

# Transverse momentum fluctuation in ultra-central Pb+Pb collision

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with Somadutta Bhatta, Jianguong Jia, Matthew Luzum, Jean-Yves Ollitrault

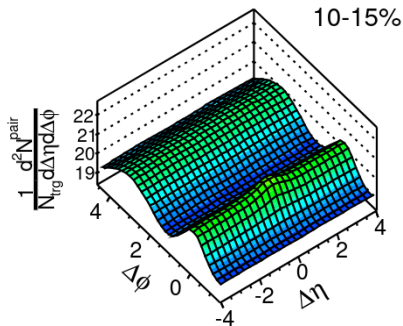
...based on arXiv:2303.15323

NA7-STRONG HFHF 2023, Giardini Naxos, Italy, October 4, 2023



## Motivation

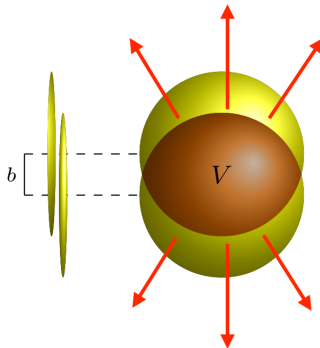
- Experimental evidence for the formation of a **little fluid** in Pb+Pb collision  $\rightarrow$  **azimuthal correlations between particles** seen in detectors.



CMS:1201.3158

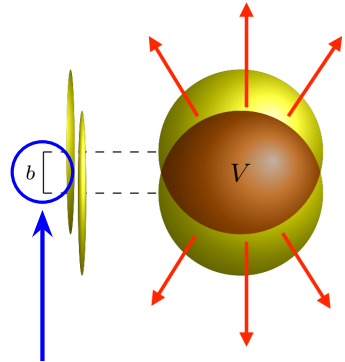
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- Evidence is **indirect** !  $\rightarrow$  azimuthal distribution of particles is **not isotropic**  $\rightarrow$  **anisotropy driven by pressure gradients** within a fluid.



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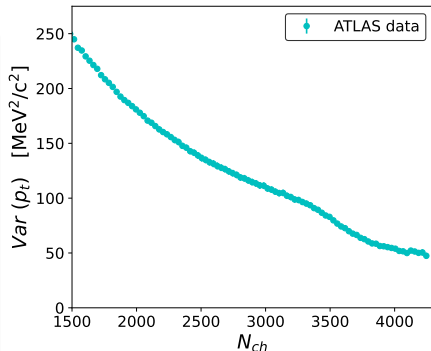
- Experimental evidence for the formation of a **little fluid** in Pb+Pb collision  $\rightarrow$  **azimuthal correlations between particles** seen in detectors.
- Evidence is **indirect !**  $\rightarrow$  azimuthal distribution of particles is **not isotropic**  $\rightarrow$  **anisotropy driven by pressure gradients** within a fluid.
- We report **more direct evidence** of **local thermalization** in Pb+Pb collisions  $\rightarrow$  **does not involve directions** of outgoing particles, but **solely their momenta**.



**b = impact parameter**  
**(important in this talk ! )**

## ATLAS data for $[p_t]$ fluctuation

- Recent ATLAS data shows multiplicity ( $N_{ch}$ ) dependence of the variance of transverse momentum per particle,  $[p_t]$ .

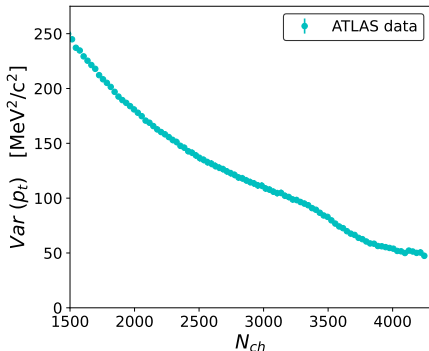


Variance of  $[p_t]$  for Pb+Pb @ 5.02 TeV  
PhysRevC.107.054910

Table 374 in <https://www.hepdata.net/record/ins2075412>

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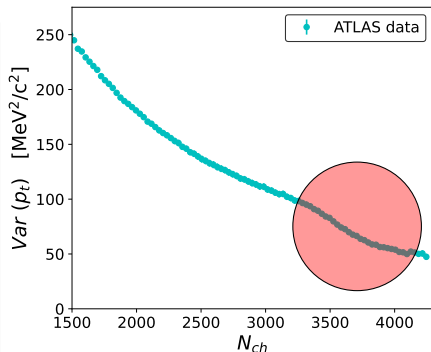


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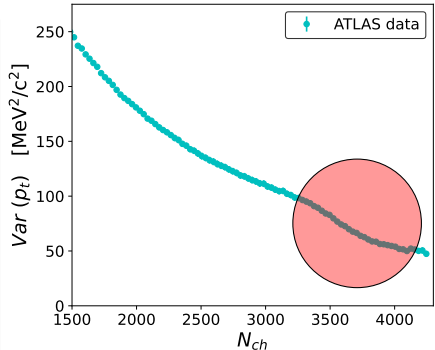


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- The relative dynamical fluctuation of  $[p_t]$  is very small  $\sim 1\%$
- Puzzling behavior in ATLAS data : steep decrease over a narrow range of  $N_{ch}$
- We will show that this is a consequence of **thermalization !**



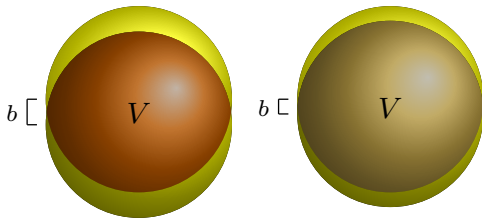
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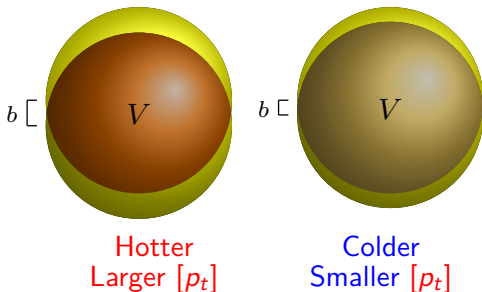
## Impact parameter ( $b$ ) is important !

- In experiment  $b$  is not known !  $\Rightarrow$   $[p_t]$  fluctuation is measured for fixed  $N_{ch}$



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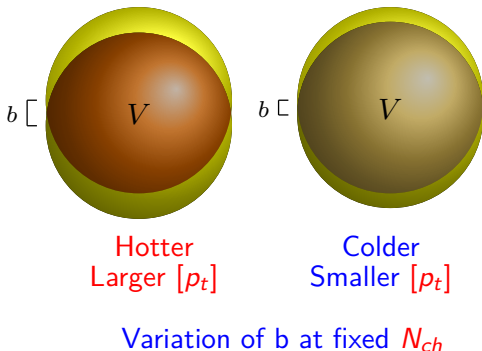
- In experiment  $b$  is not known !  $\implies [p_t]$  fluctuation is measured for fixed  $N_{ch}$
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Variation of  $b$  at fixed  $N_{ch}$

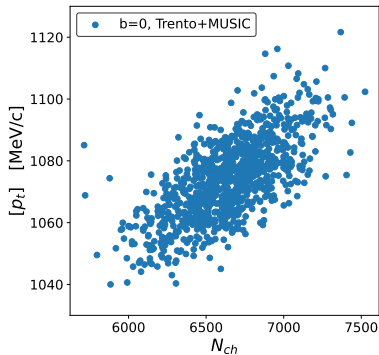
## Impact parameter ( $b$ ) is important !

- In experiment  $b$  is not known !  $\implies$   $[p_t]$  fluctuation is measured for fixed  $N_{ch}$
- Fixed  $N_{ch} \implies$  finite range of  $b$  !
- Variation of  $b$  gives a contribution to the variation of  $[p_t] \implies$  goes to 0 in ultracentral collisions !



## Hydrodynamic simulation: $b$ is known !

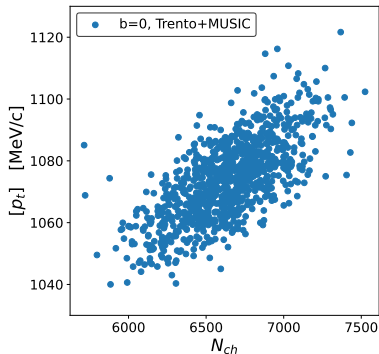
- ▶ Hydro : **assumes thermalization !**  
⇒ We simulate Pb+Pb collisions at **fixed  $b$  ( $=0$ )** with TRENTO (initial condition)+ MUSIC (hydro)



Pb+Pb @ 5.02 TeV

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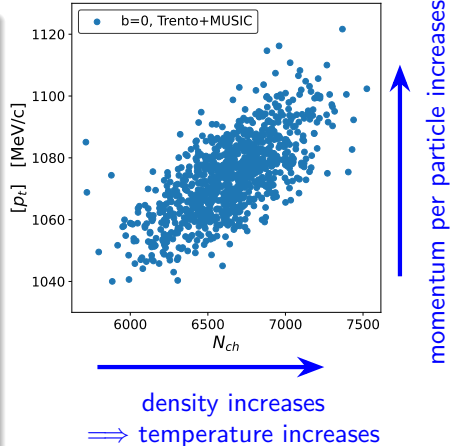
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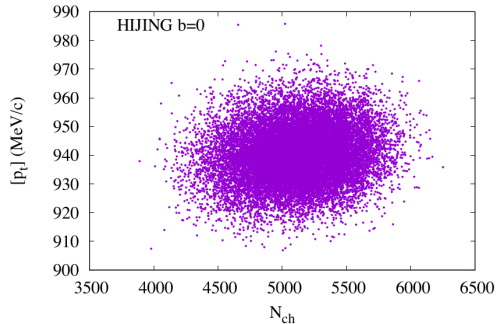
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- ▶ Fixed  $b$  ⇒ **fixed collision volume**  
Larger  $N_{ch}$  ⇒ **larger density**  
⇒ **larger temperature**  
⇒ **larger energy per particle**  
⇒ **larger  $[p_t]$**



## Comparing other models : HIJING simulation

Wang, Gyulassy, arXiv:nucl-th/9502021

- HIJING: microscopic model of HI collision  $\implies$  the system doesn't thermalize !

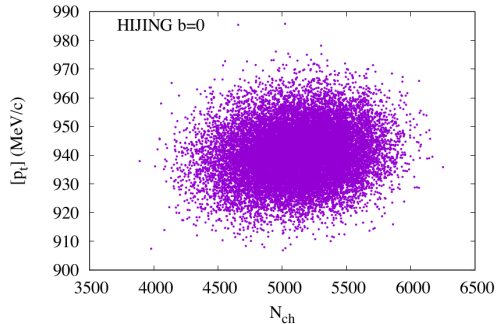


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No thermalization

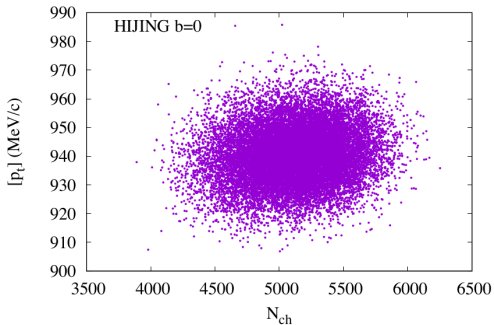
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- **Very small correlation** between  $N_{ch}$  and  $[p_t] \sim 10 \times$  smaller !!
- Hence the **correlation** is a **signature of thermalization !**



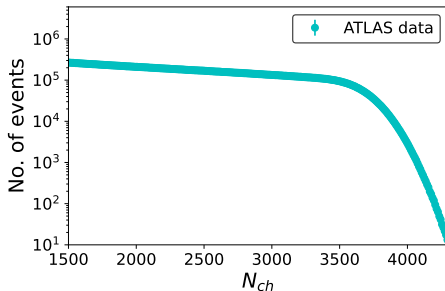
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## Bayesian reconstruction of $P(\mathbf{b} | N_{ch})$

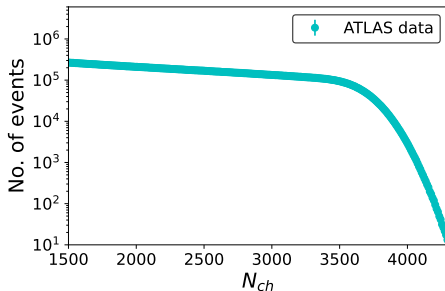
- First we solve the **inverse problem**:  
what is the distribution of  $N_{ch}$  at fixed  $\mathbf{b}$  i.e.  $P(N_{ch} | \mathbf{b})$  ?



$N_{ch}$  distribution  
for centrality classification !

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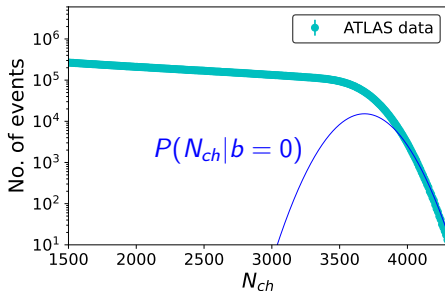
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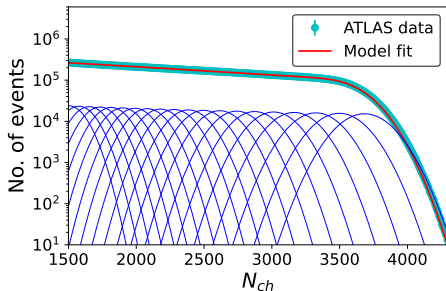
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$N_{ch}$  distribution at fixed  $\mathbf{b}$   
Gaussian assumption !

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- Fit  $P(N_{ch})$  as **sum of Gaussians**

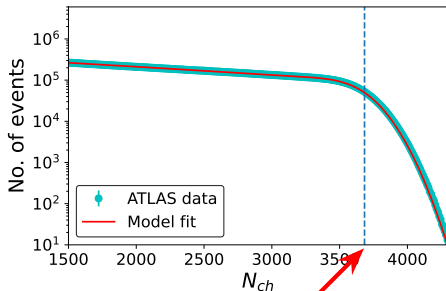


Sum of Gaussians at fixed  $\mathbf{b}$

Das, Giacalone, Monard, Ollitrault  
arXiv:1708.00081

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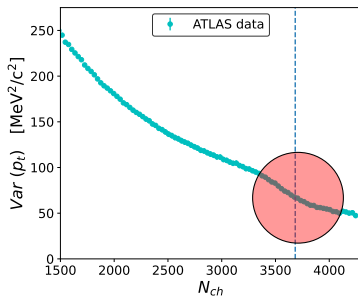
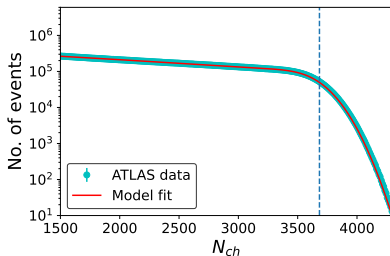
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Precise construction of knee  
 $\langle N_{ch} | \mathbf{b} = 0 \rangle$

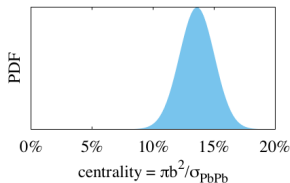
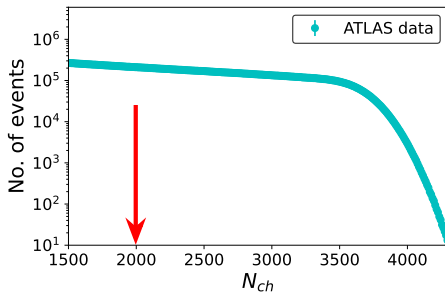
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- We precisely reconstruct the **knee** (mean  $N_{ch}$  at  $\mathbf{b}=0$ )
- The **steep fall** of the variance precisely **occur at the knee !**



## $P(b | N_{ch})$ from Bayesian reconstruction

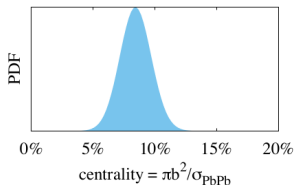
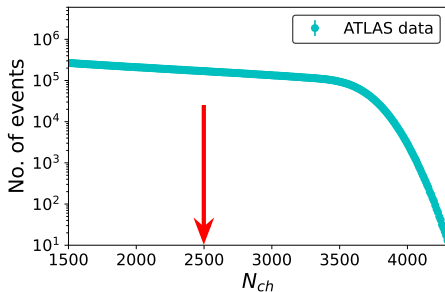
- At smaller  $N_{ch}$  the distribution  $P(b | N_{ch})$  is a full Gaussian





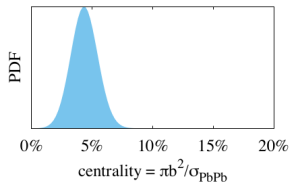
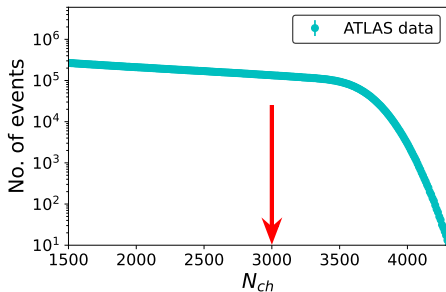
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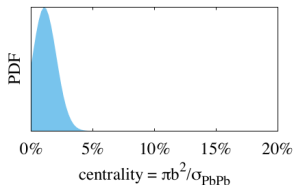
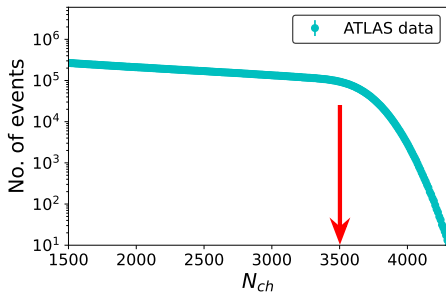
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- But as we move closer and closer to the knee,  $P(b | N_{ch})$  becomes truncated due to the limit  $b \geq 0$



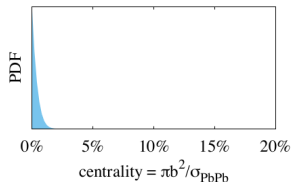
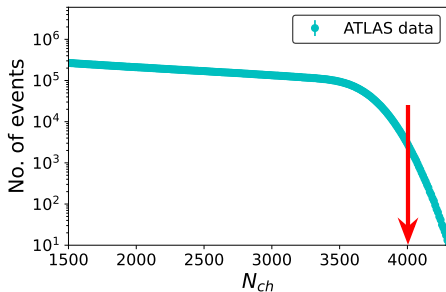
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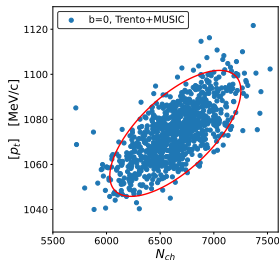
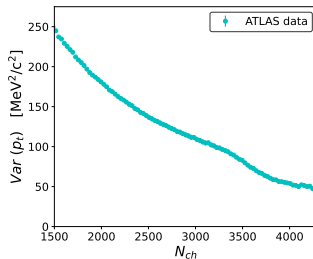
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- But as we move closer and closer to the knee,  $P(b|N_{ch})$  becomes truncated due to the limit  $b \geq 0$
- Above the knee it gets extremely truncated  $\implies$  the impact parameter fluctuation gradually disappears !



# Understanding $[p_t]$ fluctuation data : Parametrizing $P(N_{ch}, [p_t]|b)$

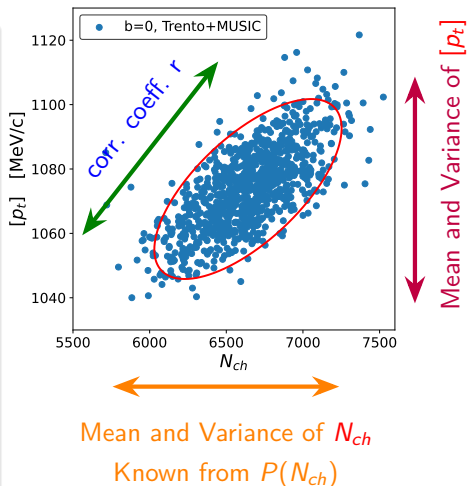
- ▶ We assume a simple 2D correlated Gaussian between  $[p_t]$  and  $N_{ch}$  at fixed impact parameter  $b$  :  $P([p_t], N_{ch}|b)$ .



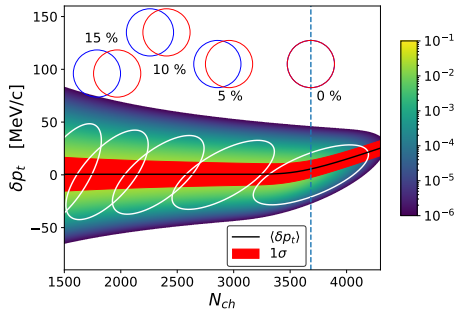


## Understanding $[p_t]$ fluctuation data : Parametrizing $P(N_{ch}, [p_t]|b)$

- ▶ We assume a simple **2D correlated Gaussian** between  $[p_t]$  and  $N_{ch}$  at fixed impact parameter  $b$  :  $P([p_t], N_{ch}|b)$ .
- ▶ The distribution has **5 parameters** : Mean and variance of  $N_{ch}$ , Mean and variance of  $[p_t]$  and correlation coefficient  $r$  between  $N_{ch}$  and  $[p_t]$ .
- ▶ Mean value of  $[p_t]$  is **constant** at fixed  $b$  and assuming it is **independent of  $b$**   $\implies$  we fit  $P(\delta p_t, N_{ch}|b)$   
 $\delta p_t = [p_t] - \langle [p_t] \rangle$



Fit result :  $P(N_{ch}, \delta p_t)$

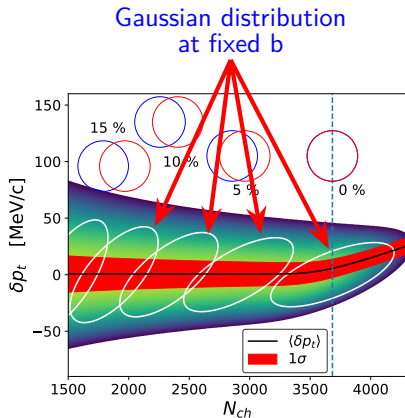


2D correlated gaussian  
distribution of  $\delta p_t$  and  $N_{ch}$



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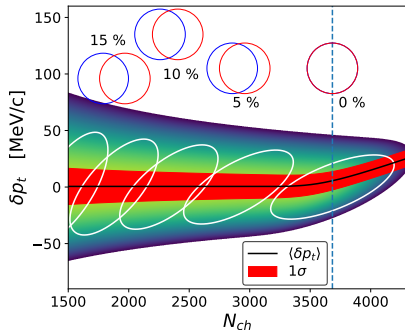
- We get,  $P(N_{ch}, \delta p_t)$   
 $= \int P(N_{ch}, \delta p_t | b) P(b) db$





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 $P(\delta p_t | N_{ch}) = \frac{P(N_{ch}, \delta p_t)}{P(N_{ch})}$   
 $\implies \text{Var}([p_t] | N_{ch})$  is the squared width of  $P(\delta p_t | N_{ch})$
- The width of  $[p_t]$  fluctuation has **two contributions** :

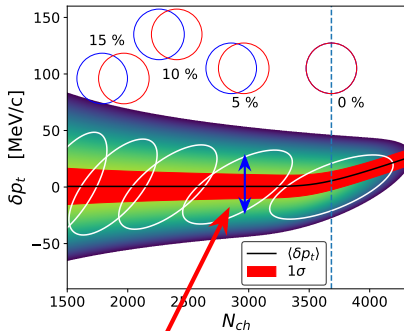


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- The width of  $[p_t]$  fluctuation has **two contributions** :
  - ❶ due to fluctuation of impact parameter  $b$
  - ❷ the true intrinsic fluctuation

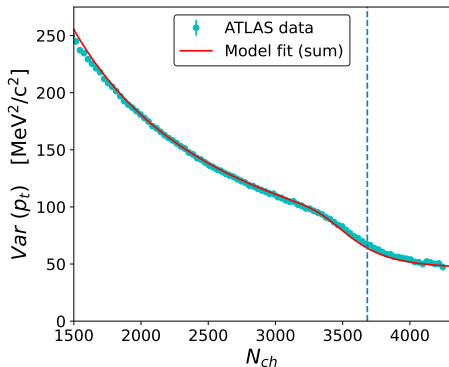


fluctuation of  $[p_t]$  at  
 fixed  $b$  and fixed  $N_{ch}$   
 (height of a single ellipse)



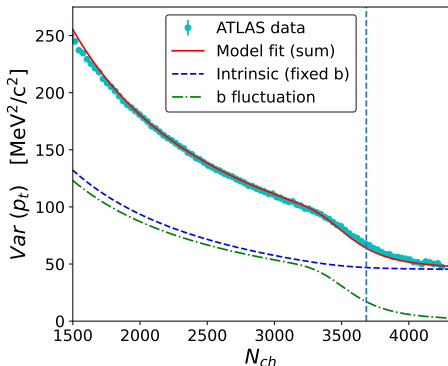
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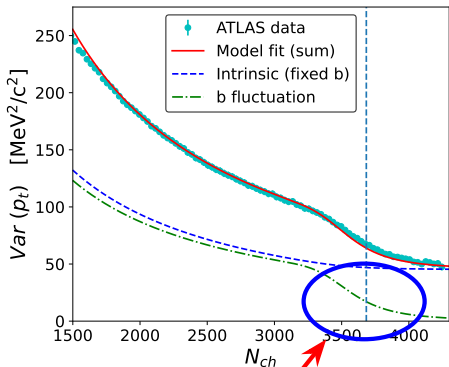
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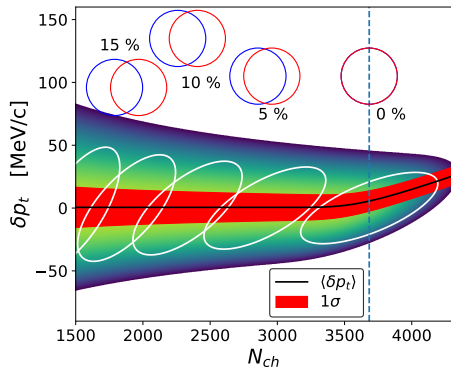
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- The contribution gradually disappears around the knee !



Contribution of b-fluctuation disappears above knee

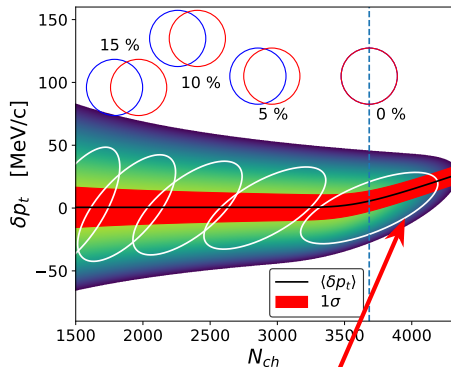
## Thermalization observed !

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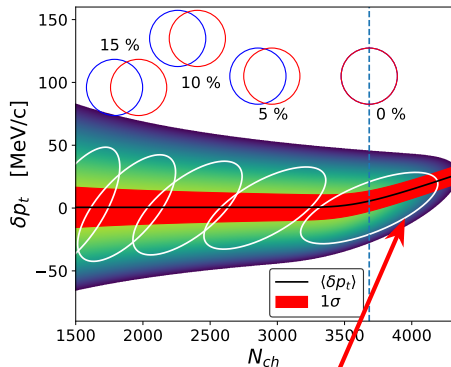
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Strong correlation between  $[p_t]$  and  $N_{ch}$  at fixed  $b$  from our model fit

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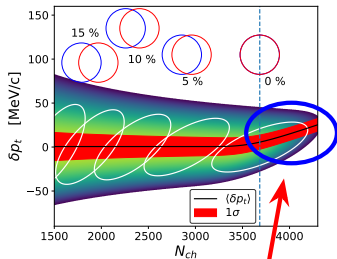
- Our model fit returns  $r = 0.676$  !
- It suggests **strong correlation** between  $[p_t]$  and  $N_{ch}$  at fixed  $b$
- Hence **thermalization is observed !**



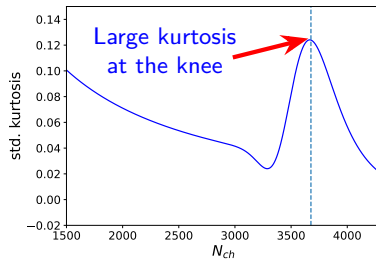
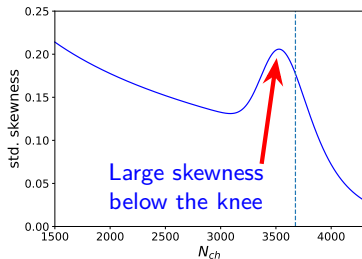
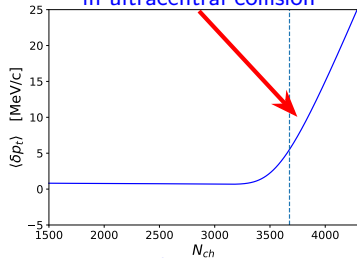
Strong correlation between  $[p_t]$  and  $N_{ch}$  at fixed  $b$  from our model fit

# Further predictions !

RS, Picchetti, Luzum, Ollitrault, arXiv:2306.09294

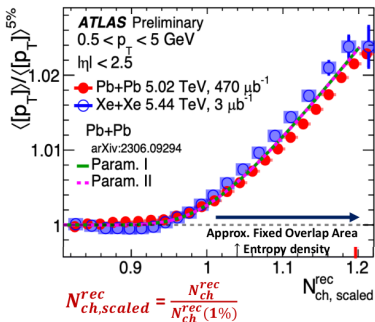


Slight increase of mean  $[p_t]$   
in ultracentral collision

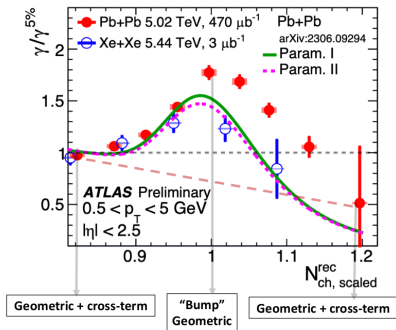


Gardim, Giacalone, Ollitrault, arXiv:1909.11609

# ATLAS Preliminary, presented in QM2023



mean  $[p_T]$



$[p_T]$ - skewness

## Summary and Outlook

- **Impact parameter fluctuation** at fixed  $N_{ch}$  plays an **important role** in ultracentral collision !
- **Two** separate contributions to  $[p_t]$  fluctuation :
  - **i** **intrinsic fluctuation** → originates from **quantum fluctuation** in the **initial state**
  - **ii** **impact parameter fluctuation** at fixed  $N_{ch}$  → **disappears in ultracentral region** → **causes the steep fall at the knee**
- Our methodology paves a way **to separate the geometrical and quantum fluctuations** → the **unique patterns of the cumulants** of  $[p_t]$  fluctuation at the ultracentral regime originates mostly due to **b-fluctuation** !
- **Transverse momentum fluctuation in ultra central collision provides a new , direct probe of the thermalization !**

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... Thank you for your attention !



$P(\delta p_t | N_{ch}, c_b)$  in terms of  $k_1$  and  $k_2$

$$P(\delta p_t | N_{ch}, c_b) = \frac{1}{\sqrt{2\pi\kappa_2(c_b)}} \exp\left(-\frac{(\delta p_t - \kappa_1(c_b))^2}{2\kappa_2(c_b)}\right)$$

$$\kappa_1(c_b) = r \frac{\sigma_{p_t}(c_b)}{\sigma_{N_{ch}}(c_b)} (N_{ch} - \overline{N_{ch}}(c_b)),$$

$$\kappa_2(c_b) = (1 - r^2) \sigma_{p_t}^2(c_b).$$

## Moments and cumulants of $[p_t]$ -fluctuation

$$\langle \delta p_t | c_b \rangle = \kappa_1,$$

$$\langle \delta p_t^2 | c_b \rangle = \kappa_1^2 + \kappa_2,$$

$$\langle \delta p_t^3 | c_b \rangle = \kappa_1^3 + 3\kappa_2\kappa_1,$$

$$\langle \delta p_t^4 | c_b \rangle = \kappa_1^4 + 6\kappa_2\kappa_1^2 + 3\kappa_2^2,$$

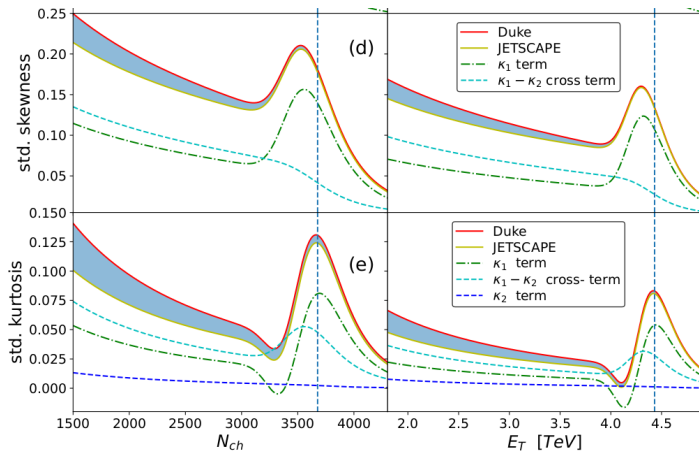
$$\langle \delta p_t \rangle = \langle \kappa_1 \rangle,$$

$$\text{Var}(p_t) = (\langle \kappa_1^2 \rangle - \langle \kappa_1 \rangle^2) + \langle \kappa_2 \rangle,$$

$$\text{Skew}(p_t) = \langle \kappa_1^3 \rangle - 3\langle \kappa_1^2 \rangle \langle \kappa_1 \rangle + 2\langle \kappa_1 \rangle^3 \\ + 3(\langle \kappa_2 \kappa_1 \rangle - \langle \kappa_2 \rangle \langle \kappa_1 \rangle),$$

$$\text{Kurt}(p_t) = \langle \kappa_1^4 \rangle - 4\langle \kappa_1^3 \rangle \langle \kappa_1 \rangle + 6\langle \kappa_1^2 \rangle \langle \kappa_1 \rangle^2 - 3\langle \kappa_1 \rangle^4 \\ + 6(\langle \kappa_2 \kappa_1^2 \rangle - \langle \kappa_2 \rangle \langle \kappa_1^2 \rangle - 2\langle \kappa_2 \kappa_1 \rangle \langle \kappa_1 \rangle) \\ + 2\langle \kappa_2 \rangle \langle \kappa_1 \rangle^2 + 3(\langle \kappa_2^2 \rangle - \langle \kappa_2 \rangle^2),$$

## Detailed structure of skewness and kurtosis



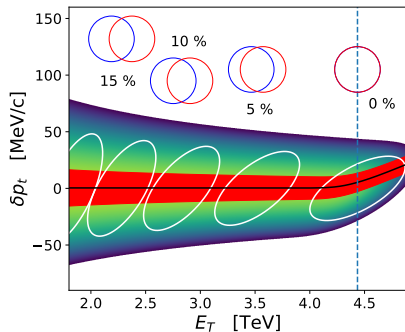
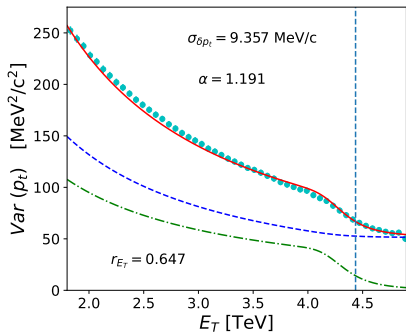
## b-dependence of the fit parameters

- We assume mean  $\langle p_t \rangle$  to be independent of  $b$
- We assume  $\text{Var}(\langle p_t \rangle)$  is a smooth function of mean multiplicity :

$$\sigma_{p_t}^2 \left( \frac{\langle N_{ch}(0) \rangle}{\langle N_{ch}(b) \rangle} \right)$$

- We also assume  $r$  to be independent of  $b$  for simplicity

# $E_T$ -dependent $[p_t]$ -fluctuation

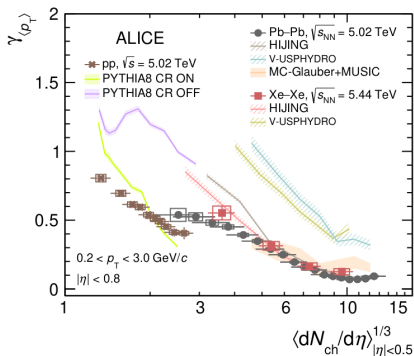


Impact parameter fluctuation is small !

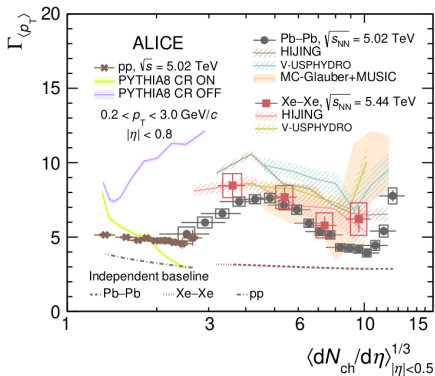


# ALICE Measurements of $[p_t]$ -skewness !

arXiv: 2308.16217



standardized  $[p_t]$ -skewness



intensive  $[p_t]$ -skewness