Equilibrium and Non-Equilibrium Dynamics in uRHIC

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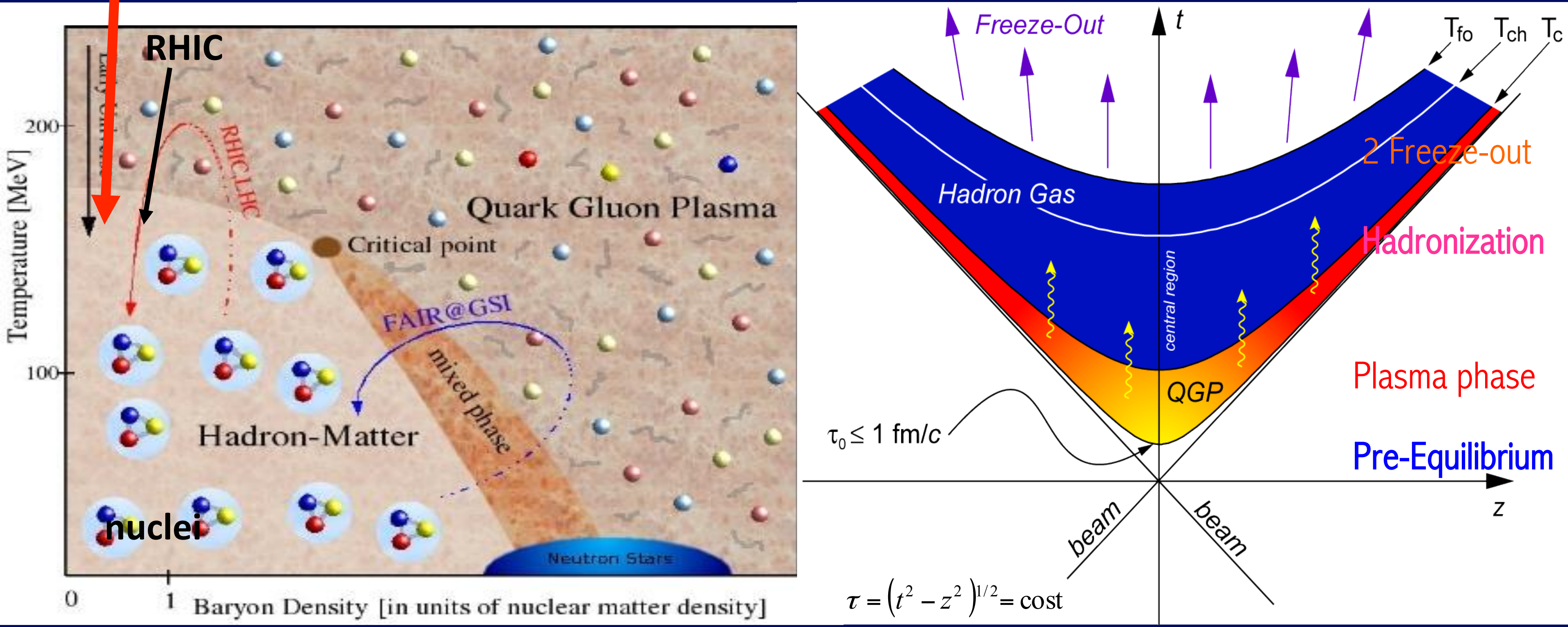
Outline

- Transport Kinetic Theory at fixed η/s :
  - Motivations and how to fix locally η/s
  - Viscous correction δf to $f(x,p)$

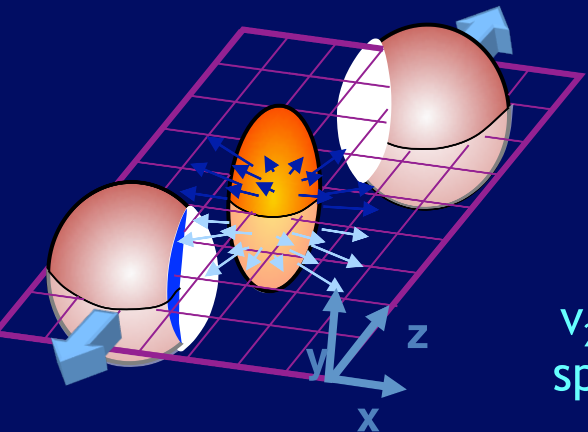
- Some results for HIC:
  - Hydro-like (equilibrium) study of $v_n(p_T)$:
    - Min. bias vs Ultra-central collisions: RHIC vs LHC
  - Impact of non-equilibrium:
    - Color Glass Condensate p_T distribution with a Qs scale going beyond ε_x and implementing also the p-space
    - From an abelian chromomagnetic E-field with negative pressure (P_L<0) -> isotropic and thermalized system

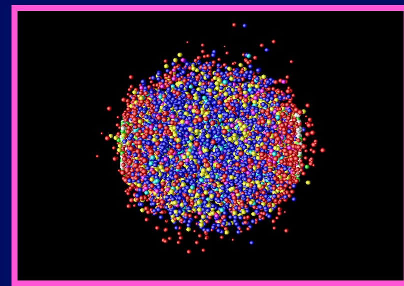
Matter from uRelativistic HIC



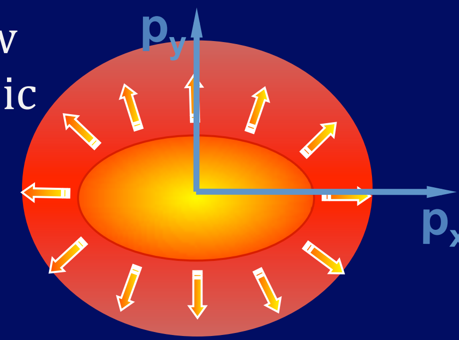
- Impact of pre-equilibrium not really known
- Initial state fluctuations are there not clear if they are those of MC-Glauber
- Impact of hadronization some hint, no self-consistent picture

Key observable: anisotropic flow ->shear viscosity η





Radial Flow
is anisotropic



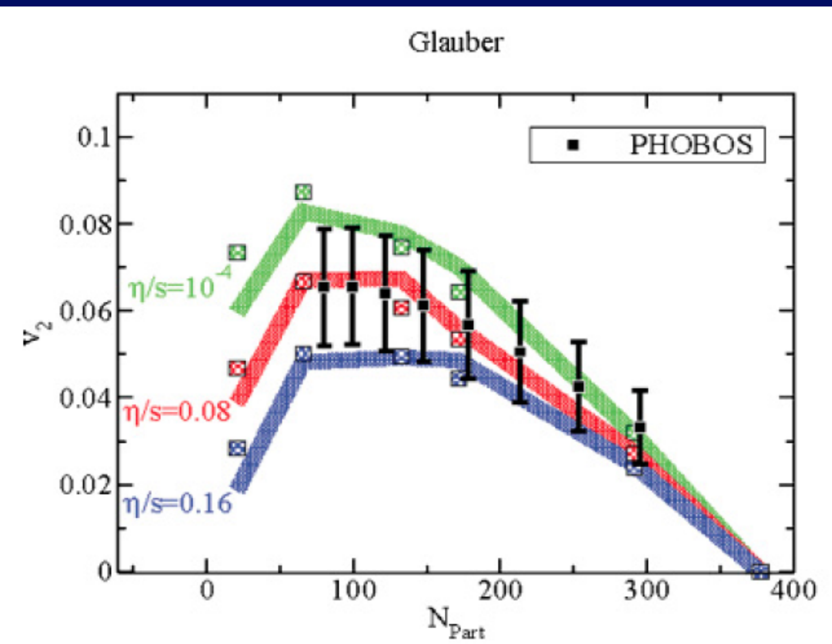
v_2/ε measures efficiency in converting space eccentricity to Momentum space

$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

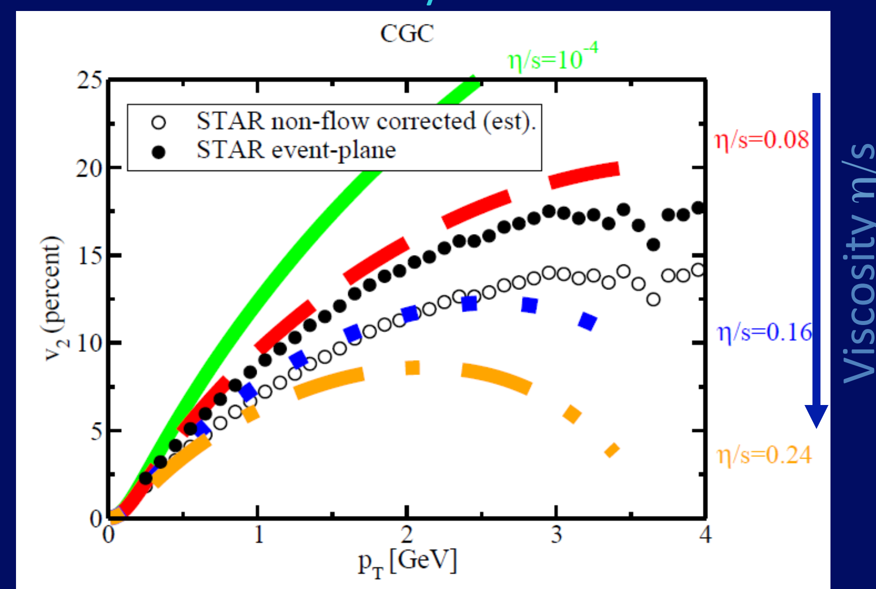
$\lambda = (\sigma\rho)^{-1}$ or η/s viscosity

EoS from lQCD

$$v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



One of the main uncertainty is the initial condition!



Viscous Hydrodynamics

Relativistic Navier-Stokes

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha)$$

but it violates causality,

II^0 order expansion needed -> Israel-Stewart

tensor based on entropy increase $\partial_\mu s^\mu > 0$

$$\pi^{\mu\nu} = \eta\nabla^{<\mu}u^{\nu>} + \tau_\pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta}\ldots\right]$$

-Dissipative correction to u^μ, T, n

-Dissipative correction to f -> $f_{eq}+\delta f_{neq}$

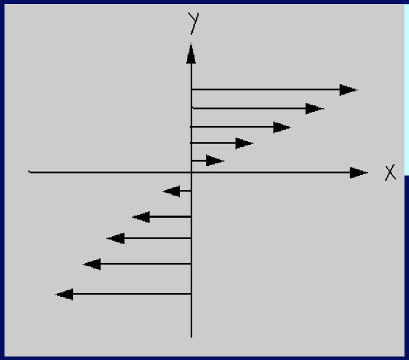
There is no one to one correspondence!

$$T^{\mu\nu}_{eq} + \delta T^{\mu\nu} \Leftarrow f_{eq} + \delta f$$

An Asantz

$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P}\frac{p_\mu p_\nu}{T^2} f_{eq}$$

- p_T~3 GeV -> $\delta f/f \approx$ 1-4
- $\Pi^{\mu\nu}(t_0) = 0$ -> discard initial non-eq (ex. minijets)
- Uncertainties from Cooper-Frye

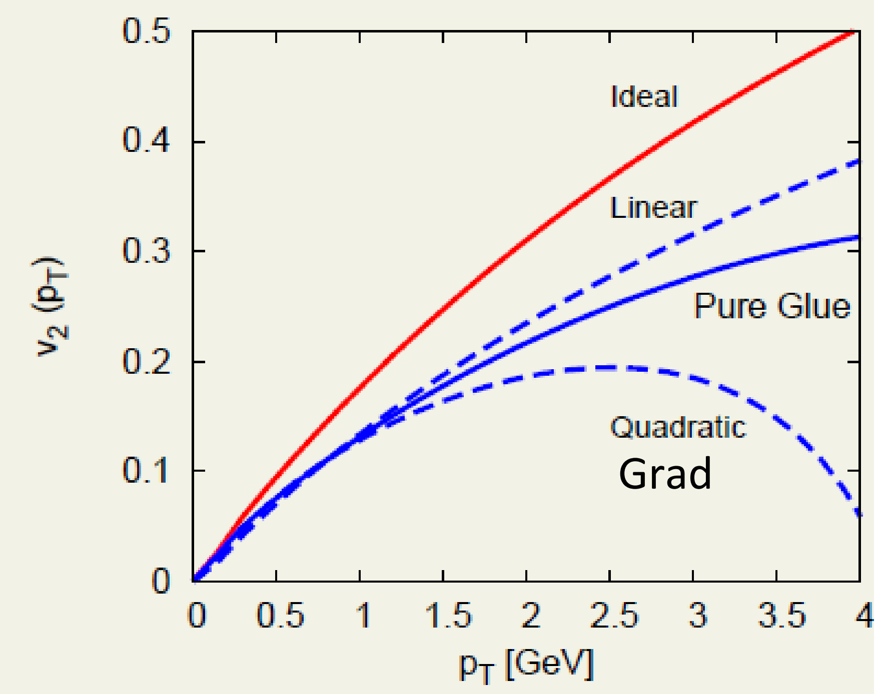


$$\frac{F_x}{A_{yz}} = -\eta\frac{\partial v_x}{\partial y}$$

τ_η, τ_ζ two parameters appears +

$\delta f \sim f_{eq}$ reduce the p_T validity range +

Full II° order has 11 transport coefficients



Relativistic Boltzmann-Vlasov approach

$$\left\{ p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^* \right]\partial^{p^*}_\mu \right\} f(x,p^*) = C[f]$$

Free streaming — Field Interaction (EoS) — Collisions -> $\eta \neq 0$

f(x,p) is the one-body distribution function

$$\begin{aligned} \mathcal{C}_{22} = & \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2) \\ & -\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2) \end{aligned}$$

- $C[f_{eq}+\delta f] \neq 0$ deviation from ideal hydro (finite λ or η/s)
- We map with C[f] the phase space evolution of a fluid at fixed η/s !

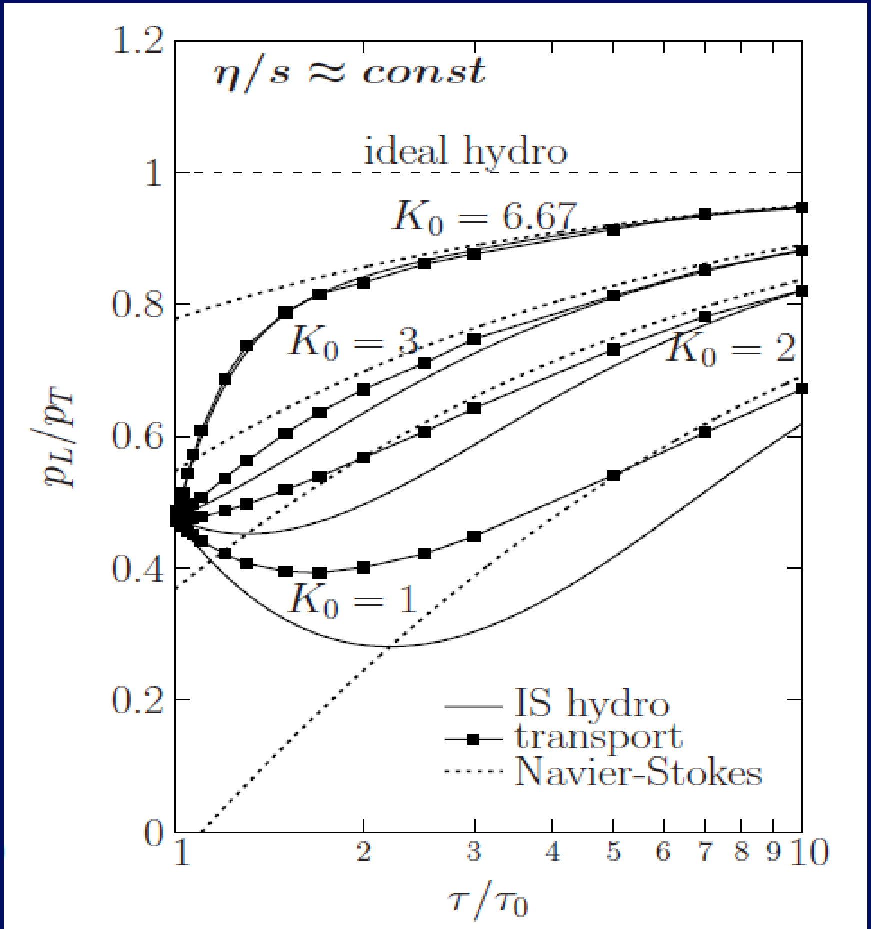
One can expand over microscopic details (2<->2,2<->3…), but in a hydro language this is irrelevant only the global dissipative effect of C[f] is important!

In fact expanding C[f] one gets viscous hydordynamics: Denicol, Rischke,…

Transport at fixed η/s vs Viscous Hydro in 1+1D

Comparison for the relaxation of pressure anisotropy P_L/P_T

Huovinen and Molnar, PRC79(2009)



Knudsen number[-1]

$$K = \frac{L}{\lambda} \rightarrow \frac{\tau}{\lambda}$$

Large K small η/s

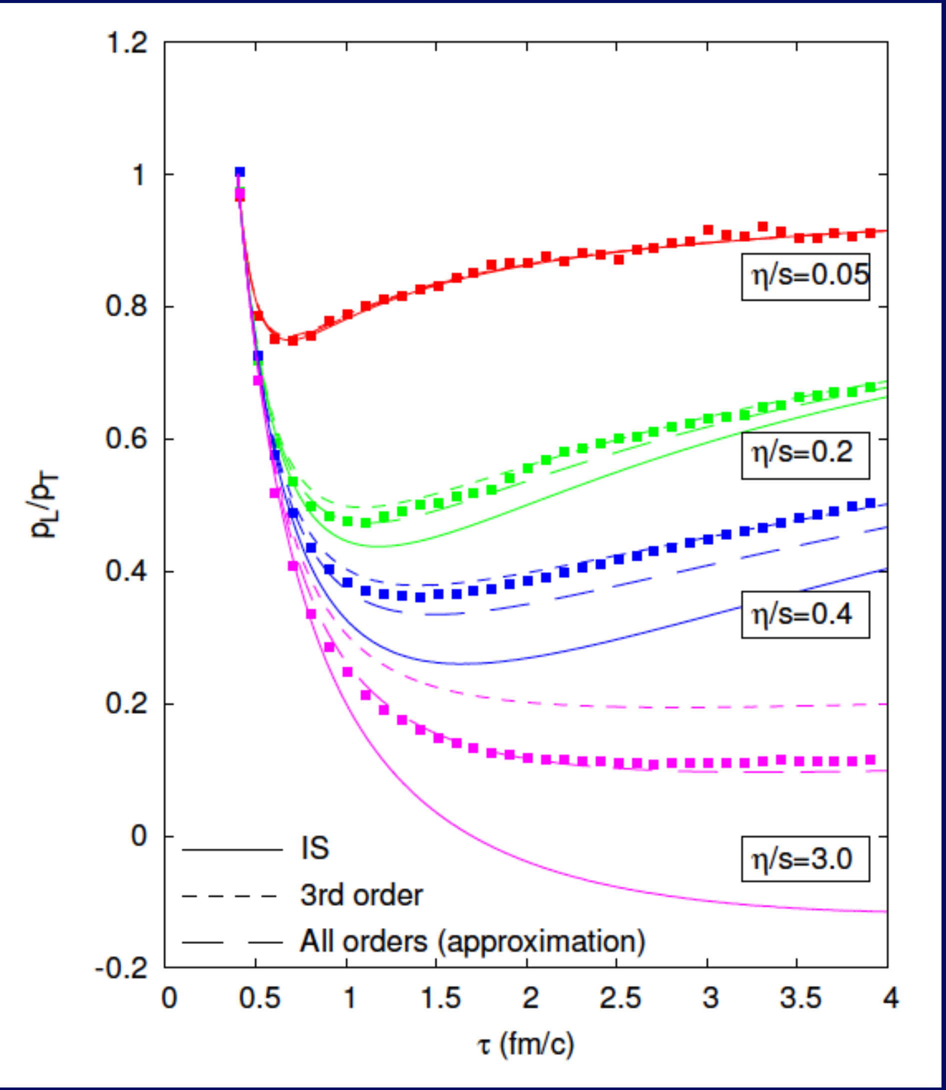
$$K_0 = \frac{1}{5}\frac{T_0 \tau_0}{\eta / s}$$

$$\frac{\eta}{s} = \frac{1}{5} T \cdot \lambda$$

In the limit of small η/s (<0.16) transport converge to viscous hydro at least for the evolution P_L/P_T

Denicol et al. have studied derivation of viscous hydro from Boltzmann kinetic theory: PRD85 (2012) 114047

Similar results from BAMPS-Frankfurt



El, Xu, Greiner, Phys.Rev. C81 (2010) 041901

- Convergency for small η/s of Boltzmann transport at fixed η/s with viscous hydro

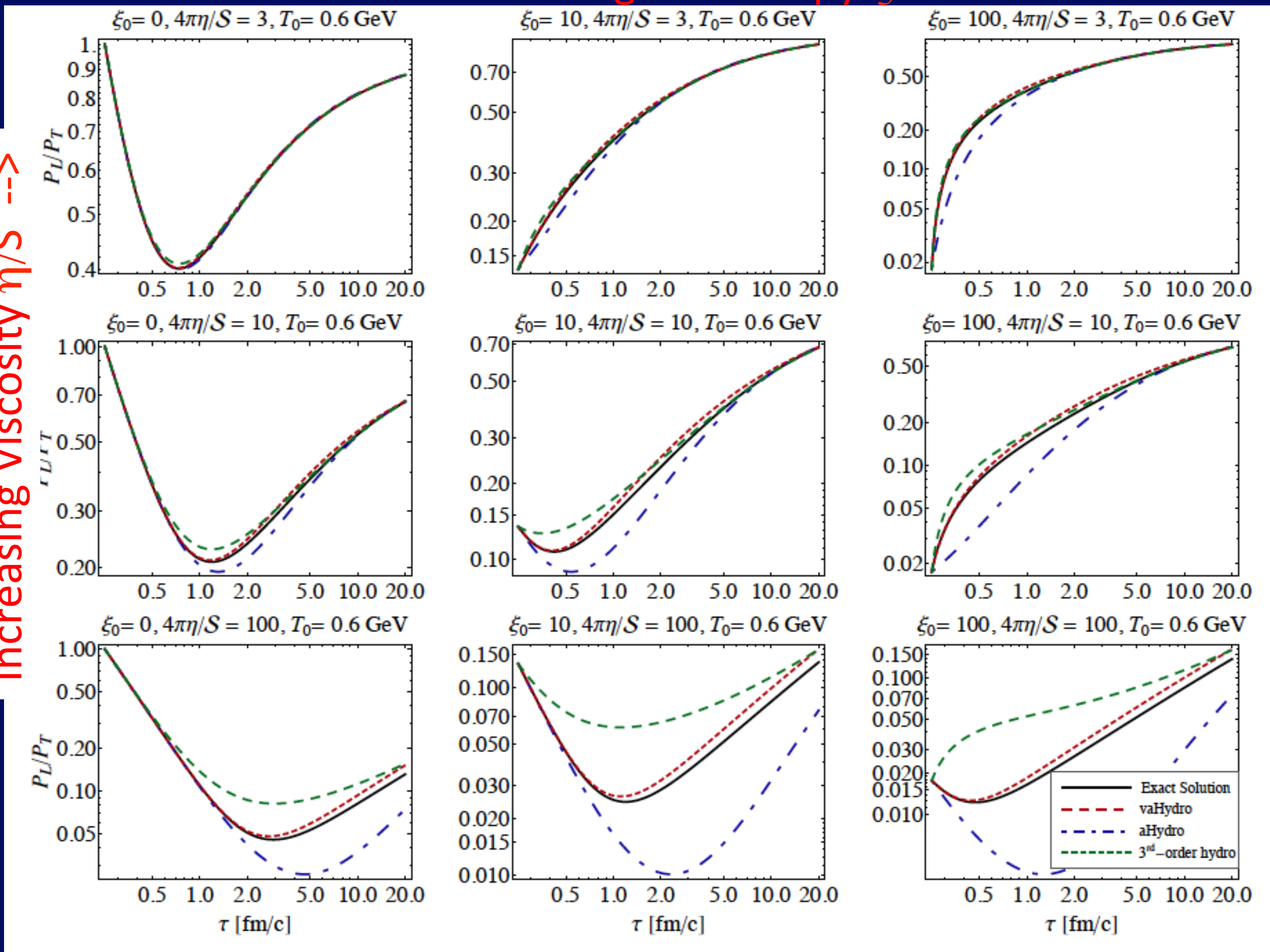
- Better agreement with 3rd order viscous hydro for large η/s

Test of vaHydro in 0+1 D –Heinz, Strickland

Use Boltzmann at fixed η/s in 1+1D to improve viscous hydro – U. Heinz (HP2015)

Increasing Anisotropy ξ -->

Increasing Viscosity η/S -->



$$f_0\left(\frac{\sqrt{\mathbf{p}_\perp^2+(1+\xi)p_z^2+m^2}}{\Lambda};\frac{\mu}{\Lambda}\right)$$

ξ long. anisotropy param.

Exact Solution means Boltzmann Eq.

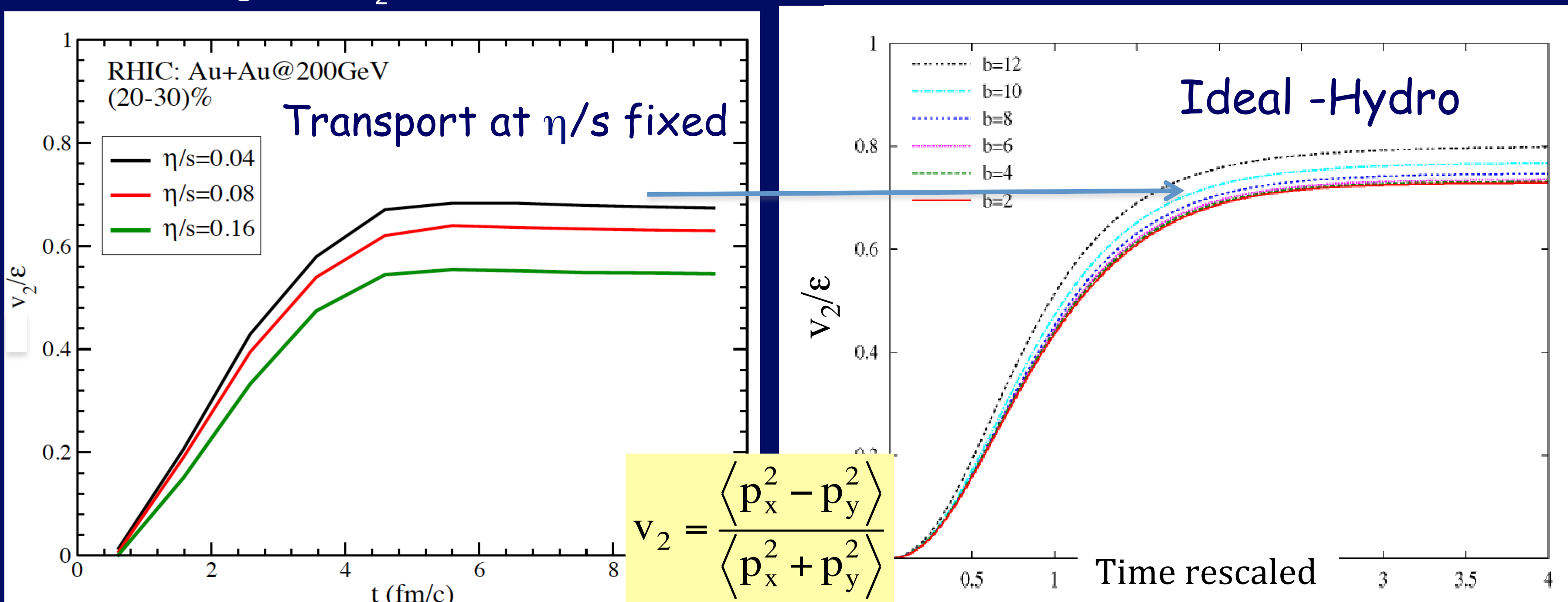
Bazow, Strickland, Heinz: arXiv:1311.6720

See also in 1+1D: Denicol et al., PRL(2014)

Test in 3+1D: v_2/ε response for almost ideal case

EoS $c_s^2=1/3$ (dN/dy tuned to RHIC)

Integrated v_2 vs time



$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

Bhalerao et al., PLB627(2005)

In the bulk the transport has an hydro v_2/ε_2 response!

Motivation for Transport approach

$$\left\{ p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^* \right] \partial^{p^*}_\mu \right\} f(x, p^*) = C[f]$$

Free streaming — Field Interaction (EoS) — Collisions -> $\eta \neq 0$

- Starting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:
  - f(x,p) out-of-equilibrium: CGC-Qs scale (beyond ε_x)
    M. Ruggieri et al., PLB727(2013)177, PRC90(2014)
  - Extract viscous correction δf to f(x,p)
    S. Plumari et al., NPA941(2015)
  - Relevant at LHC due to large amount of minijet production (high p_T)
  - Freeze-out self-consistently related to $\eta/s(T)$
- It's not a gradient expansion η/s:
  - valid also at high η/s -> LHC ($T>>T_c$) or cross-over region ($T\approx T_c$)
- Appropriate for heavy quark dynamics [J.Aichelin's talk]
  S. Das et al., PRC89 (2014)

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DISADVANTAGES?!

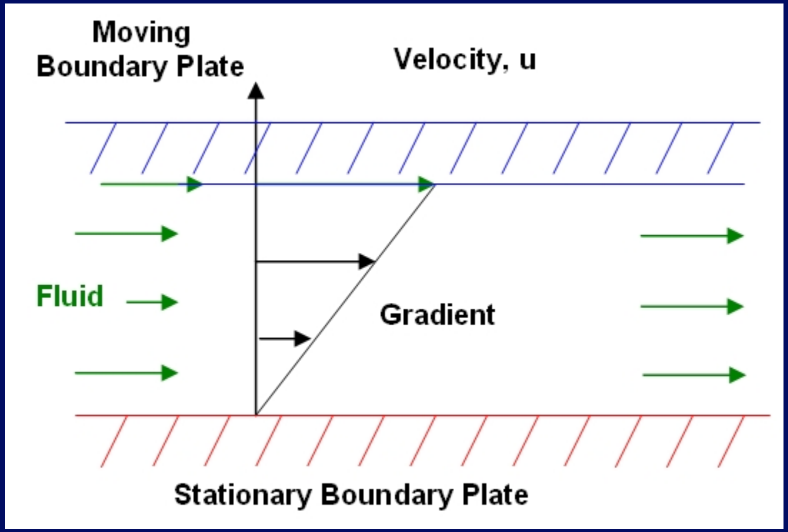
- Relaxation times fixed by kinetic theory
- Hadronization needed: coal.+frag. or SMF with CF

Part I – Kinetic Theory at fixed η/s

Instead of starting from *cross-sections and fields*,
we reverse the process starting from η/s

What is the relation η <-> σ, dσ/dΘ, M, T, ρ?

- Check η with the Green-Kubo correlator



$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$

$$\eta / s \cong \frac{1}{15} \frac{<p>}{\sigma \rho}$$?

Shear Viscosity in Box Calculation

Green-Kubo correlator

$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \left\langle \Pi^{xy}(\vec{x},t)\,\Pi^{xy}(0,0)\right\rangle$$

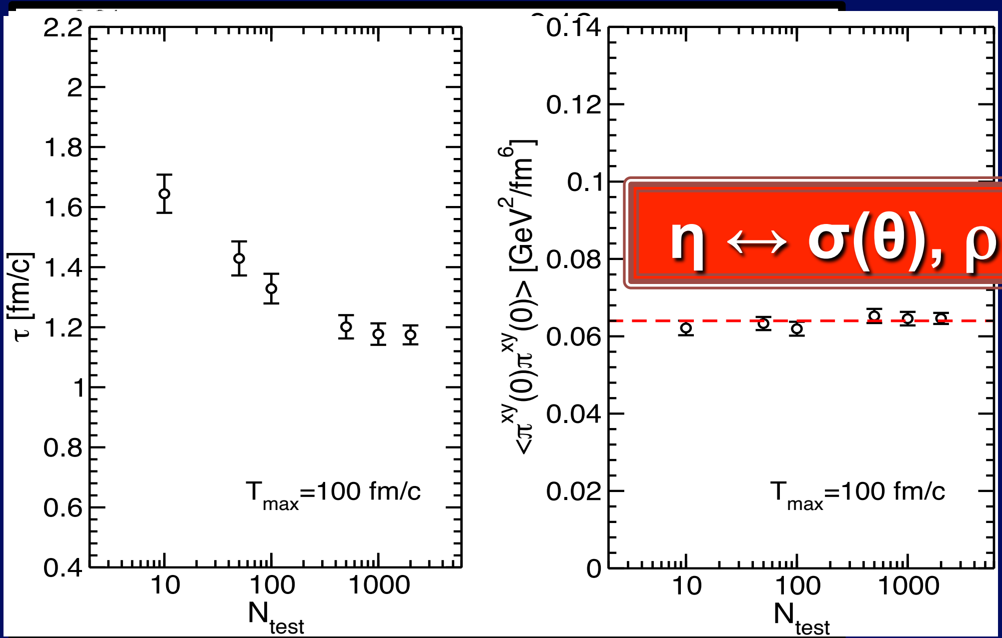
$$\left\langle \Pi^{xy}(\vec{x},t)\Pi^{xy}(0,0)\right\rangle = \left\langle \Pi^{xy}(0,0)\Pi^{xy}(0,0)\right\rangle \cdot e^{-t/\tau}$$

$$\eta = \frac{V}{T}\left\langle \pi^{xy}(0)\,\pi^{xy}(0)\right\rangle \tau$$

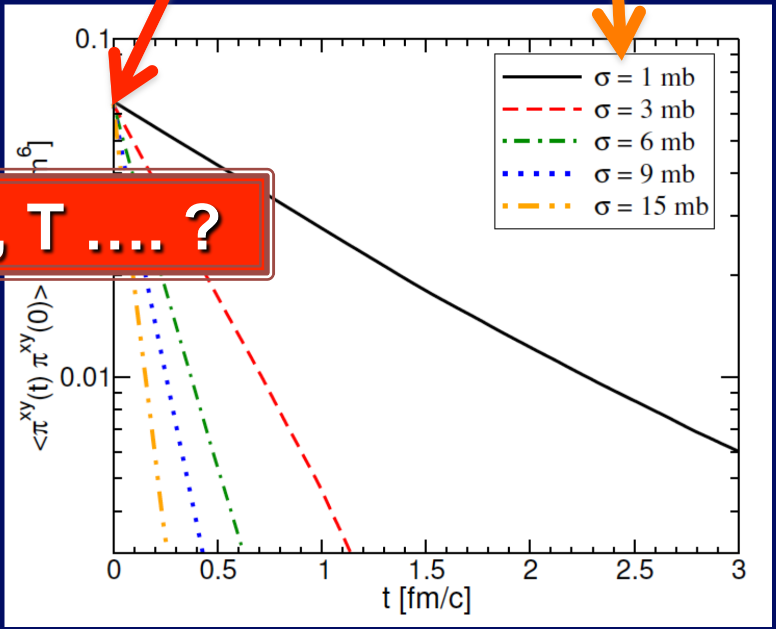
microscopic details

macroscopic observables

$$= \frac{4}{15}\frac{\varepsilon T}{V}$$



η ↔ σ(θ), ρ, M, T …. ?



S. Plumari et al., PRC86(2012);see also:
Wesp et al., Phys. Rev. C 84, 054911 (2011);
Fuini III et al. J. Phys. G38, 015004 (2011).

Needed very careful tests of convergency vs. N_{test}, Dx_{cell}, # time steps !

Non Isotropic Cross Section - $\sigma(\theta)$

Relaxation Time Approximation

$$\eta_{RTA}/s = \frac{1}{15} < p > \tau_{tr} = \frac{1}{15} \frac{< p >}{\langle h(a) \rangle \sigma_{TOT} \rho}$$

$$h(a) = 4a(1+a)\left[(2a+1)\ln(1+a^{-1}) - 2\right] \quad , a = m_D^2 / s$$

$h(a)=\sigma_{tr}/\sigma_{tot}$ weights cross section by q^2

Chapmann-Enskog (CE)

$$\eta/s = \frac{1}{15}\langle p \rangle \tau_\eta = \frac{1}{15}\frac{\langle p \rangle}{g(a)\sigma_{tot}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

g(a) correct function that fix the momentum transfer for shear motion

RTA is the one usually employed to make theroethical estimates: Gavin NPA(1985); Kapusta, PRC82(10); Redlich and Sasaki, PRC79(10), NPA832(10); Khvorostukhin PRC (2010) …

for a generic cross section:

$$\frac{d\sigma}{d\Omega} \propto \left(q^2(\theta) + m_D^2\right)^{-2}$$

m_D regulates the angular dependence

S. Plumari et al., PRC86(2012)054902

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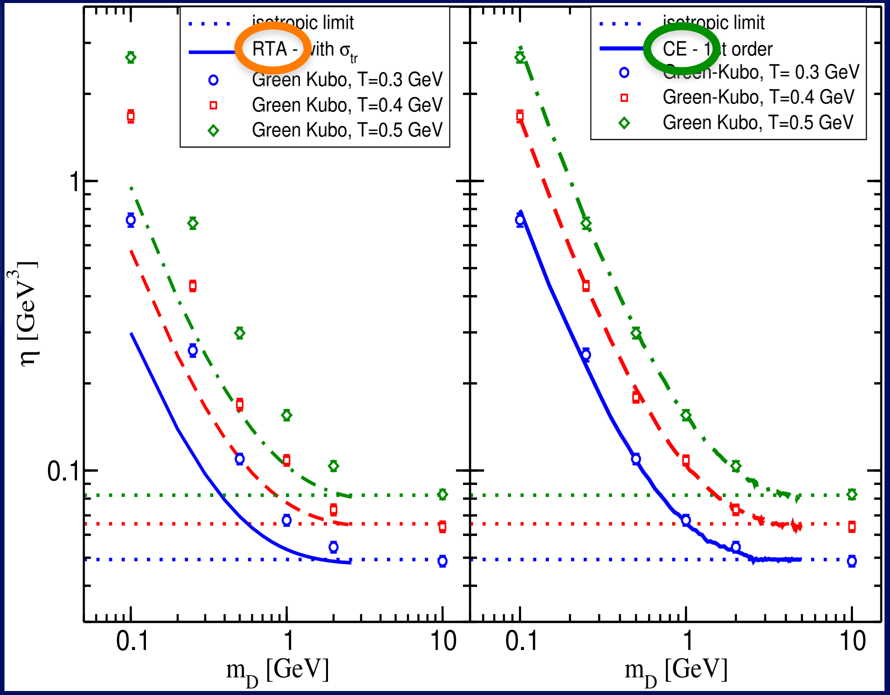
m_D regulates the angular dependence

- CE and RTA can differ by about a factor 2
- Green-Kubo agrees with CE

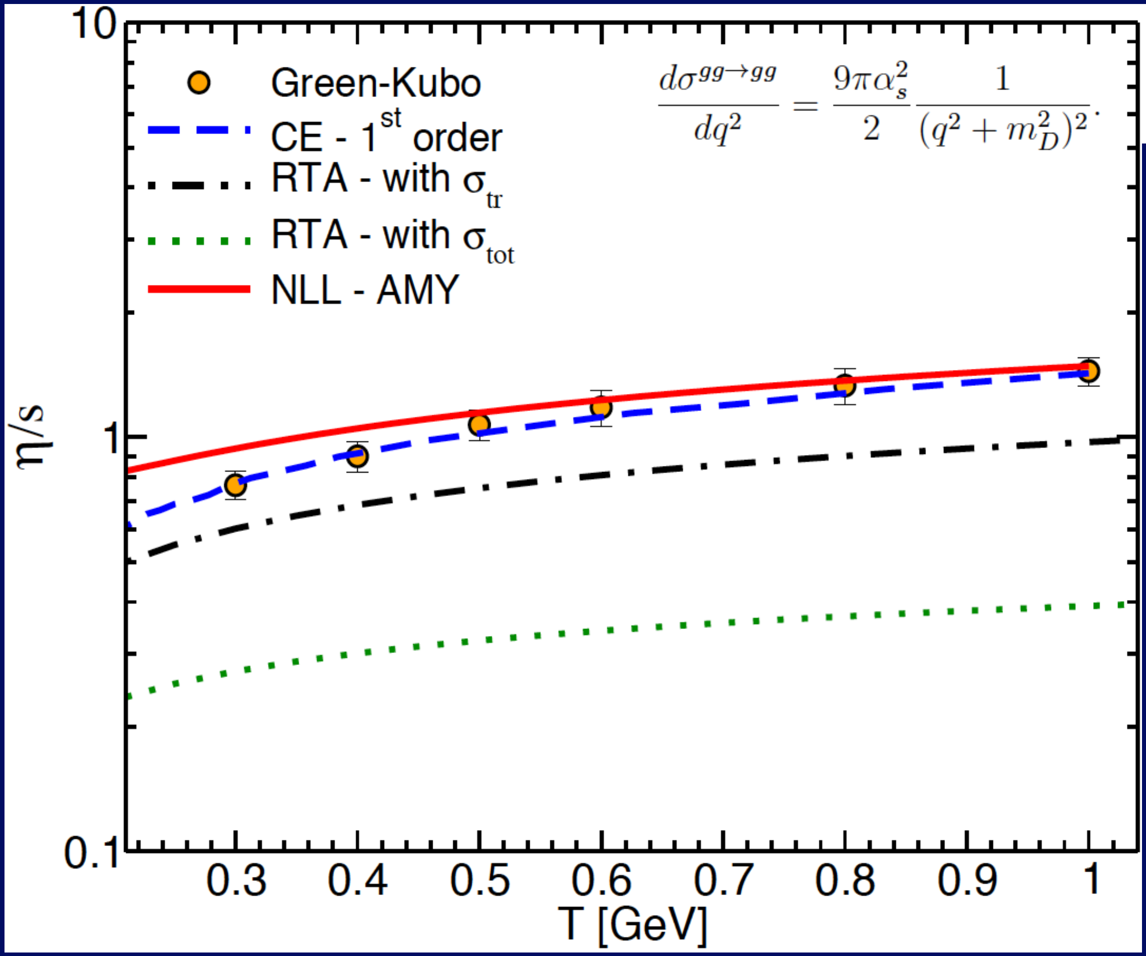
Green-Kubo in a box - σ(θ)



S. Plumari et al., PRC86(2012)054902

Viscosity of a pQCD gluon plasma

Agreement with AMY, JHEP 0305 (2003) 051



$$\alpha_s(T) = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)^2}$$

$$m_D = T\sqrt{4\pi\alpha_s}$$

Relaxation Time Approximation

$$\eta_{RTA}/s = \frac{1}{15}<p>\tau_{tr} = \frac{1}{15}\frac{<p>}{\sigma_{tr}\rho}$$

Chapmann-Enskog (CE)

$$\eta/s = \frac{1}{15}\langle p\rangle\,\tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(a)\sigma_{tot}\rho}$$

$g(a=m_D/2T)$ correct function that fix the momentum transfer for shear motion

$$0 < g(m_D/2T) < 2/3$$

∞ forward peaked | Isotropic $m_D \to \infty$

close to AMY result JHEP(2003),
but there is a significant simplification:
only direct u & t channels
with simplified HTL propagator

Simulate a fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$g(a=m_D/2T)$ correct function that fix the relaxation time for the shear motion

Transport code

$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha\rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

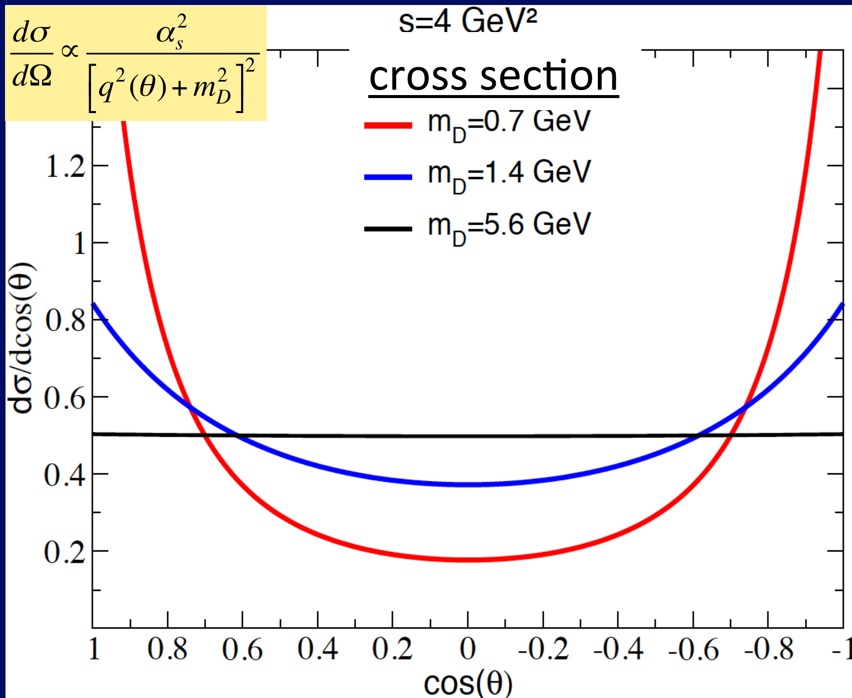
Space-Time dependent cross section evaluated locally

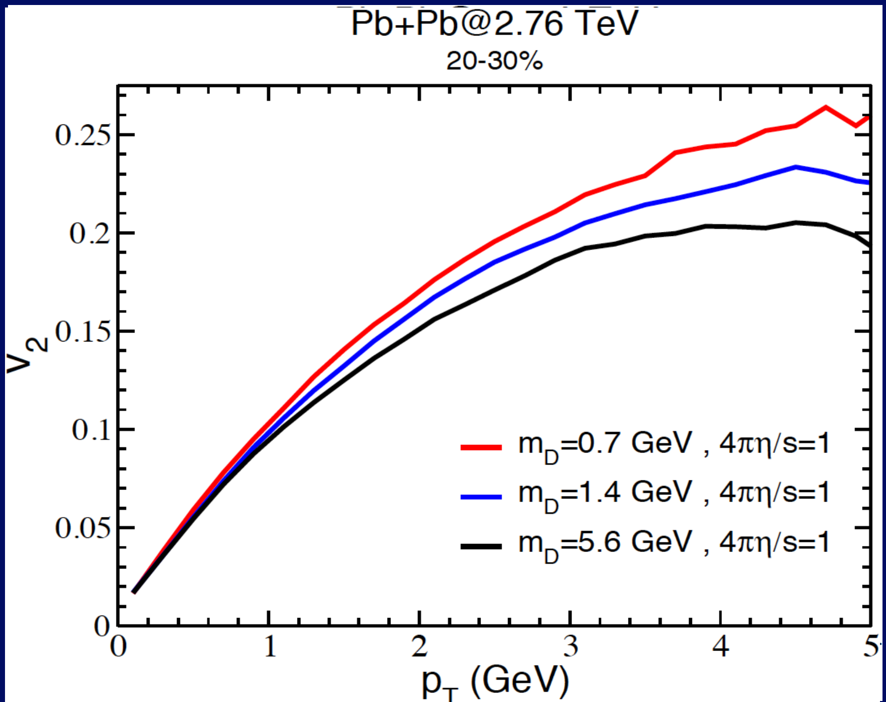
α=cell index in r-space

Viscosity fixed varying σ

G. Ferini et al., PLB670 (2009)
S. Plumari et al., PRC86(2012)

η/s or details of the cross section?





Keep same η/s means:

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta \qquad \tau_\eta^{-1} = g\left(\frac{m_D}{T}\right)\sigma_{TOT}\rho$$

$$\frac{\sigma_{TOT}(m_{1D})}{\sigma_{TOT}(m_{2D})} = \frac{g(m_{2D})}{g(m_{1D})}$$

for m_D=0.7 GeV -> factor 2 larger σ_{tot} is needed respect to isotropic case

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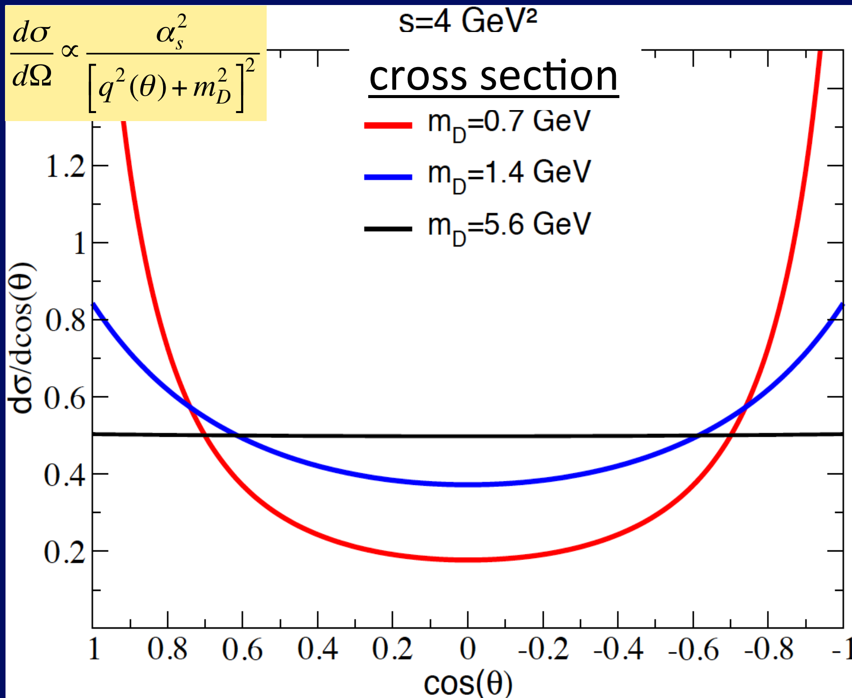

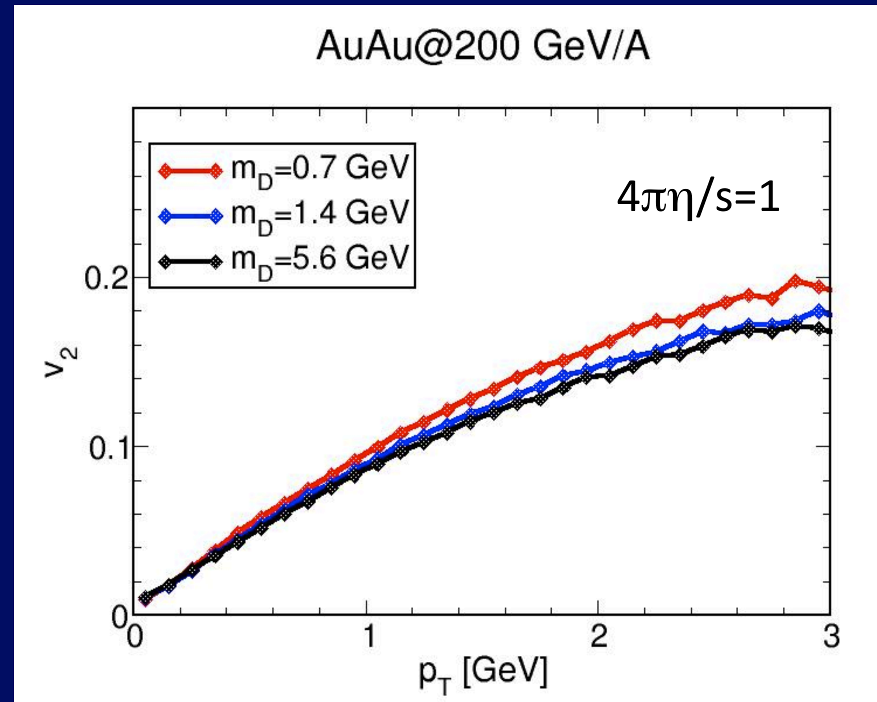

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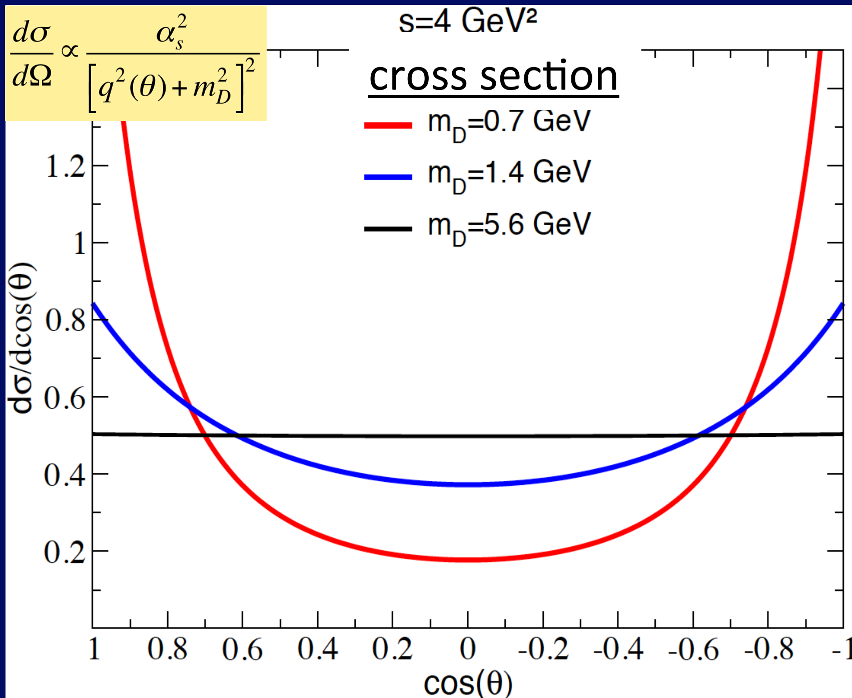
$$\tau_\eta^{-1}$$

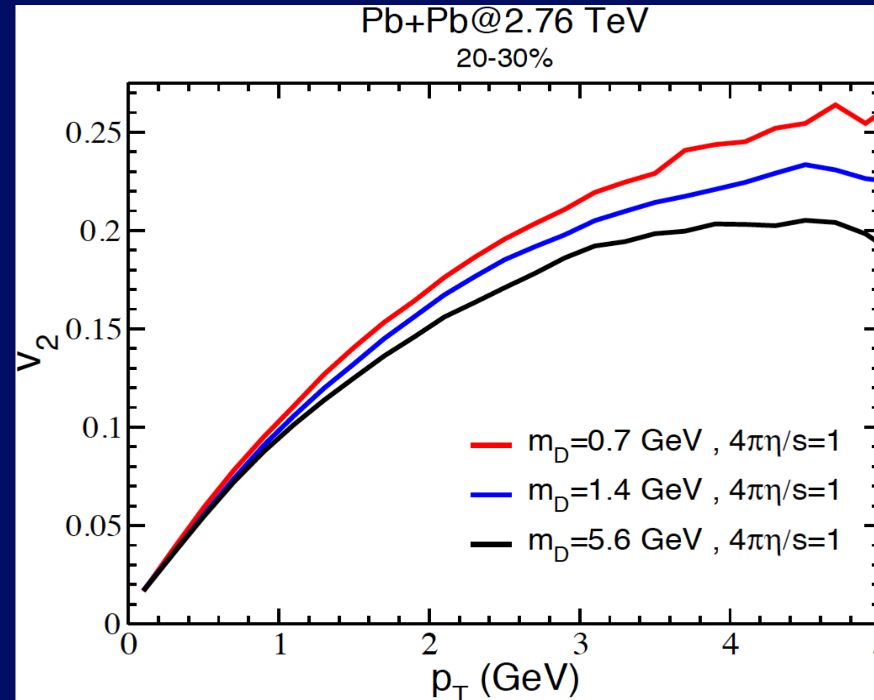
$$\frac{\sigma_{TOT}(m_{1D})}{\sigma_{TOT}(m_{2D})} = \frac{g(m_{2D})}{g(m_{1D})}$$

- ✧ η/s is really the physical parameter determining v_2 at least up to 1.5-2 GeV
- ✧ microscopic details become relevant at higher p_T
- ✧ First time η/s<-> v_2 hypothesis is verified!

➡ for m_D=0.7 GeV -> factor 2 larger σ_{tot} is needed respect to isotropic case

η/s or details of the cross section?





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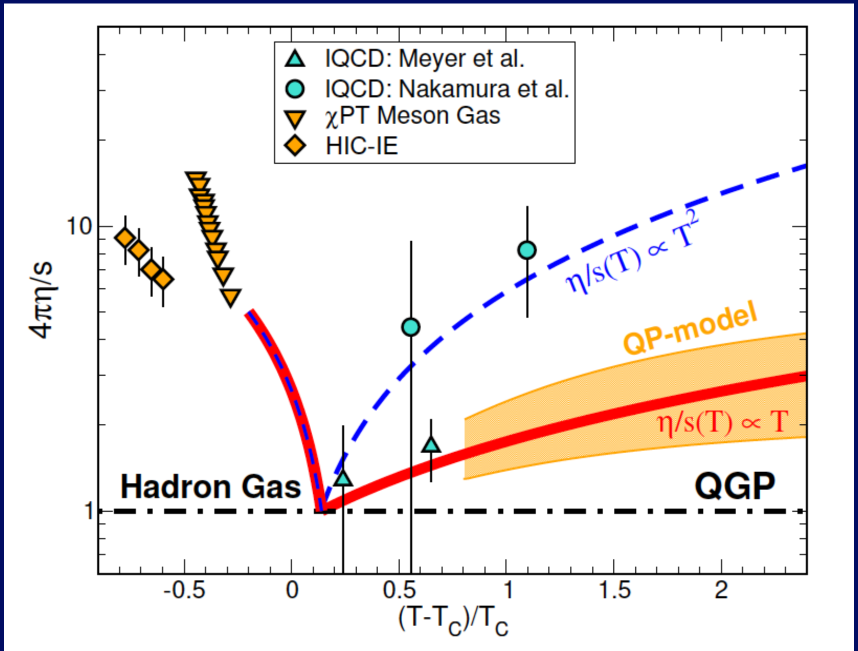
- ✧ η/s is really the physical parameter determining v_2 at least up to 1.5-2 GeV
- ✧ microscopic details become relevant at higher p_T
- ✧ First time η/s<-> v_2 hypothesis is verified!

➡ Differences arises just where in viscous hydro δf becomes relevant

$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P}\frac{p_\mu p_\nu}{T^2} f_{eq}$$

Cross section and freeze-out

Freeze-out is a smooth process: scattering rate < expansion rate

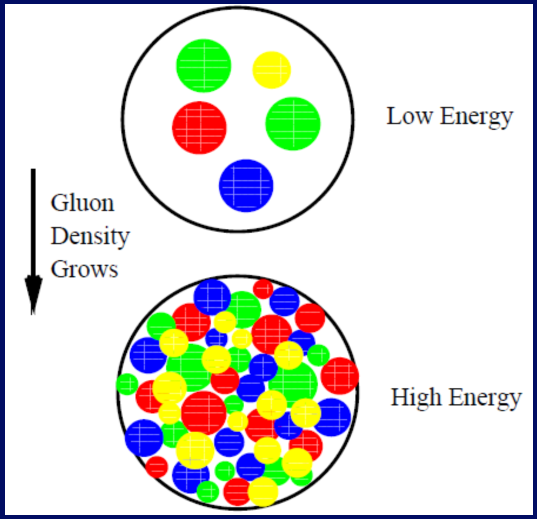


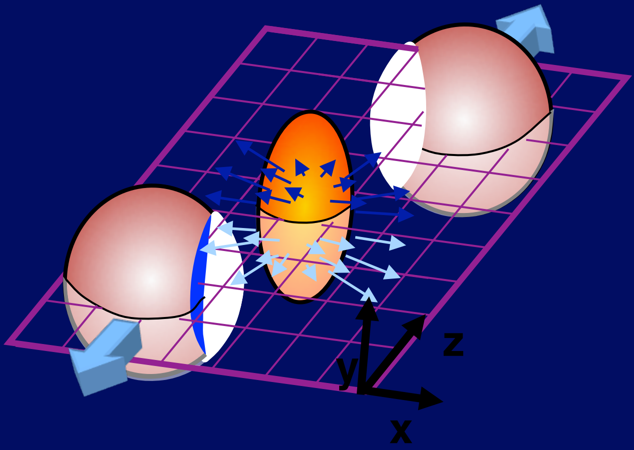
- ✓ η/s increases in the cross-over region, realizing a smooth f.o. self-consistently dependent on h/s:
- ✓ Different from hydro that is a sudden cut of expansion at some $T_{f.o}$. not related to η/s(T)

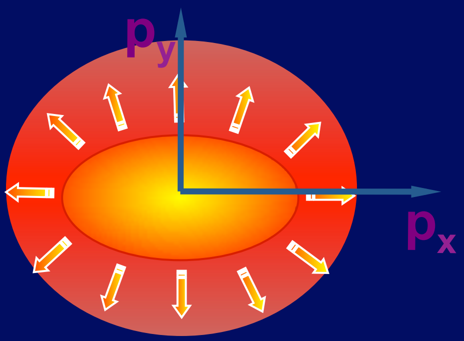
$$\sigma^* = g(a)\sigma_{tot} \approx \frac{1}{15}\frac{\bar{p}}{\rho}\frac{1}{\eta / s}$$

Part II - Transport at fixed η/s with Q_s saturation scale

What is the impact of non-equilibrium Color Glass condensate initial state?







fKLN realization of CGC

Factorization hypothesis: convolution of parton distribution functions in the parent nucleus.

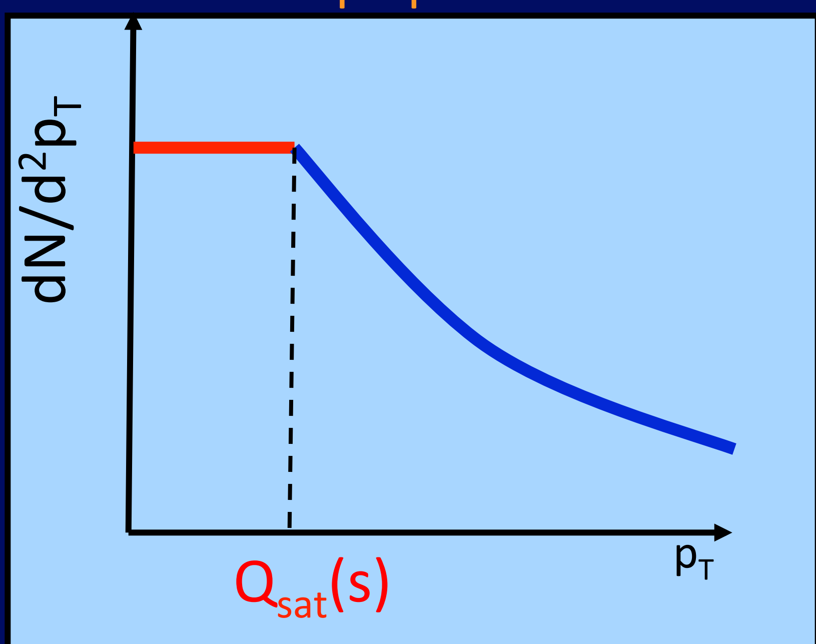
Unintegrated distribution functions (uGDFs)

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2} \int_0^{p_T} d^2k_T \alpha_s(Q^2) \times\phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times\phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

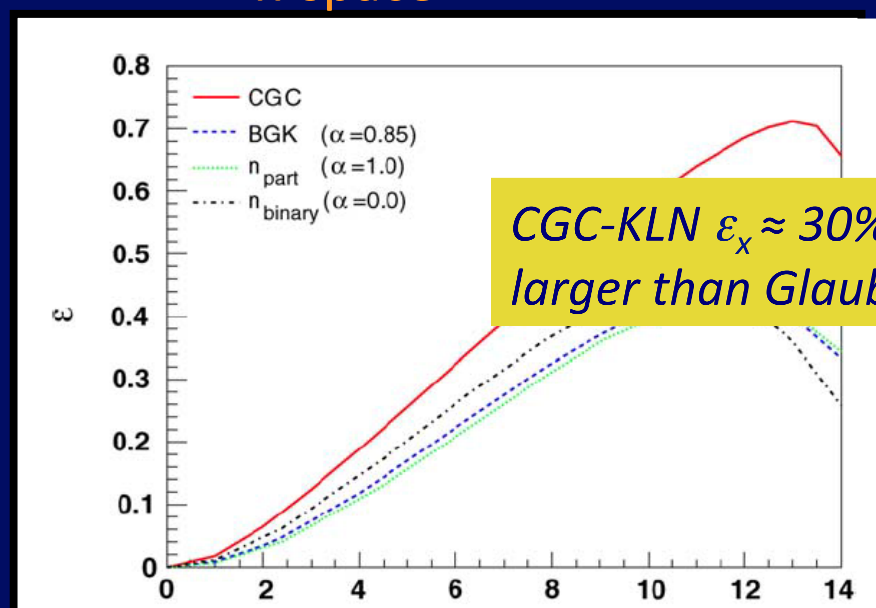
$$\phi_A(x_1, k_T^2; \boldsymbol{x}_\perp) = \frac{\kappa\, Q_s^2}{\alpha_s(Q_s^2)} \left[\frac{\theta(Q_s-k_T)}{Q_s^2+\Lambda^2} + \frac{\theta(k_T-Q_s)}{k_T^2+\Lambda^2}\right]$$

Kharzeev et al., PLB561, 93 (2003)
Nardi et al., PLB507, 121 (2001)
Drescher et al, PRC75, 034905 (2007)
Hirano et al., PRC79, 064904 (2009)
Albacete and Dumitru, arXiv:1011.5161
…

p-space



x-space



T. Hirano et al., PLB636(06)

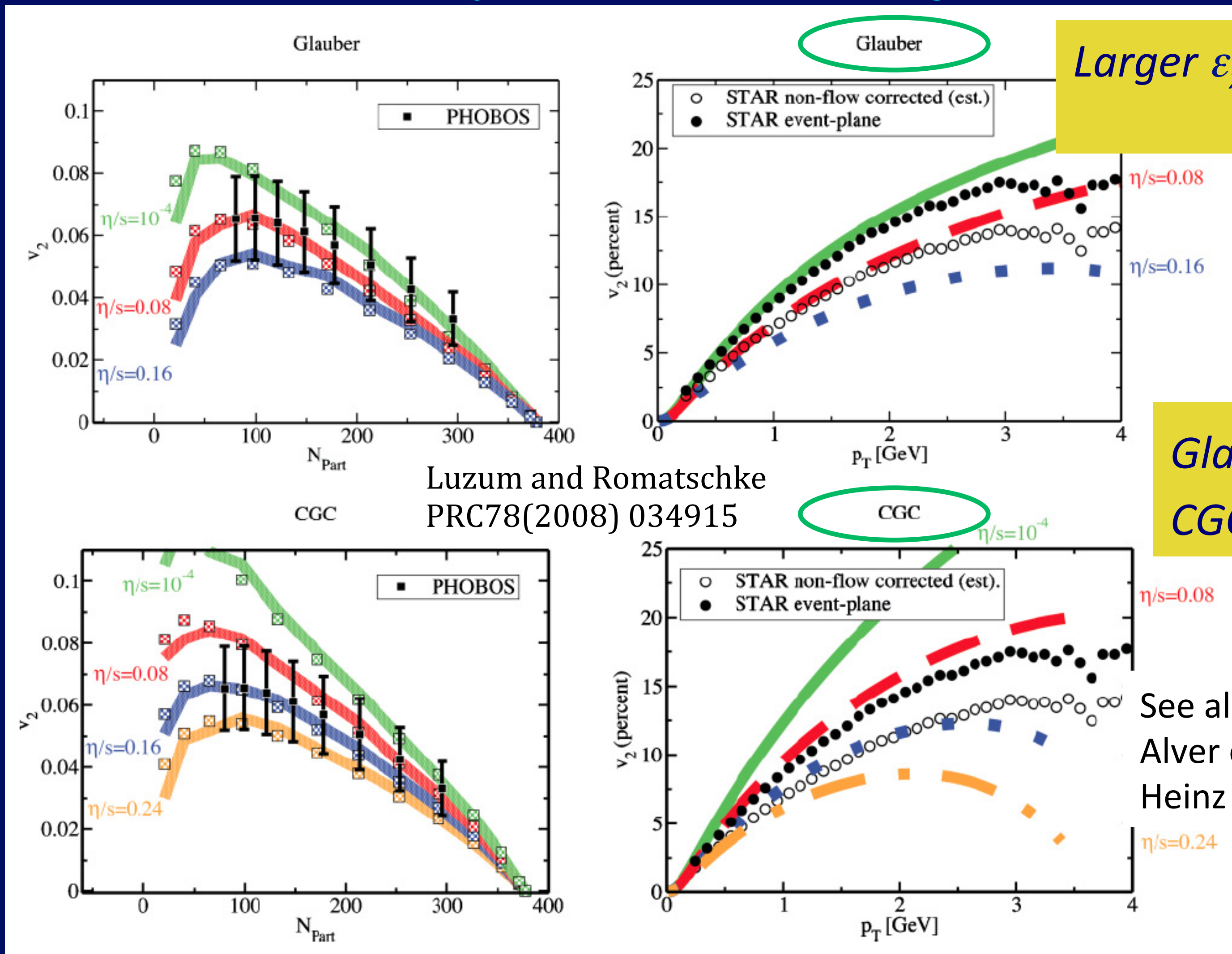
CGC-KLN $\varepsilon_x \approx$ 30% larger than Glauber

V_2 from KLN in Hydro

What means KLN in hydro?

1) r-space from KLN (larger ε_x)

2) p-space thermal at $t_0 \approx 0.6$-0.9 fm/c - No Q_s scale , We'll call it **fKLN-Th**



Luzum and Romatschke
PRC78(2008) 034915

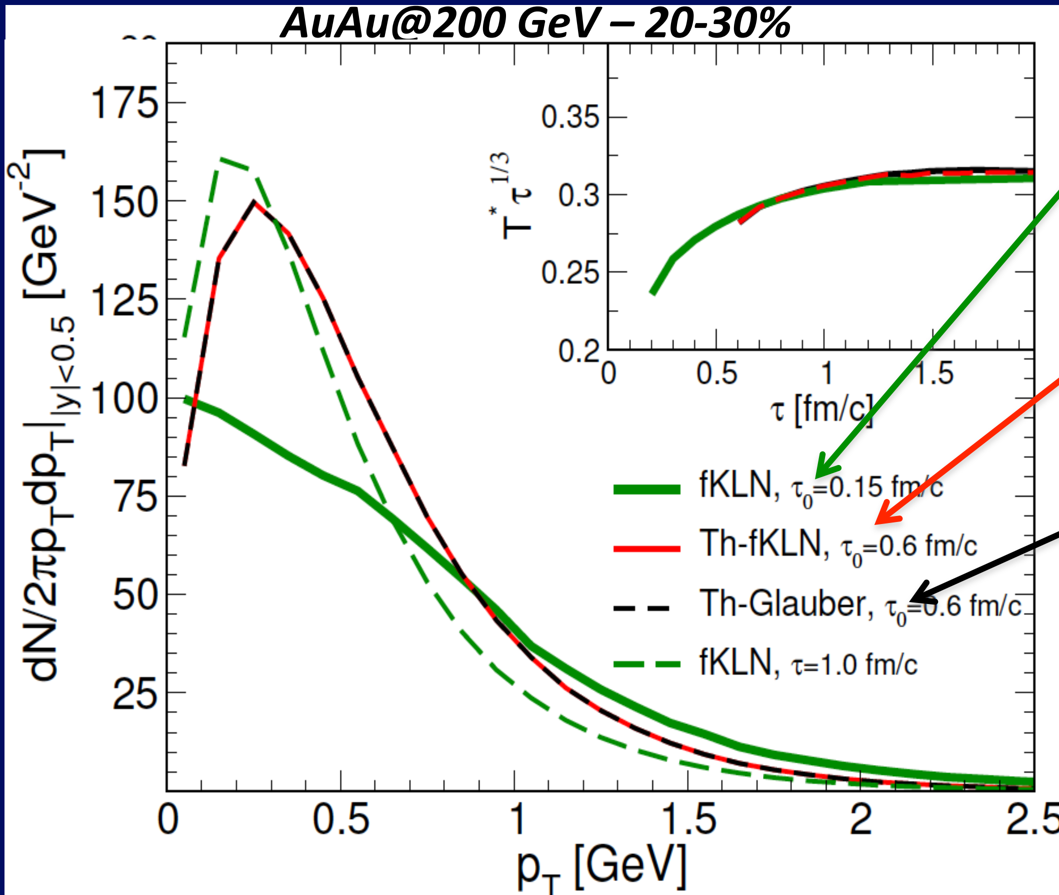
Larger ε_x - > higher η/s to get the same $v_2(p_T)$

Glauber → $\eta/s = 0.08$

CGC-KLN → $\eta/s=0.16$

See also:
Alver et al., PRC 82, 034913 (2010)
Heinz *et al.*, PRC 83, 054910 (2011)

Implementing KLN p_T distribution



Using kinetic theory
we can implement full KLN
(x & p space) - ε_x=0.34, Qs =1.4 GeV

KLN only in x space (like in Hydro)
ε_x=0.341, Qs=0 -> Th-KLN

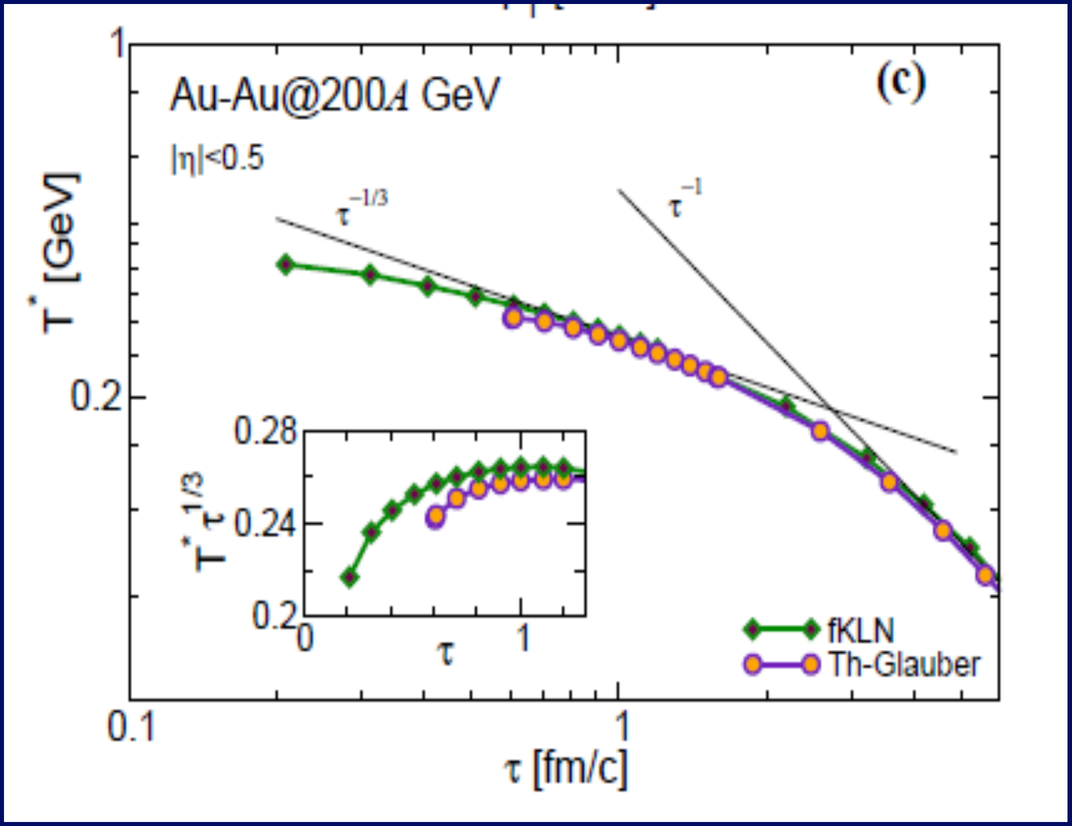
Glauber in x & thermal in p
ε_x=0.289 , Qs=0 -> Th-Glauber

M. Ruggieri *et al.*, Phys.Lett. B727 (2013) 177

Thermalization in less than 1 fm/c, in agreement with Greiner *et al.*, NPA**806**, 287 (2008).
Not so surprising: η/s is small -> large effective scattering rate -> fast thermalization.

$$\sigma_{tot} = \frac{\langle p \rangle}{\rho\, g(a)} \frac{1}{\eta/s}$$

Temperature evolution



$$T \propto \tau^{-\delta}$$

$\delta = P_L/\varepsilon$ – 1D boost invariance

$\delta = 1/3$ - 1D ideal expansion

$\delta = 1$ - 3D expansion

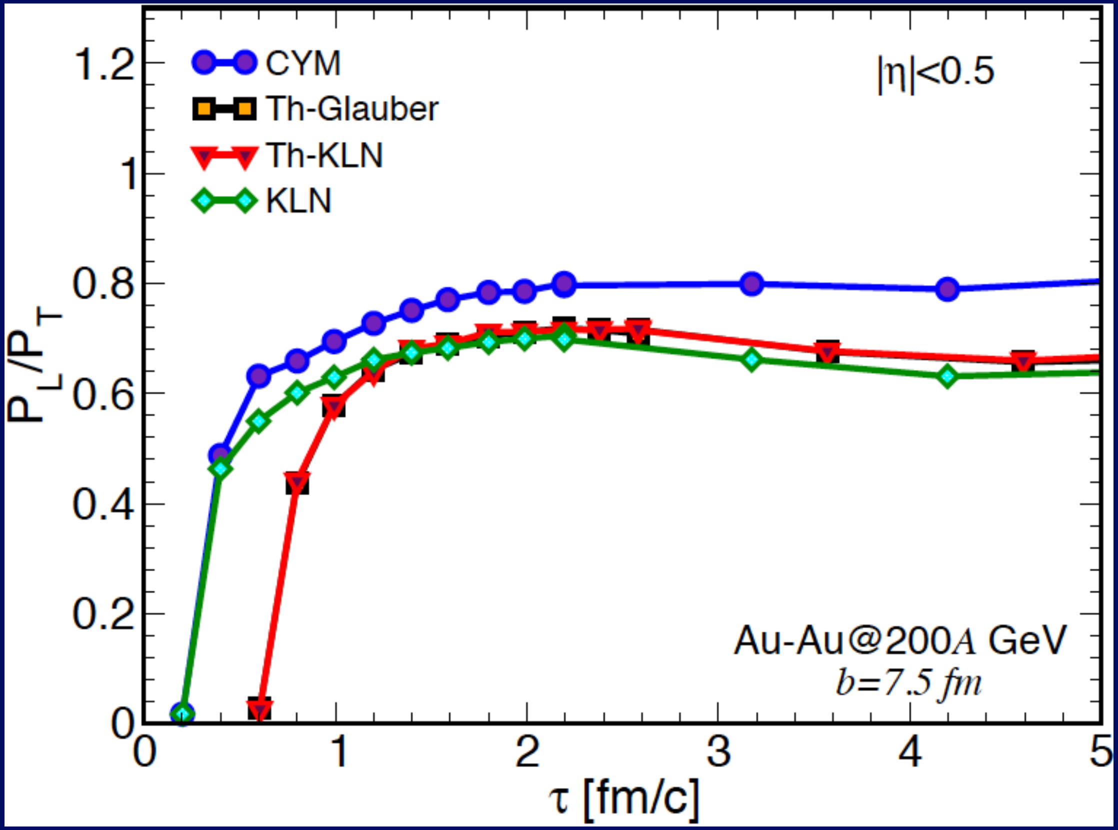
$\tau_{therm} \approx 0.8$ fm/c

M. Ruggieri et al.,PRC89 (2014) 5, 054914

T*=E/N , in the local rest frame

Longitudinal and transverse pressure

t=1/Qs≈0.1-0.2 fm/c
-> $P_L/P_T > 0$
Gelis & Epelbaum
arXiV:1307.2214

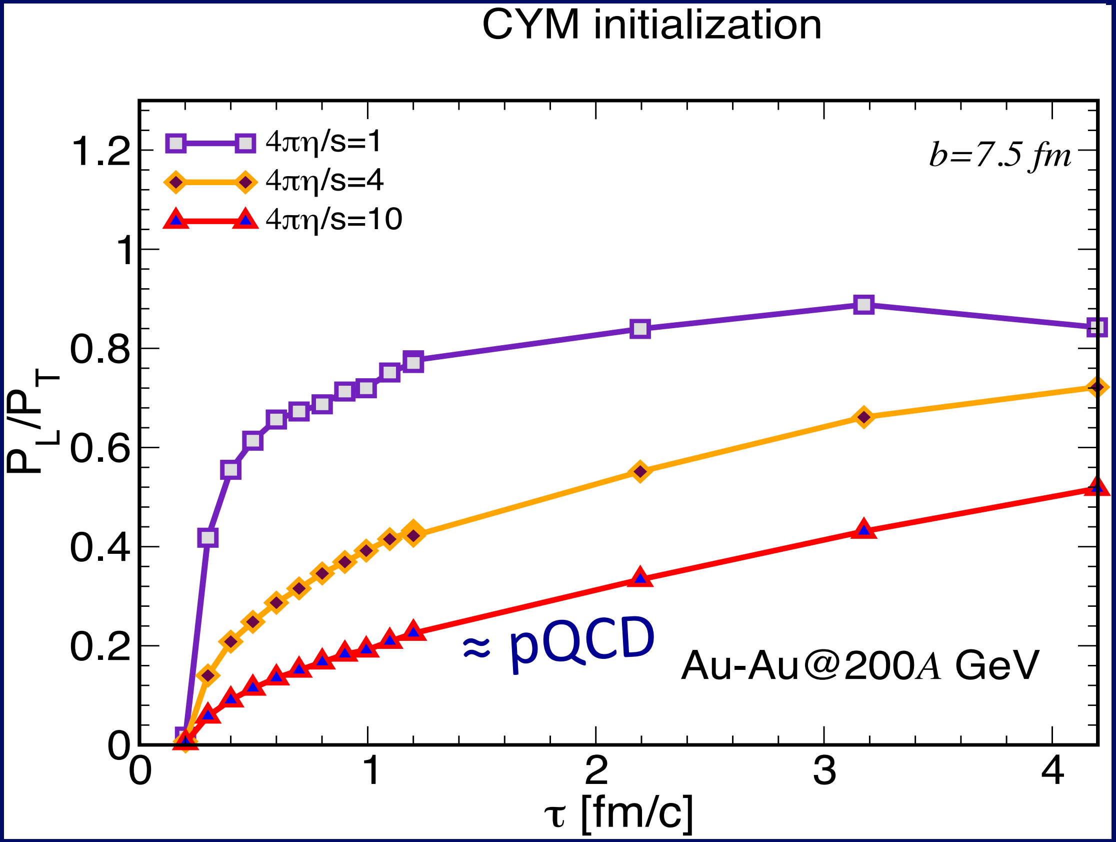


CYM (IP-Glasma)
Courtesy of B. Schenke
& R. Venugopalan

- ✧ P_L/P_T show also a very fast equilibration ($\Delta\tau_{isotr}$≈0.5 fm/c)!
- ✧ However it is not this that makes a difference for v_2:
  isotropization time quite similar for all the cases

M. Ruggieri et al.,PRC89 (2014) 5, 054914

Longitudinal and transverse pressure



70% level of isotropization

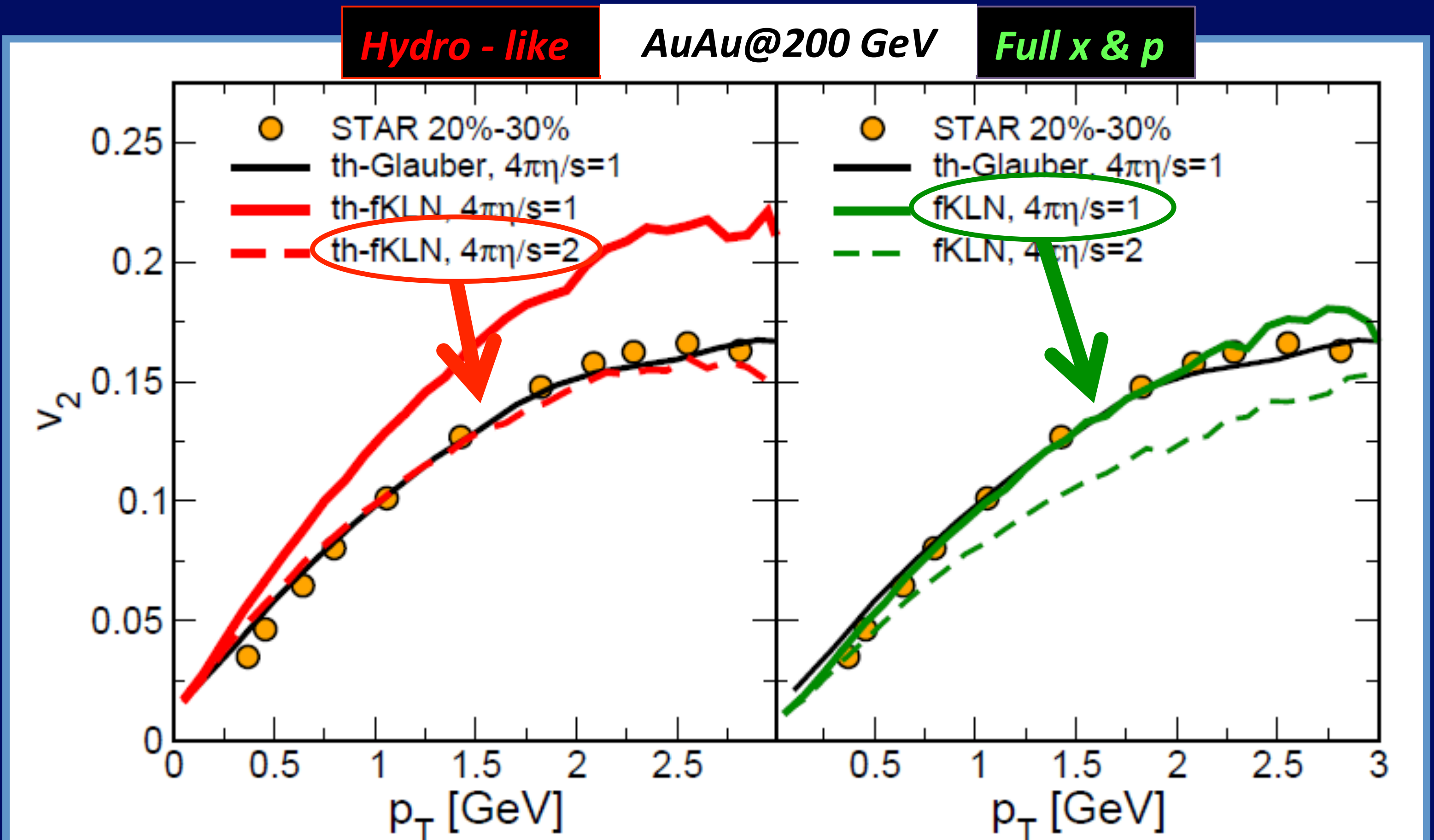
✧ For η/s > 0.3 one misses fast isotropization in P_L/P_T (τ = 2-3 fm/c)

✧ For η/s ≈ pQCD no isotropization

✧ Semi-quantitative agreement with Florkowski et al., PRD88 (2013) 034028

our is 3+1D not in relax.time but full integral but *no gauge field*

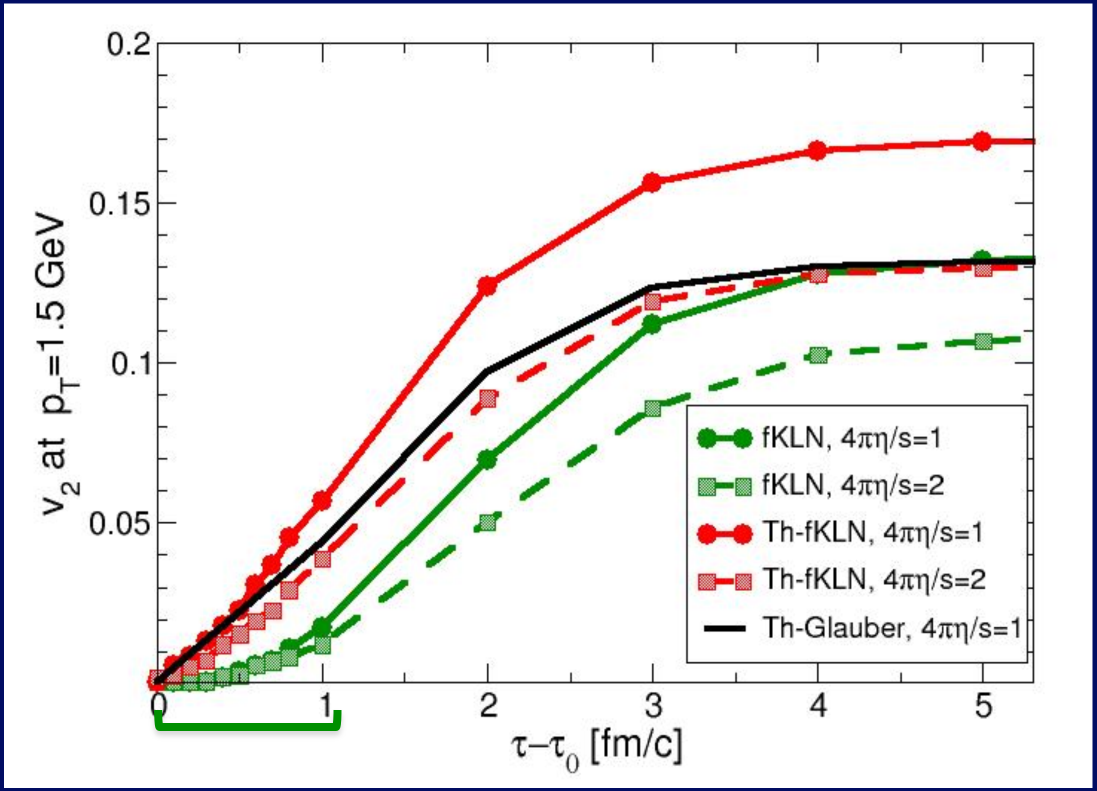
Results with kinetic theory



M. Ruggieri *et al.*, **Phys.Lett. B727 (2013) 177 -** 1303.3178 [nucl-th]

- When implementing KLN and Glauber like in Hydro we get the same of Hydro
- When implementing full KLN we get close to the data with $4\pi\eta/s = 1$:
  larger ε_x compensated by Q_s saturation scale (non-equilibrium distribution)

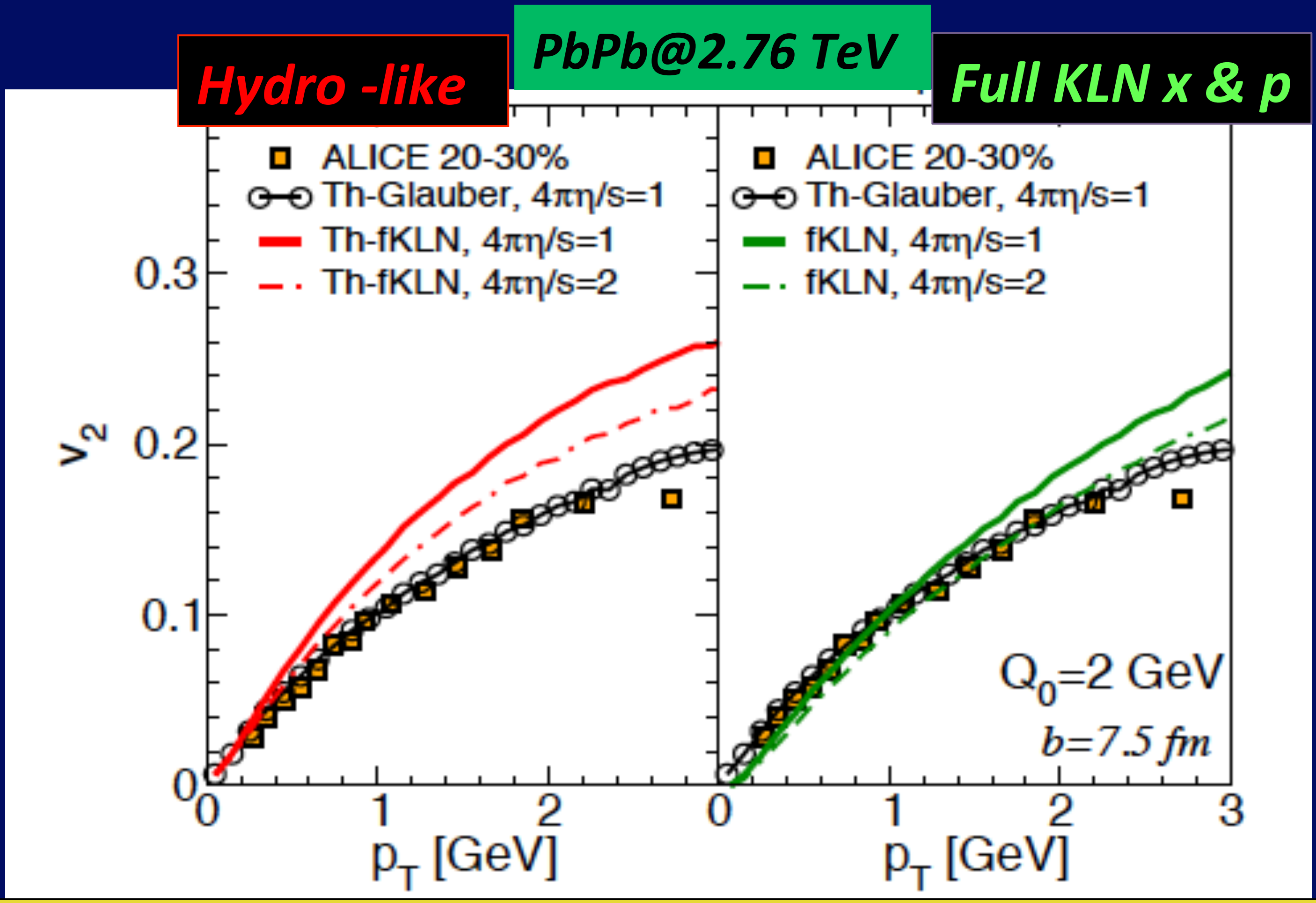
What is going on?



❖ We clearly see that when non-equilibrium distribution is implemented in the initial stage (≤ 1 fm/c) v_2 grows slowly respect to thermal one

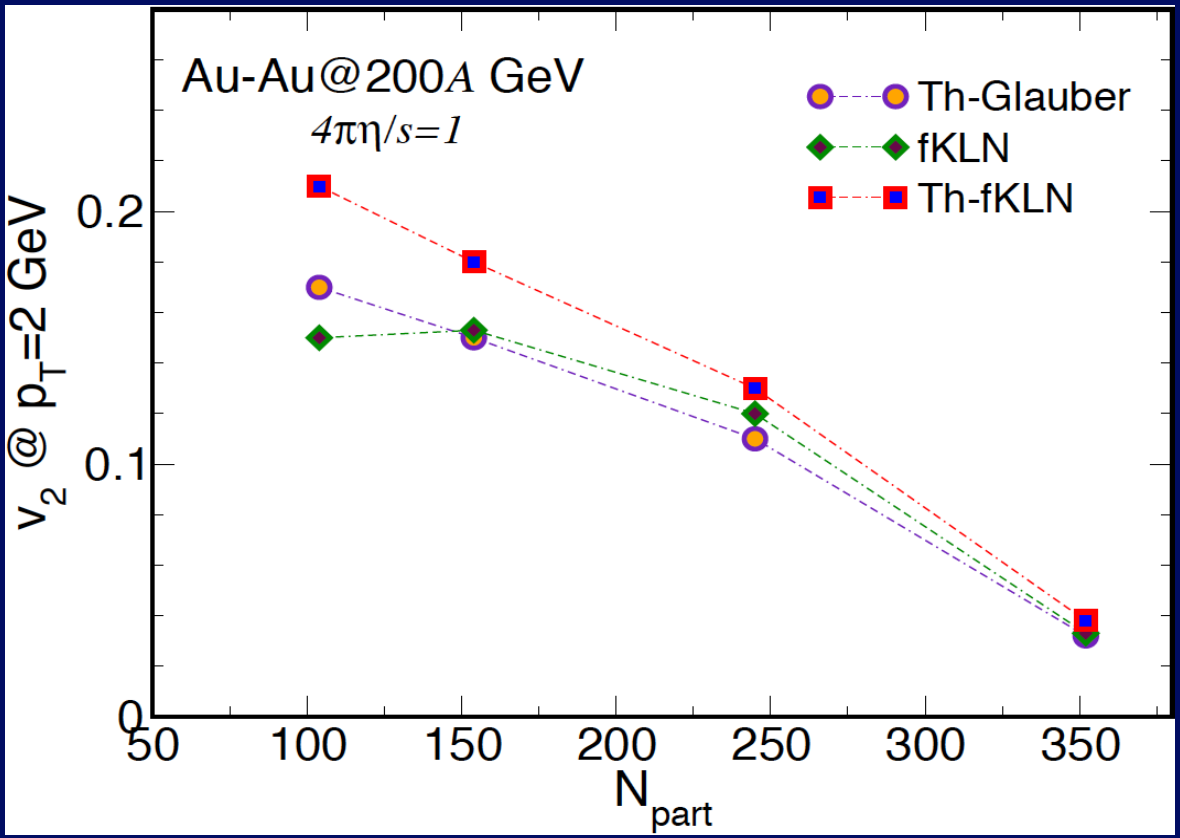
❖ Deformation of p_T distribution -> affects $v_2(p_T)$

What happens at LHC?



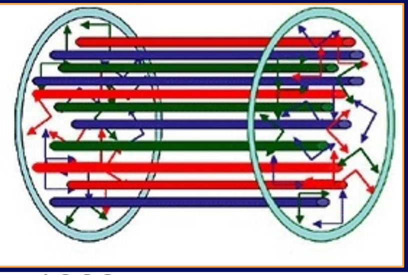
- At LHC the larger saturation Q_s (≈ 2.5 GeV):
  - $4\pi\eta/s$= 2 not sufficient to get close to the data for Th-KLN, but it is sufficient if one implements both x &p
- **Full fKLN implemention change estimate of η/s by about a factor of 3/2**

Evolution with Centrality



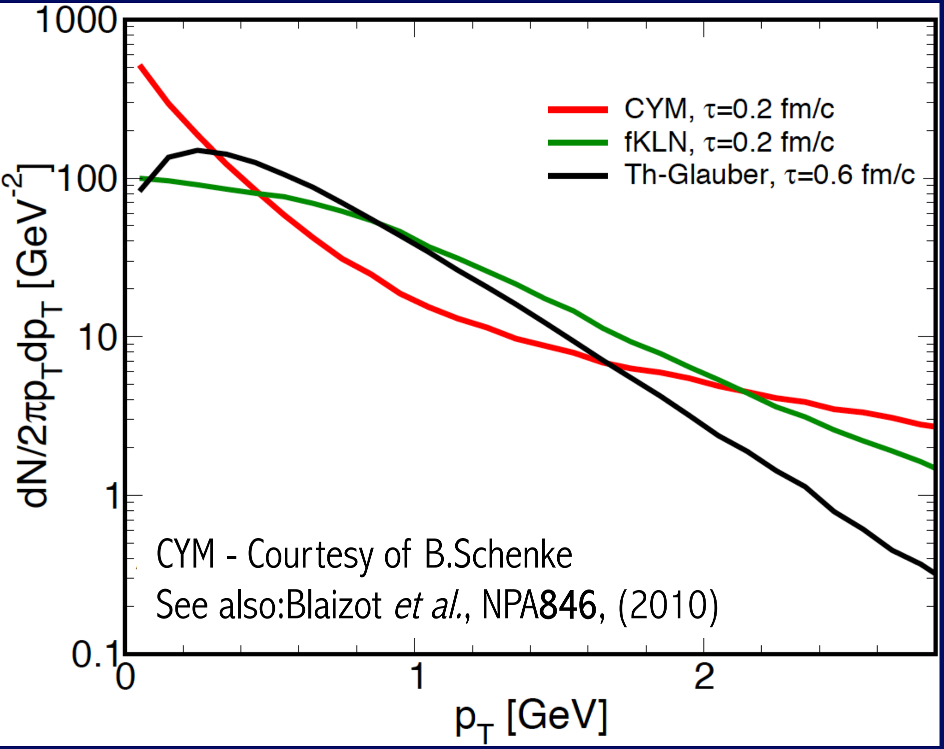
- The difference fKLN , Th-fKLN and Th-Glauber disappears at central collisions (like in hydro for Th-fKLN and Th-Glauber)
- In peripheral collisions fKLN would even be lower than Th-Glauber due to non-equilibrium impact
- Initial state fluctuation further decreae the effect by a 30-40%

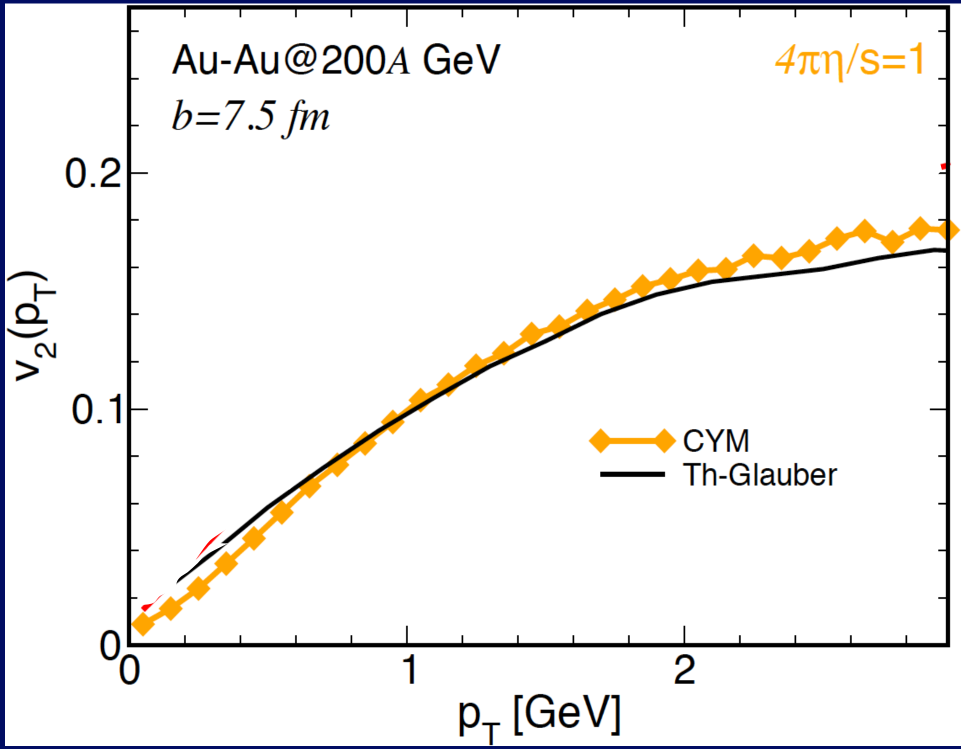
KLN vs Classic Yang-Mills



$$[D_\mu, F^{\mu\nu}] = J^\nu$$

Factorization of parton distr. funct
not valid in AA -> Classic field approach





The effect nearly disappears but indeed there is nearly No saturation!

The slope is the opposite of KLN

No real progress in the determination of η/s(T) w/o knowing initial spectra

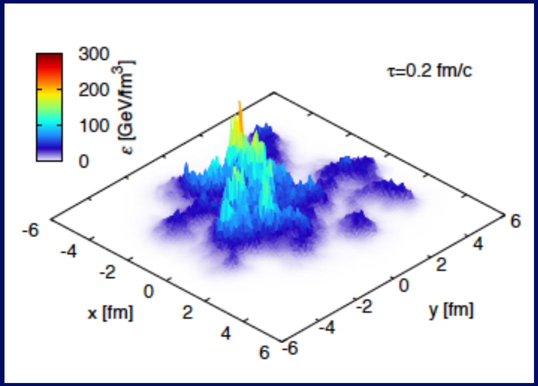
Part III – Initial State Fluctuations

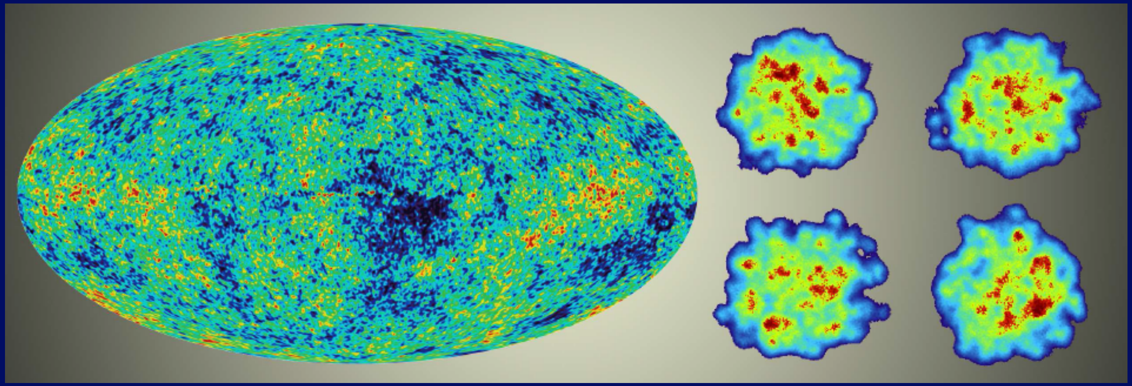
Implementing fluctuation in x-space

No issue of initial large gradients in the Boltzmann approach

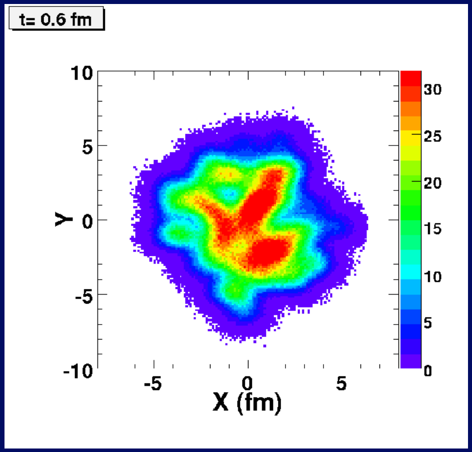
What is the impact of Initial State Fluctuations?

- Is it similar to viscous hydro?





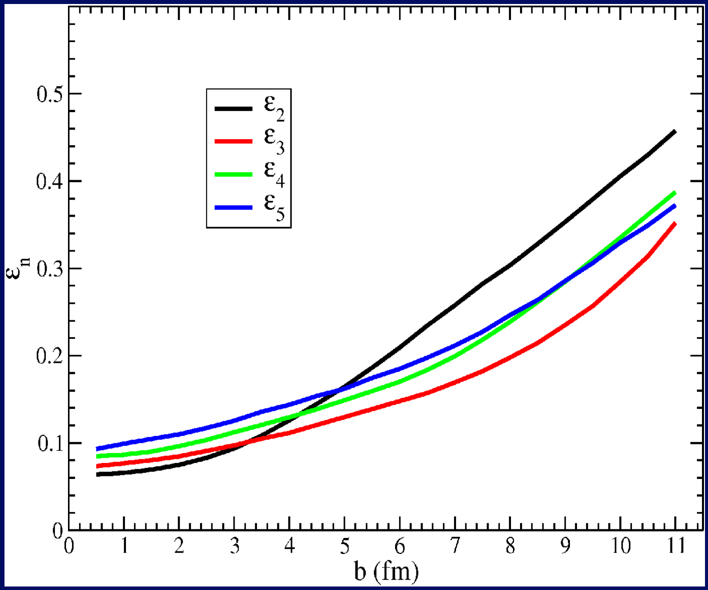
Include Initial State Fluctuations



Monte Carlo Glauber

$$\rho_{\perp} \propto \sum_{i=1}^{N_{part}} \exp\left\{-\left[\left(x-x_i\right)^2+\left(y-y_i\right)^2\right]/2\sigma^2\right\}$$

σ = 0.5 fm



$$\varepsilon_n = \frac{\left\langle r_{\perp}^n \cos\left[n(\phi-\Phi_n)\right]\right\rangle}{\left\langle r_{\perp}^n\right\rangle}$$

$$\Phi_n = \frac{1}{n}\arctan\frac{\left\langle r_{\perp}^n sen\left[n(\phi-\Phi_n)\right]\right\rangle}{\left\langle r_{\perp}^n \cos\left[n(\phi-\Phi_n)\right]\right\rangle}$$

G-Y. Qin, H. Petersen, S.A. Bass and B. Muller, PRC82,064903 (2010)
H.Holopainen, H. Niemi and K.J. Eskola, PRC83, 034901 (2011)

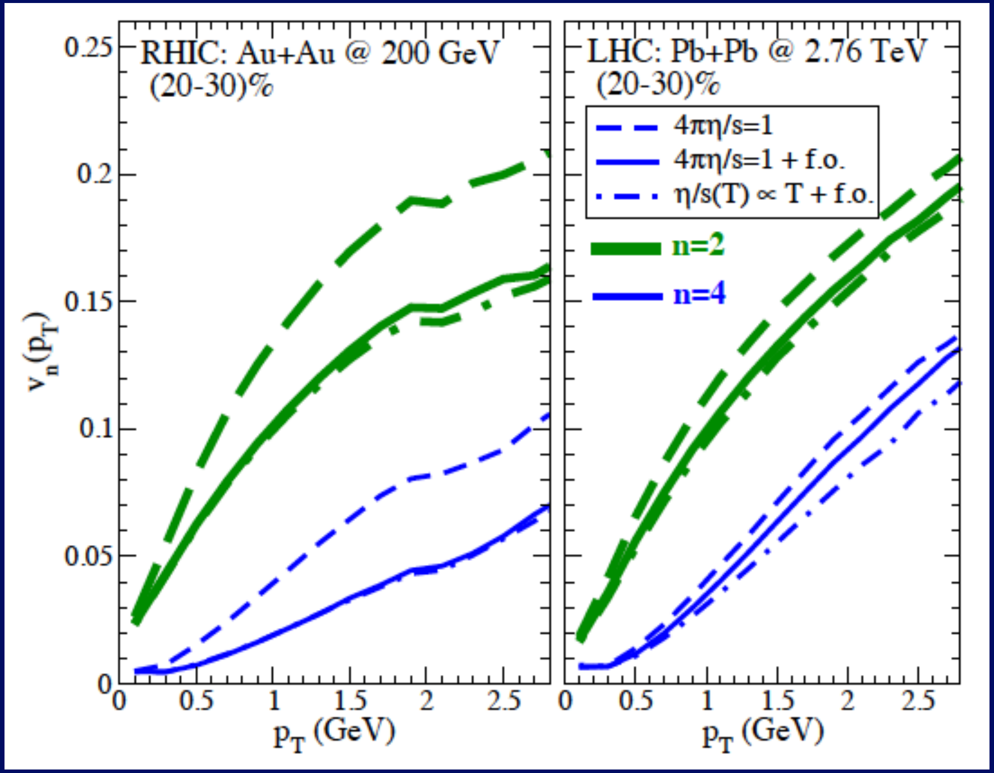
Impact of Fluctuations as in hydro:

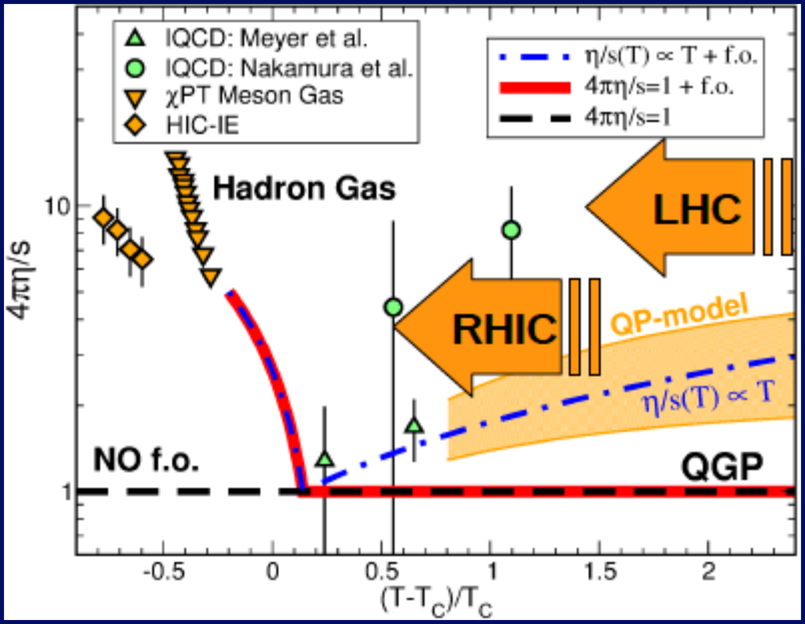
-Drecrease of $v_{2\ (15\text{-}20\%)}$

-appeareance of sizeable $v_3 \approx v_2$

-Enanhcement of v_4 about a factor 3

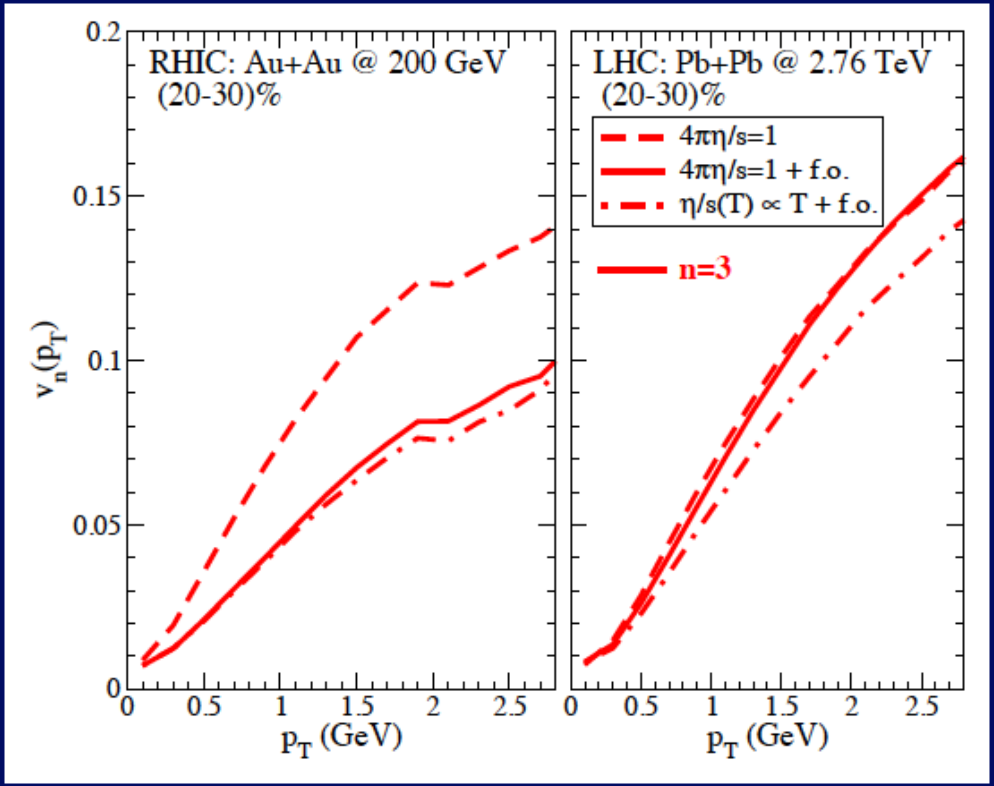
Include Initial State Fluctuations : $v_n(p_T)$ & $\eta/s(T)$

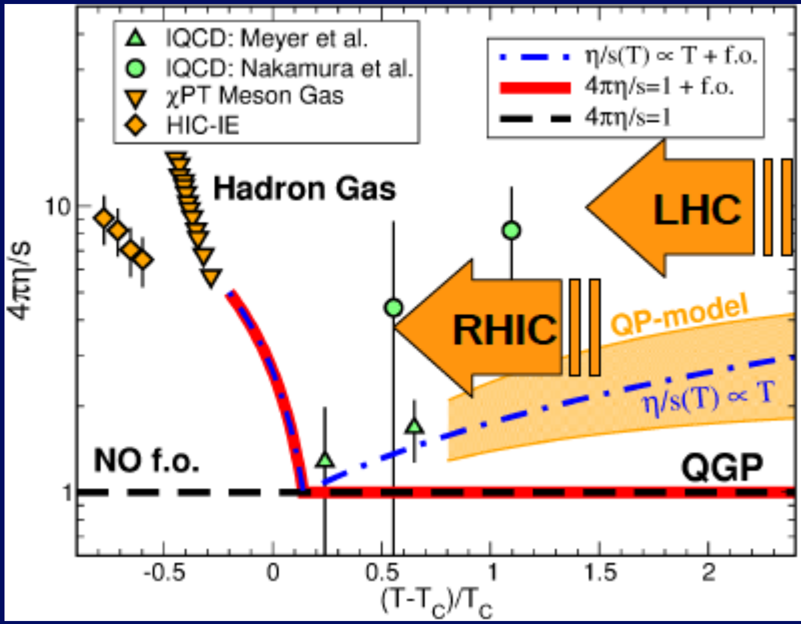




- ✓ v_n at RHIC affected by $\eta/s(T)$ in the cross-over region
- ✓ v_n at LHC determined essentially by the QGP $\eta/s(T)$
- ✓ Impact of $\eta/s(T)$ negligible at RHIC quite small LHC

Include Initial State Fluctuations : $v_n(p_T)$ & $\eta/s(T)$





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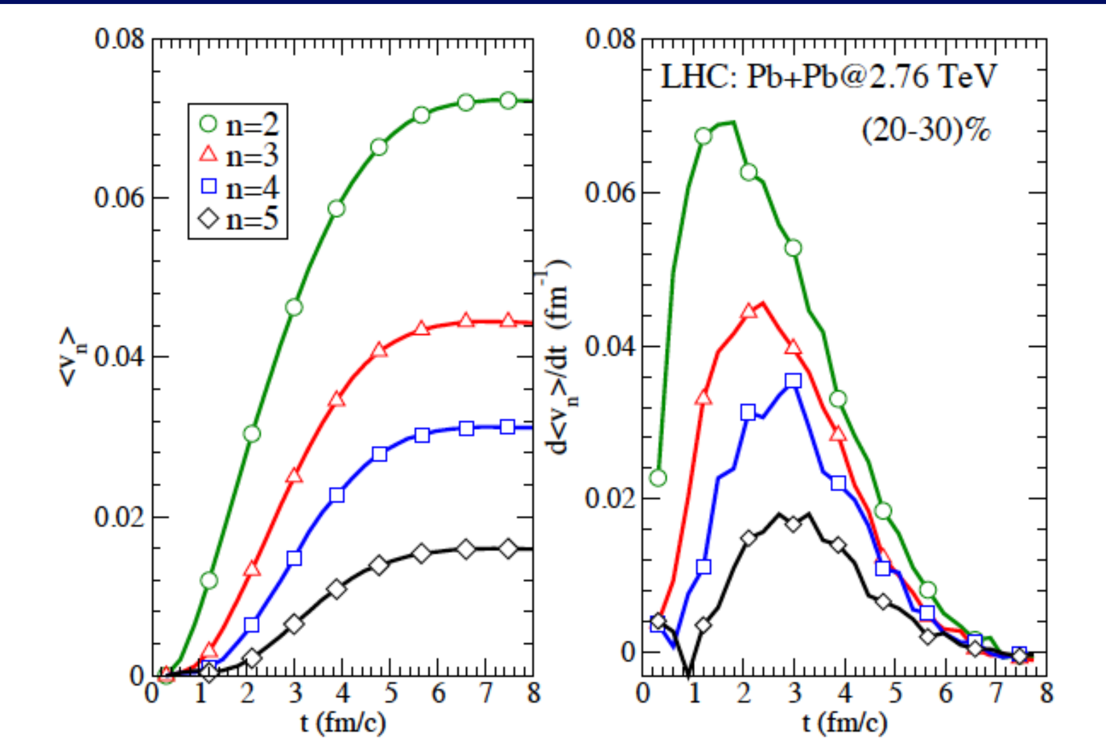
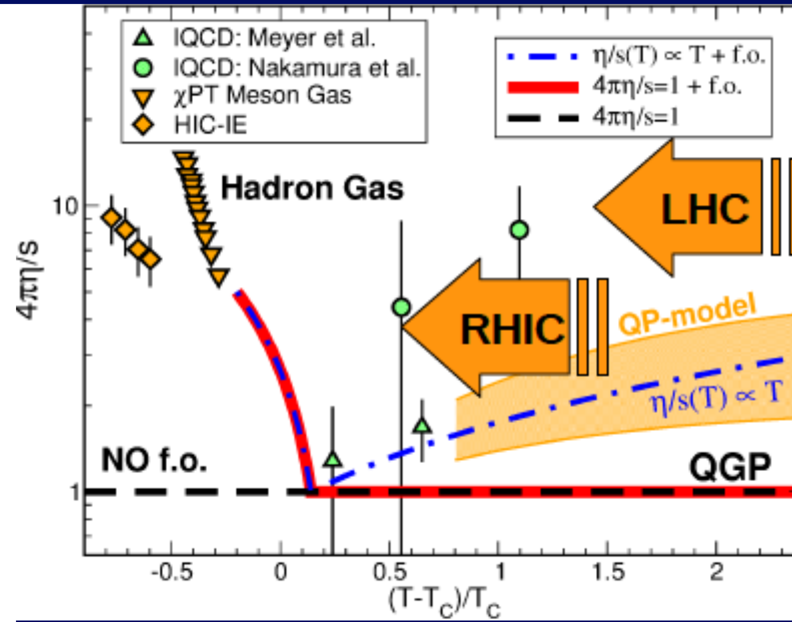
Include Initial State Fluctuations : $v_n(pT)$ & $\eta/s(T)$





Assuming

$$T \cong T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$$

T_0=3.5 T_c
τ_0=0.3 fm/c

✓ Time ordering in v_n build-up:
more sensitive to different T , hence different $\eta/s(T)$
v_2 high T≈2.2 T_c, v_4 low T≈1.5 T_c

In ultra central collision, of course viscous hydro works better:

large source, smaller surface gradients,less corona and/ or hadronic contaminations

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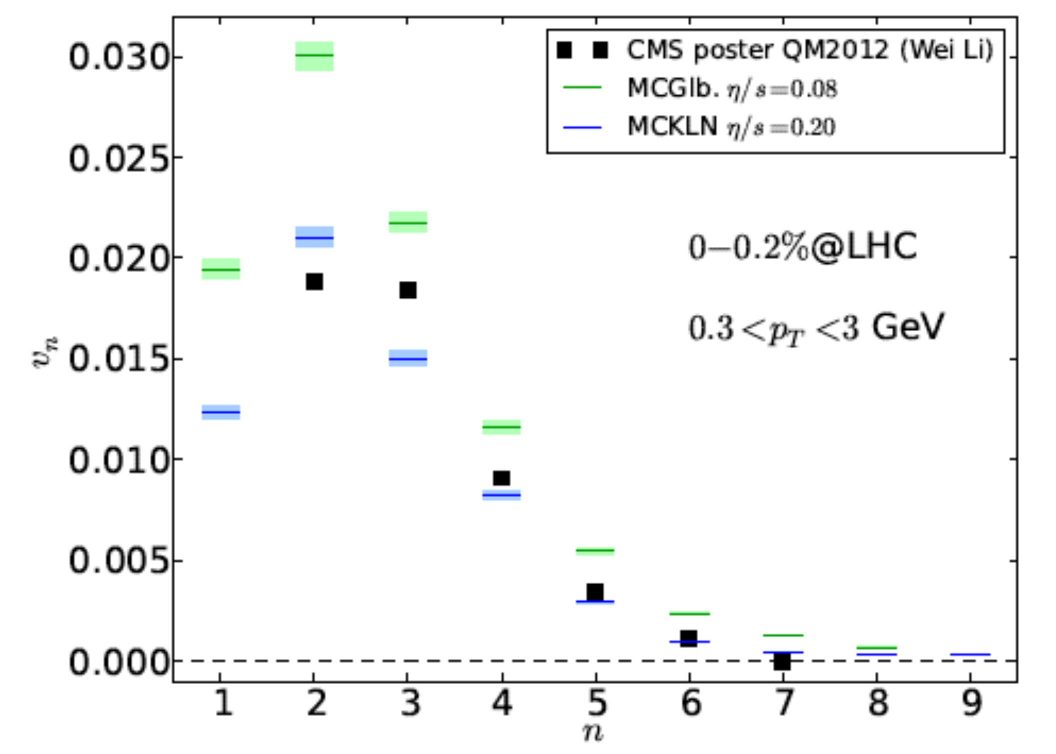
large source, smaller surface gradients,less corona and/ or hadronic contaminations



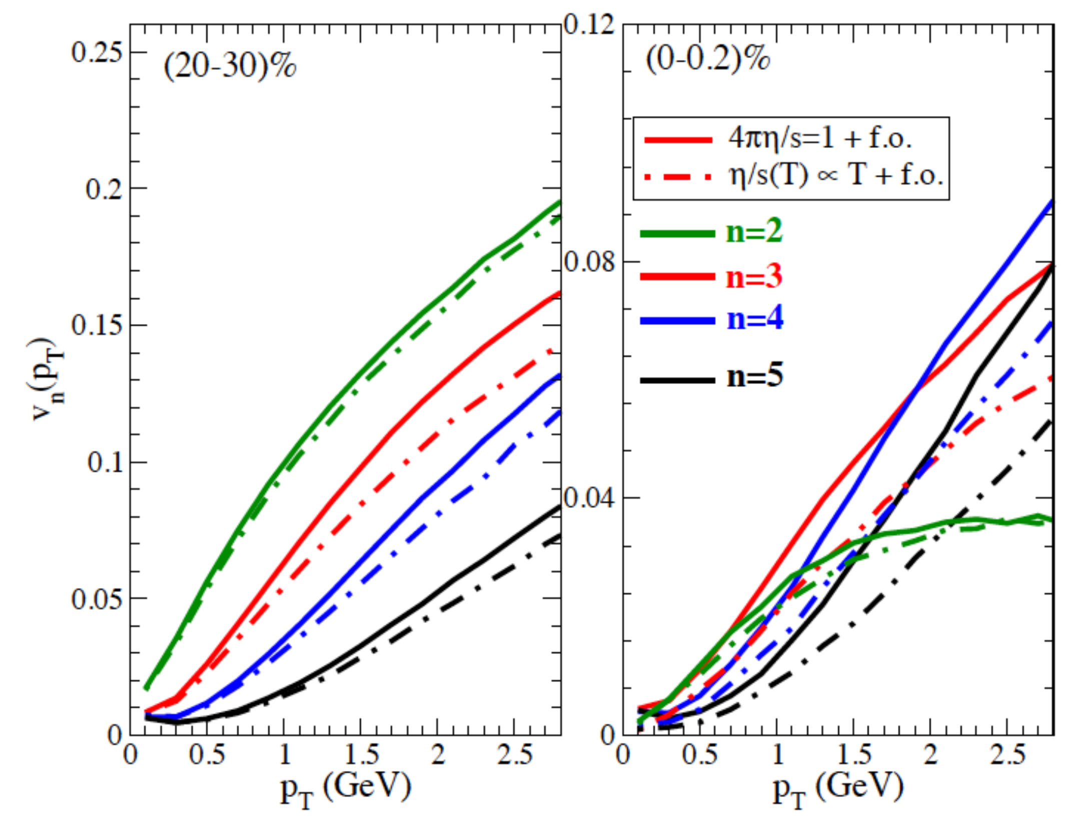
A significant failure!

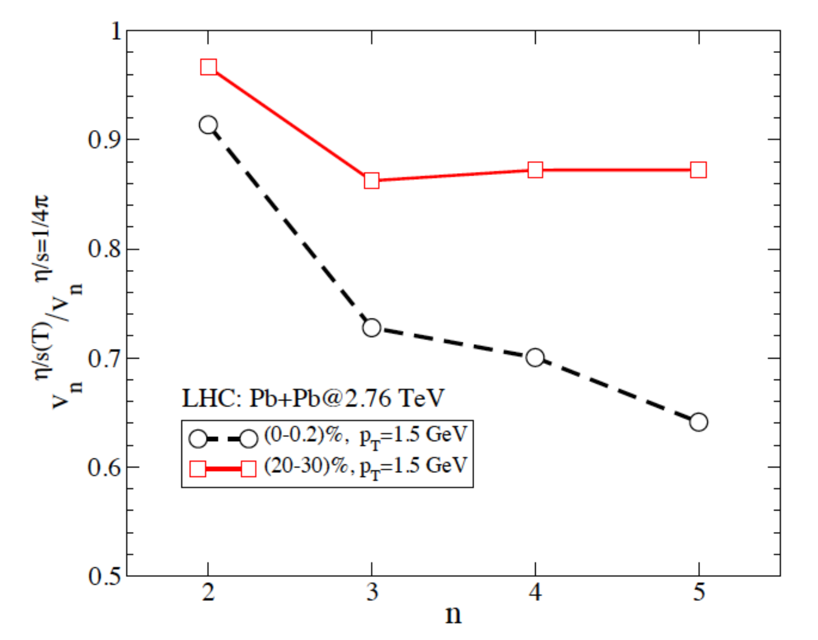
Neither MC-Glb nor MC-KLN gives the correct initial power spectrum! † R.I.P.

Is it due to our lack of knowledge about ε_n and their correlation?

or it is a non self-consistent freeze-out + lack of initial non equil.?

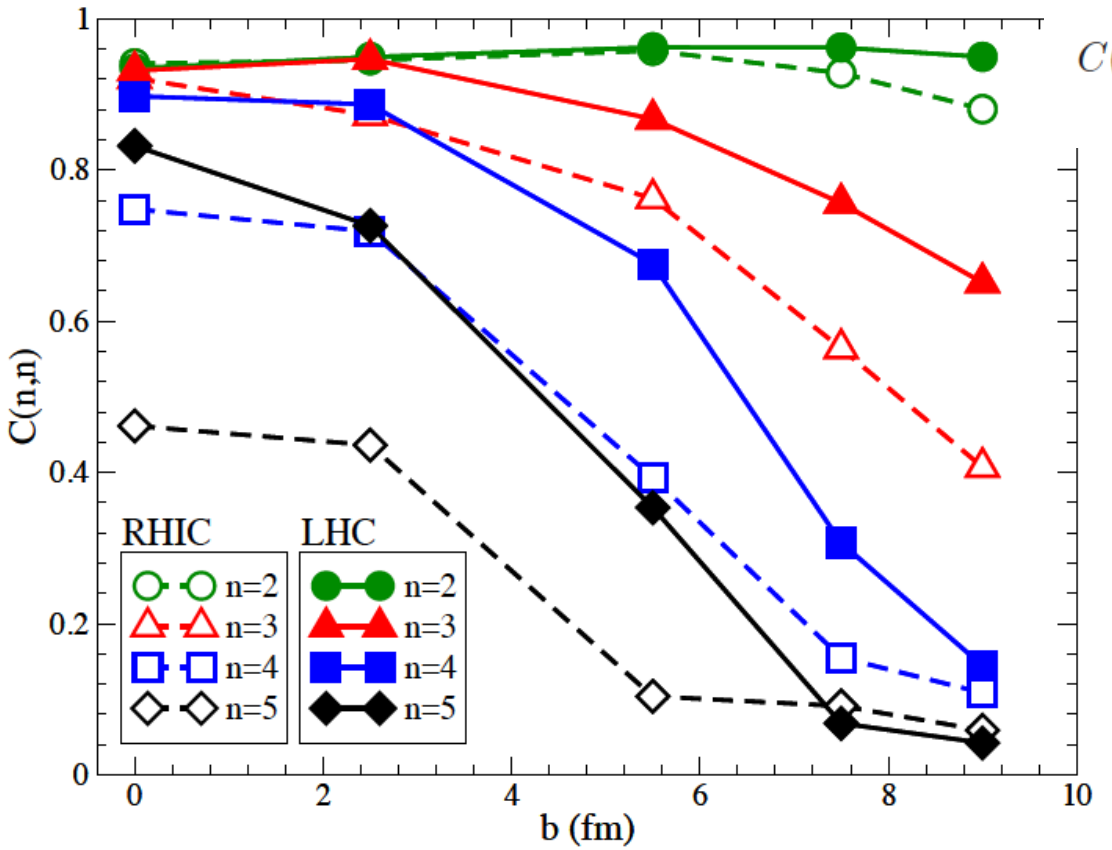
Include Initial State Fluctuations : $v_n(p_T)$ in ULTRAcentral





- ❖ For Ultra-central collisions there is a much larger sensitivity the T-dependence of η/s
- ❖ Strong saturation of $v_2(p_T)$ with p_T , while $v_n \approx p_T^{\alpha}$ seen experimentally

Correlation ε_n- v_n in ULTRAcentral



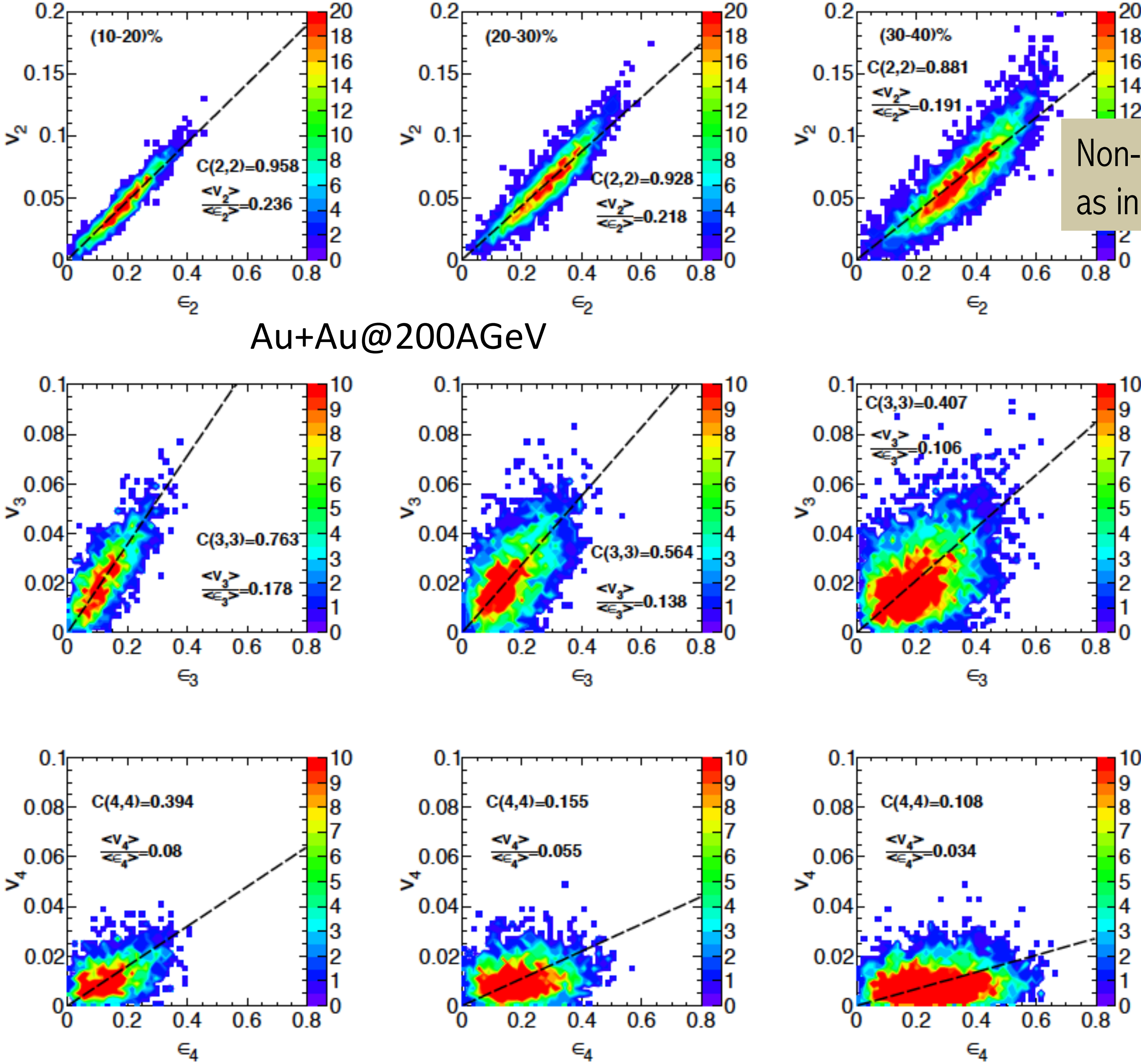
$$C(n,m) = \frac{\sum_i (\epsilon_n^i - \langle \epsilon_n \rangle)(v_m^i - \langle v_m \rangle)}{\sqrt{\sum_i (\epsilon_n^i - \langle \epsilon_n \rangle)^2 \sum_i (v_m^i - \langle v_m \rangle)^2}}$$

General agreement with hydro for LHC Niemi et al. PRC87(2013), but:

- η/s not constant (include cross-over incre
- 3+1 D not 2+1D
- σ =0.5 fm not 0.8 fm (if relevant at all!)

S. Plumari et al., arXiv:1507.05540

- At LHC larger ε_n-v_n correlation with respect to RHIC
- In ultra-central collisions al v_n has a correlation <u>larger than 0.85</u>

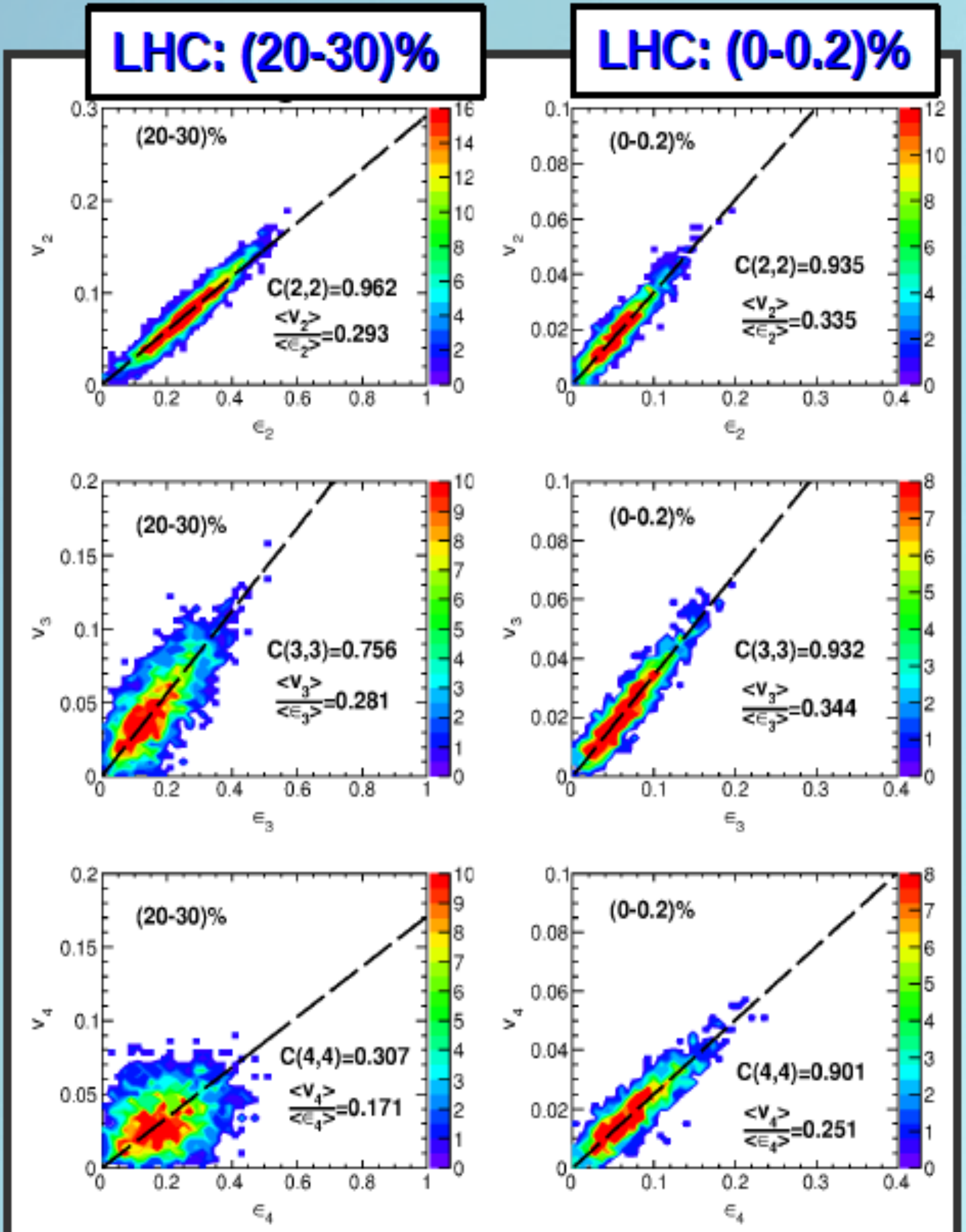


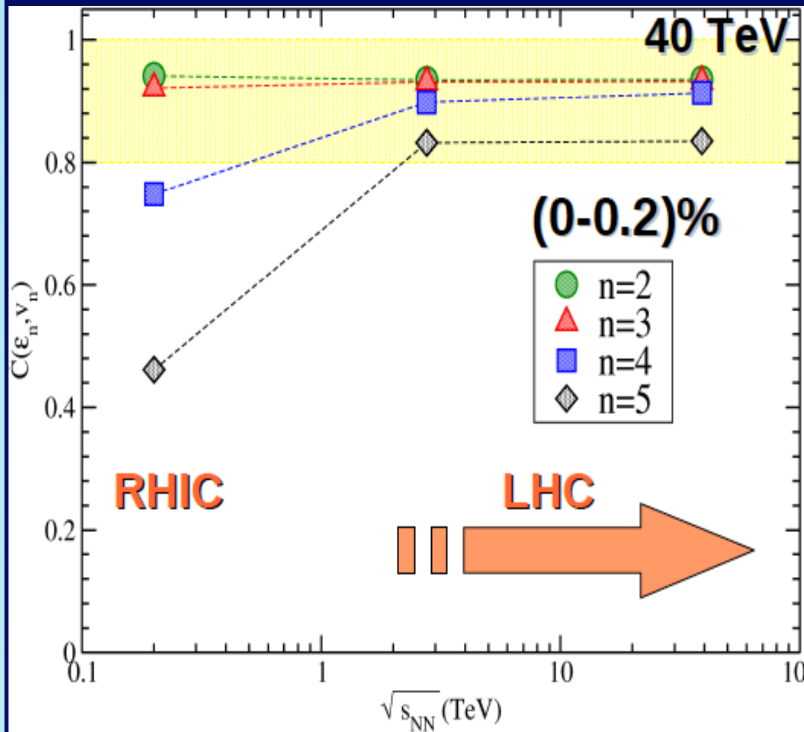
Non-linear Correlation, as in hydro: Eskola….

Correlation v_n - ε_n

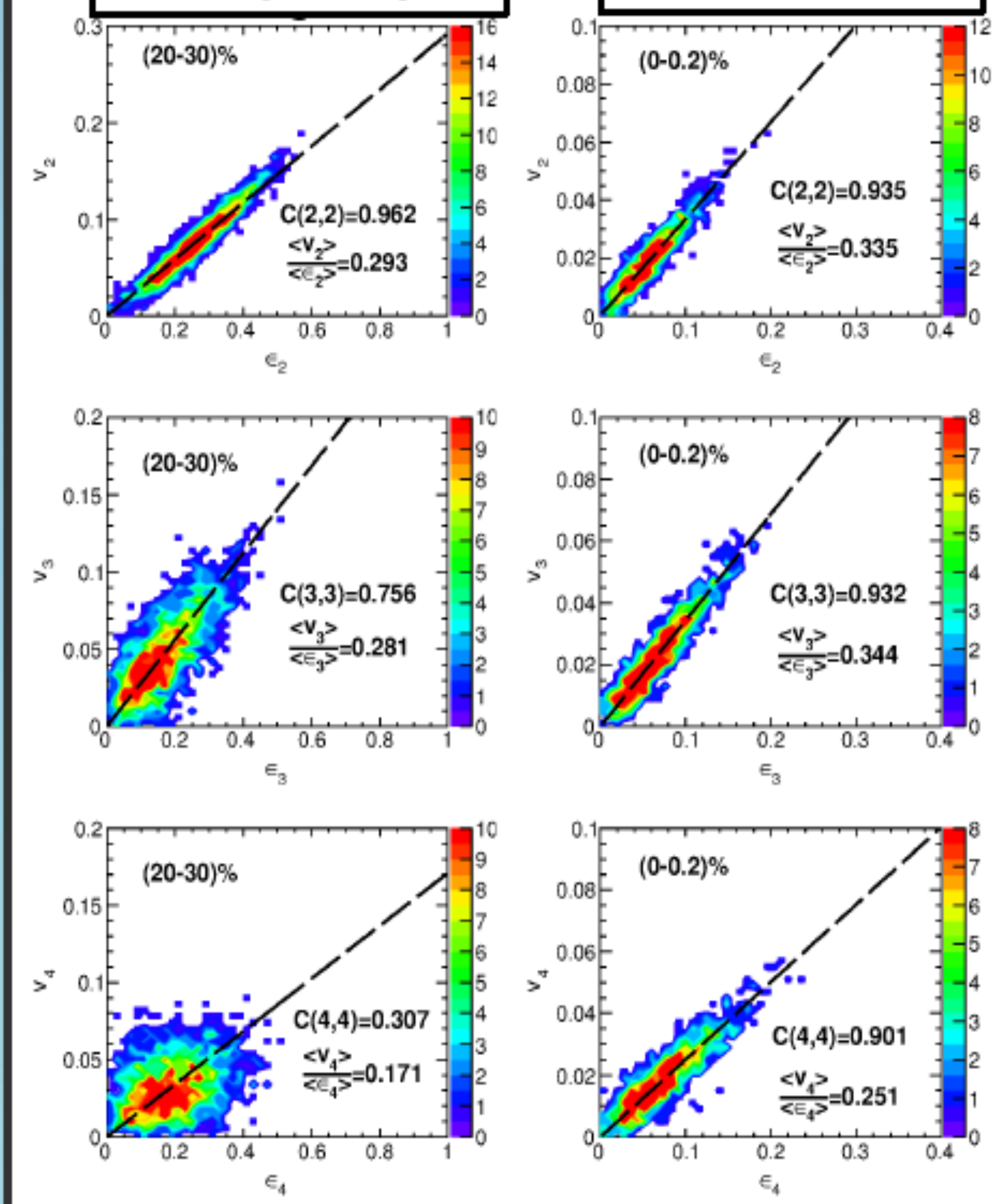
$$C(n,m)=\left\langle \frac{(v_n-\langle v_n\rangle)(\epsilon_m-\langle \epsilon_m\rangle)}{\sigma_{v_n}\sigma_{\epsilon_m}} \right\rangle$$

Ultra-central collisions
all v_n keep their correlation to ε_n

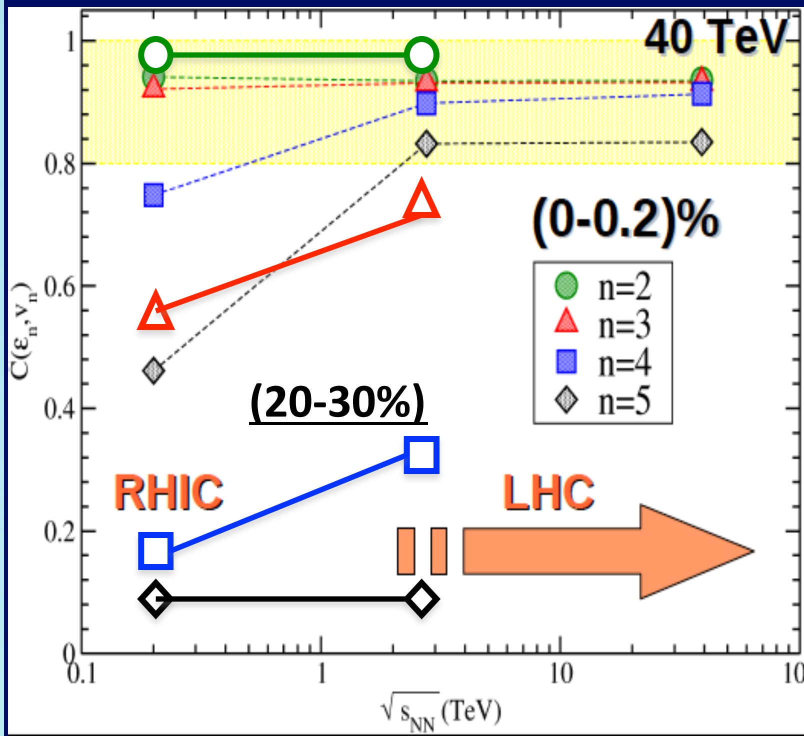




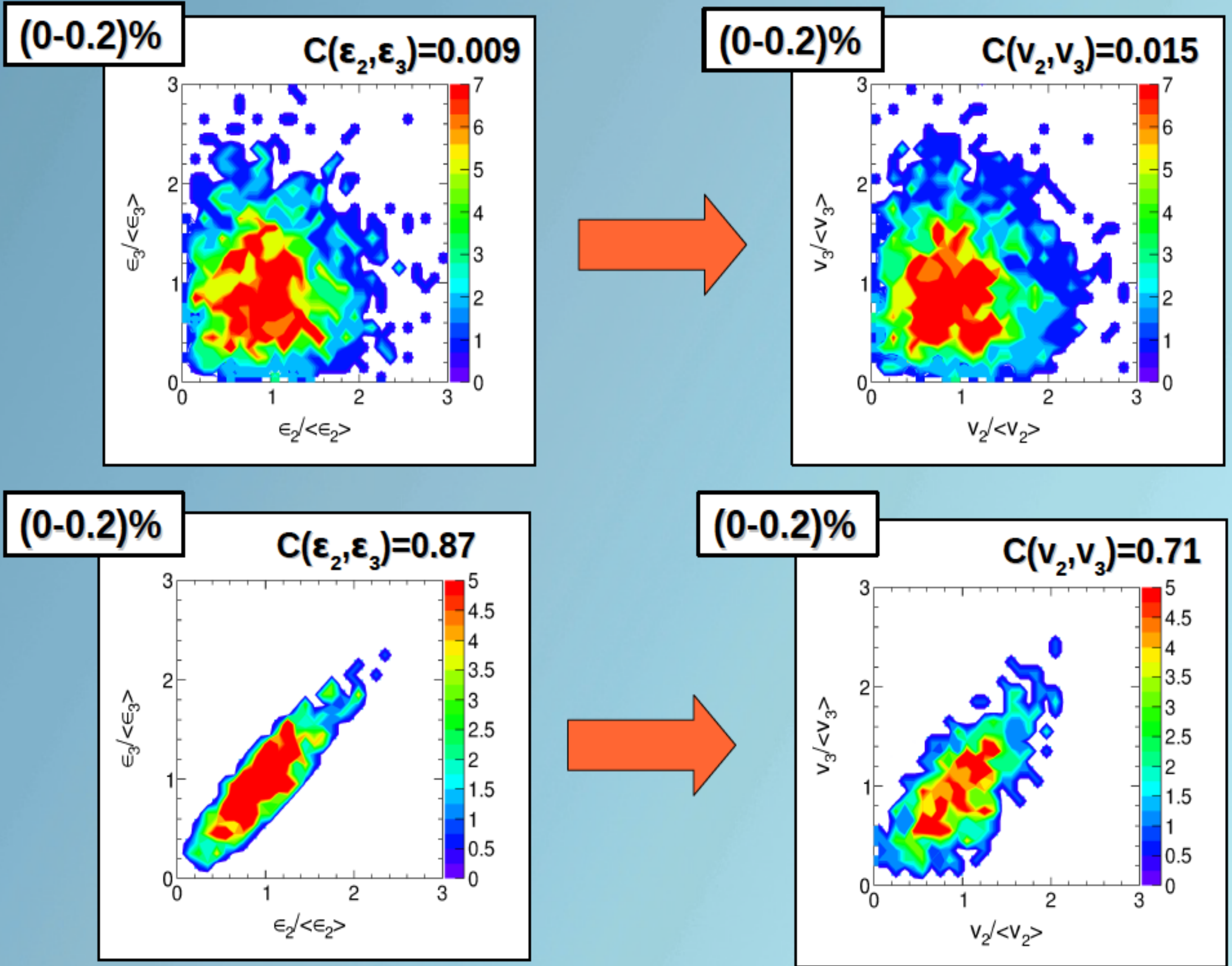
LHC: (20-30)% **LHC: (0-0.2)%**



$$C(n,m)=\left\langle \frac{(v_n-\langle v_n\rangle)(\epsilon_m-\langle \epsilon_m\rangle)}{\sigma_{v_n}\sigma_{\epsilon_m}} \right\rangle$$



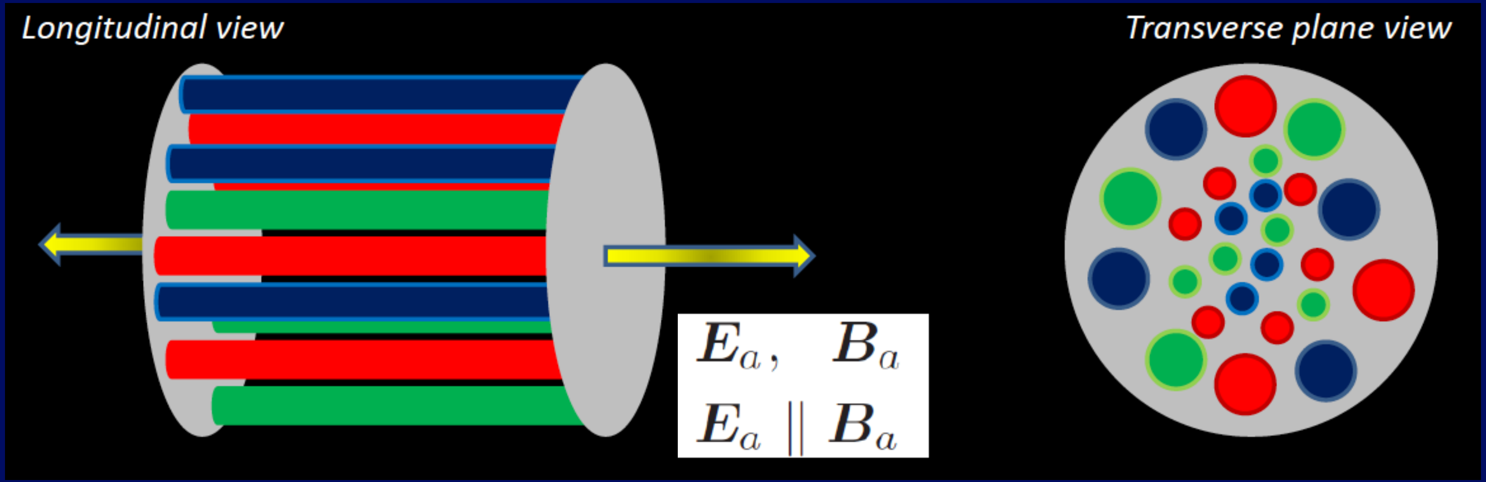
Correlations: $(\varepsilon_n,\varepsilon_m)$ vs (v_n,v_m) in (0-0.2)%



- Final correlations in (v_n,v_m) reflect initial correlations in $(\varepsilon_n,\varepsilon_m)$
- For (20-30)%: $C(v_n,v_m)=0.38$ and $C(\varepsilon_n,\varepsilon_m)=0.78$ differ a factor 2

Part IV – Color electric flux tubes -> QGP

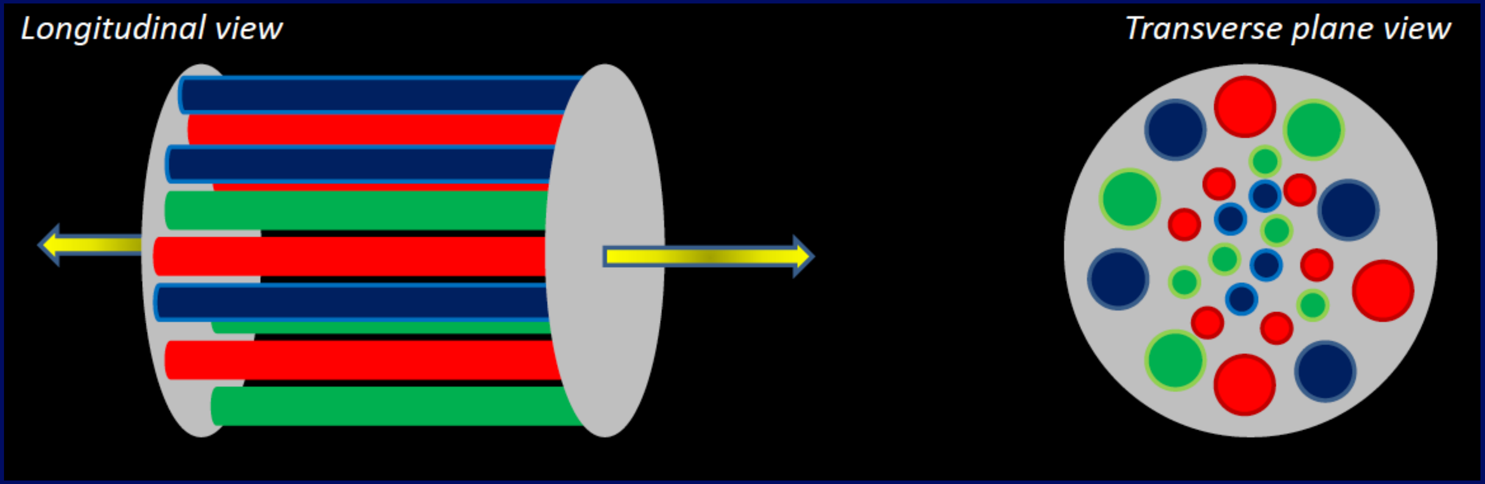
- Beyond the non-equilibrium in p_T: Longitudinal Fields decays into q,g



$$\nabla \times \boldsymbol{B} = -\boldsymbol{j}_M - \frac{\partial(\boldsymbol{E}+\boldsymbol{P})}{\partial t}$$

Initial out-of-equilibrium State

Glasma: a peculiar configuration of longitudinal color-electric and color-magnetic fields



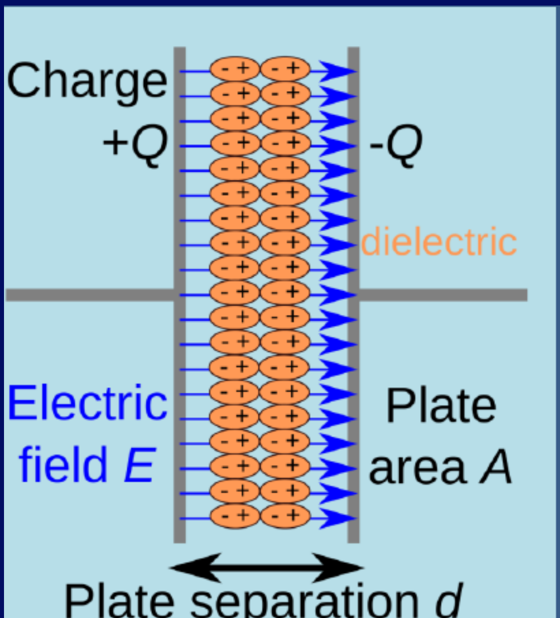
How this configuration of classical fields becomes a thermalized QGP?

A possible approach color fields decay via vacuum instability

toward pair creation (Schwinger mechanism, 1951) - [D. Blaschke talk]

Schwinger Mechanism in Electrodynamics

Vacuum with and E-field
unstable under pair creation



Quantum Effective Action of a pure electric field, has an imaginary part responsible for field instability

Vacuum Decay Probability
Per unit space-time to create electron-proton

$$\mathcal{W}(x) = \frac{e^2 E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{|eE|}\right)$$

Quantum tunneling interpretation - Casher et al. , PRD20 (1979) describe Schwinger effect as a dipole formation , $p = 2g\frac{E_T}{|g\vec{E}|}$

Once the pair pop-up charged particles propagate in real time and produce an electric current **J = σ E** – dieletric breakdown

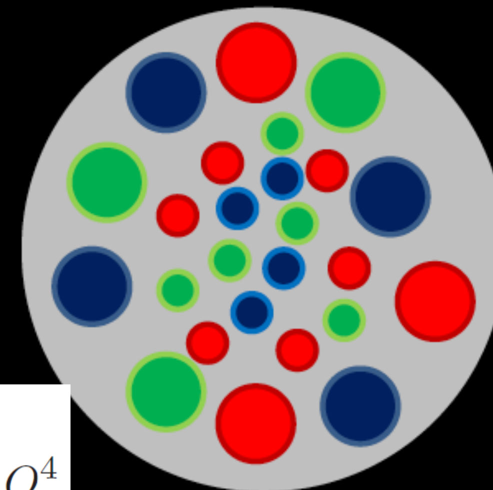
Schwinger effect in Chromodynamics

Abelian Flux Tube Model

Longitudinal view

$$\boldsymbol{E}_a\,,\quad \boldsymbol{B}_a$$
$$\boldsymbol{E}_a \parallel \boldsymbol{B}_a$$

Transverse plane view



$$R \propto 1/Q_s$$
$$\boldsymbol{E}\cdot\boldsymbol{E},\ \boldsymbol{B}\cdot\boldsymbol{B} \propto Q_s^4$$

Focus on a single flux tube:

(.) neglect color-magnetic fields;

(.) assume abelian dynamics for ***color-electric fields***;

(.) initial field is ***longitudinal;***

(.) assume ***Schwinger effect*** takes place:

Color-eletric color field decays into quark-antiquark as well as gluon pairs

Abelian Flux Tube Model

In order to permit ***particle creation*** from the vacuum we need to add a ***source term*** to the rhs of the Boltzmann equation:

$$\left(p_\mu\partial^\mu + gQ_{jc}F^{\mu\nu}p_\mu\partial^p_\nu\right) f_{jc} = p_0\frac{\partial}{\partial t}\frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

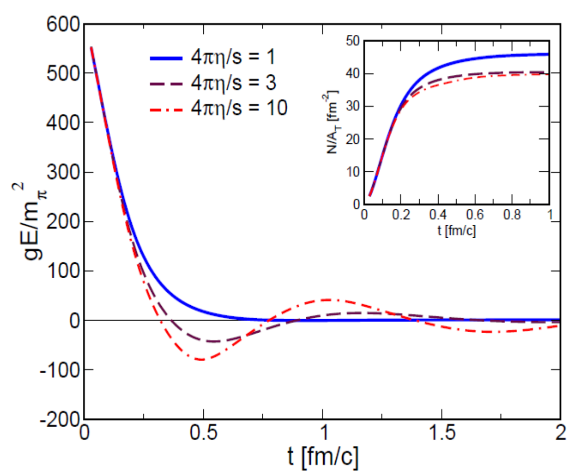
Florkowski and Ryblewski, PRD 88 (2013)

Invariant source term

Invariant source term: change of ***f*** due to particle creation in the volume at ***(x,p)***.

In our model, particles are created by means of the Schwinger effect, hence

10^{24} Volt/m



$$\frac{dN_{jc}}{d\Gamma} \equiv p_0\frac{dN_{jc}}{d^4x d^2p_T dp_z} = \mathcal{R}_{jc}(p_T)\delta(p_z)p_0$$

$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3}\left|\ln\left(1 \pm e^{-\pi p_T^2/\mathcal{E}_{jc}}\right)\right|$$

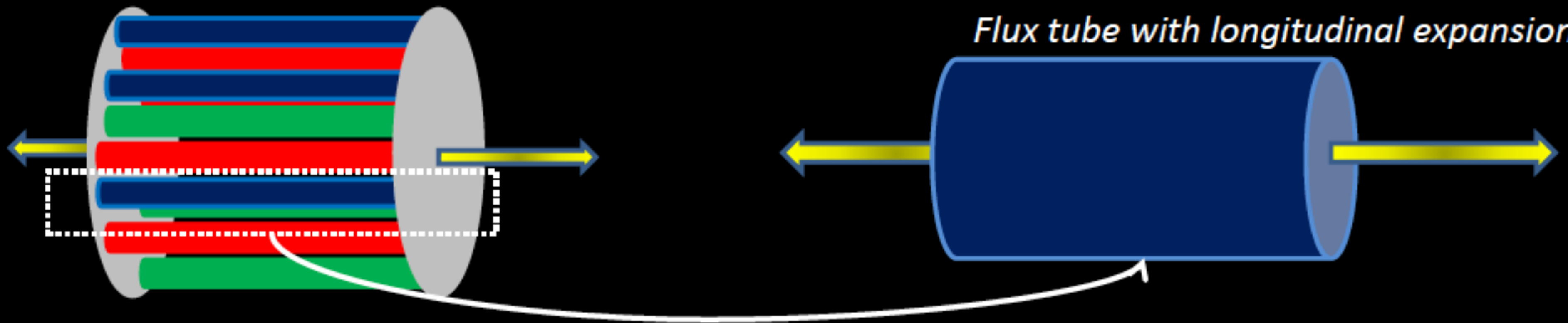
$$\mathcal{E}_{jc} = \left(g|Q_{jc}E| - \sigma_j\right)\theta\left(g|Q_{jc}E| - \sigma_j\right)$$

See also:
Gelis and Tanji, PRD 87 (2013)

$\mathcal{E}_{jc}$ effective force on pairs
Q_{jc} color flavor charges

Massless quanta

Boost invariant 1+1D expansion



$$\left(p_\mu \partial^\mu + gQ_{jc}F^{\mu\nu}p_\mu \partial^p_\nu\right) f_{jc} = p_0 \frac{\partial}{\partial t}\frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

We assume field dynamics is ***boost invariant.*** This means E=E(τ), hence independent on η:

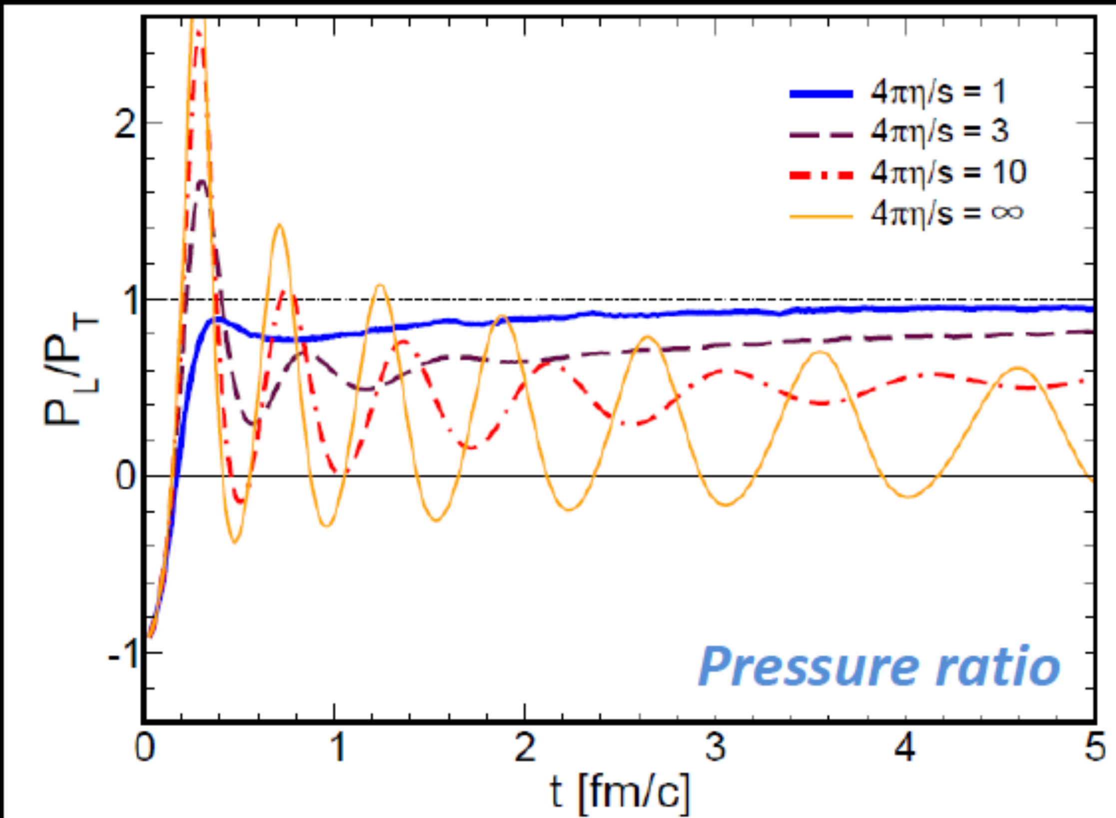
$$\left.\begin{aligned} \frac{\partial E}{\partial z} &= \rho \\ \frac{\partial E}{\partial t} &= -j \end{aligned}\right\} \qquad \frac{dE}{dt} = \rho \tanh\eta - j_M - \frac{j_D}{\cosh\eta}$$

Time derivative of dipole moment

depend on distribution functions

Link Maxwell equation to kinetic equation

Pressure isotropization



M. Ruggieri et al., arXiv: 1505.08081

$$T^{\mu\nu}_{field} = \mathrm{diag}\left(\varepsilon, P_T, P_T, P_L\right)$$
$$\propto \mathrm{diag}\left(\mathcal{E}^2, \mathcal{E}^2, \mathcal{E}^2, -\mathcal{E}^2\right)$$

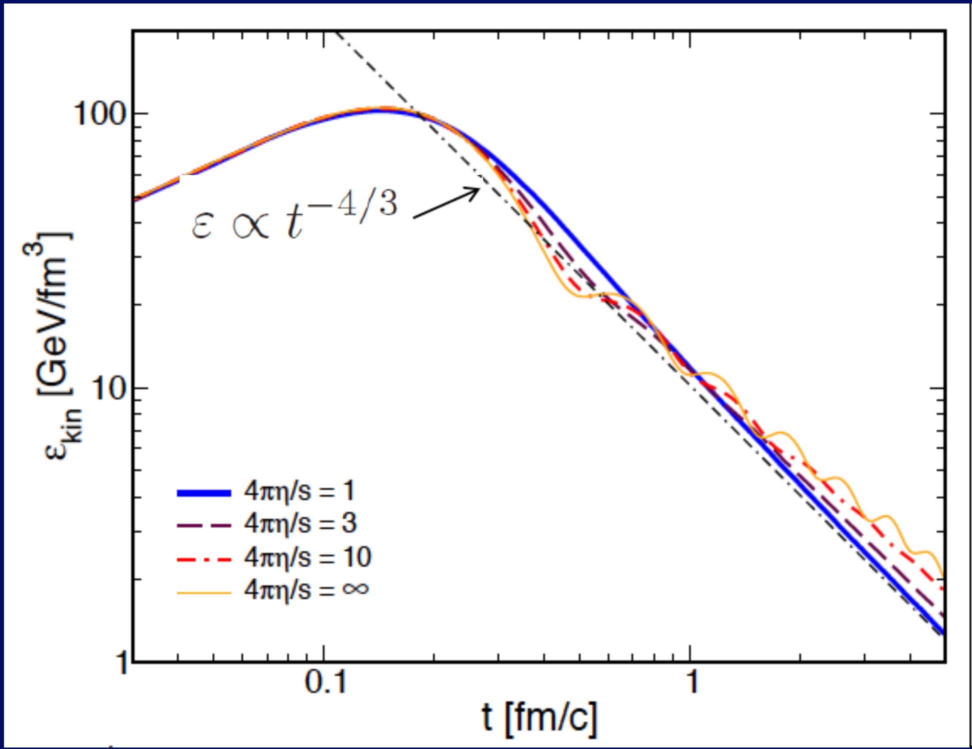
$$T^{\mu\nu}_{particles} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(\boldsymbol{x}, \boldsymbol{p})$$

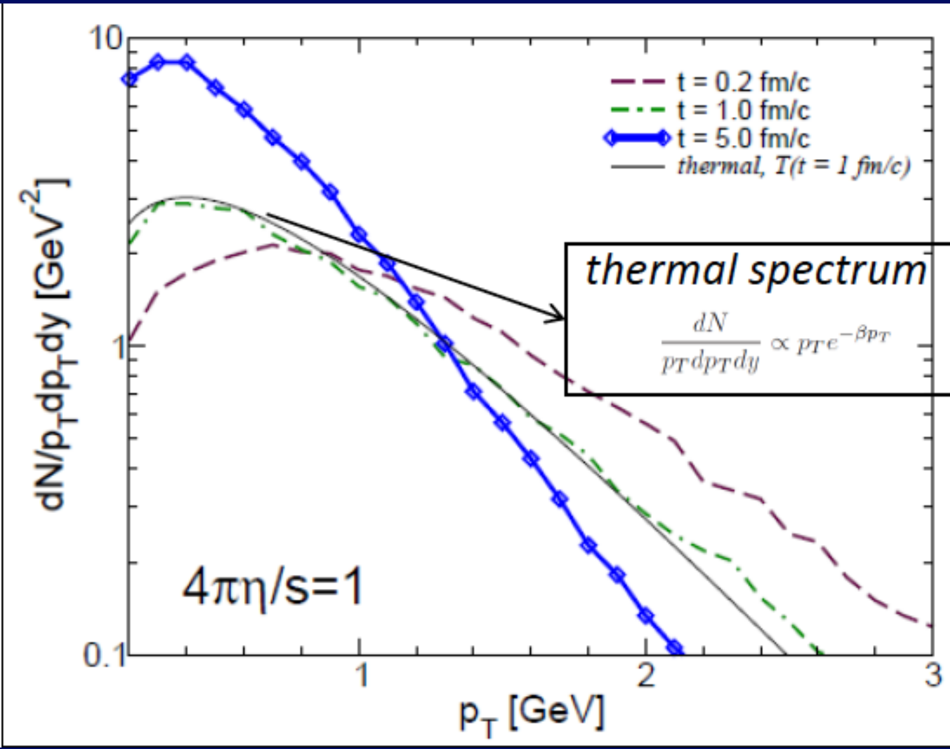
$$T^{\mu\nu} = T^{\mu\nu}_{particles} + T^{\mu\nu}_{field}$$

$$P_L = T_{zz}$$
$$P_T = \frac{T_{xx} + T_{yy}}{2}$$

- t=0 pure field with negative field P_L
- t=0.2 fm/c → $P_L > 0$ (particles pop-up) independently of η/s
- t≈0.5-1 fm/c nearly isotropization for 4πη/s<3

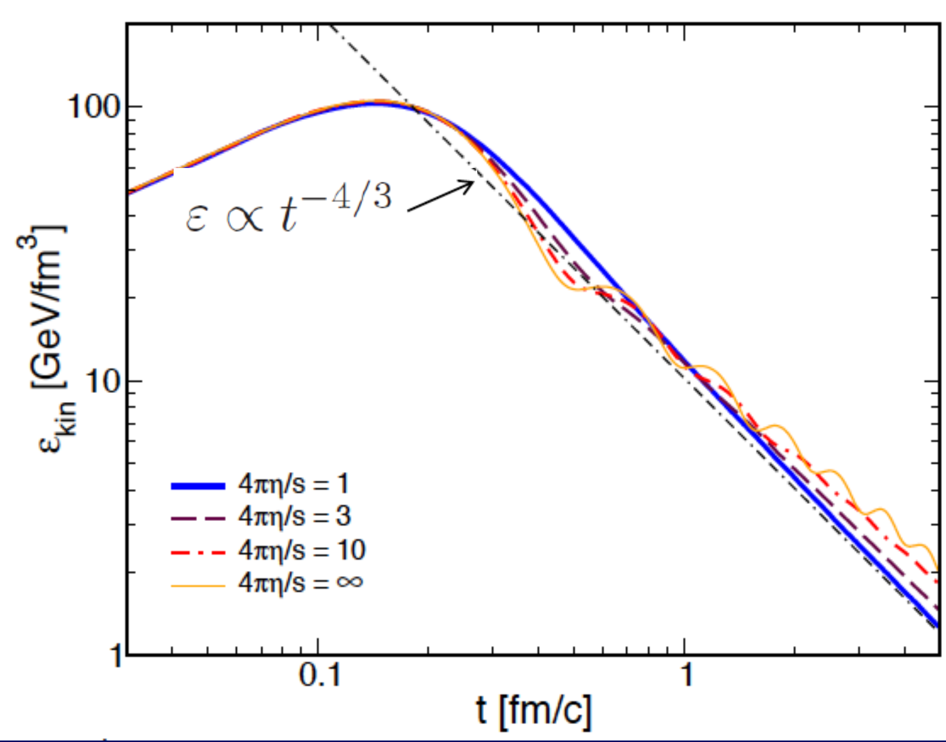
Energy Density and p_T- spectra evolution

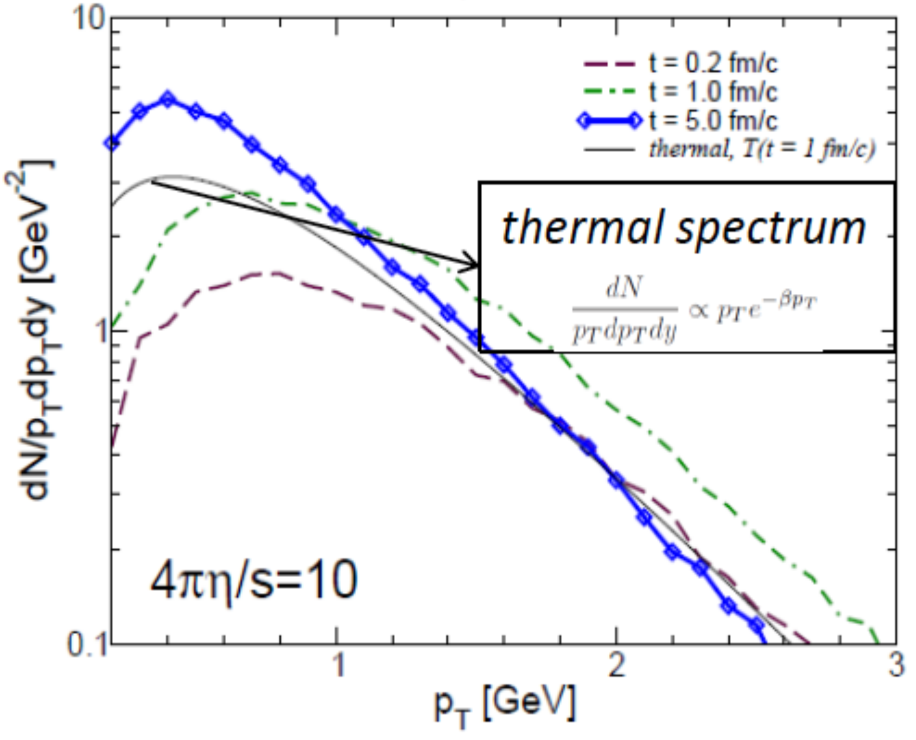




M. Ruggieri et al., arXiv: 1505.08081

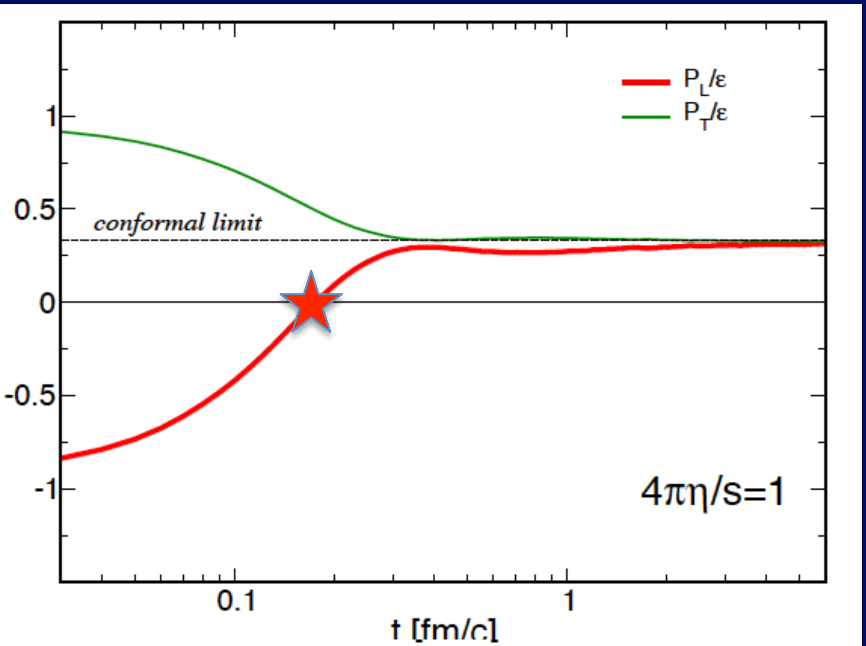
Energy Density and p_T- spectra evolution

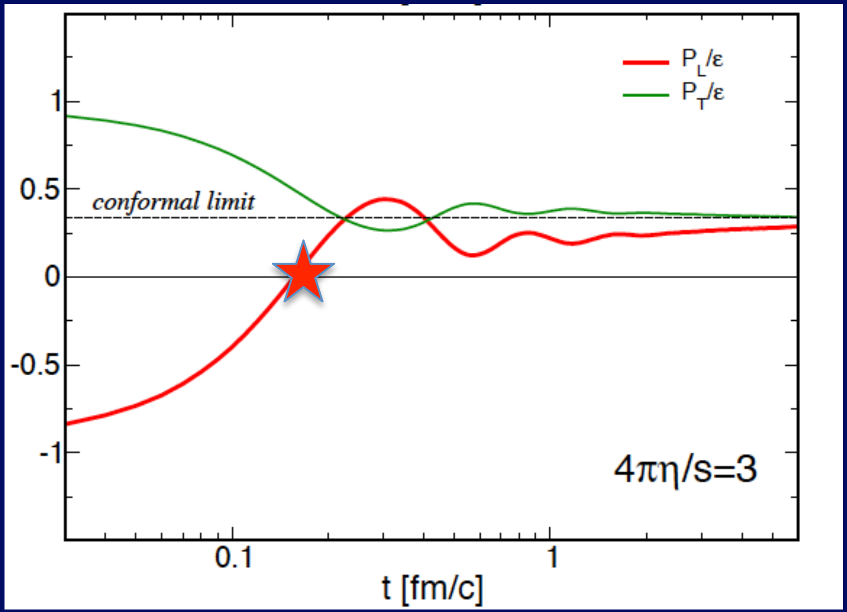


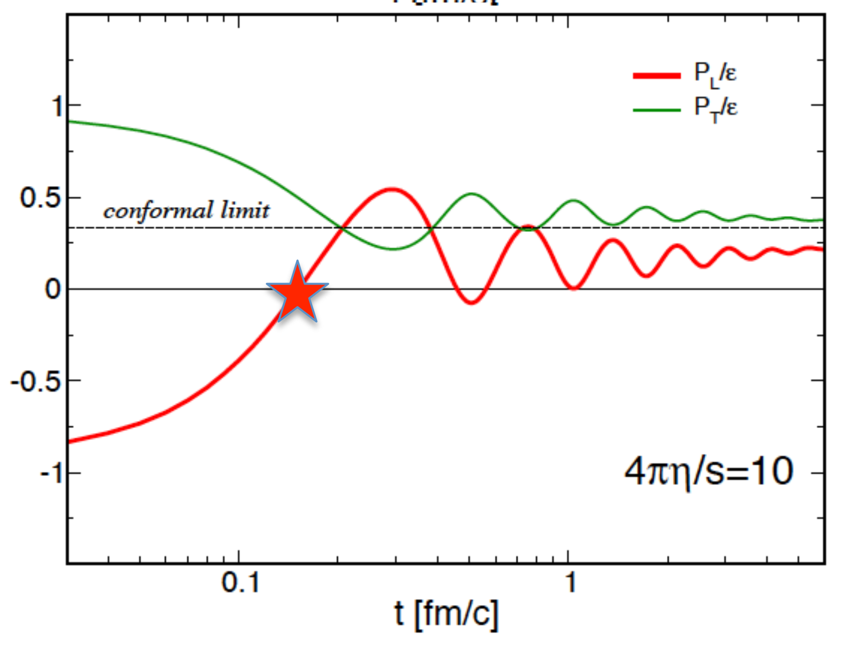


M. Ruggieri et al., arXiv: 1505.08081

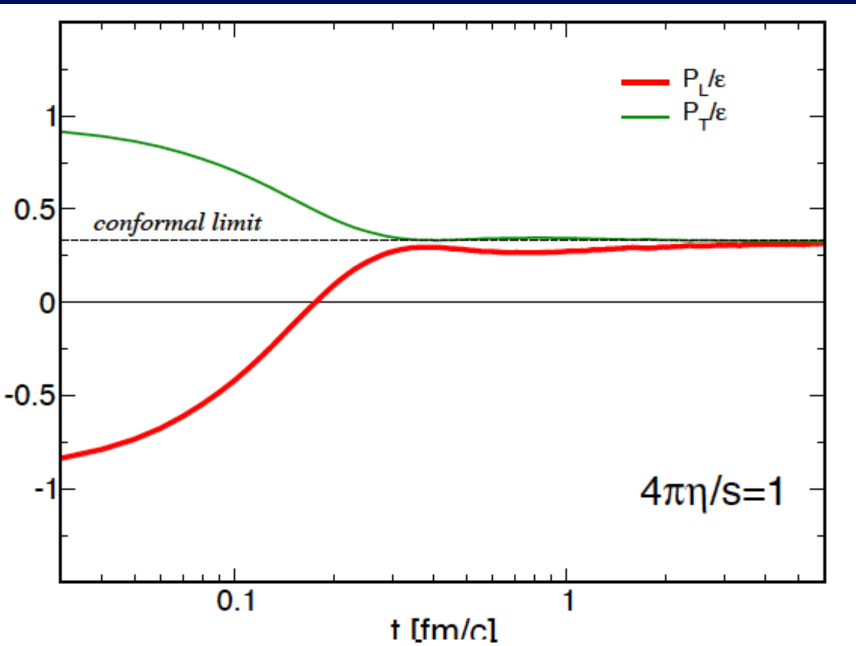
P_L/ε and P_T/ε

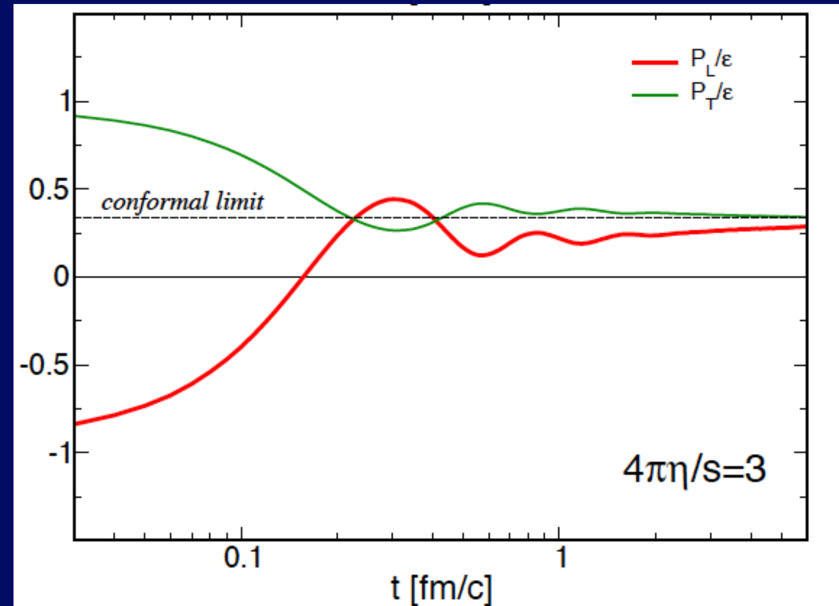


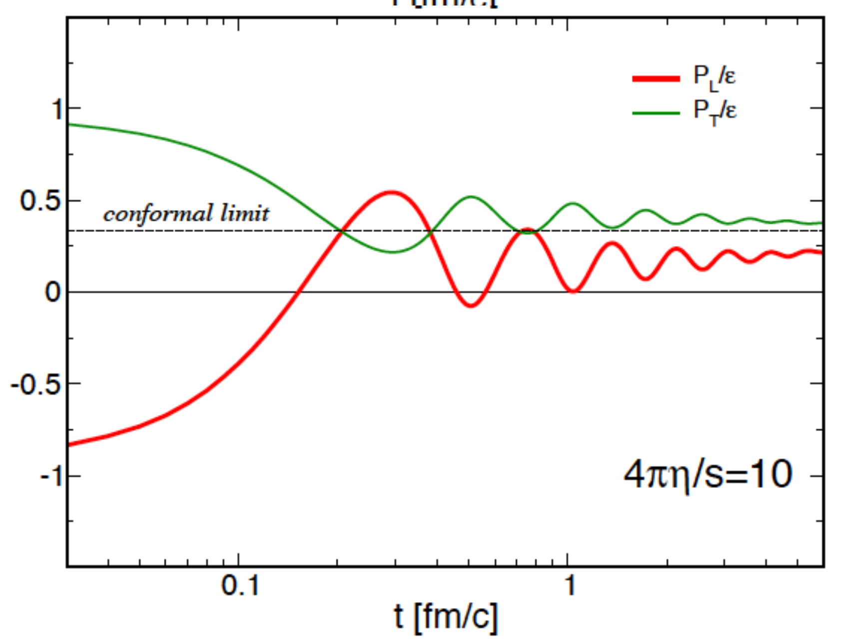




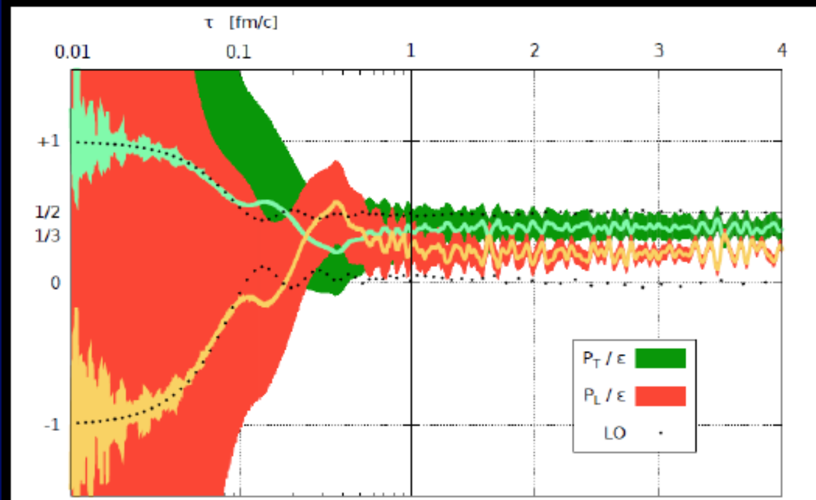
P_L/ε and P_T/ε







Classical Yang-Mills dynamics



Epelbaum and Gelis, PRL 88 (2013)

(.) Classic Yang-Mills calculation, 3+1D

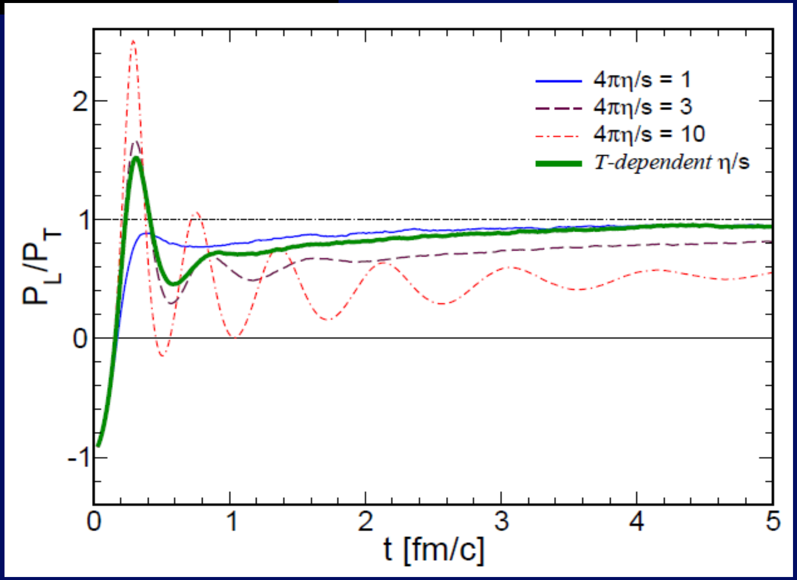
(.) Quantum fluctuations rather than Schwinger effect

Our Results:

- ✧ Initial strong color field decays in ≈0.5 fm/c
- ✧ Isotropization and thermalization achieved in t< 1fm/c
- ✧ Chemical equilibration within 1 fm/c

M. Ruggieri et al., arXiV:1502.04596

Viscous hydro regime within appropriate time scale



- ❖ Does an initial $P_L/P_T<0$ and the Some oscillations in t<1 fm/c leave any observable fingerprint?

- ❖ Need for a realistic set-up that is possible:
  - 3+1 D
  - More flux tubes + Interaction among them → pA, AA
  - Magnetic field and its decay (rot E≈0 + instabilities)

Summary

Development of kinetic at fixed $\eta/s(T)$:

- Ultra-central collisions (0-0.2 %):
  - Much larger sensitivity to $\eta/s(T)$
  - $C(\varepsilon_n,v_n)>0.9$ for al v_n at LHC (not at RHIC) → possibility to have an insight into initial $\varepsilon_n,\varepsilon_m$

Impact of non-equilibrium (in p_T + E fields):

- Non-equilibrium implied by Qs damps $v_2(p_T)$ compensating larger ε_x (data can be described by fKLN?)
- Minijets cause largely the saturation of $v_2(p_T)$ above 2 GeV, mocked up in hydro by $\delta f/f \approx p^2$
- Starting from initial fields ($P_L/P_T=-1$):
  - $P_L/P_T>0$ at $t\approx0.2$ fm/c + isotropization at $t\leq1$ fm/c for $4\pi\eta/s<3$
  - Initial P_L/P_T negative and oscillating leave some effect?

Slide from M. Ruggieri, NN2015

Pros and cons

(.) Transport theory is appropriate for studying non-equilibrium phenomena.

(.) Within a single, self-consistent theoretical framework we can follow the dynamical evolution of QGP from its early life up to final stages.

(.) It can be easily applied to ***pA*** and ***pp*** collisions.

(.) We can study the effects of the initial dynamics on observables:

Collective flows

Rapidity distributions

Particle spectra

(.) Initial field dynamics ignores the full structure of the glasma flux tubes:

Color-magnetic fields

Field fluctuations in rapidity and transverse plane

(.) The model ignores the non abelian interactions in the color field sector.

We have a good starting point at hand, quite under control, which we are improving step by step to obtain a more complete description of early stages.

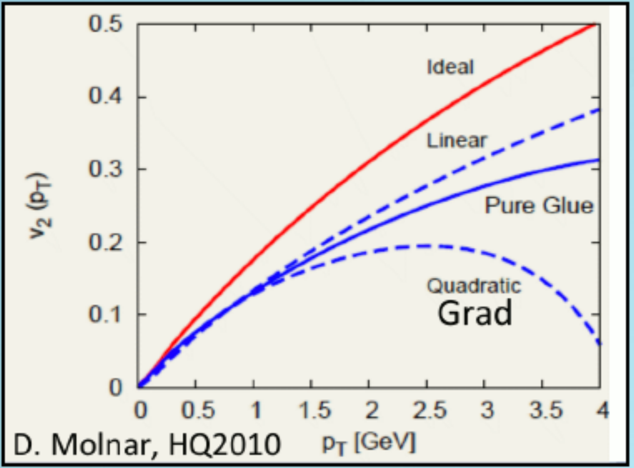
From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$

A common choice for δf – the Grad ansatz

$$\delta f \propto \Gamma_s f^{(0)} p^{\alpha} p^{\beta} \langle \nabla_{\alpha} u_{\beta} \rangle \propto p_T^2$$



BUT it doesn't care about the microscopic dynamics

In general in the limit σ→∞, f(σ) can be expanded in power of 1/σ.

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right) \quad \Rightarrow \quad v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit $f^{(0)}$, $v_n^{(0)}$ and the viscous corrections δf and δv_n solving the Relativistic Boltzmann eq for large values of the cross section σ

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

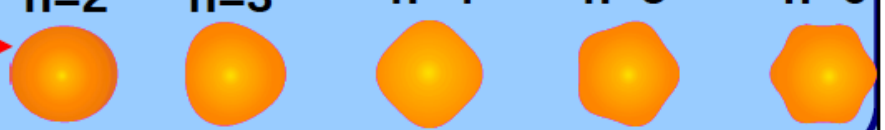
Coodinate space (x,y)

- We start with an initial azimuthally symmetric profile (optical Glauber model).
- Then we deform the initial distribution (α<<1)

$$z = x + iy \rightarrow z + \delta z \equiv z - \alpha \bar{z}^{n-1}$$

2π/n symmetry

This Creates only



Momentum space

- Thermal distribution:

$$dN/d^3p \propto \exp(-p/T)$$

- Constant distribution:

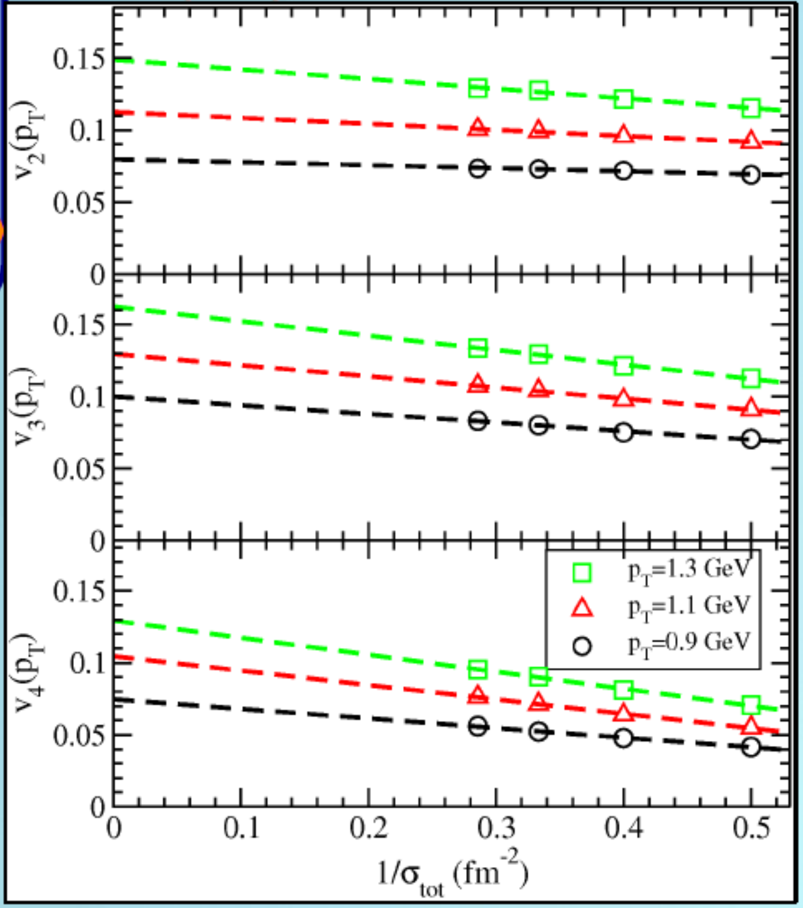
$$dN/d^3p \propto \theta(p_0 - p)$$

We assume initially the same local $T^{\mu\nu}(x)$

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right)$$

$$v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

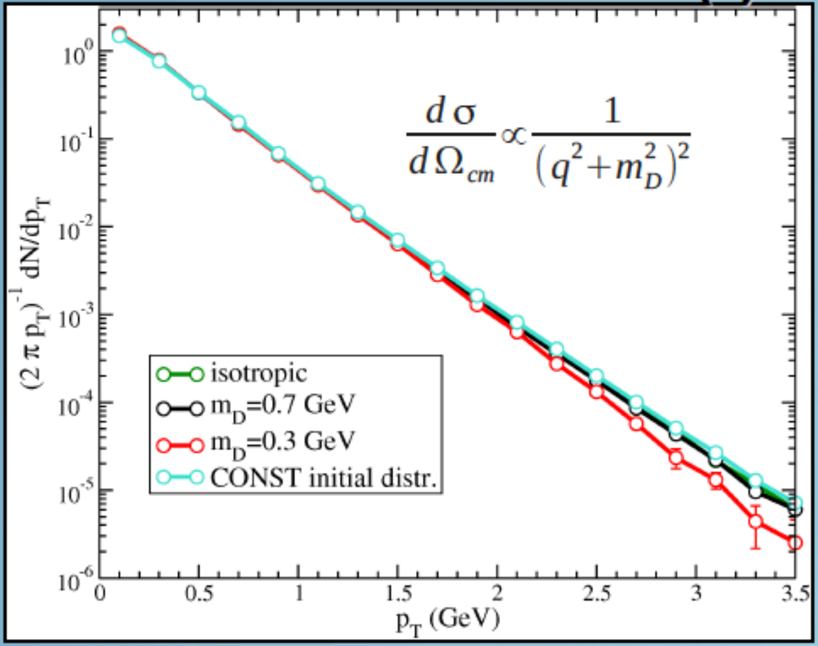
S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault NPA 941 (2015) 87

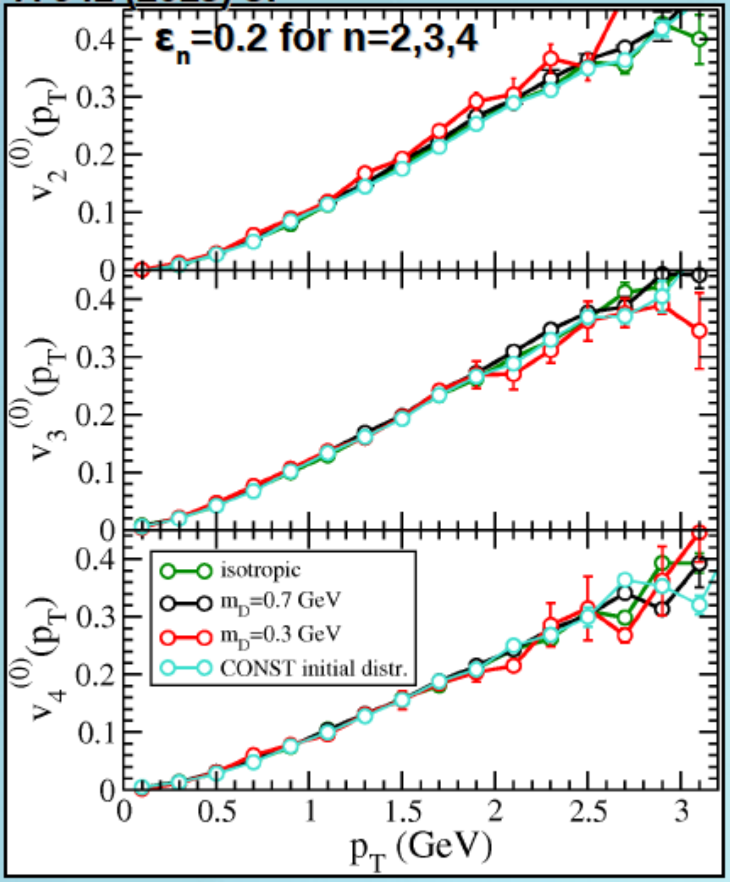


From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault
NPA 941 (2015) 87

For the same initial local $T^{\mu\nu}(x)$:





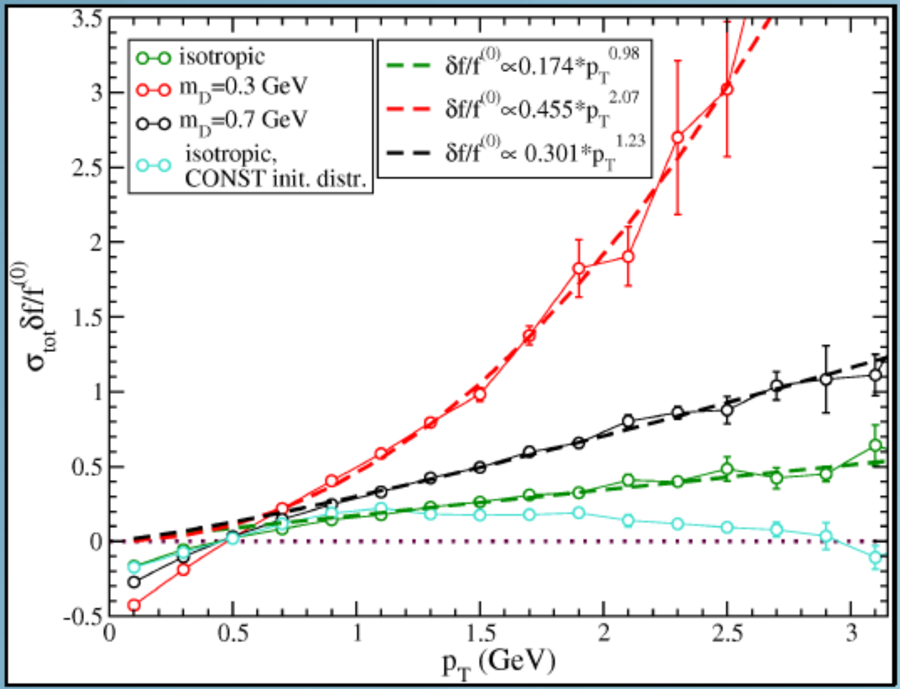
For $\sigma \to \infty$ we find the ideal Hydro limit:

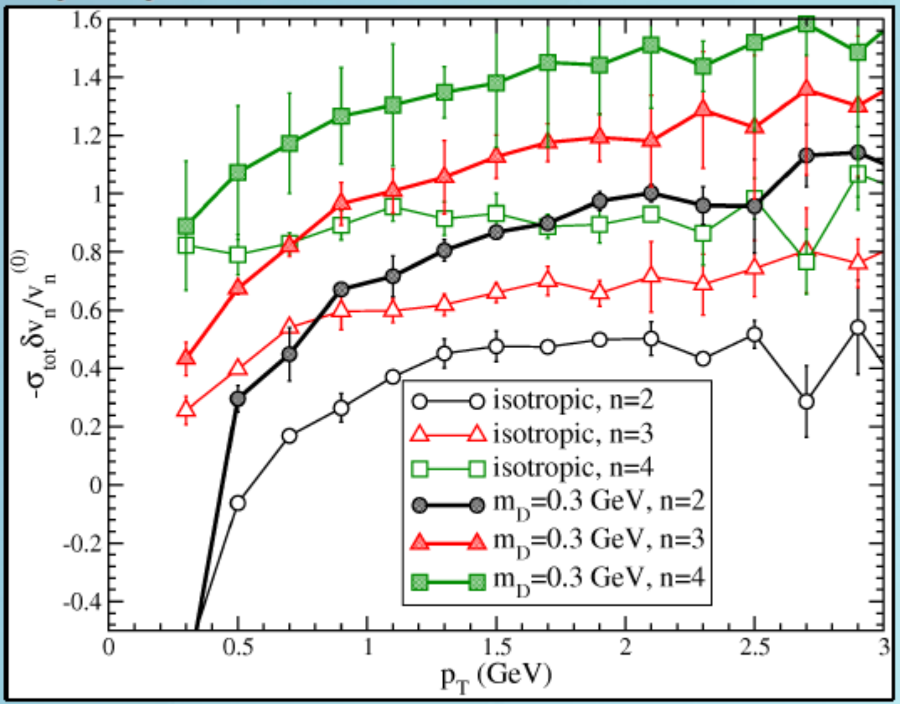
- $f^{(0)}$ is an exponential decreasing function.
- $f^{(0)}$ doesn't depends on microscopical details (i.e. mD).

- Universal behavior of $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\varepsilon_n$ is approximatively the same for all n and p_T.

From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

S.Plumari,G.L. Guardo,V. Greco,J.Y.Ollitrault NPA 941 (2015) 87





In δf and δv_n it is encoded the information about the microscopical details

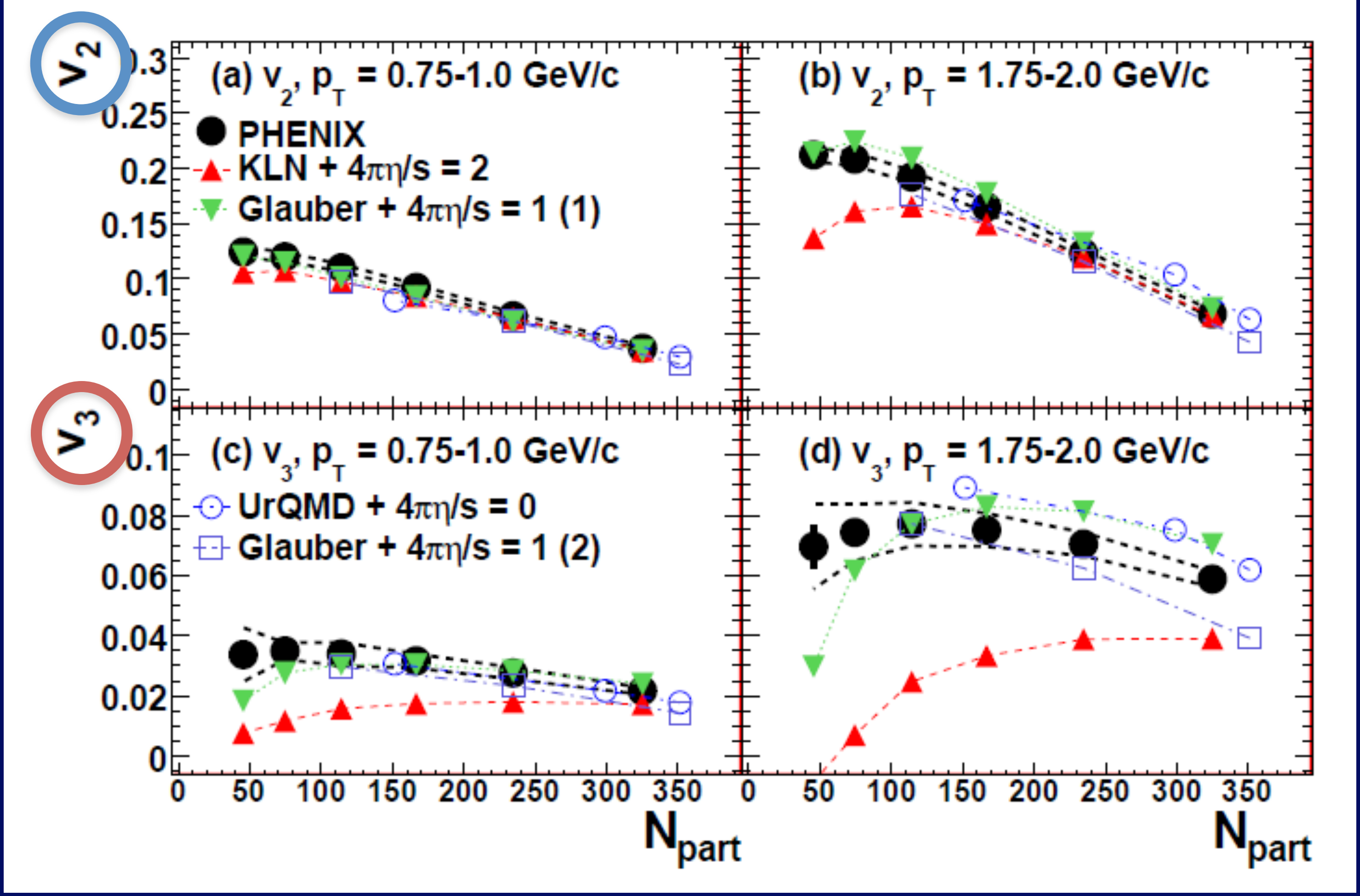
- **$\delta f(p_T)/f^{(0)} \propto p_T^{\alpha}$ with α = 1. - 2. and $\alpha(m_D)$.**

For isotropic σ similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)

- **Larger is n larger is the viscous correction to $v_n(p_T)$**
- **Scaling: for p_T > 1.5 GeV → $-\delta v_n(p_T)/v_n^{(0)} \propto n$**

Higher η/s for KLN leads to small v_3

Adare *et al.*, [PHENIX Collaboration], PRL **107**, 252301 (2011)



The value of η/s affects more higher harmonics!

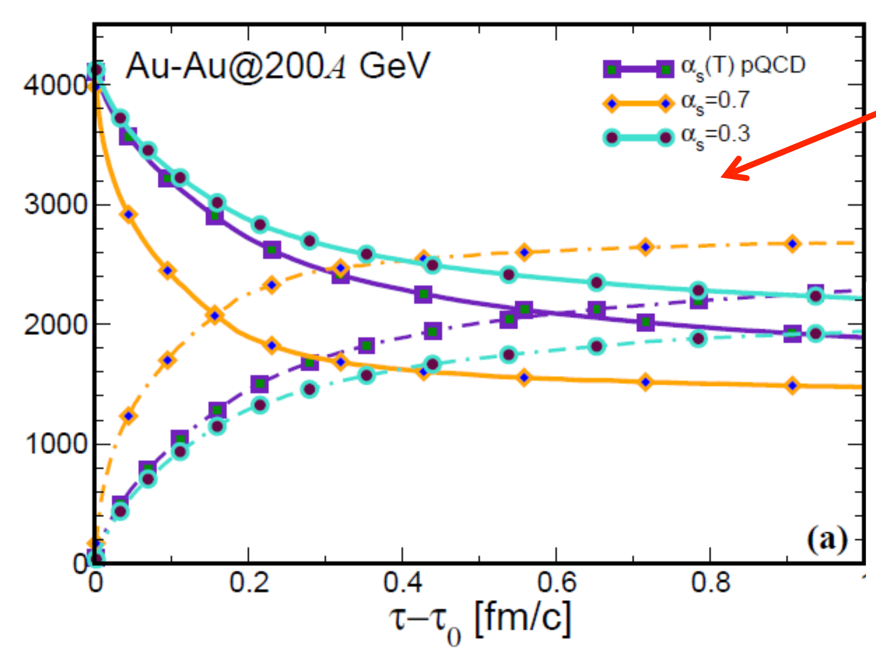
Can we discard KLN or CGC?!

Well at least before one should implement <u>both x and p space</u>

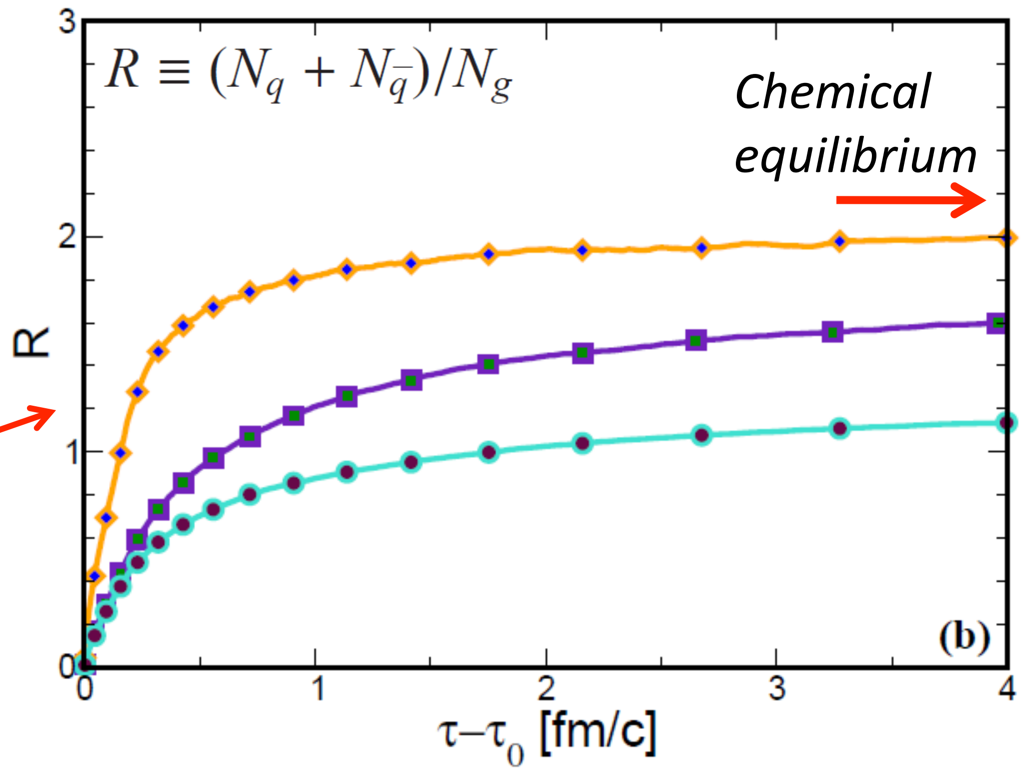
M. R. *et al.*, arXiv:1502.04596[nucl-th]
F. Scardina *et al.*, PLB**724** (2013)

Chemical equilibration of QGP

Assume no quarks in the initial stage: *how efficient QCD processes are to produce quarks*



Quark production by QCD inelastic processes is very fast



Chemical equilibrium

QGP close to chemical equilibrium

unless coupling is perturbatively small

Back-up

Simulate a fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p \rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p \rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$g(a=m_D/2T)$ correct function that fix the relaxation time for the shear motion

$0 < g(m_D/2T) < 2/3$

forward peaked

Isotropic $m_D \to \infty$

S.Plumari et al., PRC86(2012)

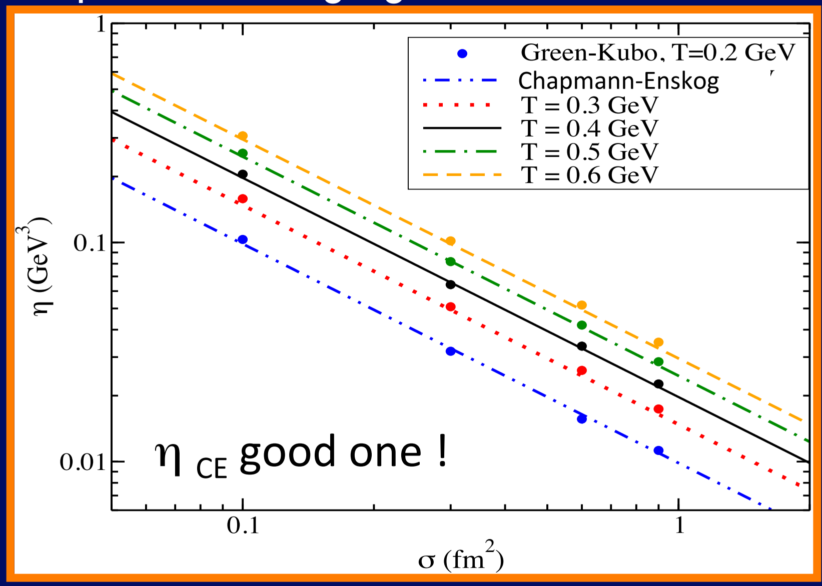
Transport code

$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha \rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

G. Ferini et al., PLB670 (2009)

Chapman-Enskog agrees with Green-Kubo

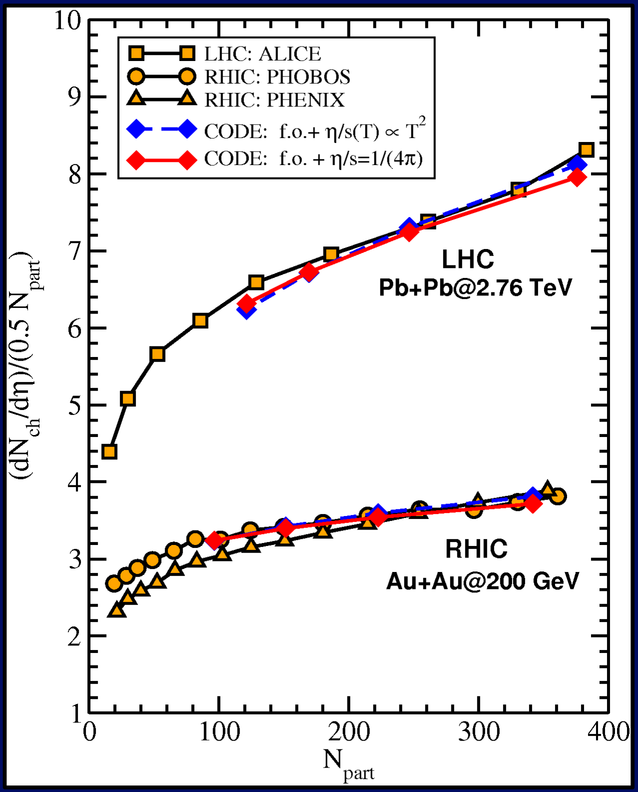


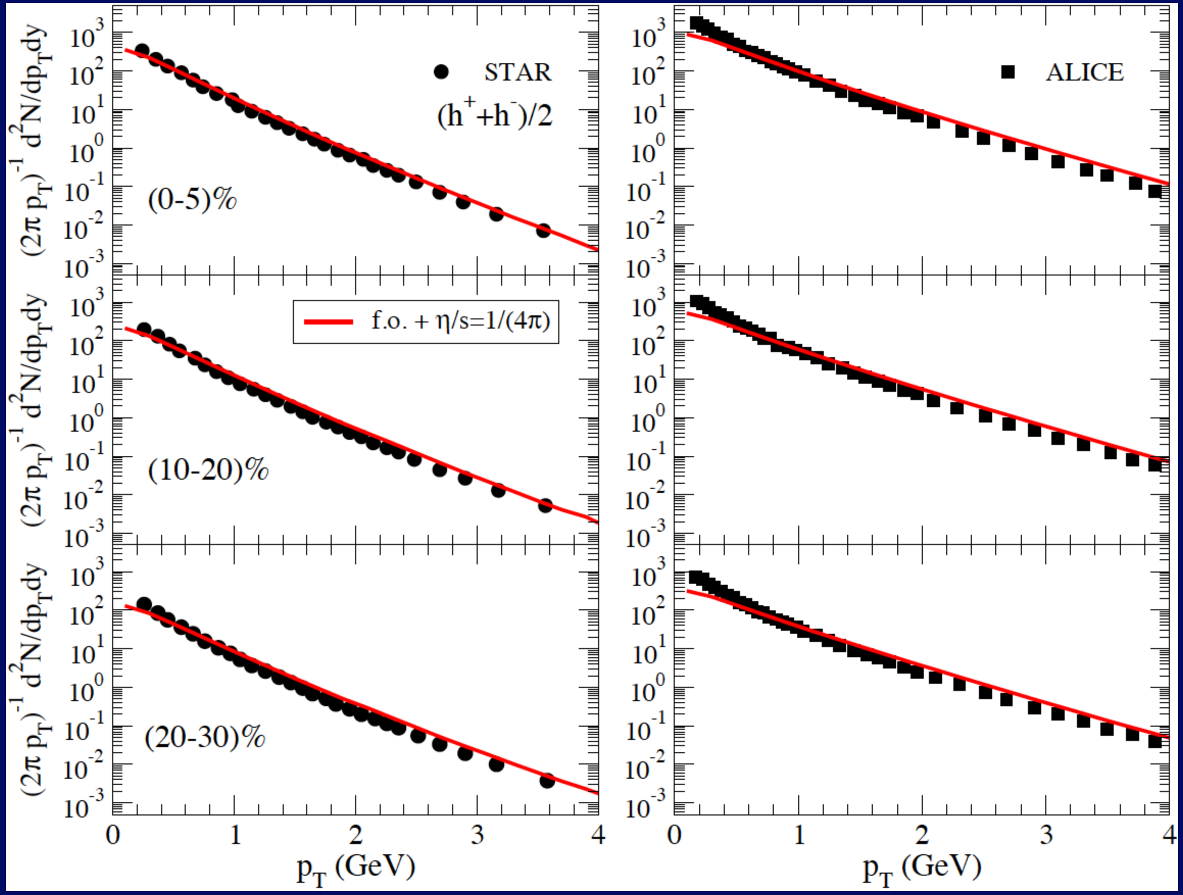
Multiplicity & Spectra

✧ r-space: standard Glauber condition

✧ p-space: Boltzmann-Juttner T_{max}=2(3) T_c [p_T<2 GeV]+ minijet [p_T>2-3GeV]

No fine tuning





Schwinger effect in Electrodynamics

Numerical estimates

Strictly speaking there is no a critical field, rather a probability for tunneling to occur. Given exponential suppression such a *probability becomes non negligible* as soon as

$$|\boldsymbol{E}| \approx m_e^2 \approx 10^{18} \text{ Volt/m}$$ *QED "critical field"*

Particles pop up is similar to dielectric breakdown.
We can compare the vacuum breakdown with typical critical fields of dielectric breakdown:

Mica: (40-100)x10^6 Volt/m

Thunderbolt: 3x10^6 Volt/m

Schwinger effect in Electrodynamics

Maxwell equations: static box

We will be interested to very simple geometrical configurations, in which

(.) Only one component of the electric field is non vanishing

(.) The electric field depends only on time and one space coordinate

$$\nabla \times \boldsymbol{B} = -j_M - \frac{\partial(\boldsymbol{E}+\boldsymbol{P})}{\partial t}$$

$\boldsymbol{P}(\boldsymbol{x},t)$ *electric dipole moment* at point (**x**,*t*)

j_M *Conduction current*
Due to charge movement

Given the symmetries of the problem:

$$\frac{dE}{dt} = -j_M - \frac{dP}{dt}$$

$$j_M = \sum_{species} g \int \frac{d^3\boldsymbol{p}}{|\boldsymbol{p}|} p_z f(|\boldsymbol{p}|, t)$$

The dipole moment is formed in the vacuum by the Schwinger effect:

$$j_D \equiv \frac{\partial P}{\partial t} = \int d^3 p g \frac{2E_T}{gE} \times \frac{dN}{d^4x d^3p}$$

Initial Conditions

✧ r-space: standard Glauber model

✧ p-space: Boltzmann-Juttner T_{max}=1.7-3.5 T_c [p_T<2 GeV]+ minijet [p_T>2-3GeV]

Discarded in viscous

We fix maximum initial T at RHIC 200 AGeV

T_{max0} = 340 MeV

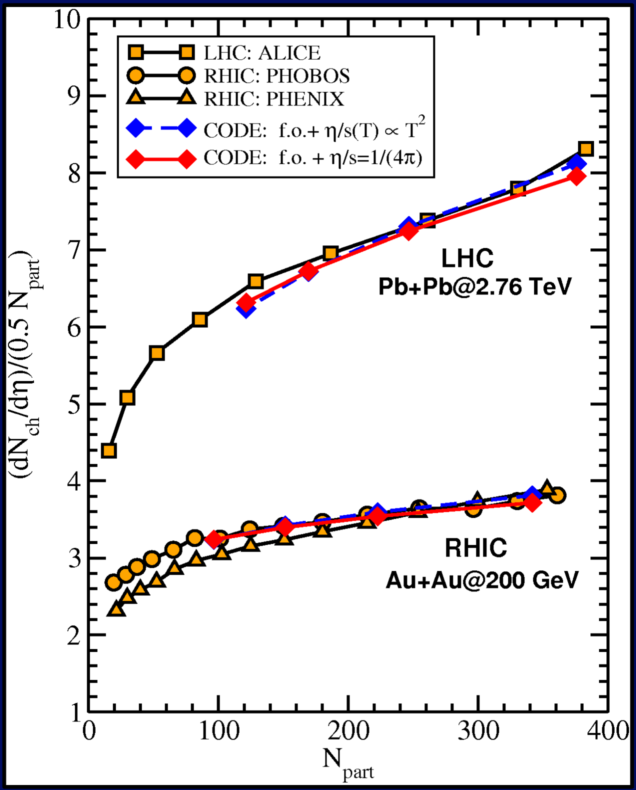
$T_0\,\tau_0$ =1 -> τ_0=0.6 fm/c

Typical hydro condition

Then we scale it according to initial ε

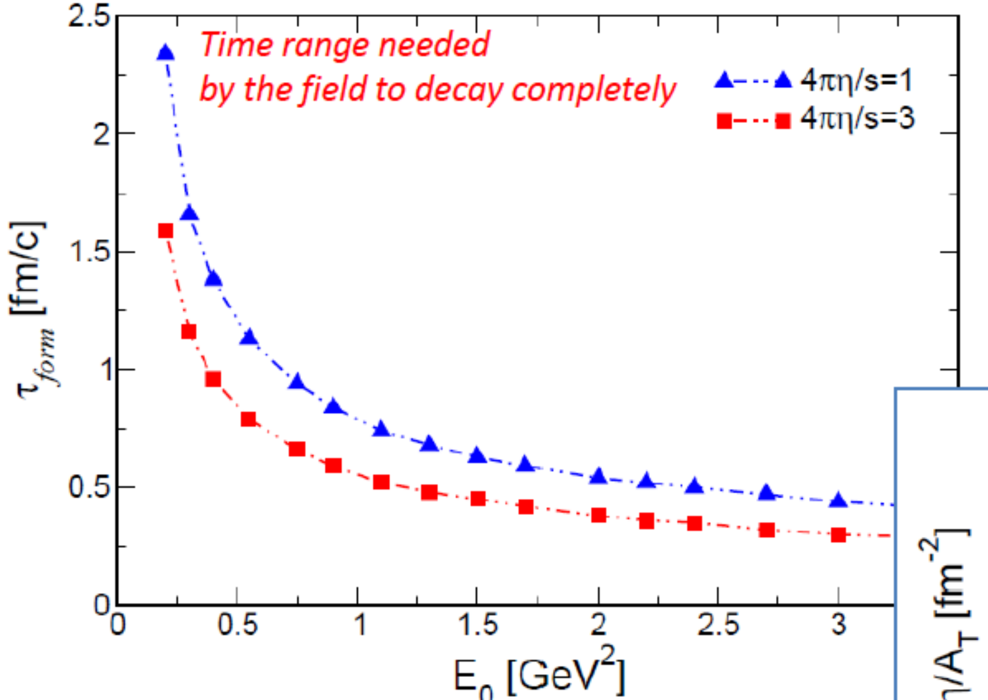
$$\frac{1}{\tau A_T}\frac{dN_{ch}}{d\eta} \propto T^3$$

	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	580 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c



Particles formation

Proper time for conversion to particles

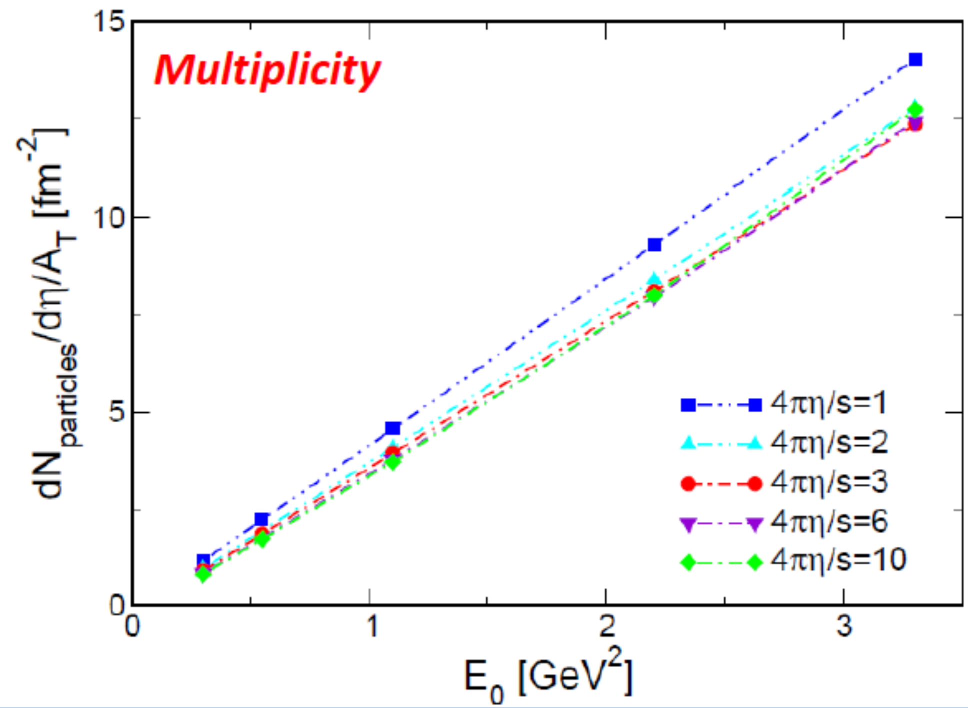


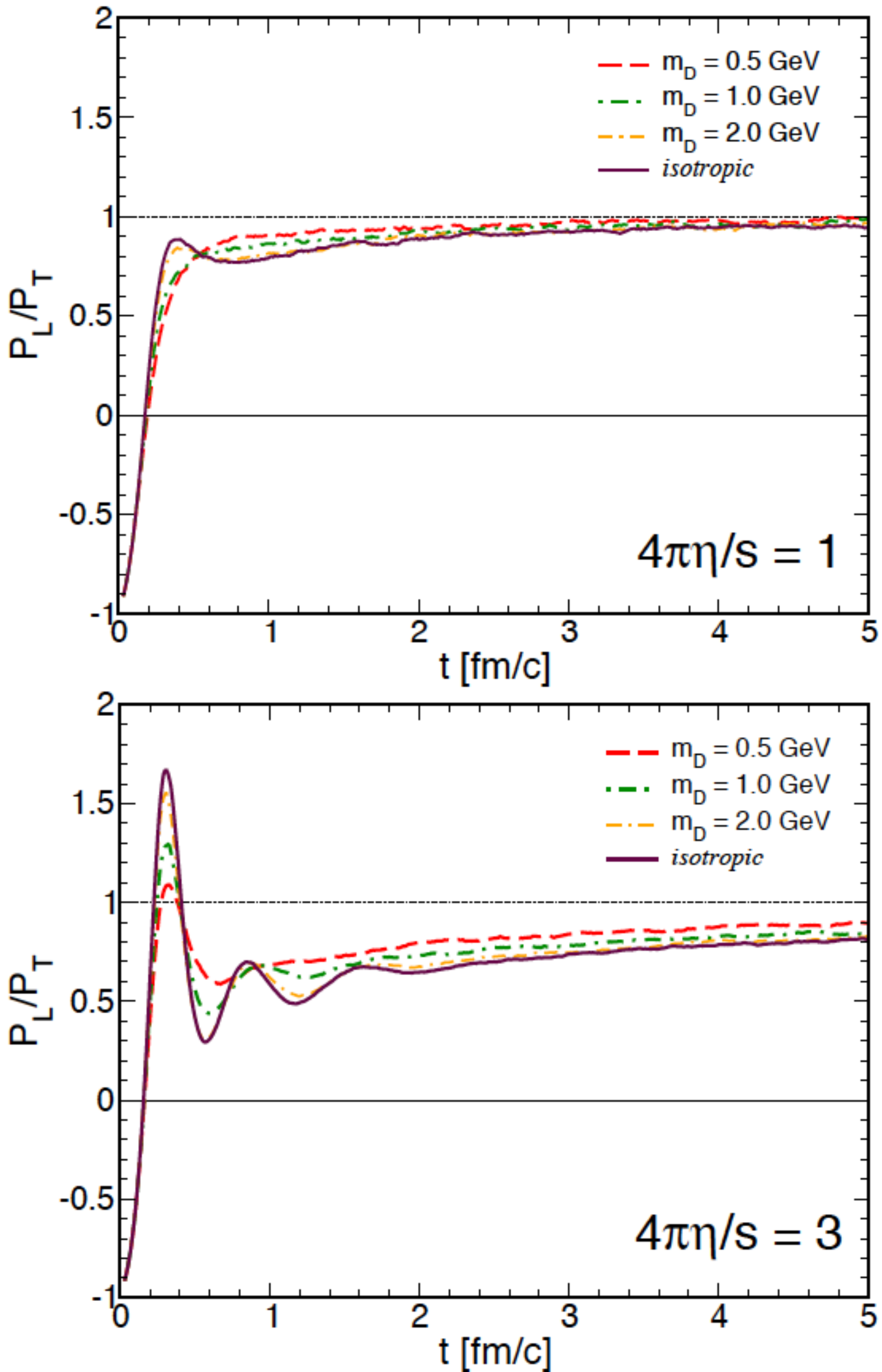
Field evolution satisfies:

$$\frac{d}{dt}\left(\frac{\boldsymbol{E}^2}{2}\right) = -\boldsymbol{j}\cdot\boldsymbol{E}$$

hence, smaller field implies slower decay.

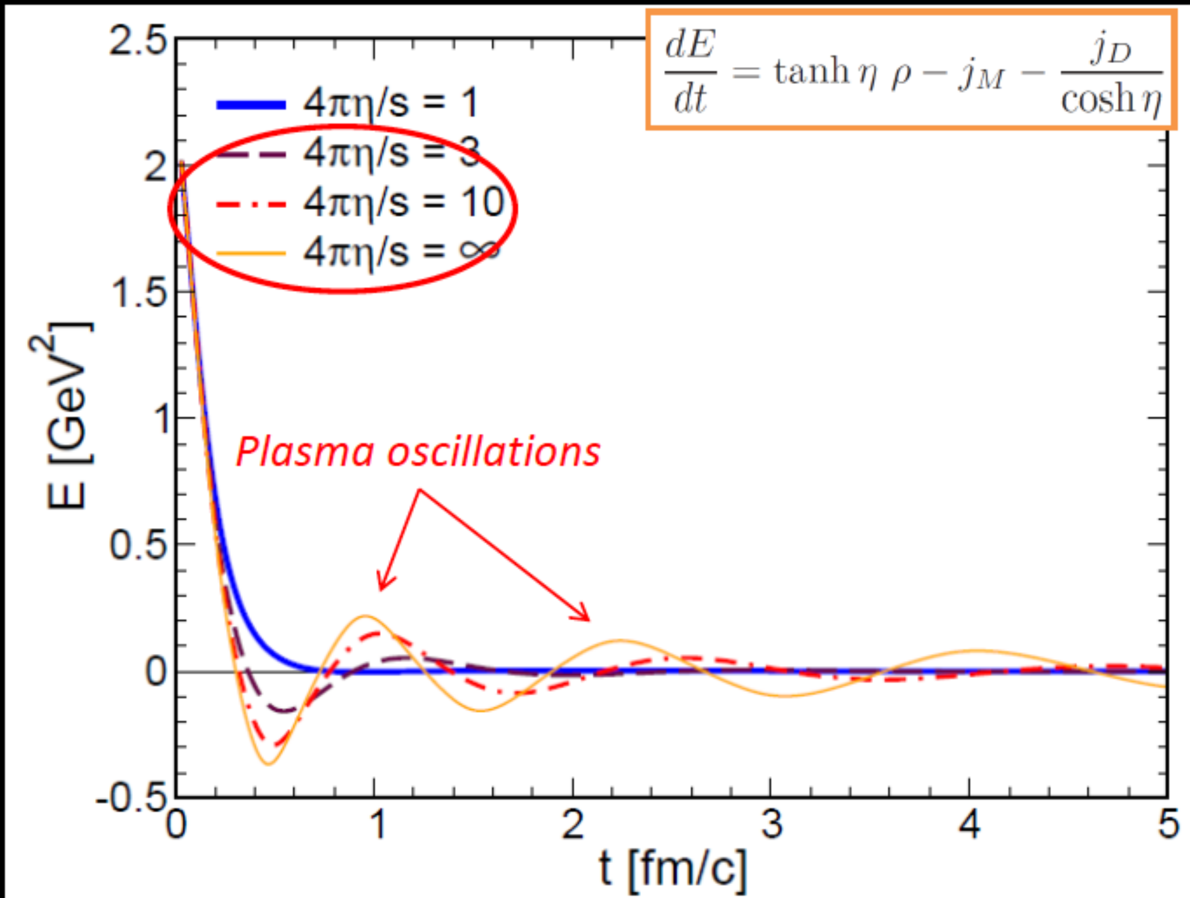
Unless initial field is very small, formation time is less than 1 fm/c





m_D = 0.5 GeV
m_D = 1.0 GeV
m_D = 2.0 GeV
isotropic
P_L/P_T
$4\pi\eta/s = 1$
t [fm/c]
m_D = 0.5 GeV
m_D = 1.0 GeV
m_D = 2.0 GeV
isotropic
P_L/P_T
$4\pi\eta/s = 3$
t [fm/c]

Field decay in 1+1D expansion



$$\frac{dE}{dt} = \tanh\eta\ \rho - j_M - \frac{j_D}{\cosh\eta}$$

ρ *electric charge density*

j_M *electric current*

j_D *polarization current*

Large η/s

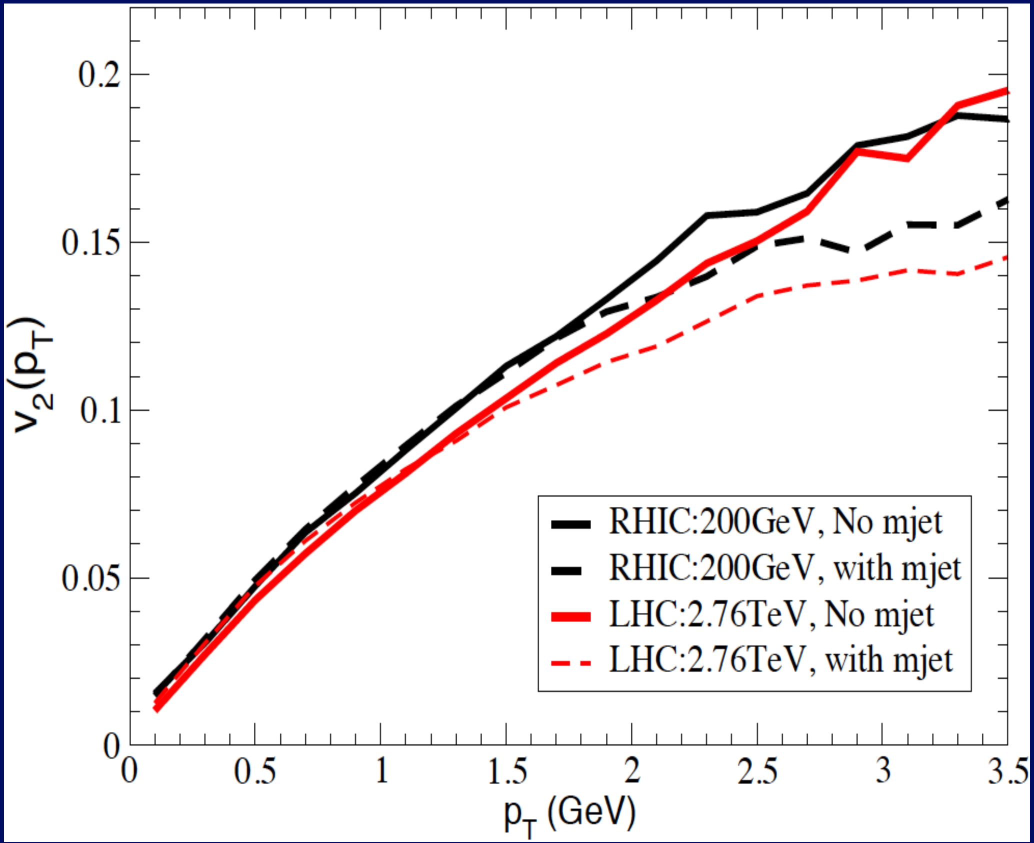
(.) Initial times dynamics faster, due to electric current:

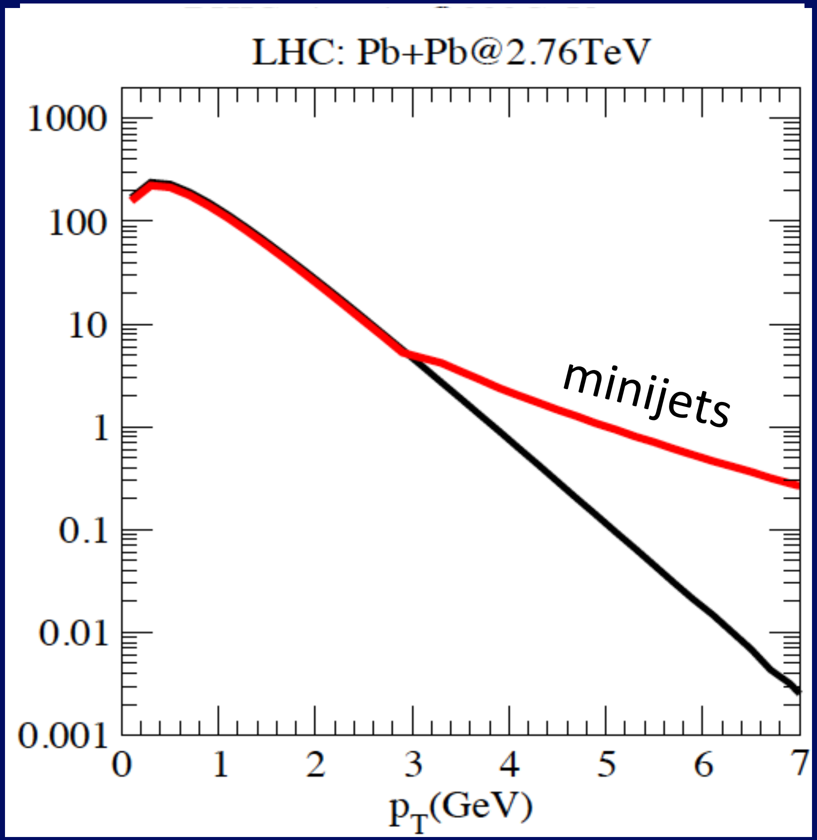
$$\frac{d}{dt}\left(\frac{\boldsymbol{E}^2}{2}\right) = -\boldsymbol{j}\cdot\boldsymbol{E}$$

Smaller coupling (i.e. smaller isotropization efficiency) favors development of conductive electric currents: the net effect is a continuous energy exchange between particles and field.

Plasma oscillations controlled by electric current

Non equilibrium at larger p_T: impact of minijets on $v_2(p_T)$





Mini-jets starts to affect $v_2(p_T)$ for p_T>1.5 GeV

Effect non-negligible. Again a flatter spectrum leads to smaller v2