Flow in Maxwell Field
Initial Velocity for Collectivity

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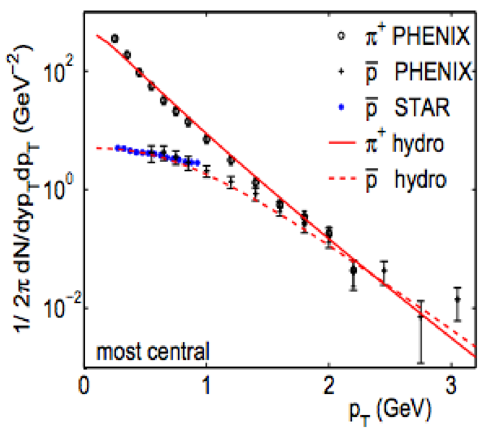
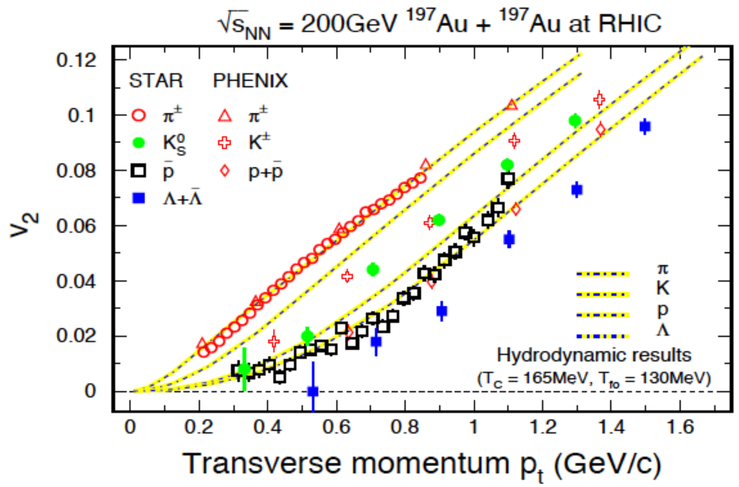
OUTLINE

- Coarse graining scale and hydrodynamic observables
- Evaluating the flow profile in PHSD
- Quantum Flow from Initial Fields ?
- Coarse-Graining in Wigner Function

- “Success” of hydrodynamics in describing the experimental observations in heavy-ion collisions



P. Kolb and U. Heinz, arXiv:nucl-th/0305084



Oldenburg M.D. (STAR Collab.), J. Phys. G 31, S437

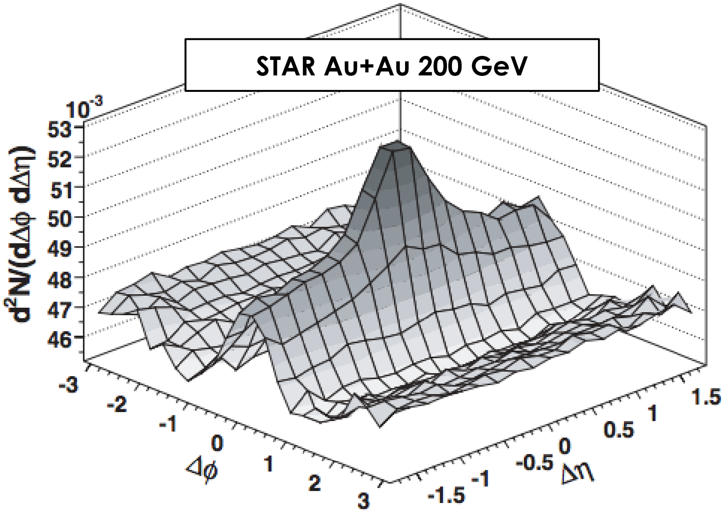
Ideal hydro:

- local thermal equilibrium
- conservation laws + equation of state

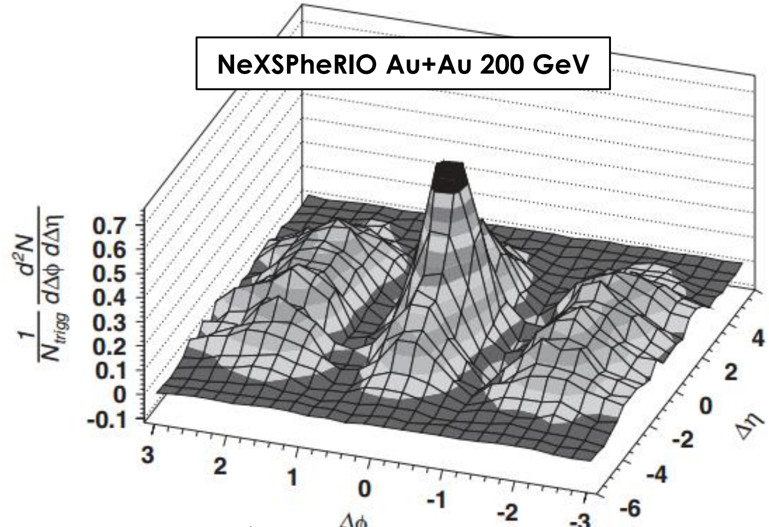
collective flow:

- hydro models can reproduce the anisotropic momentum distribution of the final particles
- the system behaves collectively (like a strongly interacting liquid)

- “Success” of hydrodynamics in describing the experimental observations in heavy-ion collisions



STAR Collab., Phys. Rev. C, 80, 064912



Takahashi J., *et al.*, Phys. Rev. Lett. 103, 242301

Event-by-event hydrodynamics:

- 2-particle correlation analysis considering inhomogeneous initial condition + hydro evolution reproduces the ridge structure observed experimentally

Some puzzles in Flow...

- It is puzzling that pA and AA data cannot be described with the same set of parameters, since one expects that the same type of fluid is created in both collisions…

I. Kozlov, M. Luzuma, G. S. Denicol, S. Jeon, Gale C, Signatures of collective behavior in small systems, arXiv:1412.3147v1

Direct Photon Puzzle

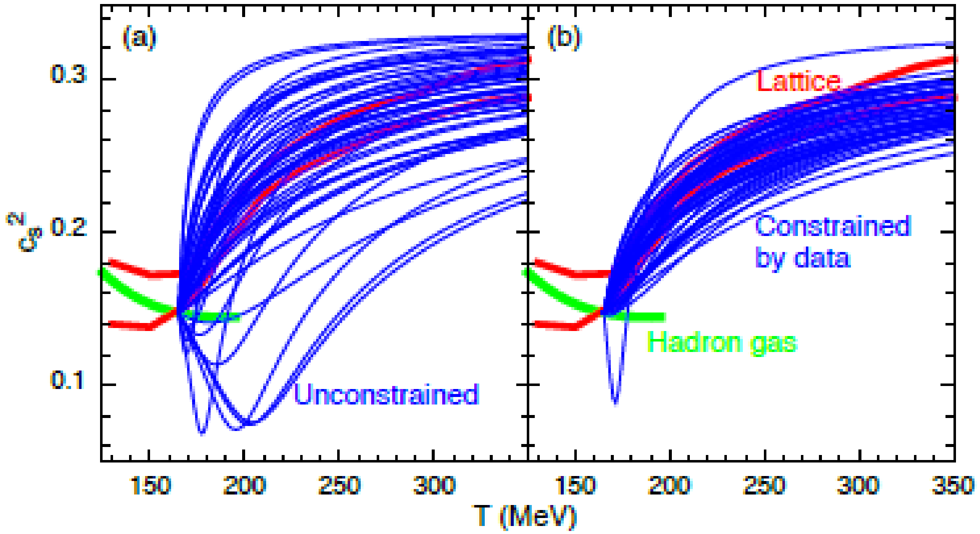
Taken from the presentation of K. Reygers,
"Ab initio approaches in many-body QCD confront heavy-ion experiments | December 15, 2014 |

- Maybe many more photons from late stage close to T_c and hadron gas phase (need large increase in HG rates) [van Hees, He, Rapp, arXiv:1404.2846]
  - Theoretical justification?
- Maybe just bremsstrahlung from the HG? ($m+m \to m+m+\gamma$, $m+B \to m+B+\gamma$) [Linnyk, Cassing, Bratkovskaya, arXiv:1311.0279]
  - Important source in PHSD transport model
- Exotic new photon source, e.g., related to large initial *B* field? [Basar, Kharzeev, Skokov., arXiv:1206.1334]
  - seems unlikely to me (centrality dependence, $\sqrt{s}$ dependence, v_3)
- Initial flow before hydro evolution starts, e.g., IPGlasma model?
  - important, but does not address the missing photon yield
- Glasma photons, i.e., large photon production in very early gluon-rich phase? [McLerran, Schenke, arXiv:1403.7462], [Klein-Bösing, McLerran, arXiv:1403.1174]
  - promising, but so far based on simplified models
  - calculations from first principles needed

Coarse graining scale of hydrodynamic modeling

"… why at all the hydrodynamic approach works so well for such a violent and almost microscopic collisional process?"

(Ph. Mota, *et al*, Eur. Phys. J A, 48, 165)



S. Pratt et al., Constraining the Equation of State of Superhadronic Matter from Heavy-Ion Collisions, Phys. Rev. Lett. 114 (2015) 202301 (v_2(Pt-weighted, 2-cenralities), HBT, Spectrum), without viscosity.

Coarse graining scale of hydrodynamic modeling

> *"… why at all the hydrodynamic approach works so well for such a violent and almost microscopic collisional process?"*
>
> (Ph. Mota, *et al*, Eur. Phys. J A, 48, 165)

- Important questions: (J. Berges, J.-P. Blaizot, F. Gelis, J. Phys. G, 39 085115 (2012))
  - What is the degree of local thermal equilibrium required for hydrodynamic behavior?
  - Can collective flow measurements provide any signal about local equilibration/isotropization?
  - What is the coarse graining scale of hydrodynamic modeling for such violent collisional process?

Hydro in Relativistic Heavy Ion Collision How Quantitatively precise ?

Uncertainties associated

- EoS, Transport Coefficients (?)
- Freezeout Mechanism (Tough)
- Initial Condition (Challenging)
- Event-by-Event vs. Ensemble Average? (To be clarified)

We need to keep some care,..

- In a Japanese popular- saying, “Typically in the following three conditions,

- in a twilight, - from far, - half-hidden by a hat

We need to keep some care,..

- In a Japanese popular- saying, “Typically in the following three conditions,

- in a twilight, - from far, - half-hidden by a hat

make a boy (girl) looking nice,…. “

(We usually see what we WANT to see)

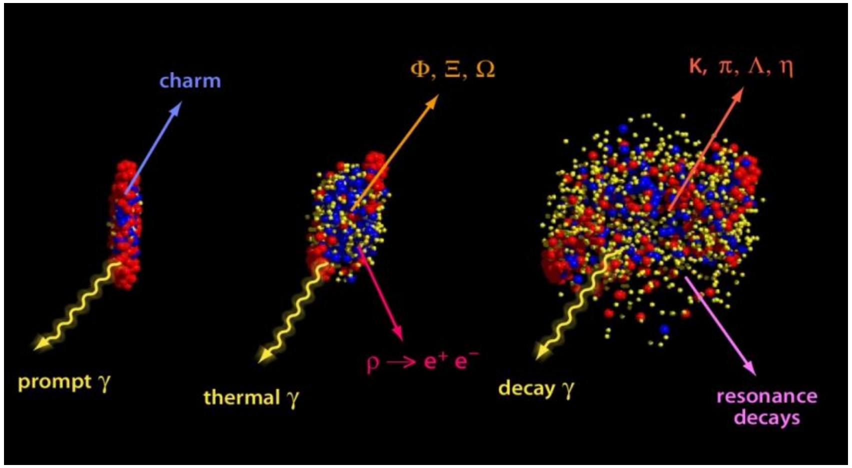
Counter-examples of Real Hydro ("Pseudo Hydro")

- Schrödinger Equation – Quantum Hydro
- Isotropic massless gas – Non Equilibrium
- Initial state correlation in free streaming case
- Event average -> Effective EoS

  :

NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

- Parton-Hadron-String Dynamics



W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;

NPA831 (2009) 215;

W. Cassing, EPJ ST 168 (2009) 3

A. Peshier, W. Cassing, PRL 94 (2005) 172301;

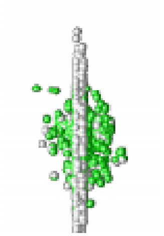
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

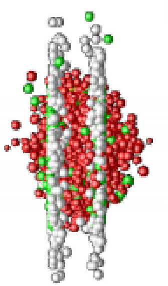
NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

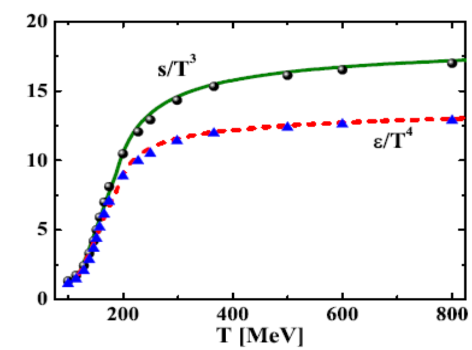
- Parton-Hadron-String Dynamics
  - Initial A+A collisions:

LUND string model



    - *String formation in primary NN collisions*
    - *String decay to pre-hadrons (B – baryons; m – mesons)*
  - Formation of QGP phase: $\varepsilon > \varepsilon_{critical}$





    - *dissolution of pre-hadrons into massive colored quarks + mean field energy*

$$B \rightarrow qqq,\ \ m \rightarrow q\bar{q} \quad \forall \quad U_q$$

    - ***Dynamical QuasiParticle Model (DQPM)***

defines quark spectral functions, i.e. masses $M_q(\varepsilon)$ and widths $\Gamma_q(\varepsilon)$ + mean field potential at a given ε (local energy density)

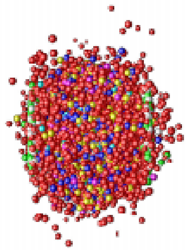
(ε related by lQCD EoS to *T* in the local cell)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;

NPA831 (2009) 215; EPJ ST 168 (2009) 3; NPA856 (2011) 162.

NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

- Parton-Hadron-String Dynamics
  - Partonic phase – QGP:



✓ *quarks and gluons = dynamical quasi-particles with off-shell spectral functions (width, mass) defined by DQPM*

✓ *self generated mean field potential for quarks and gluons U_q, U_g from DQPM*

✓ ***EoS of partonic phase**: crossover from lattice QCD (fitted by DQPM)*

✓ *(quasi-) elastic and inelastic parton-parton interactions using effective cross-section from the DQPM*

  - Hadronization:

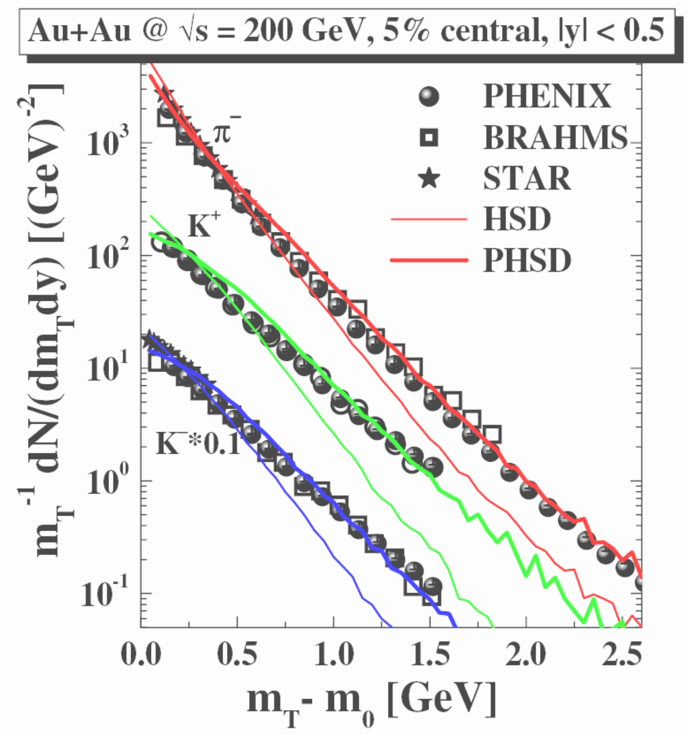
✓ *massive, off-shell (anti-)quarks with broad spectral functions hadronize to off-shell mesons and baryons or color neutral excited states – 'strings'*

$$g \rightarrow q + \bar{q}, \quad q + \bar{q} \leftrightarrow meson \ ('string')$$

$$q + q + q \leftrightarrow baryon \ ('string')$$

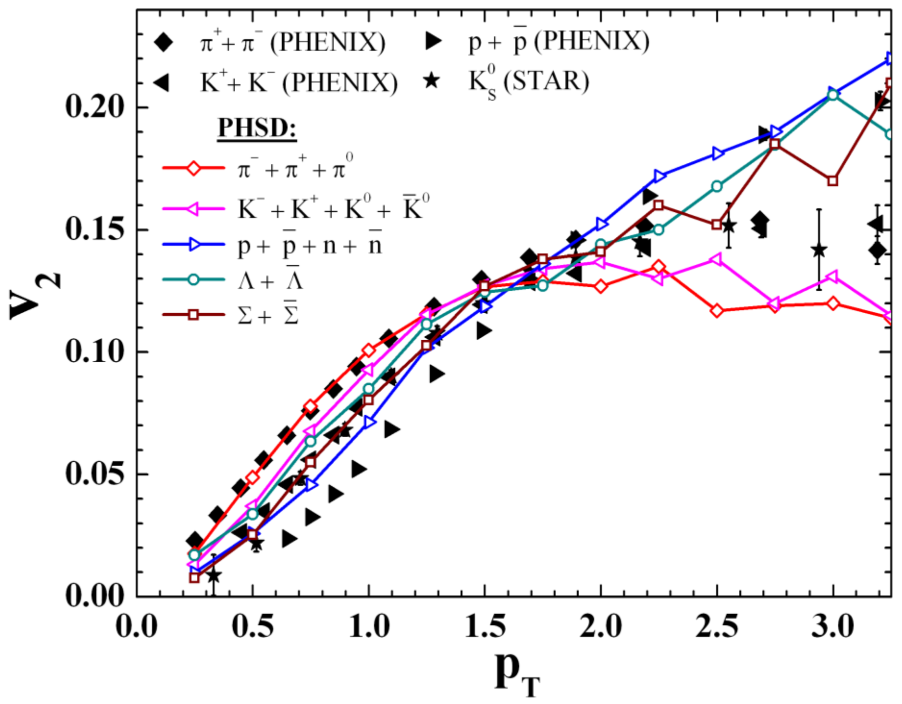
NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

- Parton-Hadron-String Dynamics



W. Cassing & E. Bratkovskaya,
NPA 831 (2009) 215

E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162



E. Bratkovskaya, W. Cassing, V. Konchakovski,
O. Linnyk, NPA856 (2011) 162

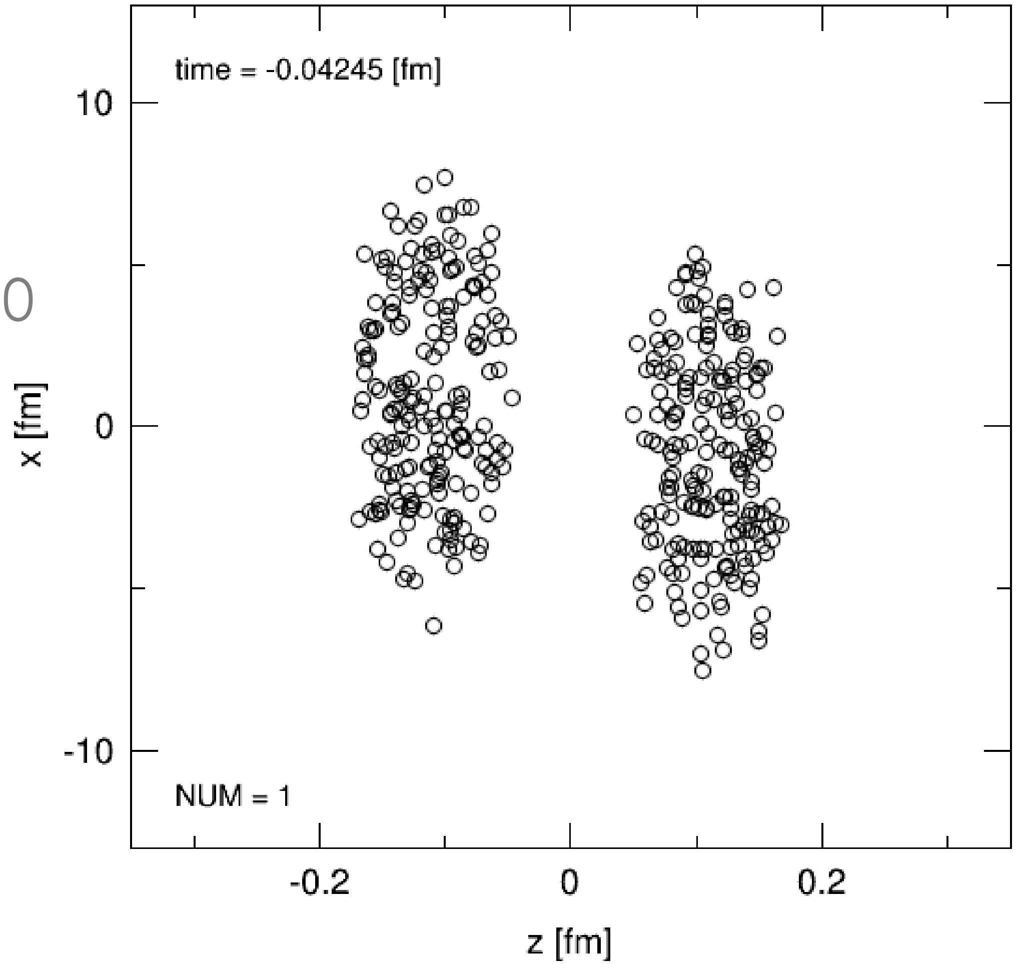
Evaluating the system evolution in PHSD

PHSD event:

- Au+Au @ 200 GeV
- b = 2 fm
- NUM: 1, 2, 5, 10, 20, 30

NUM: number of parallel events

mean-field potential for *quarks* and *gluons*



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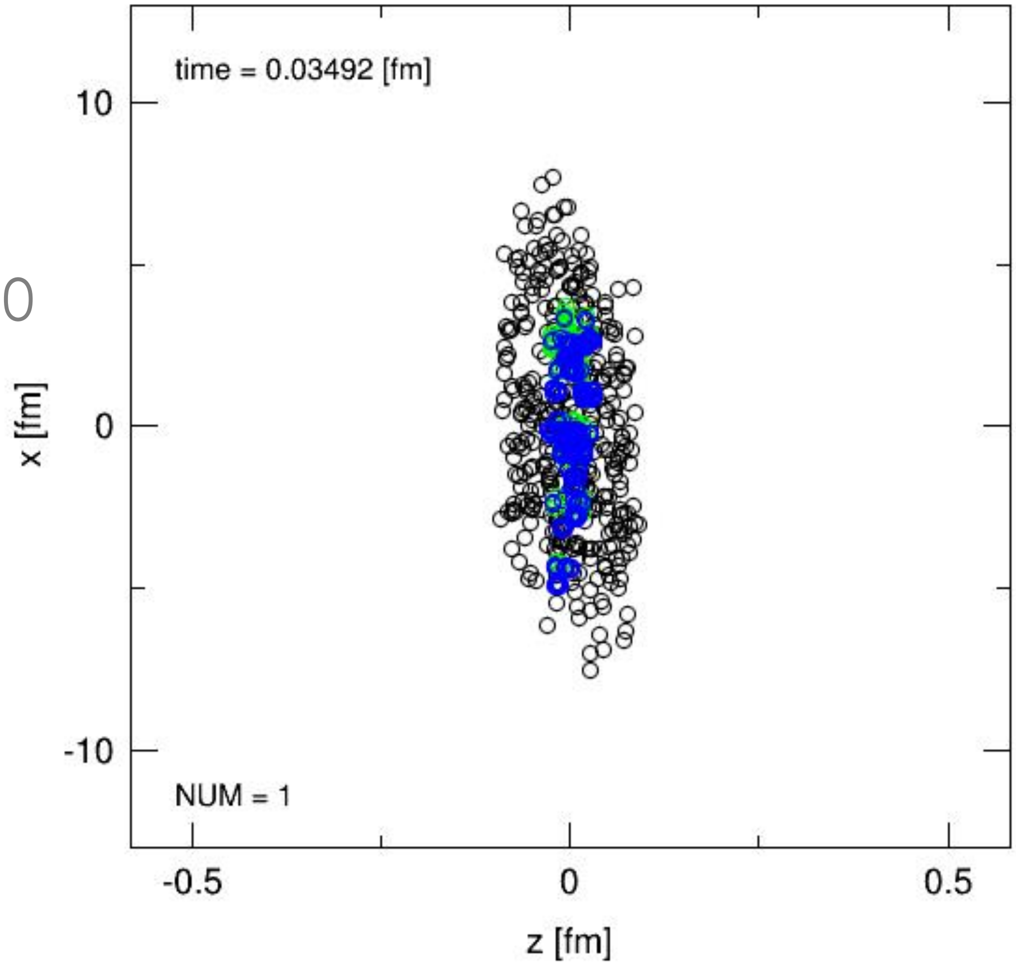

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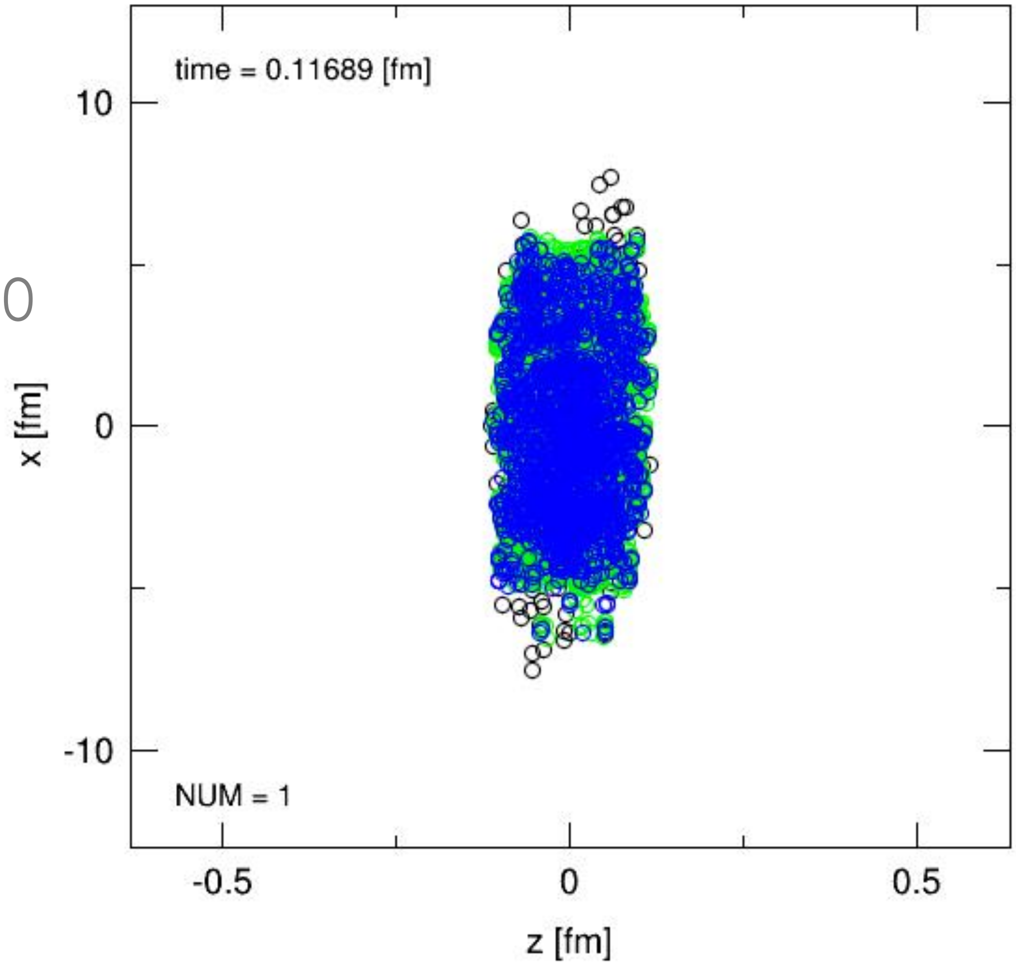

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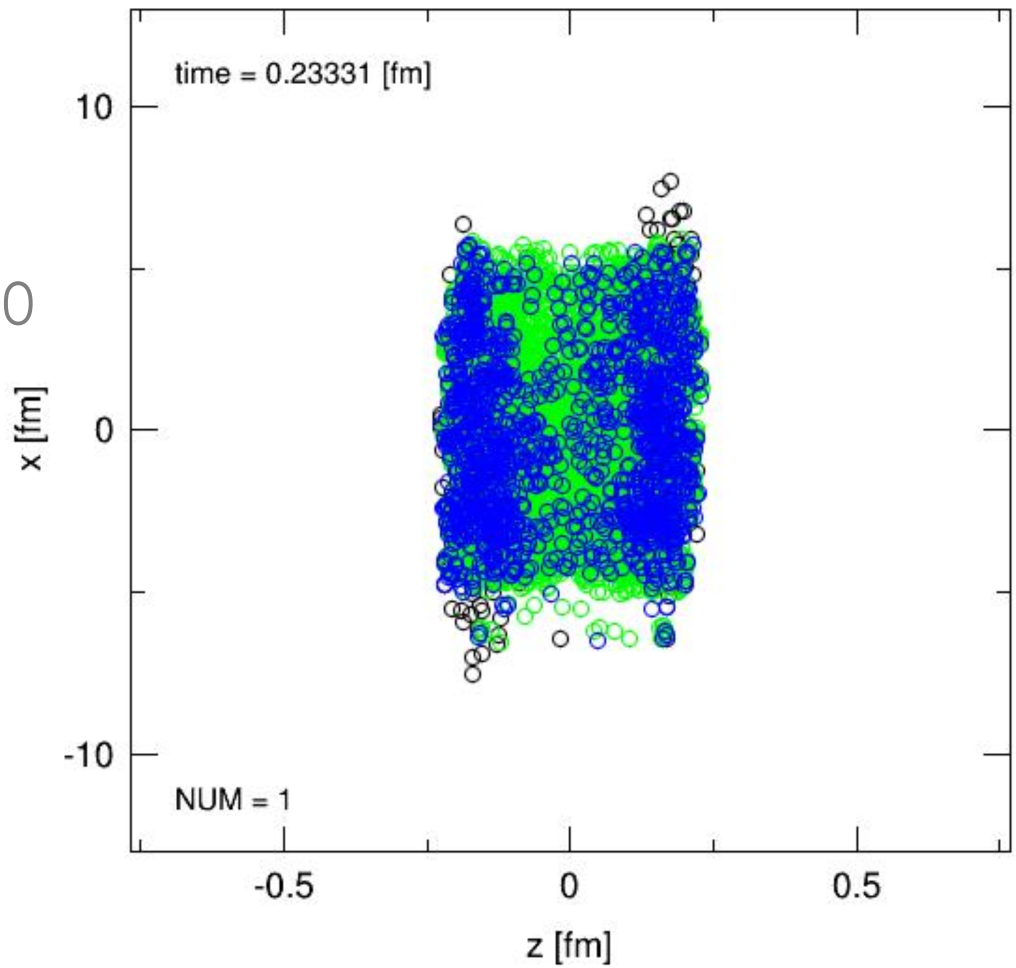

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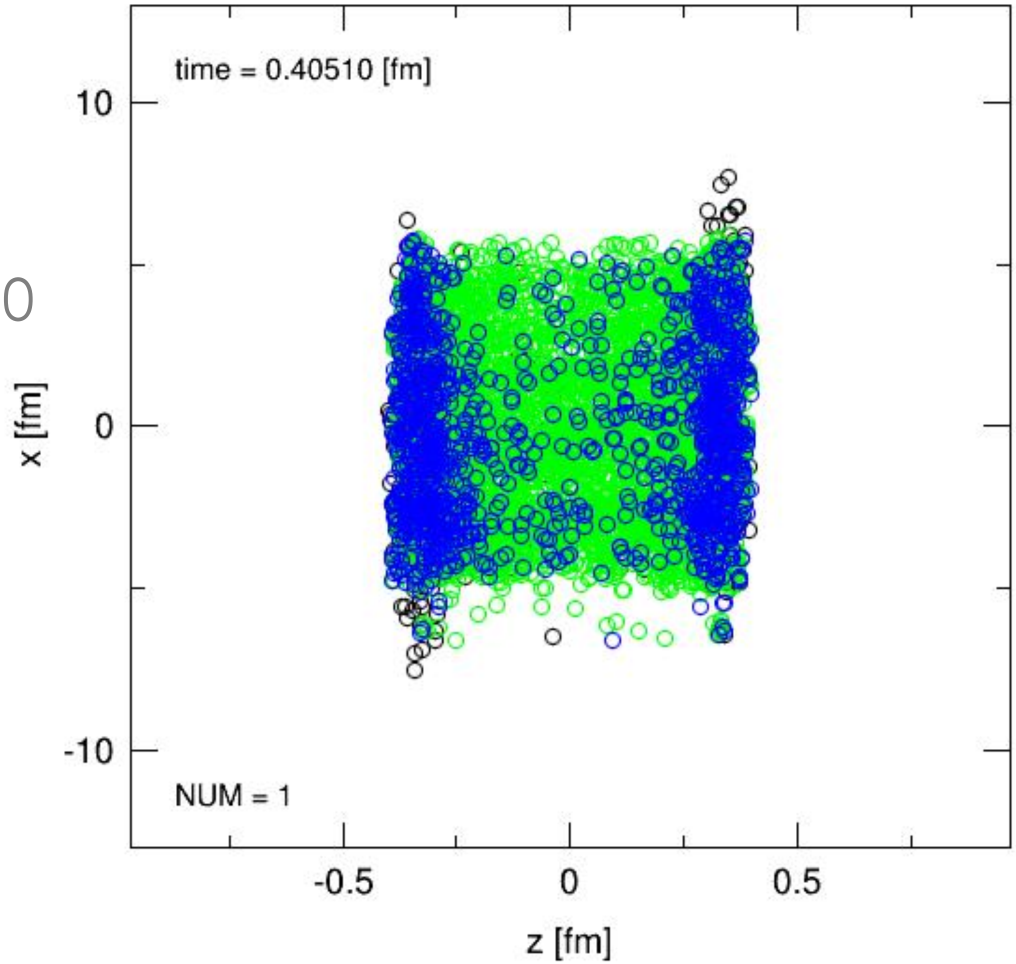

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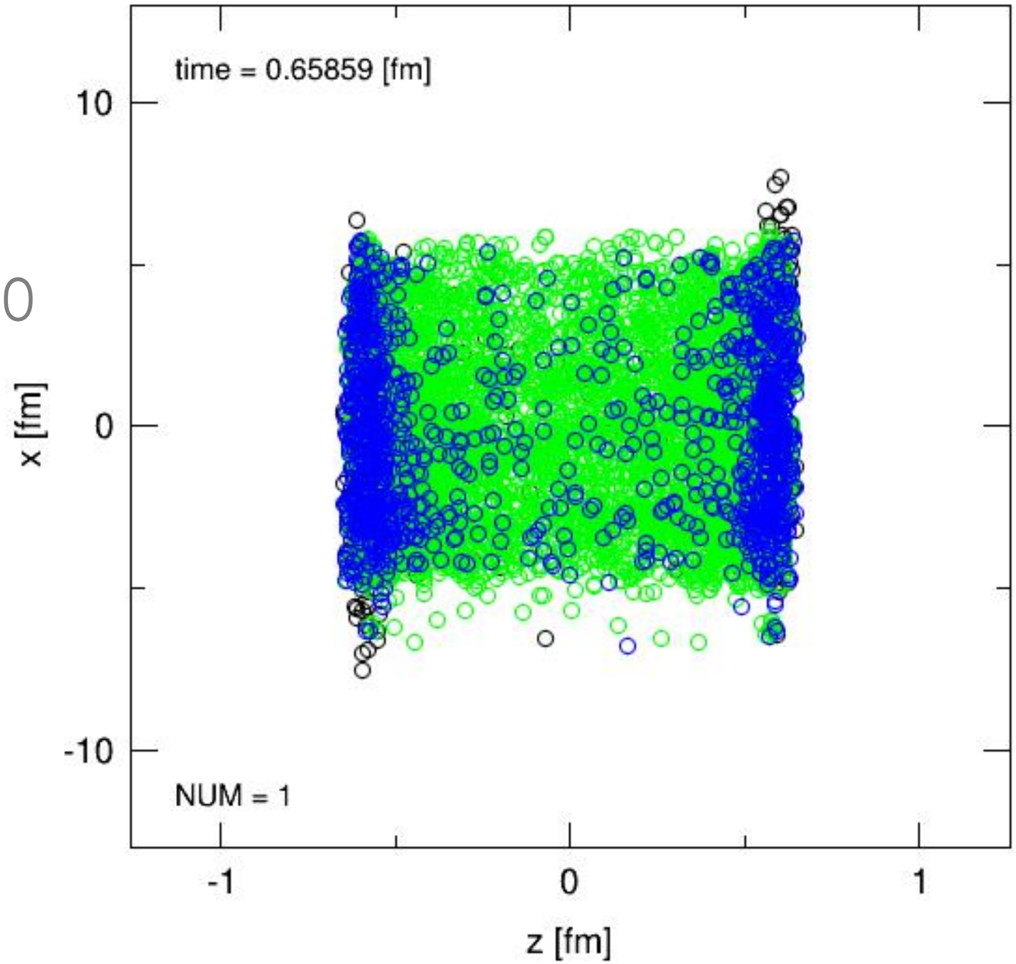

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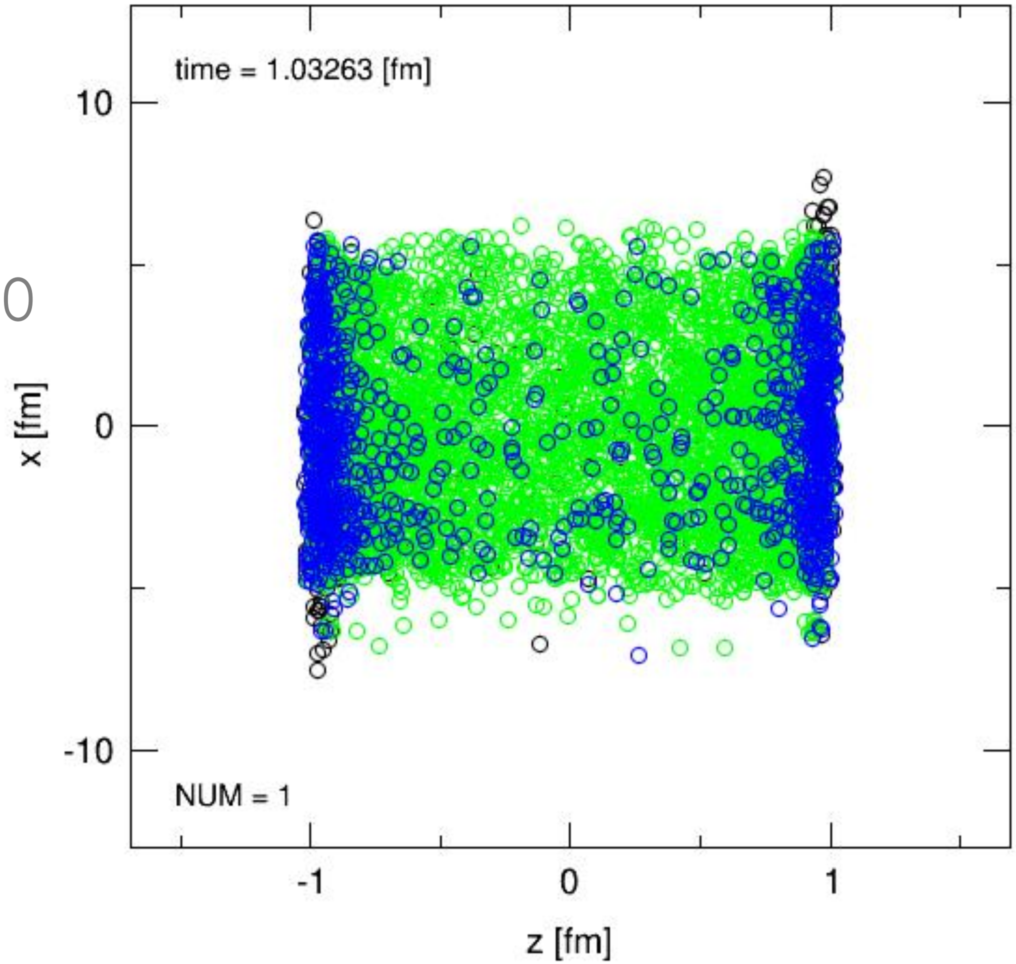

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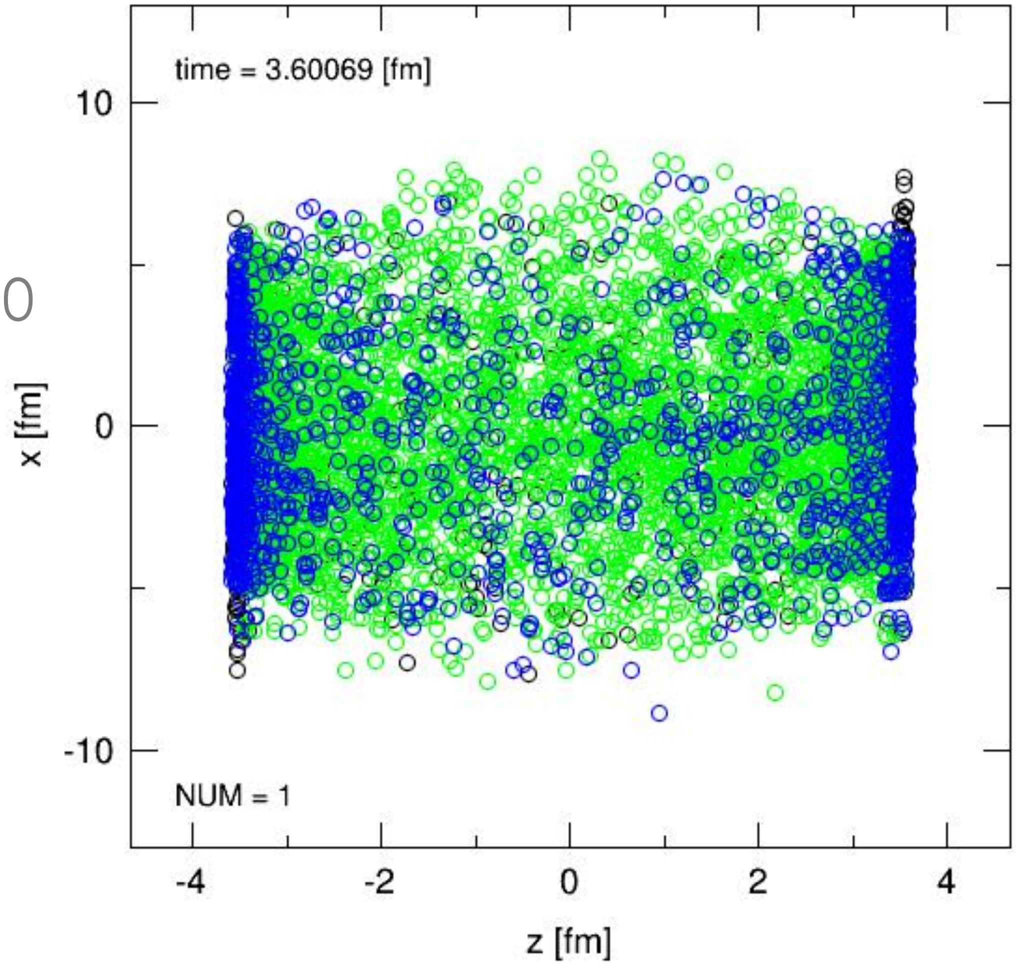

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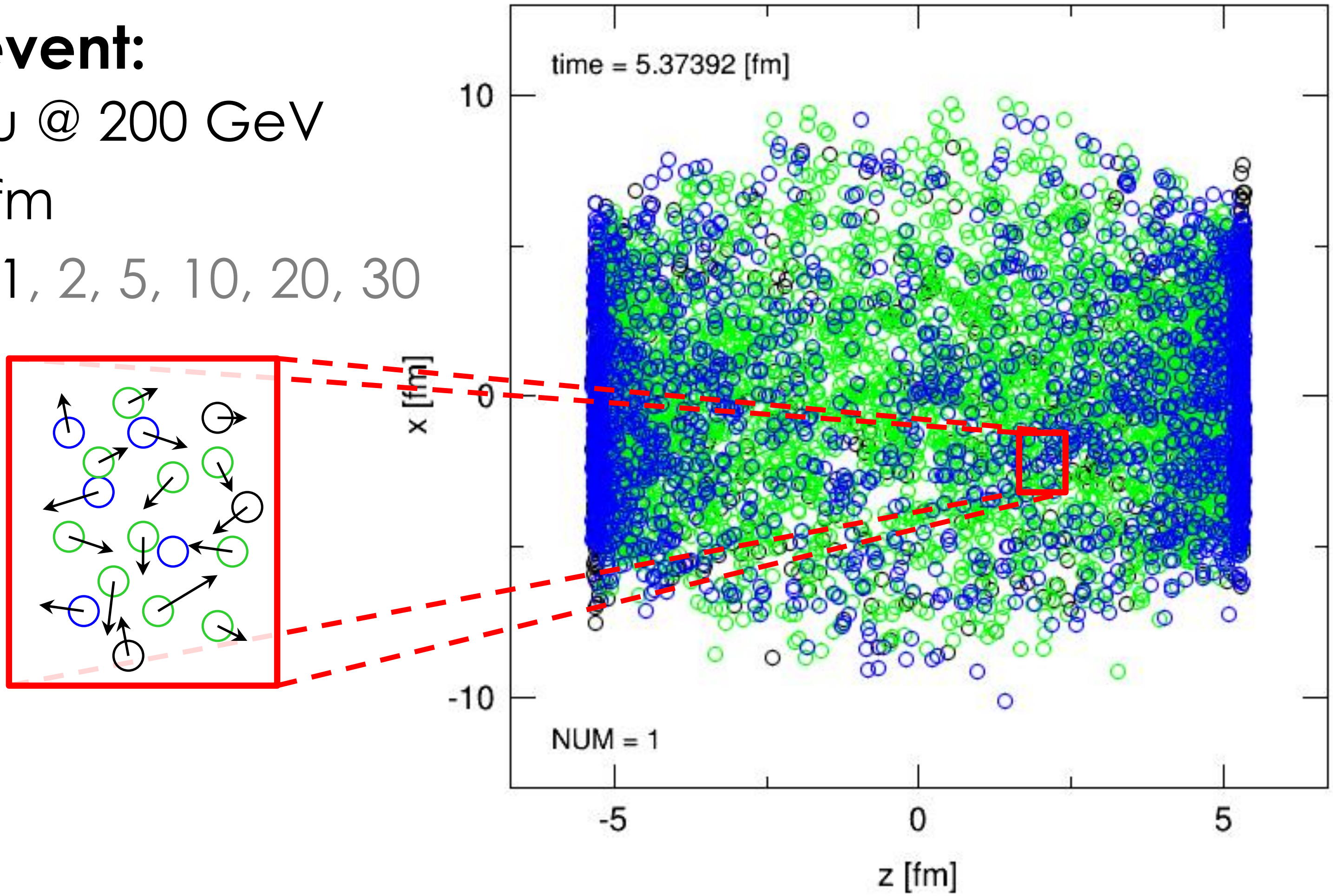

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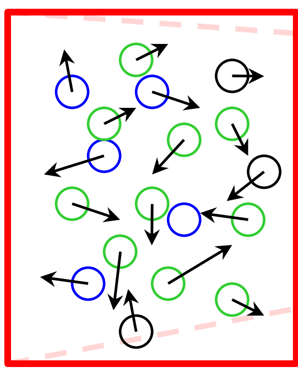
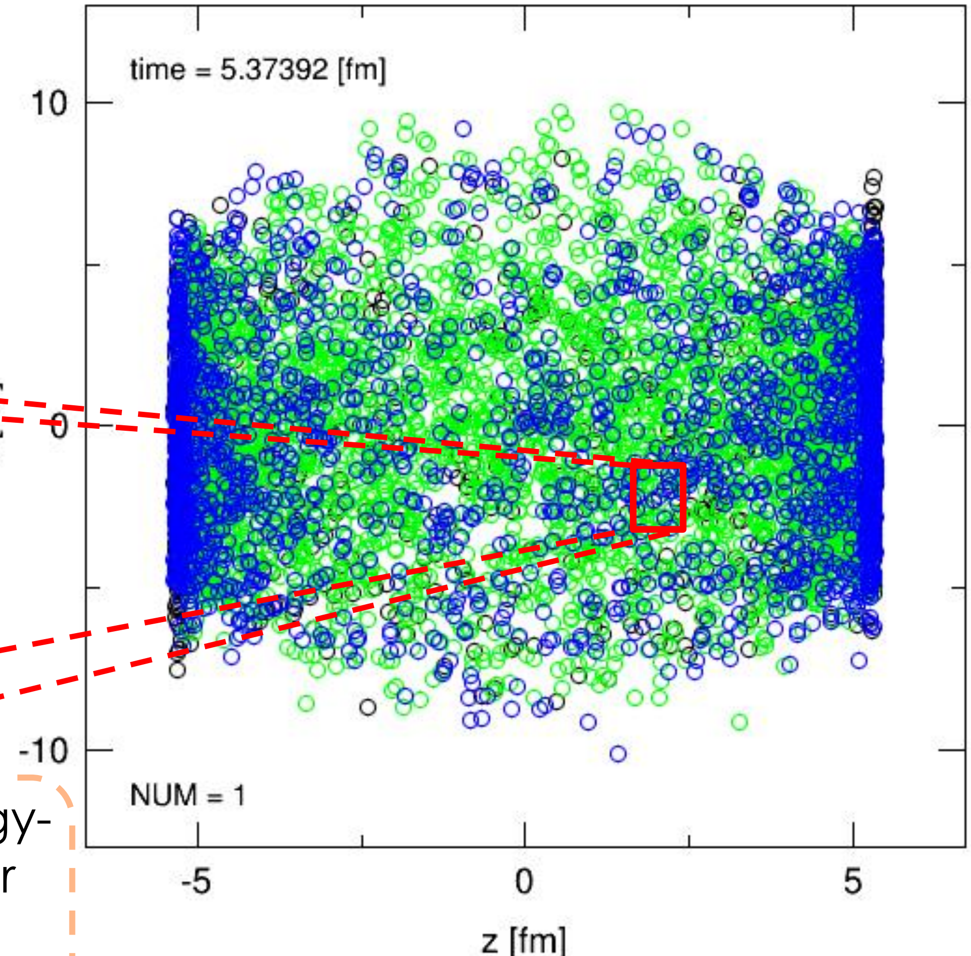




compute the energy-momentum tensor

$$T^{\mu\nu}$$

COMPUTING THE ENERGY-MOMENTUM TENSOR

$$T^{\mu\nu}(\vec{r},t) = \sum_i \frac{p_i^\mu(t) p_i^\nu(t)}{p_i^0(t)} \delta^3(\vec{r} - \vec{r}_i)$$

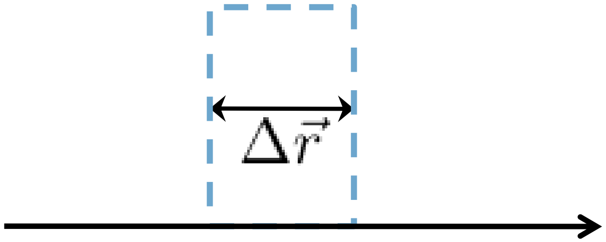
Computing the energy-momentum tensor

$$T^{\mu\nu}(\vec{r},t) = \sum_i \frac{p_i^{\mu}(t)p_i^{\nu}(t)}{p_i^0(t)}\delta^3(\vec{r}-\vec{r}_i)$$

- Replace the delta by a kernel function:

$$\delta^3(\vec{r}-\vec{r}_i) \rightarrow W(\vec{r}-\vec{r}_i(t);\Delta\vec{r})$$

rectangular box
$$\begin{cases} W = k & \text{inside a box} \\ W = 0 & \text{outside a box} \end{cases}$$



$$\left(\int W(\vec{r};\Delta\vec{r})d^3\vec{r} = 1\right)$$

Computing the energy-momentum tensor

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- Replace the delta by a kernel function:

$$\delta^3(\vec{r}-\vec{r}_i) \to W(\vec{r}-\vec{r}_i(t); \Delta\vec{r})$$

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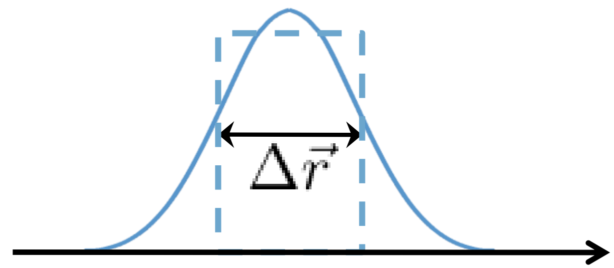


$$\int W(\vec{r}; \Delta\vec{r}) d^3\vec{r} = 1$$

- Define a smoothing (gaussian) kernel:

$$W(\vec{r}-\vec{r}_i; h_T, h_L) = A\, e^{-\frac{1}{2}\left[\left(\frac{x-x_i}{h_T}\right)^2 + \left(\frac{y-y_i}{h_T}\right)^2 + \left(\frac{z-z_i}{h_L}\right)^2\right]}$$

$h_T \to$ transverse scale

$h_L \to$ longitudinal scale

Computing the energy-momentum tensor

- Diagonalizing the energy-momentum tensor

  (solve the eigenvalue/eigenvector problem)

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} \longrightarrow \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 \\ 0 & 0 & P_2 & 0 \\ 0 & 0 & 0 & P_3 \end{pmatrix}$$

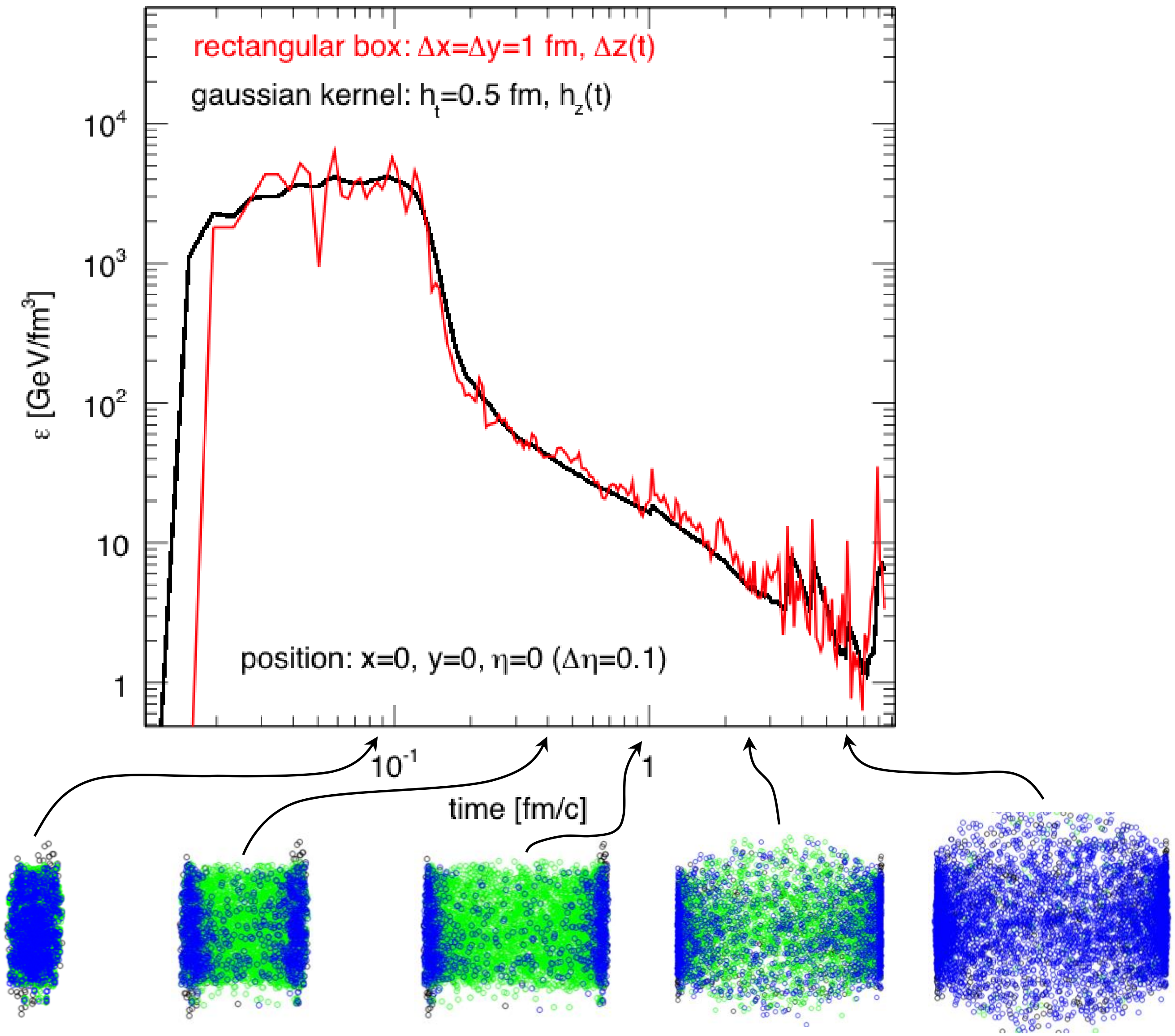
  - The four velocity $\boldsymbol{u^\mu}$ can be identified with the eigenvector associated with the eigenvalue $\boldsymbol{\varepsilon}$

    $T^{\mu\nu} u_\nu = \varepsilon u^\mu$ *(time-like eigenvector)*

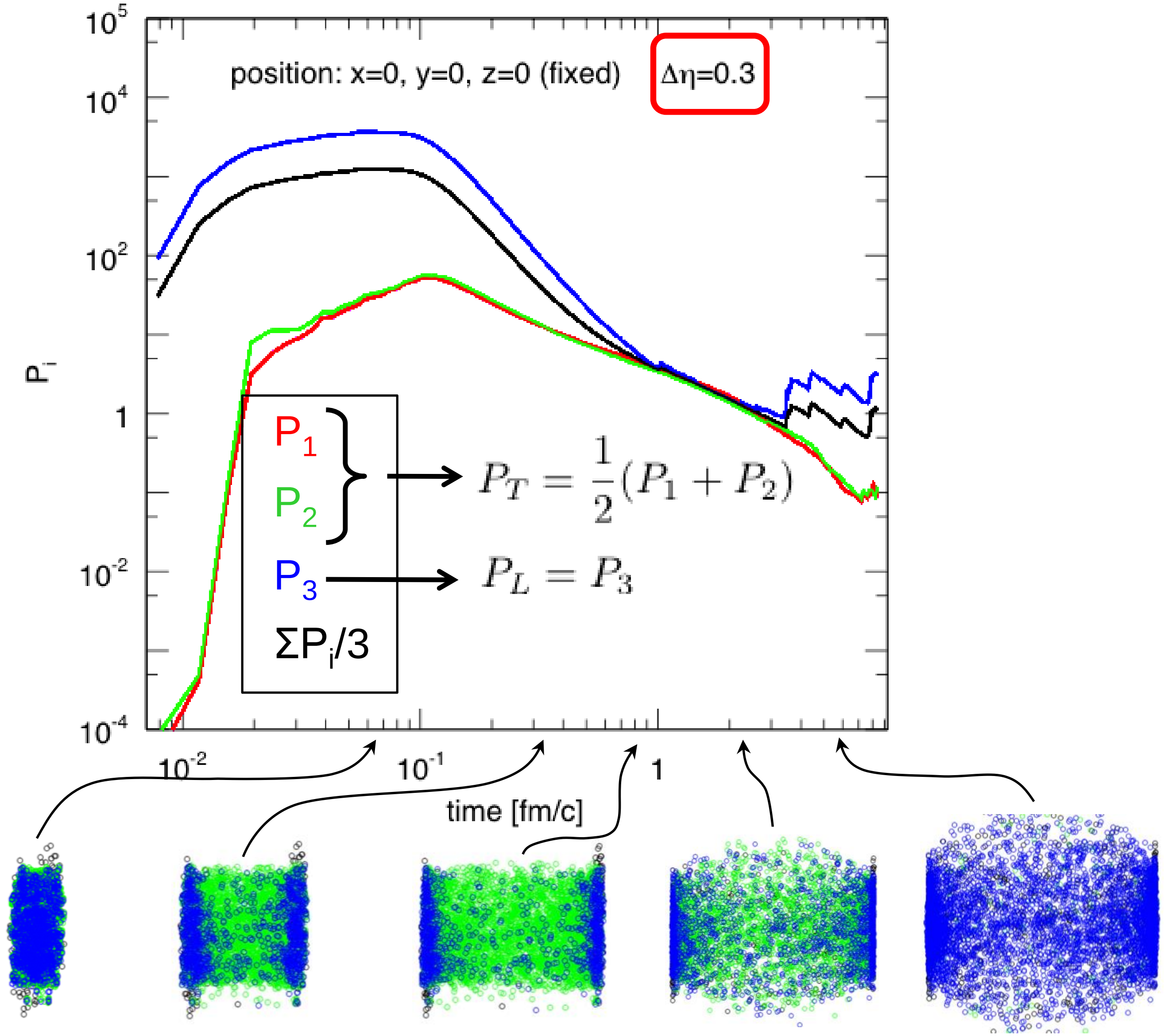
  - The flow profile:

    $\vec{\beta},\ \varepsilon,\ P$

"Rectangular box" vs "Gaussian kernel"

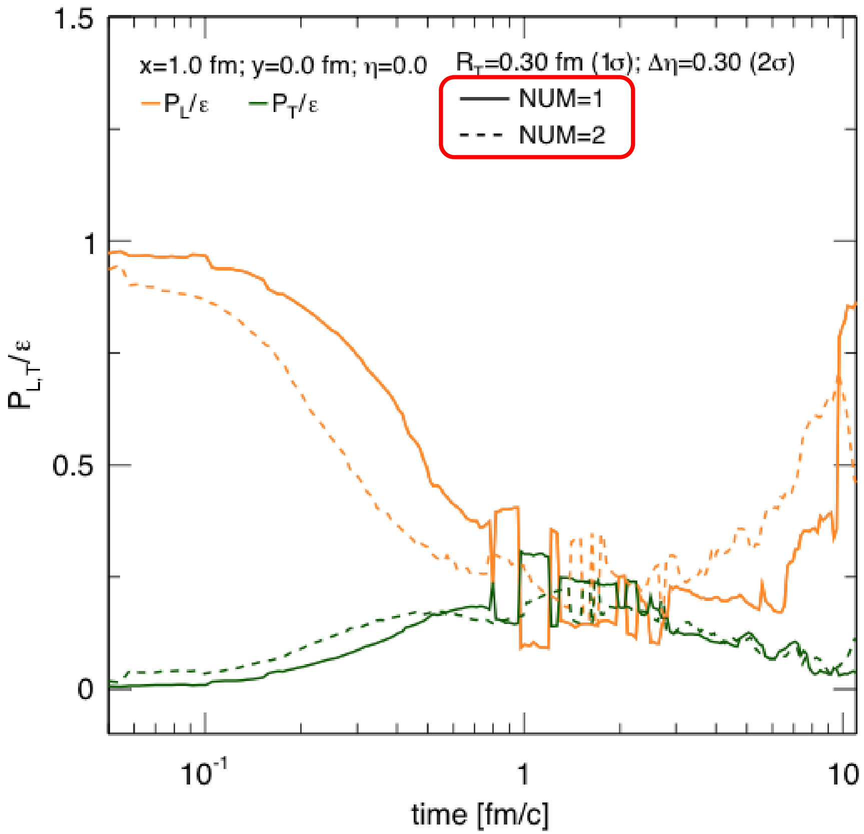


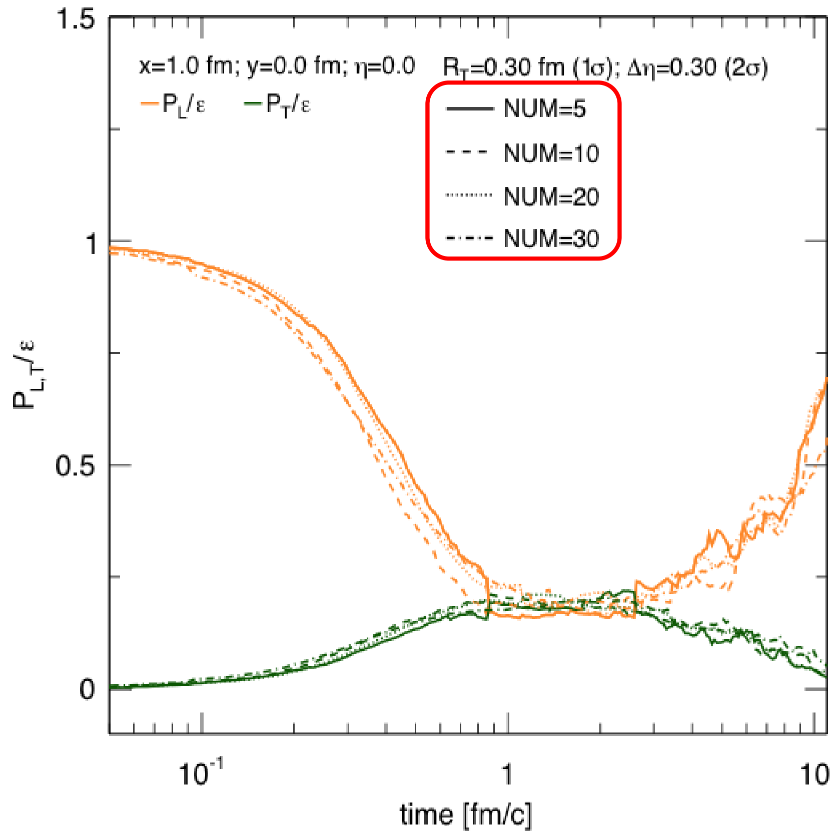
PRESSURE COMPONENTS (SPATIAL EIGENVALUES)



MEAN FIELD EFFECTS

- Varying the *number of parallel events* in PHSD





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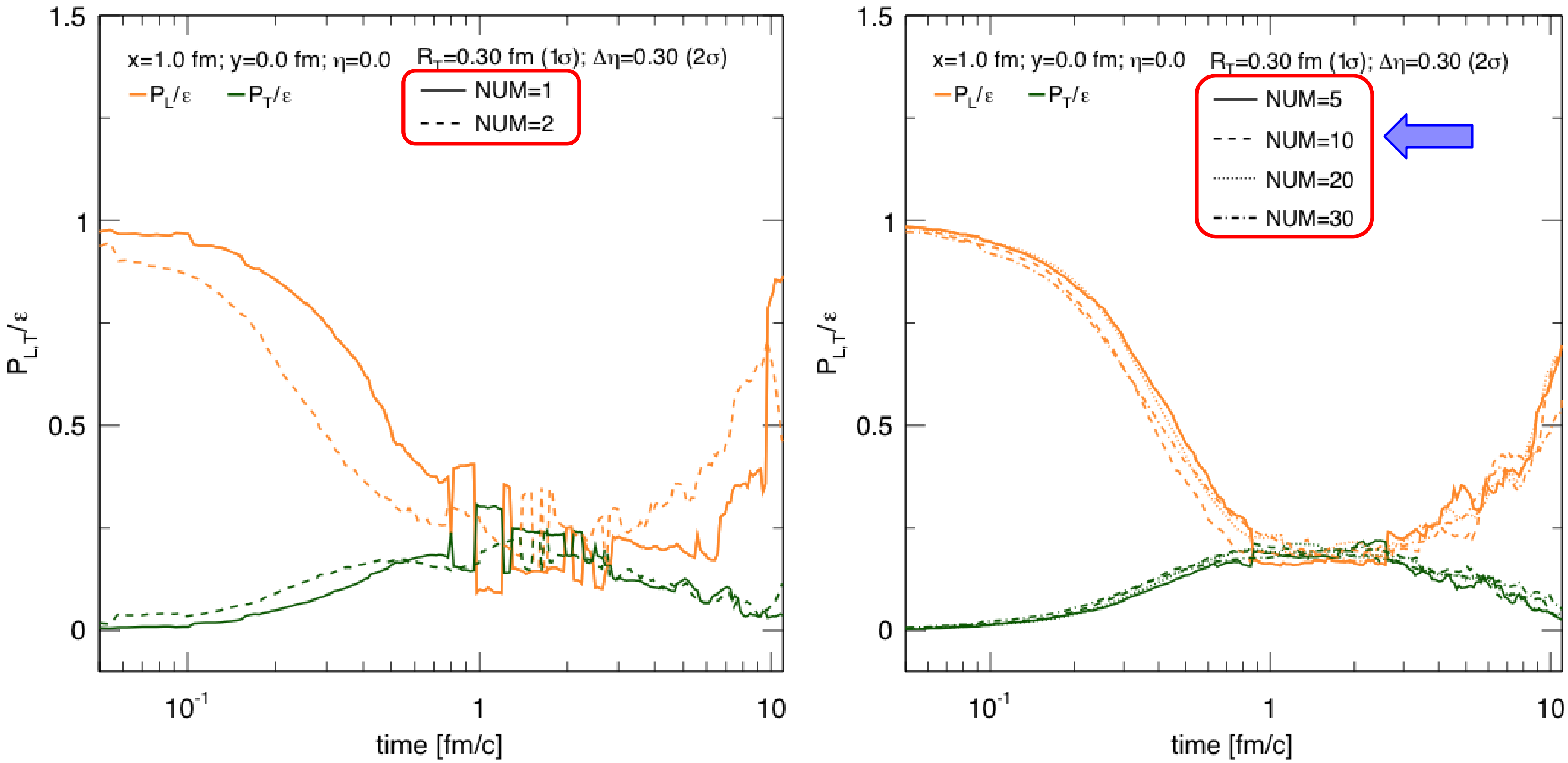
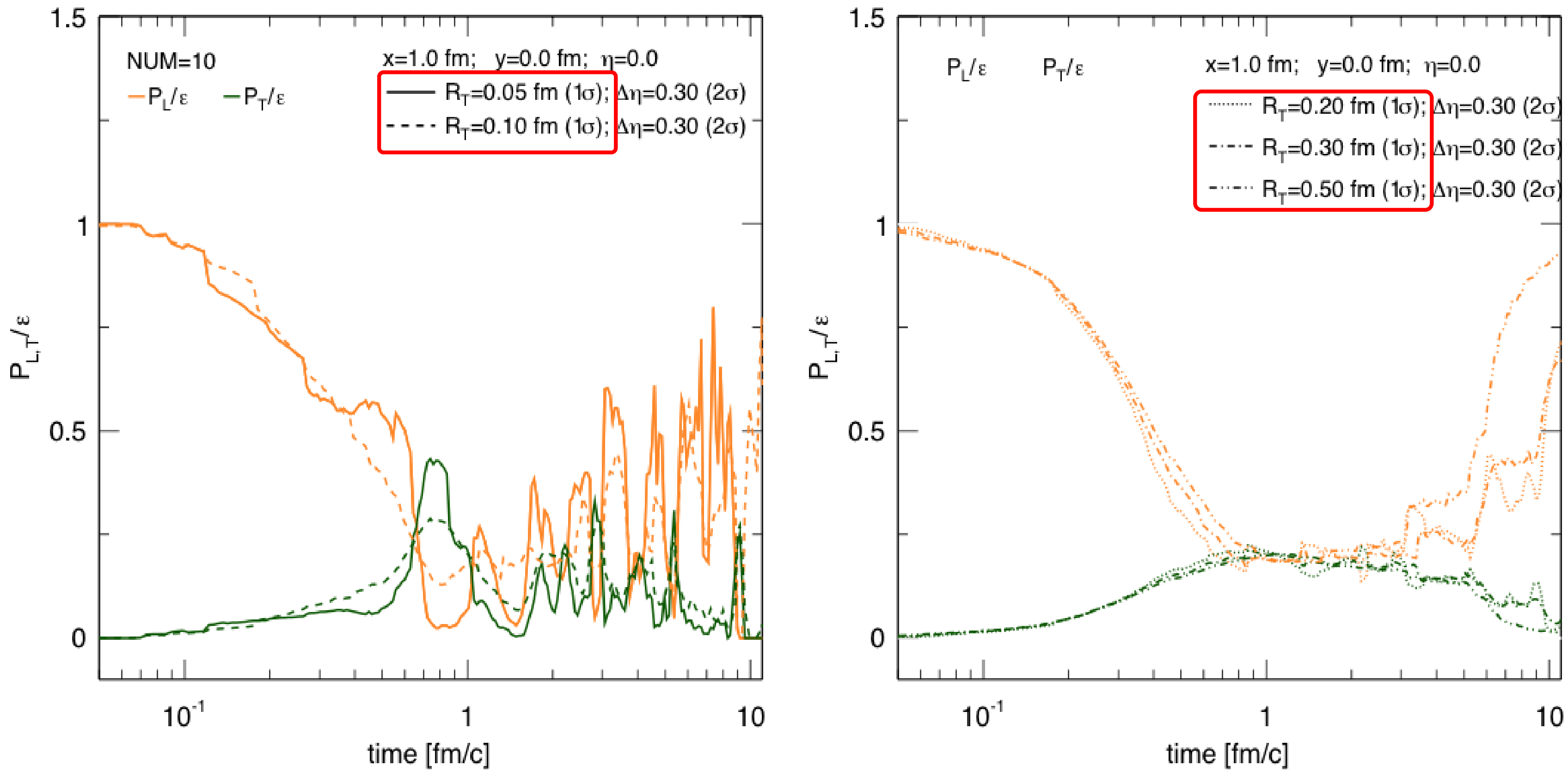


Coarse-graining scale

- Varying cell size in the transverse plane



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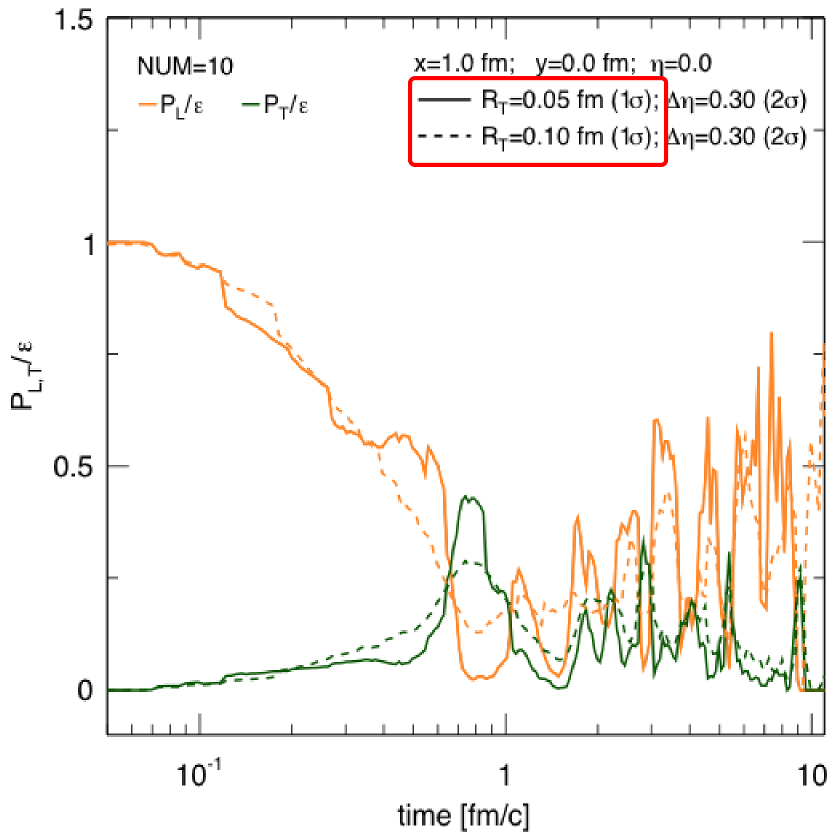
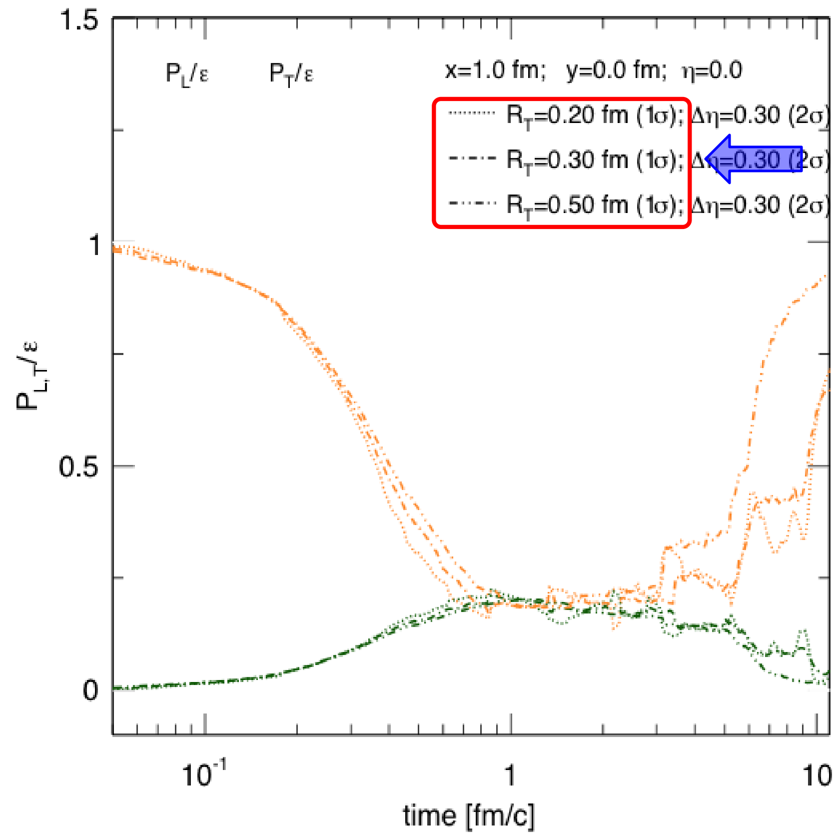
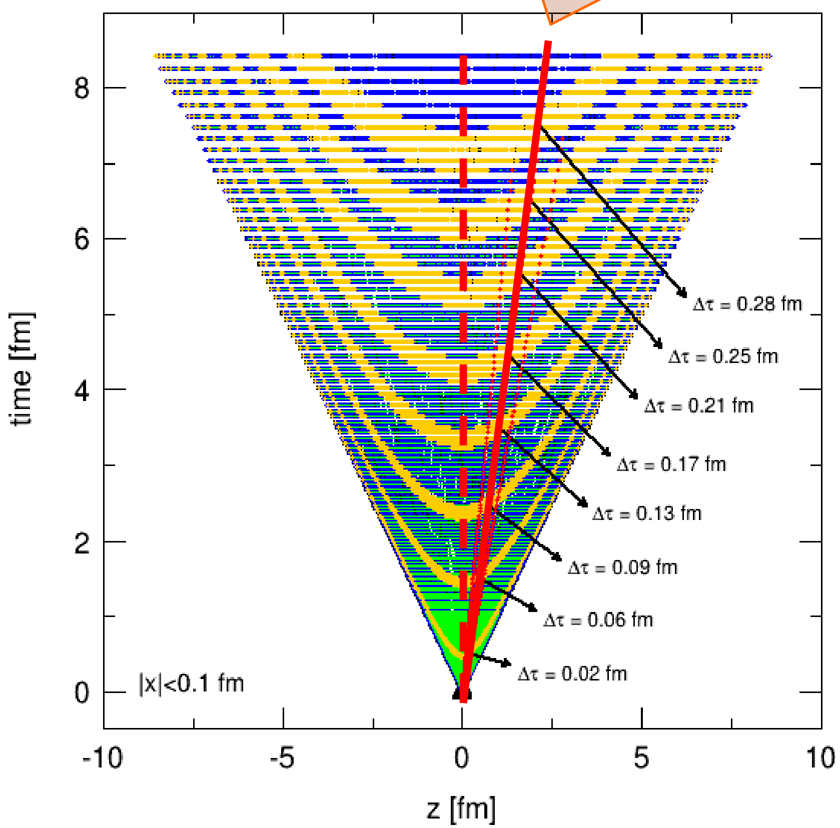
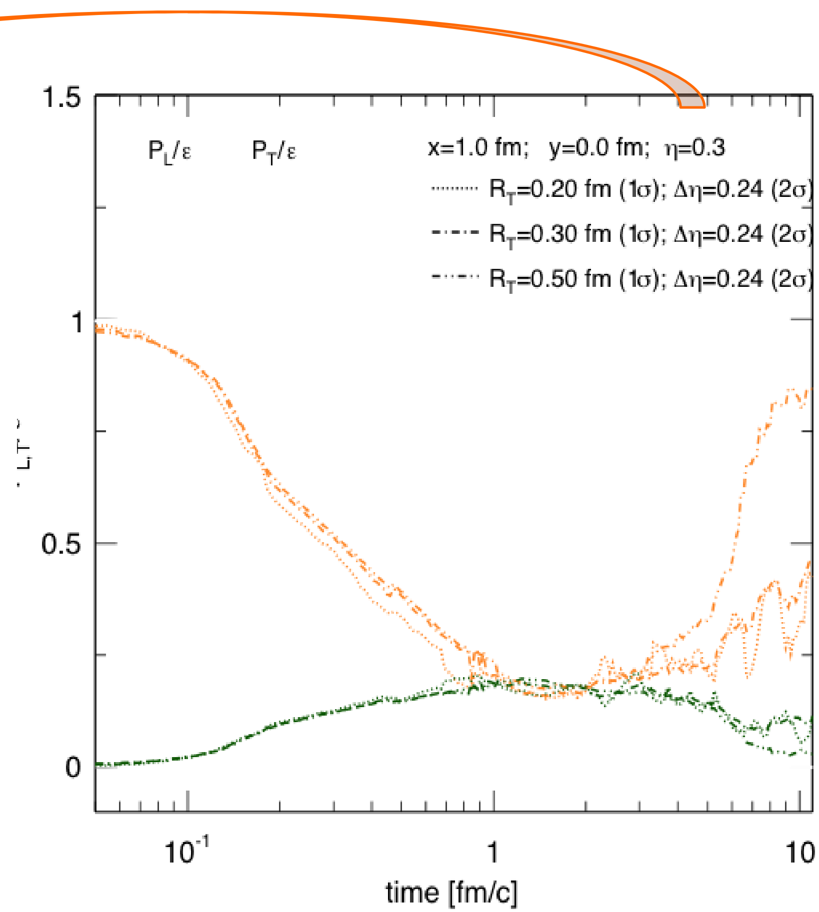




COARSE-GRAINING SCALE

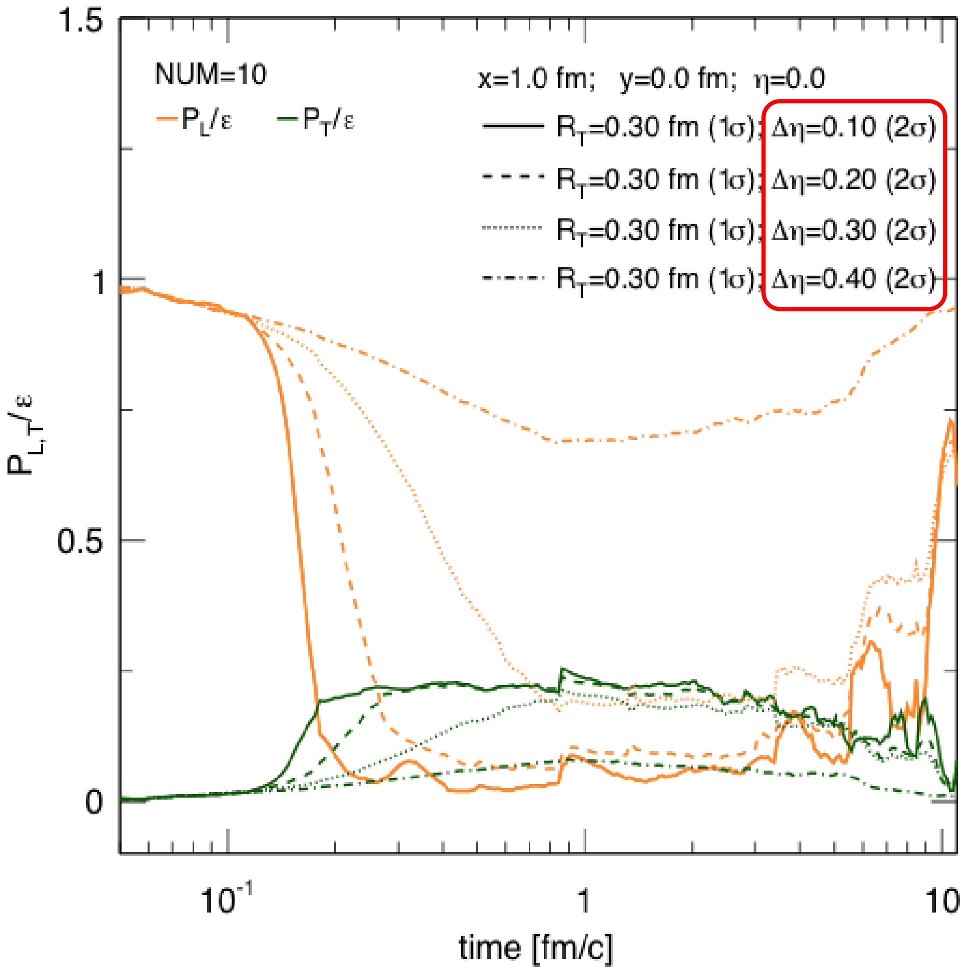
- Varying cell size in the transverse plane





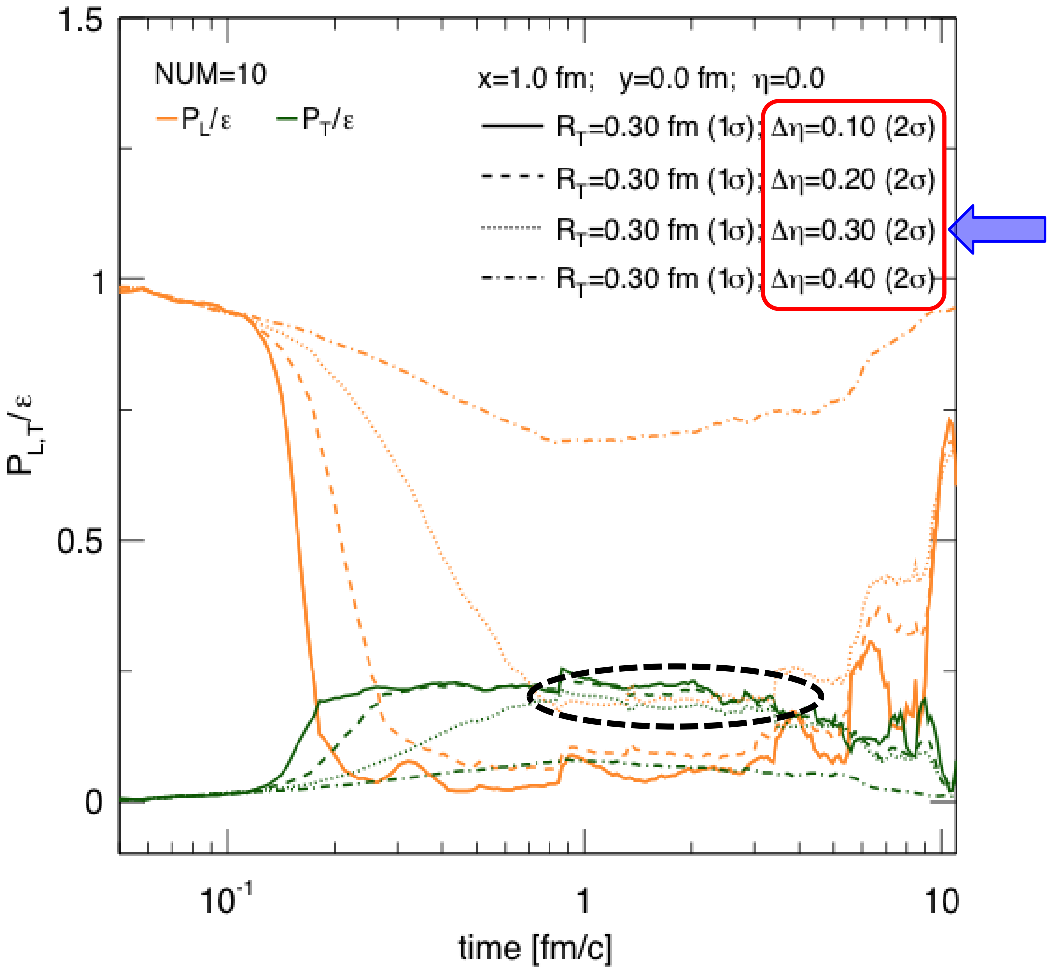
Coarse-graining scale

- Varying cell size in the longitudinal direction



COARSE-GRAINING SCALE

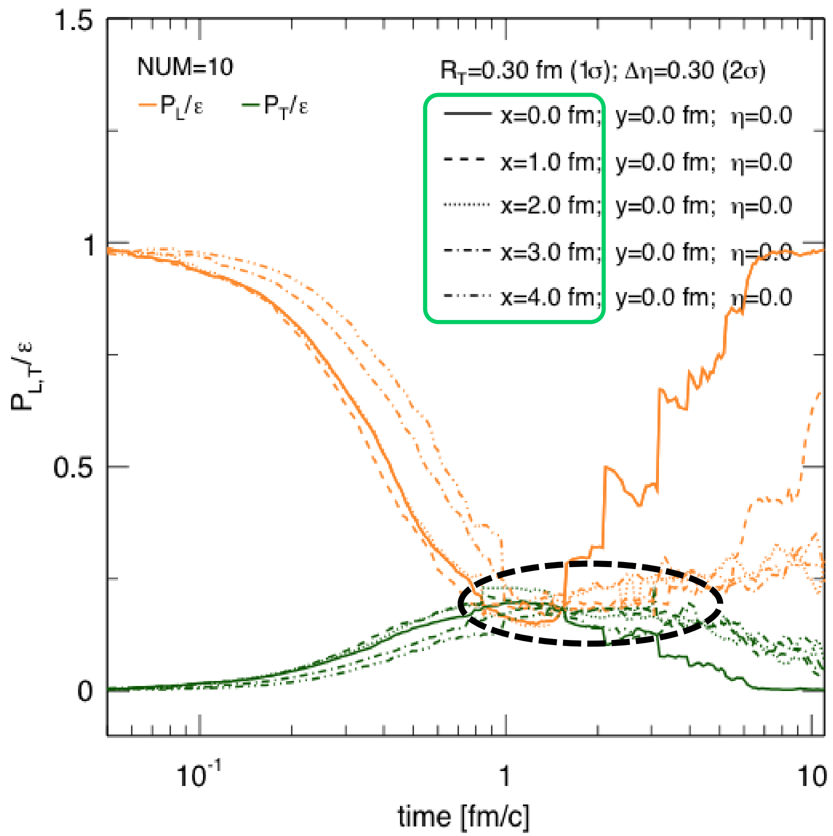
- Varying cell size in the longitudinal direction



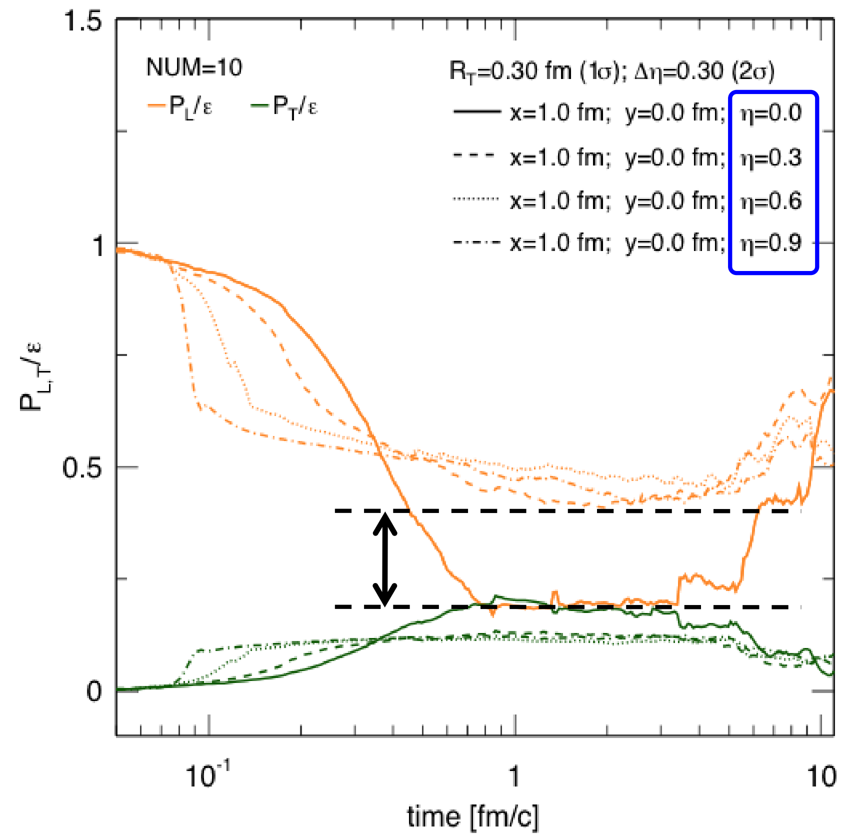
"equilibrium" seems to be achieved only for a *short period of time* and for a rather *large fluid cell*

TRANSVERSE AND LONGITUDINAL POSITION DEPENDENCE

- Varying the position of the cell along *x direction*

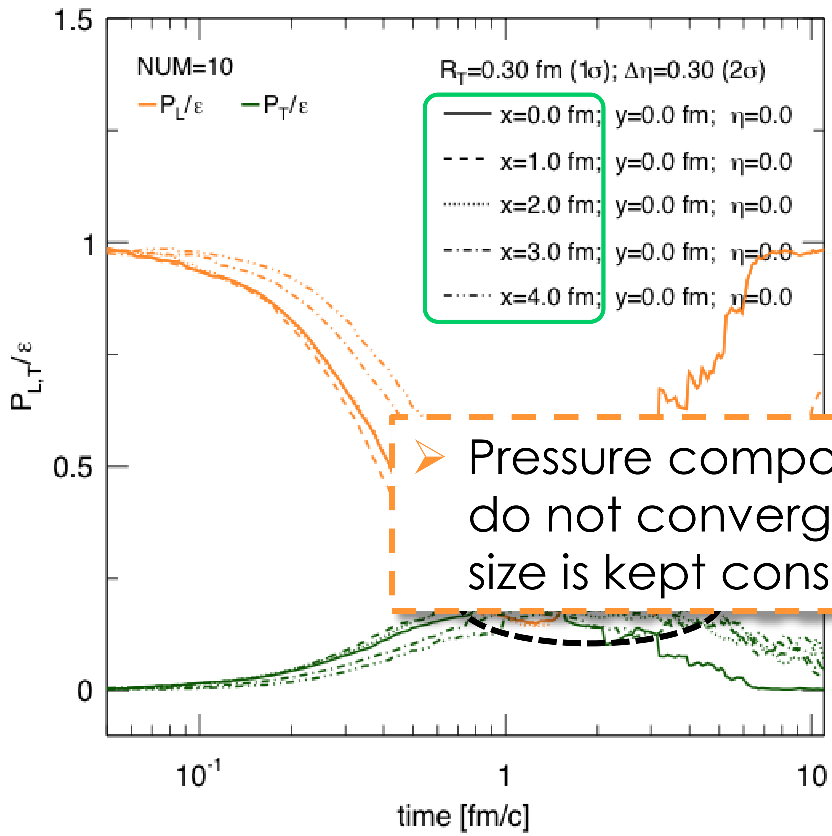


- Varying the position of the cell along *z direction*

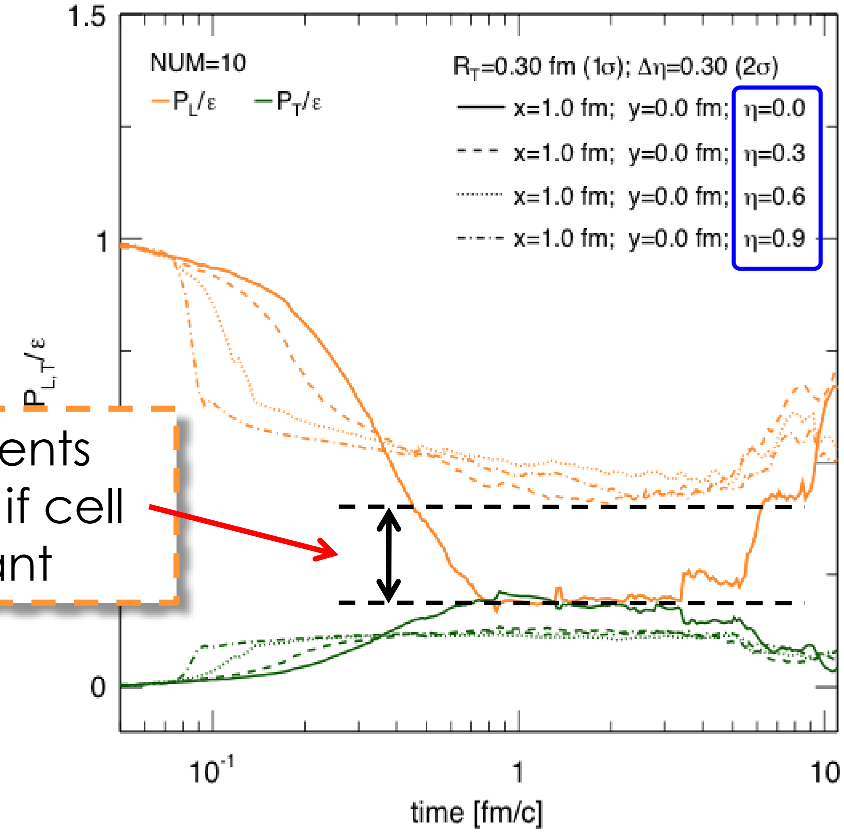


Transverse and Longitudinal position dependence

- Varying the position of the cell along *x direction*



- Varying the position of the cell along *z direction*



- Pressure components do not converge if cell size is kept constant

Initial eccentricities and flow coefficients

spatial anisotropy

$$\varepsilon_n = \frac{\langle r^n \cos\left(n\left[\phi - \Phi_n\right]\right)\rangle}{r^n}$$

$$\begin{cases} \phi = \arctan\left(y/x\right) \\ \Phi_n = \dfrac{1}{n}\arctan\left(\dfrac{\langle r^n \sin\left(n\phi\right)\rangle}{\langle r^n \cos\left(n\phi\right)\rangle}\right) \end{cases}$$

momentum anisotropy

$$v_n = \langle \cos\left(n\left[\psi - \Psi_n\right]\right)\rangle$$

$$\begin{cases} \psi = \arctan\left(p_y/p_x\right) \\ \Psi_n = \Phi_n + \dfrac{\pi}{n} \end{cases}$$

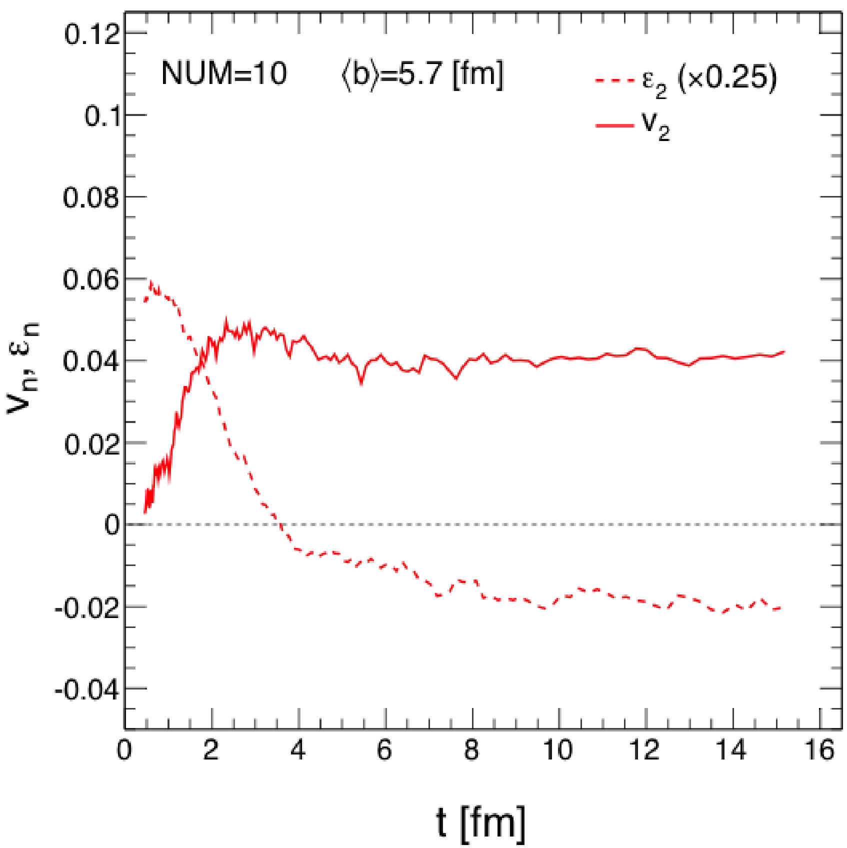
!

- Note that Ψ_n is not the event plane angle usually obtained experimentally, which is the final momentum event plane

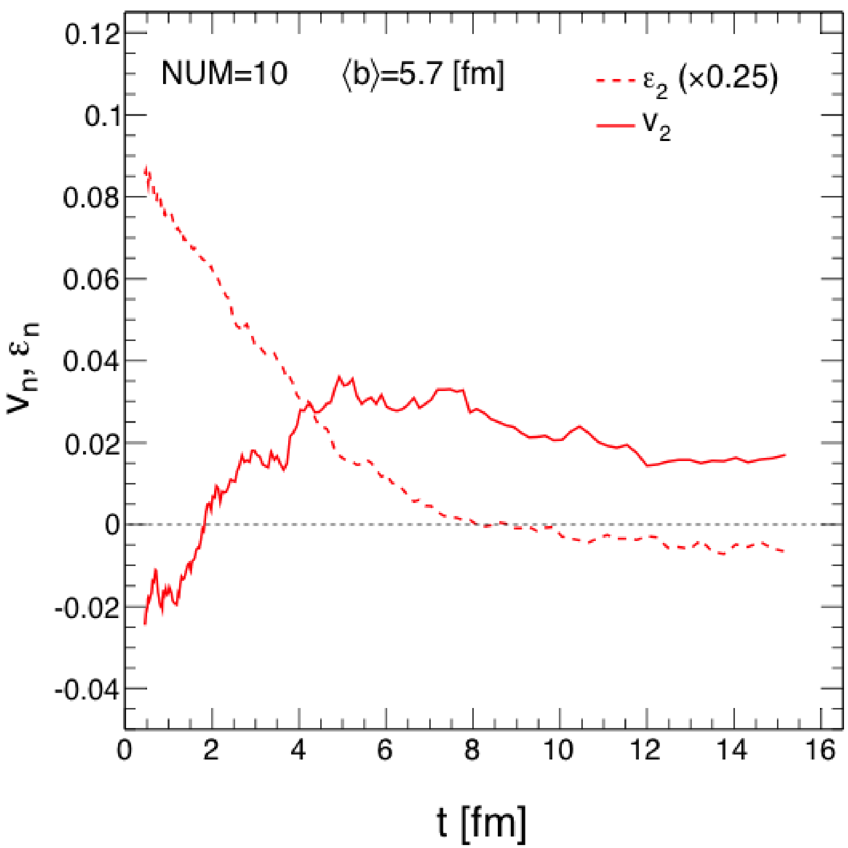
Eccentricity → Flow in PHSD

- 2^{nd} harmonic:

(single event – event A)



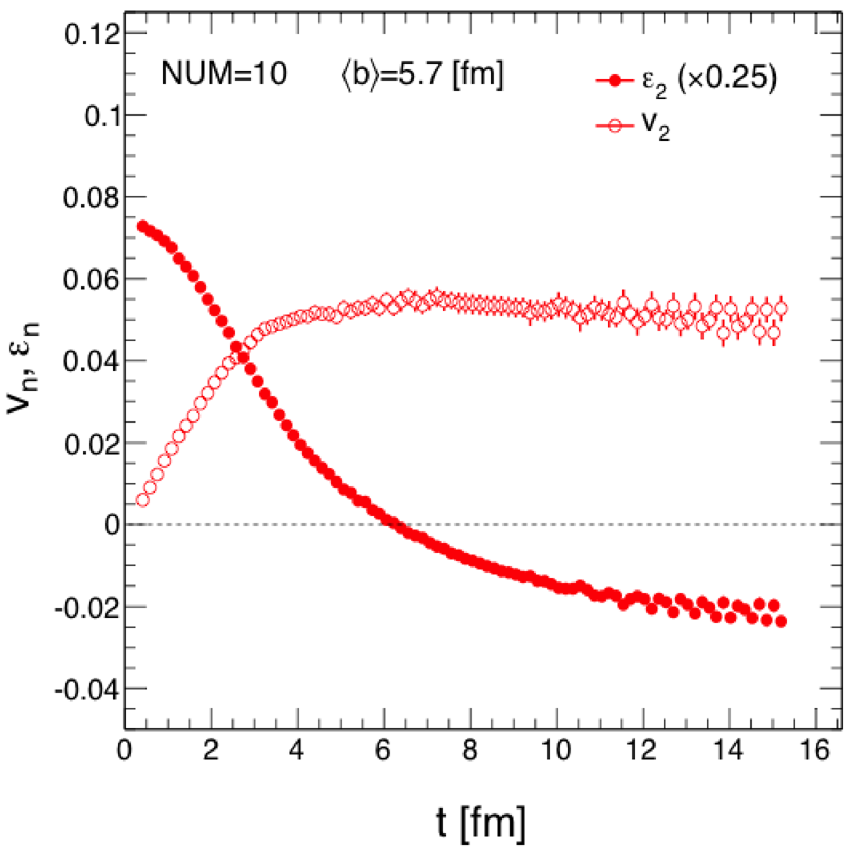
(single event – event B)



Eccentricity ➔ Flow in PHSD

- 2nd harmonic:

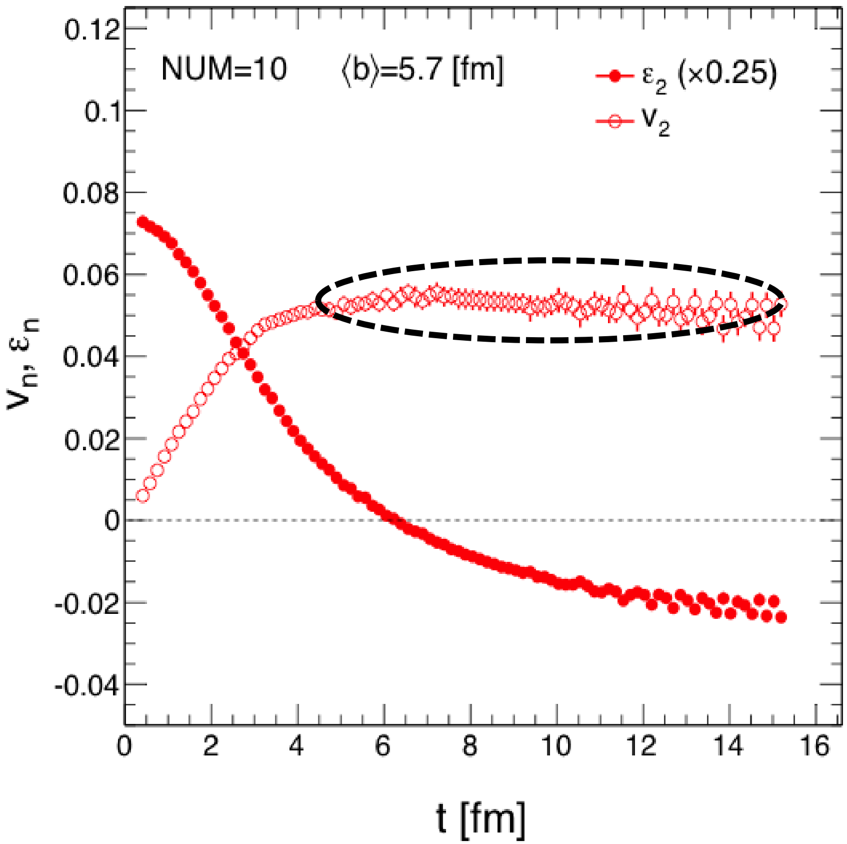
(Event average)



Eccentricity ➔ Flow in PHSD

- 2nd harmonic:

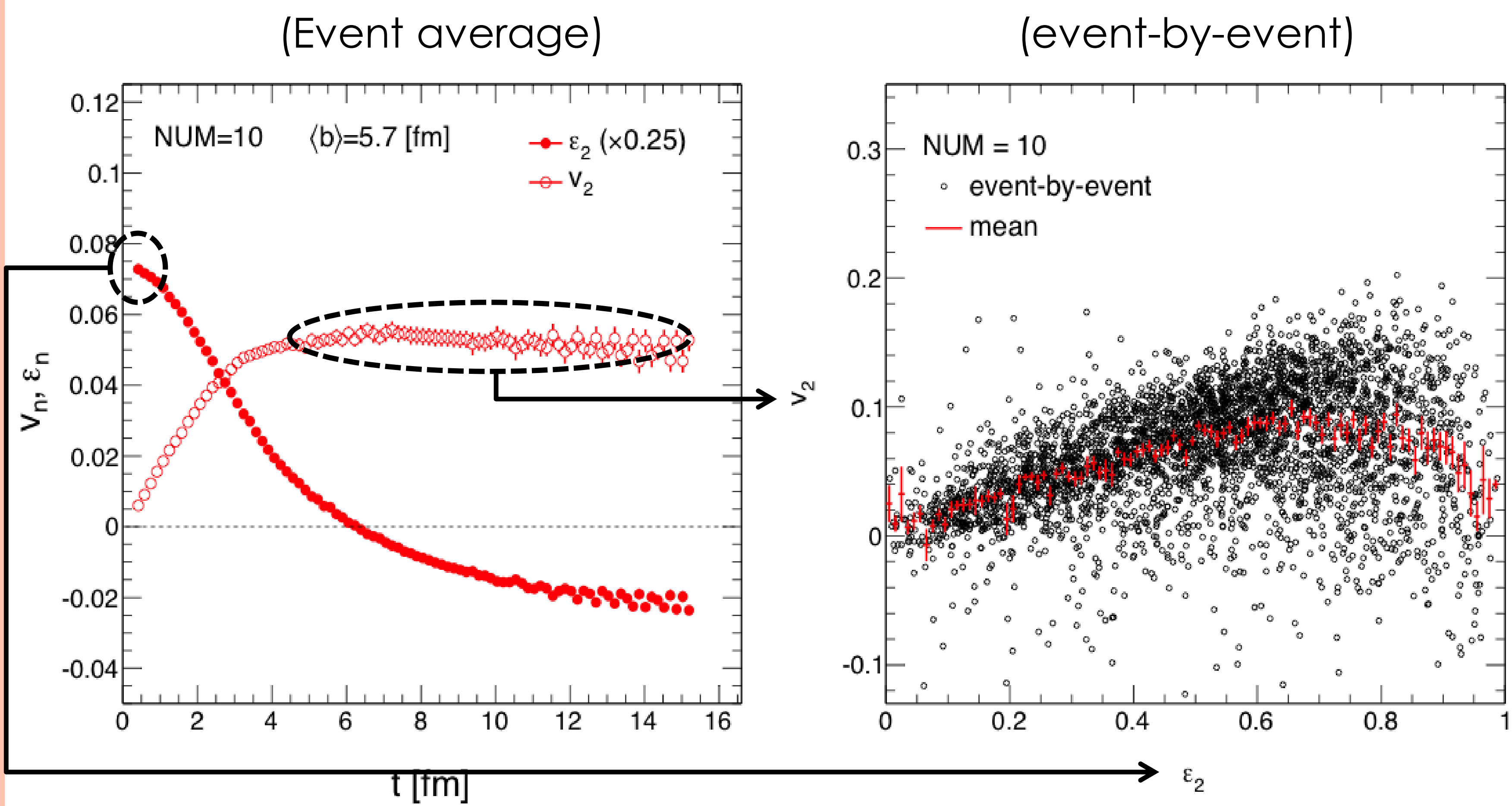
(Event average)



- Flow coefficients seem to reach asymptotic values around 5-6 fm/c

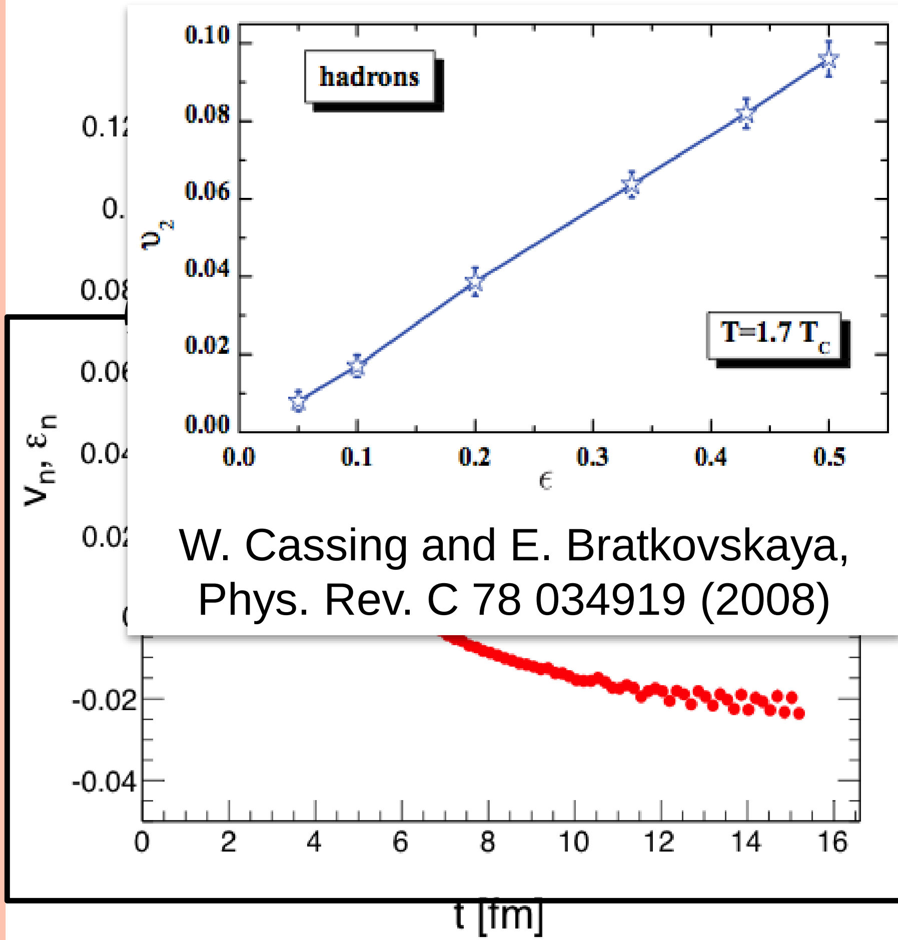
Eccentricity → Flow in PHSD

- 2nd harmonic:



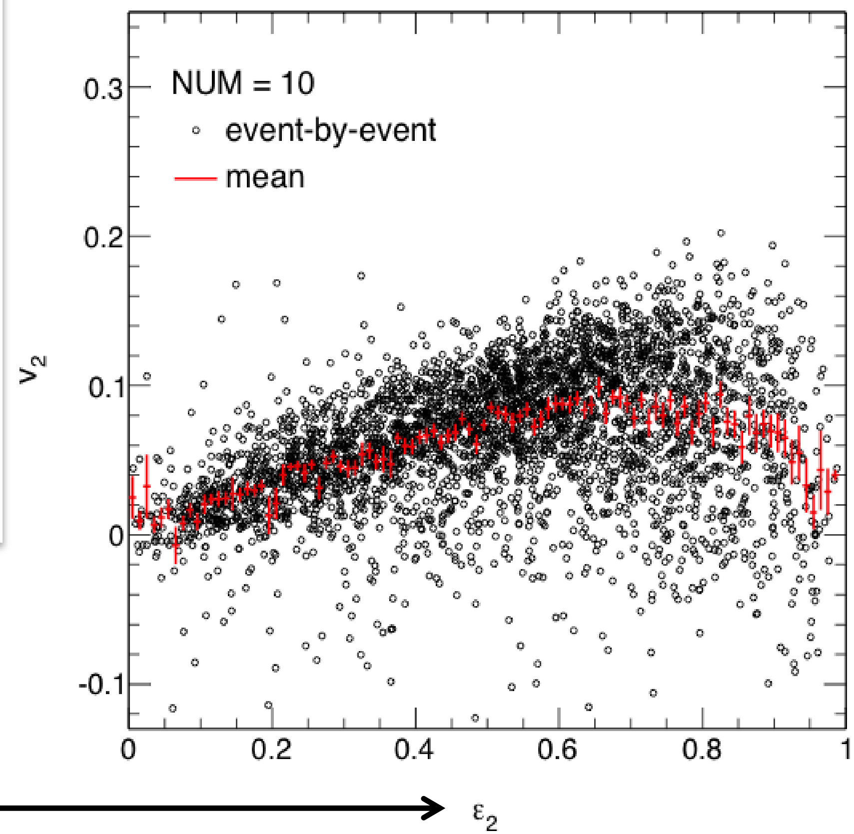
ECCENTRICITY ➔ FLOW IN PHSD

- 2^{nd} harmonic:



W. Cassing and E. Bratkovskaya, Phys. Rev. C 78 034919 (2008)

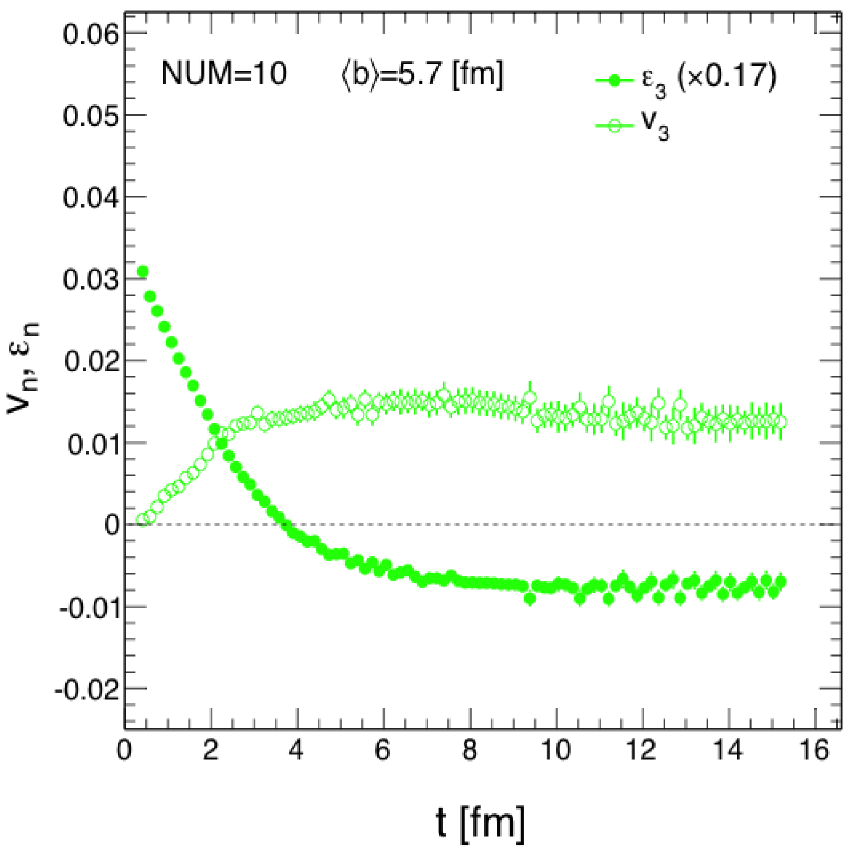
(event-by-event)



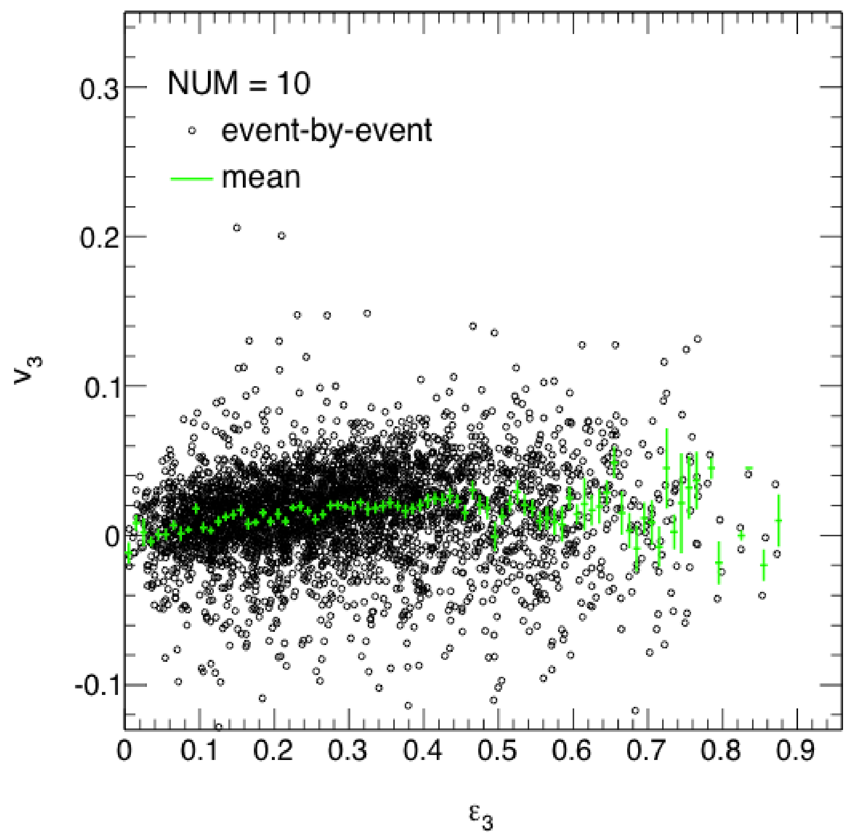
ECCENTRICITY ➔ FLOW IN PHSD

- 3rd harmonic:

(Event average)



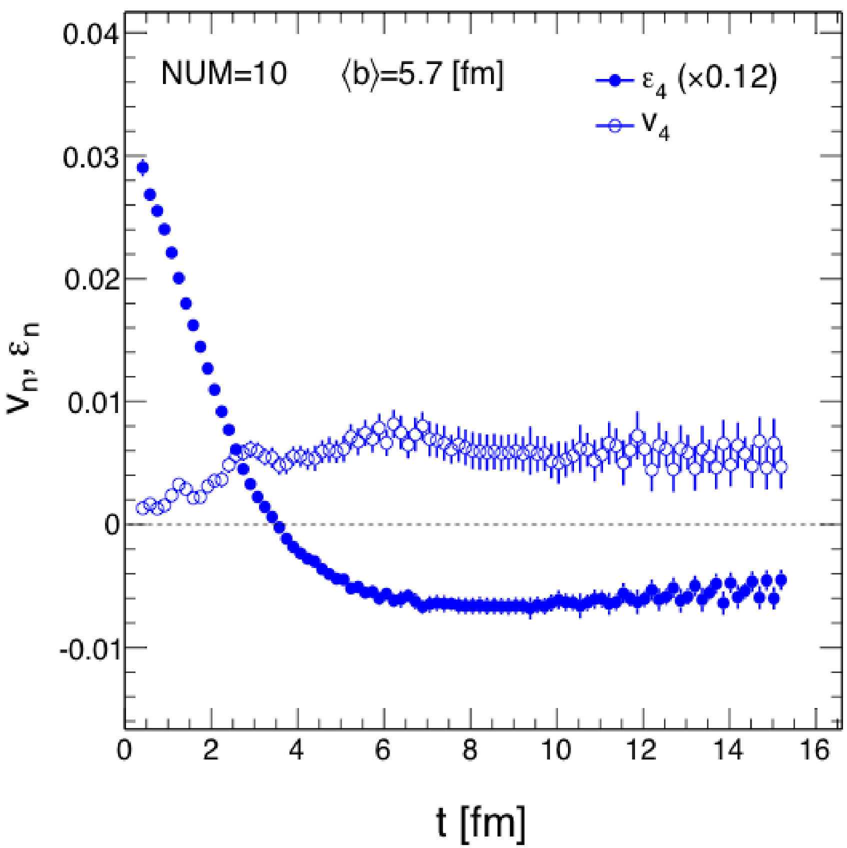
(event-by-event)



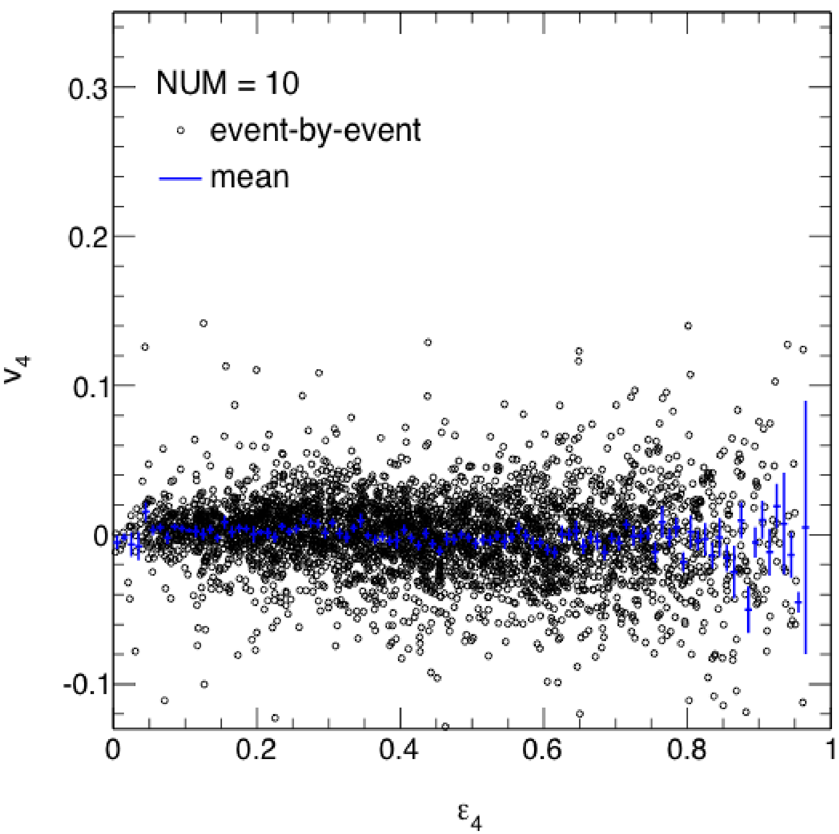
Eccentricity → Flow in PHSD

- 4^{th} harmonic:

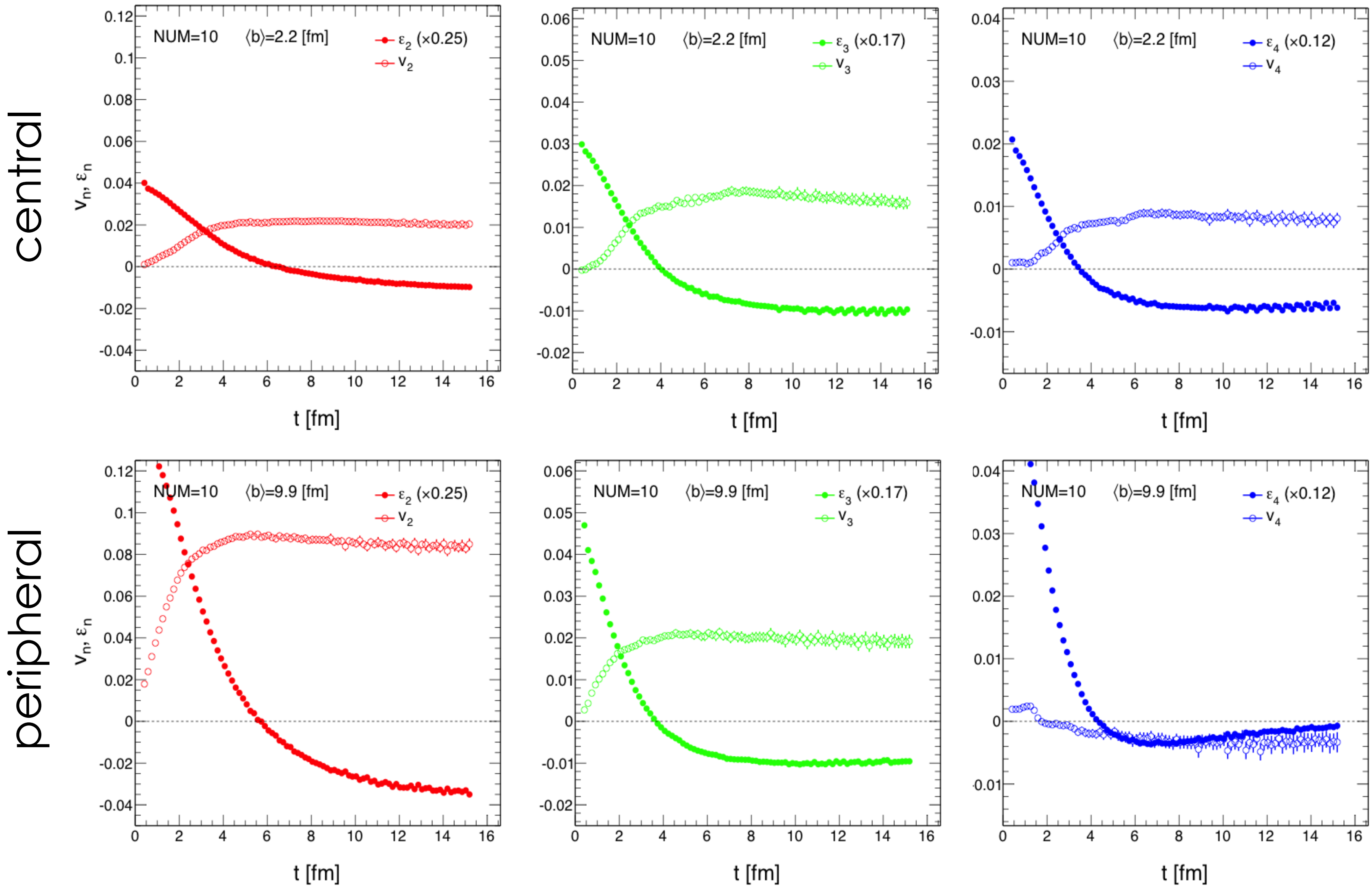
(Event average)



(event-by-event)



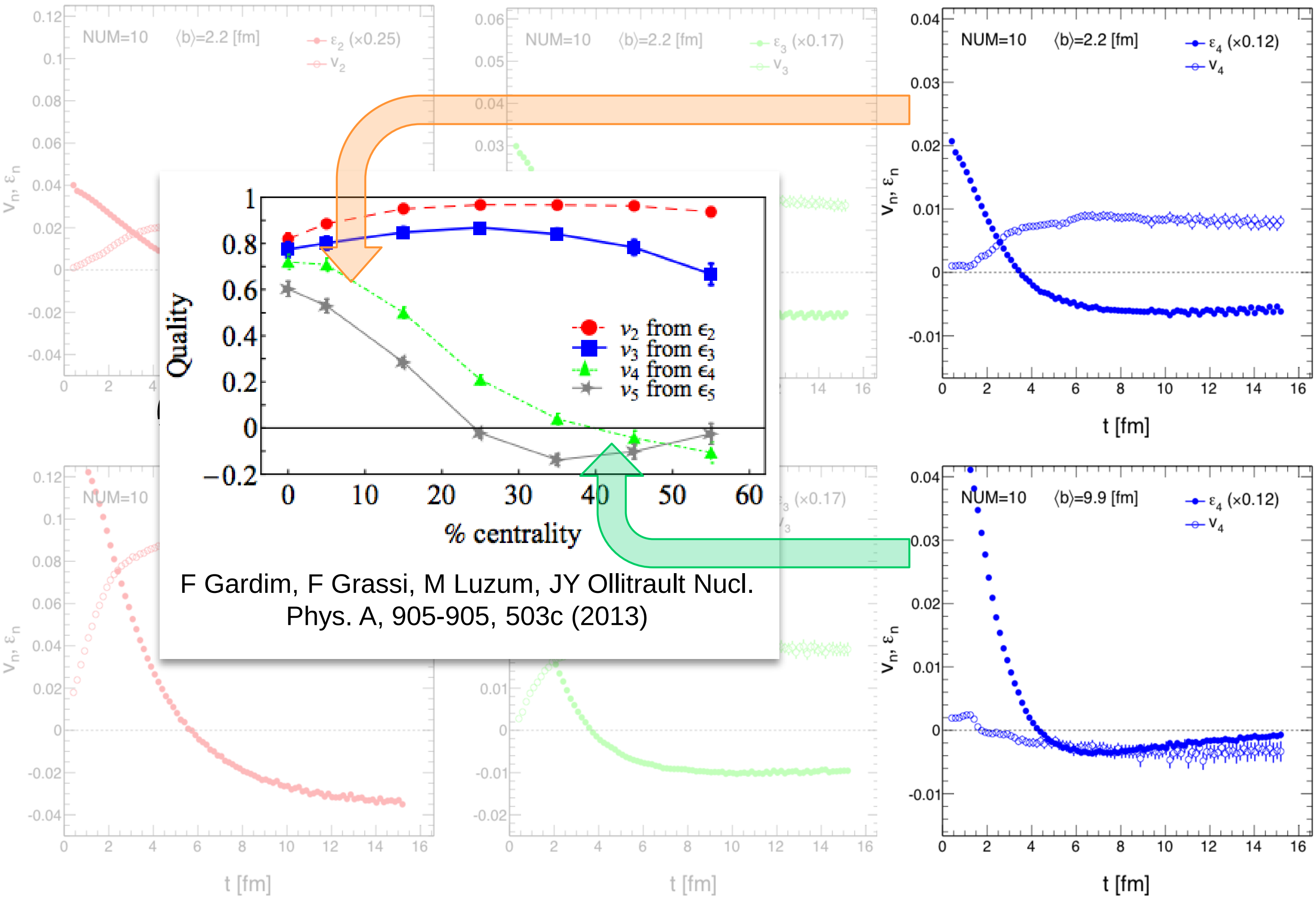
Eccentricity ➔ Flow in PHSD



ECCENTRICITY ➔ FLOW IN PHSD

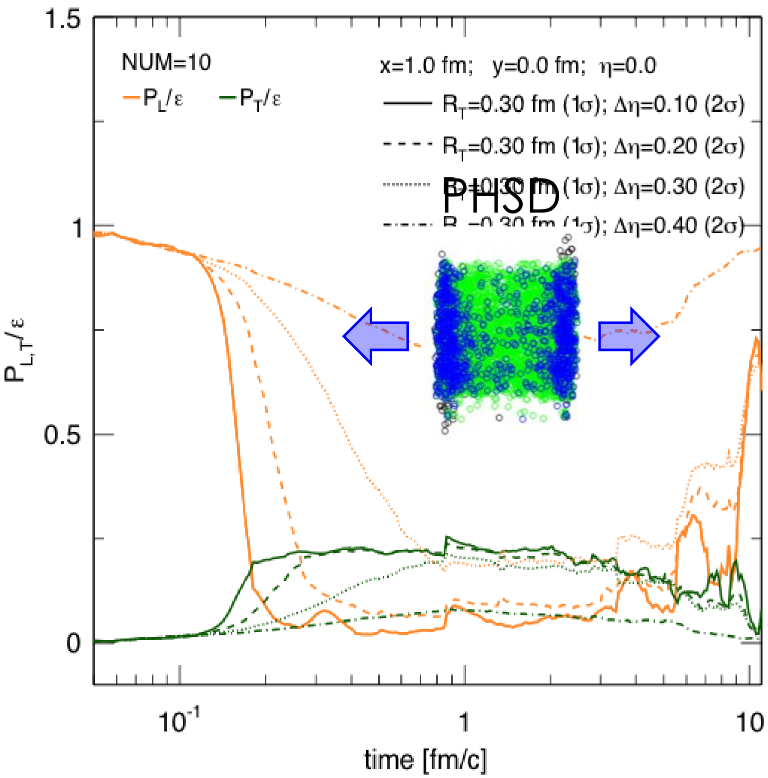
central

peripheral

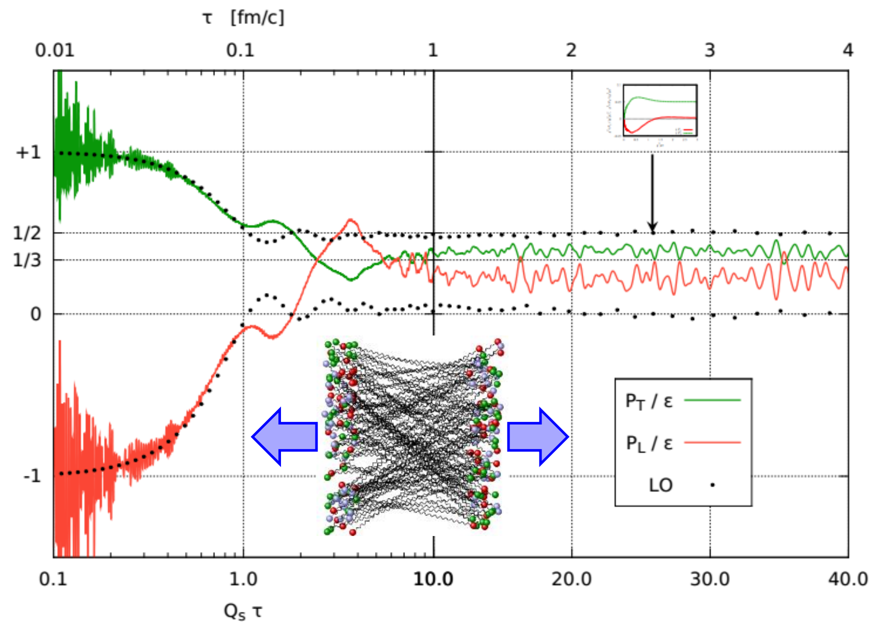


Particle vs Field pressure

"particles"



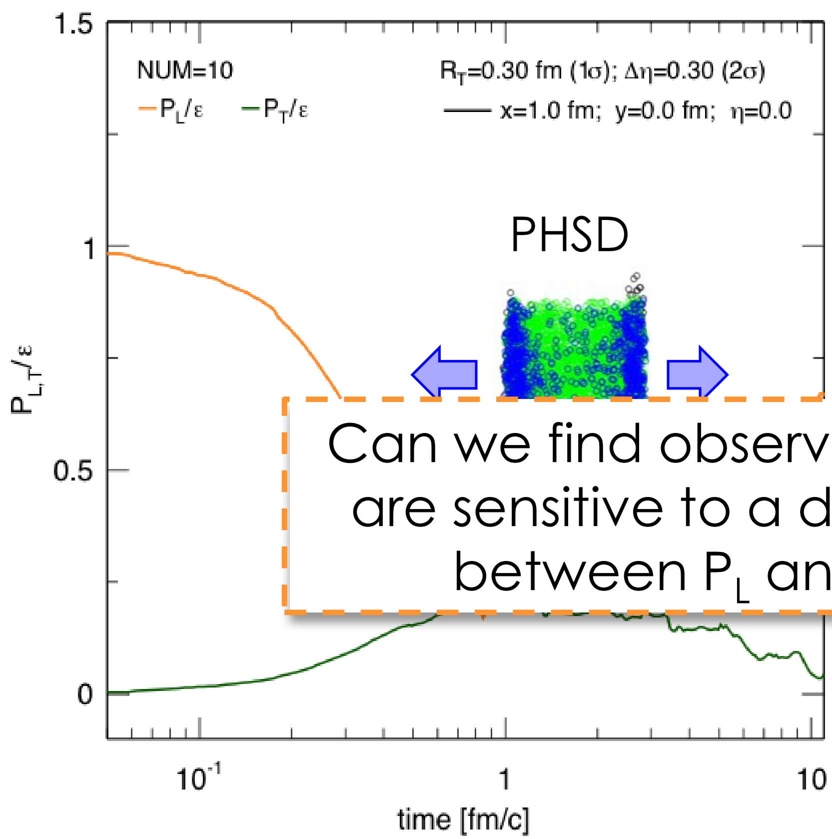
"fields"



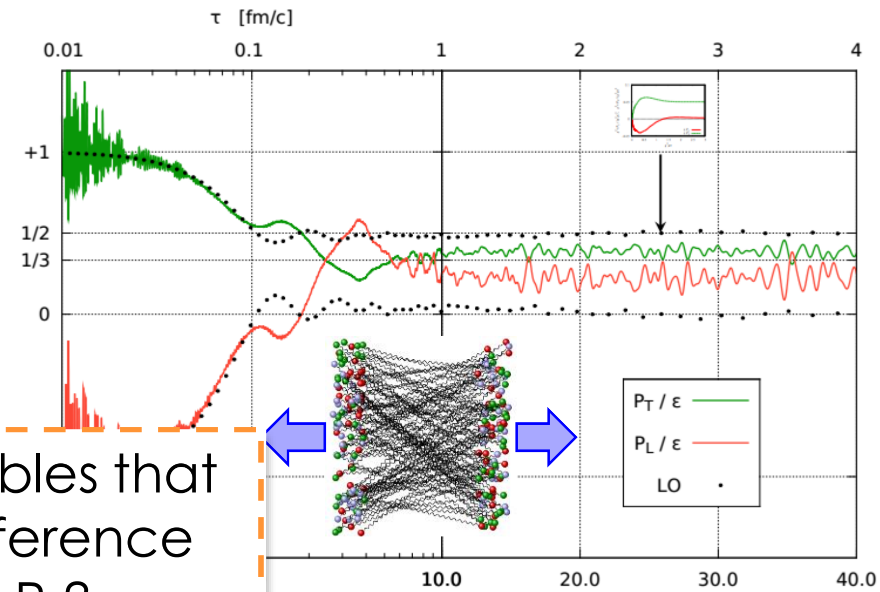
- *T. Epelbaum, QM2014*
- *T. Epelbaum, F. Gelis, PRL 111, 232301 (2013)*

Particle vs Field pressure

"particles"



"fields"



Can we find observables that are sensitive to a difference between P_L and P_T?

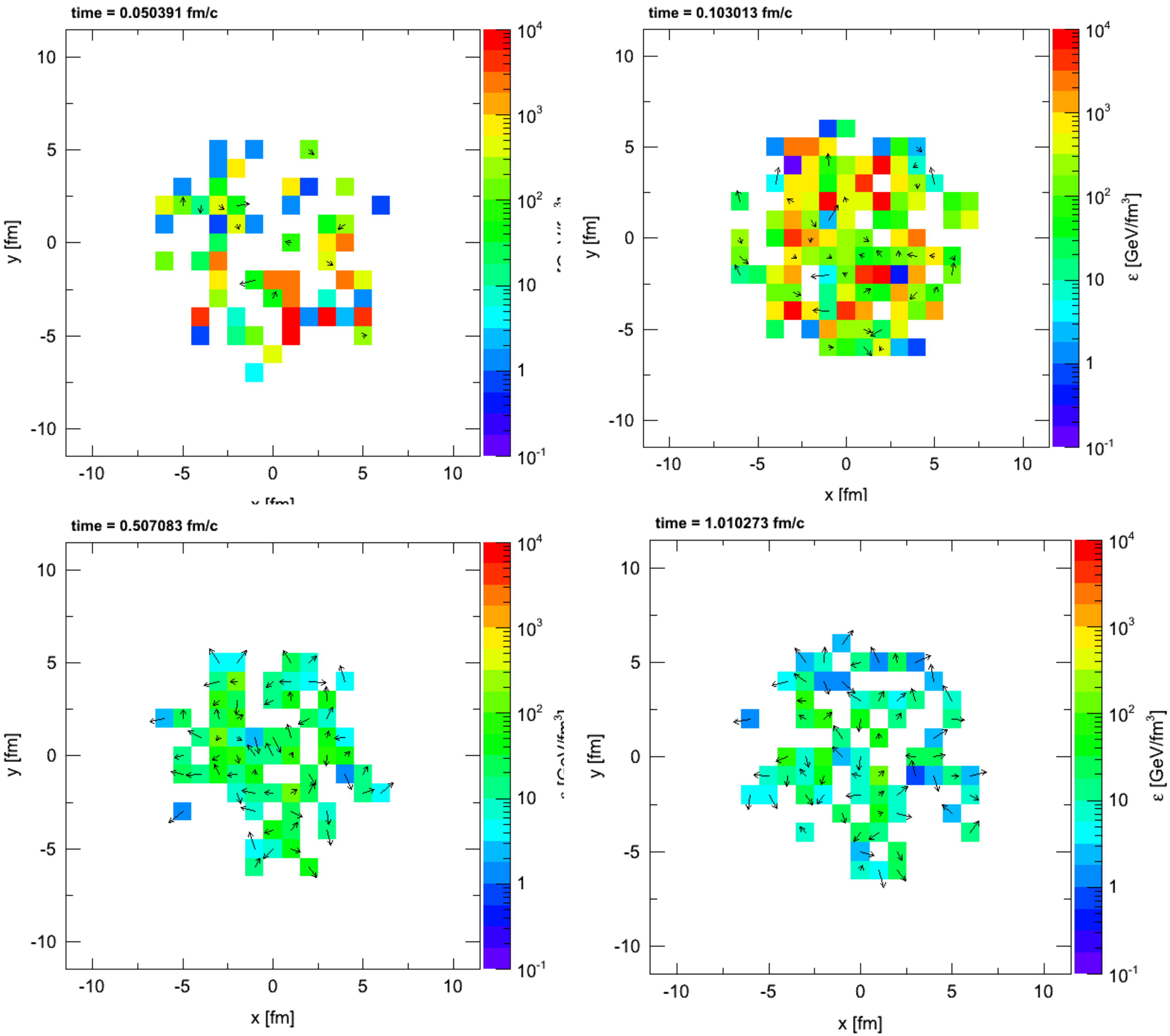
- *T. Epelbaum, QM2014*
- *T. Epelbaum, F. Gelis, PRL 111, 232301 (2013)*

Partial Summary

- ✓ PHSD model provides a convenient way to test the coarse-graining scale of hydrodynamics within a scenario of microscopic dynamics
- ✓ So far, we have observed a clear separation of the "flow profile" into longitudinal and transverse components
- ✓ Only for some very specific situations the system evolution seems to approach "equilibrium"
- ✓ On the other hand, event average eccentricities/flow coefficients seem to follow the hydrodynamic behavior in PHSD

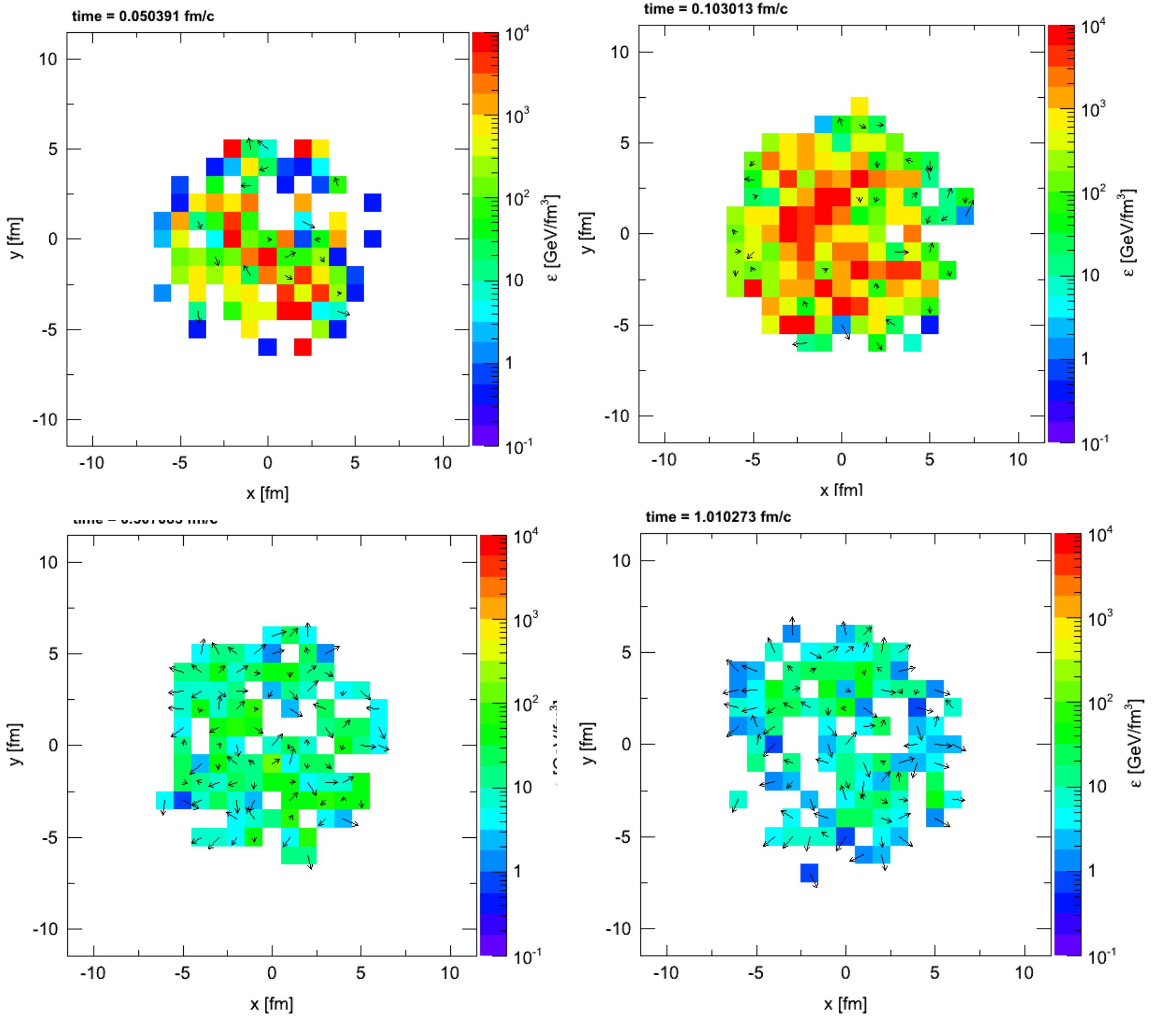
- ✓ Coarse graining may depend on Observables
- ✓ Particle vs. Field

NUM=1



time = 0.050391 fm/c
time = 0.103013 fm/c
time = 0.507083 fm/c
time = 1.010273 fm/c
x [fm]
y [fm]
ε [GeV/fm^3]
10^4
10^3
10^2
10
1
10^{-1}

NUM=2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time = 1.010273 fm/c
x [fm]
y [fm]
ε [GeV/fm^3]

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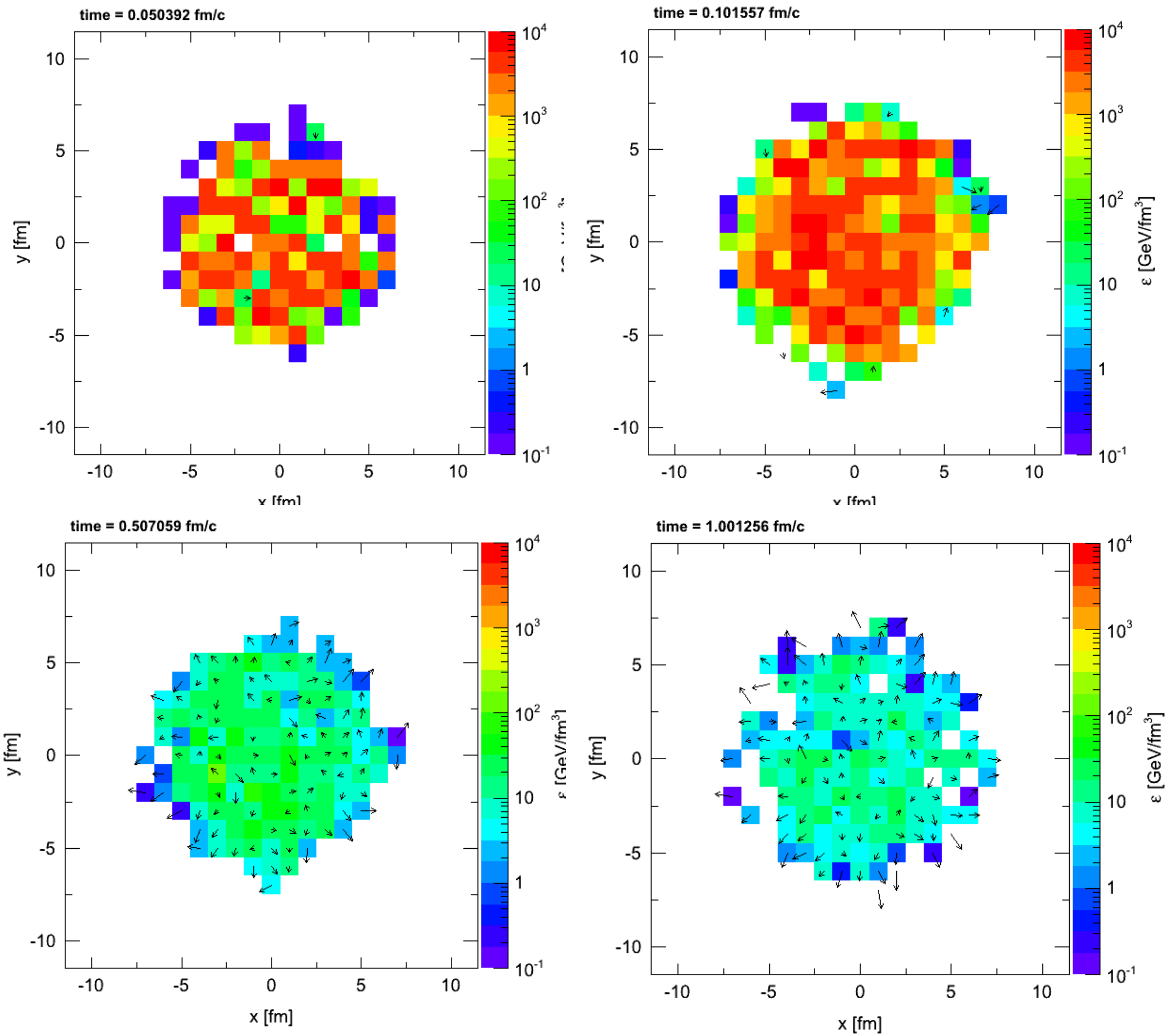
time = 0.050392 fm/c
time = 0.101557 fm/c
time = 0.507059 fm/c
time = 1.001256 fm/c
x [fm]
y [fm]
ε [GeV/fm^3]
10
5
0
-5
-10
10^4
10^3
10^2
1
10^{-1}

INTERMEZZO
WEYL PROJECTION, WIGNER FUNCTION, HUSIMI FUNCTION AND COARSE GRAINING

Weyl Projection Operator

$$\hat{\Delta}(q,p)=\frac{1}{2\pi\hbar}\int du dv\, e^{i\left\{u(q-\hat{Q})+v(p-\hat{P})\right\}/\hbar},$$

(generalization of $|q\rangle\langle q|\equiv\hat{\delta}\left(q-\hat{Q}\right)=\frac{1}{2\pi\hbar}\int du\, e^{i\left\{u(q-\hat{Q})\right\}/\hbar}$)

$$Tr\left[\hat{\Delta}(q,p)\right]=1,$$

$$Tr\left[\hat{\Delta}(q,p)\hat{\Delta}(q',p')\right]=2\pi\hbar\delta(q-q')\delta(p-p'),$$

$$O_W(q,p)=Tr\left[\hat{\Delta}(q,p)\hat{O}\right]\rightleftarrows\hat{O}=\frac{1}{2\pi\hbar}\int dqdp O_W(q,p)\hat{\Delta}(q,p)$$

$$Tr\left[\hat{A}\hat{B}\right]=\frac{1}{2\pi\hbar}\int dqdp A_W(q,p)B_W(q,p)$$

Intermezzo
Weyl Projection, Wigner Function, Husimi Function and Coarse Graining

Density Matriz

Pure state

$$\hat{\rho} = \left|\Psi\right\rangle\left\langle\Psi\right|$$

Mixed state

$$\hat{\rho} = \sum_{\alpha} \omega_{\alpha} \left|\Psi_{\alpha}\right\rangle\left\langle\Psi_{\alpha}\right|$$

Wigner Function

$$f_W(q,p) = Tr\left[\hat{\Delta}(q,p)\hat{\rho}\right] \rightleftarrows \hat{\rho} = \frac{1}{2\pi\hbar}\int dqdp\, f_W(q,p)\hat{\Delta}(q,p)$$

$$\langle O\rangle = Tr\left[\hat{\rho}\hat{O}\right] = \frac{1}{2\pi\hbar}\int dqdp\, f_W(q,p) O_W(q,p)$$

$$f_W(q,p) = \frac{1}{2\pi\hbar}\int du\, e^{iuq/\hbar}\left\langle p+\frac{1}{2}u\right|\hat{\rho}\left|p-\frac{1}{2}u\right\rangle$$

$$= \int d\nu\, e^{i\nu p/\hbar}\left\langle q-\frac{1}{2}\nu\right|\hat{\rho}\left|q+\frac{1}{2}\nu\right\rangle$$

$$= \int d\nu\, e^{i\nu p/\hbar}\psi(q-u/2)\psi^{*}(q+u/2)$$

Intermezzo
Weyl Projection, Wigner Function, Husimi Function and Coarse Graining

$$\langle O \rangle = Tr\left[\hat{\rho}\hat{O}\right] = \frac{1}{2\pi\hbar}\int dqdp\, f_W\left(q,p\right)O_W\left(q,p\right)$$

Wigner Function is not necessarily positive.

Husimi Function
Smoothing (coarse graining in q and p wih Gaussian weight,

$$f_H\left(q,p\right) = \int dq'dp'G(q-q';h)G\left(p-p';\frac{\hbar}{2h}\right)f_W\left(q,p\right)$$

$$= \left\langle q,p\left|\hat{\rho}\right|q,p\right\rangle$$

Where $$G(x,h) = \frac{h}{\sqrt{\pi}}e^{-\left(\frac{x}{h}\right)^2}$$

and $\left|q,p\right\rangle$ is the coherent state.

INTERMEZZO
WEYL PROJECTION, WIGNER FUNCTION, HUSIMI FUNCTION AND COARSE GRAINING

Still valid

$$\langle O \rangle = Tr\left[\hat{\rho}\hat{O}\right] = \frac{1}{2\pi\hbar}\int dqdp\, f_H\left(q,p\right) O_H\left(q,p\right)$$

In the limit of, for example, $h \to \infty$

$$G\left(p, \hbar/2h\right) \to \delta(p),$$

so that

$$f_H\left(q,p\right) \to \left|\varphi(p)\right|^2 = \left|\left\langle p \mid \psi \right\rangle\right|^2,$$

and the corresponding density matrix reconstructed becomes

$$\hat{\rho}_{rec} = \int dp\, \left|\varphi(p)\right|^2 \left| p \right\rangle \left\langle p \right|$$

which is a mixed state! ...

Construction of Hydrodynamics from Wigner function

P. Carruthers and F. Zachariasen
Rev. Mod. Phys., Vol. 55, No. 1, 1983

$$i\hbar\partial_t\psi\left(\vec{x},t\right)=-\frac{\hbar^2}{2m}\nabla^2\psi\left(\vec{x},t\right)+V\left(\vec{x}\right)\psi\left(\vec{x},t\right)$$

$$\partial_t\rho+\nabla\cdot\left(\rho\vec{u}\right)=0,$$

$$\partial_t\left(\rho u_i\right)+\partial_j p_{ij}=-\frac{1}{m}\partial_i V$$

with

$$\rho=\int d^3x\, f\left(\vec{x},\vec{p},t\right)$$

$$\rho\vec{u}=\int d^3p\,\frac{\vec{p}}{m}f\left(\vec{x},\vec{p},t\right),$$

$$p_{ij}=\int d^3p\,\frac{p_i p_j}{m^2}f\left(\vec{x},\vec{p},t\right).$$

Construction of Hydrodynamics from Wigner function

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This is equivalent to that of E. Madelung, 1926, Z. Phys. 40, 322.

$$i\hbar\partial_t\psi\left(\vec{x},t\right)=-\frac{\hbar^2}{2m}\nabla^2\psi\left(\vec{x},t\right)+V\left(\vec{x}\right)\psi\left(\vec{x},t\right)$$

$$\partial_t\rho+\nabla\cdot\left(\rho\vec{u}\right)=0,$$

$$\partial_t\vec{u}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{m}\nabla V-\frac{1}{\rho}\nabla\left(\frac{1}{\sqrt{\rho}}\nabla^2\ln\sqrt{\rho}\right)$$

with

$$\rho=|\psi\left(\hat{x},t\right)|^2$$

$$\vec{u}=\frac{\hbar}{m}\mathrm{Im}(\ln\psi\left(\hat{x},t\right))$$

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Quantum pressure

$$\partial_t\rho+\nabla\cdot\left(\rho\vec{u}\right)=0,$$

$$\partial_t\vec{u}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{m}\nabla V-\frac{1}{\rho}\nabla\left(\frac{1}{\sqrt{\rho}}\nabla^2\ln\sqrt{\rho}\right)$$

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Rev. Mod. Phys., Vol. 55, No. 1, 1983

E. Madelung, 1926, Z. Phys. 40, 322.

Takabayashi-Wallstrom

$$i\hbar\partial_t\psi\left(\vec{x},t\right) = -\frac{\hbar^2}{2m}\nabla^2\psi\left(\vec{x},t\right) + V\left(\vec{x}\right)\psi\left(\vec{x},t\right)$$

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$$\vec{u} = \frac{\hbar}{m}\mathrm{Im}(\ln\psi\left(\hat{x},t\right))$$

When exists a large inhomogeniety, the quantum pressure may generate a collective flow...

P. Carruthers and F. Zachariasen
Rev. Mod. Phys., Vol. 55, No. 1, 1983

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$$\partial_t\left(\rho u_i\right) + \partial_j p_{ij} = -\frac{1}{m}\partial_i V$$

with

$$\rho = \int d^3x\, f\left(\vec{x},\vec{p},t\right)$$

$$\rho\vec{u} = \int d^3p\, \frac{\vec{p}}{m} f\left(\vec{x},\vec{p},t\right),$$

$$p_{ij} = \int d^3p\, \frac{p_i p_j}{m^2} f\left(\vec{x},\vec{p},t\right).$$

Maxwell Equation in Schrödinger Form

Peter Holland, *Proc. R. Soc. A* 2005 **461**,

1st step:

rewrite the Maxwell Eqs.

$$\nabla \times \vec{E} = -\partial_t \vec{B}, \qquad \nabla \times \vec{B} = \frac{1}{c^2}\partial_t \vec{E},$$

$$\nabla \cdot \vec{E} = 0, \qquad \nabla \cdot \vec{B} = 0.$$

as

$$i\hbar\partial_t \vec{F} = -ic\left(\vec{\Sigma}\cdot\nabla\right)\vec{F}$$

where

$$\vec{F} \equiv \sqrt{\frac{\varepsilon_0}{2}}\left(\vec{E} + ic\vec{B}\right)$$

and $\vec{\Sigma}$ is (3x3) matrix, corresponding to the Pauli matrix for spin 1, satisfying

$$\vec{\Sigma}\times\vec{\Sigma} = i\hbar\vec{\Sigma}$$

together with the initial condition

$$\nabla \cdot \vec{F}\left(t=0\right) = 0.$$

Maxwell Equation in Schrödinger Form

Peter Holland, *Proc. R. Soc. A* 2005 **461**,

2nd step:

rewrite this Dirac-like Eqs.
in the Cartesian base into the
spherical base,

$$i\hbar\partial_t\vec{F} = -ic\left(\vec{\Sigma}\cdot\nabla\right)\vec{F}$$

$$i\hbar\partial_t\vec{G} = -ic\left(\vec{J}\cdot\nabla\right)\vec{G},$$

where

$$\vec{G} = \begin{pmatrix} G_{+1} \\ G_0 \\ G_{-1} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -F_x + iF_y \\ \sqrt{2}F_z \\ F_x + iF_y \end{pmatrix}$$

and J is the matrix given by

$$J_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\; J_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},\; J_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

MAXWELL EQUATION IN SCHRÖDINGER FORM

P. Holland, *Proc. R. Soc. A* 2005 **461**,

3rd step:

Introduce the derivative operator whose matrix elements coincide with those of J,

$$J_x \to \hat{M}_x = i\hbar\left(\cos\beta\,\partial_\alpha - \sin\beta\cot\alpha\,\partial_\beta + \sin\beta\,\mathrm{cosec}\,\alpha\,\partial_\gamma\right),$$

$$J_y \to \hat{M}_y = i\hbar\left(-\sin\beta\,\partial_\alpha - \cos\beta\cot\alpha\,\partial_\beta + \cos\beta\,\mathrm{cosec}\,\alpha\,\partial_\gamma\right),$$

$$J_z \to \hat{M}_z = i\hbar\partial_\beta,$$

where **α**=(α,β,γ) are Euler angles (in the convention of Holland) in the sense that

$$\int d\Omega\, u_m^{*}(\vec{\alpha})\hat{M}_i u_m(\vec{\alpha}) = (J_i)_{mn},\quad m,n=-1,0,1$$

with

$$u_1(\vec{\alpha}) = \frac{\sqrt{3}}{4\pi}\sin\alpha\, e^{-i\beta},\; u_0(\vec{\alpha}) = i\sqrt{\frac{3}{2}}\frac{1}{2\pi}\cos\alpha,\;\; u_{-1}(\vec{\alpha}) = \frac{\sqrt{3}}{4\pi}\sin\alpha\, e^{i\beta}$$

and

$$d\Omega = \sin\alpha\, d\alpha\, d\beta\, d\gamma,\quad \alpha\in[0,\pi],\;\; \beta,\gamma\in[0,2\pi]$$

MAXWELL EQUATION IN SCHRÖDINGER FORM

P. Holland, *Proc. R. Soc. A* 2005 **461**,

4th step:

Introduce the scalar wavefunction

$$\psi\left(x,\vec{\alpha},t\right)=\sum_{m=1,0,-1} G_m\left(x,t\right)u_m\left(\alpha\right)$$

the Schrödinger equation in R^3 X SO(3) space,

$$i\hbar\partial_t\psi\left(x,\vec{\alpha},t\right)=-ic\sum_i \hat{M}_i\partial_i\psi\left(x,\vec{\alpha},t\right)$$

reduces to the Maxwell equation.

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P. Holland, *Proc. R. Soc. A* 2005 **461**,

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reduces to the Maxwell equation.

Hydrodynamic form ?

$$\rho=\int d^3x\, f\left(\vec{x},\vec{p},t\right)$$

$$\rho\vec{u}=\int d^3p\,\frac{\vec{p}}{m}\,f\left(\vec{x},\vec{p},t\right),$$

$$p_{ij}=\int d^3p\,\frac{p_i p_j}{m^2}\,f\left(\vec{x},\vec{p},t\right).$$

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P. Holland, *Proc. R. Soc. A* 2005 **461**,

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reduces to the Maxwell equation..........................

Particle interpretation with the quantum pressure ?

MAXWELL EQUATION IN HYDRO FORM

P. Holland, *Proc. R. Soc. A* 2005 **461**,

Starting point:

Schrödinger equation in R^3 X SO(3) space, with second order differencial in the space,

$$i\hbar\partial_t\psi\left(x,\vec{\alpha},t\right)=-\hbar c\sum_i\hat{\lambda}_i\partial_i\psi\left(x,\vec{\alpha},t\right)$$

where $\hat{\lambda}_i=i\hbar\hat{M}_i$ are real differential operators with respect to the three Euler angles $\vec{\alpha}$.

Energy density:

$$\frac{\varepsilon_0}{2}\left(\vec{E}^2+c^2\vec{B}^2\right)=\int d^3\Omega\left|\psi(\vec{x},\vec{\alpha})\right|^2,$$

and the Poynting vector

$$\frac{\varepsilon_0}{2}\left(\vec{E}^2+c^2\vec{B}^2\right)=\int d^3\Omega\left|\psi(\vec{x},\vec{\alpha})\right|^2,$$

Maxwell Equation in Hydro Form

P. Holland, *Proc. R. Soc. A* 2005 **461**,

Second step:

Following Madelung, write

$$\psi\left(x,\vec{\alpha},t\right)=\sqrt{\rho}\,e^{iS/\hbar},$$

Then we get

$$\frac{\partial S}{\partial t}+\frac{c}{\hbar}\left(\vec{\hat{\lambda}}\cdot\nabla_x\right)S+Q=0,$$

and

$$\frac{\partial\rho}{\partial t}+\frac{c}{\hbar}\nabla_x\cdot\left(\rho\vec{\hat{\lambda}}\right)S+\frac{c}{\hbar}\vec{\hat{\lambda}}\cdot\left(\rho\nabla S\right)=0.$$

where

$$Q=-c\hbar\left(\vec{\hat{\lambda}}\cdot\nabla_x\right)\ln\sqrt{\rho},$$

MAXWELL EQUATION IN HYDRO FORM

P. Holland, *Proc. R. Soc. A* 2005 **461**,

Like Schrödinger Equation´s case, we need Takabayashi-Wallstrom constraints,

$$i\hbar\partial_t\psi\left(x,\vec{\alpha},t\right) = -\hbar c\sum_i \hat{\lambda}_i\partial_i\psi\left(x,\vec{\alpha},t\right)$$

$$\frac{\partial S}{\partial t} + \frac{c}{\hbar}\left(\vec{\hat{\lambda}}\cdot\nabla_x\right)S + Q = 0,$$

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$$\psi\left(x,\vec{\alpha},t\right) = \sqrt{\rho}\,e^{iS/\hbar}, \quad Q = -c\hbar\left(\vec{\hat{\lambda}}\cdot\nabla_x\right)\ln\sqrt{\rho},$$

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Maxwell Equation in Hydro Form

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Circulation should be quantized !!

Maxwell Equation in Hydro Form

P. Holland, *Proc. R. Soc. A* 2005 **461**,

1. Maxwell´s Equations can be written in the form of hydrodynamic flow in R^3 X SO(3) space with the quantum pressure (which is not visible in the original equation).
2. Energy-momentum tensor can be obtained as the coarse grained over the angular state (What happens for the generalized Husimi states?)
3. Circulations are quantized. Vortexes in polarization.
4. How to introduce the Gauge transformation?

SUMMARY

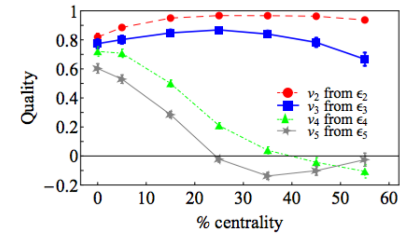
- How to describe "Particulization" from the Intense Initial Field consistently ? (Analogy to Quantum Optics)
- Effects of Initial Velocity Field for the Initial Condition ?
- To find the initial condition for hydromodel, several levels of Coarse Graining is necessary (Total wavefunction to the single-particle states, localization in space, etc). All of them introduce the mixed states and may affect the momentum distribution,..
- Can non-Abelian field be described in the form of Schrödinger form? Inhomonegeities in Quantum Pressure affects the initial momentum distribution?

In 60´s...

Where have all the flowers gone...

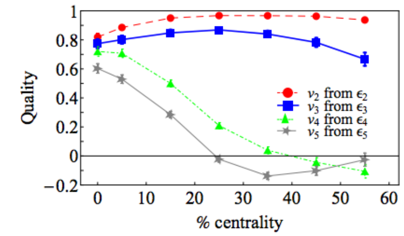
Now ...

Where have all the flows come ...



Now in PHSD and CGC-Glasma...

Where have all the flows come ...



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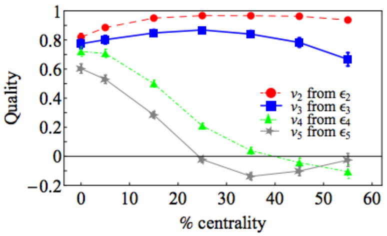

I am sorry ...

And thank you for Helea, Jörg and Marcus ...