Dynamical Modeling of Fluctuations at the QCD Phase Transition in Heavy-Ion Collisions

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The goal...

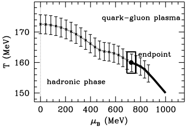
... is to understand the phase structure and the phase diagram of QCD theoretically and experimentally.

Make the connection between QCD thermodynamics (LQCD) and heavy-ion collisions.

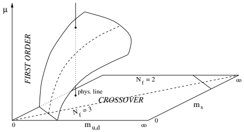
https://news.uic.edu/collider-reveals-sharp-change-from-quark-soup-to-atoms

From the theory side...

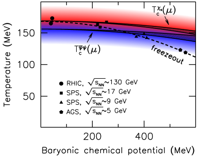
- Lattice QCD calculations:



Wuppertal-Budapest (2002)



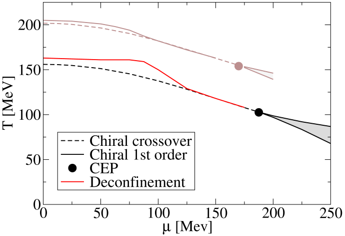
F. Karsch et al. (2004)



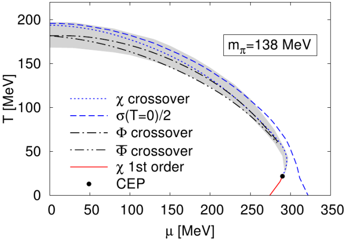
Wuppertal-Budapest (2011)

+ newer approaches to circumvent the sign problem!

- Functional methods of QCD:



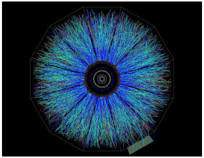
C. Fischer, J. Luecker, PLB**718** (2013)



T. Herbst, J. Pawlowski, BJ. Schaefer PRD**88** (2013)

From the experimental side...

- One of the main goals of heavy-ion collisions is to understand the phase structure of hot and dense strongly interacting matter.



ALICE PHENIX

FAIR SHINE NA61 NICA

- Can we experimentally produce a deconfined phase with colored degrees of freedom?
- What are the properties of this phase?
- What is the nature of the phase transition between deconfined and hadronic phase?

Challenges for the BES II

- Need good dynamical models.
- Need good input.
- Need good observables.
- Need good data.

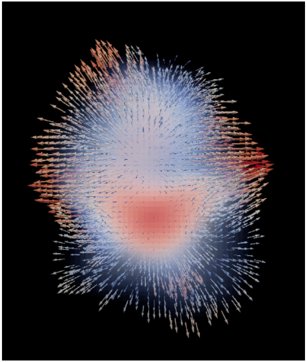
Challenges for the BES II

- Need good dynamical models.
  Initial state, coupling to FD, propagation of fluctuations, coupling to hadrons, ...
- Need good input.
  Equation of state, transport coefficients, ...
- Need good observables.
  Large scale simulations, sensitivity analysis, statistical tools, ...
- Need good data.
  Efficiency corrected, smaller error bars, 14.5 GeV, different particle species, ...

Dynamics of heavy-ion collisions

Systems created in heavy-ion collisions are

- short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical!



plot by H. Petersen, madai.us

Indications that we might still be able to learn about thermodynamic properties:

- success of fluid dynamics ($\Rightarrow$ local thermalization) with input from LQCD (EoS)
- success of statistical model and HRG analysis of particle yields and fluctuations

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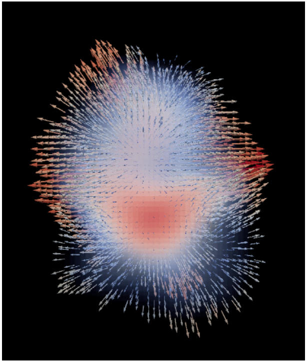

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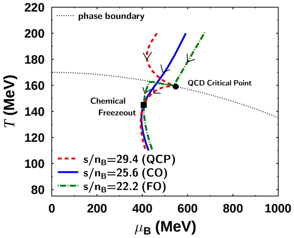
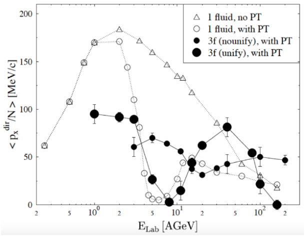
Phase transitions in fluid dynamics

- Conceptually, studying phase transitions in fluid dynamics is really simple!
- ⇒ Just need to know the equation of state and transport coefficients!



C. Nonaka, M. Asakawa PRC**71** (2005)



J. Brachmann et al. PRC**61** (2000)

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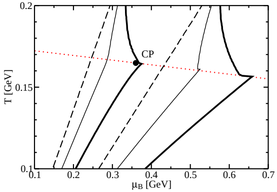
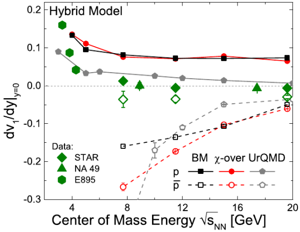


M. Bluhm, B. Kampfer CPOD 2006



J. Steinheimer et al. PRC**89** (2014)

- No clear sensitivity on the equation of state in observables.
- BUT at the phase transition: fluctuations matter! Including fluctuations in fluid dynamics is more challenging...

Fluctuations at the phase transition

At a critical point

- correlation length of fluctuations of the order parameter diverges $\xi \to \infty$
- fluctuations of the order parameter diverge: $\langle \Delta\sigma^n \rangle \propto \xi^\alpha$ with higher powers of divergence for higher moments
- mean-field studies in Ginzburg-Landau theories, beyond mean-field: renormalization group
- relaxation time diverges $\Rightarrow$ critical slowing down!

$\Rightarrow$ fluctuations in equilibrated systems!

... and a first-order PT:

- at T_c coexistence of two stable thermodynamic phases
- metastable states above and below T_c $\Rightarrow$ supercooling and -heating
- nucleation and spinodal decomposition in nonequilibrium
- domain formation and large inhomogeneities

$\Rightarrow$ fluctuations in nonequilibrium!

... but also at the crossover:

- remnant of the $O(4)$ universality class in the chiral limit.

$\Rightarrow$ fluctuations in equilibrated systems!

Ref. by many people: M. Stephanov, E. Shuryak, K. Rajagopal, L. Csernai, J. Randrup, I. Mishustin, C. Sasaki, B. Friman, K. Redlich et al.

The Kurtosis

The kurtosis is a measure of the deviation of fluctuations from Gaussian statistical fluctuations.

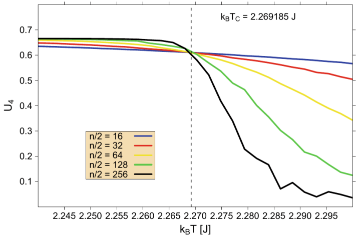
$$
\begin{aligned}
\langle \Delta X_i \Delta X_j \Delta X_k \Delta X_l \rangle \sim & \langle \Delta X_i \Delta X_j \rangle \langle \Delta X_k \Delta X_l \rangle \\
& + \langle \Delta X_i \Delta X_k \rangle \langle \Delta X_j \Delta X_l \rangle \\
& + \langle \Delta X_i \Delta X_l \rangle \langle \Delta X_j \Delta X_k \rangle
\end{aligned}
$$

$\Rightarrow \langle \Delta X^4 \rangle - 3\langle \Delta X^2 \rangle^2 = 0$ in the Gaussian approximation.

compare to *Binder cumulant* for eg. 2d Ising model:

$$
U = 1 - \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2}
$$

$= 0 + \mathcal{O}(1/V)$ in symmetric phase
$= U^* = 2/3$ at $T = T_c$
$= 2/3 + \mathcal{O}(1/V)$ in the broken phase



T. Preis et al., JCP228 (2009)

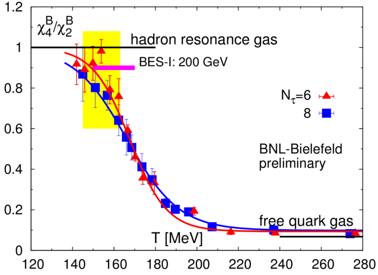
Kurtosis in lattice QCD

fluctuations of conserved charges B, Q, S can be expressed in terms of generalized susceptibilities

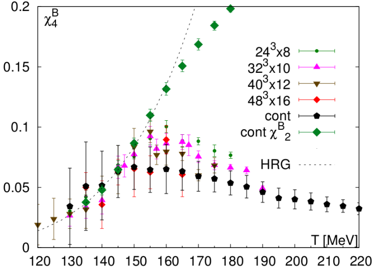
$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k}(p/T^4)}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\mu=0} ; \frac{p}{T^4} = \frac{\ln Z}{VT^3} , \hat{\mu} = \frac{\mu}{T}$$

kurtosis $\kappa_B = \chi_4^B/(\chi_2^B)^2 \rightarrow$ studied as $\kappa_B \sigma_B^2 = \chi_4^B/\chi_2^B$ with variance $\sigma_B^2 = \chi_2^B$

at zero density:



BNL-Bielefeld Coll., F. Karsch, BNL-meeting



WB Coll., R. Bellwied et al., 1507.04627

Kurtosis in lattice QCD - finite baryon density

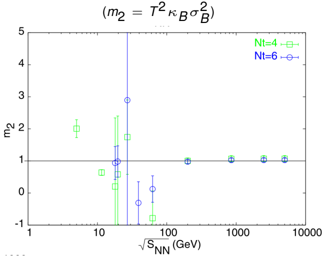
- Taylor expansion:

  (for $\mu_S = \mu_Q = 0$)

$$\kappa_B \sigma_B^2 = \frac{\chi_4^B + \frac{1}{2}\chi_6^B \hat{\mu}_B^2 + \ldots}{\chi_2^B + \frac{1}{2}\chi_4^B \hat{\mu}_B^2 + \ldots}$$

  requires knowledge of 6^{th}-order susceptibility

  valid within radius of convergence of $\ln Z$ - Taylor expansion

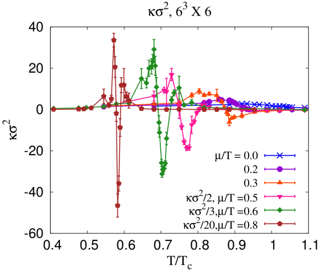


R.V. Gavai, S. Gupta, PLB 696, 459 (2011)

- strong coupling lattice QCD:

  chiral limit calculation of netbaryon-number fluctuations

  oscillatory behavior in higher-order ratios around 2^{nd}-order phase transition boundary



T. Ichihara et al., 1507.04527

The Kurtosis in transport models

- Transport models take the microcanonical nature of individual particle scatterings into account.
- Baryon-number conservation limits fluctuations of net-baryon number.

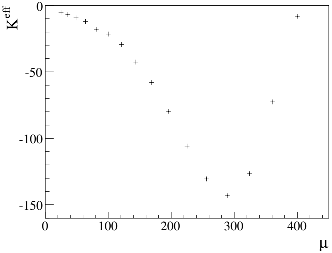
$$P_\mu(N, C) = \mathcal{N}(\mu, C) e^{-\mu} \frac{\mu^N}{N!} \quad \text{on} \quad [\mu - C, \mu + C]$$

μ: the expectation value of the original Poisson distribution, $\mathcal{N}(\mu, C)$: normalization factor, $C > 0$: cut parameter

$$C = \alpha\sqrt{\mu}\left(1 - \left(\frac{\mu}{N_{\rm tot}}\right)^2\right) \quad \text{with} \quad \alpha = 3,\ N_{\rm tot} = 416.$$

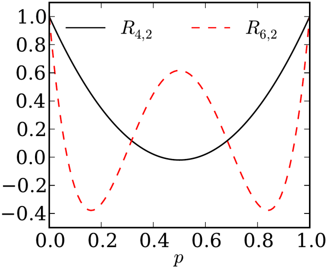
- An increase of the average net-baryon number does not lead to stronger fluctuations.
- At the upper limit of $N_{\rm tot} = 416$ the distribution changes to a δ-function ($K_\delta^{\rm eff} = 0$).

MN et al. Eur.Phys.J. C72 (2012)

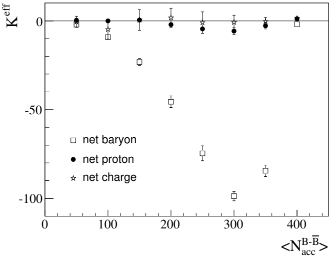


The Kurtosis in UrQMD

- Same qualitative behavior of the net-baryon kurtosis as expected from the toy model.
- For small net-baryon numbers in the acceptance, the values of net-baryon, net-proton and net-charge kurtosis are compatible with values of 0 − 1.



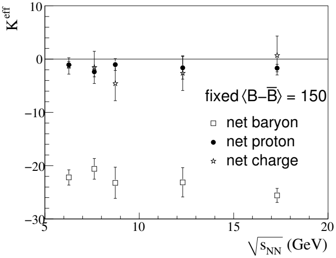
A. Bzdak et al. PRC87 (2013)



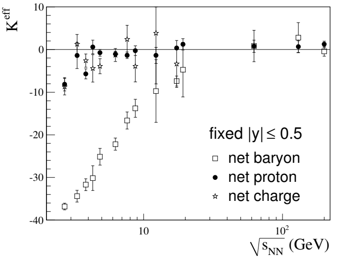
- Much larger effect than expectation from binomial distribution ⇒ volume fluctuations?
- Recent UrQMD calculations by J. Steinheimer give the same result with much smaller error bars!

The Kurtosis in UrQMD

- adapting the rapidity window to fix the mean net-baryon number
- net-baryon effective kurtosis does not show an energy dependence



- fixed rapidity cut
- the net-baryon number varies with $\sqrt{s}$
- for lower $\sqrt{s}$ K^{eff} becomes increasingly negative
- at $E_{\mathrm{lab}} = 2A\mathrm{GeV}$: $\langle N_{B-\bar{B}} \rangle \simeq 240$

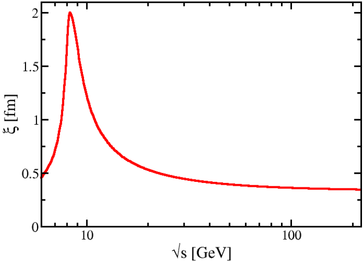


The Kurtosis in thermal models + critical fluctuations

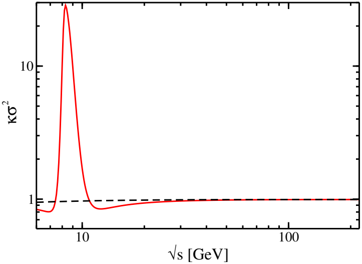
- Sigma field fluctuations: $\kappa_4 = \langle \delta\sigma^4 \rangle_c = \frac{6T}{V} \left(2(\lambda_3 \xi)^2 - \lambda_4 \right) \xi^8$ M. Stephanov, PRL102 (2009)
- Sigma field couples to the protons via: $g_p p \sigma \bar{p} \Rightarrow m_p \to m_p + g_p \Delta\sigma$

$\Rightarrow$ Fluctuations in net-protons: $\langle (\delta N_{p-\bar{p}})^4 \rangle_c = \langle (\Delta N_{p-\bar{p}})^4 \rangle_c + \langle (V\delta\sigma)^4 \rangle_c \cdot I^4_{p-\bar{p}}$.

$\sqrt{s}$ dependence of the correlation length from 3d Ising model:



net-proton, HRG + critical fluctuations



- Possibility to study resonance decay + regeneration and isospin randomization effects!

work in progress with M. Bluhm (NCSU)

Nonequilibrium chiral fluid dynamics (NχFD)

IDEA: combine the dynamical propagation of fluctuations at the phase transition with fluid dynamical expansion!

(model-independent is nice, but in the end some real input is needed...)

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{\rm q}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu} \,, \quad \partial_\mu N_{\rm q}^\mu = 0$$

$\Rightarrow$ includes a stochastic source term!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC **84** (2011); PLB **711** (2012); JPG **40** (2013)
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013); NPA**925** (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG **41** (2014)

Dynamical slowing down

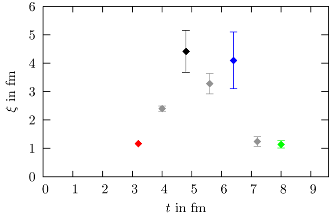
Phenomenological equation: $\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)})$
with input from the dynamical universality class $\Rightarrow \xi \sim 1.5 - 2.5$ fm
B. Berdnikov and K. Rajagopal, PRD **61** (2000))

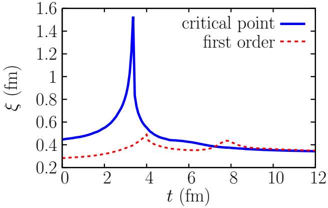
$$G(r) = \int \mathrm{d}^3x\mathrm{d}^3y \langle \sigma(x) - \sigma_0 \rangle \langle \sigma(y) - \sigma_0 \rangle$$
$$\sim \exp(-r/\xi)$$

Assume σ_0 is the volume averaged field.

From the curvature of V_{eff}:

$$\langle \xi^2 \rangle = \langle 1/m_\sigma^2 \rangle = \left\langle \left(\frac{\mathrm{d}^2 V_{\mathrm{eff}}}{\mathrm{d}\sigma^2} \right)^{-1} \right\rangle$$





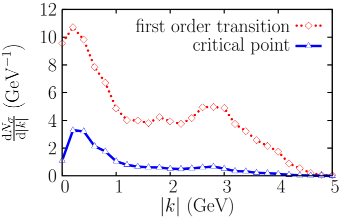
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

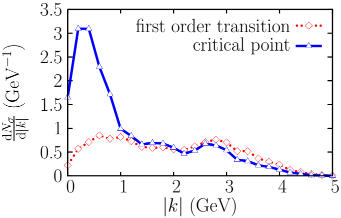
Definition of ξ in inhomogeneous systems involves averaging!

$\Rightarrow$ Similar magnitude of $\xi \sim 1.5 - 3$ fm!

Dynamics versus equilibration

- Static box with temperature quench to $T < T_c$.
- Fluctuations of the order parameter:



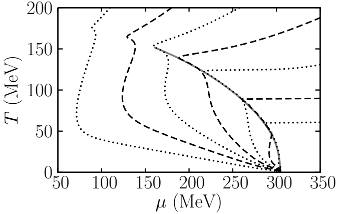


- Strong enhancement of the intensities for a first-order phase transition during the evolution.
- Strong enhancement of the intensities for a critical point scenario after equilibration.

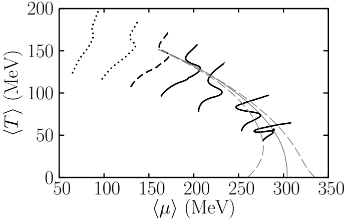
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

Trajectories and isentropes at finite μ_B

Isentropes in the PQM model

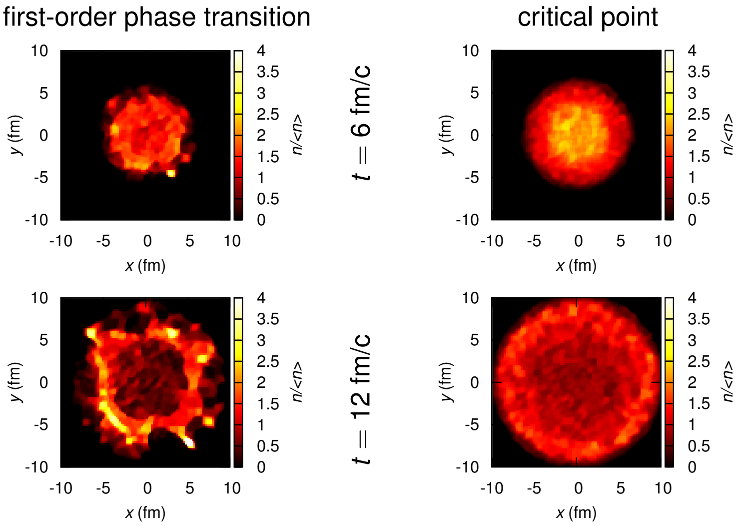


Fluid dynamical trajectories



- Fluid dynamical trajectories similar to the isentropes in the crossover region.
- No significant features in the trajectories left of the critical point.
- Right of the critical point: trajectories differ from isentropes and the system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

Bubble formation in net-baryon density

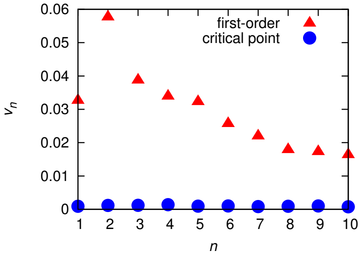


C. Herold, MN, I. Mishustin, M. Bleicher, arxiv:1304.5372

Bubble formation in net-baryon density

Fourier-decomposition of $n_B(x, y)$
$\rightarrow$ quantifies strong enhancement of first-order PT versus critical point/crossover.

not (yet) in momentum space!

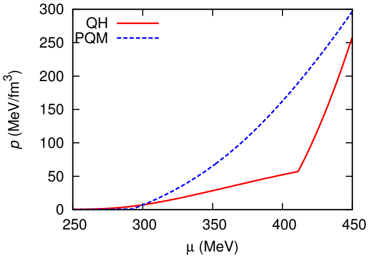


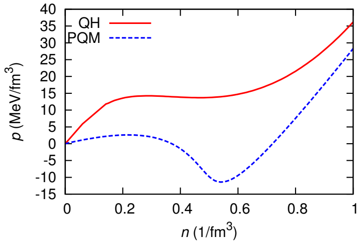
Can we expect experimental evidences for the first-order phase transition from bubble formation?

- Do the irregularities survive when a realistic hadronic phase is assumed?
- A strong pressure could transform the coordinate-space irregularities into momentum-space Fourier-coefficients of baryon-correlations $\Rightarrow$ enhanced higher flow harmonics at a first-order phase transition? Very eos dependent!

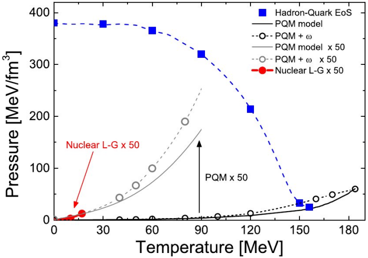
work in progress with C. Herold

EoS: PQM versus QH





- Below μ_c, $p \approx 0$ in PQM, while it still decreases in HQ model and $p < 0$ can arise in PQM!
- Several eos lead to similar pressures at $\mu_B \approx 0$, but differ at large μ_B.
- With coexistence between dense quark matter and compressed nuclear matter (HQ-EoS) : $\partial p_c / \partial T < 0$
- From effective models, like PNJL, PQM etc.: $\partial p_c / \partial T > 0$



J. Steinheimer, J. Randrup, V. Koch PRC**89** (2014)

SU(3) chiral quark-hadron model

- Hadronic SU(3) non-linear sigma model including quark degrees of freedom

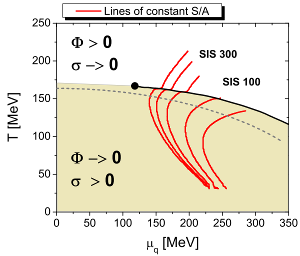
$$\mathcal{L} = \sum_i \bar{\psi}_i(i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i)\psi_i + 1/2(\partial_\mu\sigma)^2 - U(\sigma,\zeta,\omega) - \mathcal{U}(\ell)$$

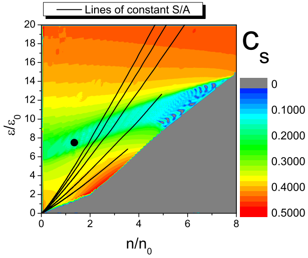
and effective masses generated by

$$M_q = g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell)$$
$$M_B = g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{qB}\ell^2$$

V. Dexheimer, S. Schramm, PRC**81** (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC**88** (2013)

- hadrons are included as quasi-particle degrees of freedom
- yields a realistic structure of the phase diagram and phenomenologically acceptable results for saturated nuclear matter:

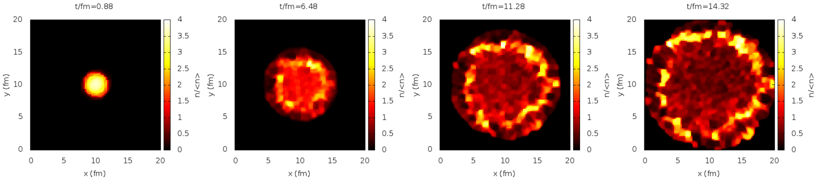




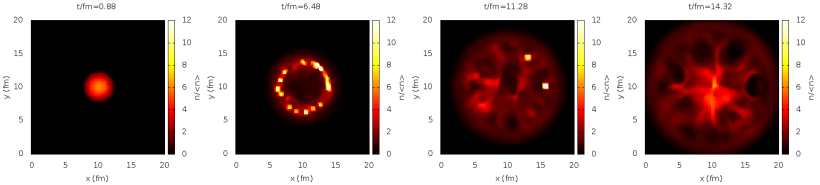
J. Steinheimer, V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm, H. Stoecker, PRC**81** (2010)

PQM vs. QH model - stability of droplets

PQM EoS



QH EoS

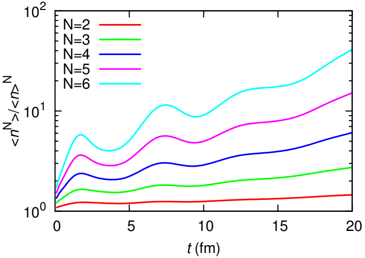


- Dynamical and stochastic droplet formation at the phase transition and subsequent decay in the hadronic phase.

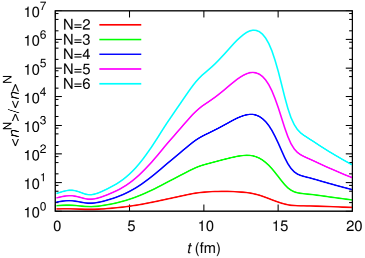
PQM vs. QH model - moments of netbaryon density

Define normalized moments of the net-baryon density distribution as:

$$\langle n^N \rangle = \int \mathrm{d}^3 x n(x)^N P_n(x) \quad \text{with} \quad P_n(x) = \frac{n(x)}{\int \mathrm{d}^3 x n(x)}$$



PQM EoS



QH EoS

- Infinite increase in the PQM.
- Increase in the HQ model around the phase transition followed by a rapid decrease due to pressure in the hadronic phase!
- REMEMBER: We started with smooth initial conditions and all inhomogeneities are formed dynamically!

And the critical point?

- At $\mu_B \neq 0$ σ mixes with the net-baryon density n (and e and $\vec{m}$)
- In a Ginzburg-Landau formalism:

$$V(\sigma, n) = \int \mathrm{d}^3x (\sum_m (a_m \sigma^m + b_m n^m) + \sum_{m,l} c_{m,l} \sigma^m n^l) - h\sigma - jn$$

- $V(\sigma, n)$ has a flat direction in $(a\sigma, bn)$ direction
- Equations of motion (including symmetries in $V(\sigma, n)$):

$$\partial_t^2 \sigma = -\Gamma \delta V / \delta\sigma + \ldots$$
$$\partial_t n = \gamma \vec{\nabla}^2 \delta V / \delta n + \ldots$$

- two time scales (with $D \to 0$ at the critical point)

$$\omega_1 \propto -i\Gamma a$$
$$\omega_2 \propto -i\gamma D \vec{q}^2$$

- The diffusive mode becomes the critical mode in the long-time dynamics. These fluctuations need to be included at the critical point!

H. Fuji, M. Ohtani PRD**70** (2004); M. Stephanov, D. Son PRD**70** (2004)

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq}$$

$$N^{\mu} = N^{\mu}_{\rm eq}$$

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}} + \Delta N^{\mu}_{\mathrm{visc}}$$

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Stochastic viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} + \mathrm{I}^{\mu}$$

Fluid dynamical fluctuations

The noise terms are such that averaged quantities exactly equal the conventional quantities:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} \qquad \text{with} \quad \langle \Xi^{\mu\nu} \rangle = 0$$
$$\langle N^{\mu} \rangle = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} \qquad \text{with} \quad \langle \mathrm{I}^{\mu} \rangle = 0$$

The two formulations will, however, differ when one calculates correlation functions:

$$\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$$
$$\langle N^{\mu}(x) N^{\mu}(x') \rangle$$

Fluid dynamical fluctuations

In linear response theory the retarded correlator

- $\langle T^{\mu\nu}(x)T^{\mu\nu}(x')\rangle$ gives the viscosities and
- $\langle N^{\mu}(x)N^{\mu}(x')\rangle$ the charge conductivities

via the dissipation-fluctuation theorem (Kubo-formula)!

It means that when dissipation is included also fluctuations need to be included!

CAUTION: If nonlinearities are included fluid dynamical fluctuations contribute to the transport coefficients!

$\Rightarrow$ absolut lower limit for the effective viscosity!

$\Rightarrow$ non-analytic contribution to τ_π, breakdown of gradient expansion!

P. Kovtun, G. D. Moore, P. Romatschke, PRD**84** (2011); C. Chafin, T. Schäfer, PRA**87** (2013); P. Romatschke, R. E. Young, PRA**87** (2013)

Fluid dynamical fluctuations

- Linearized fluid dynamical equations: small fluctuations $\bar{e} + \delta e$, $\bar{p} + \delta p$ and δv^i with: $\delta T^{00} = \delta e$ and $\delta T^{ij} = m^i = (\bar{e} + \bar{p}) v^i = \bar{w} v^i$

$$\begin{aligned} \partial_t \mathbf{m}_\perp + \eta/\bar{w}\mathbf{k}^2 \mathbf{m}_\perp &= 0 \\ \partial_t \delta e + i\mathbf{k} \cdot \mathbf{m}_{||} &= 0 \\ \partial_t \mathbf{m}_{||} + i v_s^2 \mathbf{k} \delta e + \gamma_v \mathbf{k}^2 \mathbf{m}_{||} &= 0 \end{aligned}$$

- retarded Green's function for δe and $\mathbf{m}_{||}$:

$$G^{\mathrm{ret}}_{ab}(\omega, \mathbf{k}) = \frac{\bar{w}}{\omega^2 - v_s^2 \mathbf{k}^2 + i\omega\gamma_s \mathbf{k}^2} \begin{pmatrix} \mathbf{k}^2 & \omega|\mathbf{k}| \\ \omega|\mathbf{k}| & v_s^2 \mathbf{k}^2 - i\omega\gamma_s \mathbf{k}^2 \end{pmatrix}$$

- including the transverse momentum density:

$$G^{\mathrm{ret}}_{m_i, m_j}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \frac{\eta \mathbf{k}^2}{i\omega - \gamma_\eta \mathbf{k}^2} + \frac{k_i k_j}{\mathbf{k}^2} \frac{\bar{w}(v_s^2 \mathbf{k}^2 - i\omega\gamma_s \mathbf{k}^2)}{\omega^2 - v_s^2 \mathbf{k}^2 + i\omega\gamma_s \mathbf{k}^2}$$

- Kubo-formulas for viscosities:

$$\eta = -\frac{\omega}{2\mathbf{k}^2} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \Im G^{\mathrm{ret}}_{m_i m_j}(\omega, \mathbf{k} \to 0)$$

$$\zeta + \frac{4}{3}\eta = -\frac{\omega^3}{\mathbf{k}^4} \Im G^{\mathrm{ret}}_{ee}(\omega, \mathbf{k} \to 0)$$

Fluid dynamical fluctuations

$$\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu 0}(x)\Xi^{\mu 0}(x')\rangle^S = -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu 0}(x)T^{\mu 0}(x')\rangle^S$$

$$= \int \frac{\mathrm{d}\omega}{2\pi}\int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} e^{-i\omega(t-t')} \times$$

$$\times \left(\omega^2 \underbrace{G^S_{ee}(\omega,\mathbf{k})}_{\text{FDT}} - 2\omega|\mathbf{k}| \underbrace{G^S_{em_{||}}(\omega,\mathbf{k})}_{\text{FDT}} + \mathbf{k}^2 \underbrace{G^S_{m_{||}m_{||}}(\omega,\mathbf{k})}_{\text{FDT}} \right)$$

$$G^S_{ab}(\omega,\mathbf{k}) = -\frac{2T}{\omega}\Im G^{\mathrm{ret}}_{ab}(\omega,\mathbf{k})$$

$$= 0$$

$$\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu i}(x)\Xi^{\mu j}(x')\rangle^S = -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu i}(x)T^{\mu j}(x')\rangle^S$$

$$= 2T\left[\left(\zeta + \frac{4}{3}\eta\right)\partial_i\partial_j + \eta(\delta_{ij}\nabla^2 - \partial_i\partial_j)\right]\delta^4(x-x')$$

Then boost to arbitrary frame:

Fluid dynamical fluctuations

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} + \mathrm{I}^{\mu}$$

with

$$\langle \Xi^{\mu\nu}(x)\Xi^{\alpha\beta}(x')\rangle = 2T[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - 2/3\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}]\delta^4(x - x')$$

- In second-order fluid dynamics there are relaxation equations for $\Xi^{\mu\nu}$:

$$u^\gamma \partial_\gamma \Xi^{\langle\mu\nu\rangle} = -\frac{\Xi^{\mu\nu} - \xi^{\mu\nu}_{\rm gauss}}{\tau_\pi}$$

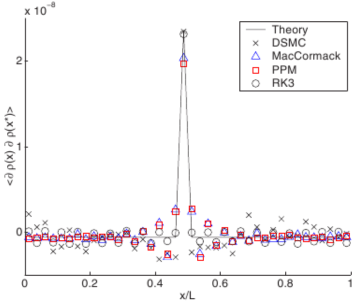
- In white noise approximation and ignoring bulk viscosity ($\zeta = 0$):

$$\langle \xi^{\mu\nu}_{\rm gauss}(x)\xi^{\alpha\beta}_{\rm gauss}(x')\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x - x')$$

Fluid dynamical fluctuations

- In a numerical treatment → discretization: $\langle \xi^2 \rangle \propto \frac{1}{\Delta V}$
- $\Rightarrow$ large fluctuations from cell to cell $\Rightarrow$ coarse-graining, smearing, etc. compare to expectations from equilibrium and MC kinetic theory!

- Example: non-relativistic Navier-Stokes + fluctuations
- 1d, dilute gas, periodic boundary conditions



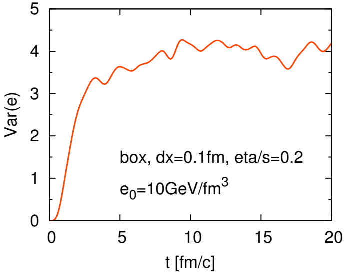
J. Bell, A. Garcia, S. Williams, PRE**76** (2007)

- Different algorithms treat fluctuations differently, third-order methods seem to work best.

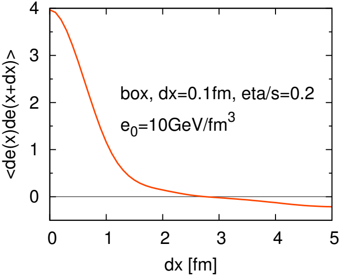
Fluid dynamical fluctuations

- Static box with periodic boundary conditions in relativistic $3 + 1$d fluid dynamics
  based on $3 + 1$d viscous fluid dynamical code by Y. Karpenko.
- Noise correlated over 1 fm^3

time evolution of the variance $\langle \delta e^2 \rangle$:



$\langle \delta e(x) \delta e(x + dx) \rangle$ correlation function:

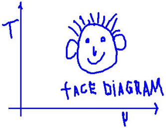


- Average energy(-momentum) conserved within 5%.
- Variance of the energy density fluctuations are approximately $30 - 40\%$ of what is expected in a grandcanonical ensemble.

work in progress with M. Bluhm and Th. Schäfer (NCSU)

Conclusions

- Fluctuation data from heavy-ion collisions at finite μ_B can only be understood with dynamical models of the phase transition!
- In NχFD, effects like critical slowing down and droplet formation can be observed.
- PQM-like EoS do not include pressure in hadronic phase, droplets remain stable.
- In HQ-like EoS: droplets form dynamically at the phase transition, then decay.
- Some more effort is needed for studying event-by-event critical fluctuations...
- Next steps: particle production in NχFD and (net-baryon) fluid dynamical fluctuations.

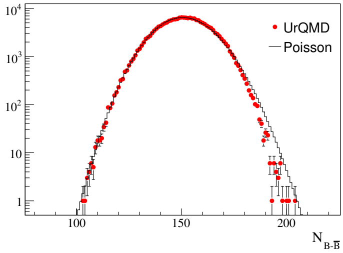


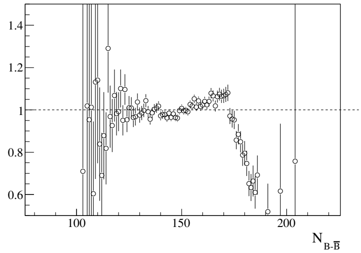
backup

Net-baryon number distribution in UrQMD

- central Pb+Pb collisions at $E_{\rm lab} = 20A$GeV
- fit to a Poisson distribution
- shoulders are enhanced
- tails are cut

$\Longrightarrow$ decrease from $K^{\rm eff}_{\rm Poisson} = 1$ to $K^{\rm eff}_{\rm UrQMD} = -22.2$

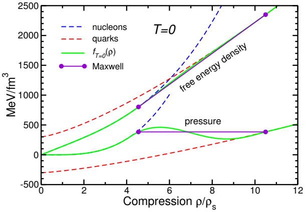


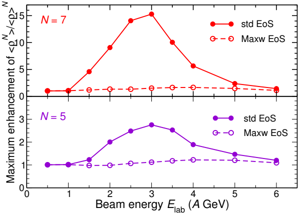


ratio of UrQMD to Poisson distribution

Comparison

- Nonequilibrium construction of the EoS from QGP and hadronic matter:

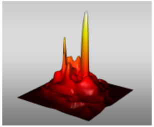




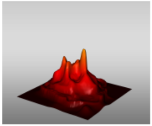
J. Steinheimer, J. Randrup, PRL **109** (2012), PRC **87** (2013)

- Significant amplification of **initial** density irregularities

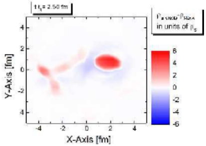
With phase transition:



Without phase transition:



Density enhancement:



plot by V. Koch

- BUT: deterministic evolution of the system $\Rightarrow$ No inhomogeneities for smooth initial conditions!