Recent news from quarkonia

Pol B Gossiaux, SUBATECH (NANTES)

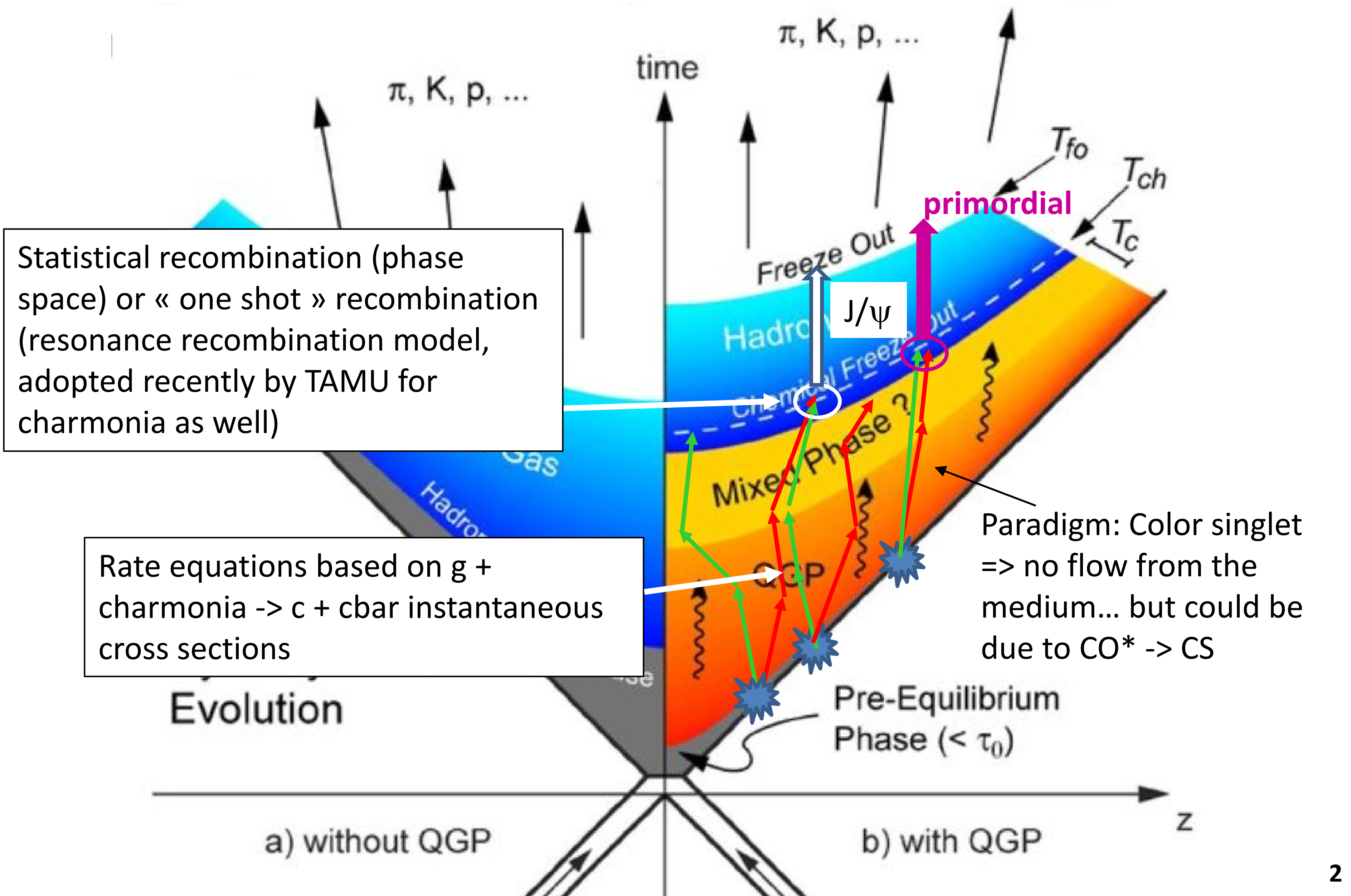
2nd Workshop of the Network NA7-HF-QGP of the European program "STRONG-2020" and the 'HFHF Theory Retreat 2023'

28 September – 4 October 2023
Giardini Naxos, Sicily, Italy

With Thierry Gousset, Roland Katz, Stéphane Delorme, Jean-Paul Blaizot & Aoumeur Daddi

and Pays de la Loire

Quarkonia as a possible probe of the QGP



Statistical recombination (phase space) or « one shot » recombination (resonance recombination model, adopted recently by TAMU for charmonia as well)

Rate equations based on g + charmonia -> c + cbar instantaneous cross sections

Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

The best working horse today: Rate equations

$$\frac{dN(t)}{dt} = -\Gamma(T(t))\,(N(t) - N^{\rm eq}(T(t)))$$

Loss

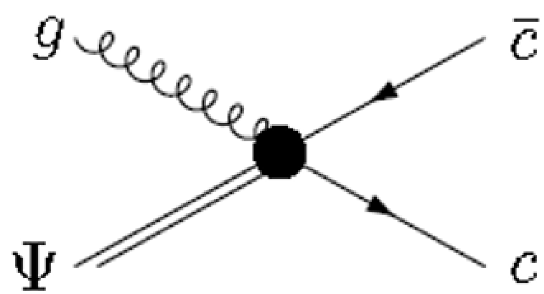
Gain

Statistical limit (canonical) assumed

For instance, Bhanot-Peskin gluo-dissociation

$$\sigma_{J/\psi}(\omega) = A_0 \frac{(\omega/\epsilon_{J/\psi} - 1)^{3/2}}{(\omega/\epsilon_{J/\psi})^5}$$

ω: gluon energy in the quarkonium rest frame



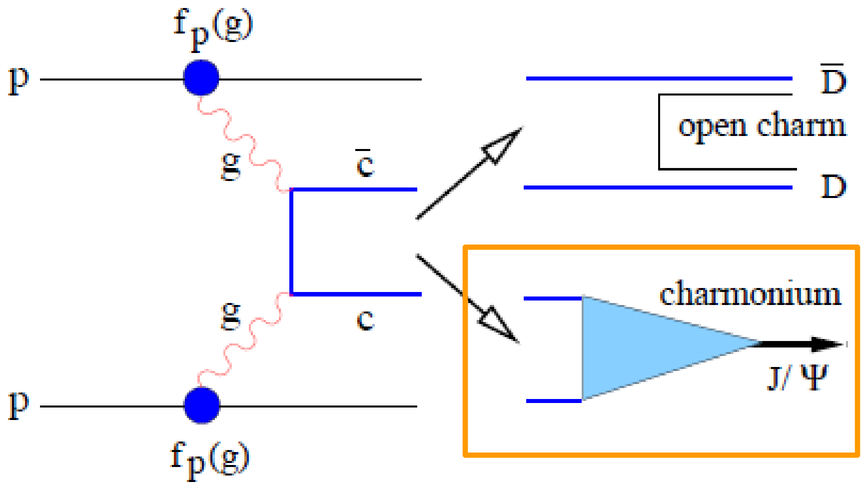
$$\Gamma_\Psi(T) \sim \int d^3k_g \sigma_\Psi(\omega) f_{BE}(T,\omega)$$ Dissociation rate

$$N_{\rm final} = N_0 \times \underbrace{e^{-\int_{t_0}^{+\infty} \Gamma_\Psi(T(t)){\rm d}t}}_{R_{AA}}$$ If just suppression

Various states are still decoupled in their evolution… while in principle, one could have some gluon-induced "conversion" (not implemented in any model to my knowledge)

Decoupled production of various HF mesons

Picture behind transport theory :

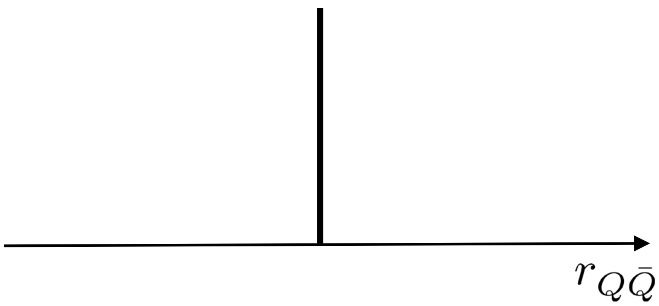


Open heavy flavor and quarkonia assumed to be uncorrelated

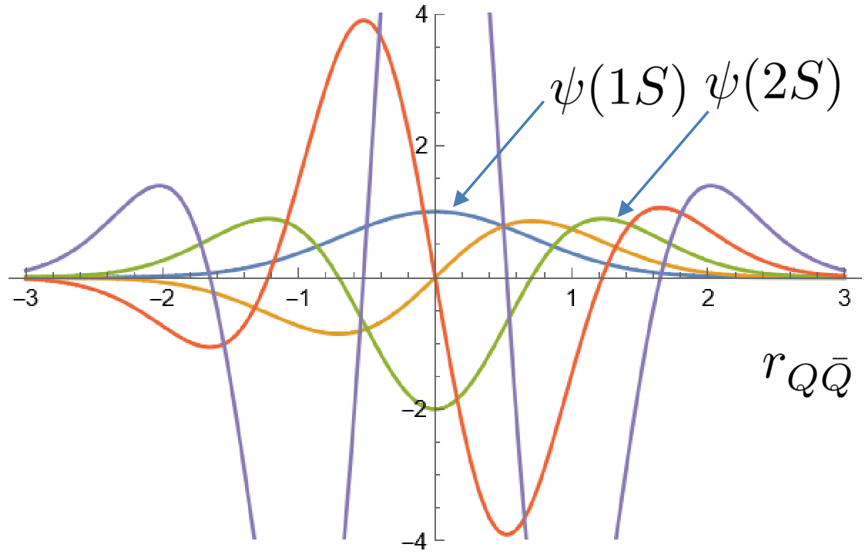
Formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Quantum coherence at « early » time

$$\psi_{\rm in}(\vec{r}_{Q\bar{Q}}) \propto \delta^{(3)}(\vec{r}_{Q\bar{Q}})$$



$$= \Sigma$$



Initial compact state : δ

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2$

Coherence

Neglect of coherence

$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle\psi_j|r^2|\psi_i\rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle\psi_i|r^2|\psi_i\rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Crucial to include coherence !

N.B. : one can model this effect by phenomenological formation time, but lack of control

How can we restore the quantumness of Quarkonia treatment (in interaction with some environment) ?

- Statistical mechanics is about averages… averaging wave functions makes no sense.

Stochastic-like Hamiltonians and ensemble average on the various realizations… even possibly integrating dissipative terms (Schroedinger Langevin with R. Katz)

Like : less computer demanding

Dislike : cannot be rigorously derived from fundamental principles

XOR

Deals with density matrix for the full state (quarkonia + environment)

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$|\psi\rangle = |\psi_{\mathrm{QGP}}\rangle \otimes |\psi_{Q\bar{Q}}\rangle$$

Possible to make statistical averages on the environment (« tracing out ») while still preserving all

Evolution equation for $\rho_{Q\bar{Q}}(x_Q, x_{\bar{Q}}, x'_Q, x'_{\bar{Q}})$

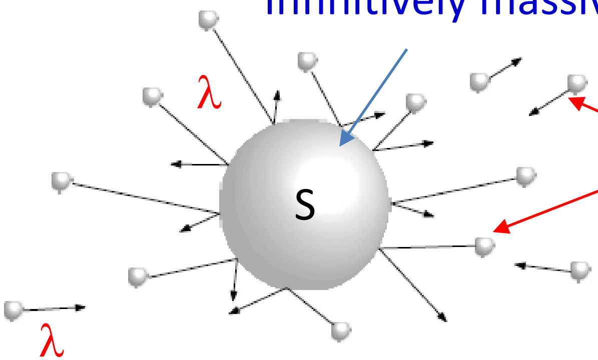
Quantum Master Equation

Like : rigorously derived from fundamental principles

Dislike : computer demanding… currently for several pairs

Decoherence from system-env. interaction

Quantitative model :



$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$$

Reduced density matrix $\rho_S(\mathbf{x},\mathbf{x}')$

⇕

Wigner density $W_S(\mathbf{X} = \frac{\mathbf{x}+\mathbf{x}'}{2}, p)$

Fourier conjugate of $\mathbf{x} - \mathbf{x}'$

$$\frac{\partial \rho_S(\mathbf{x},\mathbf{x}',t)}{\partial t} = -F(\mathbf{x}-\mathbf{x}')\rho_S(\mathbf{x},\mathbf{x}',t)$$

Decoherence factor:

$$F(\mathbf{x}-\mathbf{x}') = \int dq\rho(q)v(q)\int \frac{d\hat{n}d\hat{n}'}{4\pi}\left(1 - e^{iq(\hat{n}-\hat{n}')\cdot(\mathbf{x}-\mathbf{x}')/\hbar}\right)\underbrace{|f(q\hat{n},q\hat{n}')|^2}_{\frac{d\sigma}{d\Omega(\hat{n},\hat{n}')}}$$

Short wave length (λ << Δx)

$$F(\mathbf{x}-\mathbf{x}') = \Gamma_{\text{tot}}$$

Total collision rate

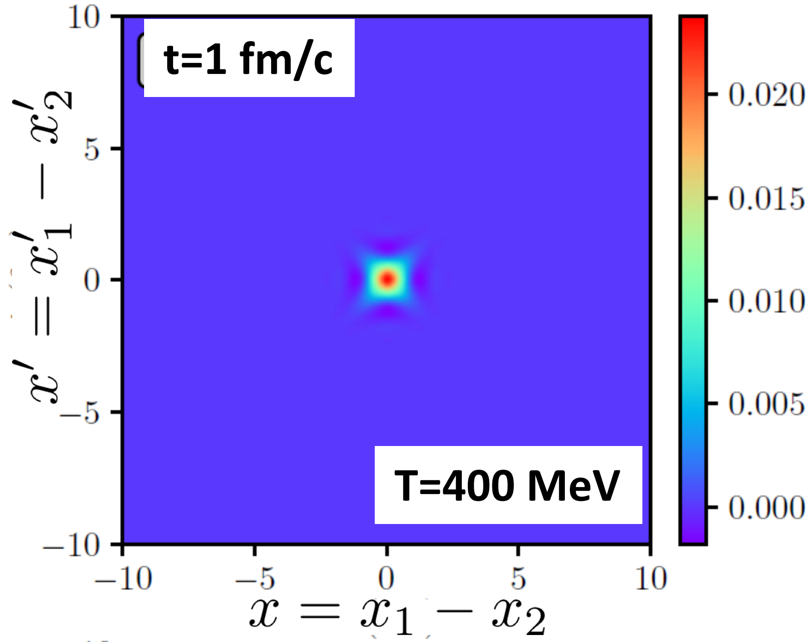
Long wave length (λ >> Δx)

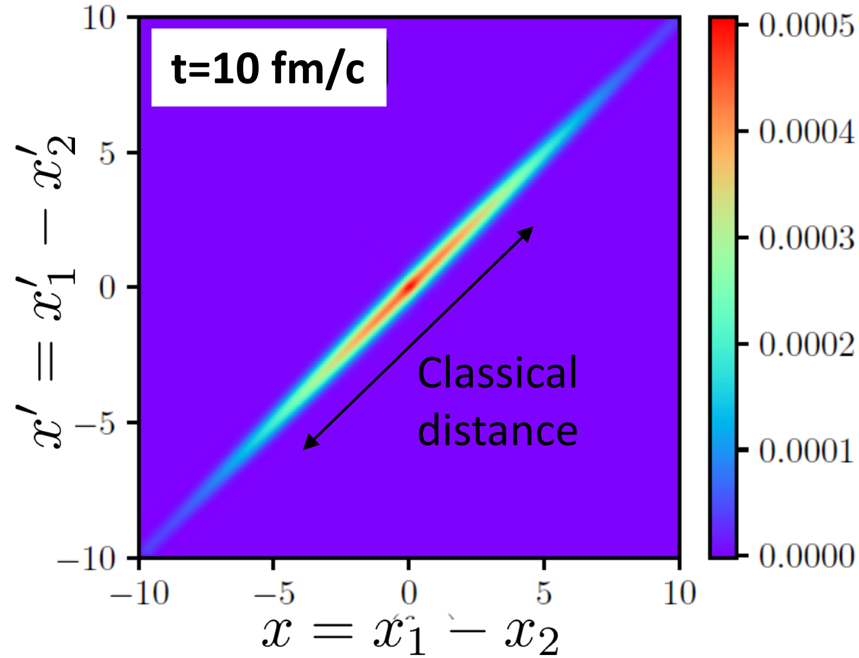
$$F \approx \int dq\rho(q)v(q)q^2\sigma_{\text{transp}}(q) \times (\mathbf{x}-\mathbf{x}')^2 \approx \kappa(\mathbf{x}-\mathbf{x}')^2$$

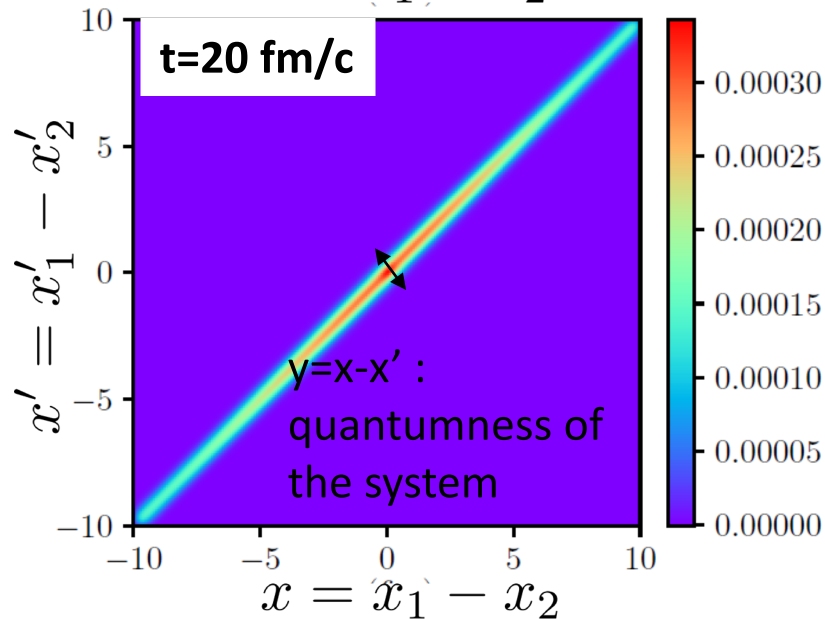
Suppresses coherence at large x-x' : classicalization

For small objects, coherence can be preserved over long times (several "cycles")

Decoherence from system-env. interaction







$$\rho_S(x,x',t) \sim \rho_S(x,x',0)e^{-\Lambda(x-x')^2 t}$$

- Compactification along the short diagonal = « classicalization »
- $t_d \sim \frac{1}{\kappa(\Delta x)^2} \sim \frac{1}{TM\eta_D(\Delta x)^2}$

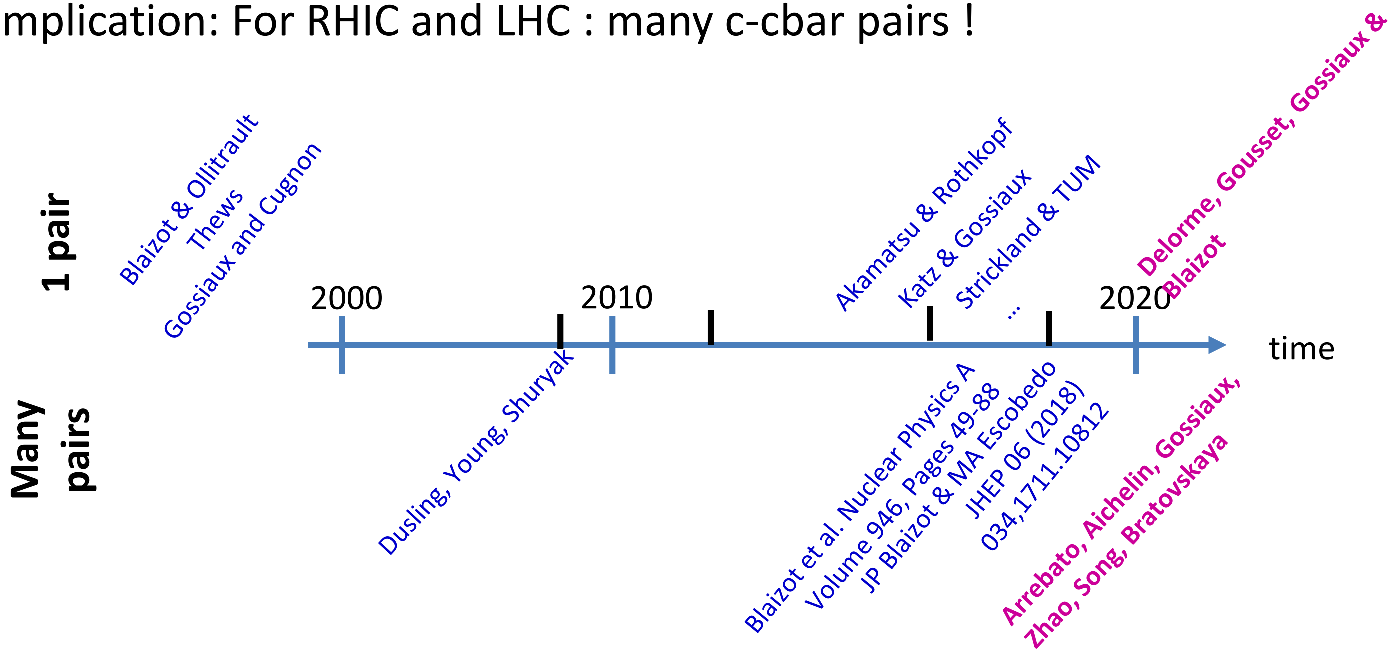
Einstein relation

Single part. relaxation rate

$$t_d \sim \frac{\tau_R^{\text{single}}}{\frac{1}{\lambda_{th}^2}(\Delta x)^2} \sim \tau_R^{\text{single}} \times \left(\frac{\lambda_{\text{th}}}{\Delta x}\right)^2$$

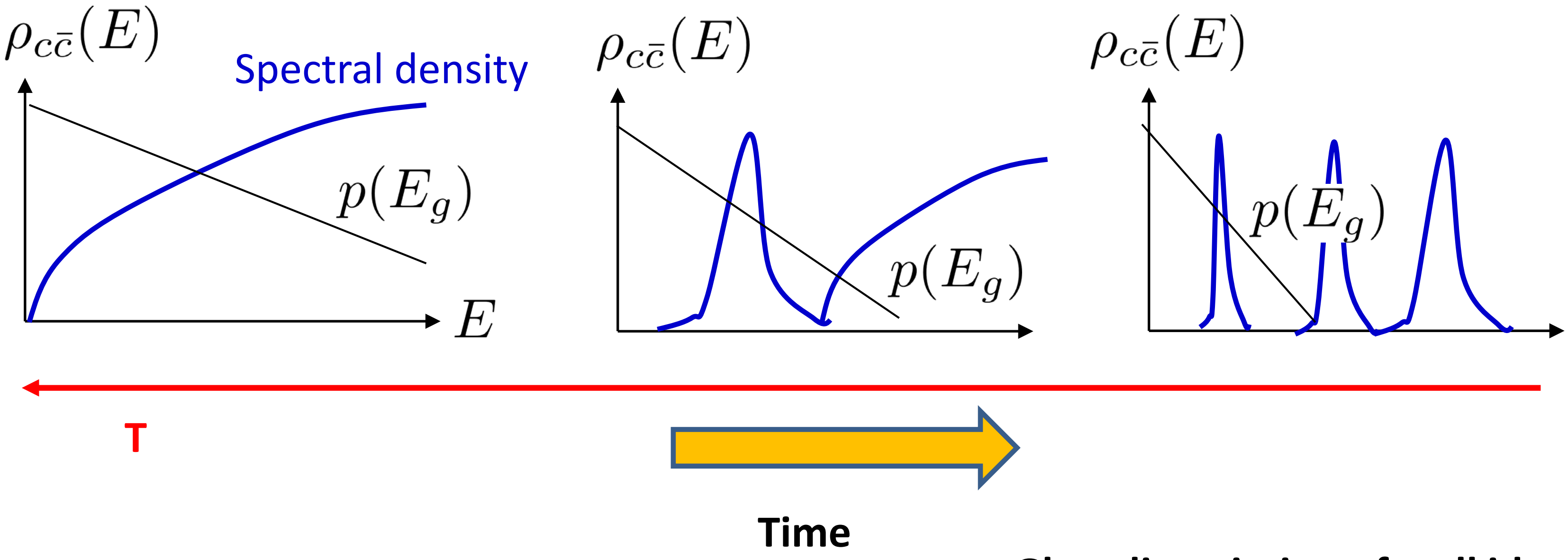
Other motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for **« from small to large »**
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs => Semi-Classical Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

Charmonia in a microscopic theory
Several regimes / effects



Multiple scattering on quasi free states

Gluo-dissociation of well identified levels by scarce “high-energy” gluons (dilute medium => cross section ok)

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Yet, still a need to define the equivalent of a formation – dissociation rate

Two types of dynamical modelling

(and a 3rd class of its own: statistical hadronization)

$m_D \gg E_{\text{bind}}$ — Quantum Brownian Motion

$m_D \sim E_{\text{bind}}$

$m_D \ll E_{\text{bind}}$ — Quantum Optical Regime

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

?

- Well identified resonances
- Time long enough wrt quantum decoherence time

Good description with transport models (TAMU, Tsinghua, Duke)

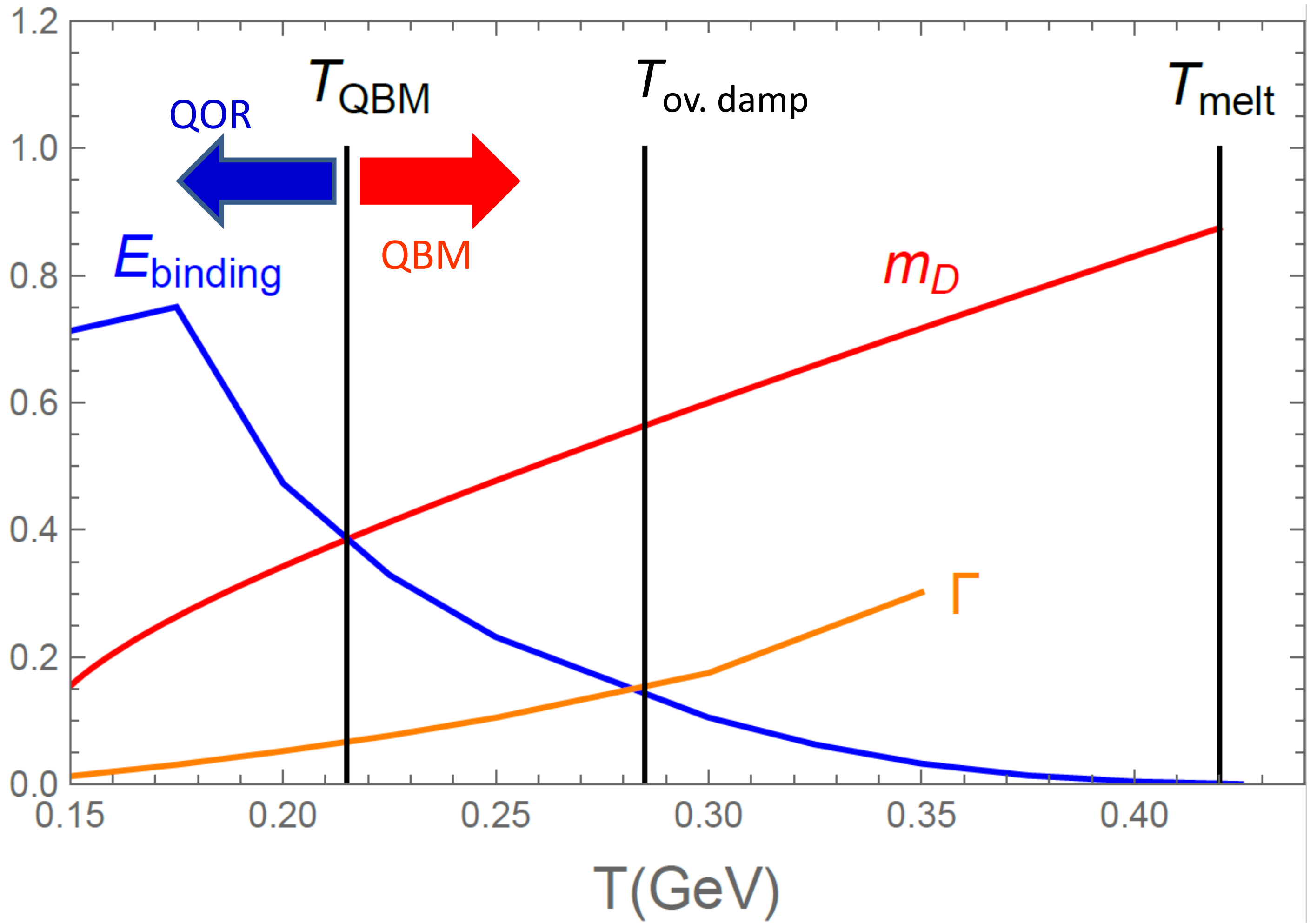
Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : $\exp(-E_n/T)$ (theorem)

SC Approx: rate equations

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these model is an important prerequisite !!!

Two types of dynamical modelling



Numbers extracted from potential described in Phys. Rev. D 101, 056010 (2020)

Blaizot-Escobedo Quantum Master Equation

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}] \xrightarrow[\text{representation}]{\text{Interaction}} i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

Coulomb gauge

Free Quark Hamiltonian

Plasma Hamiltonian

Quark-Plasma Interactions…

$$\mathcal{D}^I(t) = \mathcal{U}_I(t, t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t, t_0)$$

Average over plasma d.o.f + rapid environment hypothesis

$$H_1 = -g\int_{\boldsymbol{r}} A_0^a(\boldsymbol{r})n^a(\boldsymbol{r})$$

No magnetic term (NR)

color charge density of the heavy particles

Generic Linblad – like QME on $\mathcal{D}_Q$

… treated as a perturbation

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^{t}\mathrm{d}t'\int_{\boldsymbol{x}\boldsymbol{x}'}\Big([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}')$$
$$+[\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\Big)$$

$\Delta^>, \Delta^<$ Time ordered HTL gluon propagators

Blaizot-Escobedo Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations => decohence, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price of extra subleading terms* :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

* As well as another time discretization

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$\mathcal{L}_3$

* As well as another time discretization

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Mean field hamiltonian

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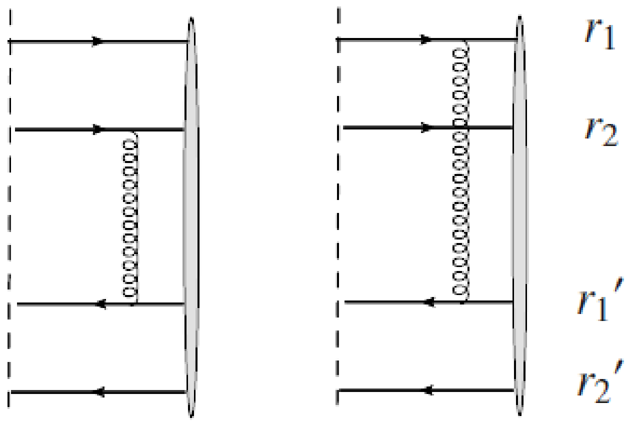
$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

QED-like vs genuine QCD case

Genuine QCD

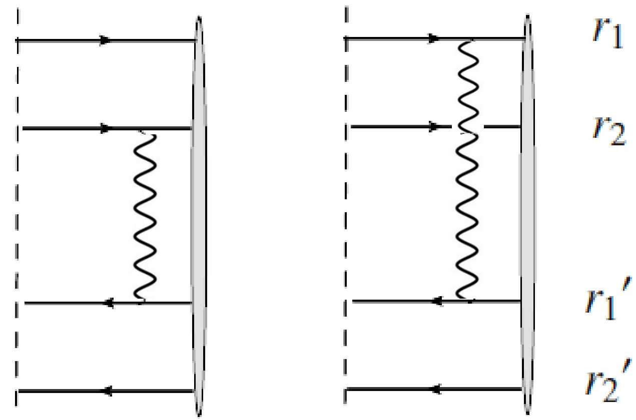


- Scattering from gluons change the color respresentation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- No binding potential in the octet chanel => « large » energy gap

QED-like



- Scattering from photons do not change the Casimir : s <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \end{pmatrix}$$

- Usual 1S <-> 1P transitions between bound states.

B-E Quantum Master Equation: QED-like case

- For the relative motion (2 body):

$$\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}'_1 - \vec{x}'_2\end{array}\right\} \qquad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2nd order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\left\{\begin{array}{l}\mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}}\end{array}\right.$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \to \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

B-E Quantum Master Equation: QCD case

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

octet density matrix

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0\\ \mathcal{D}_8\end{pmatrix}$ Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Example of the $\mathcal{D}_s$ evolution (after semi-classical expansion, i.e power series in y=s-s') :

$$\begin{aligned}(D_s|\mathcal{L}|\mathcal{D}) = &\left(2i\frac{\nabla_r\cdot\nabla_y}{M} + i\frac{\nabla_{\mathcal{R}}\cdot\nabla_{\mathbf{Y}}}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s\\ &-2C_F\Gamma(\boldsymbol{r})(D_s-D_o)\\ &-\frac{C_F}{4}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o\right)\\ &-C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o\\ &+\frac{C_F}{2MT}\left[\nabla^2W(0)-\nabla^2W(\boldsymbol{r})-\boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\right](D_s-D_o)\\ &-\frac{C_F}{2MT}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_o\right)\\ &-\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}D_o.\end{aligned}$$

Our ongoing projects

Our Goals:

- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $Q\bar{Q}$ pair without performing the Semi-Classical approximation:
    - Evolution of the density matrix
    - Evolution of states probabilities over time
    - Singlet-octet transitions
    - Color relaxation time
    - …
- Understand the asymptotic limit of the QME
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)
- Possibly design improved algorithm for intermediate temperatures

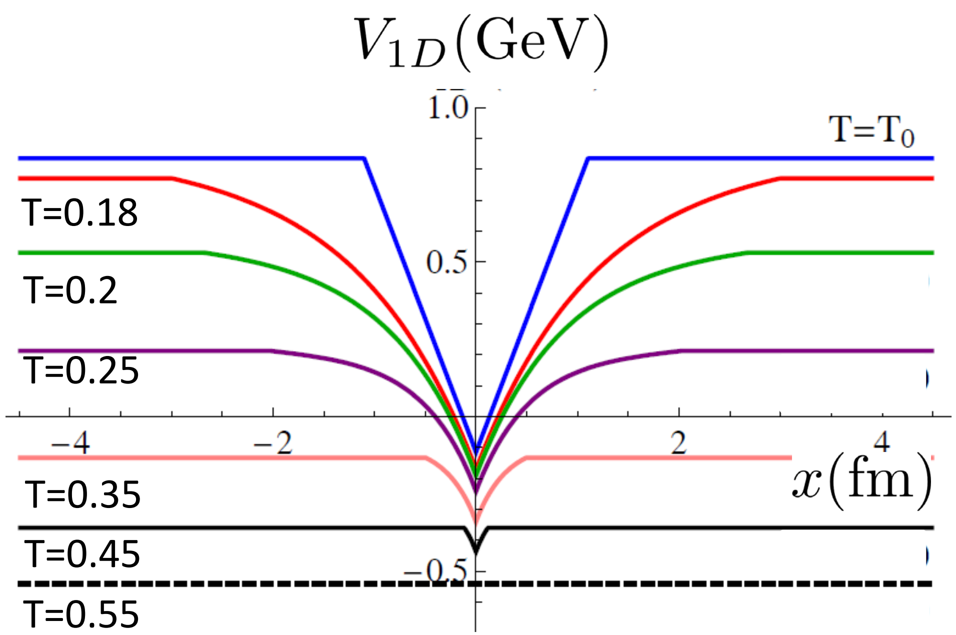
Further implementation features

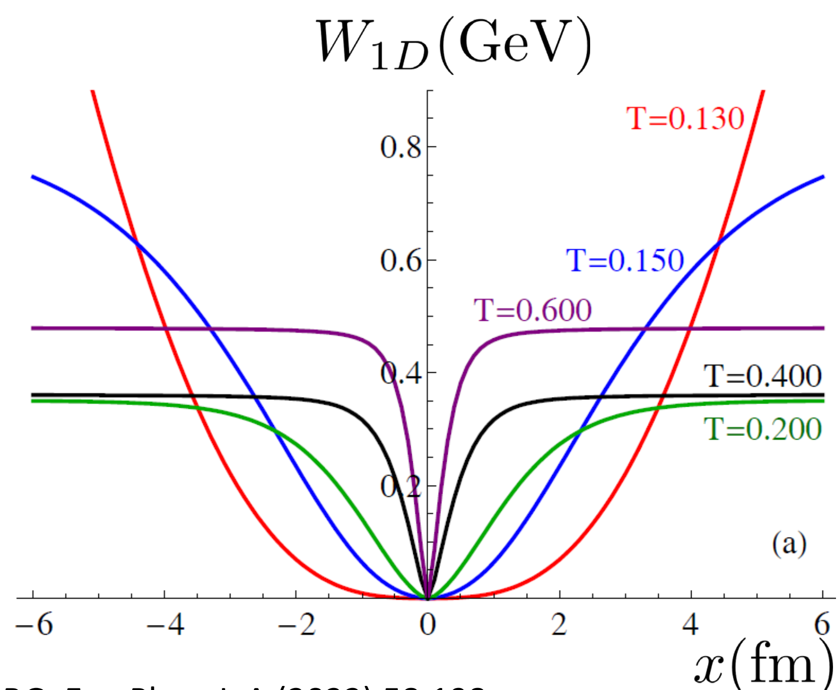
➢ 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths)





1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Further implementation featu…

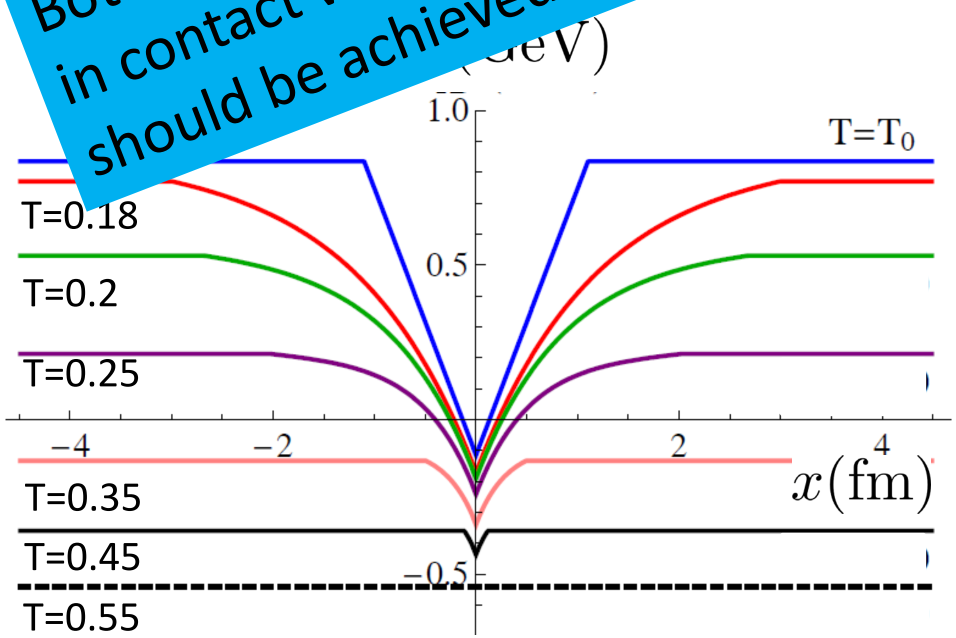
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and s…

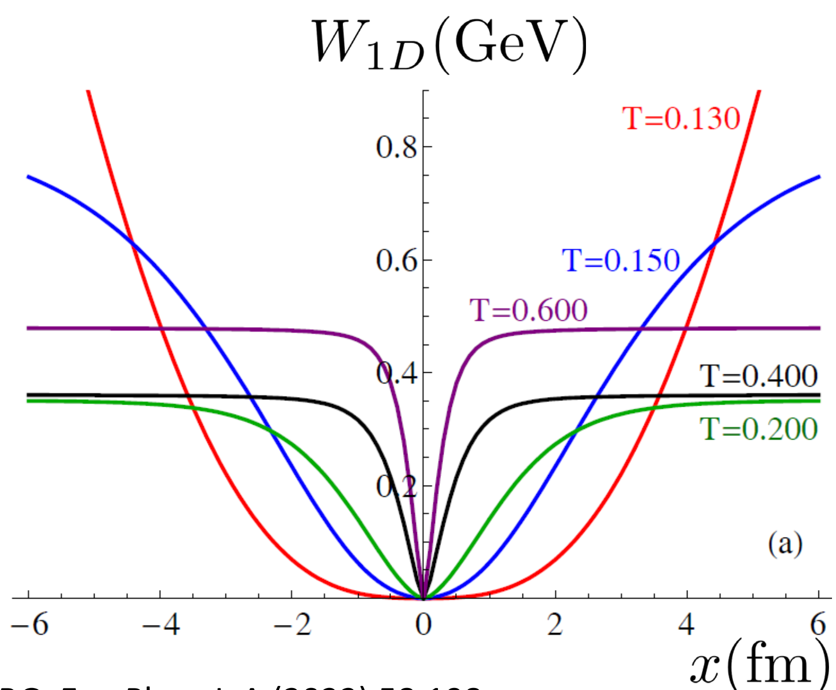
!!! Not the radial decompositio… …bersome

Even states will be c… …d states will be considered a…

Need t… …e potential V + i W (based on 3D lQCD … …D mass spectra and decay widths)

Both regimes where heavy quarks are close together and far apart in contact with the heat « reservoir » => correct thermalisation should be achieved



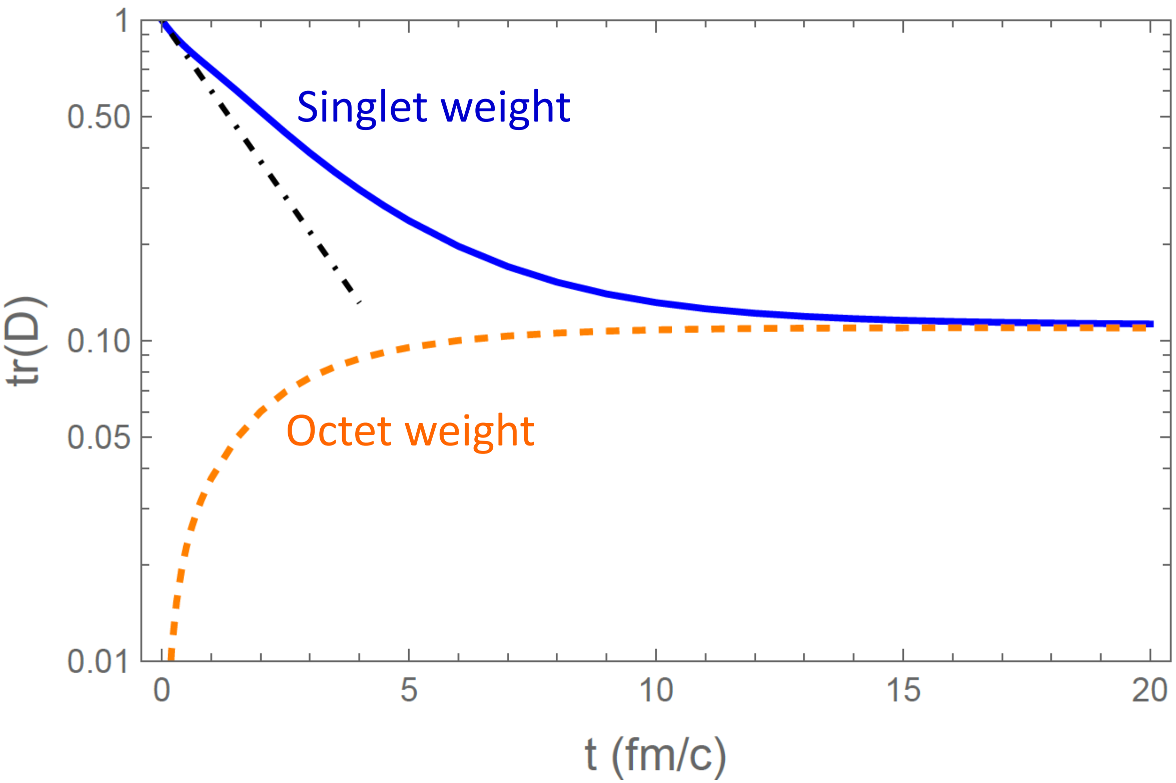


1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Some selected results for 1 c-cbar system

Color Dynamics : Singlet – octet probabilities:

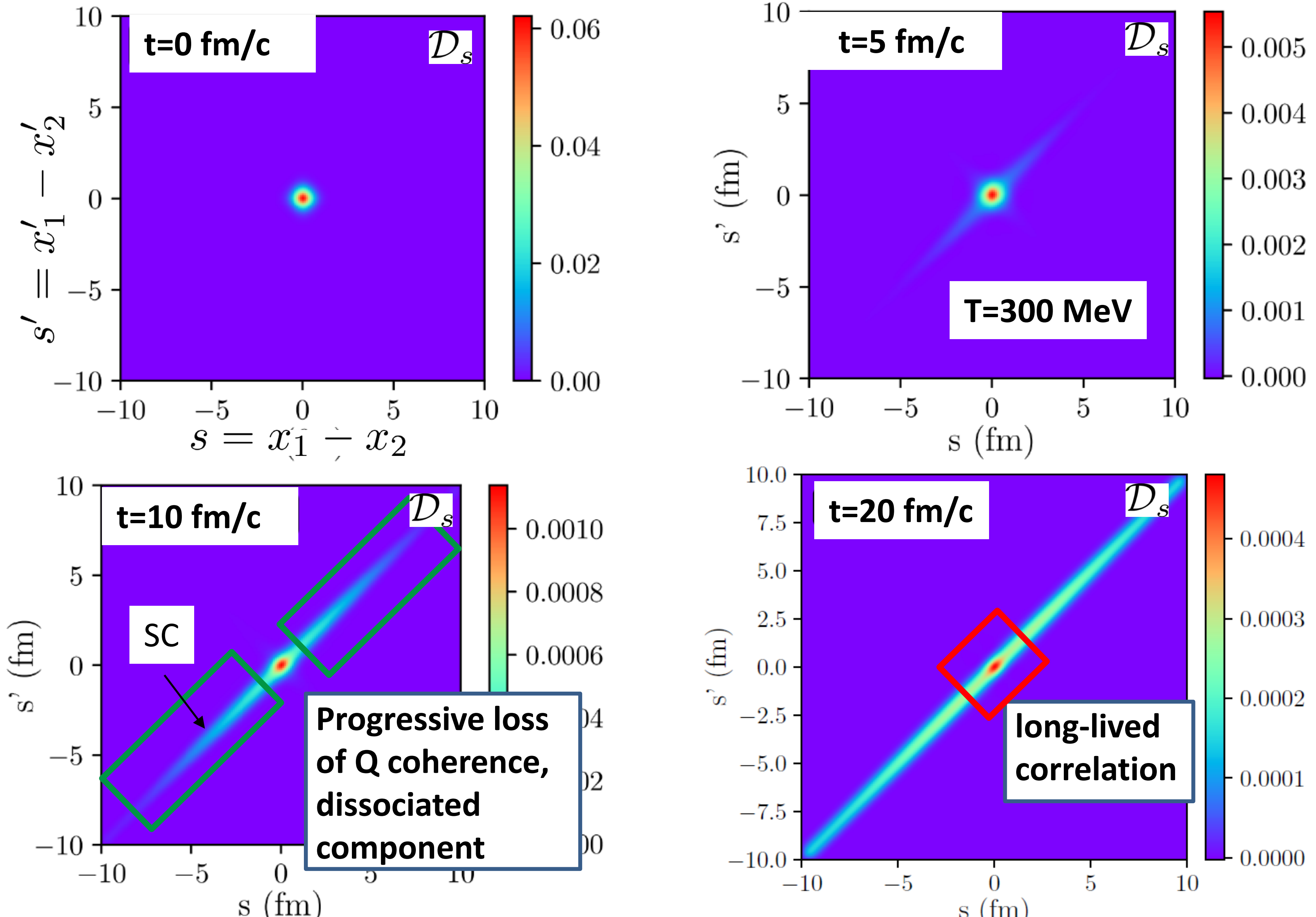
- Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\mathrm{eq}} = D_o^{\mathrm{eq}} = \frac{1}{9}$ $\quad (1+8) \times \frac{1}{9}$



- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.
- C-cbar can interact with the surrounding QGP as an octet => energy loss

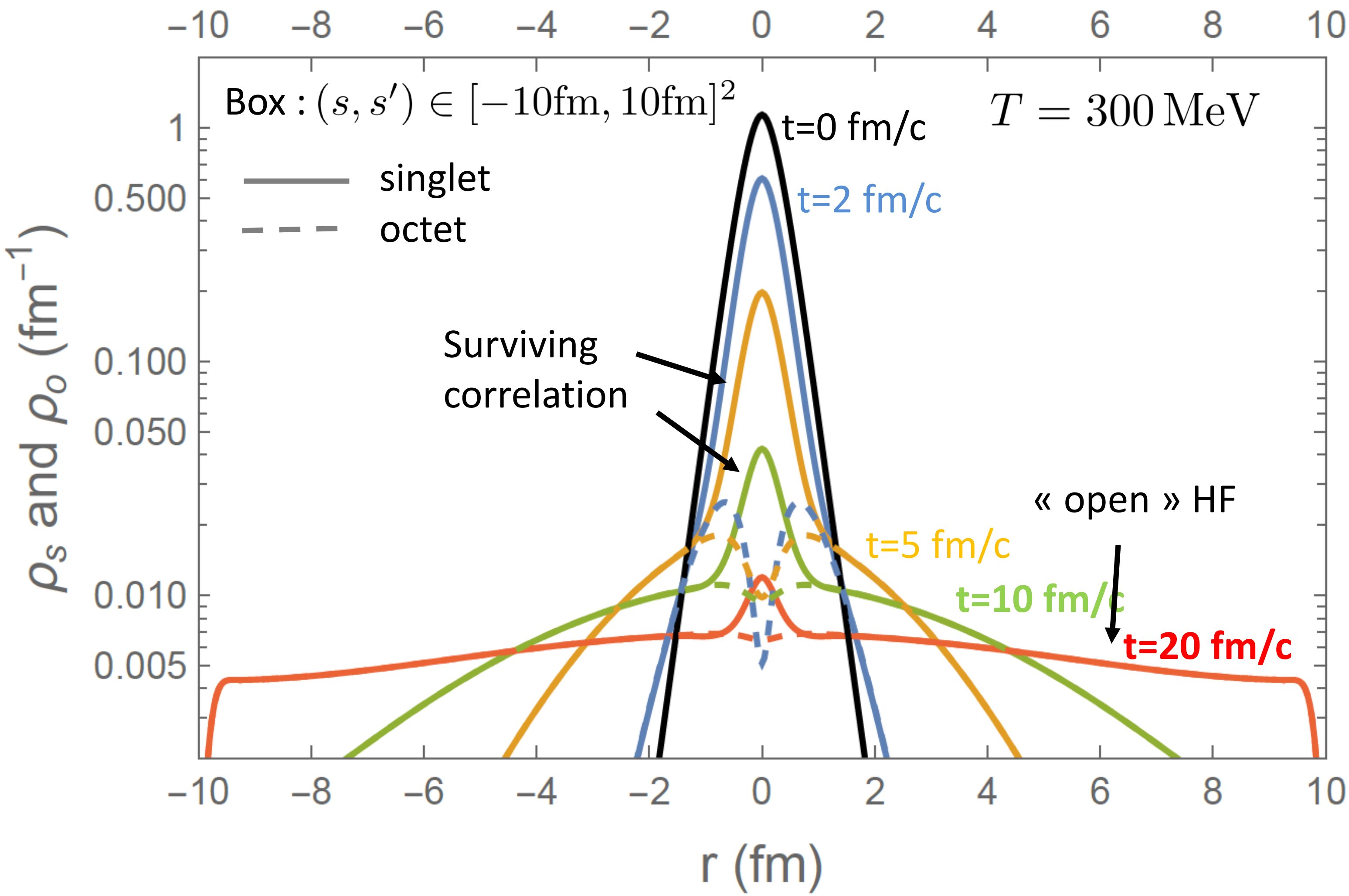
Evolution of the Density matrix

1S singlet initial state:



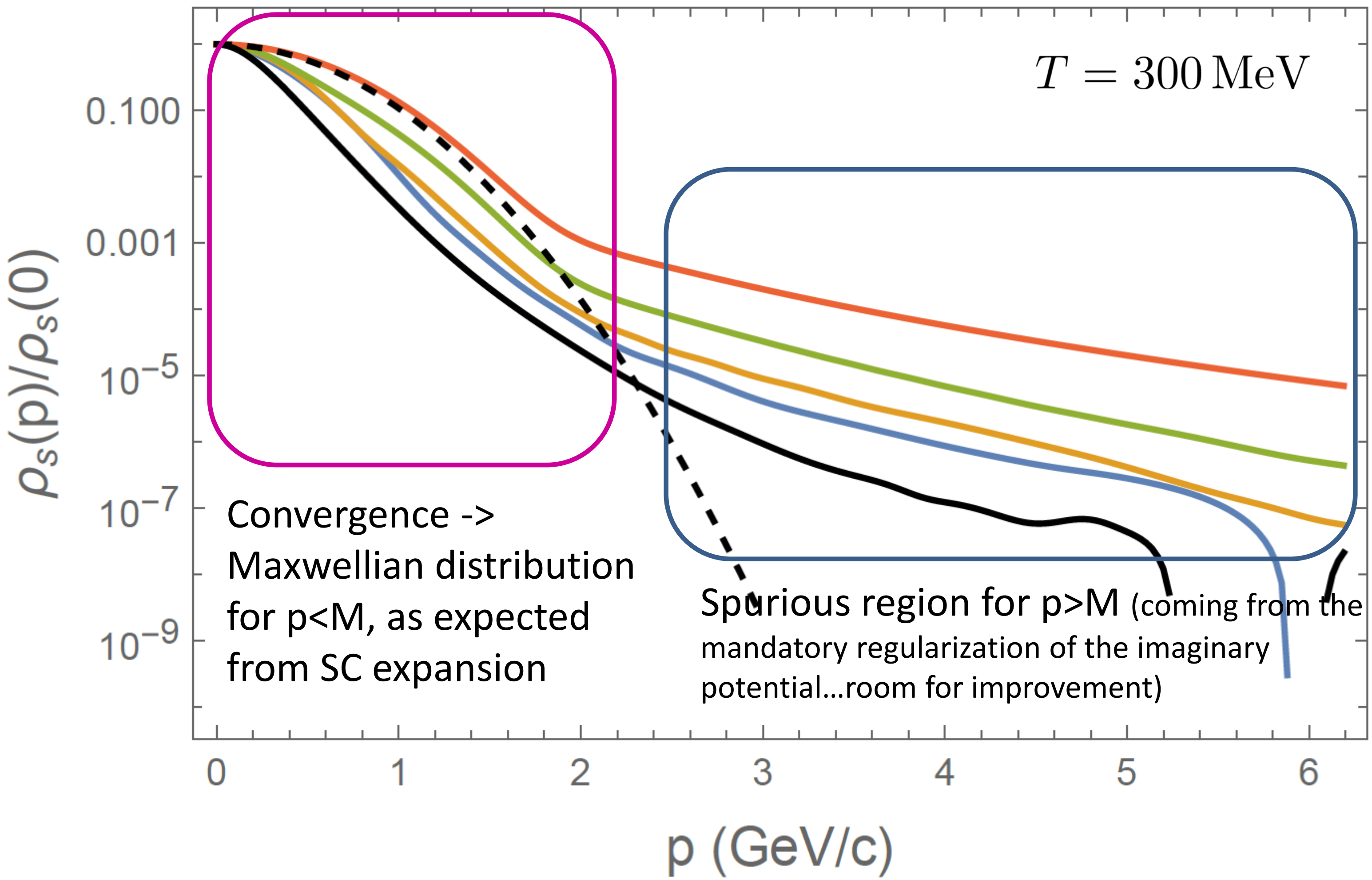
Evolution of the spatial density

1S singlet initial state:



Some c-cbar stay at intermediate distance ("recombination") … remaining peak in the asymptotic distribution

Evolution of the momentum density

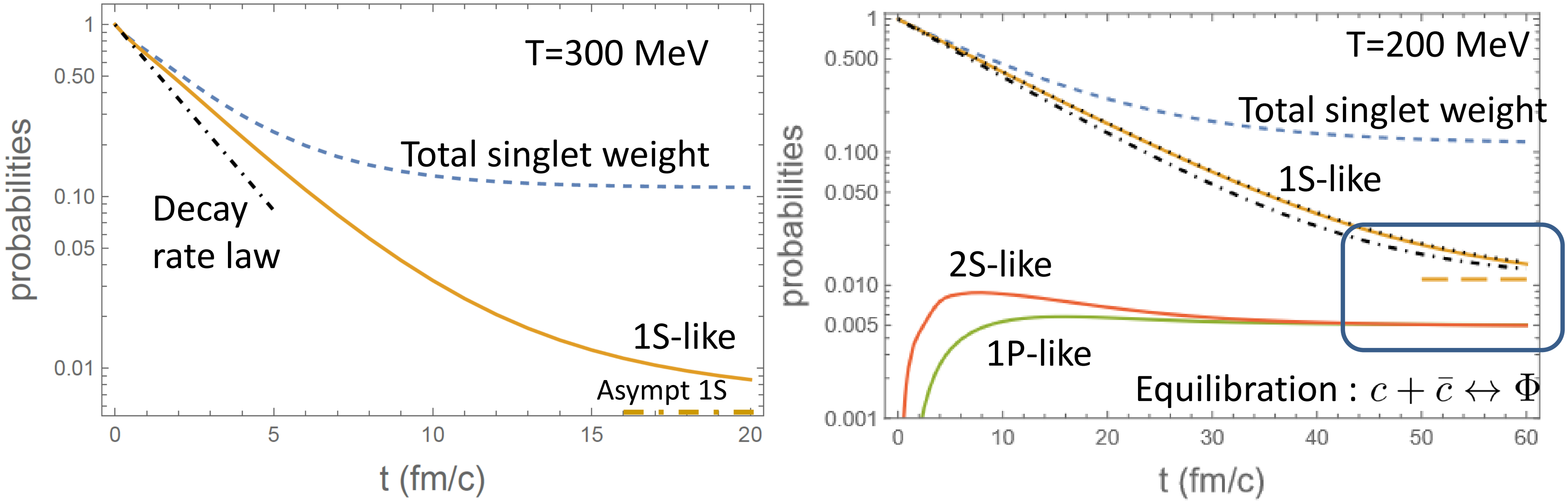


Mostly sensitive to the distribution at large relative distance

Results for projection on local states

« local states » = eigenstates of the screened potential at a given T (<> vacuum states)

$$p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$$

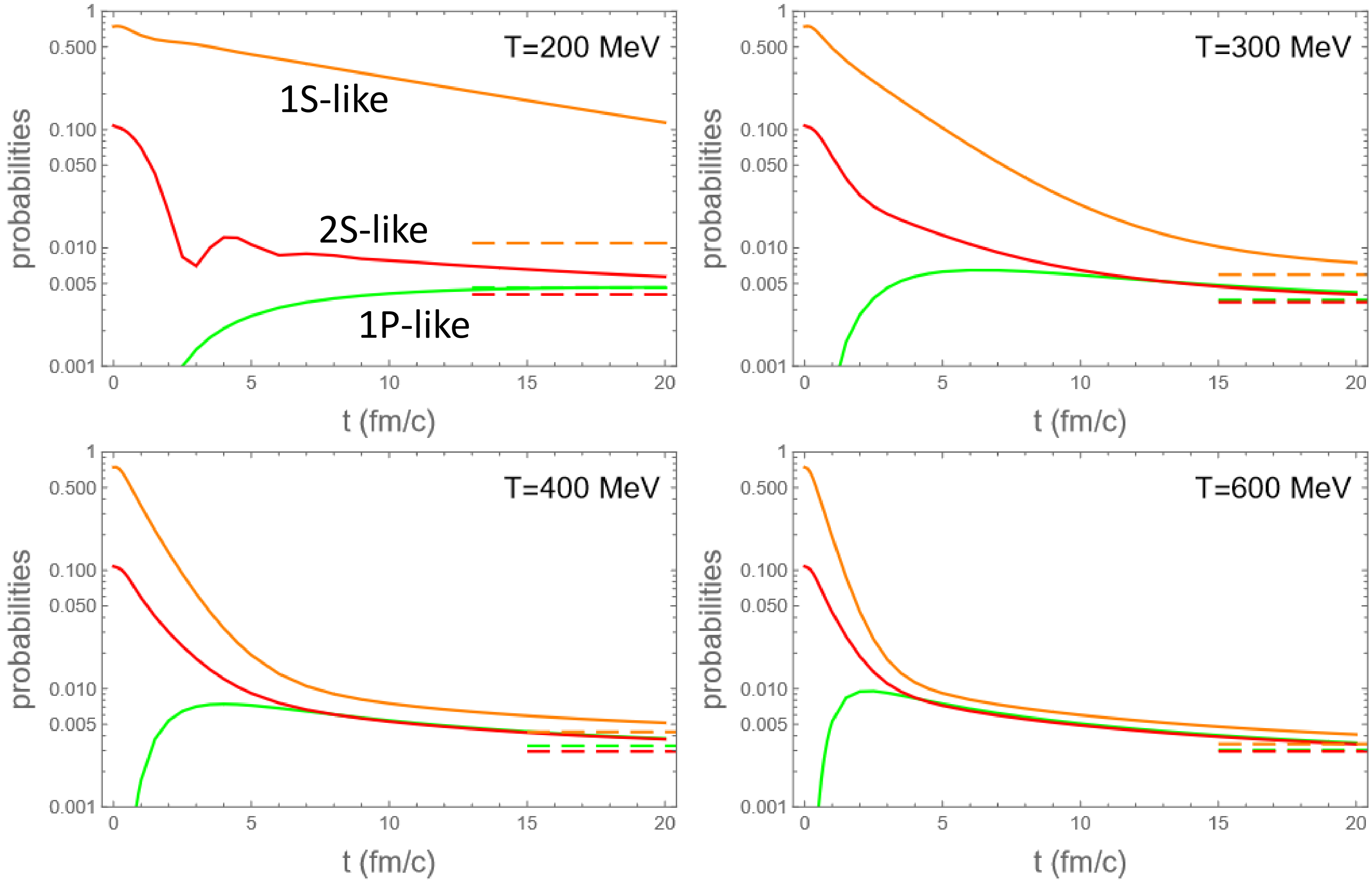


- At small times, $\mathcal{L}_3 \ll \mathcal{L}_2$ fluctuations dominate… higher state repopulation
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states.

- 1S evolution at small time well described by decay rate law (decay rate can be calculated within the QME)
- 1P and 2S generated from 1S show a more complex behavior, not governed by their own decay rate !!!

Results for projection on vacuum states

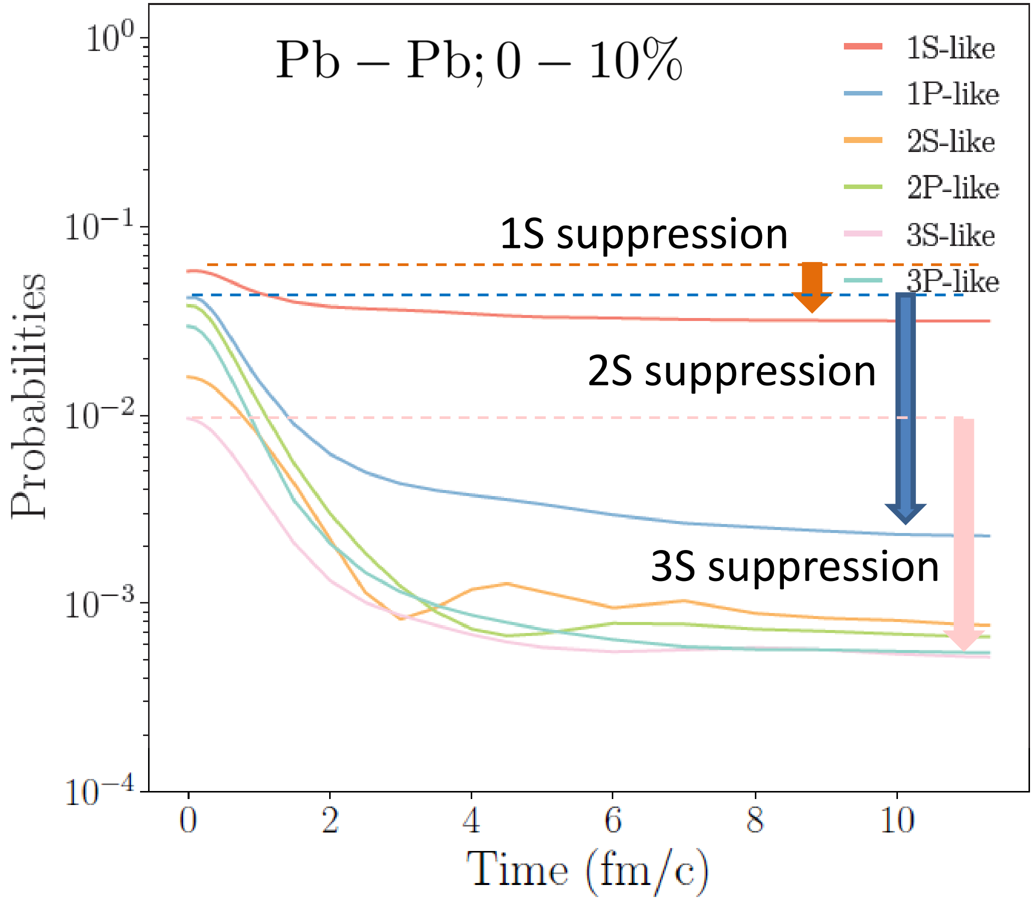
$$p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$$

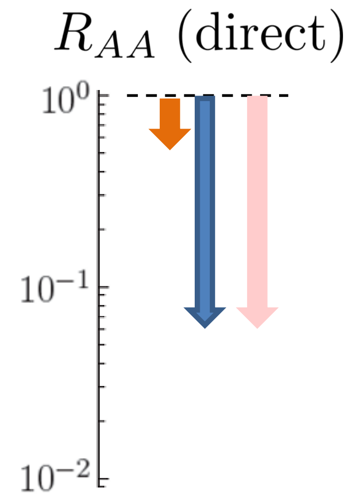


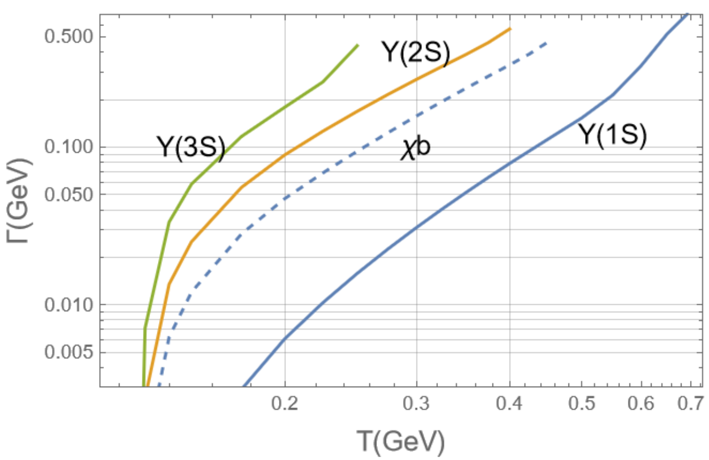
- Natural evolution for 1S-like suppression, from low to high energy
- Excited states partly driven by the ground state at later time.

Contact with experiment (b-bar)

➢ Calculation of bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.







➢ Similar RAA for Υ(3S) and Υ(2S) although 3S decay rate >> 2S decay rate

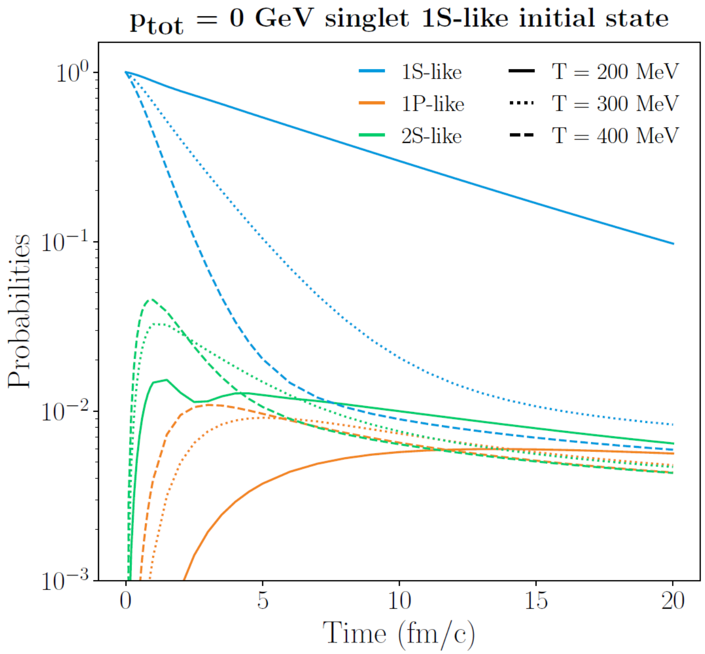
➢ **See Stephane Delorme's talk at Hard Probe 2023 for more details.**

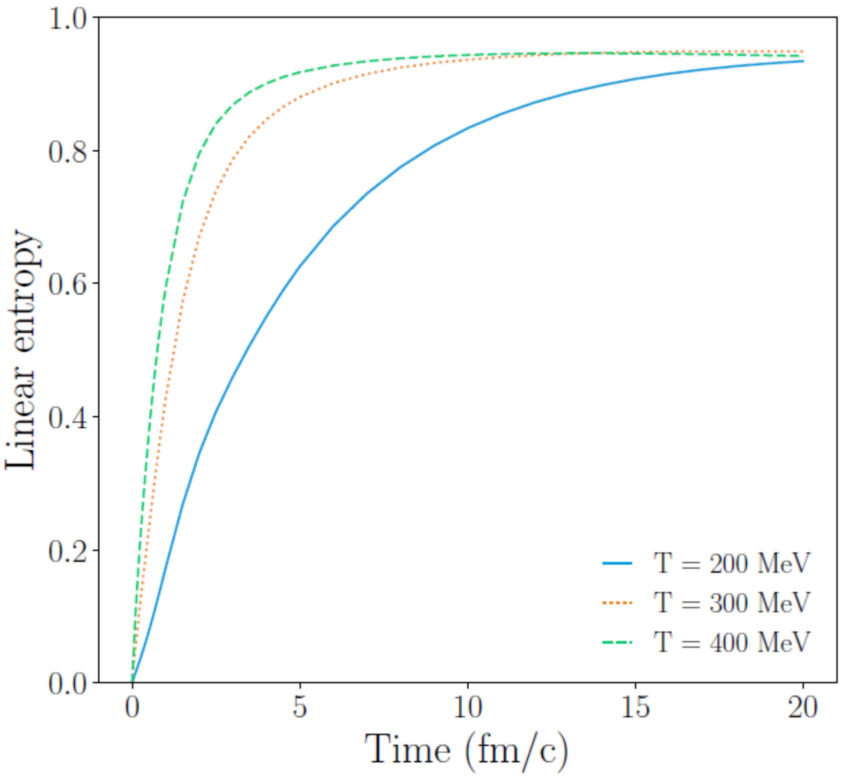
Results for Linear quantum entropy

$$S_L = \mathrm{Tr}\hat{\rho} - \mathrm{Tr}\hat{\rho}^2 = 1 - \mathrm{Tr}\hat{\rho}^2$$

De Boni, J. High Energ. Phys. (2017) 2017: 64

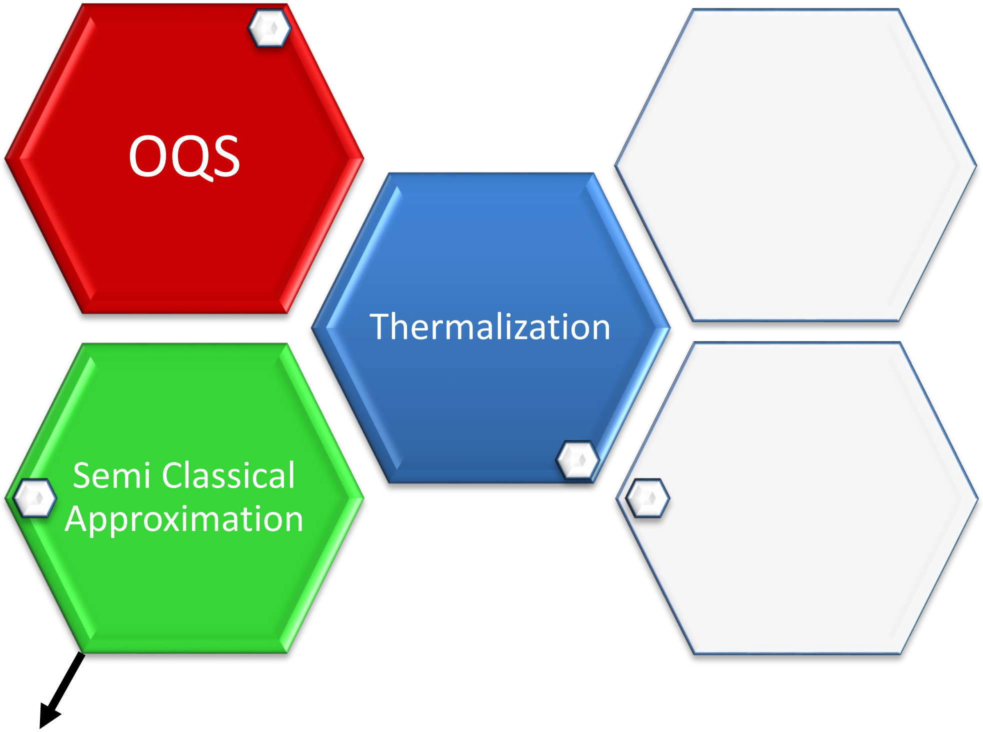
(results for QED like evolution)





- Suppression and decoherence appear to happen on the same time scale…
- … does not seem in favour of applying classical rate equations (to be investigated further)

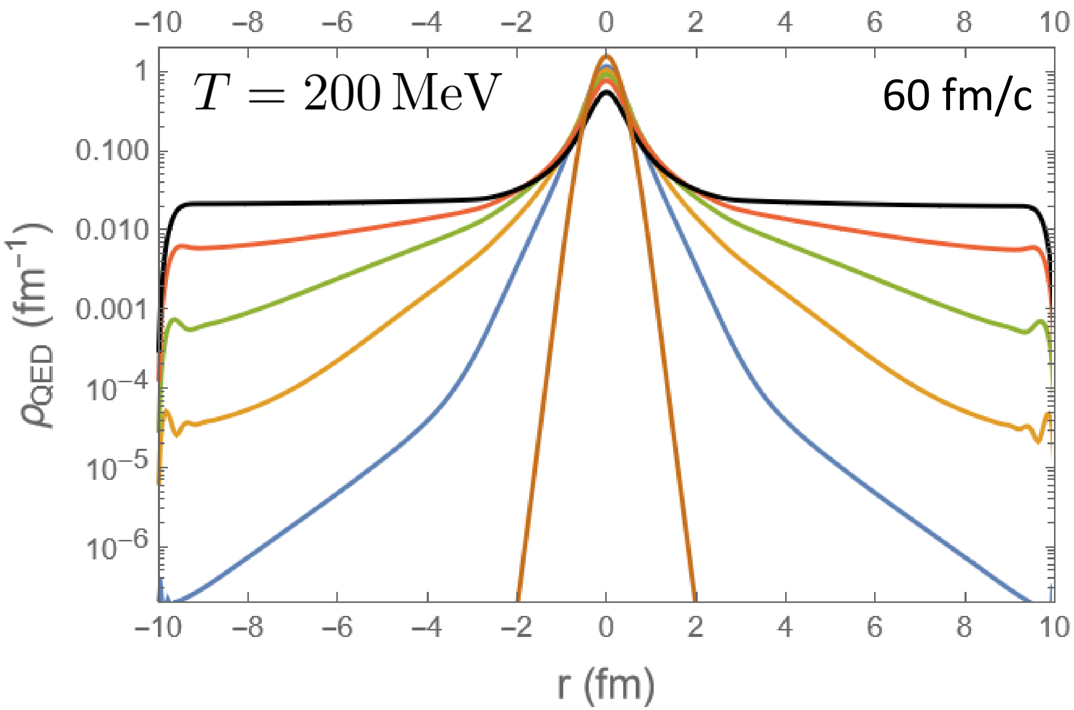
Two closely related topics



Used in recent transport approaches (Blaizot-Escobedo, Arrebato et al)

Asymptotic distributions / quarkonium weights

➢ First, looking at the QED-like case.



➢ In the simulation, some peak survives in the density, at small $c\bar{c}$ distance

➢ This peak is in direct correlation with the charmonium weight

➢ The asymptotic states may not be reached in realistic AA collisions, but controlling/understanding them is important :

- Sanity check of the models
- Privileged link with lQCD spectral distribution (evaluated in this limit)

B-E Quantum Master Equation: QED-like case

- For the relative motion (2 body):

$$\left.\begin{aligned}\vec{s} &= \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' &= \vec{x}_1' - \vec{x}_2'\end{aligned}\right\} \qquad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2nd order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases}\mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}}\end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\}\to\{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

Asymptotic distribution: QED-like

- First, looking at the QED-like case.

$$\frac{d}{dt}\mathcal{D}_s = \mathcal{L}[\mathcal{D}_s] \Rightarrow \mathcal{L}[\mathcal{D}_s^{\text{asymp}}] = 0$$

- Even simpler case : Semi classical approximation : $W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$

(Wigner representation)

2 lines calculation :

$$\mathcal{L}_2 + \mathcal{L}_3 \to \frac{\mathcal{H}(r)+\mathcal{H}(0)}{2}\partial_p\left[\frac{\partial_p}{2} + \frac{p}{m_Q T}\right]W(r,p) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T}}$$

$$\mathcal{L}_0 + \mathcal{L}_1 \to \left(-\frac{p\,\partial_r}{m_Q} + \partial_r V(r)\partial_p\right) \Rightarrow W(r,p) \propto e^{-\frac{p^2}{m_Q T} - \frac{V(r)}{T}}$$

Asymptotic distribution: QED-like

- First, looking at the QED-like case.

$$\frac{d}{dt}\mathcal{D}_s = \mathcal{L}[\mathcal{D}_s] \Rightarrow \mathcal{L}[\mathcal{D}_s^{\mathrm{asymp}}] = 0$$

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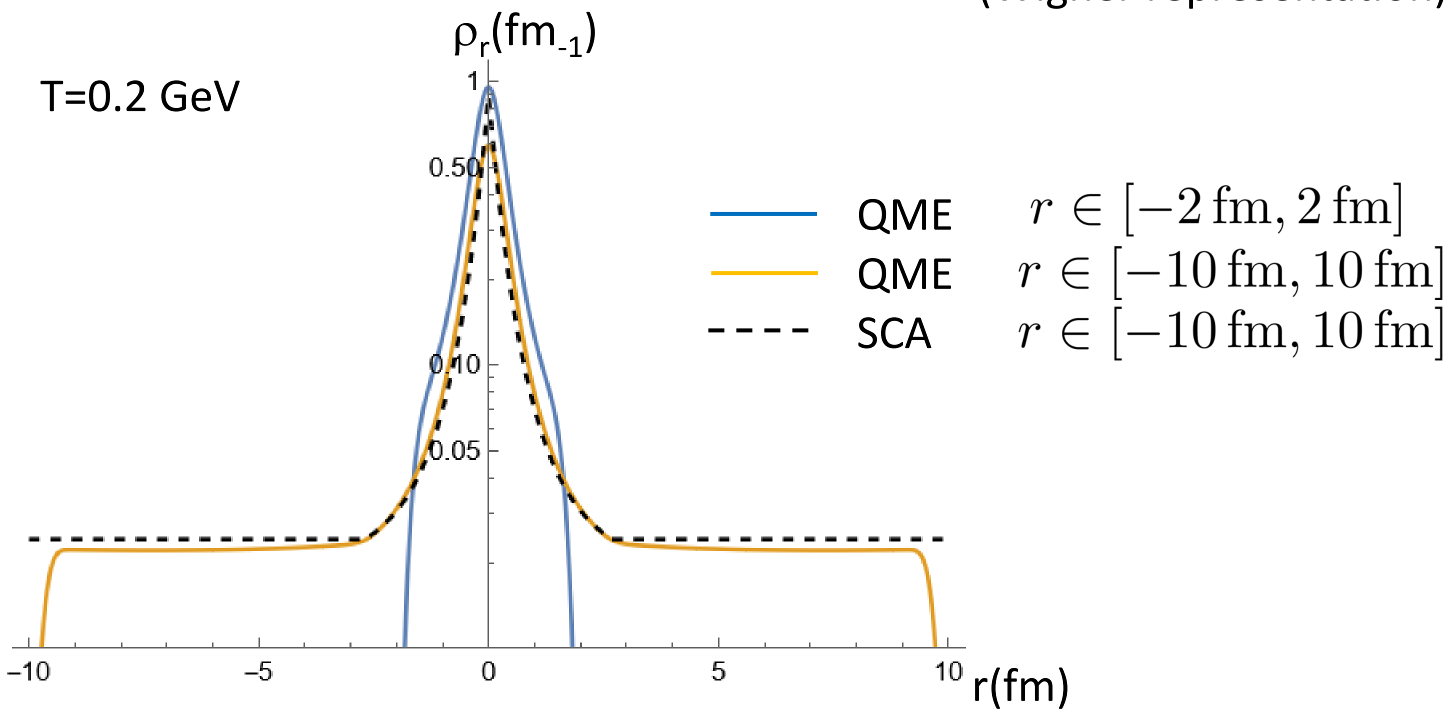
(Wigner representation)



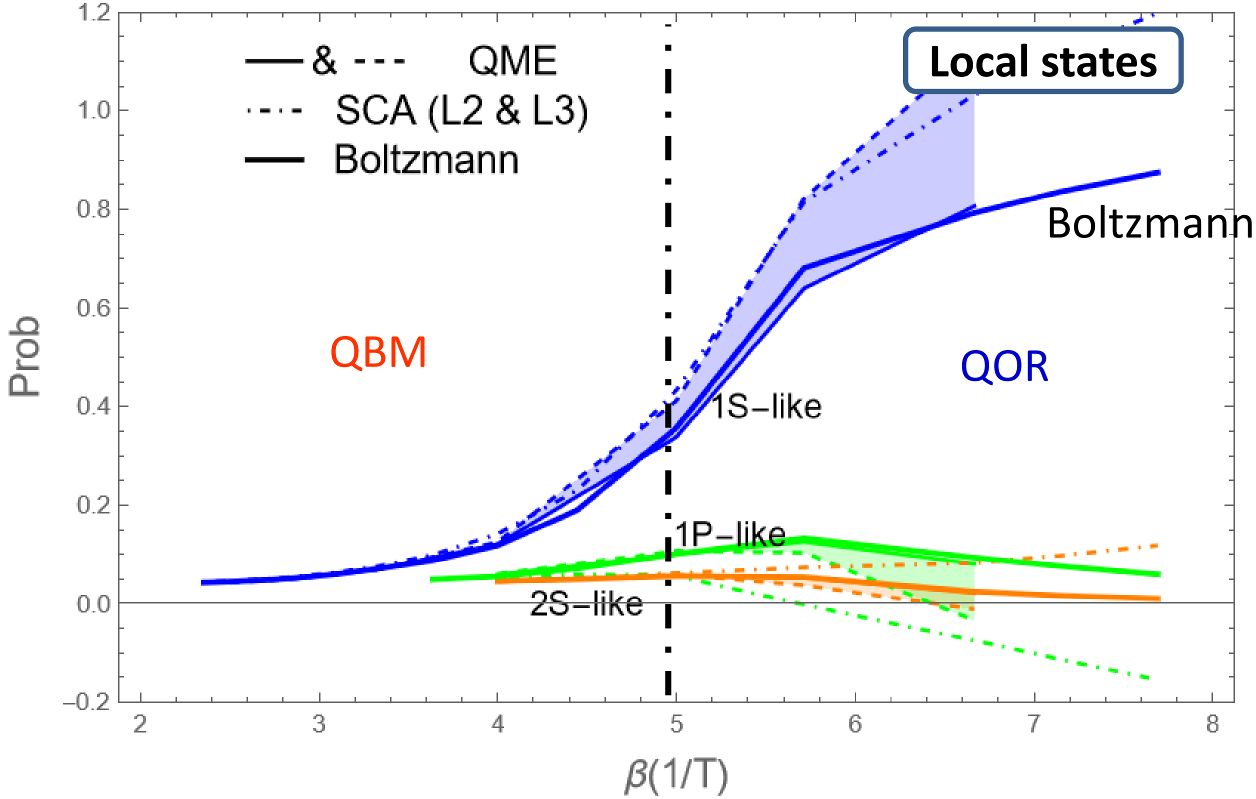
- Peak, independently of box size
- Pretty well described by SC relation (apart around the origin):

Quarkonium-like weights

- First, looking at the QED-like case.

SCA : $\mathrm{prob}_n = \langle n | e^{-\frac{V(\frac{x+x'}{2})}{T} - \frac{MT(x-x')^2}{(2\hbar c)^2}} | n \rangle$

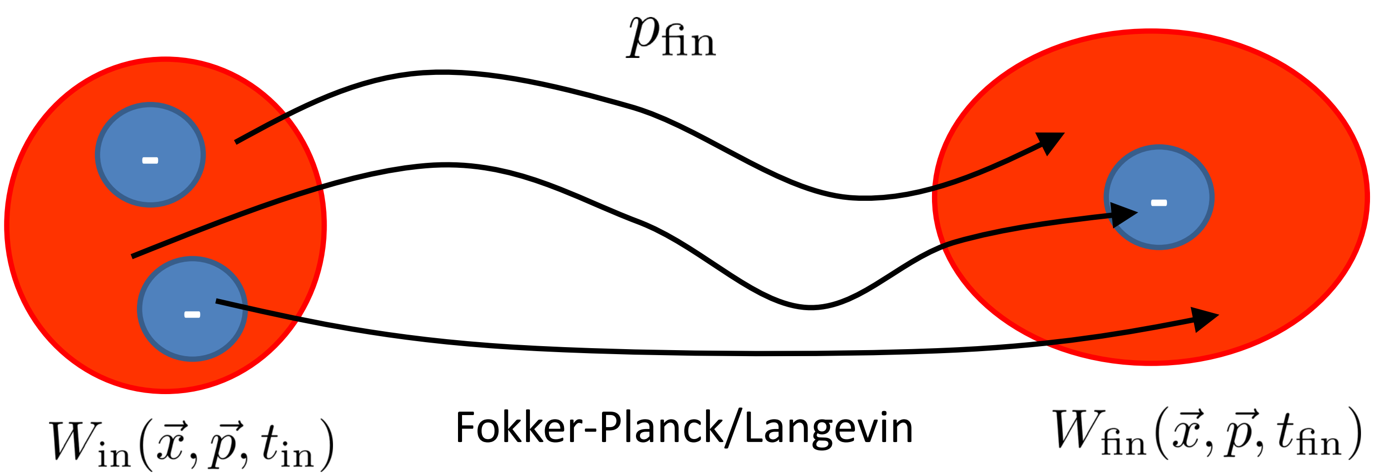
Boltzmann : $\mathrm{prob}_n = \frac{e^{-E_n/T}}{Z}$



- Good agreement between QME & SC in the deep QBM regime (expected)
- Good agreement between QME & Boltzmann Ansatz, even in the QOR !!! Not expected at all.
- SCA : not succesfull for 1P-like state

Quantum vs SC dynamics

- SCA : linear mapping



- Several aspects :
  - Temperature
  - Initial state
  - Property considered

Always positive defined even if W_{in} and W_{fin} are not positive defined

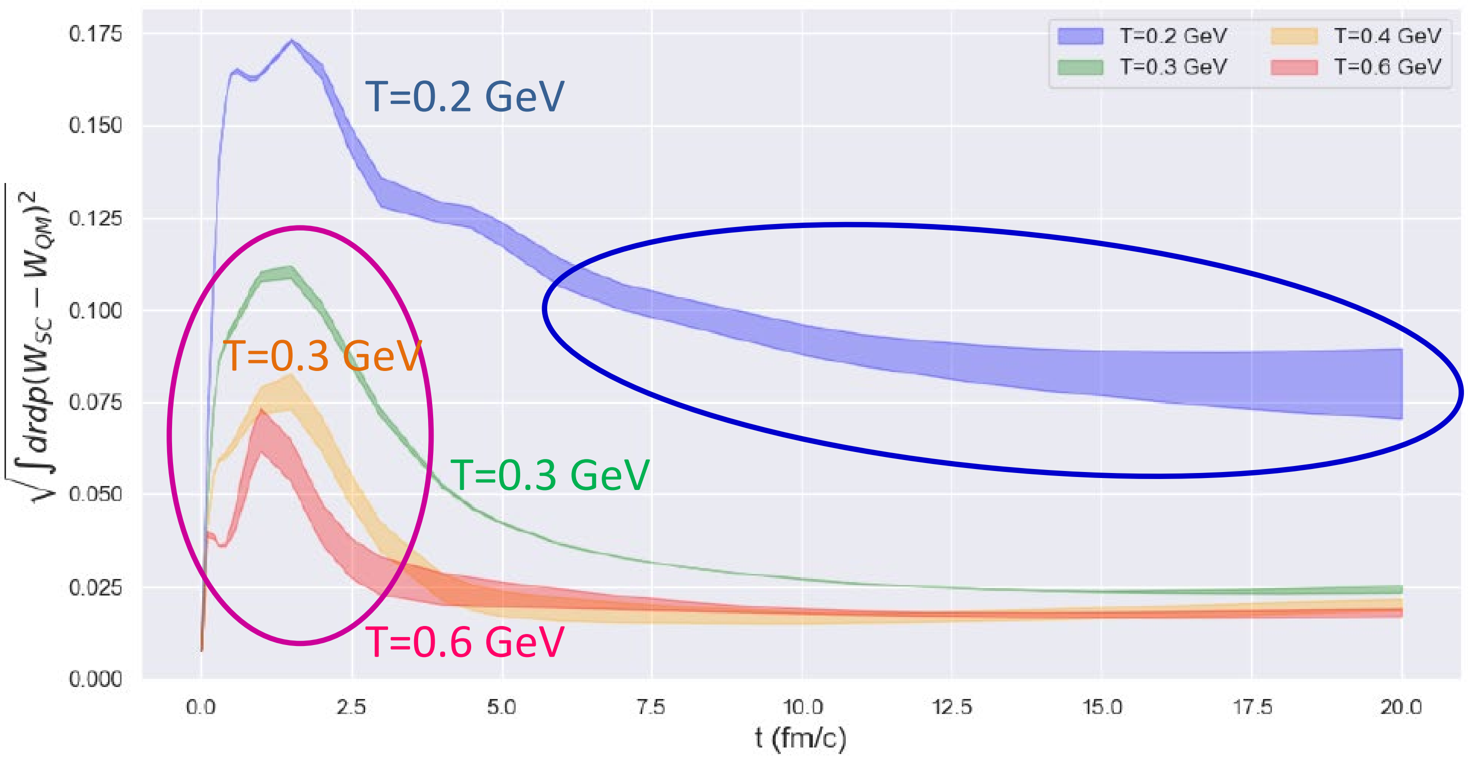
In the following : only a limited set of results; manuscript to come soon

- First looking at global difference :

$$d = \sqrt{\int \mathrm{drdp}\,(W_{QM} - W_{SC})^2}$$

Quantum vs SC dynamics (QED-like)

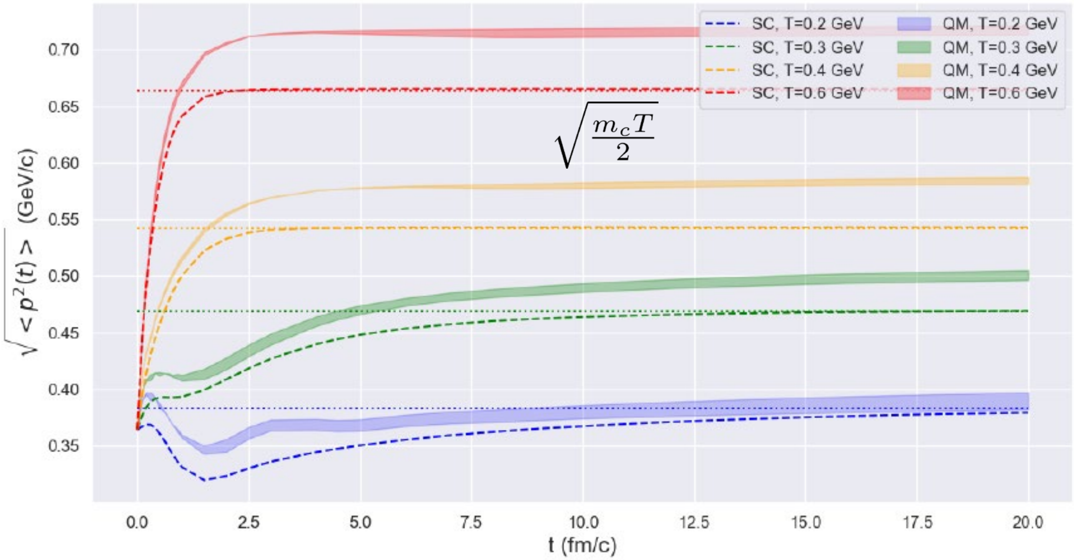
Initial state : 1S (vacuum like)



- Band : 2 implementations of the OQS (with and without L4 term)
- Rise and fall of the deviation for all temperatures ≥ 300 MeV
- As expected, deviations larger for smaller temperatures, where quantum corrections should be implemented
- For T=200 MeV, important long lasting deviation, mostly due to differences in the asymptotic ρ_r.

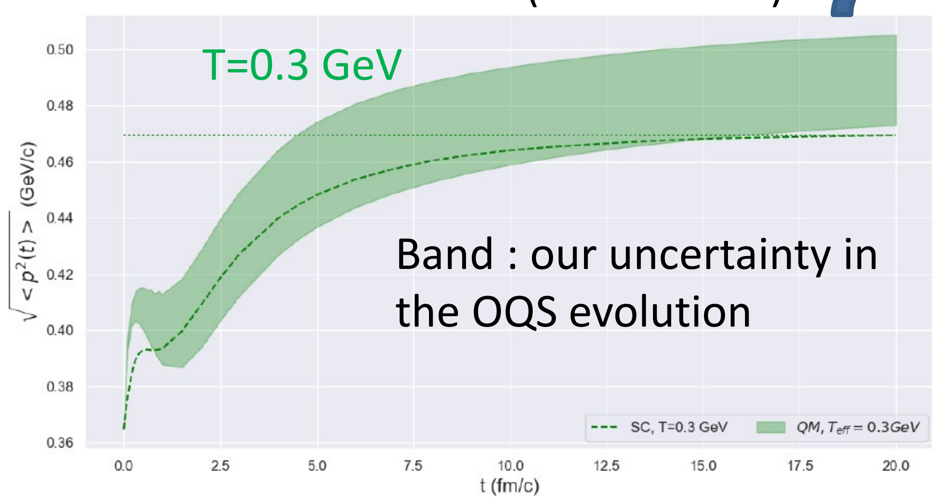
Quantum vs SC dynamics (QED-like)

Initial state : 1S (vacuum like)

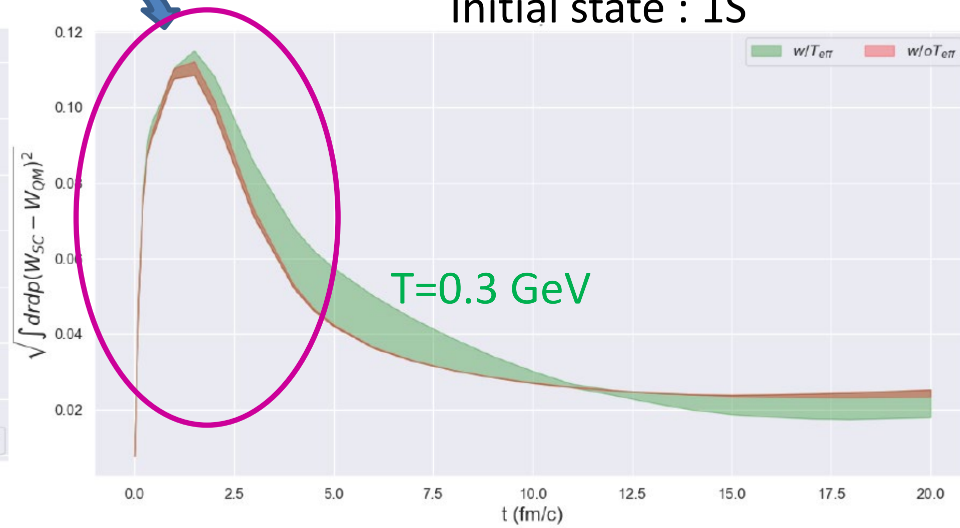


- Spurious effect $<p^2>$ in the OQS does not perfectly converge towards the correct $\sqrt{\frac{m_c T}{2}}$ value
- Pragmatic prescription consider a T_{eff} ($\approx$ 0.85 T) leading to the good asympt. p-distribution
- Still some deviation seen in the norm difference => robust

Initial state : 1S (vacuum like)

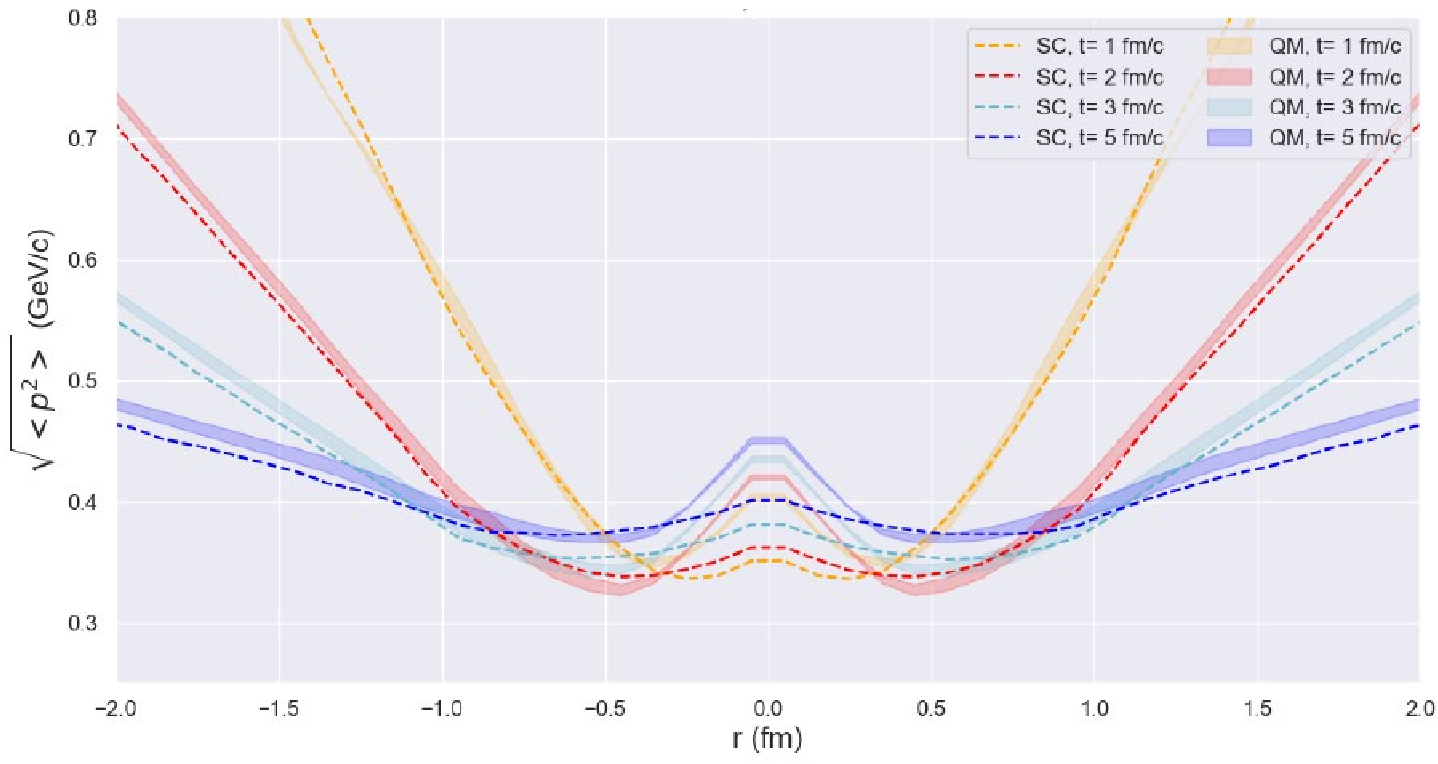


Initial state : 1S



Quantum vs SC dynamics (QED-like)

- More local quantities: <p²> vs r (in the Wigner sense)

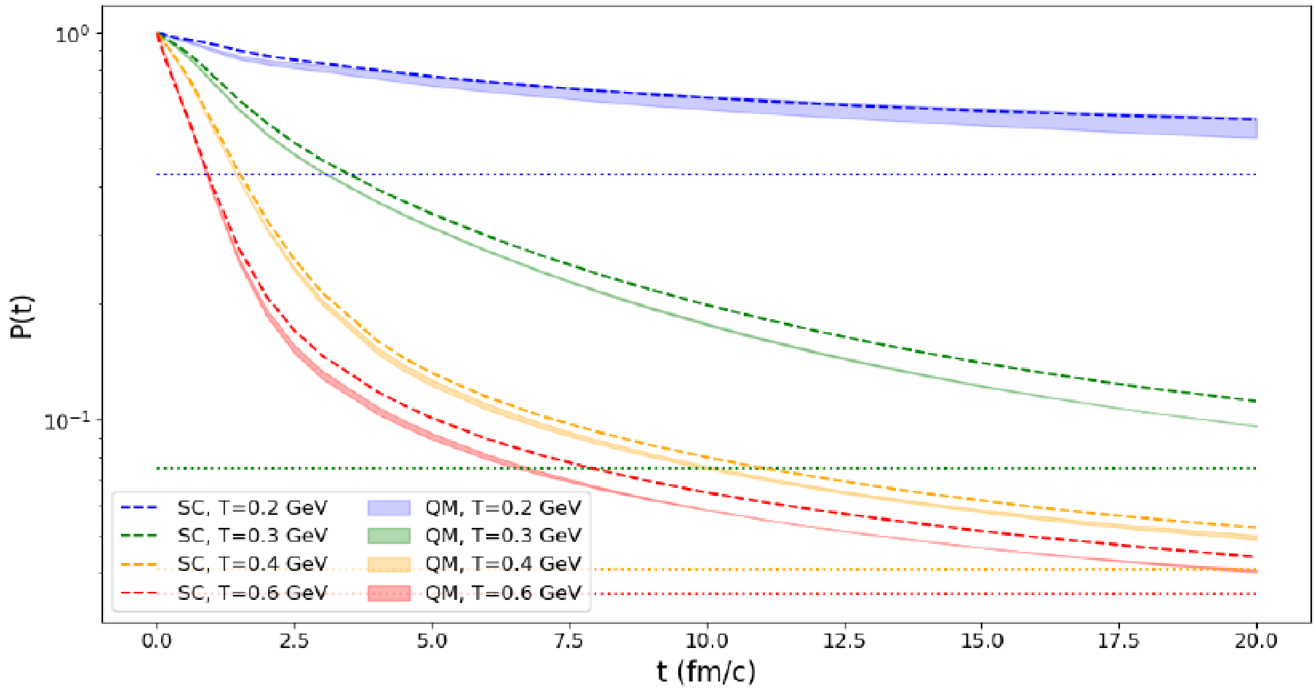


- Nearly perfect agreement at large distance
- More structure seen in the genuine QM calculation at close ccbar distance

Quantum vs SC dynamics (QED-like)

- Probabilities

Survival of 1S state probability

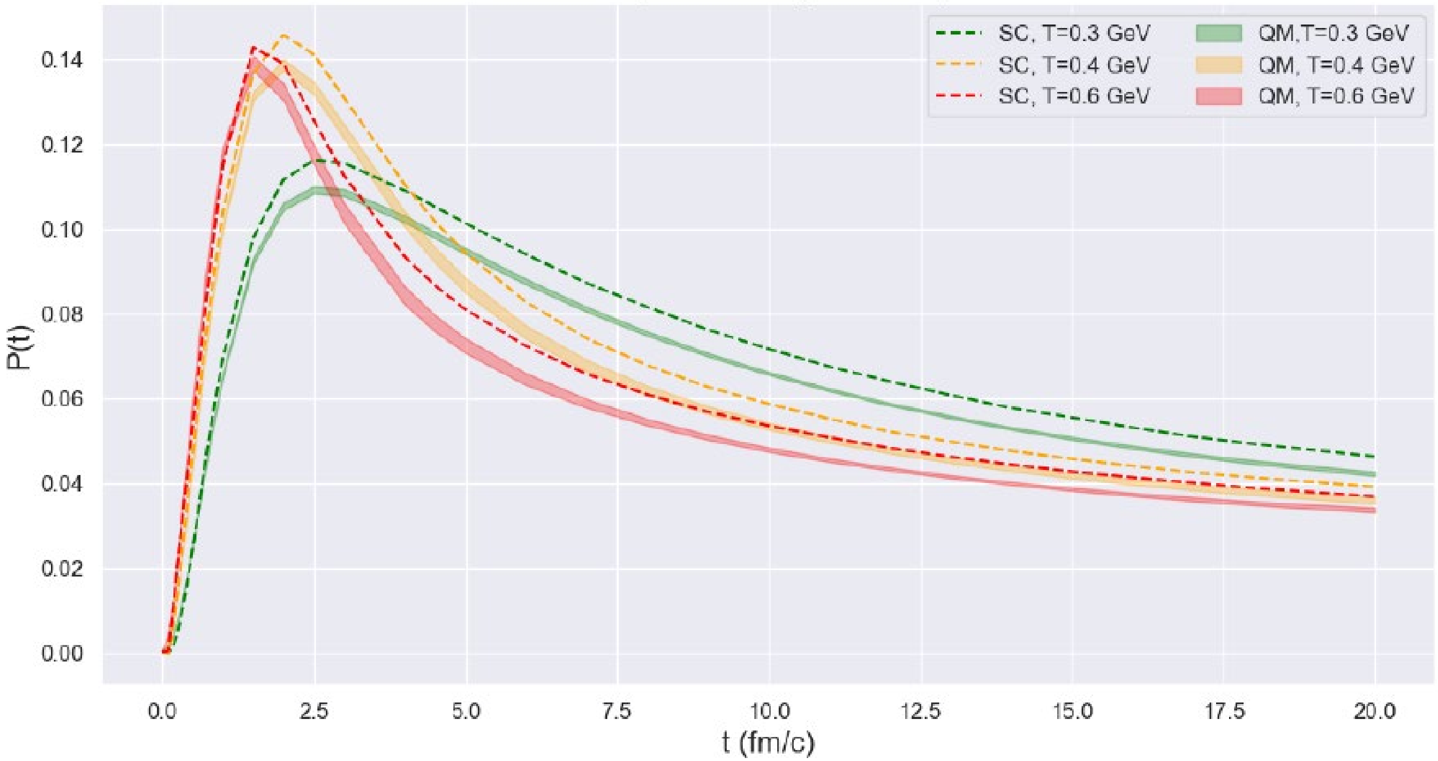


- Pretty good agreement for the survival probability, whatever the temperature

Quantum vs SC dynamics (QED-like)

➢ Probabilities

Generation of 2S state from 1S initial state

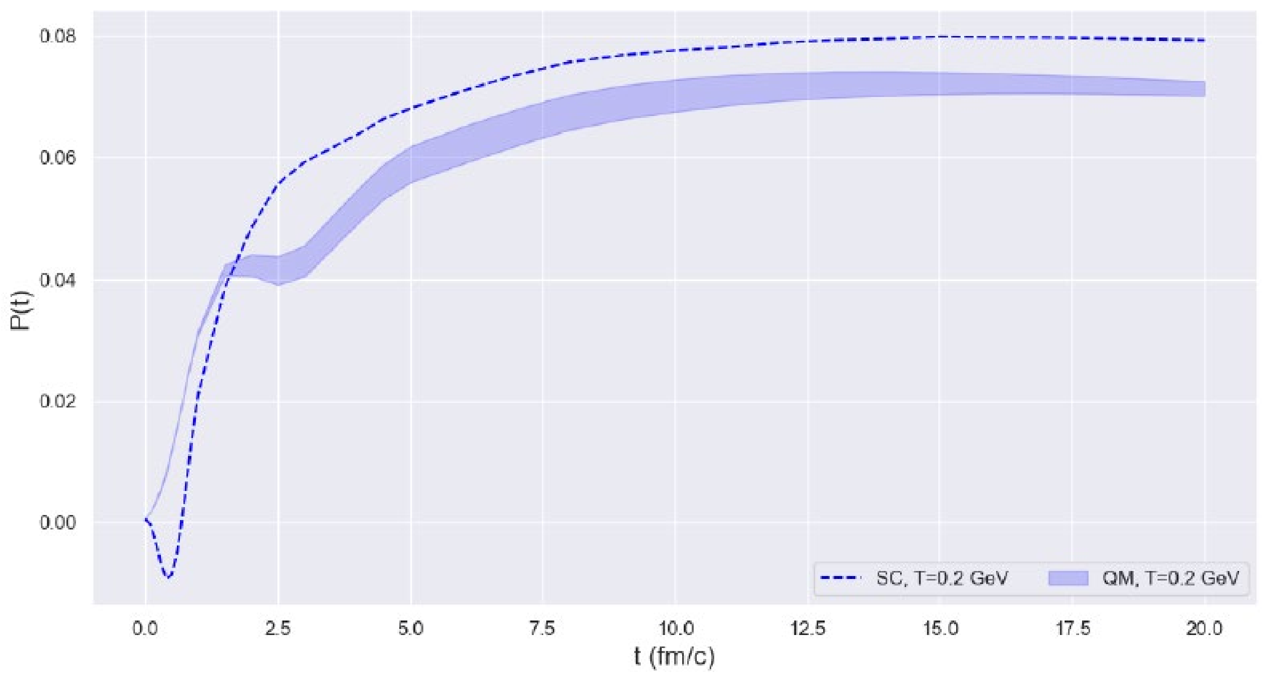


➢ Still pretty good comparison between the QM and the SCA for T ≥ 300 MeV

Quantum vs SC dynamics (QED-like)

- Probabilities

Generation of 2S state from 1S initial state

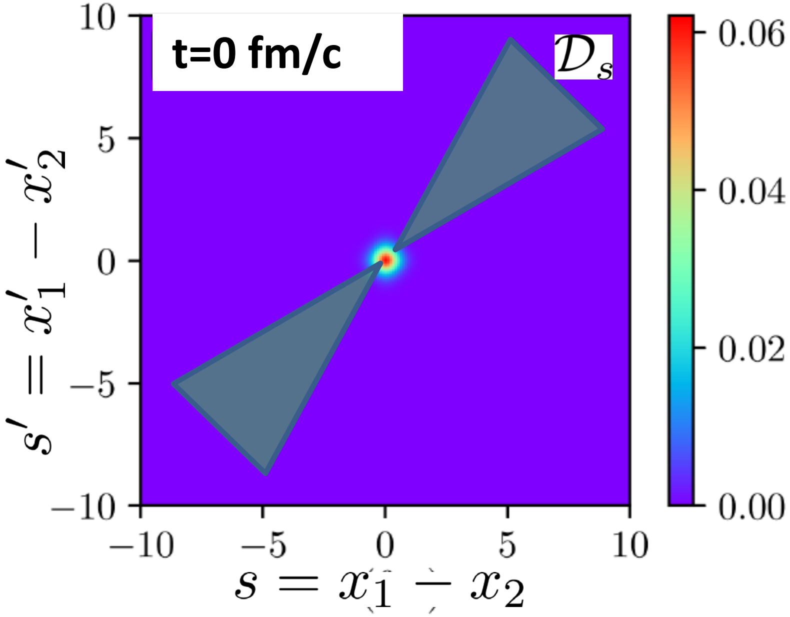


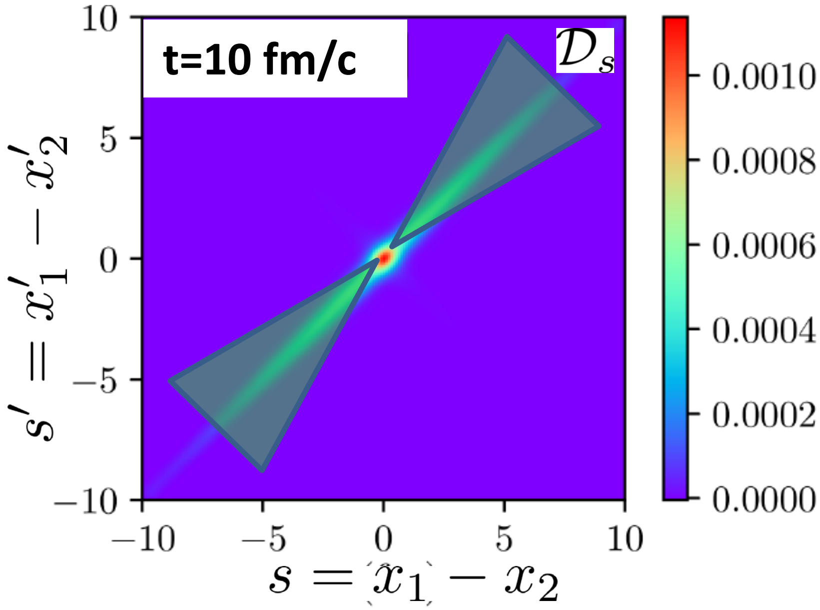
- Still pretty good comparison between the QM and the SCA for T ≥ 300 MeV
- More questionnable for T=200 MeV (lower boundary of the QBM regime), especially at early times

Quantum vs SC dynamics (QED-like)

- The most problematic term of the linblad operator: $V(\vec{s}) - V(\vec{s}\,') \approx \vec{\nabla} V(\vec{r}) \cdot \vec{y}$

  Valid for $\|\vec{y}\| \lesssim \|\vec{r}\|$



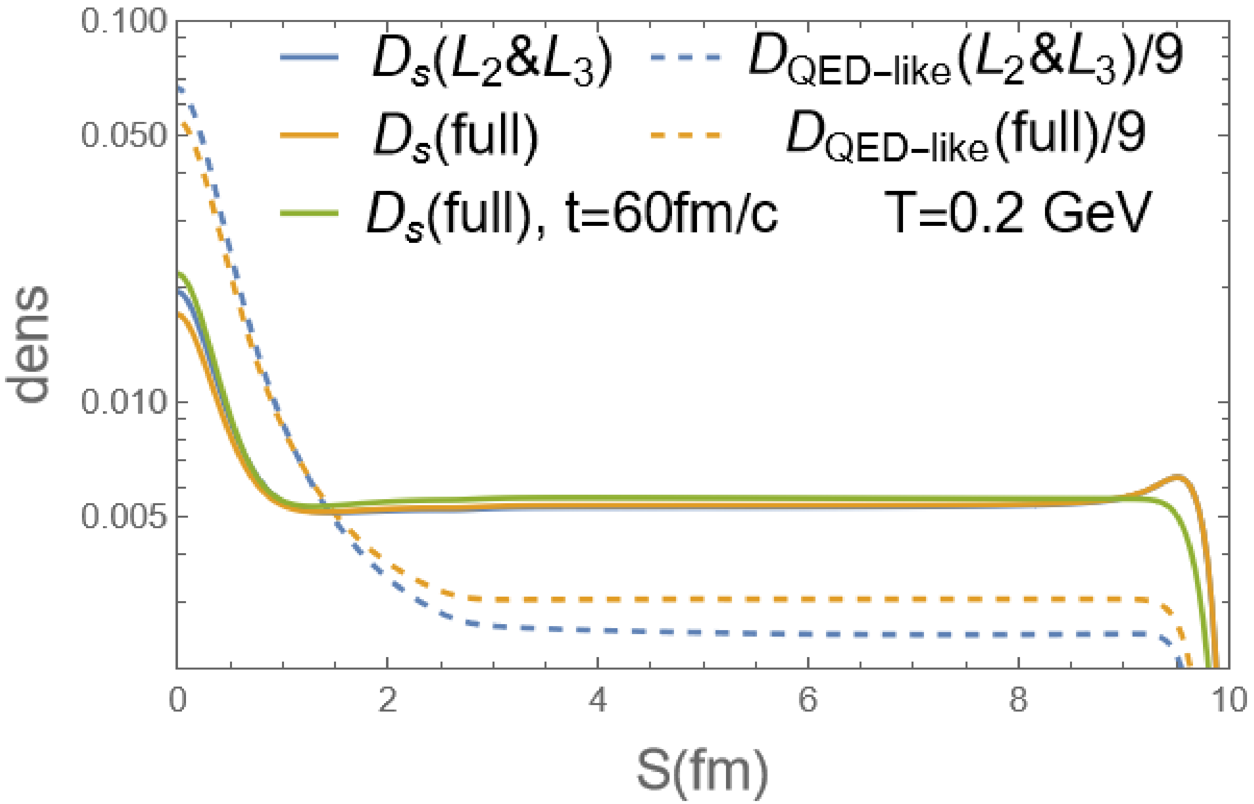


- A initial time, this condition is satisfied for at most 50% of the dynamical space…
- … But it is the range of validity increase with time and with the « classicalization »
- The initial high temperature may help in this respect

Asymptotic distribution: QCD

- Now the genuine QCD case
- Solving $\mathcal{L}_{ss} \cdot \mathcal{D}_s = -\mathcal{L}_{so} \cdot \mathcal{D}_o$

With thermalized Ansatz for D_o

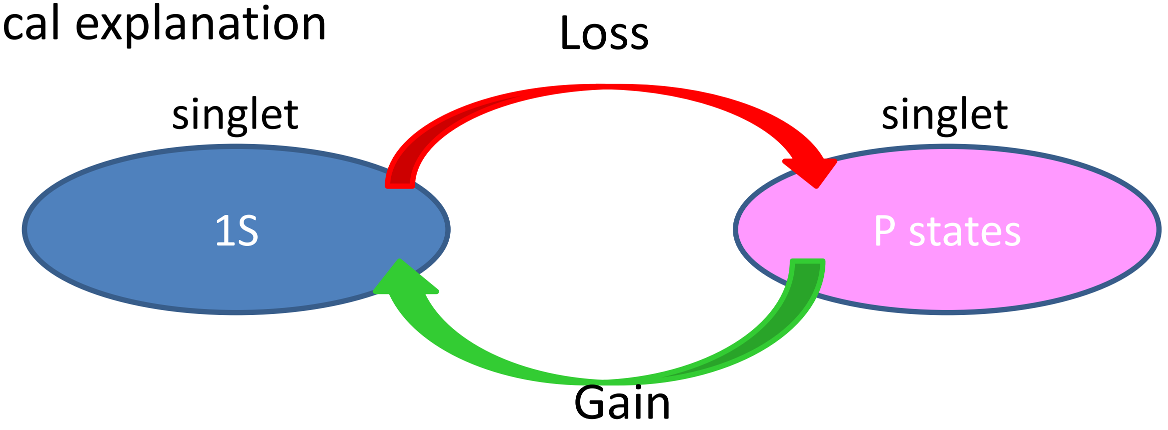


- The peak around origin is reduced in the case of QCD !
- Quarks more deconfined than in QED-like case.
- Reminder : discussion specific to the QBM regime (not the QOR)

Asymptotic distribution: QCD

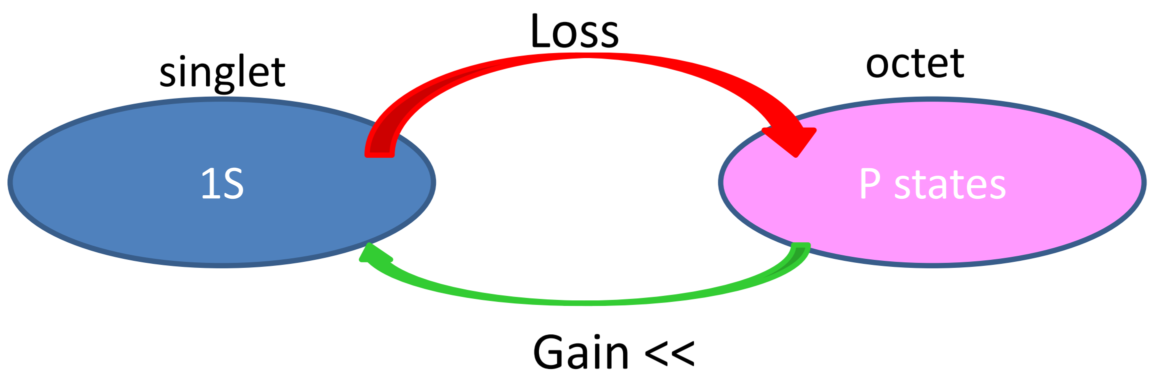
- A possible physical explanation

QED – like :



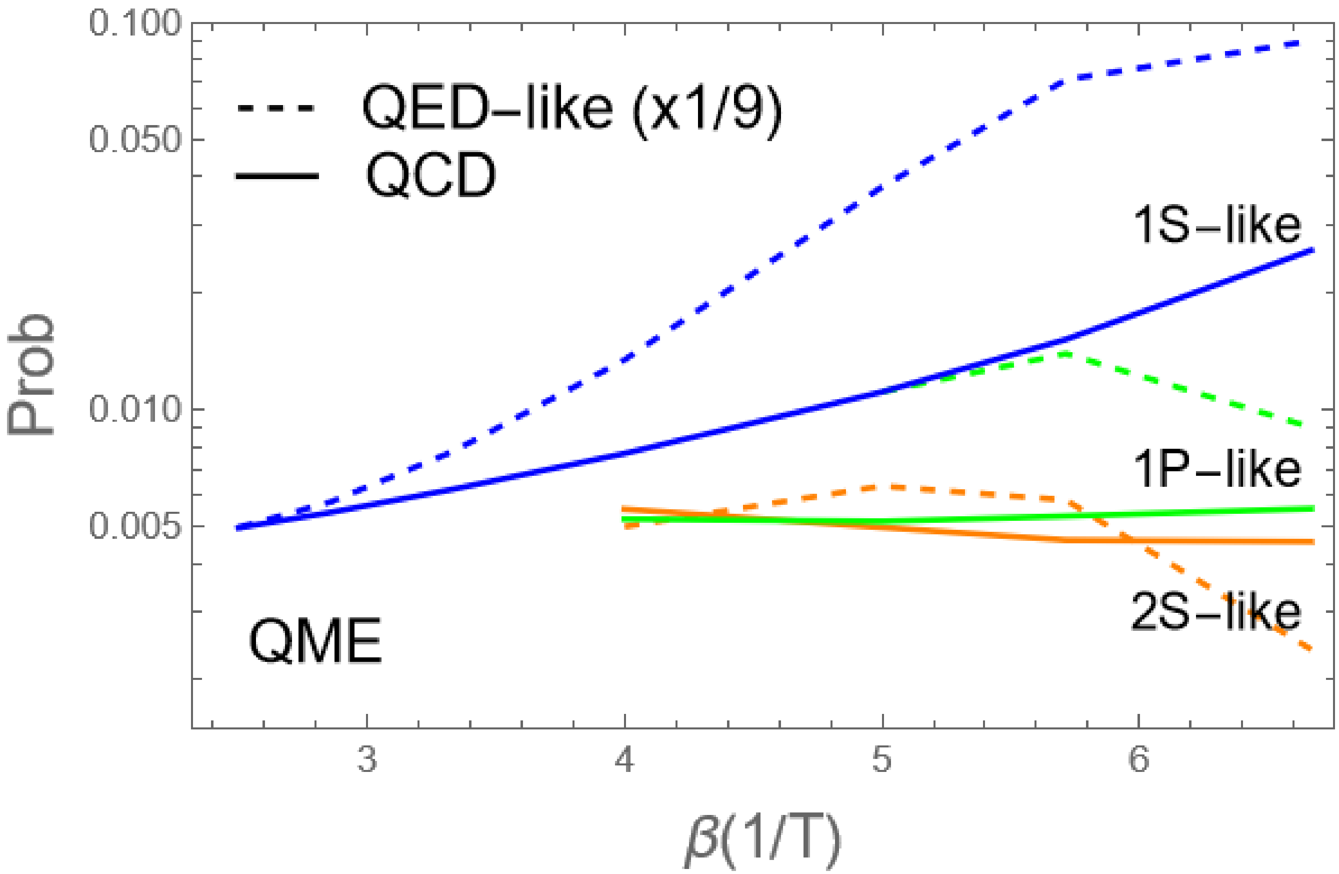
As the binding potential also acts on P-states (singlet), both densities can increase together when a real potential is applied (gain and loss terms keep α)

QCD :



As there is no potential in the octet chanel, the Q and Qbar quarks have a tendancy to fly apart fast (the asymtotic octet density is found indeed flat).
Hence, the gain term does not increase α to the loss term and the equilibrium limit is displaced wrt QED-like

Quarkonium weights



And of course, a lot of questions…

- Is this result correct or the sign of some illness in the QME ? (or the author's mind)
- Can this result be understood by rephrasing things with usual rate equation ?
- Can we put some mathematical modelling on it ?

Conclusions and prospects

- Illustration of a QME solved exactly, with some interesting features and a first (not so bad) contact towards experiment using EPOS4 profiles

- Novel feature : discussion of the asymptotic limit of this equation, both for the QED-like and QCD cases… raising some questions to be addressed in a near feature…

- Exact solution of the QME compared with semi-classical solution (adopted in some microscopic models) for the simpler QED-like case; pretty encouraging, although some difference is seen for the lowest T of the QBM regime. Genuine QCD case should be also tested.

Positivity

- For instance: Equations for the QED-like plasma in 1D :

$$\frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D} = \frac{i}{M}(\hbar c)^2(\partial_s^2 - \partial_{s'}^2)\mathcal{D} - i[V(s) - V(s')]\mathcal{D}$$
$$+\left[2W(0) - W(s) - W(s') - 2W\left(\tfrac{s-s'}{2}\right) + 2W\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^2}{4MT}\left[2W''(0) - W''(s) - W''(s') - 2W''\left(\tfrac{s-s'}{2}\right) + 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$-\frac{(\hbar c)^2}{4MT}\left[2W'(s)\partial_s + 2W'(s')\partial_{s'} + 2W'\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) - 2W'\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D} \qquad \mathcal{L}'_4$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[2W''''(0) + W''''(s) + W''''(s') - 2W''''\left(\tfrac{s-s'}{2}\right) + 2W''''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W'''(s)\partial_s + 4W'''(s')\partial_{s'} - 4W'''\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) + 4W'''\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W''(0)(\partial_s^2 + \partial_{s'}^2) + 4W''(s)\partial_s^2 + 4W''(s')\partial_{s'}^2 + 8W''\left(\tfrac{s-s'}{2}\right)\partial_s\partial_{s'} + 8W''\left(\tfrac{s+s'}{2}\right)\partial_s\partial_{s'}\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}p_{\mathrm{tot}}^2\left[-2W''(0) + W''(s) + W''(s') + 2W''\left(\tfrac{s-s'}{2}\right) - 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$

- Indeed subleading in 1/T expansion
- No higher derivatives on D than the 2nd one => still a FP equation in the semi-classical limit.
- Higher derivatives of the imaginary potential W => possible UV divergences => need for some regularization.