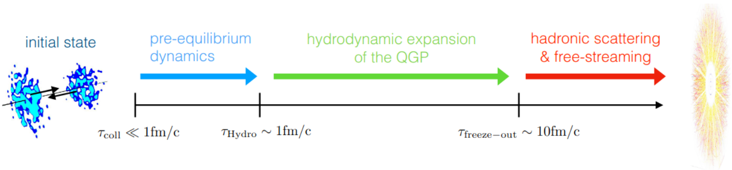
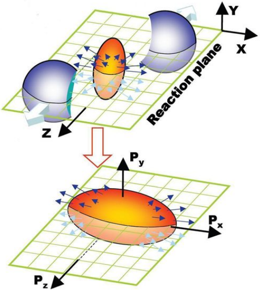
System size dependence of pre-equilibrium and applicability of hydrodynamics in heavy-ion collisions

Clemens Werthmann
in Collaboration with Victor Ambruș and Sören Schlichting
based on PRD 107, 094013 and PRL 130, 152301 and WiP

University of Wrocław

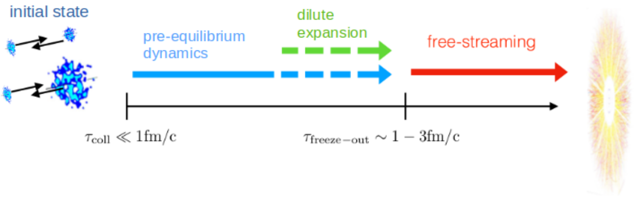


- early stage requires non-equilibrium description, but system quickly equilibrates
- strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables:
  $\mathbf{v_n}$, $\langle p_T \rangle$, particle yields, ...



Hiroshi Masui (2008)

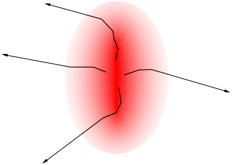
Small systems



Very dilute, hydrodynamics not necessarily applicable

- still collective behaviour is observed!

  Nagle, Zajc Ann.Rev.Nucl.Part. 68 (2018) 211



collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration

- interpolate between free streaming at small opacities and hydrodynamics at large opacities!

Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics for transverse flow observables

Model and Setup

- microscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y)$$

  - boost invariance
  - initialized with vanishing longitudinal pressure and no transverse momentum anisotropies

- time evolution: Boltzmann equation in conformal relaxation time approximation

$$p^\mu \partial_\mu f = C_{\text{RTA}}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\text{eq}}) \ , \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

results will depend only on initial state and opacity

Parametric dependencies

- dimensionless parameter: opacity $\sim$"total interaction rate"

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi}R\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

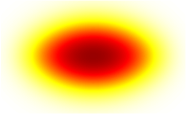
  - encodes dependencies on viscosity, transverse size and energy scale

- our initial condition:
  average profiles for centrality classes of Pb+Pb at $5.02\ \mathrm{TeV}$

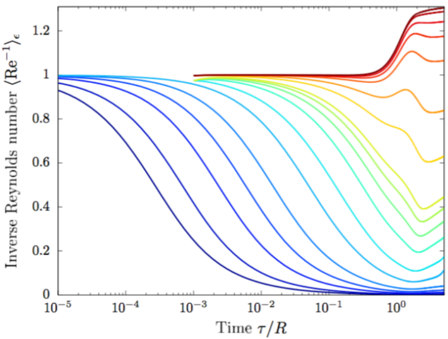
  Borghini, Borrell, Feld, Roch, Schlichting, Werthmann PRC 107 (2023), 034905

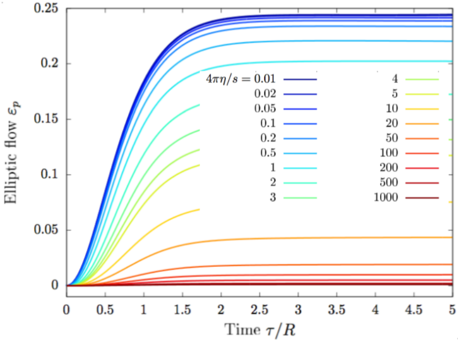
  - for fixed profile, vary $\hat{\gamma}$ via η/s: $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

30-40%

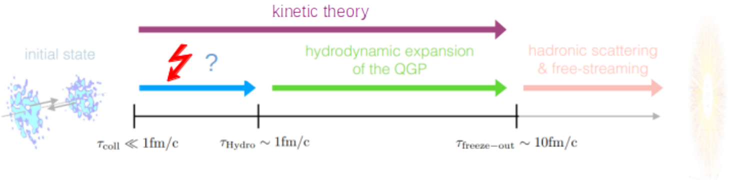


Time evolution in different systems

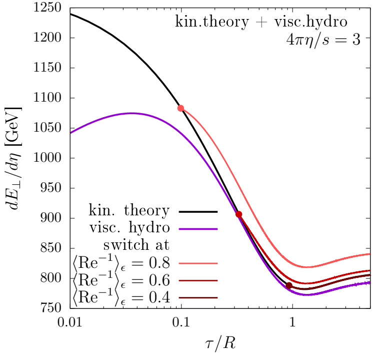




- $\mathrm{Re}^{-1} = \left(\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{e^2}\right)^{1/2}$ measures relative size of non-equilibrium effects
  - equilibration timescale strongly depends on opacity; smaller systems take longer to equilibrate
- elliptic flow on similar timescales; continuously varying strength of response



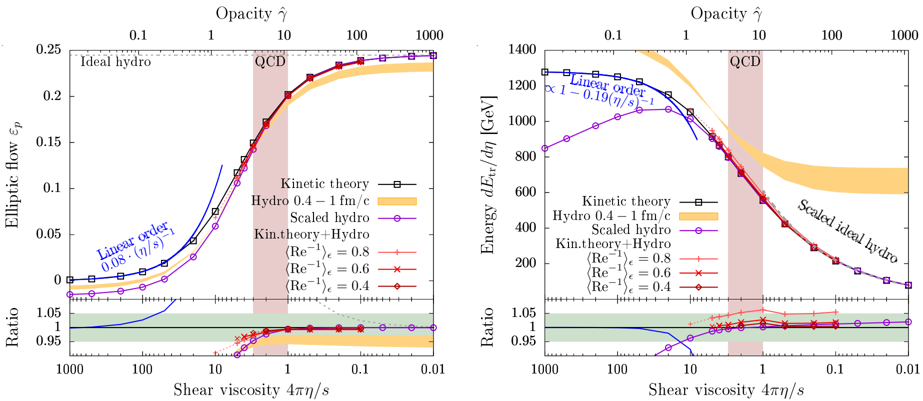
- want to find the regime where hydrodynamics agrees with kinetic theory
- caveat: even at large opacities, naive hydrodynamics does not accurately describe pre-equilibrium
  - how to find a meaningful setup to compare to?



- scaled hydro: change initial condition in anticipation of different early time behaviour
- hybrid simulations: switching from kinetic theory to hydrodynamics after Re^{-1} has dropped to a specific value
  - later switch $\Rightarrow$ more accurate results

Comparison of improved hydro schemes with kin. theory

CRC-TR 211



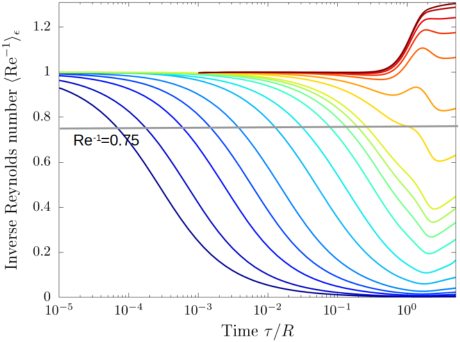
- naive hydro is off; scaled hydro accurate if $\hat{\gamma} \gtrsim 4$
- Hybrid kin. theory scheme can improve on scaled hydro at intermediate opacities
- later switching improves agreement: accurate on 5% level if $\mathrm{Re}^{-1} < \mathrm{Re}_c^{-1} \sim 0.75$

Characterize dynamics by timescales of:

transition to hydrodynamic behaviour

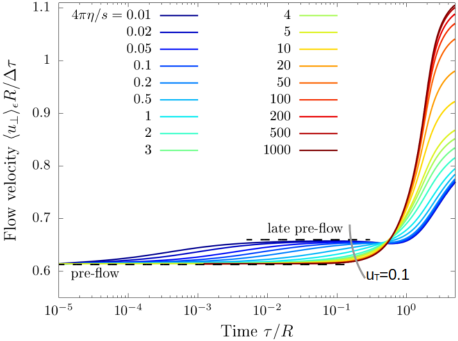
and onset of transverse expansion

- hydro applicable for $\mathrm{Re}^{-1} \lesssim 0.75$



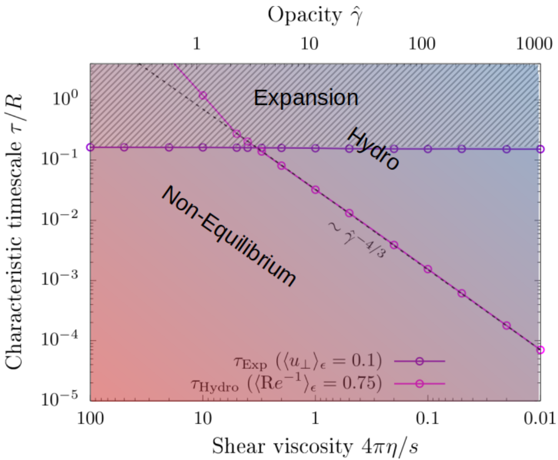
- takes longer for smaller systems; some systems never equilibrate enough!

- measured by buildup of flow: $u_\perp \sim 0.1$



- almost independent of opacity: $\tau_{\mathrm{Exp}} \sim 0.2R$

Regime of applicability of hydrodynamics



- upper area: regime of transverse expansion
- Hydro applicable in top right corner
- $\hat{\gamma} \lesssim 3$: hydro does not become applicable before onset of transverse expansion!

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\mathrm{Pb+Pb}: \; \hat{\gamma} \overset{30-40\%}{\sim} 5.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

$$\mathrm{p+Pb}: \; \hat{\gamma} \overset{\mathrm{min.bias}}{\sim} 1.5 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \overset{\mathrm{high\ mult.}}{\lesssim} 2.7$$

very high multiplicity events approach regime of applicability, but do not reach it

$$\mathrm{p+p}: \; \hat{\gamma} \overset{\mathrm{min.bias}}{\sim} 0.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4}$$

far from hydrodynamic behaviour

Hydrodynamics in real collision systems

Taking the criterion of $\hat{\gamma} \gtrsim 3$ seriously, what does this mean for the applicability of hydrodynamics to "real-life" collisions?

$$\mathrm{Pb+Pb}: \; \hat{\gamma} \overset{30-40\%}{\sim 5.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

$$\mathrm{O+O}: \; \hat{\gamma} \overset{30-40\%}{\sim 2.2} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{1.13\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{55\,\mathrm{GeV}}\right)^{1/4} \sim \overset{70-80\%}{1.4} - \overset{0-5\%}{3.1}$$

probes transition region to hydrodynamic behaviour

$$\mathrm{p+Pb}: \; \hat{\gamma} \overset{\text{min.bias}}{\sim 1.5} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \overset{\text{high mult.}}{\lesssim 2.7}$$

very high multiplicity events approach regime of applicability, but do not reach it

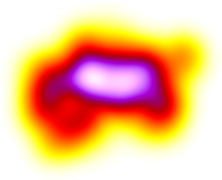
$$\mathrm{p+p}: \; \hat{\gamma} \overset{\text{min.bias}}{\sim 0.7} \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4}$$

far from hydrodynamic behaviour

- TRENTO initial conditions for two different collision systems
  Moreland, Bernhard, Bass PRC 92 (2015) 011901(R)
- pre-generated nucleon positions to account for correlations like α-clustering

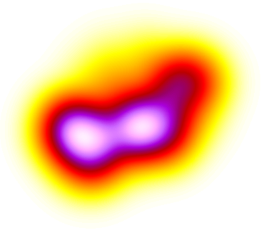
Pb+Pb 2.76 TeV

Alvioli, Drescher, Strikman PLB 680 (2009) 225



O+O 7 TeV

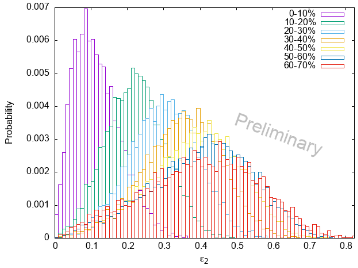
Loizides, Nagle, Steinberg SoftwareX 1-2 (2015) 13



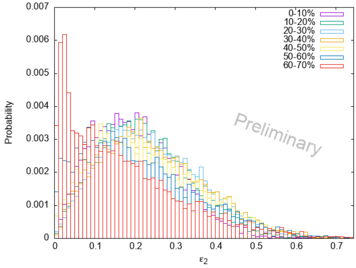
(example profiles from 20-30% centrality class)

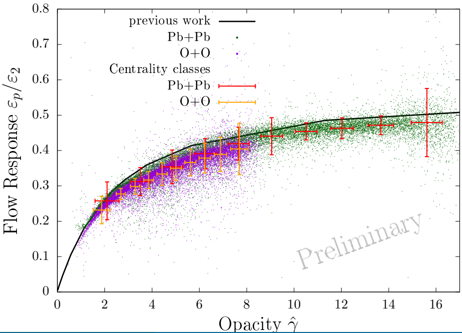
Event-by-event flow responses

Pb+Pb



O+O

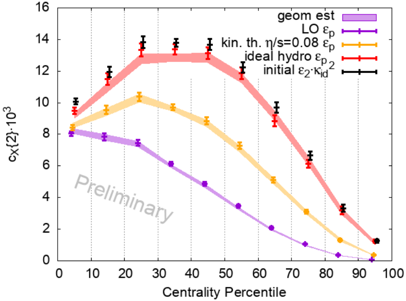


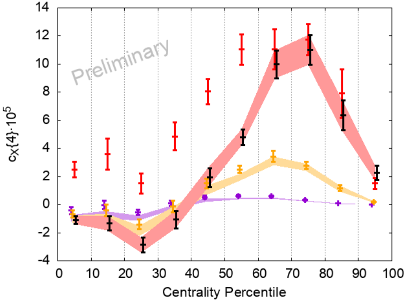


- significant difference in geometry
  - Pb+Pb: visible effect of mean geometry
  - O+O: mostly fluctuation (?)
- flow response still mostly depends on $\hat{\gamma}$

Flow cumulants in O+O

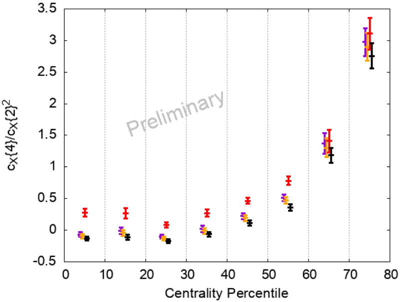
CRC–TR 211





$$\langle (\epsilon_p)^n \rangle = \langle (\kappa \epsilon_2)^n \rangle = \bar{\kappa}^n \langle (\epsilon_2)^n \rangle + \ldots$$

- flow fluctuations dominated by avg. response to geometry fluctuations
- no $\hat{\gamma}$-dependence in ideal hydro $\Rightarrow$ same fluctuation curve
- ratio eliminates avg. response



Summary

- kinetic theory description of transverse flow on whole range in system size

- comparison to hydrodynamics: accurate at 5% level if $\mathrm{Re}^{-1} \lesssim 0.75$

- small systems (p+p, p+Pb): transverse expansion faster than equilibration $\Rightarrow$ hydro not applicable!
  - O+O covers transition regime to hydro behaviour

Outlook:

- looking for observables indicating degree of hydrodynamization in event-by-event simulations

Backup

In theoretical descriptions:

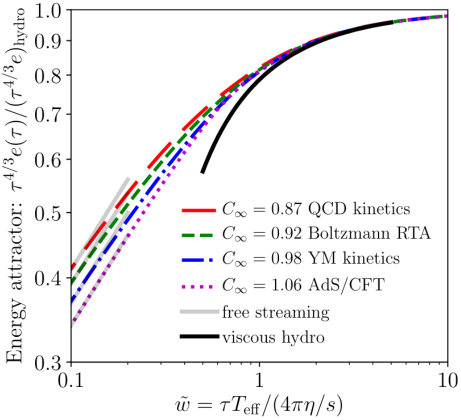
$$v_n = \kappa_{n,n} \cdot \epsilon_n$$

- Flow can be compared to experiment
- Response depends on the dynamical model
- Initial state geometry is poorly constrained in small systems

Varying initial condition in order to fit flow measurements will mask inaccuracies in the description of the dynamical response!

What might happen when going beyond RTA?

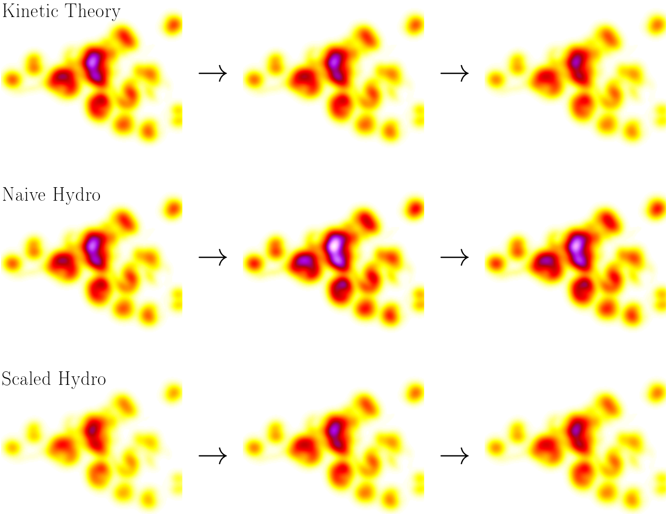
- more complex kernels will introduce further parameter dependence, but opacity dependence might still be "leading order approximation"
- in Bjorken flow, equilibration happens in very similar ways across different model descriptions:



Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

Early time longitudinal cooling and scaled hydro

evolution of τe:

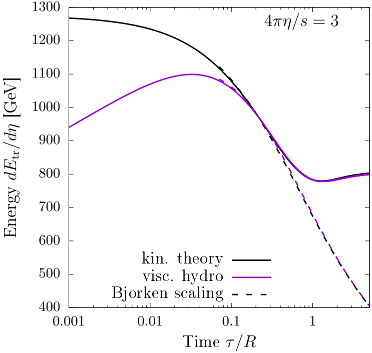


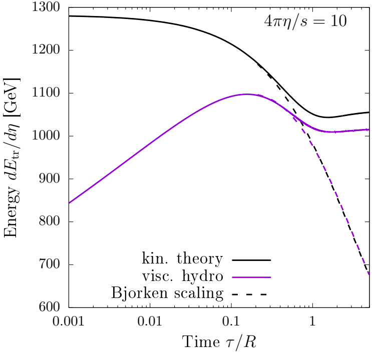
$\tau = 3 \cdot 10^{-6}$fm $\quad$ $\tau = 8 \cdot 10^{-4}$fm $\quad$ $\tau = 3 \cdot 10^{-3}$fm

(times for $4\pi\eta/s = 0.05$)

Initializing on the local attractor

- accuracy depends on timescale separation of pre-equilibrium and transv. expansion

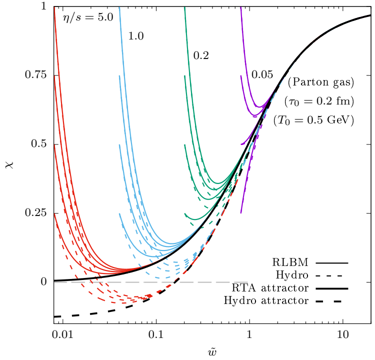


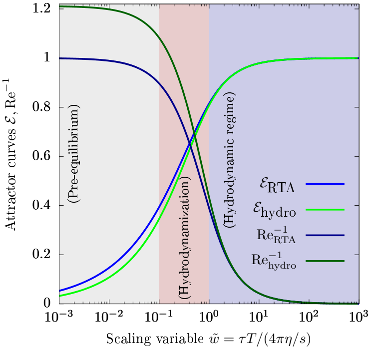


Bjorken flow attractor

- longitudinal boost-invariant Bjorken flow exhibits universal behaviour
- time evolution curves converge to an attractor curve when expressed via the scaling variable $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$
  $\Rightarrow$ expressed via universal scaling functions
  $\chi(\tilde{w}) = p_L/p_T, \quad \mathcal{E}(\tilde{w}) \propto \tau^{4/3} e, \quad f_{E_\perp}(\tilde{w}) \propto \tau^{1/3} \frac{dE_\perp}{dy}, \ldots$

Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301





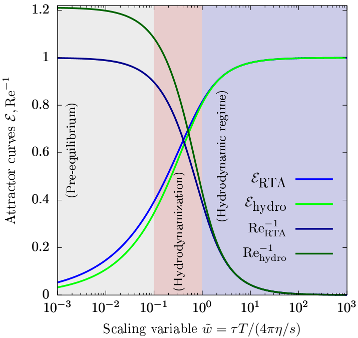
Ambruș, Bazzanini, Gabbana, Simeoni, Succi, Tripicione, arXiv:2201.09277

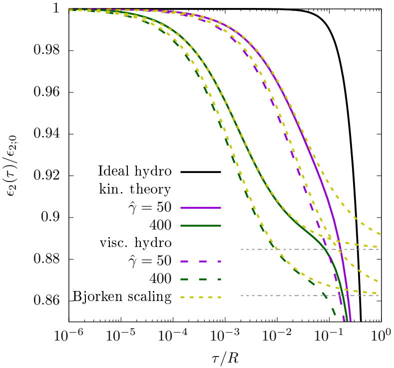
Early time eccentricity decrease

CRC-TR 211

- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$

    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3} e = \text{const.}$, $\tau^{1/3} \frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3} e \sim \tilde{w}^\gamma$





- inhomogeneous cooling changes energy density profile

Bjorken flow universal attractor curve in scaling variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$:

$$\epsilon(\tau)\tau^{4/3} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\, C_\infty\, \mathcal{E}(\tilde{w})\,,$$

$$\tau^{1/3} \frac{\mathrm{d}E_\perp}{d^2\mathbf{x}_\perp \mathrm{d}\eta} = (4\pi\eta/s)^{4/9} a^{1/9} (\epsilon\tau)_0^{8/9}\, C_\infty\, f_{E_\perp}(\tilde{w})$$

- using $\epsilon = aT^4$, recast first eq. into self consistency eq. for $\tilde{w}$
- use this togehter with initial cond. for $\epsilon\tau$ to relate differentials of $\mathrm{d}\tilde{w}$ and $\mathrm{d}x_\perp$
- integrate second equation to find scaling of $\mathrm{d}E_\perp/\mathrm{d}\eta$
- use $\frac{(4\pi\eta/s)^4 a}{dE_\perp^0/d\eta\ R} = \frac{1}{\pi}\left(\frac{4\pi}{5\hat{\gamma}}\right)^4$ to identify $\hat{\gamma}$

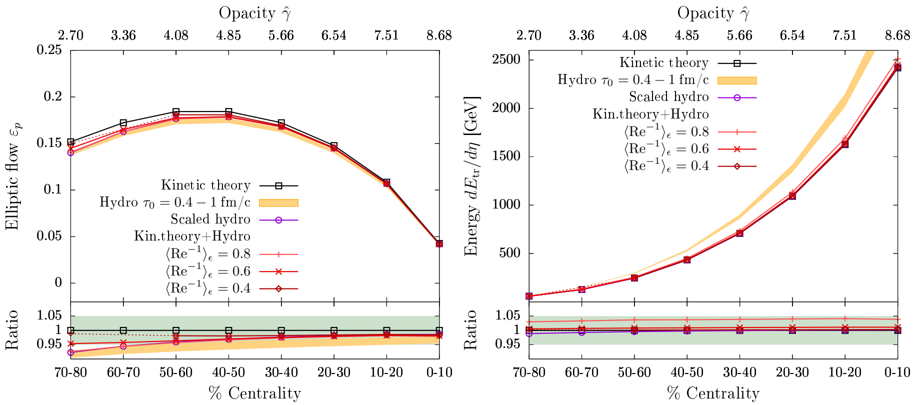
$$\frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9}{2}\left(\frac{4\pi}{5\hat{\gamma}}\right)^4 \left(\frac{R}{\tau}\right)^3 \int_0^{\tilde{w}(\tau,\mathbf{x}_\perp=0)} \frac{\tilde{w}^3 d\tilde{w}}{\mathcal{E}(\tilde{w})}\left[1 - \frac{\tilde{w}}{4}\frac{\mathcal{E}'(\tilde{w})}{\mathcal{E}(\tilde{w})}\right] f_{E_\perp}(\tilde{w})\,,$$

$$\tilde{w}(\tau, \mathbf{x}_\perp = 0) = \left(\frac{5\hat{\gamma}}{4\pi}\right)^{8/9}\left(\frac{\tau}{R}\right)^{2/3}\left[C_\infty \mathcal{E}(\tilde{w})\right]^{1/4}$$

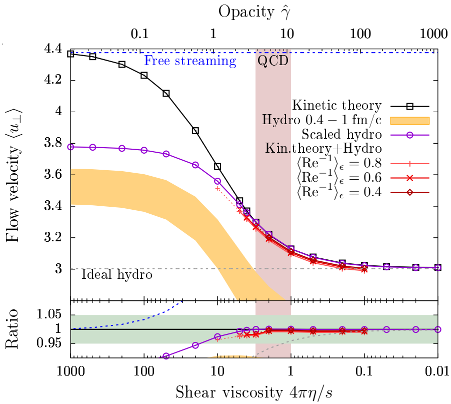
Limits of this scaling law:

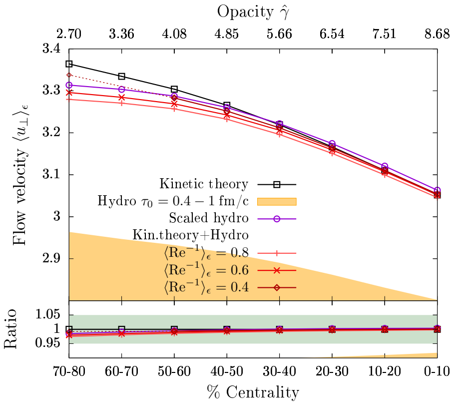
- $\hat{\gamma}\left(\frac{\tau}{R}\right)^{3/4} \ll 1 \Rightarrow \tilde{w} \ll 1 \ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx f_{E_\perp}(\tilde{w}) \approx C_\infty^{-1}\tilde{w}^{4/9} \ \Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = 1$
- $\hat{\gamma}^{3/4}\left(\frac{\tau}{R}\right) \gg 1 \ \Rightarrow\ \tilde{w} \gg 1 \ \Rightarrow\ \mathcal{E}(\tilde{w}) \approx 1, f_{E_\perp} \approx \frac{\pi}{4}$
  $\Rightarrow\ \frac{dE_\perp/d\eta}{dE_\perp^0/d\eta} = \frac{9\pi}{32}\left(\frac{4\pi}{5\hat{\gamma}}\right)^{4/9}\left(\frac{R}{\tau}\right)^{1/3} C_\infty$

Centrality dependence



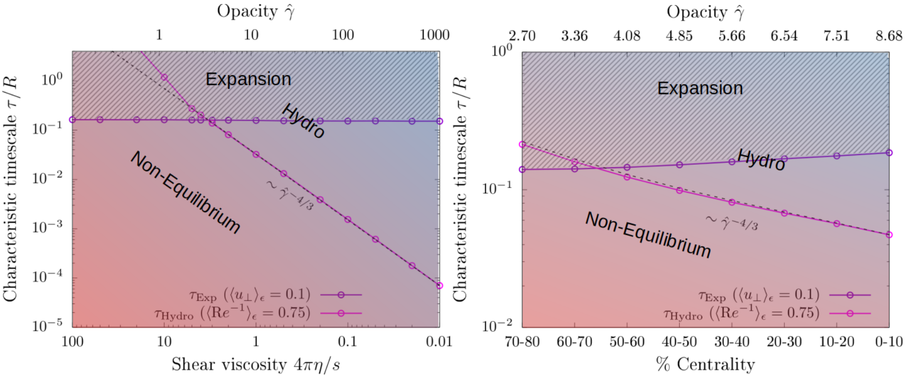
Transverse flow velocity





Hydrodynamization in viscosity and centrality dependence

CRC–TR 211



- transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro appicable when $\mathrm{Re}^{-1} < \mathrm{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\mathrm{Hydro}}/R \approx 1.53\, \hat{\gamma}^{-4/3} \left[(\mathrm{Re}_c^{-1})^{-3/2} - 1.21 (\mathrm{Re}_c^{-1})^{0.7} \right]$$

- hydrodynamization before transv. Expansion for $\hat{\gamma} \gtrsim 3$