Dynamical evolution of the Chiral Magnetic Effect

Applications to the QGP

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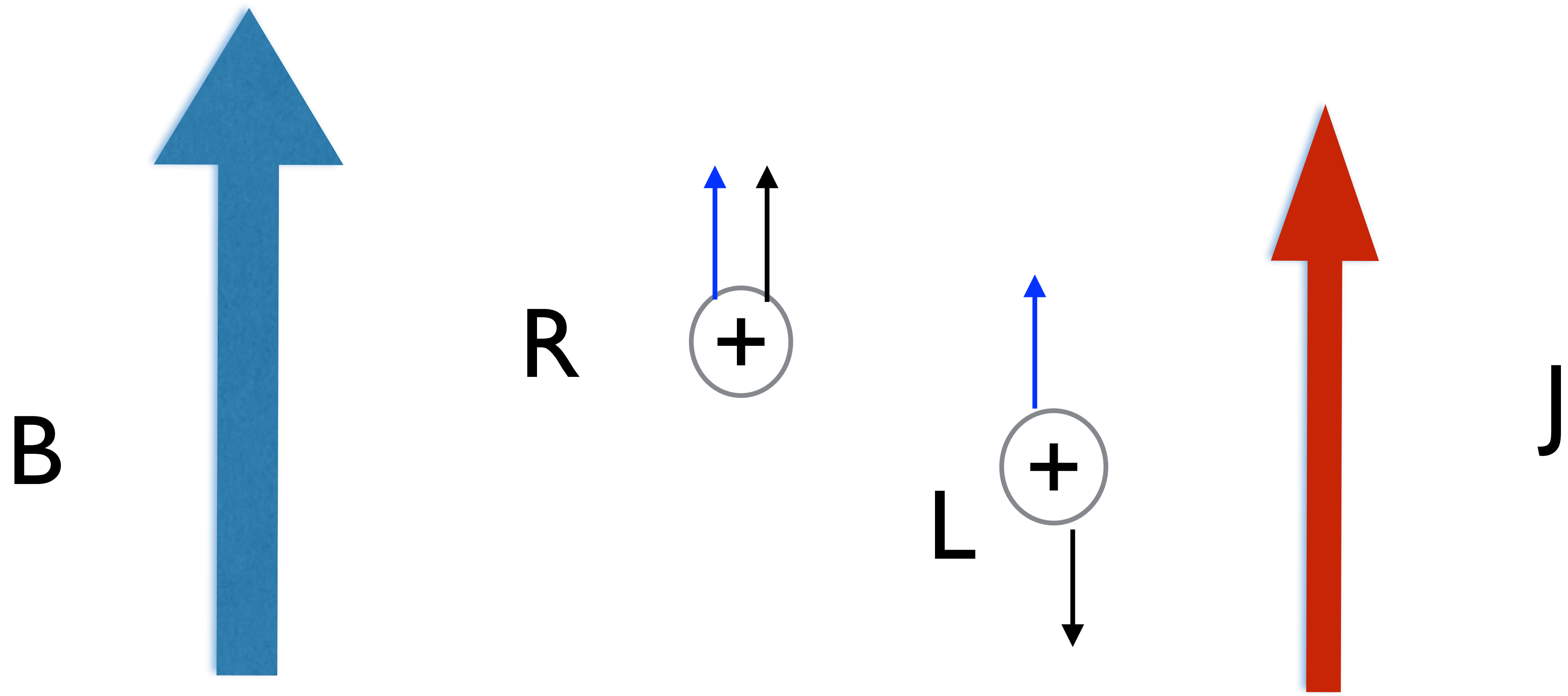
Outline

- Chiral Magnetic Effect
- Chiral Transport Equation
- Anomalous Maxwell Equations
- Applications to HIC and the QGP

Chiral Magnetic Effect



In a B a misbalance in the population of L/R handed fermions leads to an e.m. current I I to B

$$\mathbf{J} = \frac{e^2\,\mu_5}{4\pi^2}\,\mathbf{B}$$

Chiral Magnetic Effect

- First seen in EW (Vilenkin, 80)
- Discussed in the framework of HIC Kharzeev, McLerran, Fukushima, Warringa, '08
- Discussed in AdS/CFT Yee, Landsteiner et a,l, etc
- Studied in the lattice Buividovich et al; M Abramczyk et al
- Derived in hydrodynamics Son and Surowka
- Derived in kinetic theory (Son and Yamamoto, Stephanov and Yin, CM and Torres-Rincon)
- Observed in Dirac semimetals Kharzeev et al, '15

All these ideas here discussed are relevant for condensed matter physics

Discovery of Weyl semimetals and Weyl fermions as quasiparticles

Hasan et al, '15; Weng et al,

Chiral Transport Equation

Son and Yamamoto, '12; Stephanov and Yin, '12

In a collisionless case

$$\frac{\partial f_p}{\partial t} + (1 + e\hbar \mathbf{B}\cdot\boldsymbol{\Omega})^{-1}\left\{\left[\tilde{\mathbf{v}} + e\hbar\,\tilde{\mathbf{E}}\times\boldsymbol{\Omega} + e\hbar\,\mathbf{B}(\tilde{\mathbf{v}}\cdot\boldsymbol{\Omega})\right]\cdot\frac{\partial f_p}{\partial \mathbf{r}} + e\left[\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + e\hbar\boldsymbol{\Omega}\,(\tilde{\mathbf{E}}\cdot\mathbf{B})\right]\cdot\frac{\partial f_p}{\partial \mathbf{p}}\right\} = 0$$

where

$$\tilde{\mathbf{E}} = \mathbf{E} - \frac{1}{e}\frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{r}}$$

$$\tilde{\mathbf{v}} = \frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{p}}$$

$$\boldsymbol{\Omega} = \frac{\mathbf{p}}{2p^3}$$

One can reproduce the chiral anomaly equation

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{j} = -e^2 \hbar \int \frac{d^3 p}{(2\pi\hbar)^3} \left(\mathbf{\Omega} \cdot \frac{\partial f_p}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B}$$

In a thermal plasma: take into account both particles/ antiparticles to correctly reproduce the chiral anomaly

$$f_p^{R,L} = \frac{1}{\exp\left[\frac{1}{T}\left(p - \mu_{R,L}\right)\right] + 1}$$

$$\bar{f}_p^{L,R} = \frac{1}{\exp\left[\frac{1}{T}\left(p + \mu_{R,L}\right)\right] + 1}$$

$$\partial_\mu j_A^\mu = \frac{e^2}{2\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} \qquad \partial_\mu j_V^\mu = 0$$

The chiral transport equation can be deduced simply by computing (for m=0) the first quantum corrections to the classical eqs. of motion

FW diagonalization

EFT methods

Semiclassical chiral transport equation

Foldy-Wouthuysen Diagonalization

- The Dirac eq. for a free fermion mixes particles and antiparticles d.o.f.
- FW found a representation where these can be separated, through a canonical transformation

exact for the free theory

approx. for an interacting theory

$$H\psi = i\frac{\partial\psi}{\partial t} \qquad H' = UHU^\dagger \qquad \psi' = U\psi$$

$$\mathrm{H}_0 = \alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R})) + \beta m + eA_0(\mathbf{R})$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

At order $\mathcal{O}(\hbar^0)$

$$U = \frac{E + m + \beta\alpha\cdot(\mathbf{P} - e\mathbf{A}(\mathbf{R}))}{\sqrt{2E(E+m)}}$$

$$H_D = UH_0U^\dagger = \beta E + eA_0(\mathbf{R})$$

$$E \equiv \sqrt{(\mathbf{P} - e\mathbf{A}(\mathbf{R}))^2 + m^2}.$$

At order $\mathcal{O}(\hbar)$ $[R_i, P_j] = i\hbar\delta_{ij}$

Gosselin, Berard and Mohrbach 2007

Give a prescription to deal with products of R, P

Keep unitarity; project over the diagonal

Rotate all operators

$$\mathbf{r} = \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{R}\,U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{R} + \mathcal{P}(\mathcal{A}_R)\ ,$$
$$\mathbf{p} = \mathcal{P}[U(\mathbf{P},\mathbf{R})\,\mathbf{P}\,U^\dagger(\mathbf{P},\mathbf{R})] = \mathbf{P} + \mathcal{P}(\mathcal{A}_P)$$

$$\mathcal{P}(\mathcal{A}_{R_i}) = -\hbar\frac{E[\mathbf{\Sigma}\times(\mathbf{P}-e\mathbf{A})]_i}{2E^2(E+m)}\ , \qquad \mathcal{A}_{P^i} = e\ \nabla_{R^i}A_k(\mathbf{R})\mathcal{A}_{R^k}$$

$$\Sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}$$

In terms of the rotated variables

$$H_D = \beta\left(E - \frac{e\hbar\mathbf{\Sigma}\cdot\mathbf{B}}{2E} - \frac{e\mathbf{L}\cdot\mathbf{B}}{E}\right) + eA_0(\mathbf{r})$$

$$E = \sqrt{(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2 + m^2}$$

$$\mathbf{L} = \tilde{\mathbf{p}} \times \mathcal{P}(\mathcal{A}_{\mathbf{R}}) = \hbar\,\frac{\tilde{\mathbf{p}} \times (\tilde{\mathbf{p}} \times \mathbf{\Sigma})}{2E(E+m)} \qquad \tilde{\mathbf{p}} \equiv \mathbf{p} - e\mathbf{A}(\mathbf{r})$$

Gauge invariance kept at order of accuracy

The new variables are non canonical

$$\begin{aligned} [r_i, r_j] &= i\hbar^2 G_{ij} = -i\hbar^2 \epsilon_{ijk} G_k \\ [\tilde{p}_i, \tilde{p}_j] &= ie\hbar F_{ij} + ie^2\hbar^2 F_{ik} F_{jm} G_{km} \,, \\ [r_i, \tilde{p}_j] &= i\hbar\delta_{ij} + ie\hbar^2 F_{jk} G_{ik} \end{aligned}$$

$$\mathbf{G}(\tilde{\mathbf{p}}) = \frac{1}{2E^3}\left(m\mathbf{\Sigma} + \frac{(\mathbf{\Sigma}\cdot\tilde{\mathbf{p}})\tilde{\mathbf{p}}}{E+m}\right)$$

Massless fermions

$$\tilde{\mathbf{p}} \to \mathbf{p}$$

$$\mathbf{G} = \lambda \mathbf{\Omega} \,, \qquad \mathbf{\Omega} = \frac{\mathbf{p}}{2p^3} \qquad \lambda = \frac{\Sigma \cdot \mathbf{p}}{p}$$

Fermion dispersion law in an B field is modified

$$\epsilon^{\pm}_{\mathbf{p}} = \pm p \left(1 - e\hbar\, \lambda \frac{\mathbf{B} \cdot \mathbf{p}}{2p^3} \right)$$

Semiclassical equations of motion (e.g. right-handed)

$$\begin{aligned} \dot{\mathbf{p}} &= -\frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{r}} + e(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B}) \,, \\ \dot{\mathbf{r}} &= \frac{\partial \epsilon^{+}_{\mathbf{p}}}{\partial \mathbf{p}} - \hbar(\dot{\mathbf{p}} \times \mathbf{\Omega}) \end{aligned}$$

Linear response analysis

Electromagnetic current obtained in a thermal plasma, with chiral misbalance

$$J^\mu(k) = \Pi_+^{\mu\nu}(k) A_\nu(k) + \Pi_-^{\mu\nu}(k) A_\nu(k)$$

$$\Pi_+^{\mu\nu}(k) = -m_D^2 \left(\delta^{\mu 0}\delta^{\nu 0} - \omega \int_v \frac{v^\mu v^\nu}{v \cdot k} \right)$$

$$\Pi_-^{\mu\nu}(k) = \frac{c_E e^2}{2\pi^2} i\epsilon^{\mu\nu\alpha\beta} k^2 k_\beta \int_v \frac{v_\alpha}{(v \cdot k)^2}$$

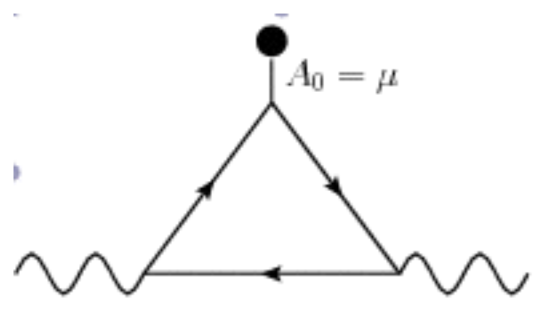
$$m_D^2 = e^2 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right) \qquad c_E = -\mu_5/2 \qquad \mu_5 = \mu_R - \mu_L$$

Both pieces (+/-) agree with the non-anomalous/anomalous Feynman diagrams computed in the HTL/HDL approximation

Laine, `05

$$S^{+}_{\rm HTL} = -\frac{m_D^2}{4}\int_{x,v} F_{\alpha\mu}(x)\,\frac{v^\alpha v^\beta}{(v\cdot\partial)^2}\,F_\beta{}^\mu(x)$$

$$S^{-}_{\rm HTL} = \frac{c_E e^2}{4\pi^2}\int_{x,v}\partial^2\widetilde{F}_{\alpha\mu}(x)\,\frac{v^\alpha v^\gamma}{(v\cdot\partial)^3}\,F_\gamma{}^\mu(x)$$



Kinetic theory provides a framework to treat in a local way also the anomalous HTL effects (energy density, etc …)

Transport theory provides a perfect framework to study the dynamical evolution of the system, where different anomalous effects can be taken into account

Including collisions, in the RTA

$$C^{\mathrm{RTA}}[f_p] = -\frac{1}{\tau}(f_p - f_p^{eq})$$

Solve the dynamics for time scales larger than the relaxation time

$$\mathbf{J} = \sigma \mathbf{E} + \sum_{s=1}^{N_s} \frac{e_s^2\, \mu_5}{4\pi^2}\, \mathbf{B}$$

Anomalous Maxwell Equations

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma}\nabla^2\mathbf{B} + \frac{C\alpha\mu_5}{\pi\sigma}\nabla\times\mathbf{B} - \frac{1}{\sigma}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{2C\alpha}{\pi}\frac{1}{V}\int d^3x\,\mathbf{E}\cdot\mathbf{B} = -\frac{C\alpha}{\pi}\frac{d\mathcal{H}}{dt}$$

$$C \equiv \sum_{s=1}^{N_s}\frac{e_s^2}{e^2}$$

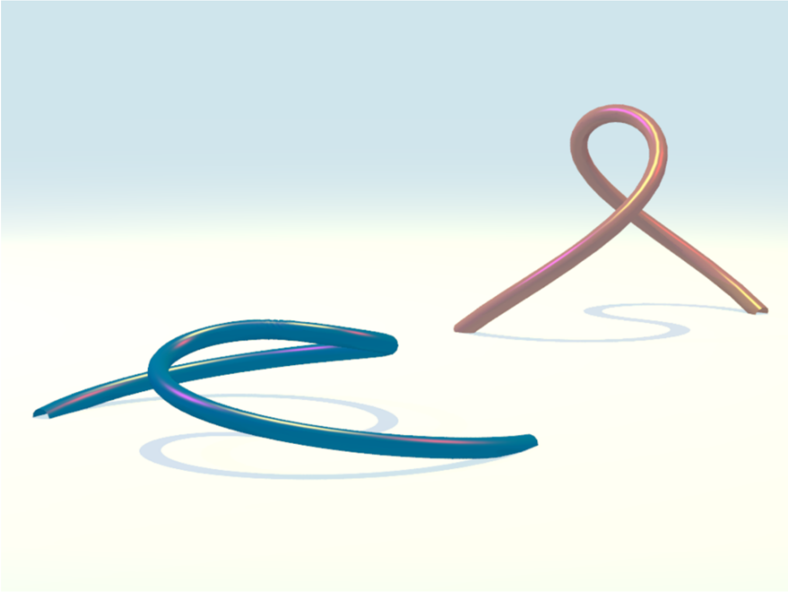
Linked dynamical evolution of magnetic fields and chiral fermion imbalance

Magnetic Helicity Density

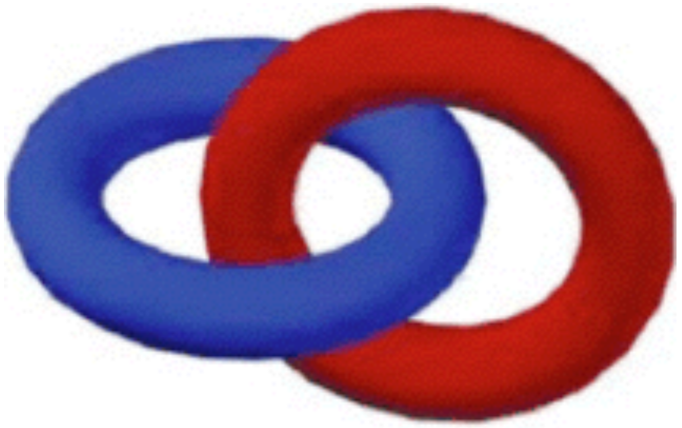
(or Chern-Simons number)

$$\mathcal{H}(t) = \frac{1}{V}\int_V d^3x\, \mathbf{A}\cdot\mathbf{B}$$

gives a measure of a non-trivial topology of the B lines



gauge invariant if B=0 on ∂V (or B n=0)



$H_M = \int \mathbf{A}\cdot\mathbf{B}dV = 2\phi\cdot\phi$

$$\Phi = \int \mathbf{B}\cdot\mathbf{dS}$$

$$\Phi = \int \mathbf{B}\cdot\mathbf{dS}$$

$$\int \mathbf{A}\cdot\mathbf{B}\,dV = \int A_1 dl_1 \int B_1 dS_1 + \int A_2 dl_2 \int B_2 dS_2$$

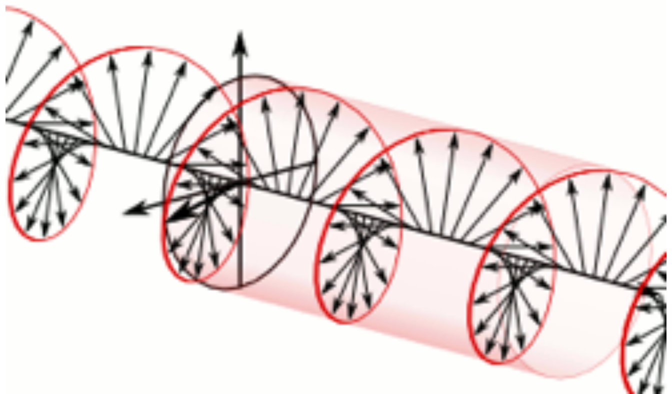
Magnetic helicity
gives a measure of magnetic linkage

In Fourier modes, using vector polarization vectors describing circular polarized waves $(\mathbf{e}_+, \mathbf{e}_-, \hat{\mathbf{k}})$

$$\mathbf{B}_\mathbf{k} = B^+_\mathbf{k} \mathbf{e}_+ + B^-_\mathbf{k} \mathbf{e}_-$$

$$\mathcal{H} = \frac{1}{V} \int \frac{d\Omega_\mathbf{k}}{4\pi} \int dk \frac{k}{2\pi^2} \left(|B^+_\mathbf{k}|^2 - |B^-_\mathbf{k}|^2\right)$$

it gives account of an asymmetry of L(+)/R(-) polarized fields



(Integrated) Anomaly Equation

Expresses a conservation law of total helicity

$$\frac{d}{dt}\left(n_5 + \frac{C\alpha}{\pi}\mathcal{H}\right) = 0$$

chiral fermion imbalance can be converted into magnetic helicity and vice versa

$$\frac{1}{\sigma}\frac{\partial^2 B^{\pm}_{\mathbf{k}}}{\partial t^2} + \frac{\partial B^{\pm}_{\mathbf{k}}}{\partial t} = -\left(\frac{1}{\sigma}k^2 \mp \frac{C\alpha\mu_5 k}{\pi\sigma}\right) B^{\pm}_{\mathbf{k}} \ ,$$

$$\frac{dn_5}{dt} = -\frac{C\alpha}{V}\int\frac{d\Omega_{\mathbf{k}}}{4\pi}\int dk\,\frac{k}{2\pi^2}\frac{d}{dt}\left(|B^{+}_{\mathbf{k}}|^2 - |B^{-}_{\mathbf{k}}|^2\right) - \Gamma_f n_5 \ .$$

helicity flipping rate

Note: L/R handed polarized fields evolve differently with fermion chiral imbalance!

(circular dichroism)

We will solve the dynamics for both B and n5 assuming $t \ll 1/\Gamma_f$

Small frequencies $\omega \ll \sigma$

Analytical solutions can be found

$$B^{\pm}_{\mathbf{k}} = B^{\pm}_{\mathbf{k},0} \exp\left[-\frac{(k^2 \mp K_p k)}{\sigma} t\right]$$

$$K_p \equiv \frac{1}{t}\int_0^t d\tau \frac{C\alpha\mu_5(\tau)}{\pi}$$

At high T $\quad n_5 \propto T^2 \mu_5$

$$\mu_5(t) = \mu_{5,0} \exp\left(-\Gamma_f t\right) - c_{\Delta} C\alpha[\mathcal{H}(t) - \mathcal{H}(0)]$$

Assume an initial monochromatic helicity

$$|B^+_{\mathbf{k},0}|^2 = |B^+_0|^2 \delta(k - k_0)$$

$$\mathcal{H}(t) = h_0 \exp\left[\frac{2k_0}{\sigma}\left(K_p - k_0\right) t\right]$$

Chiral magnetic instability $k < \frac{C\alpha\mu_5}{\pi}$

B grows till reaching the tracking solution

$$\mu_{5,\mathrm{tr}} = \frac{\pi k_0}{C\alpha} \qquad t_{\mathrm{inst},h} \approx \frac{\pi\sigma}{2k_0 C\alpha(\mu_{5,0} - \mu_{5,tr})}$$

Several modes: inverse cascade phenomenon H transferred from the highest to lowest modes

used in cosmological scenarios

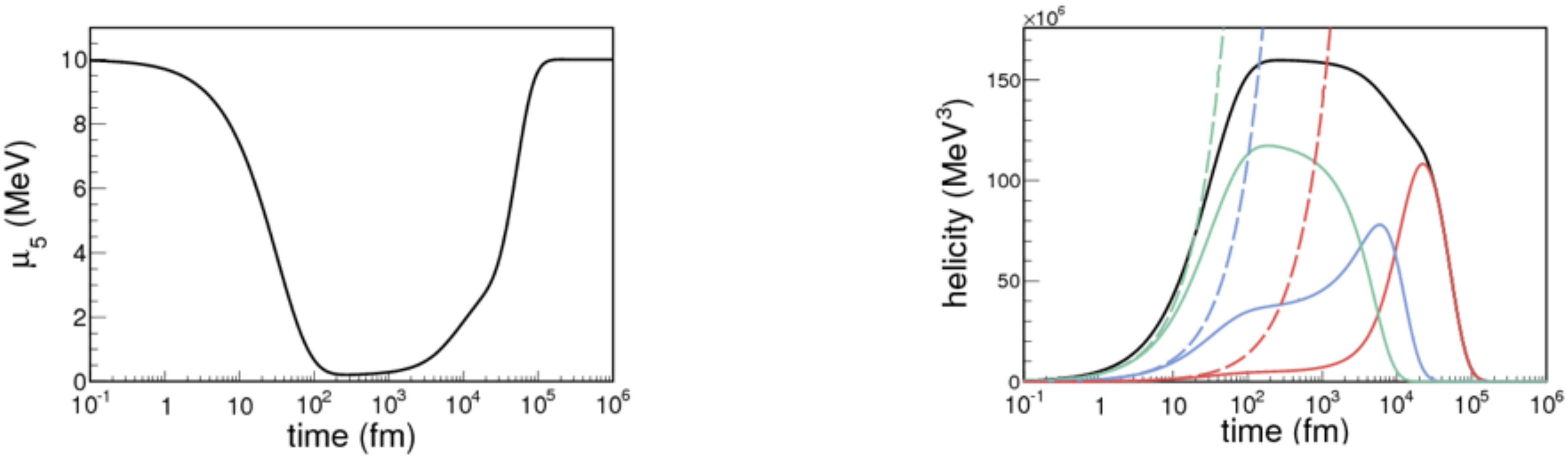
Assume initial monochromatic B, but no initial helicity

$$\mathcal{H}(t) = 2\tilde{h}_0 \exp\left[-\frac{2k_0^2}{\sigma}t\right] \sinh\left(\frac{2k_0 K_p}{\sigma}t\right) \qquad \tilde{h}_0 = \frac{|\tilde{B}_0|^2 k_0}{2\pi^2 V}$$

At short t the helicity grows!

(no chiral instability)

with 3 FM



Large frequencies $\omega \gg \sigma$

Only analytical solutions for constant chiral imbalance

$$B^{\pm}_{\mathbf{k}} = \frac{1}{2} B^{\pm}_{\mathbf{k},0} \left[e^{-\frac{\sigma t}{2}\left(1+\sqrt{\phi_{\pm}}\right)} \left(1 - \frac{1}{\sqrt{\phi_{\pm}}}\right) + e^{-\frac{\sigma t}{2}\left(1-\sqrt{\phi_{\pm}}\right)} \left(1 + \frac{1}{\sqrt{\phi_{\pm}}}\right) \right]$$

$$\phi_{\pm} \equiv 1 - \frac{4k}{\sigma^2}\left(k \mp \frac{C\alpha\mu_5}{\pi}\right)$$

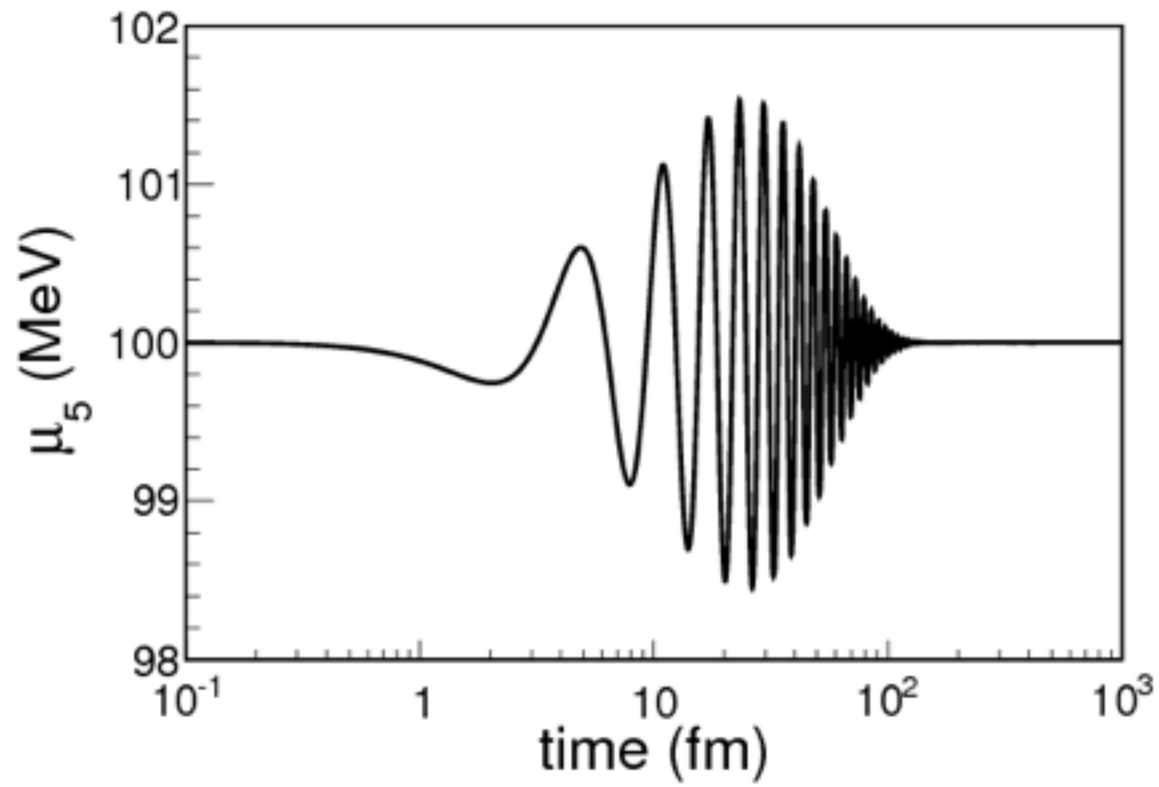
Chiral magnetic instability $k < \frac{C\alpha\mu_5}{\pi}$

Otherwise, the solutions are oscillatory at short time

$$B^{\pm}_{\mathbf{k}} = B^{\pm}_{\mathbf{k},0} e^{-\frac{\sigma t}{2}} \left[\cos\frac{\sigma \Delta_{\pm} t}{2} + \frac{1}{\Delta_{\pm}} \sin\frac{\sigma \Delta_{\pm} t}{2} \right] \qquad \Delta_{\pm} \equiv \sqrt{|\phi_{\pm}|}$$

Monochromatic B

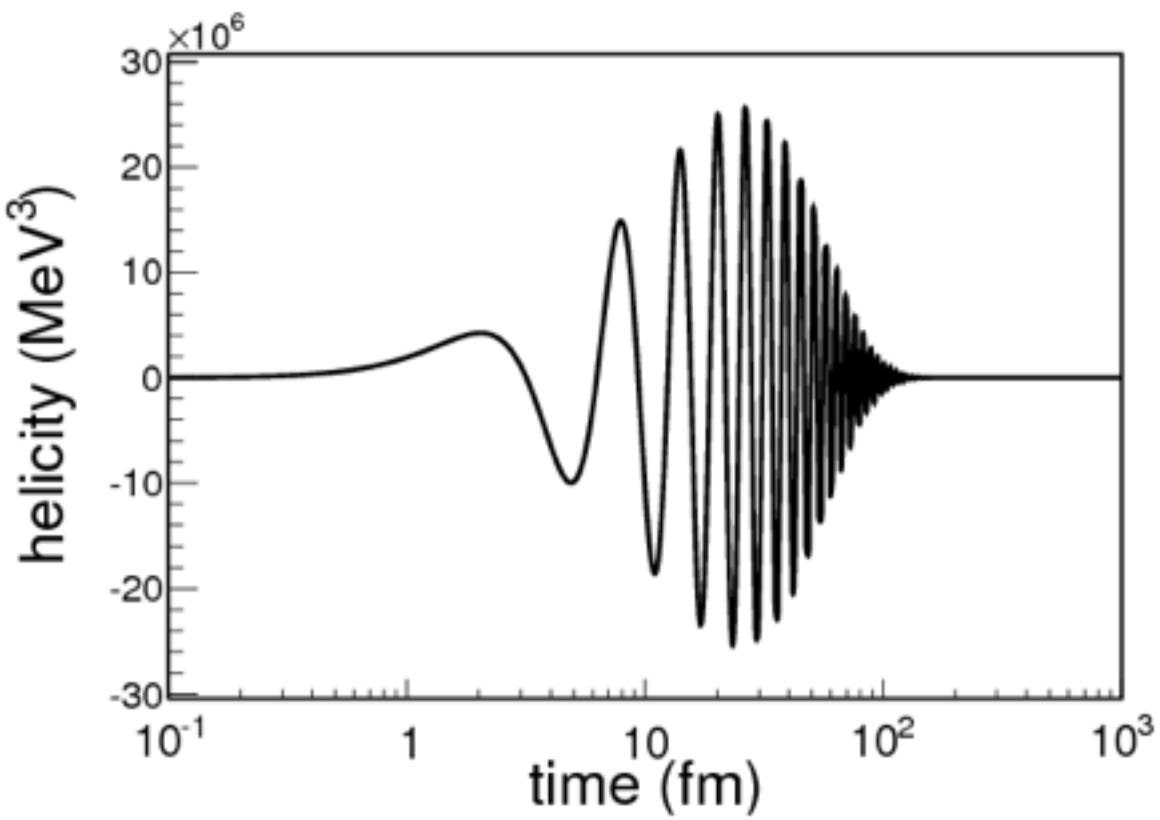
$k_0 = 100\,\mathrm{MeV}$



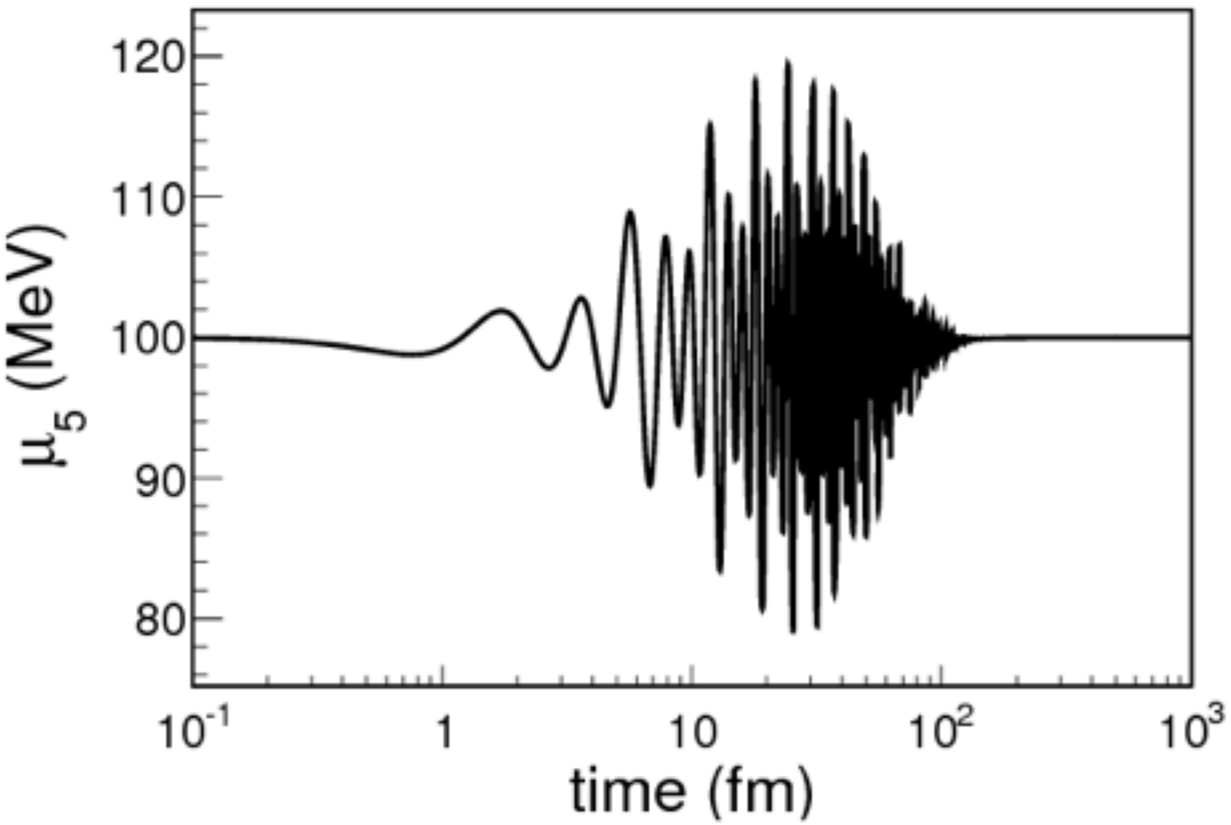
$\sigma = 5\,\mathrm{MeV}$

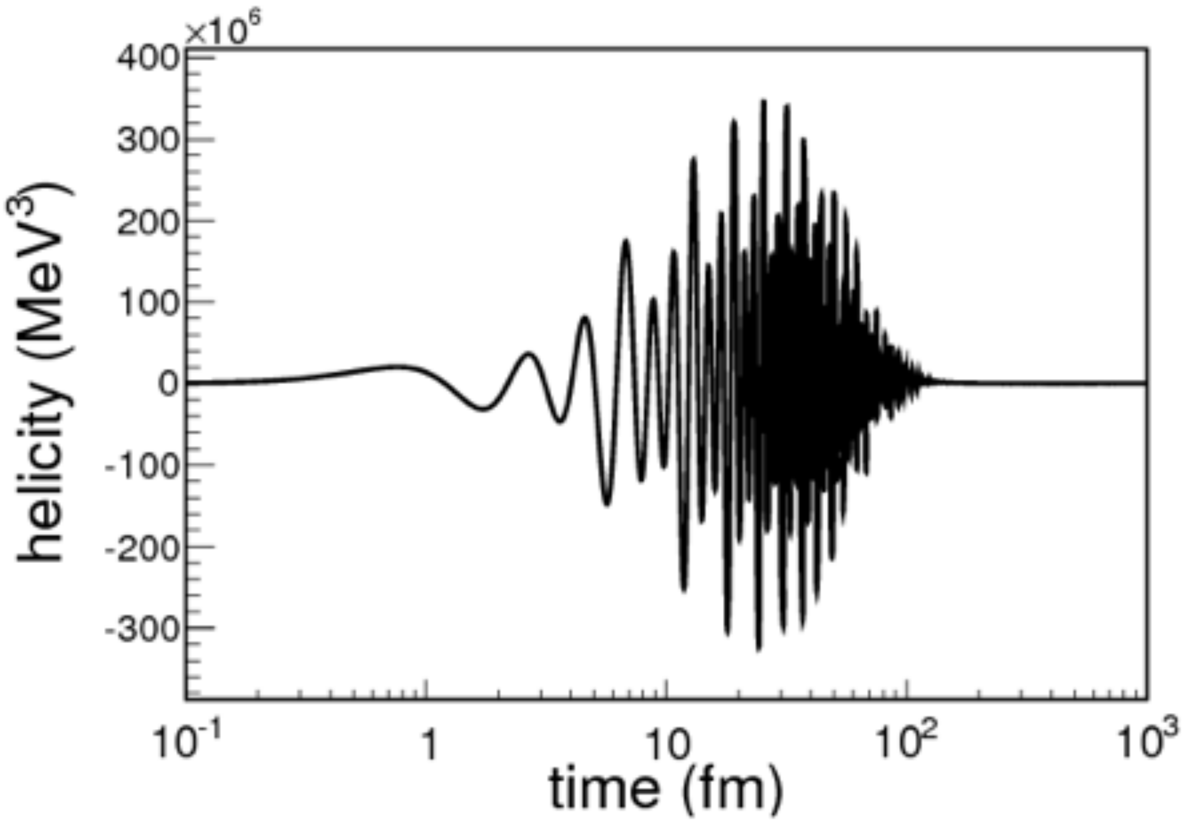
$T = 225\,\mathrm{MeV}$

$R = 10\,\mathrm{fm}$



Several modes: interference



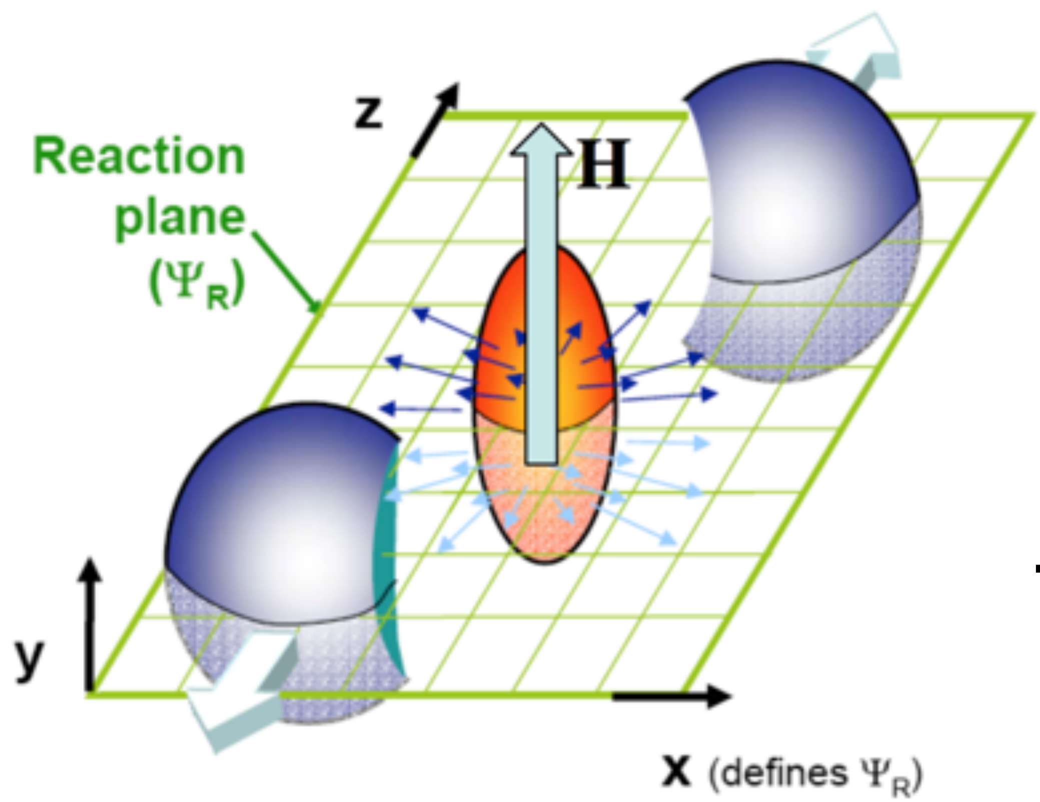


$$k_0 = 100, 200, 300\,\mathrm{MeV}$$

All modes decay at

$$t \sim 1/\sigma \sim 40\,\mathrm{fm}$$

Applications to QGP



Kharzeev

$$e|\mathbf{B}| \sim (1-10)m_\pi^2$$

Topological properties of QCD might create a chiral fermion imbalance

Ground state of QCD: different vacua with non-Abelian Chern Simons number

$$\partial_\mu j_A^\mu = \frac{2C\alpha}{\pi}\mathbf{E}\cdot\mathbf{B} + \frac{\alpha_s N_f}{\pi}\mathbf{E}_a\cdot\mathbf{B}_a$$

Helicity flipping rate

We can safely ignore it in HIC

We have computed rates associated to Compton scattering (with non vanishing mass)

$$e^-_R \gamma_L \leftrightarrow e^-_L \gamma_R$$

$$\Gamma_f = \frac{3}{2\pi^3}\alpha^2 \frac{m^2}{T}\gamma(y^2)$$

$$\tau_f^{QED} = \Gamma_f^{-1} \simeq \begin{cases} 5.7\cdot 10^4 & \text{fm (for } m = 5 \text{ MeV)}\ , \\ 6.5\cdot 10^4 & \text{fm (for } m = 100 \text{ MeV)}\ . \end{cases}$$

$$\tau_f^{QCD} = \tau_f^{QED}\, \frac{9\alpha^2}{2\alpha_s^2} \simeq \begin{cases} 123 & \text{fm (for } m = 5 \text{ MeV)}\ , \\ 140 & \text{fm (for } m = 100 \text{ MeV)}\ . \end{cases}$$

Expect similar results for qq scattering

Toy model

Sphere R = 10 fm $\mu_5 \sim 100\,\mathrm{MeV}$

$$n_5 = N_c N_f \frac{\mu_5}{24\pi^2}\left(\mu_5^2 + 12\mu_V^2 + 4\pi^2 T^2\right)$$

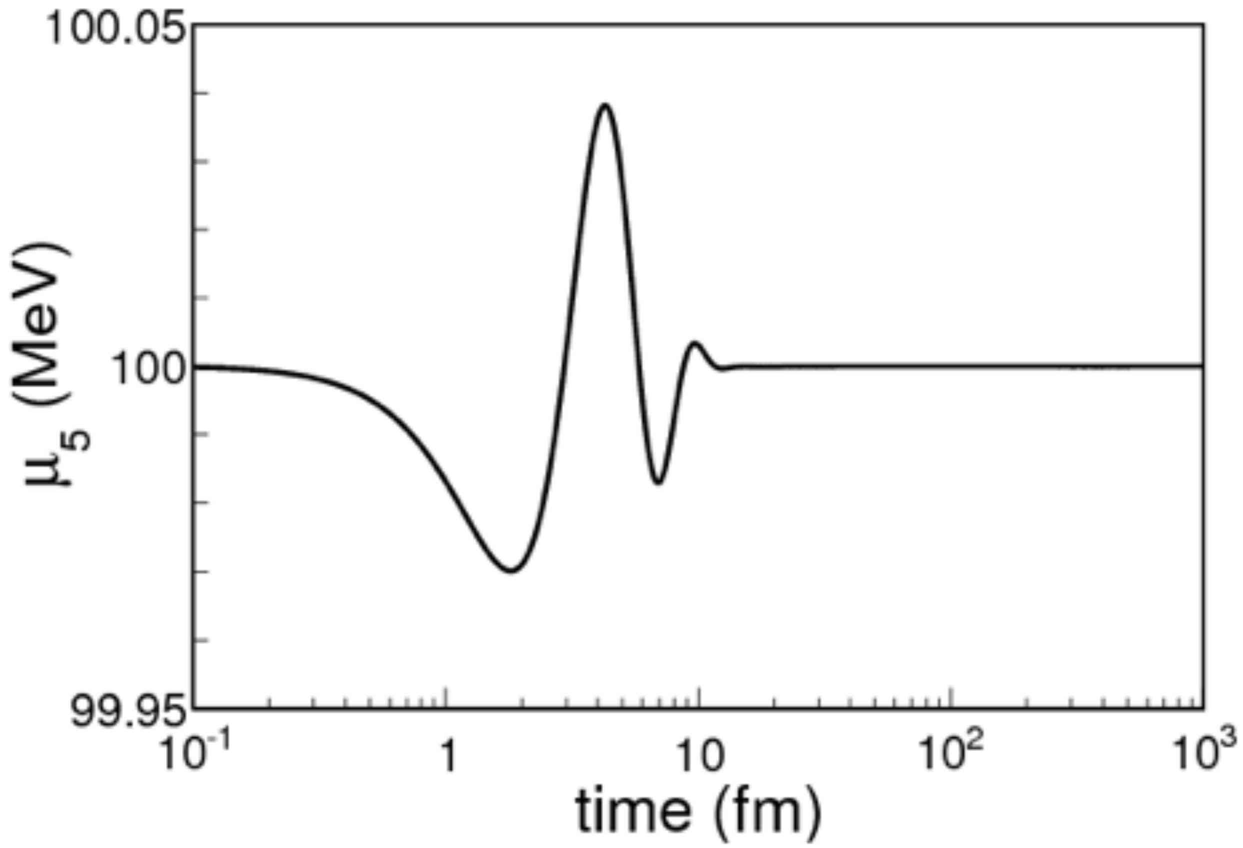
$$\sigma = 0.0224T \qquad T = 225\,\mathrm{MeV}\ , \qquad \sigma \sim 5.4\,\mathrm{MeV}$$

Magnetic field of Gaussian shape

$$B_{\mathbf{k},0} = b_0 \exp\left[-\frac{1}{2}\left(\frac{k-k_0}{\kappa}\right)^2\right]$$

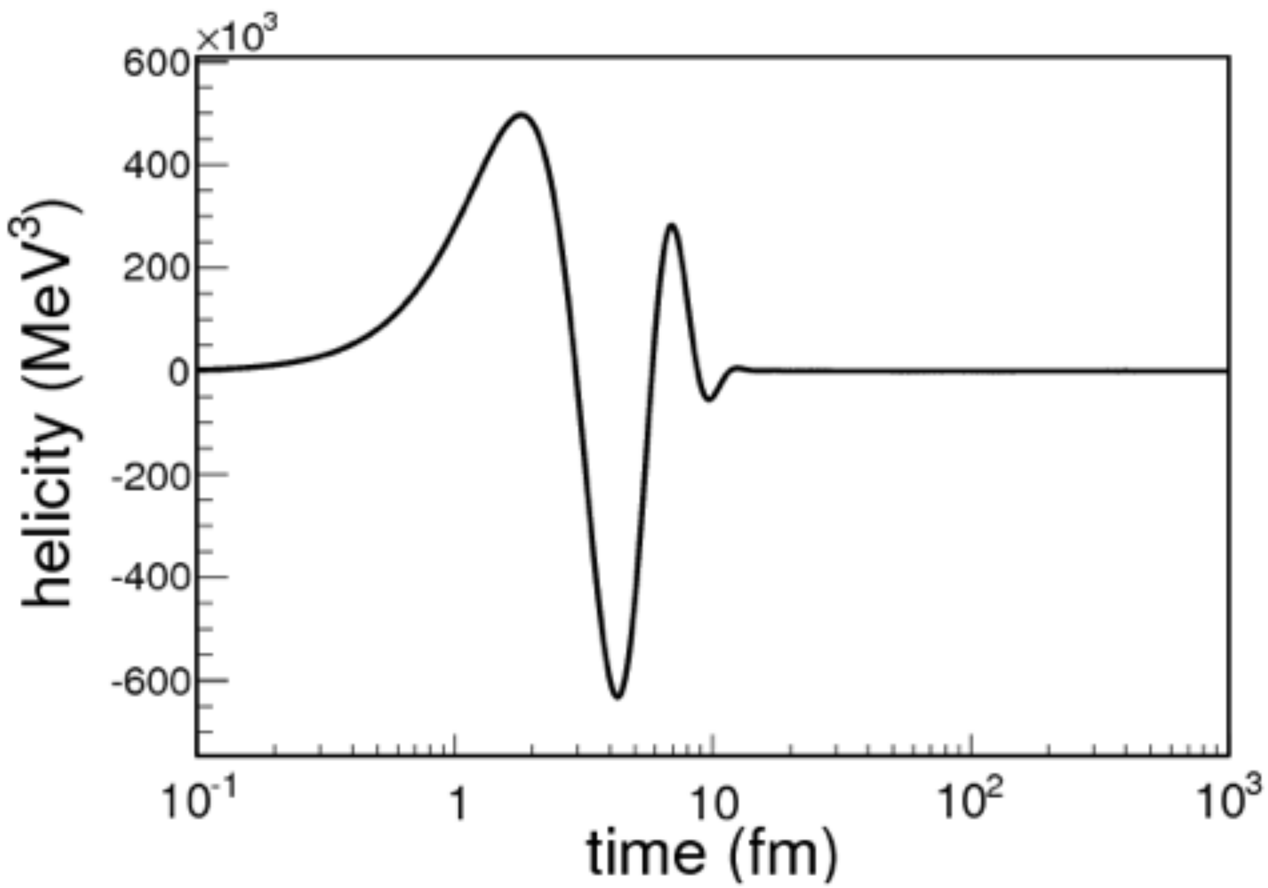
Note $k > \frac{1}{R} \sim 20\,\mathrm{MeV} \gg \sigma$

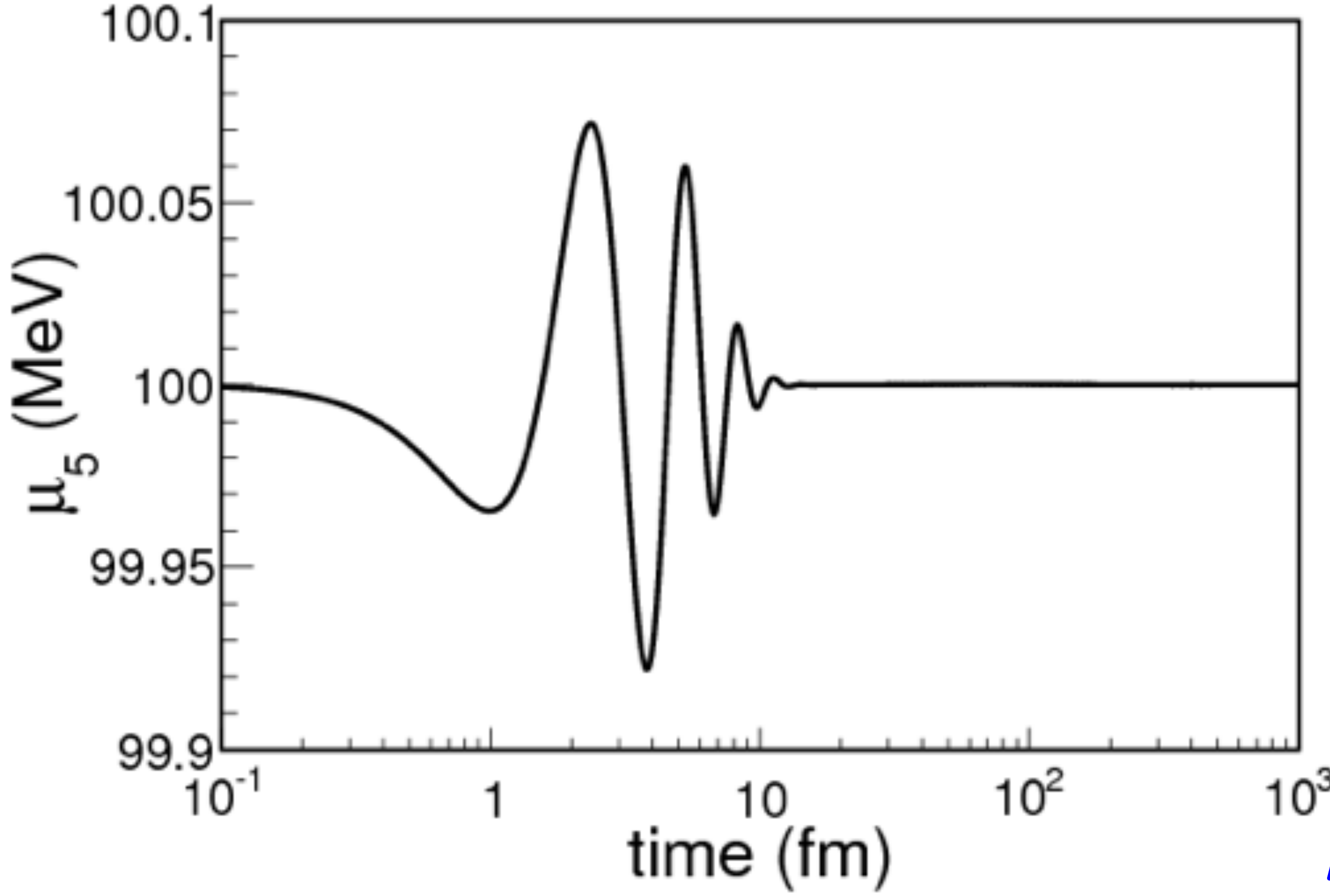
A chiral instability very unlikely to occur $k < \frac{C\alpha\mu_5}{\pi}$



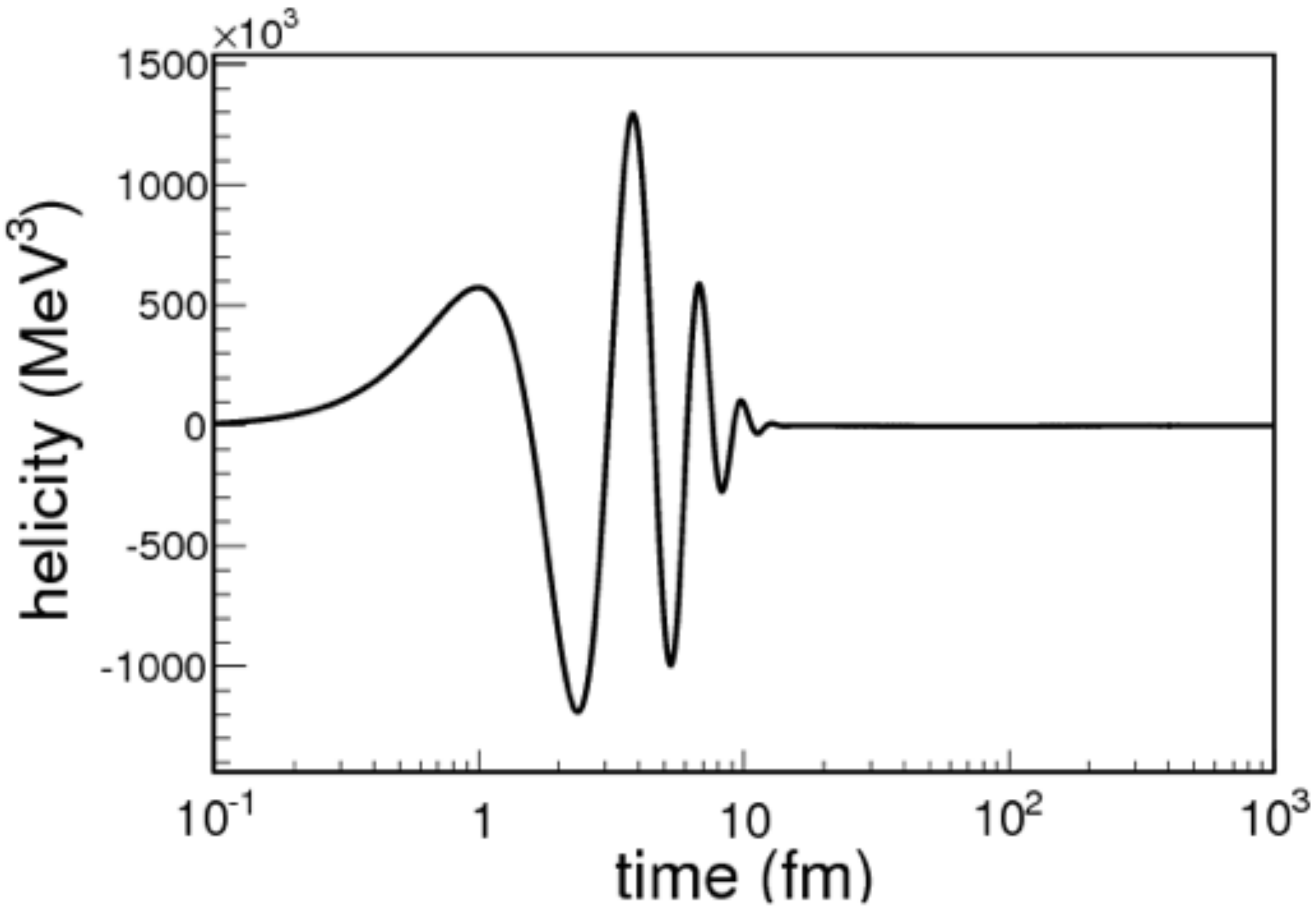
$$e|\mathrm{B}_0| = m_\pi^2$$

$$k_0 = 100\,\mathrm{MeV}\ , \qquad \kappa = 40\,\mathrm{MeV}$$

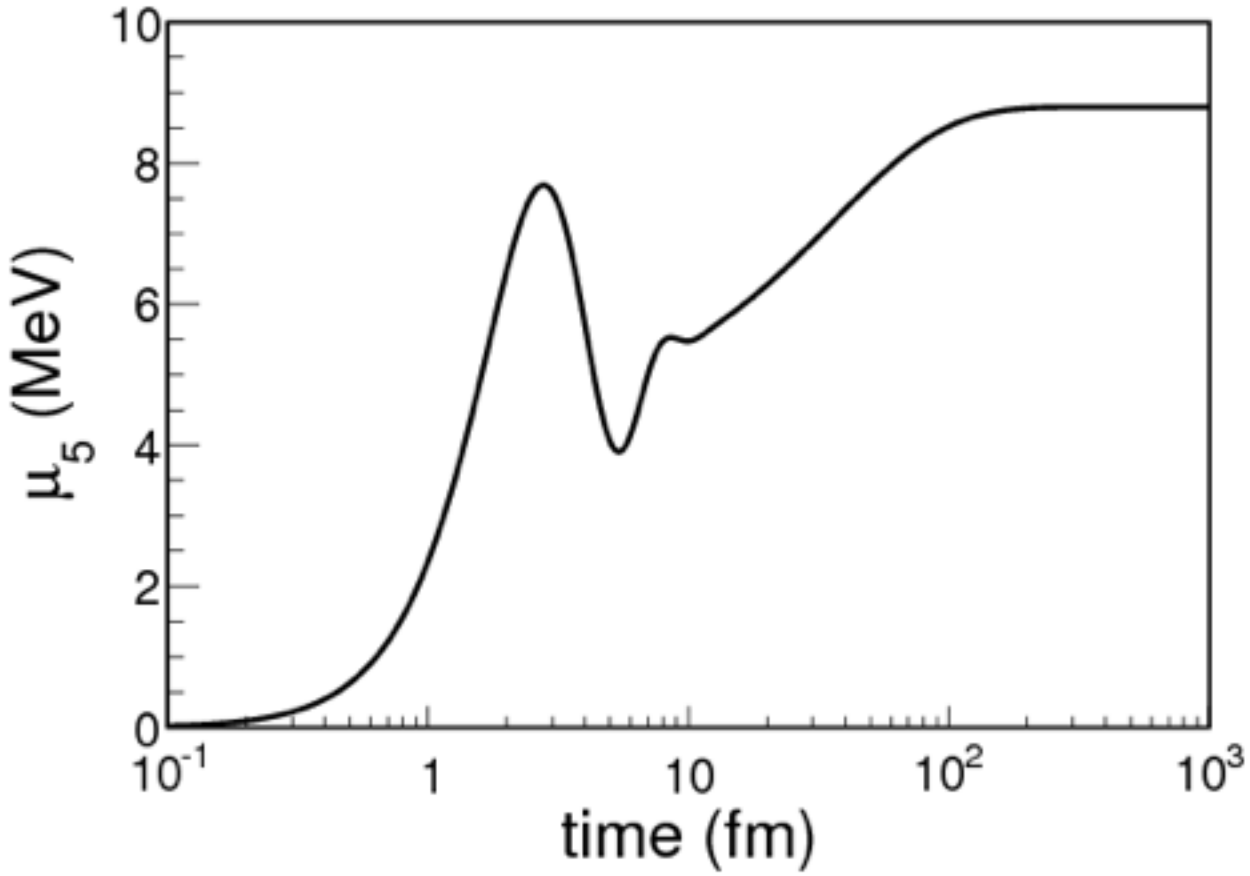




$k_0 = 200\,\mathrm{MeV}$, $\kappa = 40\,\mathrm{MeV}$

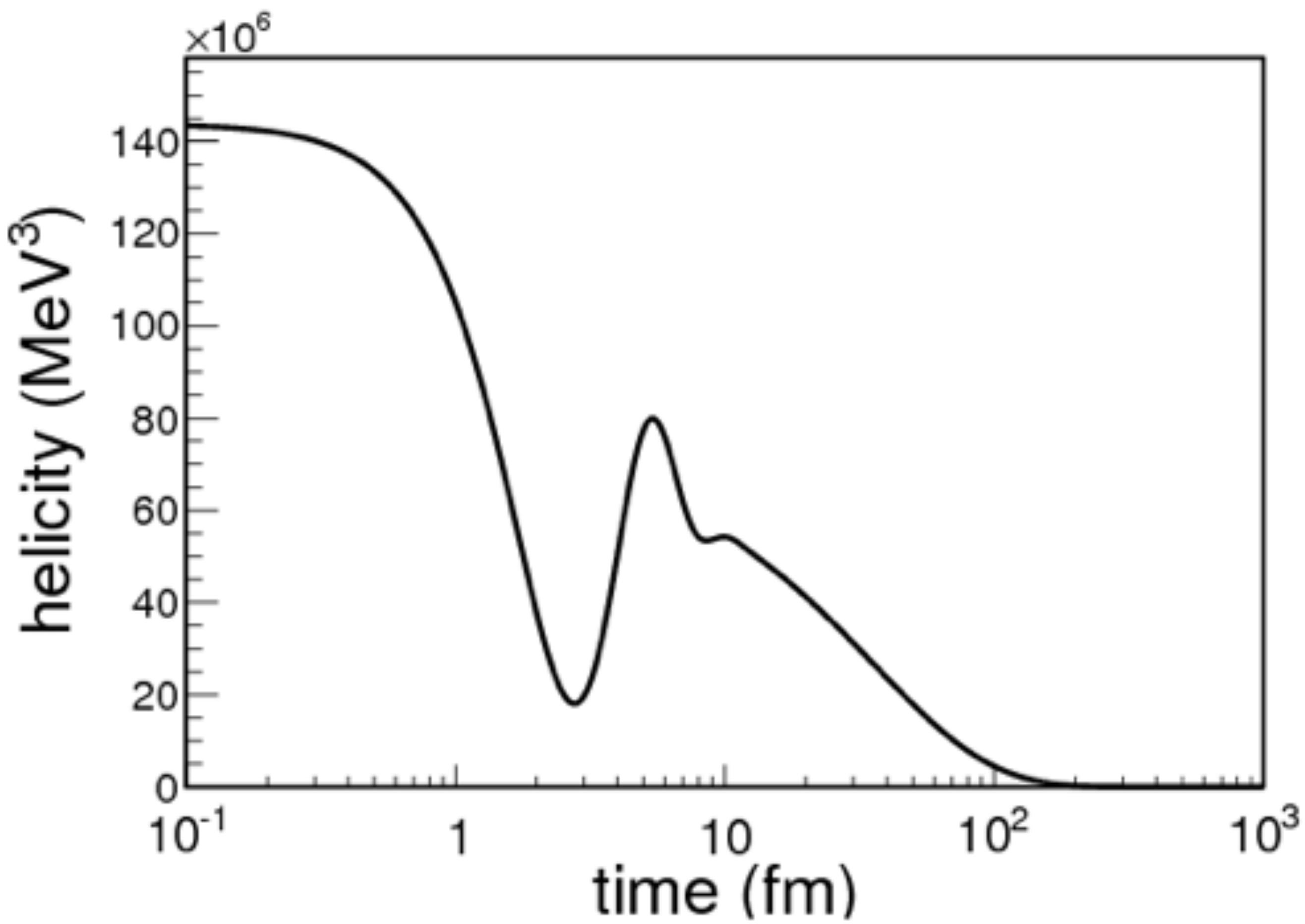


Generation of fermion chiral imbalance



$$e|\mathrm{B}_0| = m_\pi^2$$

$$k_0 = 100\,\mathrm{MeV}\ , \qquad \kappa = 40\,\mathrm{MeV}$$



Anomalous hydrodynamics for HIC

The effects just discussed should have an impact on the (anomalous) hydrodynamics relevant for HIC

Look into the azimuthal charged particle distribution functions the existence of these effects would result in the presence of P-odd harmonics. Study correlations

Conclusions

- Chiral transport equation includes quantum corrections that allow us to study anomalous effects, such as the CME; needed for NED
- Anomalous Maxwell's equations: magnetic helicity and chiral fermion imbalance linked
- HIC and QGP: presence of chiral imbalance results in generation of magnetic helicity; this should affect the event-by-event hydrodynamics