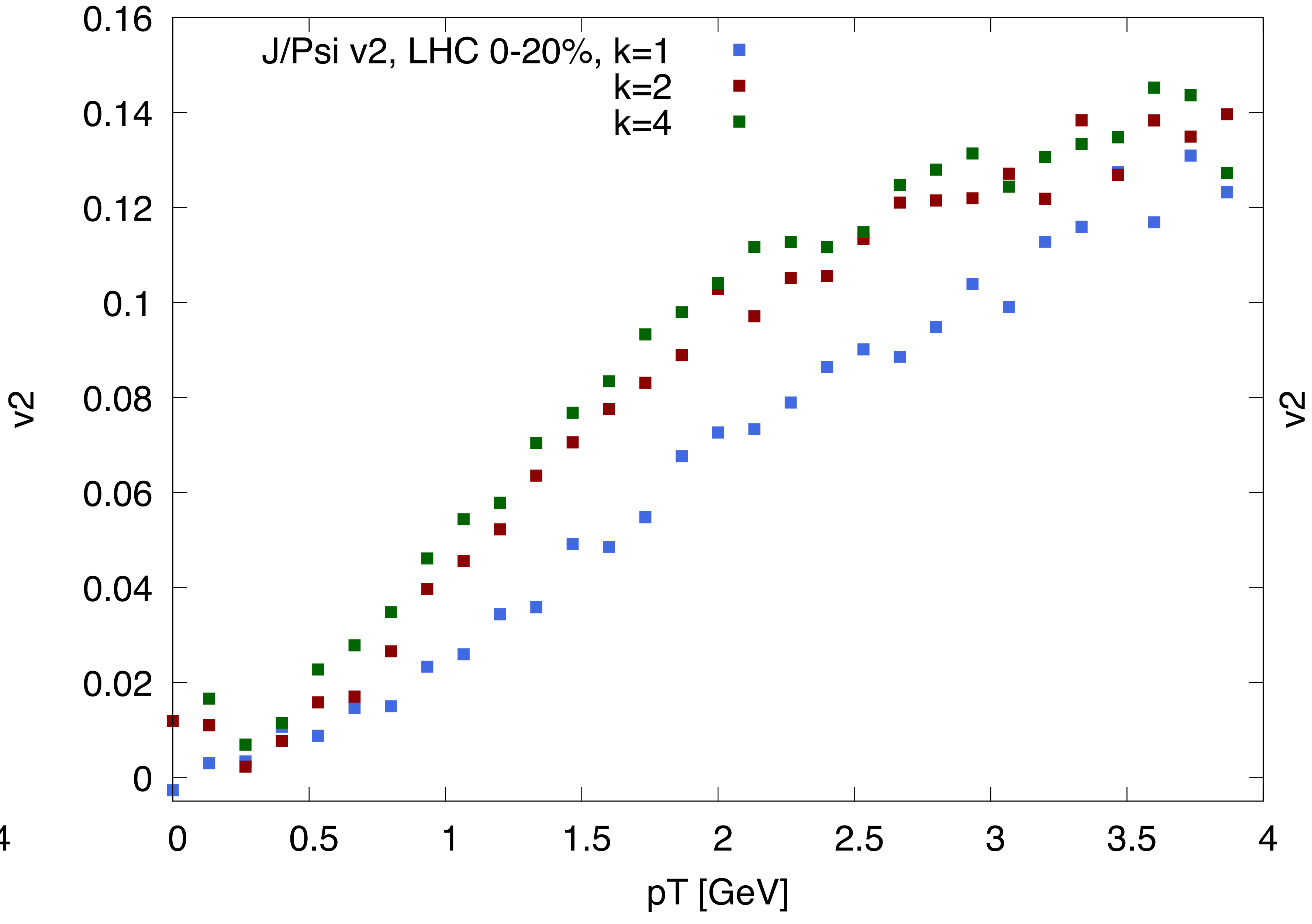


LHC, 0-20% Centrality



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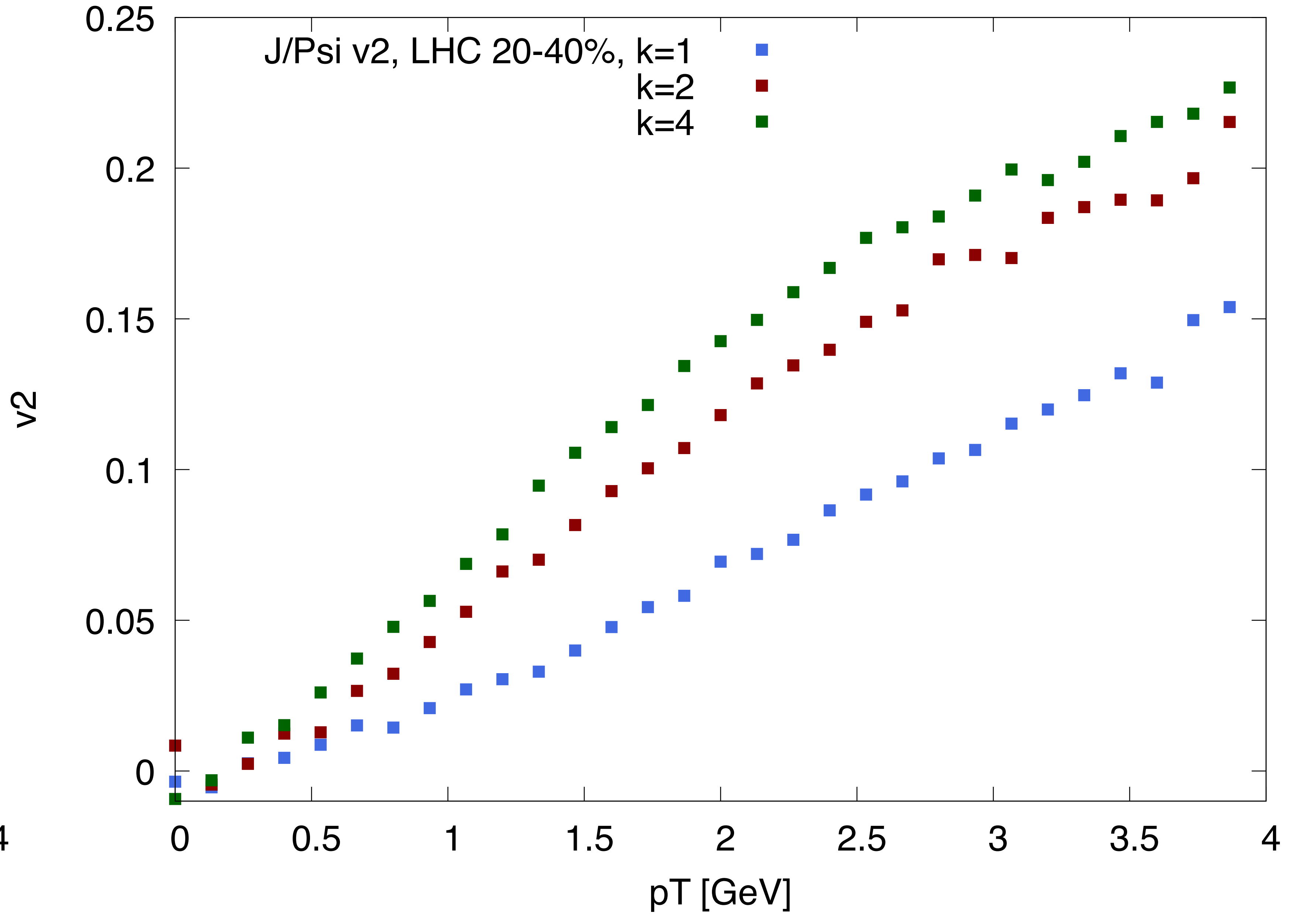


Outlook

Possible extensions:

- different heavy-quark potentials
- Initial momentum distribution from PYTHIA ⟶ include primordial J/ψ
- Number of initial heavy-quark pairs according to experimental data
- Nuclear modification factor R_{AA}

Backup

Langevin simulations

- First half of coordinate update step

$$\vec{r}_{c,i+\frac{1}{2}} = \vec{r}_{c,i} + \frac{\vec{p}_{c,i}}{2E_c}\Delta t$$

- Calculation of Potential for Momentum Update

$$\overrightarrow{F}(\vec{r}_{c,i+\frac{1}{2}}, \vec{r}_{\bar{c},i+\frac{1}{2}})\Delta t$$

- Boost to Medium Rest Frame

$$p_i^* = p_i - \gamma\beta_i E + (\gamma - 1)\frac{\beta_i}{\vec{\beta}^2}\vec{\beta}\vec{p}, \quad i = 1,2,3$$

Langevin simulations

- Analytic form of momentum update step:

$$dp_j = -\gamma p_j dt + \sqrt{dt} C_{jk} \rho_k$$

- Stochastic process dependent on specific choice of the momentum argument of the covariance matrix $\boldsymbol{C}_{jk}$

- Determination of momentum argument in $\boldsymbol{C}_{jk}$:

$$C_{jk} \to C_{jk}(t, \boldsymbol{x}, \boldsymbol{p} + \xi d\boldsymbol{p})$$

➡ $\xi = 0, \frac{1}{2}, 1$ for pre-point, midpoint and post-point realisation

- In this work: post-point scheme,

$$C_{jk} \to C_{jk}(t, \boldsymbol{x}, \boldsymbol{p} + d\boldsymbol{p})$$

Langevin simulations

- momentum update:

$$dp_j = -\gamma p_j dt + \sqrt{2dtD(|\boldsymbol{p} + d\boldsymbol{p}|)}\rho_j = -\gamma p_j dt + \sqrt{2\gamma m T dt}\rho_j$$

➡ Two-step computation:

I. Calculation of dp_j of pre-point scheme, $dp_j = -\gamma p_j dt + \sqrt{2dtD(p)}\rho_j$

II. Use result for argument $|\boldsymbol{p} + d\boldsymbol{p}|$ of D to evaluate the second part of the postpoint momentum update, $dp_j^{diff} = \sqrt{2dtD(|\boldsymbol{p} + d\boldsymbol{p}|)}\rho_j$

III. Complete momentum update: $dp_j = dp_j^{drag} + dp_j^{diff}$ with dp_j^{drag} from I.

Langevin simulations

- Boost back to computational frame
- Complete momentum update:

$$\vec{p}_{c,i+1} = \vec{p}_{c,i} + \overrightarrow{F}(\vec{r}_{c,i+\frac{1}{2}}, \vec{r}_{\bar{c},i+\frac{1}{2}})\Delta t - \gamma \vec{p}_{c,i}\Delta t + \sqrt{2mT\gamma\Delta t}\rho$$

- Second half of coordinate update step:

$$\vec{r}_{c,i+1} = \vec{r}_{c,i+\frac{1}{2}} + \frac{\vec{p}_{c,i+1}}{2m_c}\Delta t$$

Description of the Model

Elliptic Fireball: Momentum distribution

- parametrisation fitting charm-quark spectra from PYTHIA

➡ Intitial momentum distribution given by

$$\frac{1}{2\pi p_T dp_T} = \frac{(A_1 + p_T^2)^2}{(1 + A_2 \cdot p_T^2)^{A_3}}$$

- with the parameters $A_1 = 0.5$, $A_2 = 0.1471$, $A_3 = 21$

Potential of the Heavy Quarks

‣Formalism to describe heavy quarks in Abelian plasma by Blaizot et al.

‣Idea: effective theory of non-relativistic HQs in plasma of relativistic particles

‣Influence functional in infinite-mass limit and large time limit:

$$\Phi[\boldsymbol{Q}] \simeq g^2(t_f - t_i) \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} (1 - exp[i\boldsymbol{k}(\boldsymbol{r} - \bar{\boldsymbol{r}})]\Delta(0,\boldsymbol{k}))$$

➡Interpretation as complex potential

$$\mathscr{V}(\boldsymbol{r}) = -g^2\left[V(\boldsymbol{r}) - V_{ren}(0)\right] - ig^2\left[W(\boldsymbol{r}) - W(0)\right]$$

Potential of the Heavy Quarks

- Real part:

$$V(\boldsymbol{r}) = -\Delta^R(0,\boldsymbol{r}) = -\int \frac{d\boldsymbol{k}}{(2\pi)^3} e^{i\boldsymbol{k}\boldsymbol{r}} \Delta^R(\omega = 0,\boldsymbol{k})$$

- Imaginary part:

$$W(\boldsymbol{r}) = -\Delta^<(0,\boldsymbol{r}) = -\int \frac{d\boldsymbol{k}}{(2\pi)^3} e^{i\boldsymbol{k}\boldsymbol{r}} \Delta^<(0,\boldsymbol{k})$$

with the propagator $\Delta(0,\boldsymbol{r}) = \Delta^R(0,\boldsymbol{r}) + i\Delta^<(0,\boldsymbol{r})$

Heavy Quarks in Abelian Plasma

- Complex potential for $c\bar{c}$-pair after evaluation of integrals:

$$\mathcal{V}(\boldsymbol{s}) = -\frac{g^2}{4\pi}m_D - \frac{g^2}{4\pi}\frac{\exp(-m_D s)}{s} - i\frac{g^2 T}{4\pi}\phi(m_D s)$$

With $\phi(x) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$

- Drag and diffusion coefficients derived from potential from the second derivative of $\boldsymbol{W}$:

$$\mathcal{H}_{\alpha\beta}(\boldsymbol{s}) = \frac{\partial^2 W(\boldsymbol{s})}{\partial r_\alpha \partial r_\beta},$$

and using $g^2\mathcal{H}(0)_{\alpha\beta} = 2MT\gamma\delta_{\alpha\beta}$

Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

Elliptic Fireball

- Extension to 3D-flow field with longitudinal component and finite rapidity:

➡ Superimpose of model using boost-invariant Bjorken flow

- Ansatz for 4-velocity with $t = \tau \cosh(\eta)$, $z = \tau \sinh(\eta)$:

$$u^\mu(\tau, \boldsymbol{r}_\perp, \eta) = f(\tau, \boldsymbol{r}_\perp) \begin{pmatrix} \tau \cosh(\eta) \\ \boldsymbol{g}_\perp(\tau, \boldsymbol{r}_\perp) \\ \tau \sinh(\eta) \end{pmatrix}$$

➡ 3D-velocity:

$$\boldsymbol{v} = \frac{\boldsymbol{u}}{u^0} = \left(\frac{\boldsymbol{g}_\perp}{\tau \cosh(\eta)}, \tanh(\eta) \right)$$

Elliptic Fireball

- Resulting transversal velocity at mid rapidity ($\eta = 0$): $\boldsymbol{v}_{\perp|\eta=0} = \dfrac{\boldsymbol{g}_\perp}{\tau}$
- Extension to by combining the two previous equations:

$$\boldsymbol{v}_\perp(\tau, \boldsymbol{r}_\perp, \eta) = \frac{1}{\cosh(\eta)} \boldsymbol{v}_\perp(\tau, \boldsymbol{r}_\perp, \eta = 0) = \frac{\tau}{t} \boldsymbol{v}_\perp(\tau, \boldsymbol{r}_\perp, \eta = 0)$$

- 3D-flow field:

$$v_x = \frac{\tau}{t} v_b(\tau) \cos(v) \frac{r}{r_B}, \quad v_y = \frac{\tau}{t} v_a(\tau) \sin(v) \frac{r}{r_B}, \quad v_z = \tanh(\eta)$$

- Initial momentum distribution of heavy quarks in the fireball given by parametrization fitting charm-quark spectra from PYTHIA
- Initial spatial distribution according to Glauber model

Elliptic Fireball

Parametrisation of hadronic freeze-out

- differential momentum spectrum of a particle:

$$\frac{dN}{p_T dp_T d\phi_p dy} = \frac{2g}{(2\pi)^3}\tau_f \; m_T \; e^{\frac{\mu}{T_f}} \int r dr \int d\phi_s K_1(m_T, T, \beta_T) e^{\frac{p_T}{T_f \sinh(\rho(r,\phi_s)} \cos(\phi_p - \phi_b))}$$

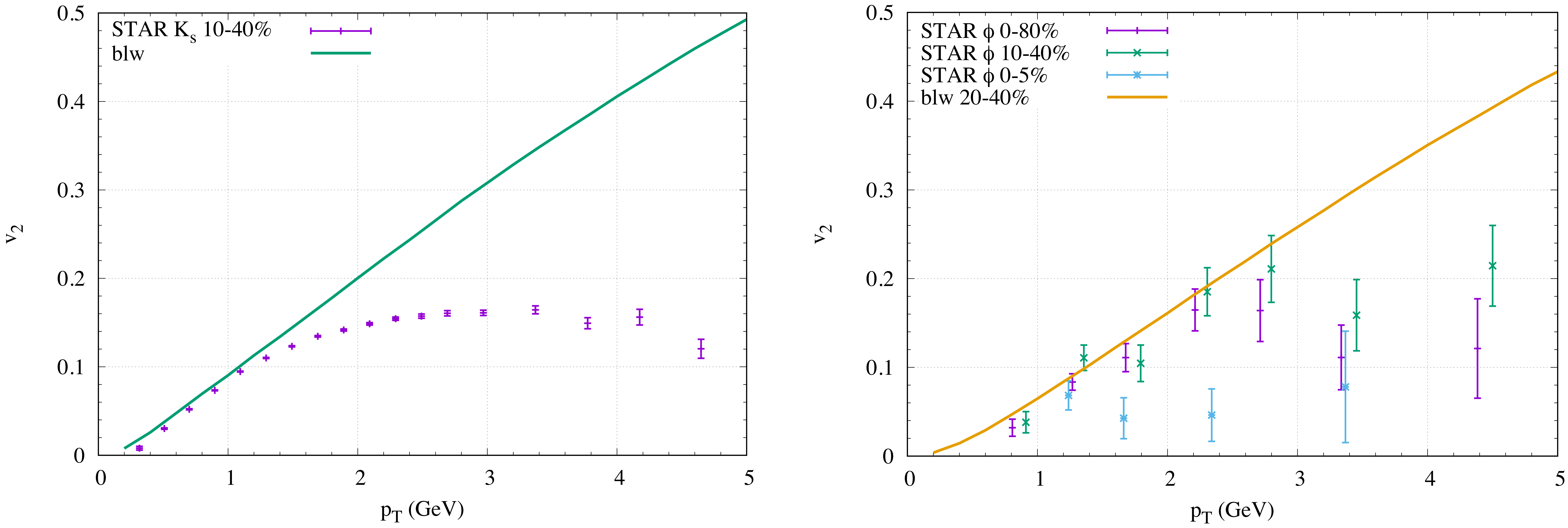
- T_f: freeze-out temperature, ϕ_b: azimuthal angle of the boost, K_1: Bessel function
- transverse rapidity $\rho(r, \phi_s)$: function of radius r and spatial azimuthal angle ϕ_s
- Elliptic flow:

$$v_2(p_T) = \frac{\int_0^{2\pi} d\phi_p \cos(2\phi_p) \frac{dN}{p_T dp_T d\phi_p dy}}{\int_0^{2\pi} d\phi_p \frac{dN}{p_T dp_T d\phi_p dy}}$$

Parametrization of the Fireball

RHIC (20-40%), v_2

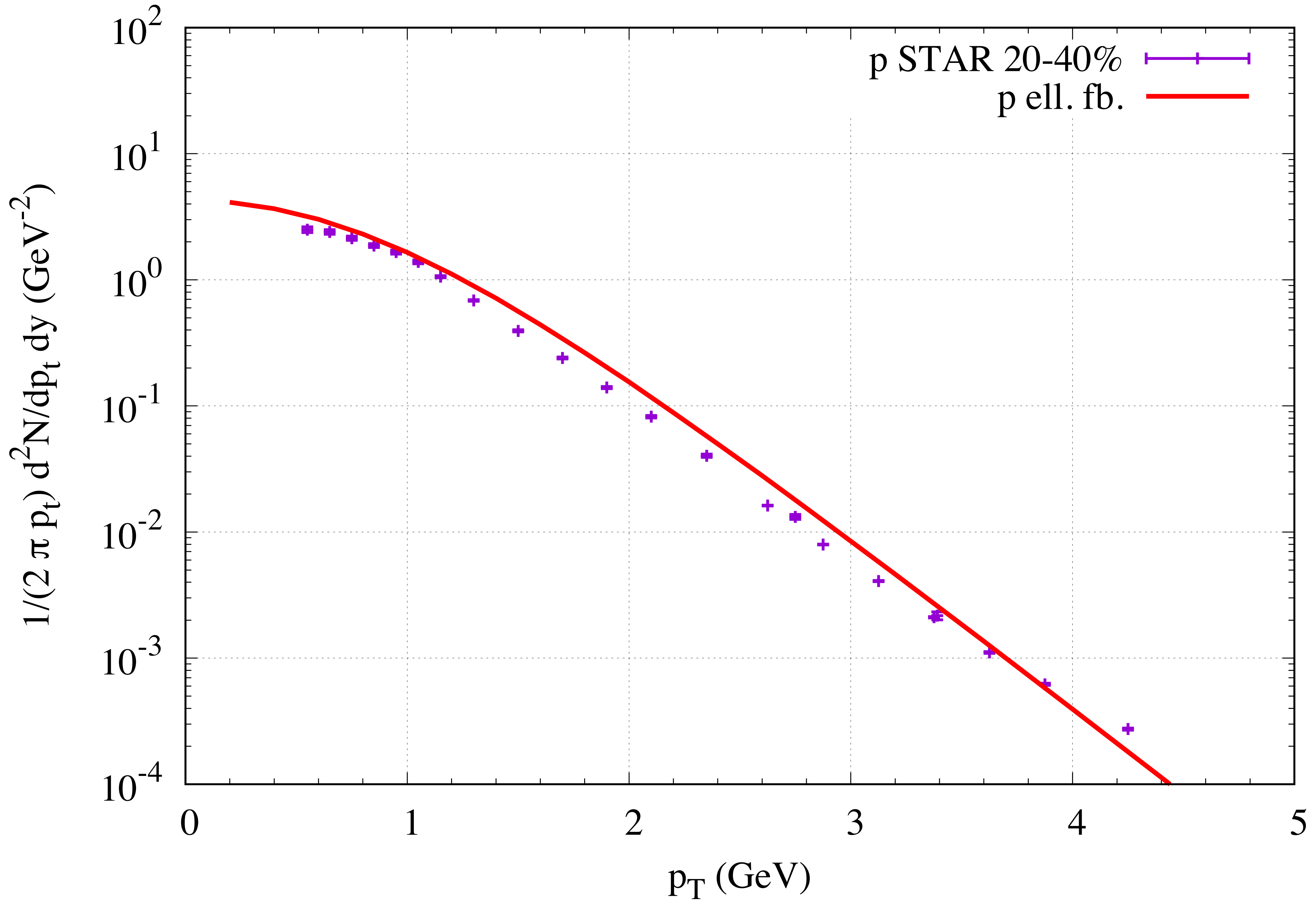
- Choice of parameters in fireball model by fitting results to experimental data
- Elliptic flow v_2 of K_S and ϕ from STAR

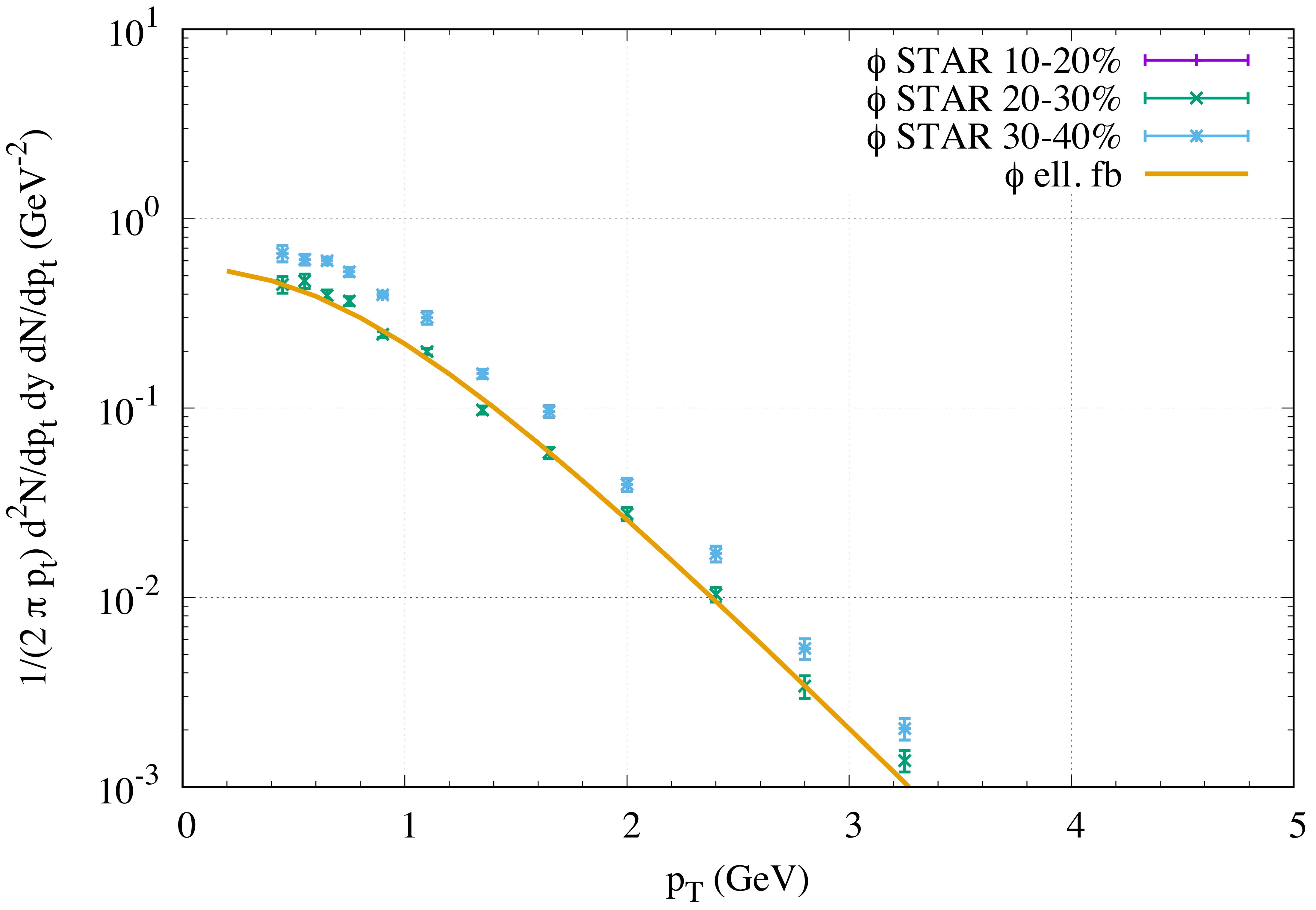


Parametrization of the Fireball

RHIC (20-40%), p_T

- Choice of parameters in fireball model by fitting results to experimental data
- p_T -spectra of p and ϕ from STAR

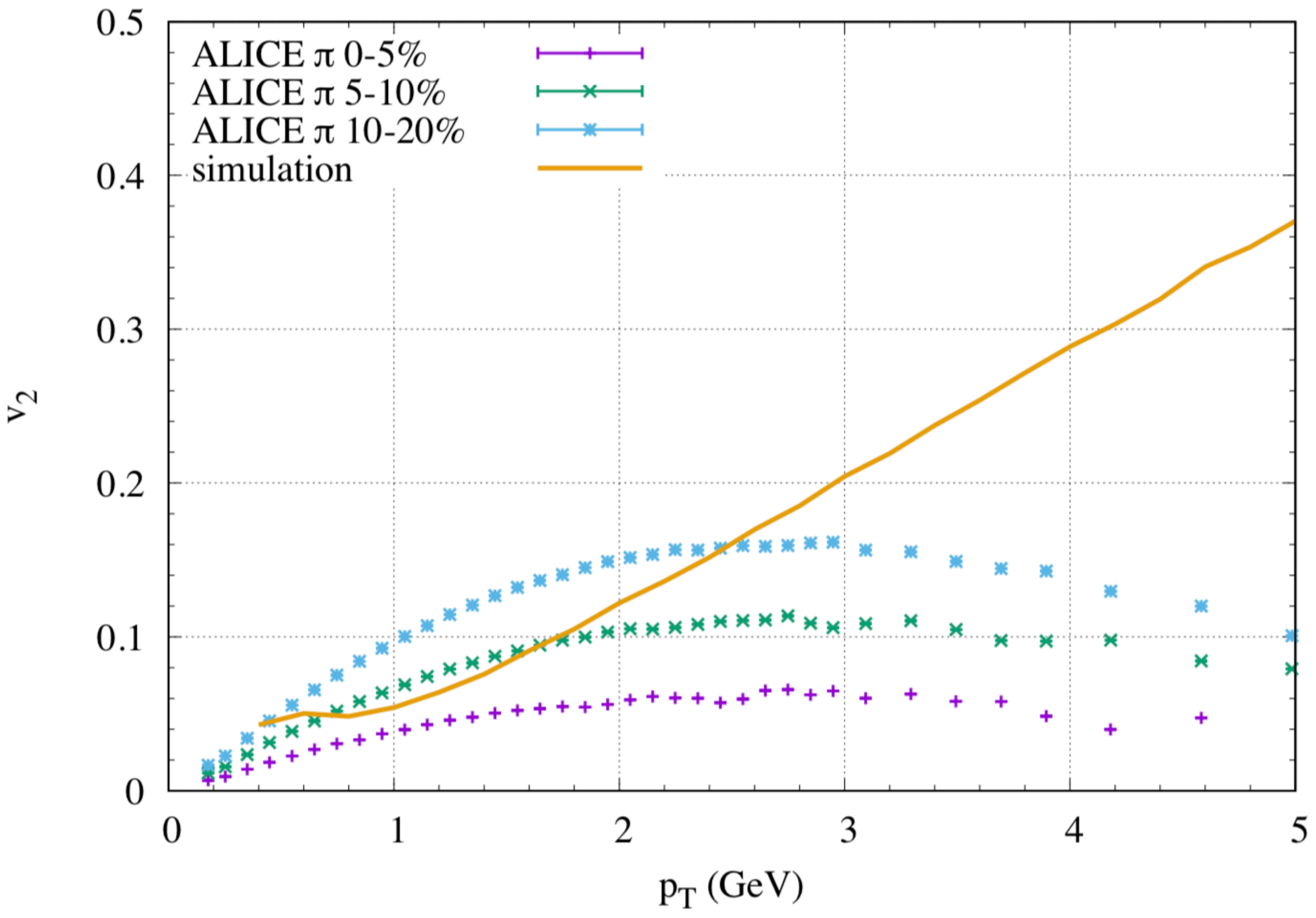


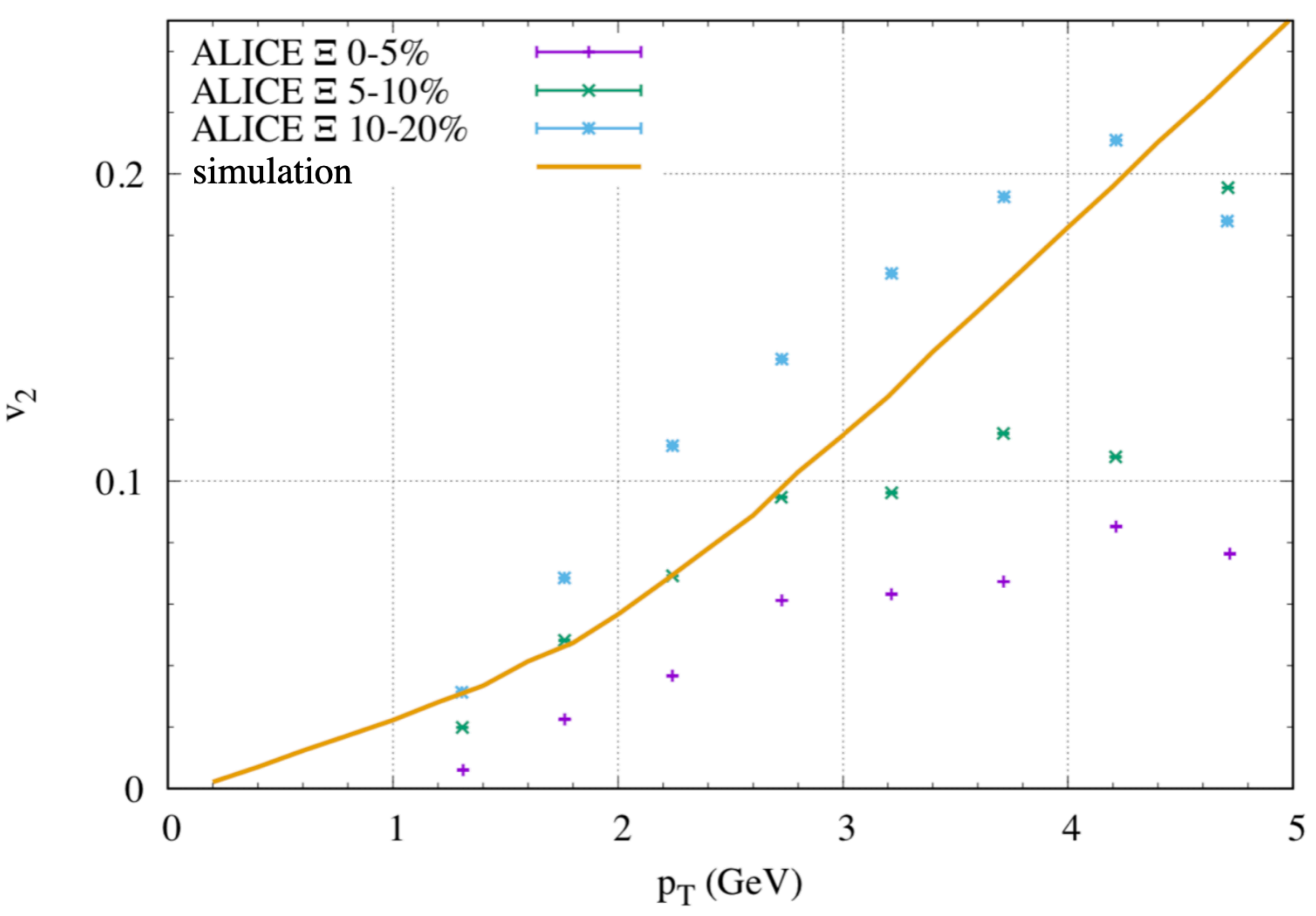


Testing the Model

Parametrization of the Fireball (LHC, 0-20%)

- Comparison of elliptic flow spectra from simulation to data from ϕ and Ξ from ALICE

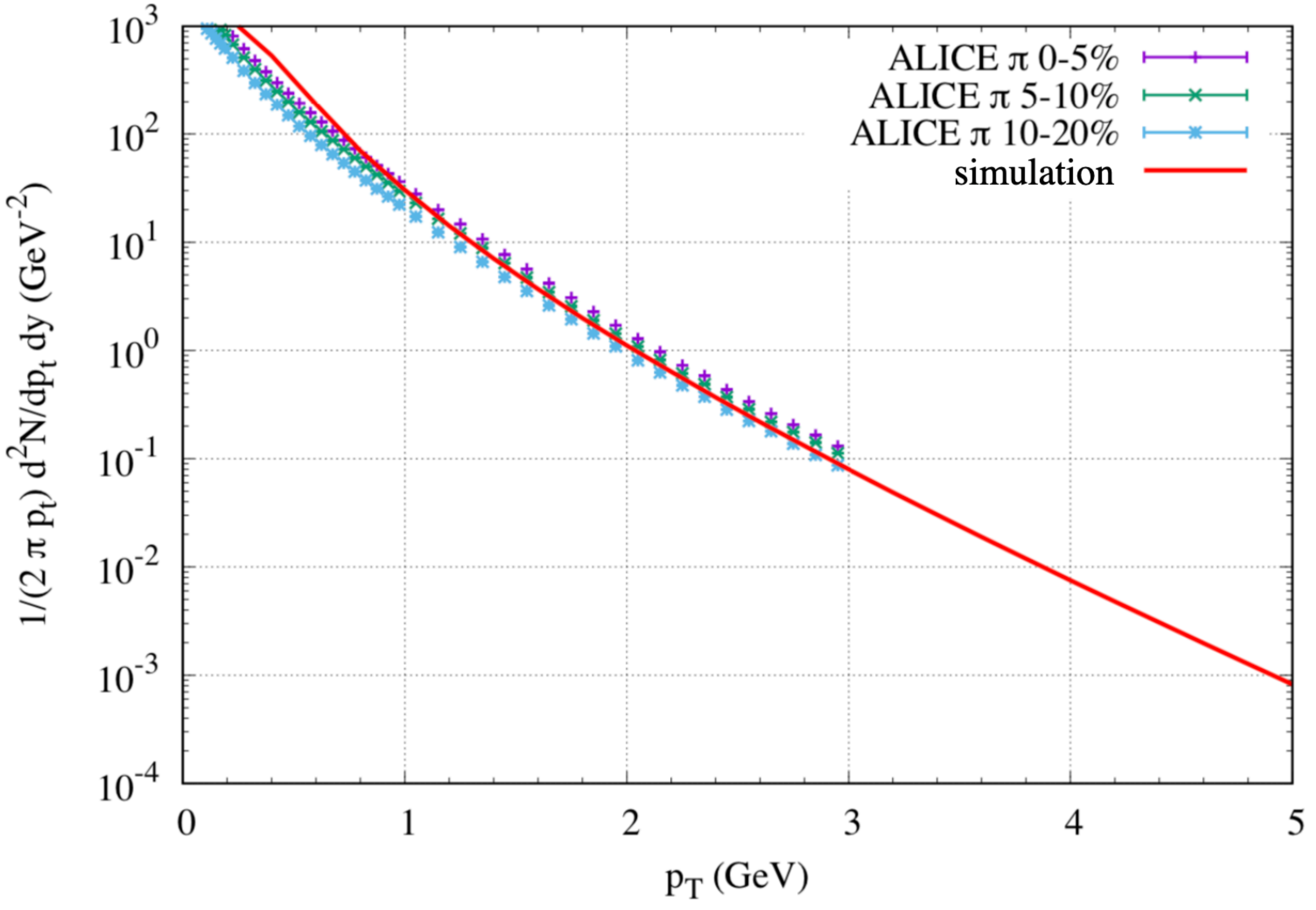


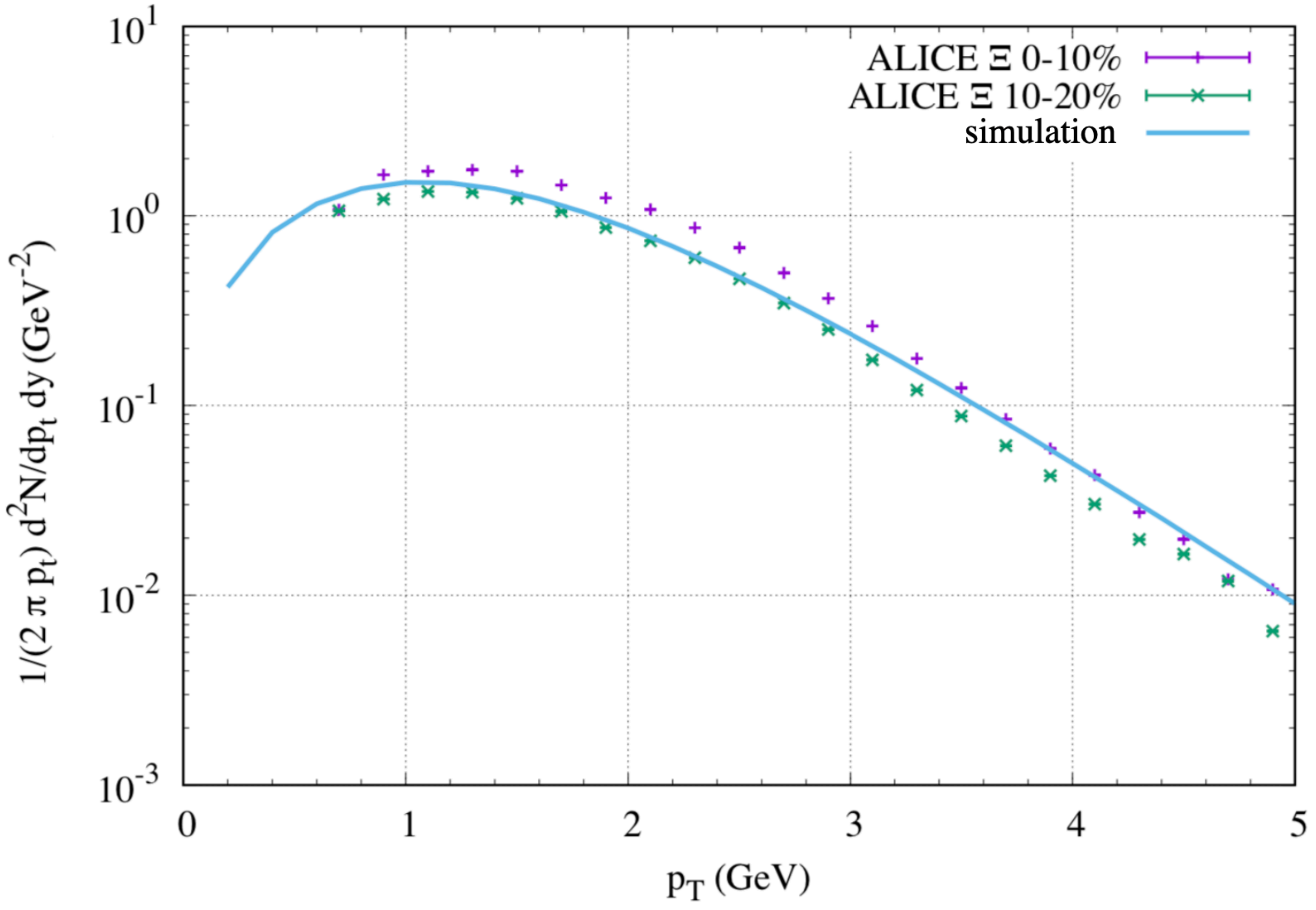


Testing the Model

Parametrization of the Fireball (LHC, 0-20%)

- Comparison of p_T - spectra from simulation to data from π and Ξ from ALICE

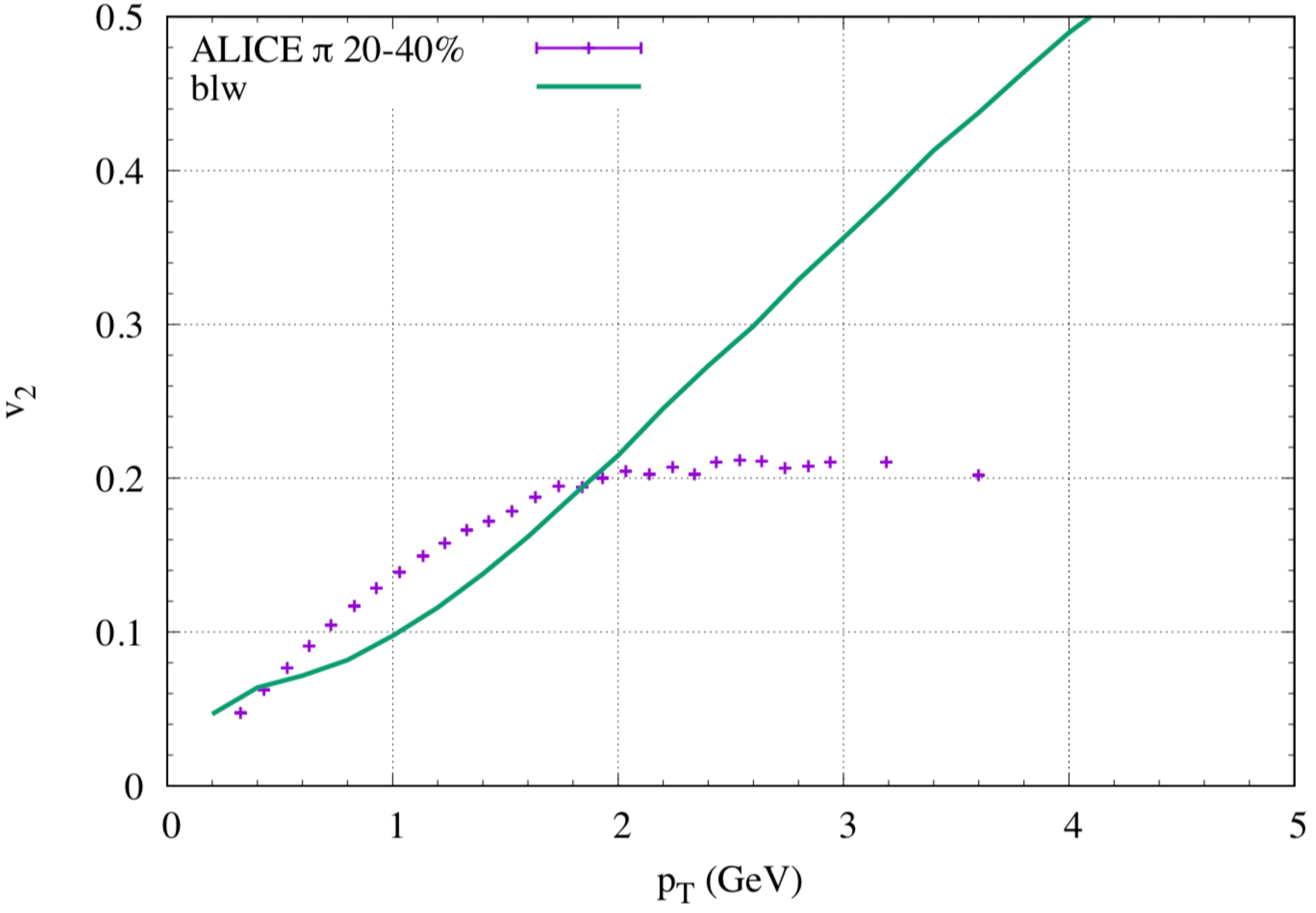


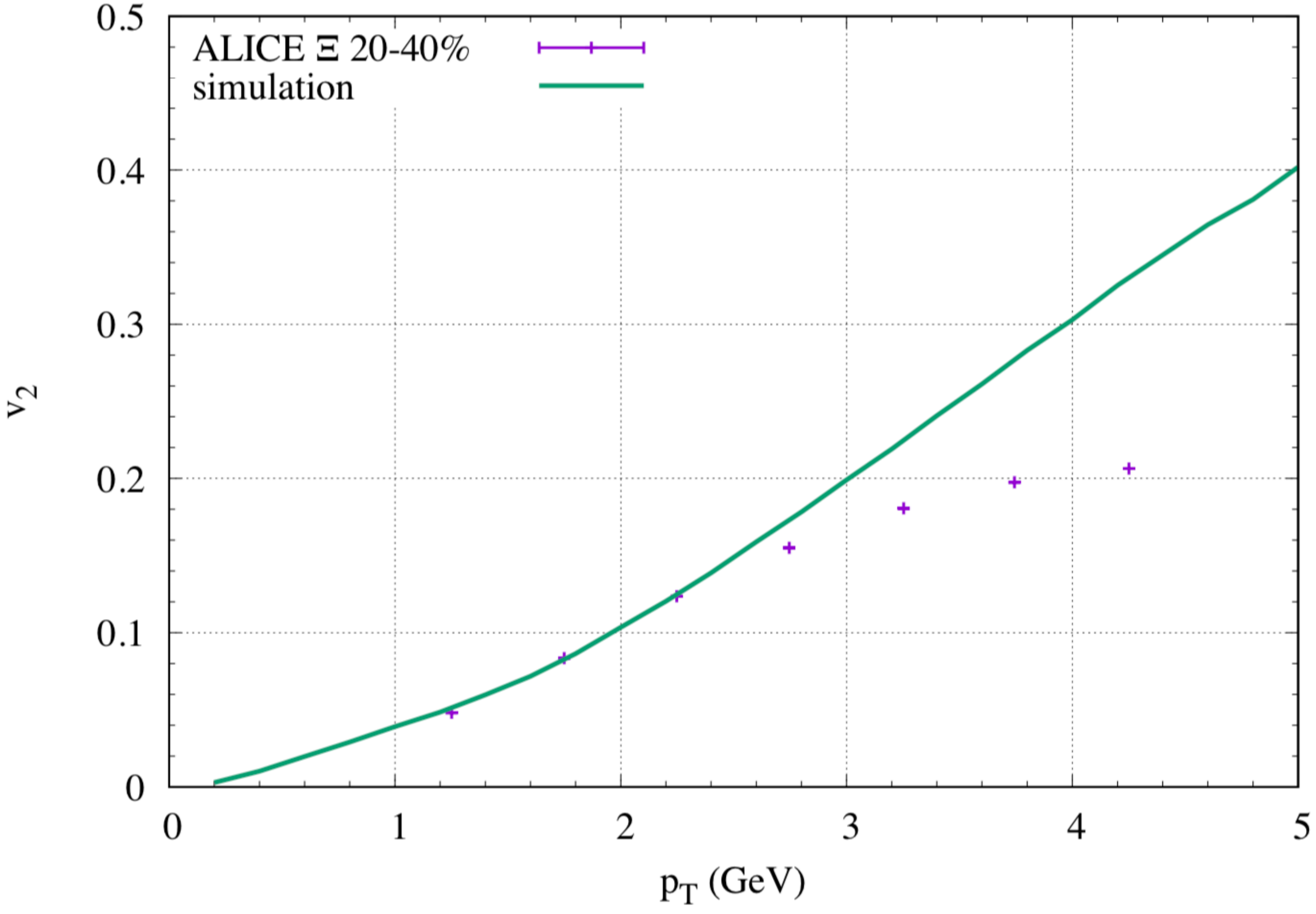


Testing the Model

Parametrization of the Fireball (LHC, 20-40%)

- Comparison of elliptic flow spectra from simulation to data from π and Ξ from ALICE

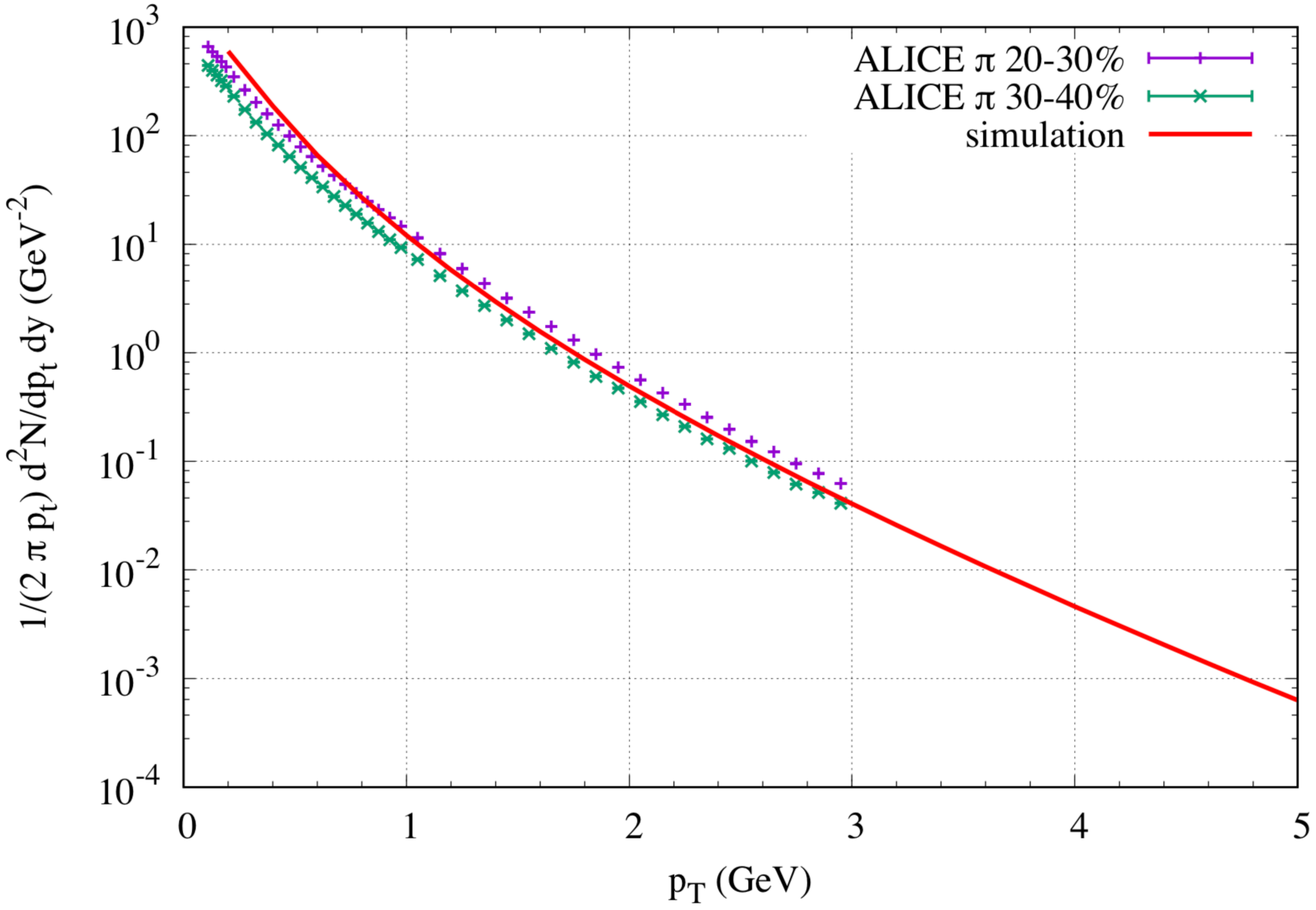


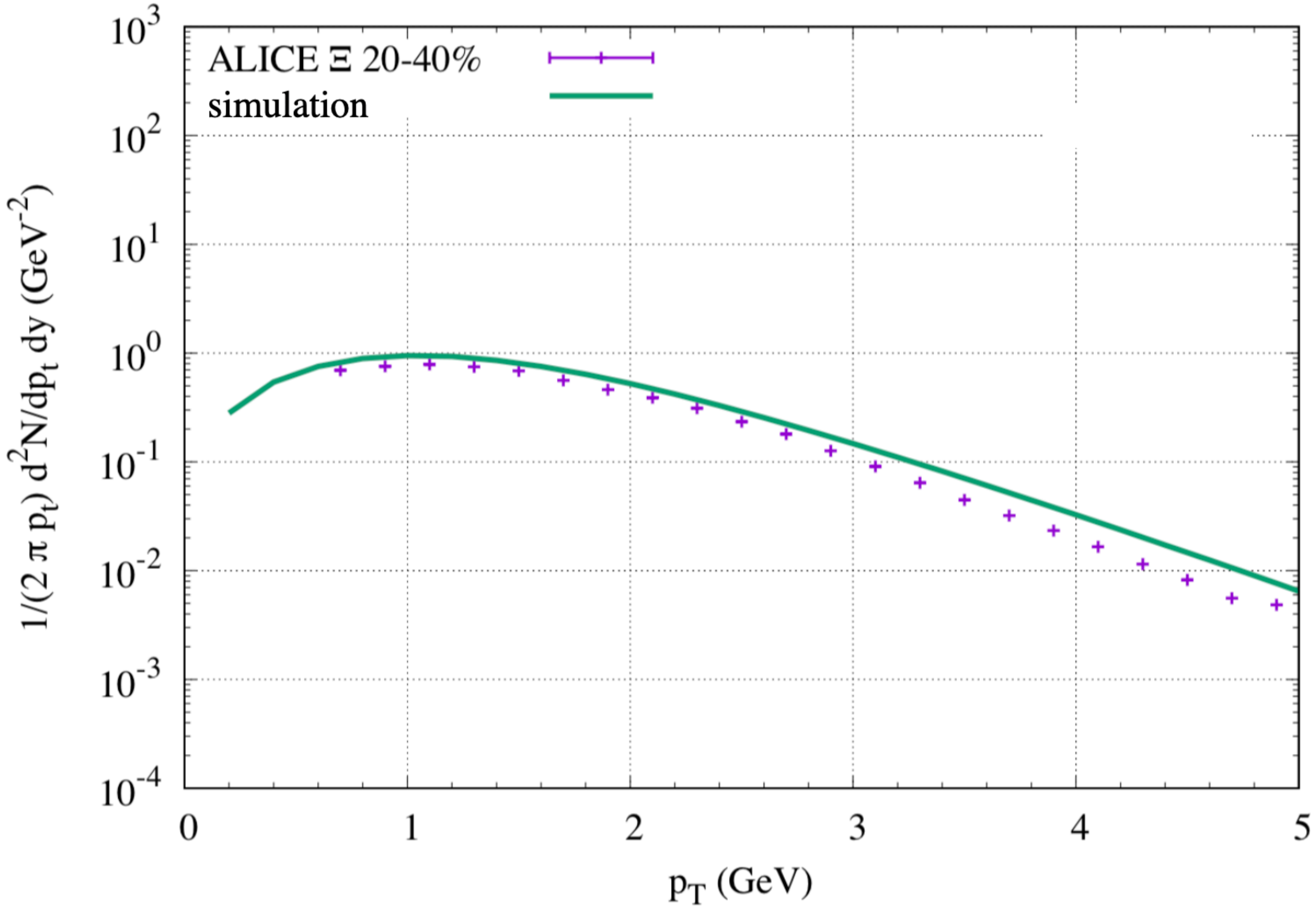


Testing the Model

Parametrization of the Fireball (LHC, 20-40%)

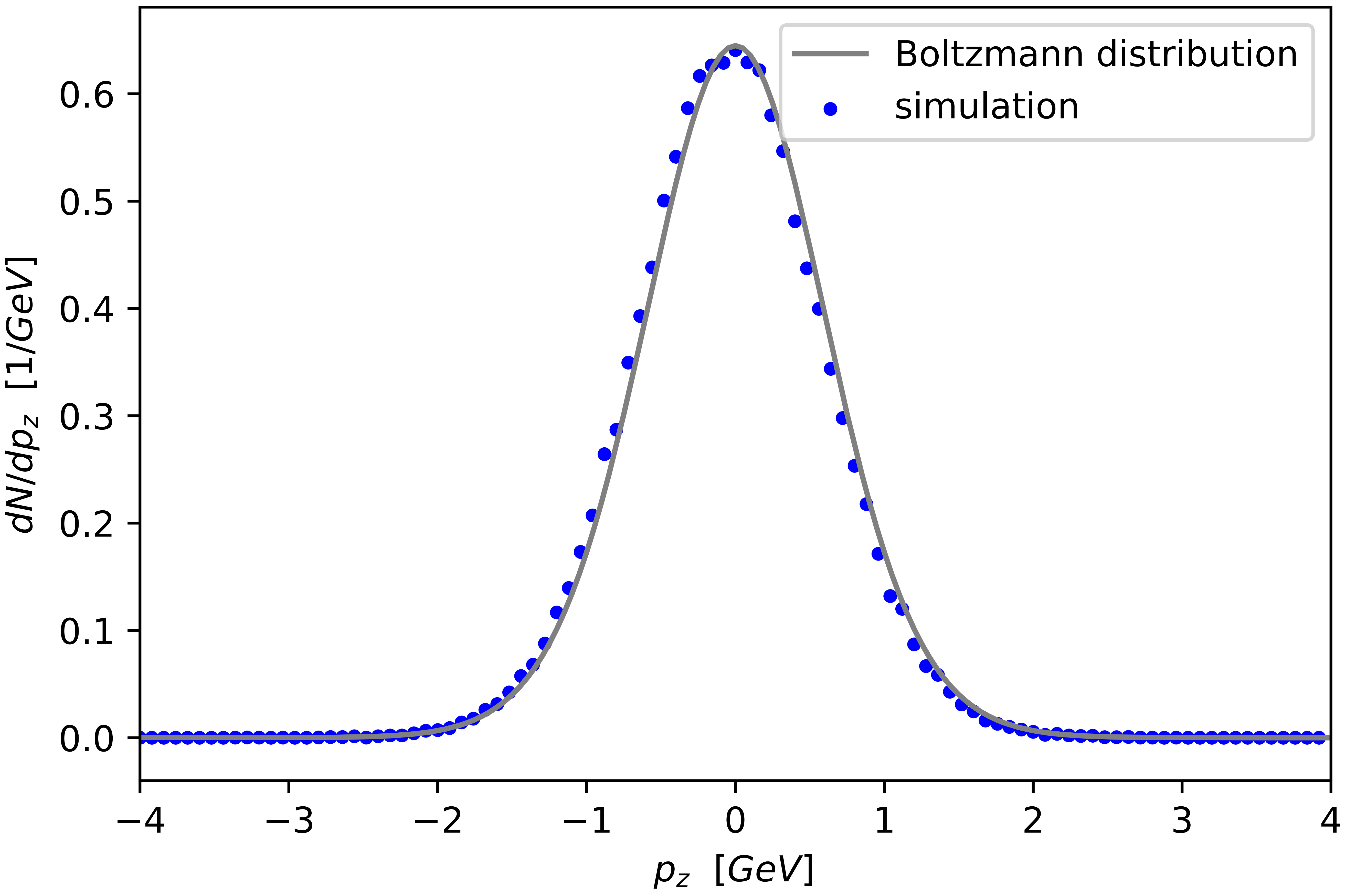
- Comparison of p_T - spectra from simulation to data from π and Ξ from ALICE





Testing the Model

Equilibrium Conditions in Box Calculations

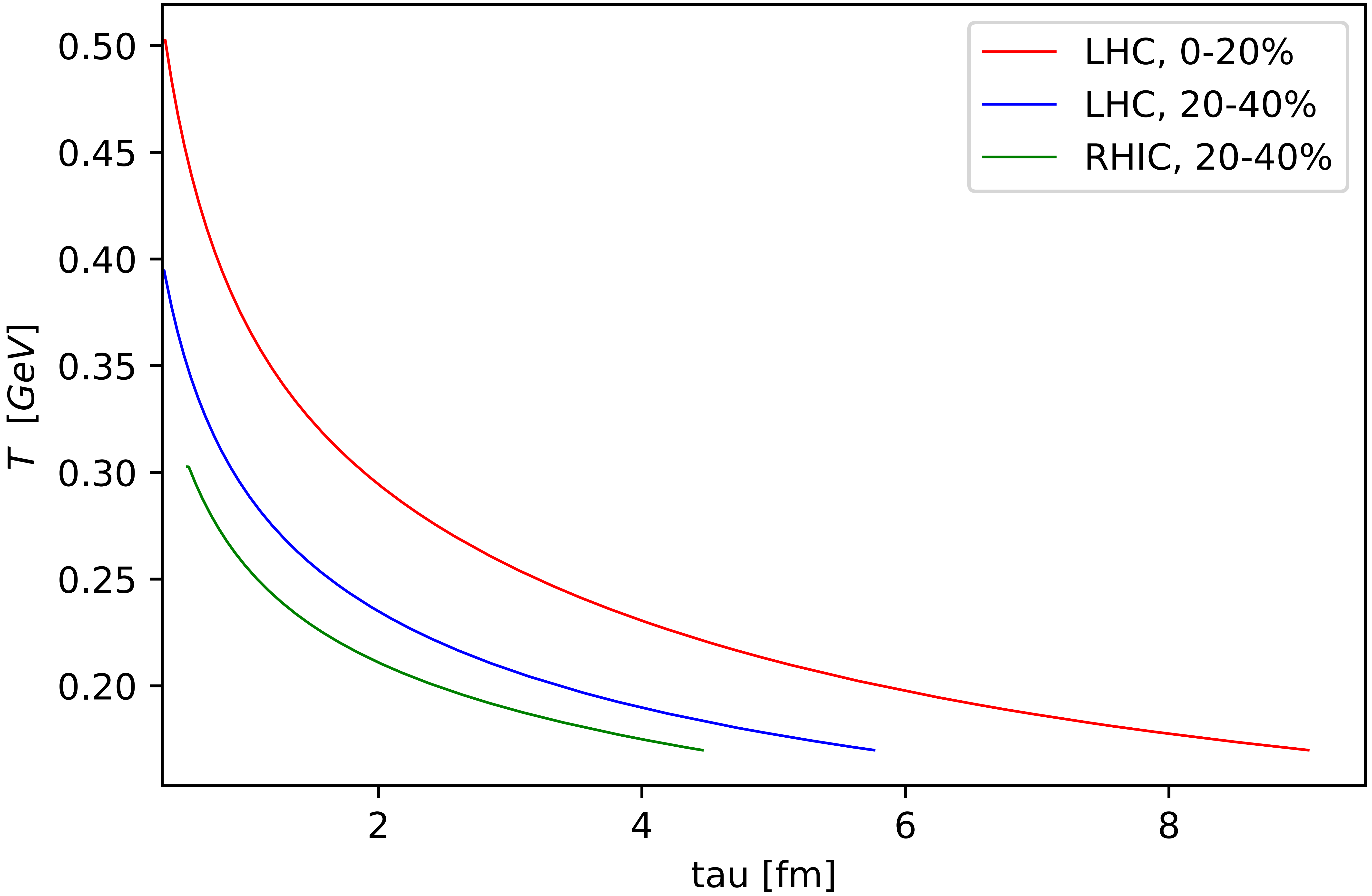


- Single $c\bar{c}$-pair in box calculation with $T = 180\ MeV$ and $m_c = 1.8\ GeV/c^2$
- Momentum distribution in equilibrium limit:

$$f_{eq}(\boldsymbol{p}) \propto \exp\left(-\frac{\boldsymbol{p}^2}{2MT}\right)$$

Testing the Model

Temperature of the Fireball



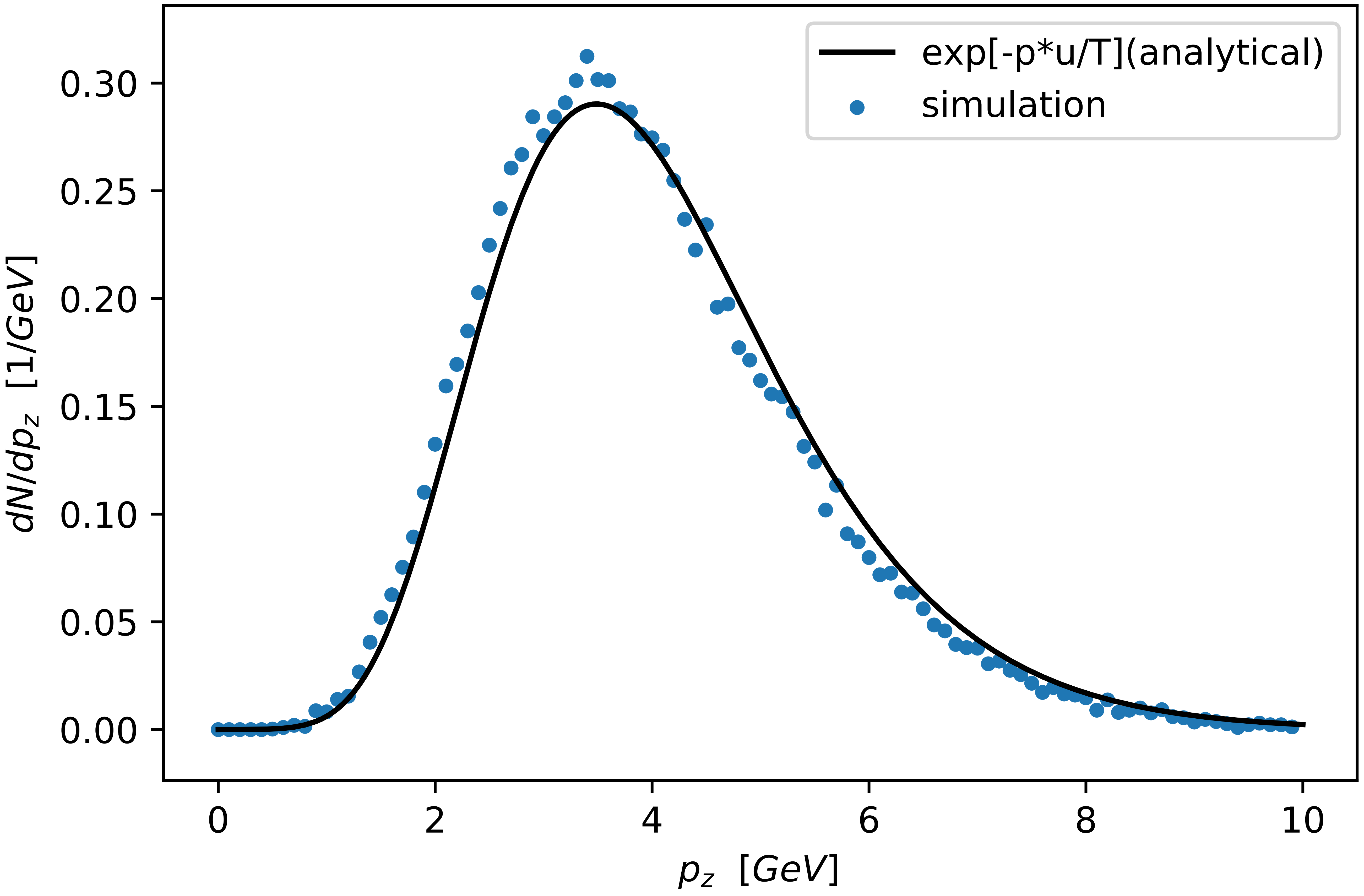
- Sequential freeze-out
- $T_{ch} = 160\ MeV$
- Isentropic expansion towards kinetic freeze-out

➡ Extrapolation to temperature in QGP-phase

- Exponential decrease of T until T_{ch}

Lorentz Boost to Moving Medium

p_z distribution



- Single $c\bar{c}$-pair in box calculation with $T = 180\ MeV$ and $m_c = 1.5\ GeV/c^2$
- constant flow-field $\boldsymbol{v} = (0, 0, 0.9)$
- Boltzmann-Jüttner distribution:

$$f_{eq}(\boldsymbol{p}) \propto \exp\left(-\frac{E(\boldsymbol{p})}{T}\right)$$

Relative energy of a $c\bar{c}$-pair

Energy distribution in equilibrium

- Relative energy of $c\bar{c}$-pair:

$$E_{rel} = E_c + E_{\bar{c}} + V(|\boldsymbol{r}_c - \boldsymbol{r}_{\bar{c}}|) - E_{tot}$$

$$= \sqrt{m_c^2 + \boldsymbol{p_c}^2} + \sqrt{m_{\bar{c}}^2 + \boldsymbol{p}_{\bar{c}}{}^2} + V(r, T) - \sqrt{(m_c + m_{\bar{c}})^2 + (\boldsymbol{p}_c + \boldsymbol{p}_{\bar{c}})^2}$$

- In com-system ($(\boldsymbol{p}_c + \boldsymbol{p}_{\bar{c}}) = \boldsymbol{0}$) equivalent to

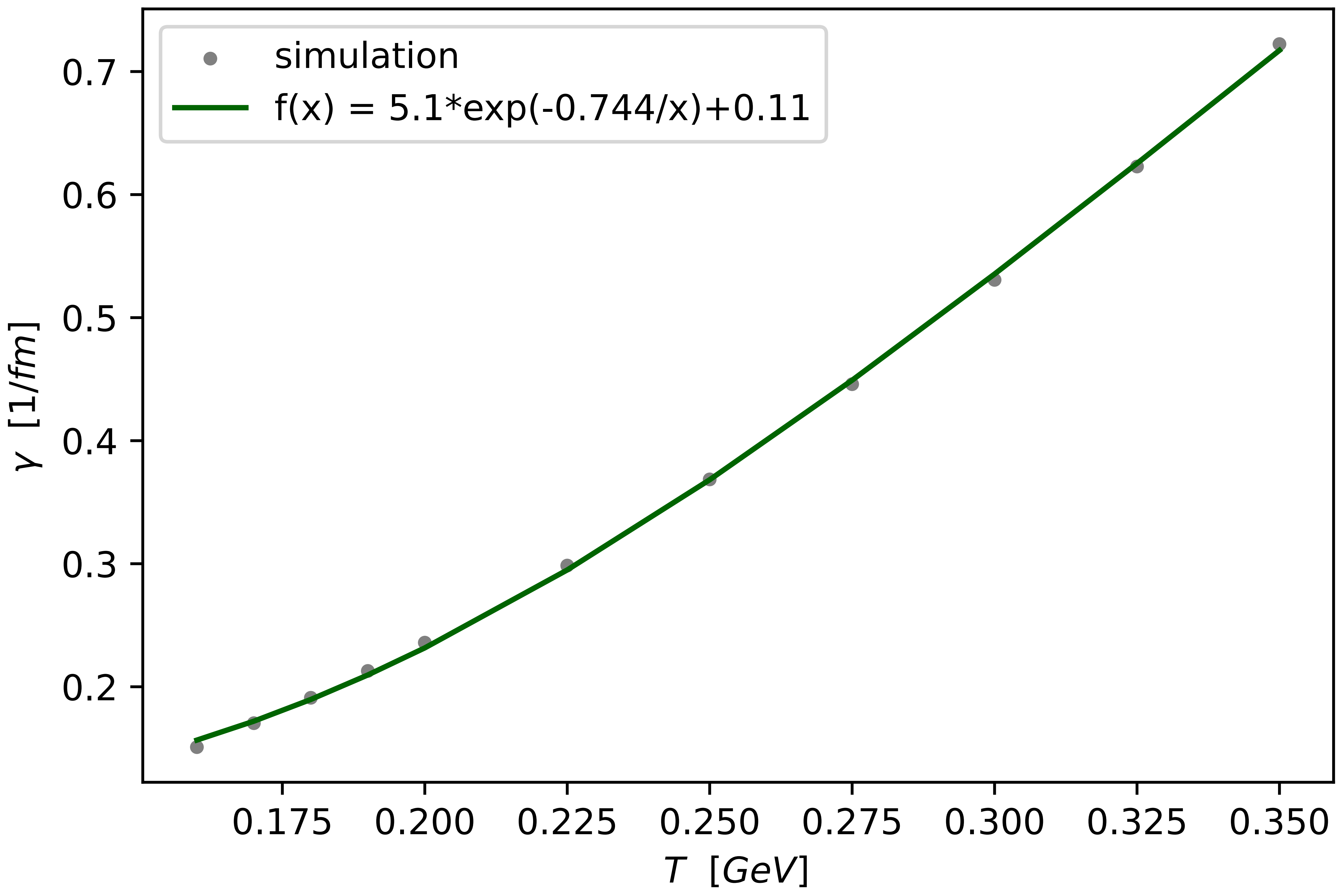
$$E_{rel} = m_{0,cms} + V(r, T) - (m_c + m_{\bar{c}})$$

With $p^{\mu}_{tot}\ p_{\mu,tot} = (E^c + E^{\bar{c}})^2 - (\boldsymbol{p_c} + \boldsymbol{p}_{\bar{c}})^2 = m^2_{0,cms}$

Box Calculations

Drag Coefficient, Equilibration Time and Temperature

Drag coefficient γ as a function of Temperature



$\tau_{eq} = 1\gamma$ as a function of Temperature