Anisotropic Flow; ad nauseam

Raimond Snellings

Universiteit Utrecht

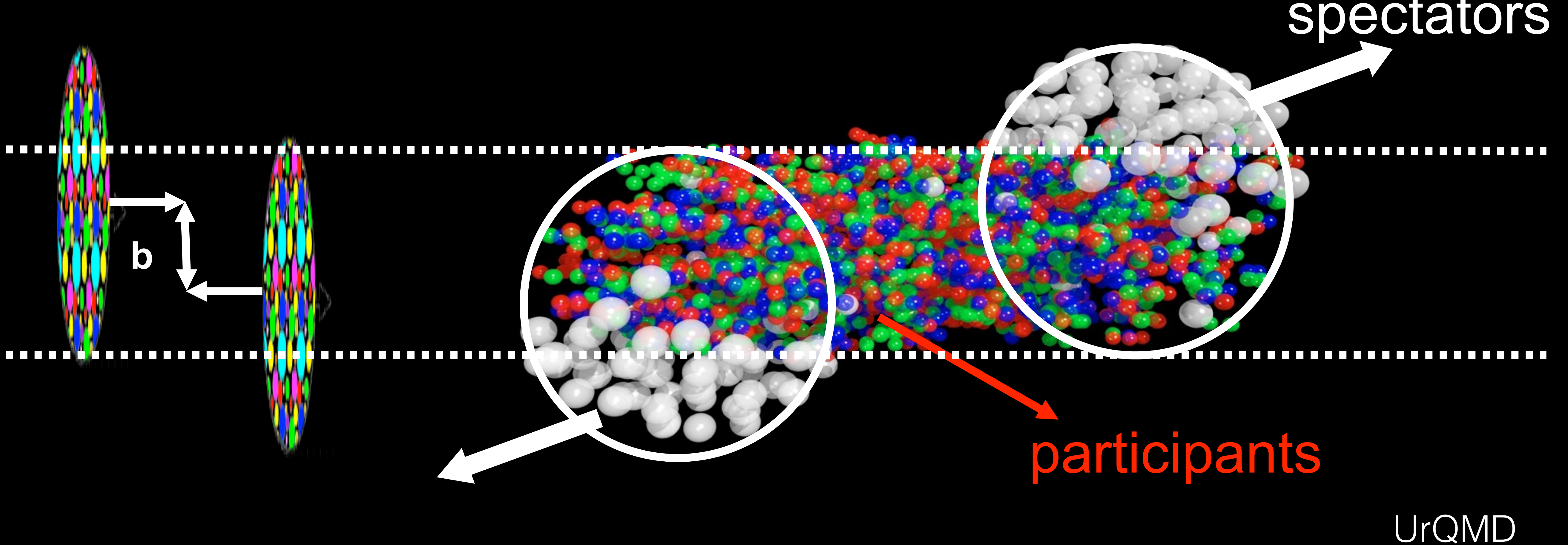
NIKHEF

4th International Symposium on Non-equilibrium Dynamics
30-08 — 05-09-2015
Giardini Naxos, Sicily,Italy

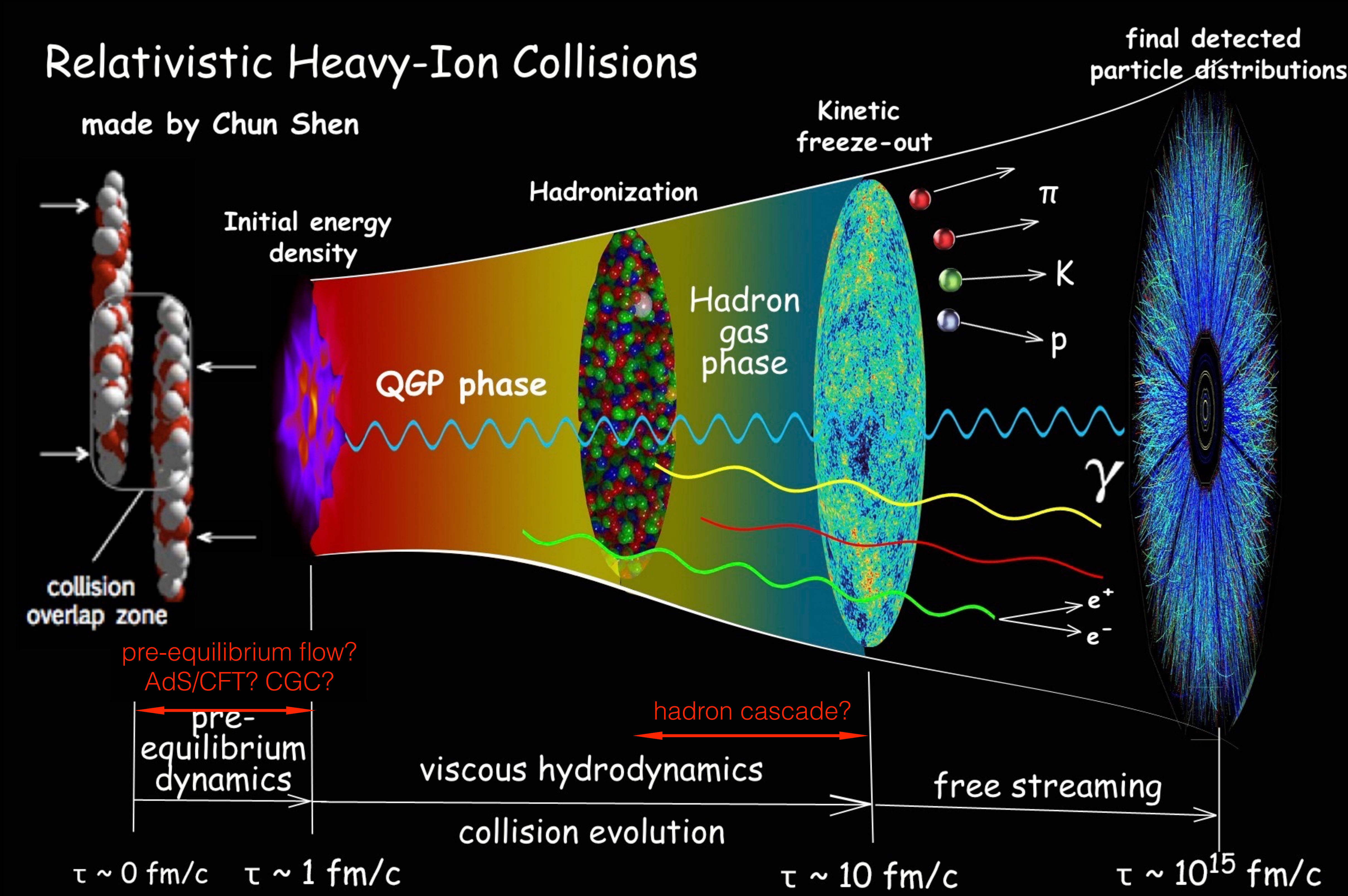
Outline

- initial spatial distributions and the response of the system
- integrated elliptic flow
  - EoS, Knudsen number and η/s
- p_t-differential elliptic flow
  - identified particles
- other harmonics
- what do we measure?
- v_n probability distributions
- rapidity dependence

A Heavy-Ion Collision

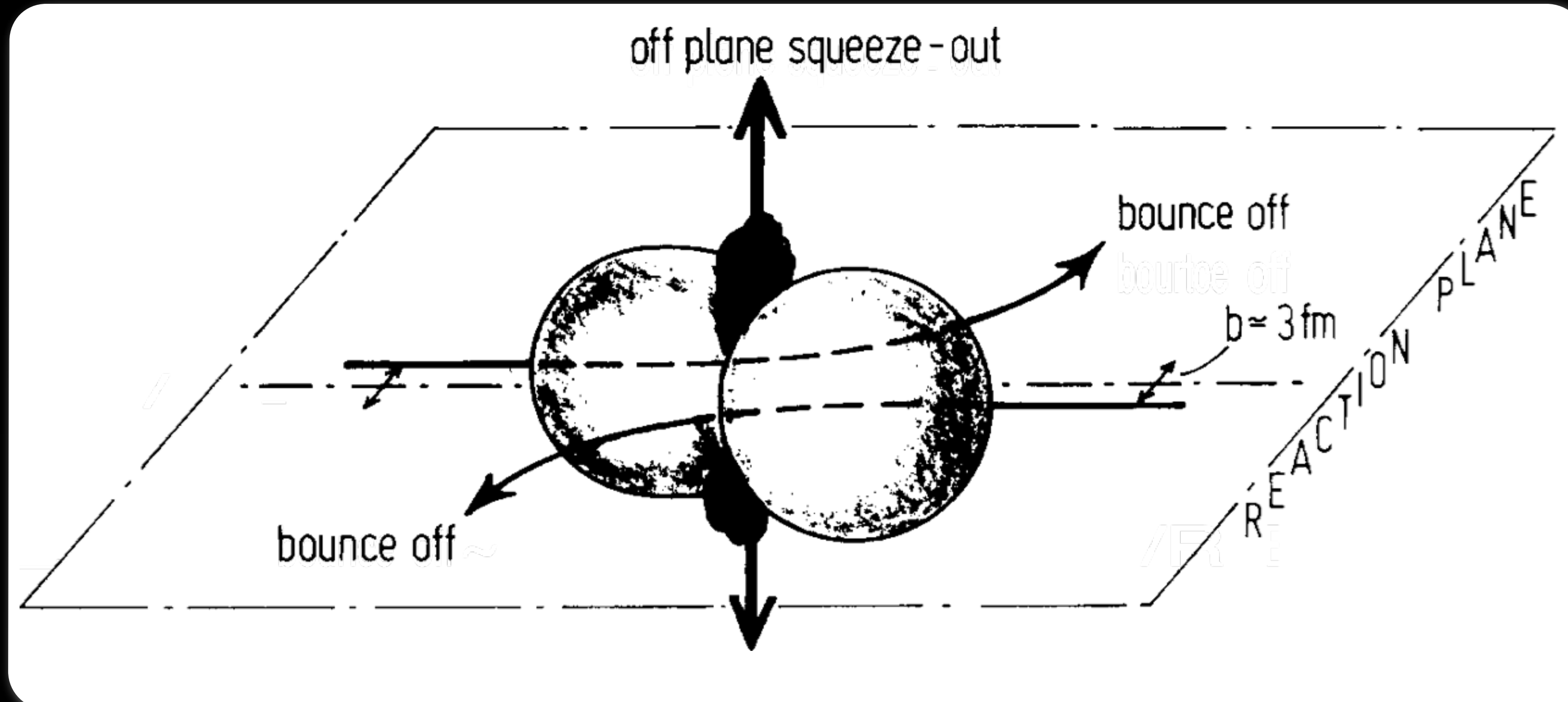


Our current picture



A long long time ago

H. Stocker and W. Greiner



Jean-Yves Ollitrault; PRD 46 (1992)

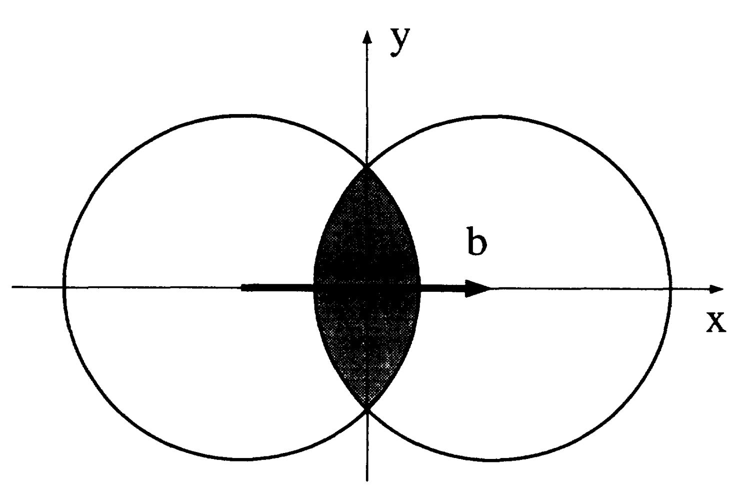
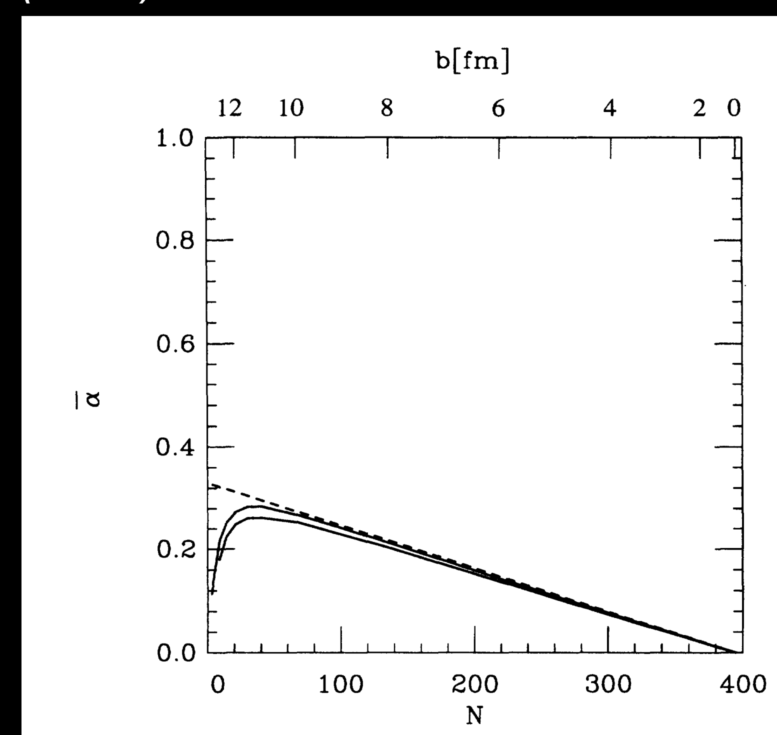
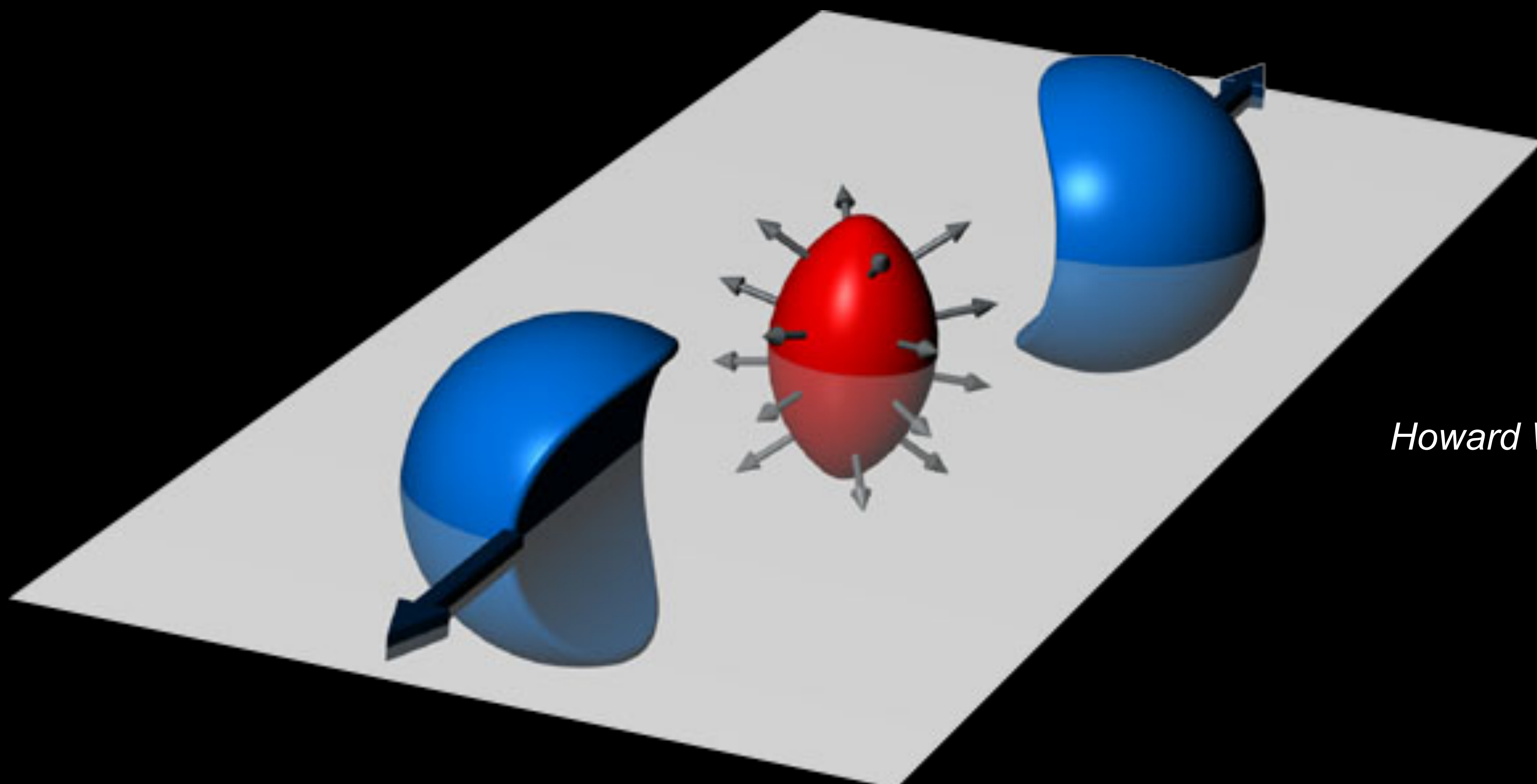


FIG. 1. Peripheral collision viewed in the transverse plane. **b** is the impact parameter. The shaded area corresponds to the region where particles are created in the central rapidity region. Outside this region is the vacuum.

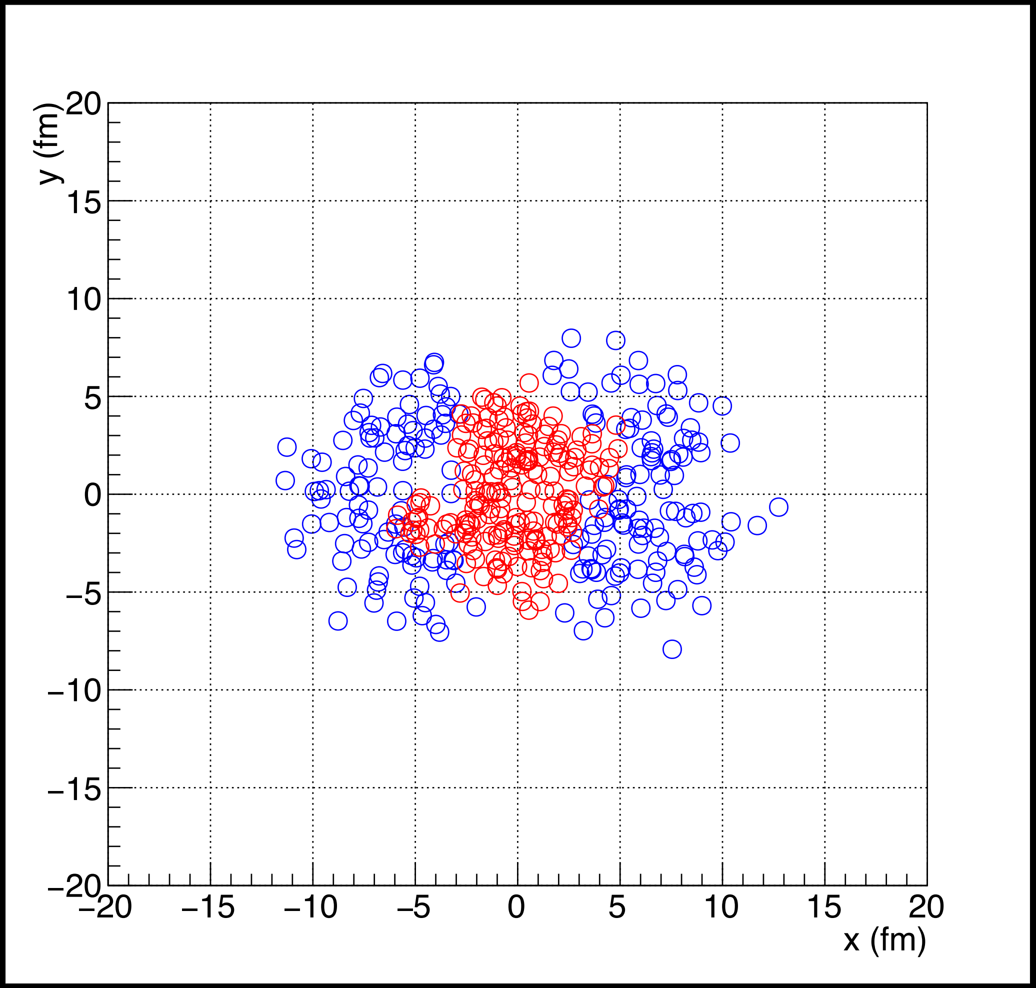


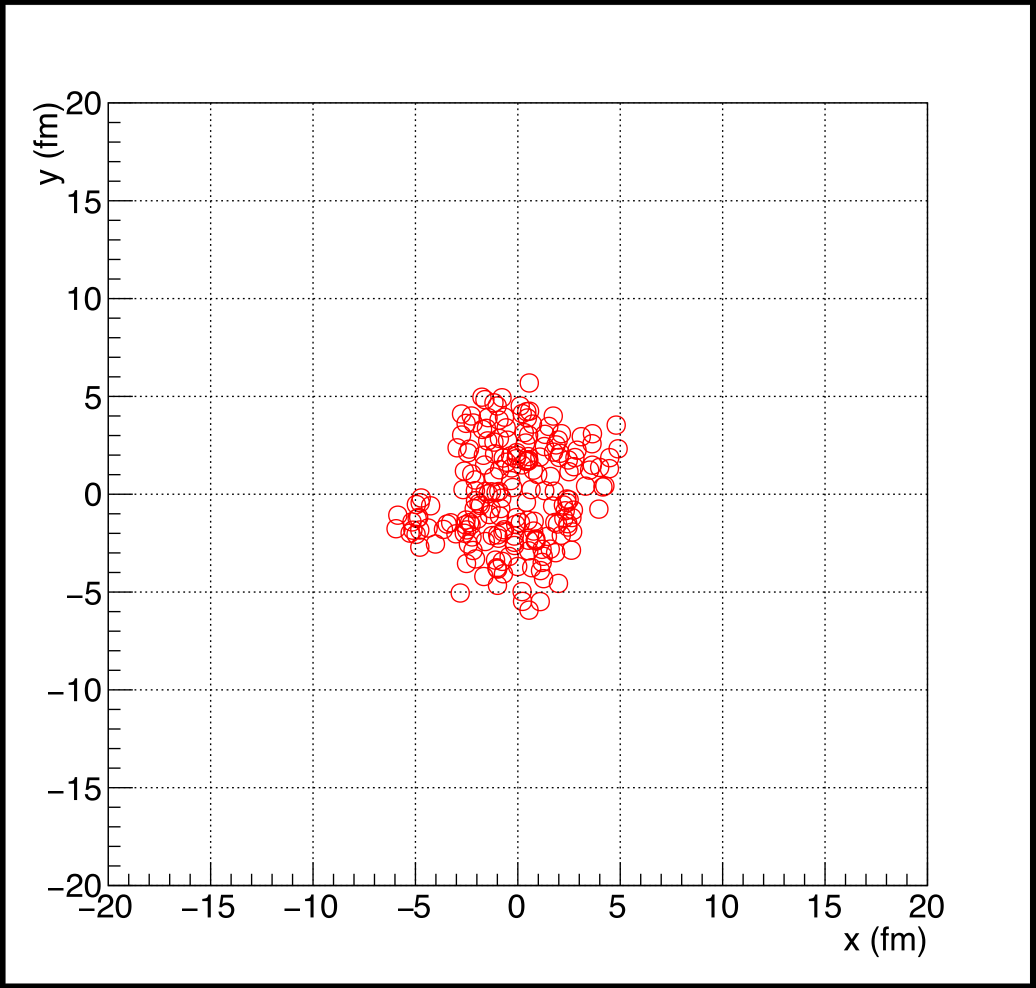
The Classical Picture



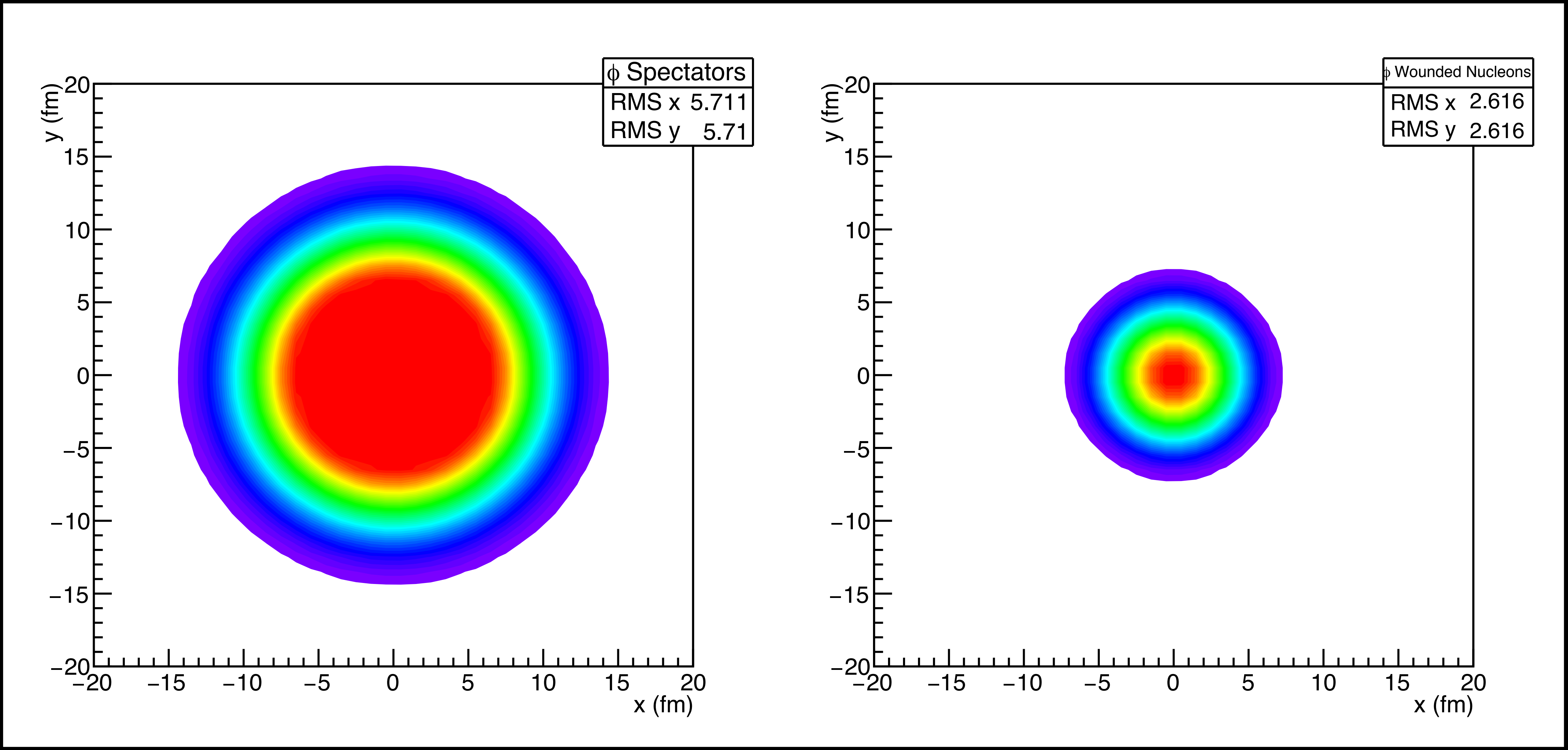
Howard Wieman

A Single Collision

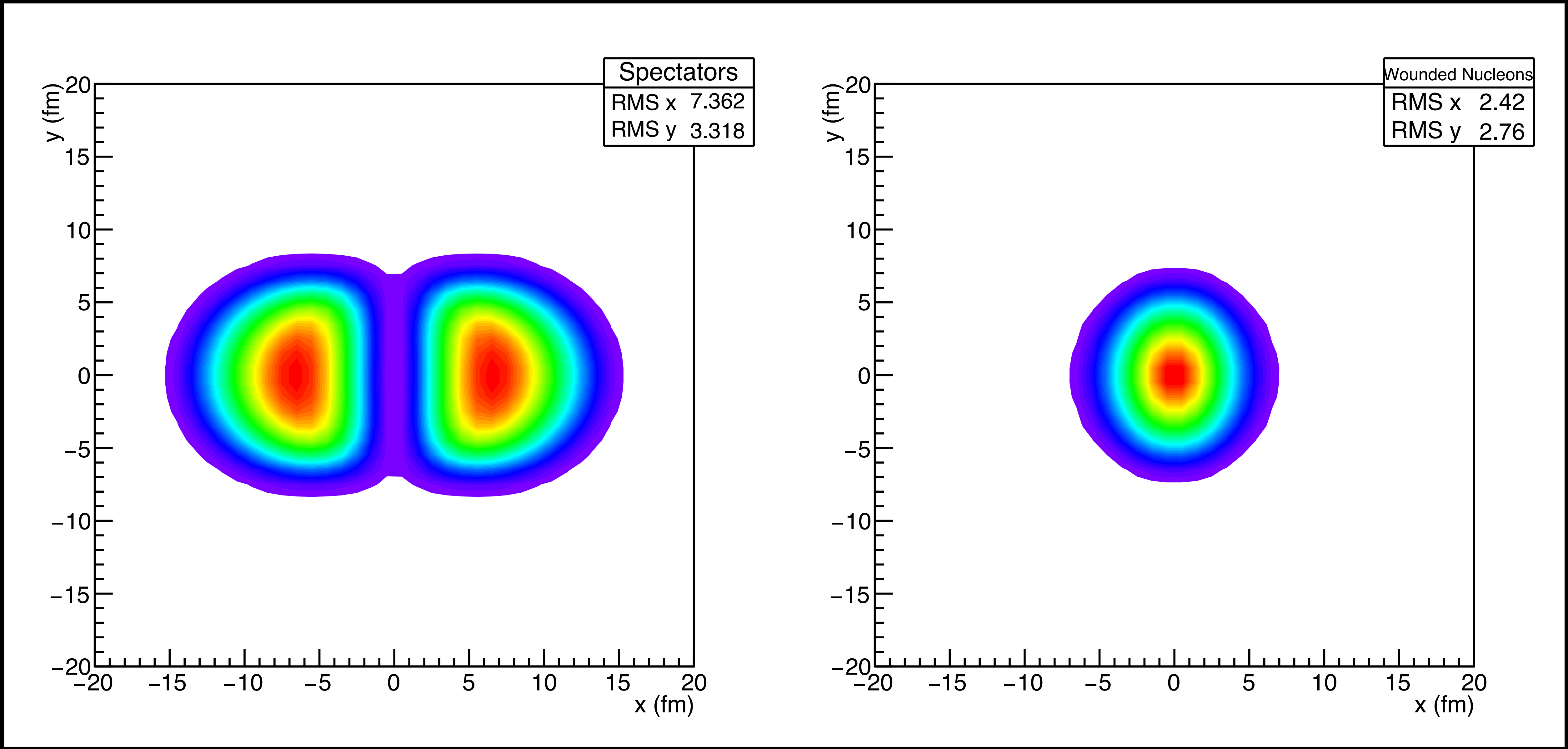




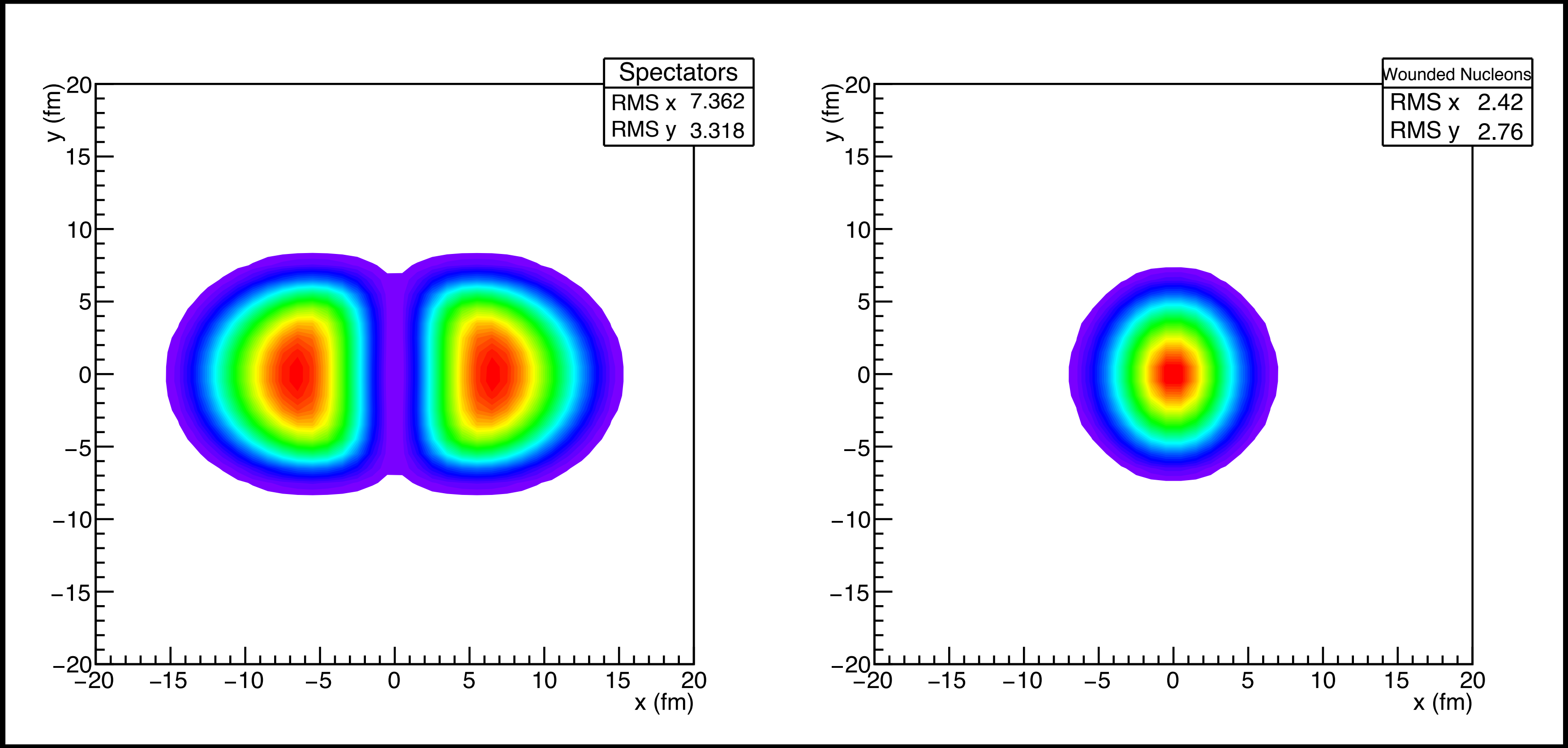
Many Collisions in the Lab Frame

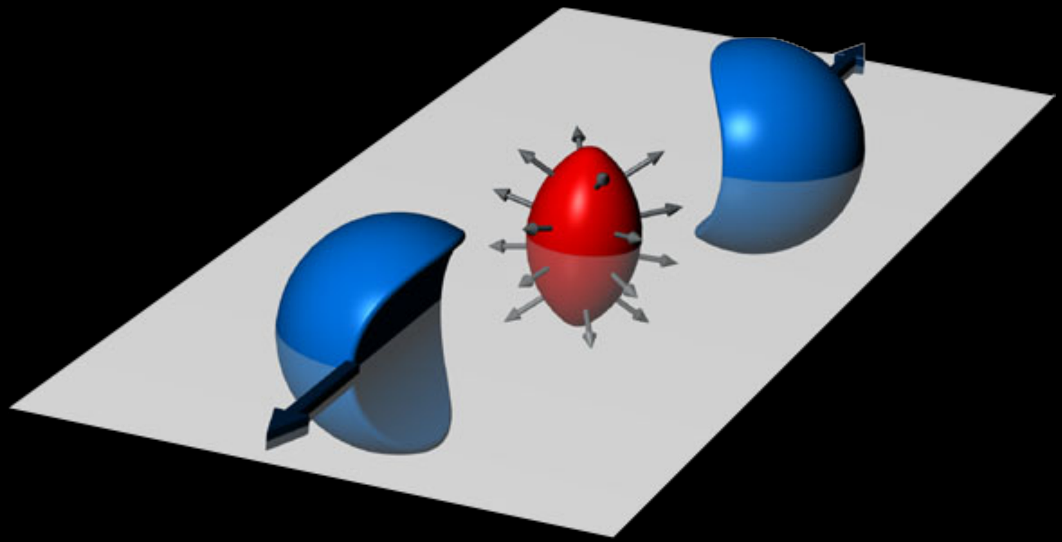


Many Collisions versus the Reaction Plane



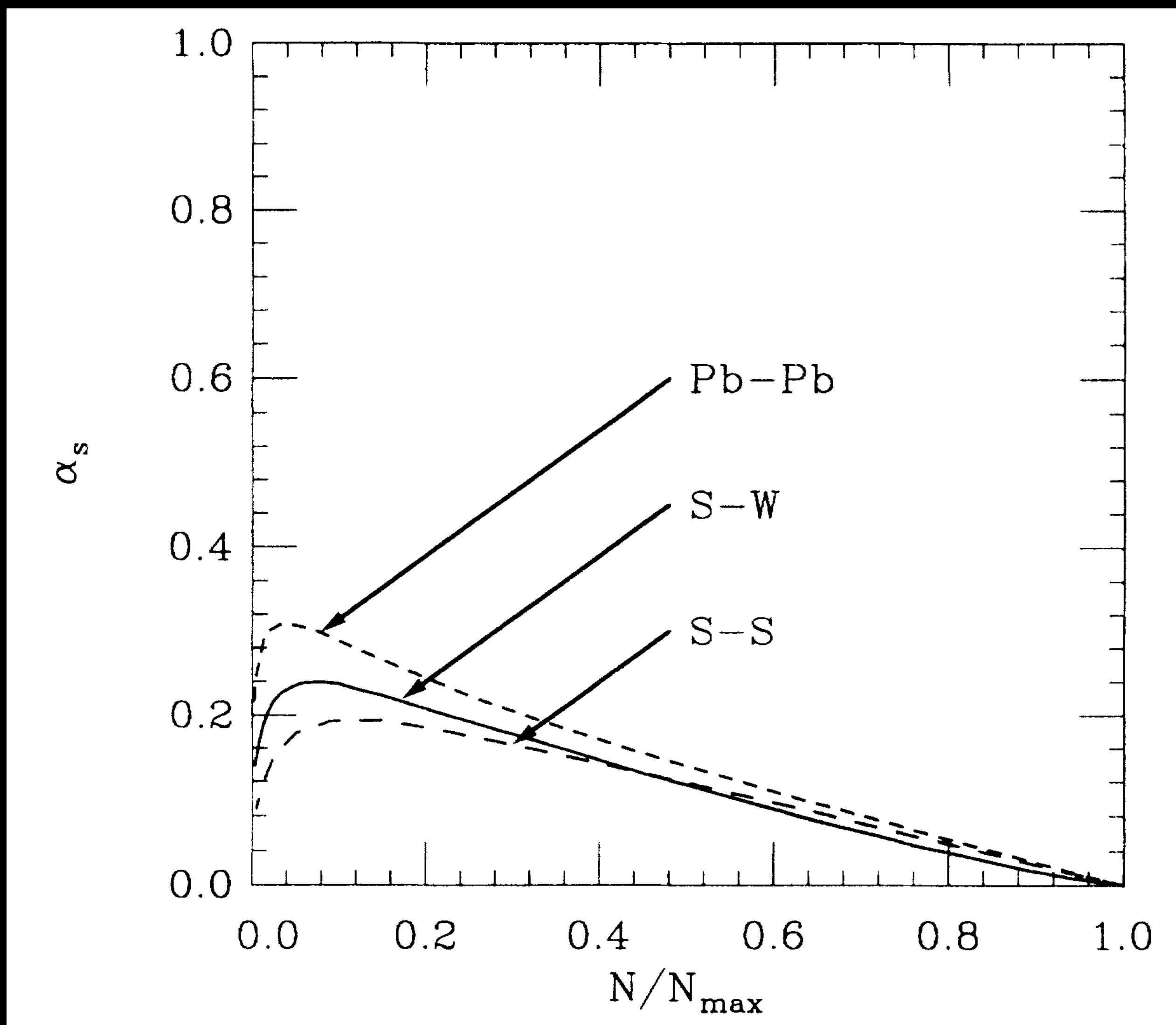
A long long time ago

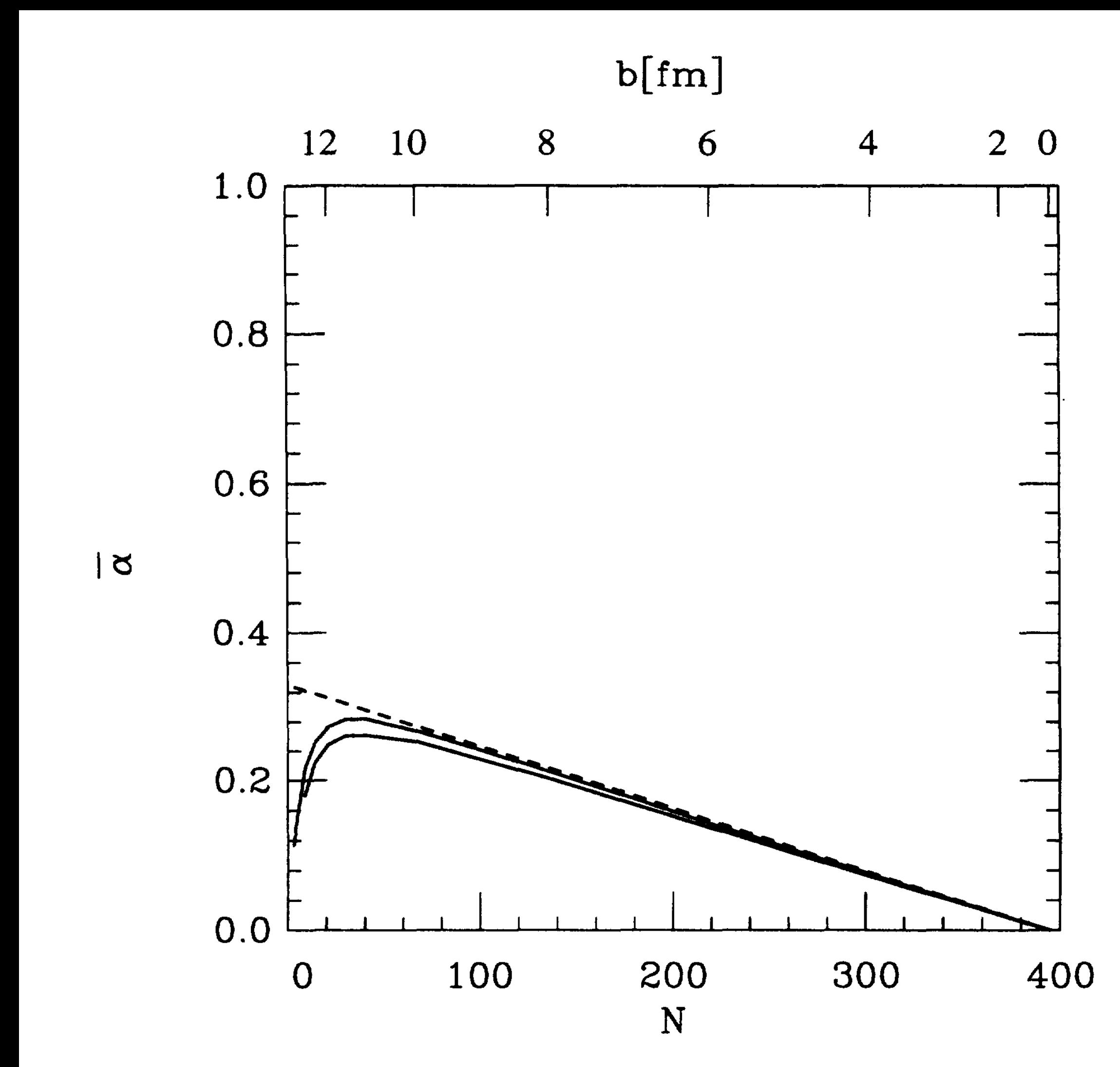




A long long time ago

Jean-Yves Ollitrault; PRD 46 (1992)

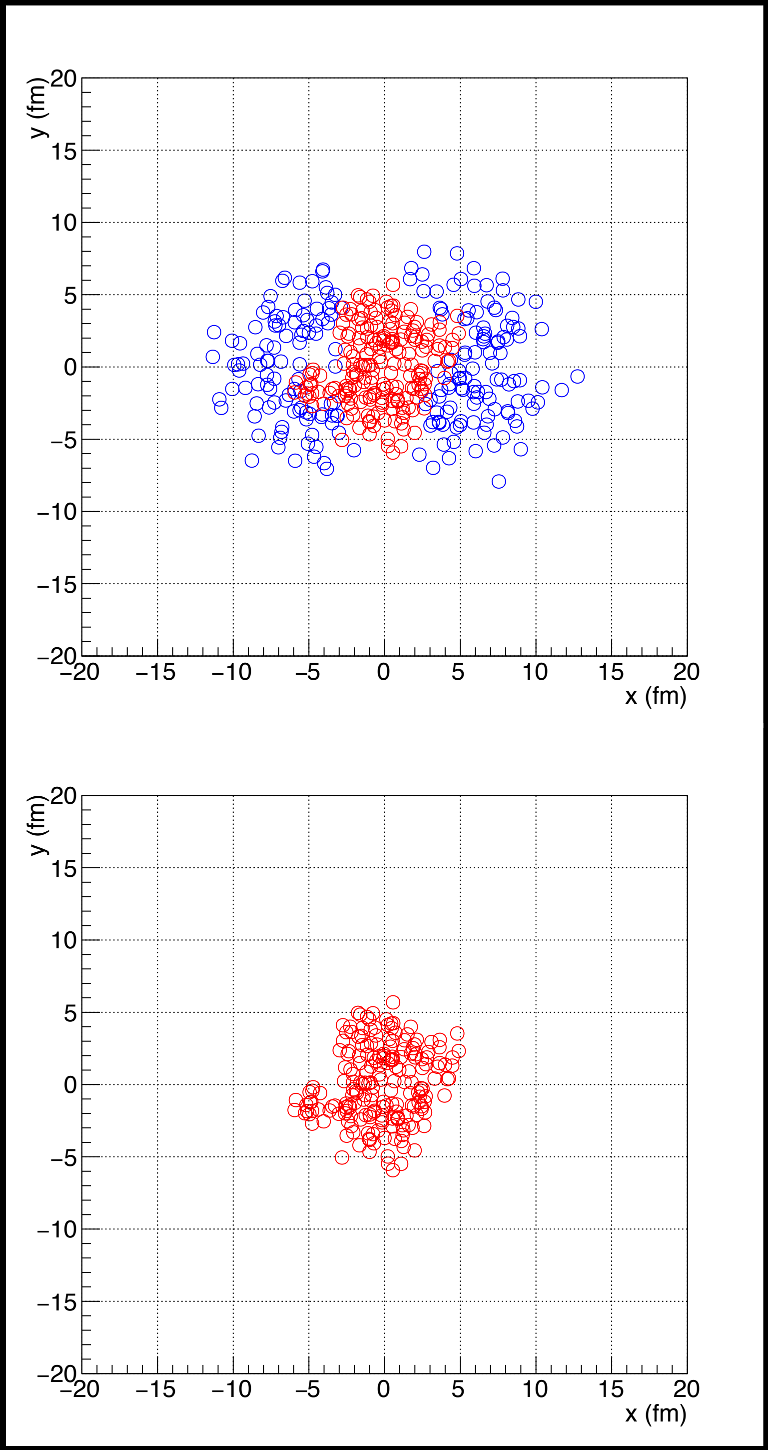


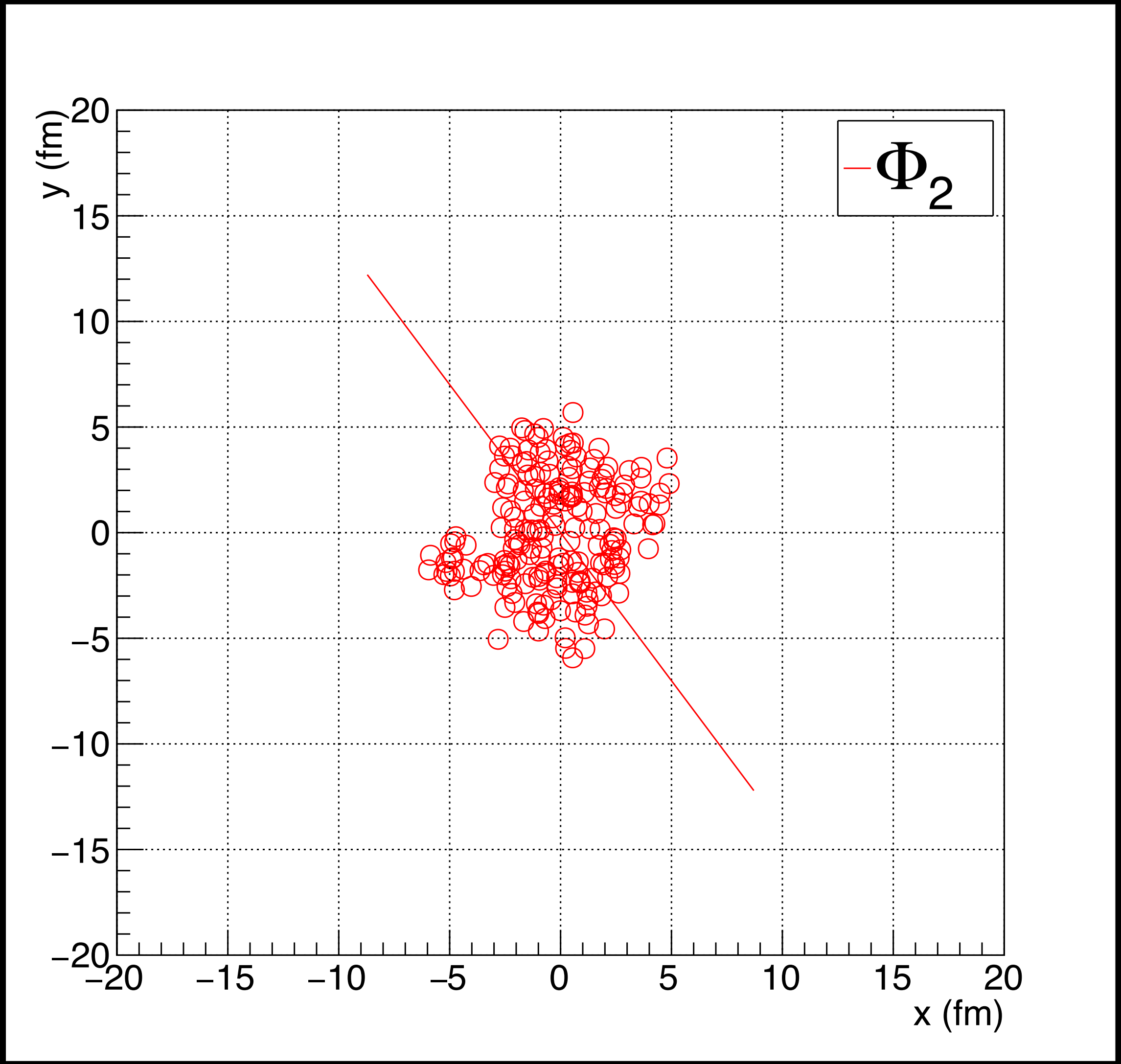


$$v_n \propto \varepsilon_n$$

sensitive to the EoS

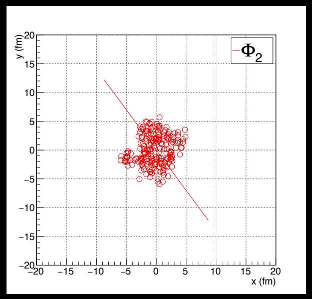
Symmetry Plane



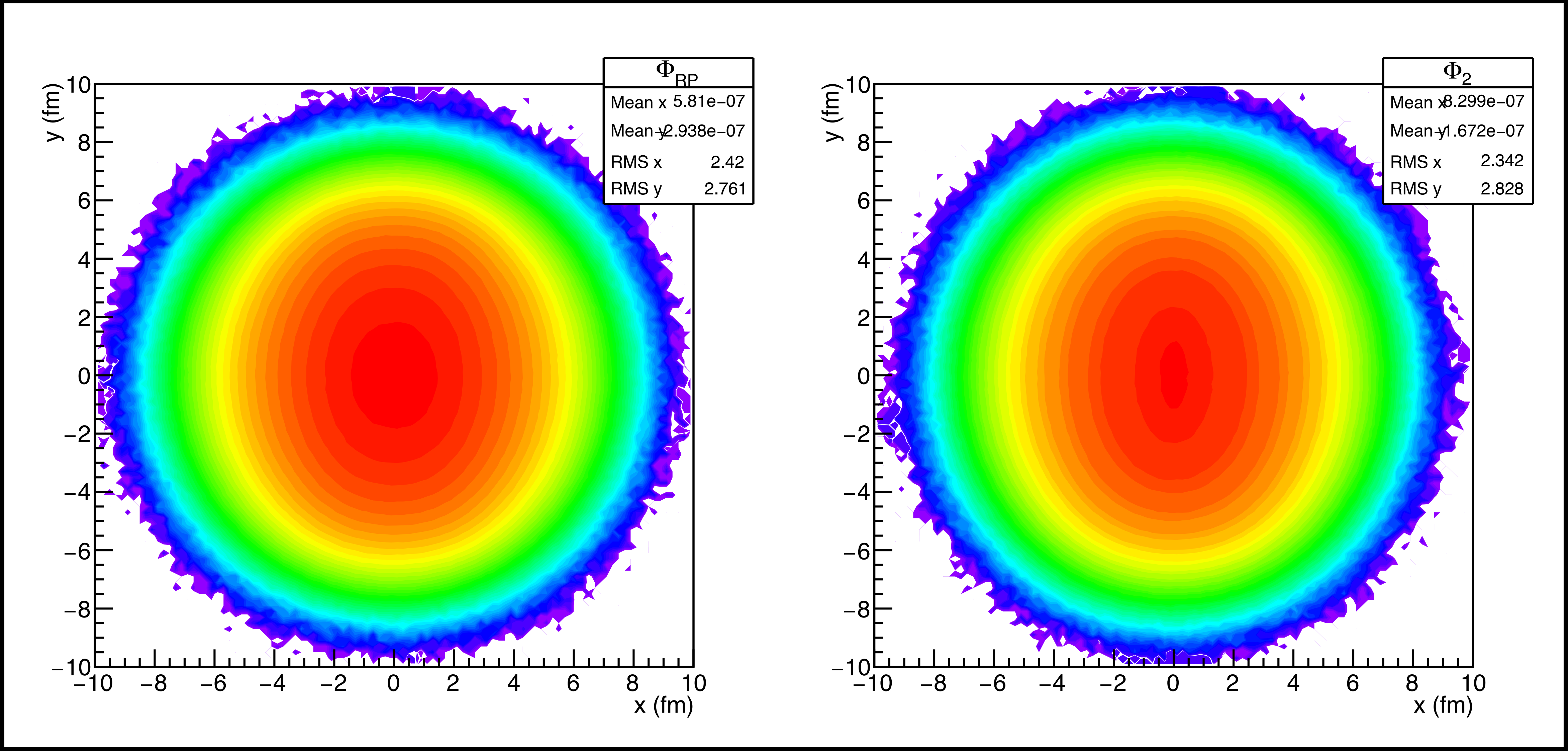


Using the particles produced we (experimentalists) determine, due to the fluctuations, a symmetry plane which is different than the Reaction Plane

$$v_n \propto \varepsilon_n$$

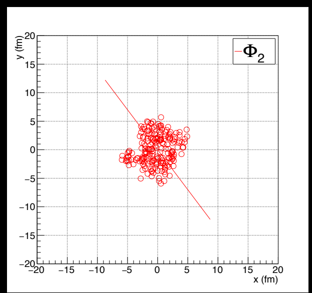


Symmetry Planes



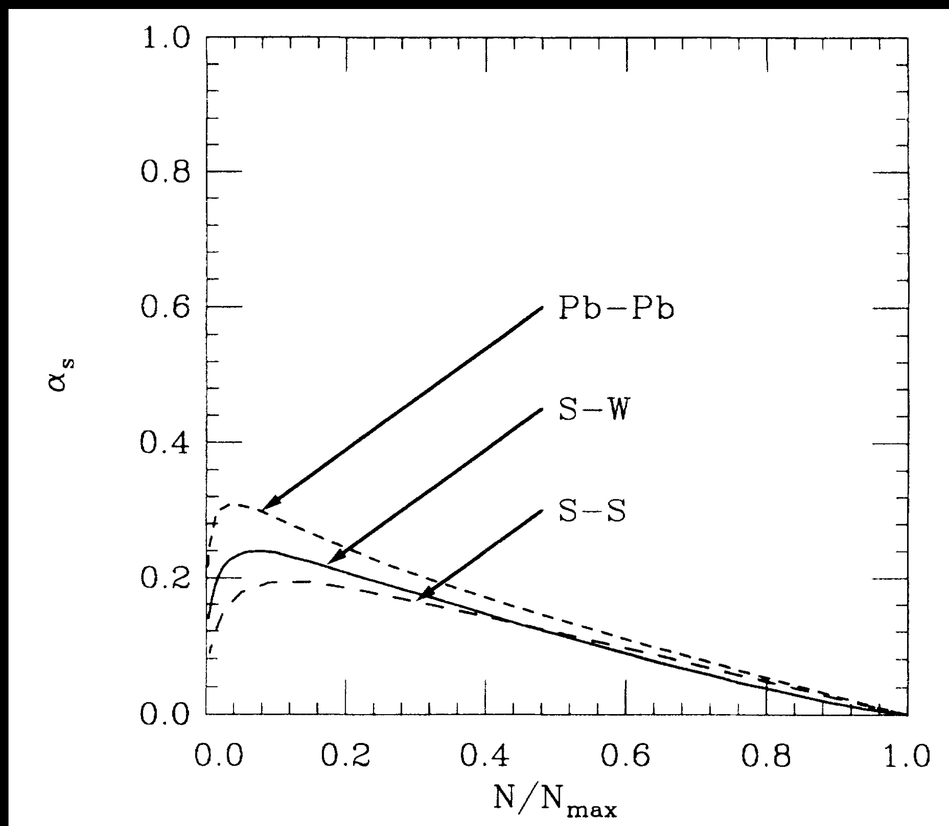
The asymmetry of the system is larger versus this symmetry plane

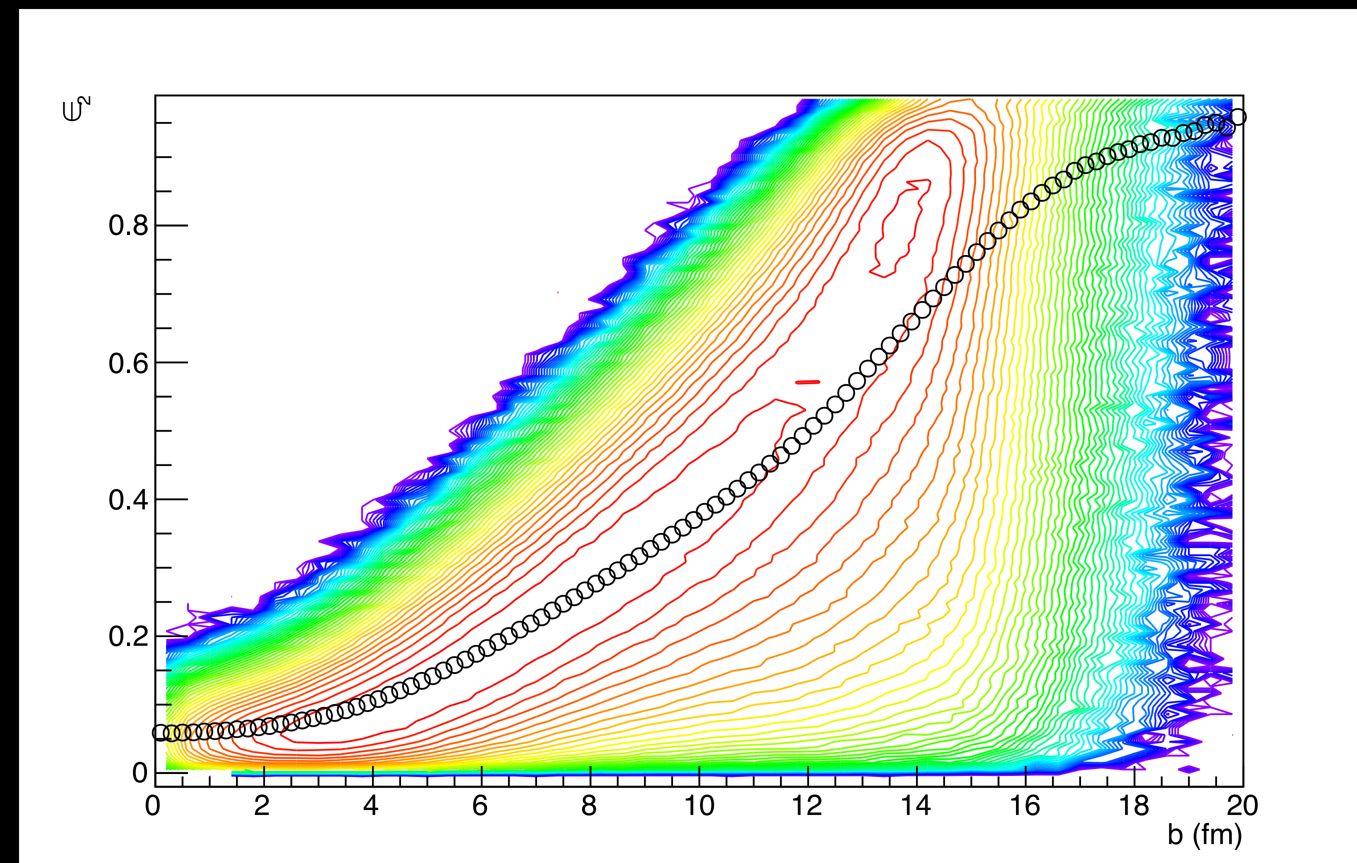
$$v_n \propto \varepsilon_n$$



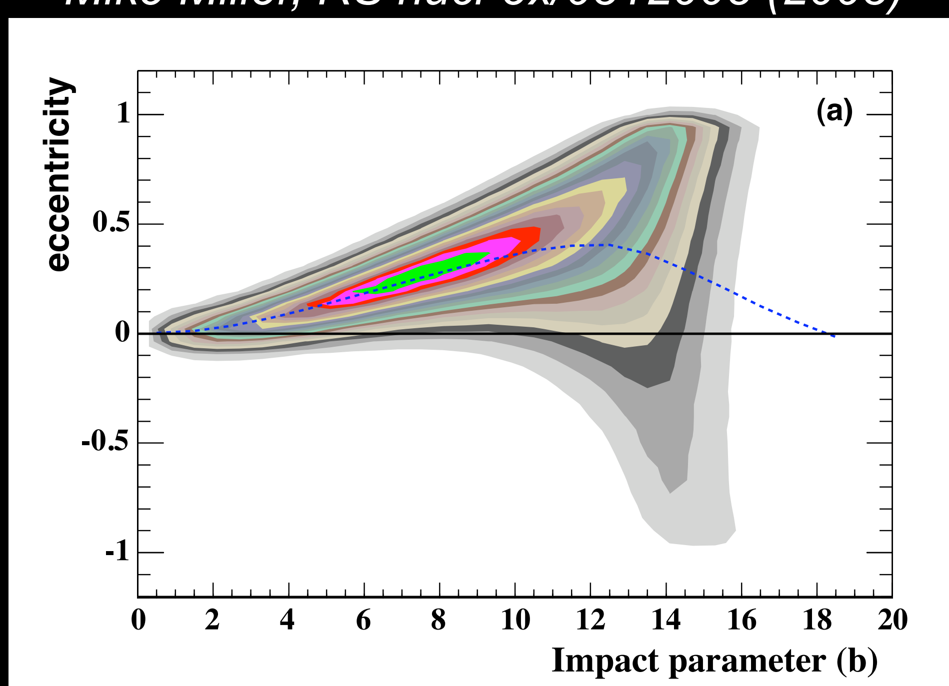
Fluctuations

Jean-Yves Ollitrault; PRD 46 (1992)





Mike Miller, RS nucl-ex/0312008 (2003)

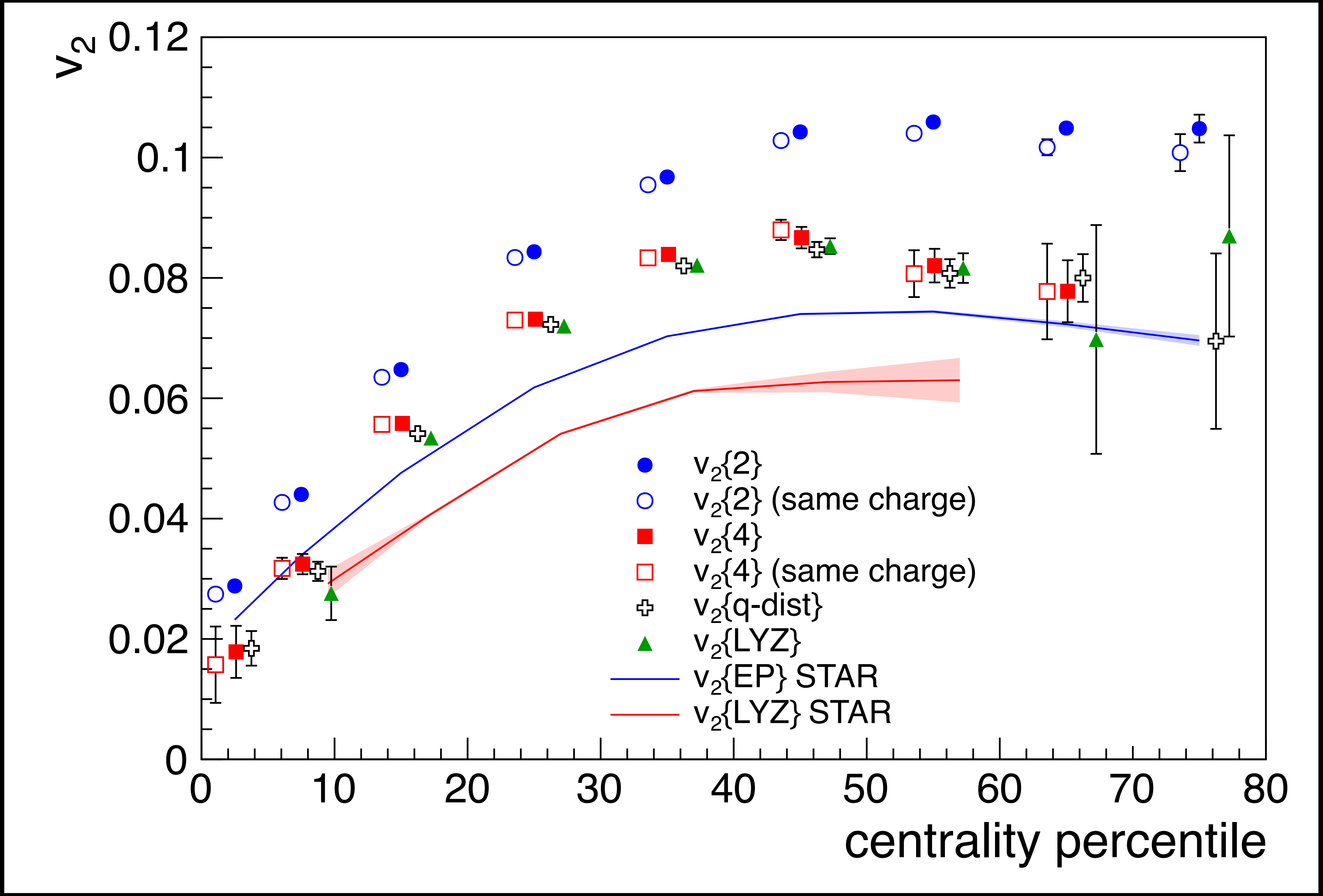


The asymmetry is larger and even non-zero for perfectly central collisions

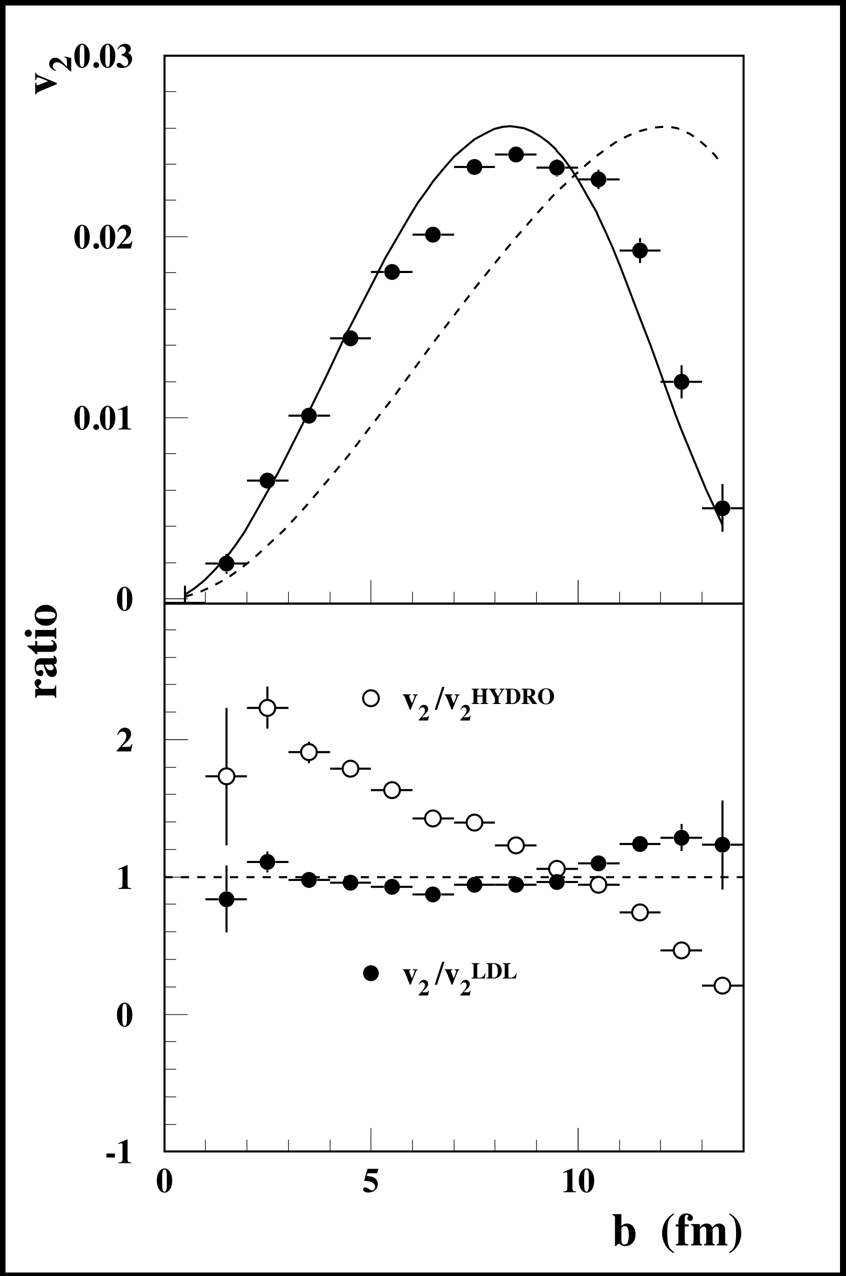
This asymmetry in coordinate space is though to be responsible, due to e.g. final state interactions, for the observed anisotropy in particle production

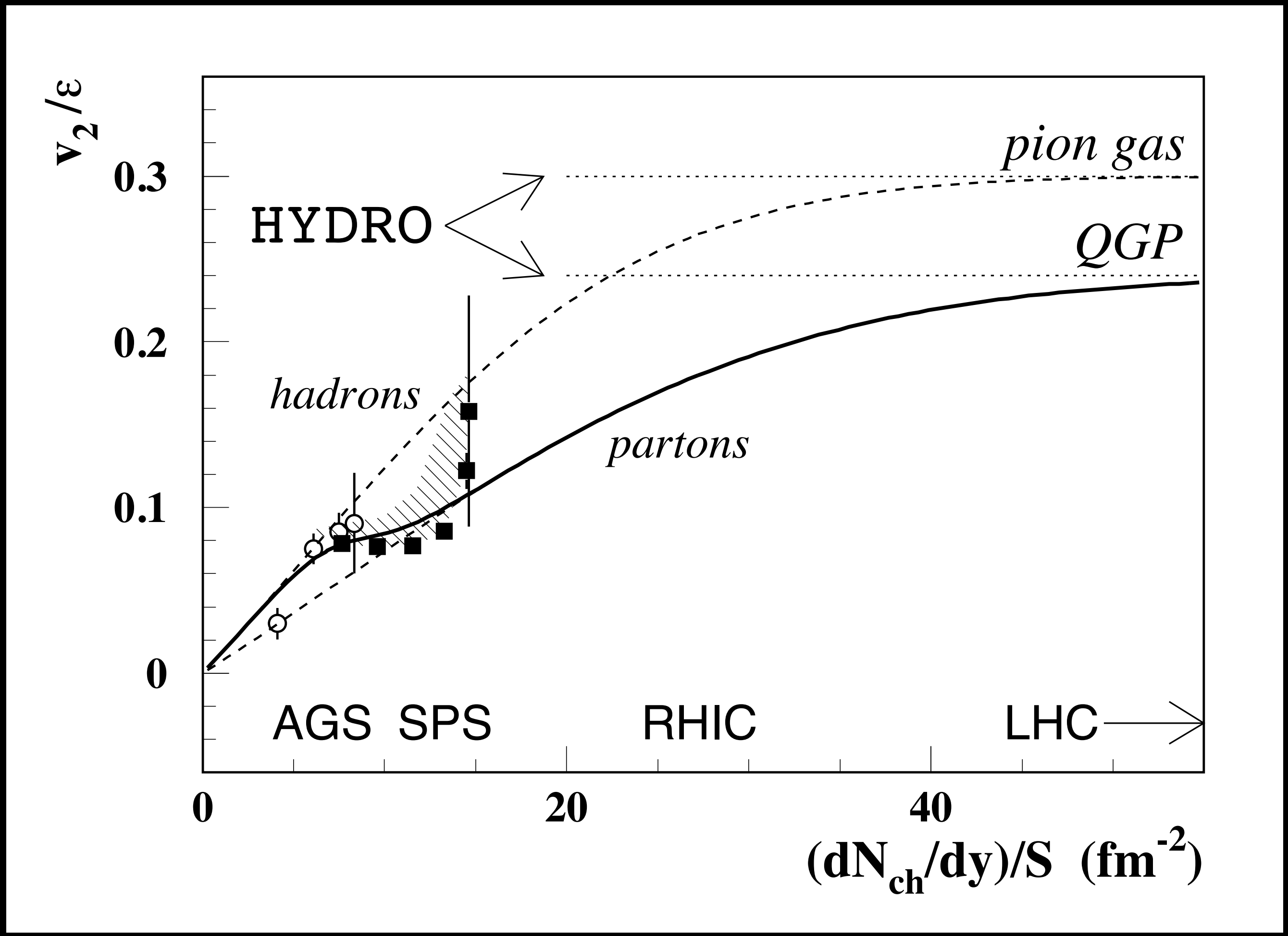
$$v_n \propto \varepsilon_n$$

Integrated v_2



Integrated v_2 and the eccentricity

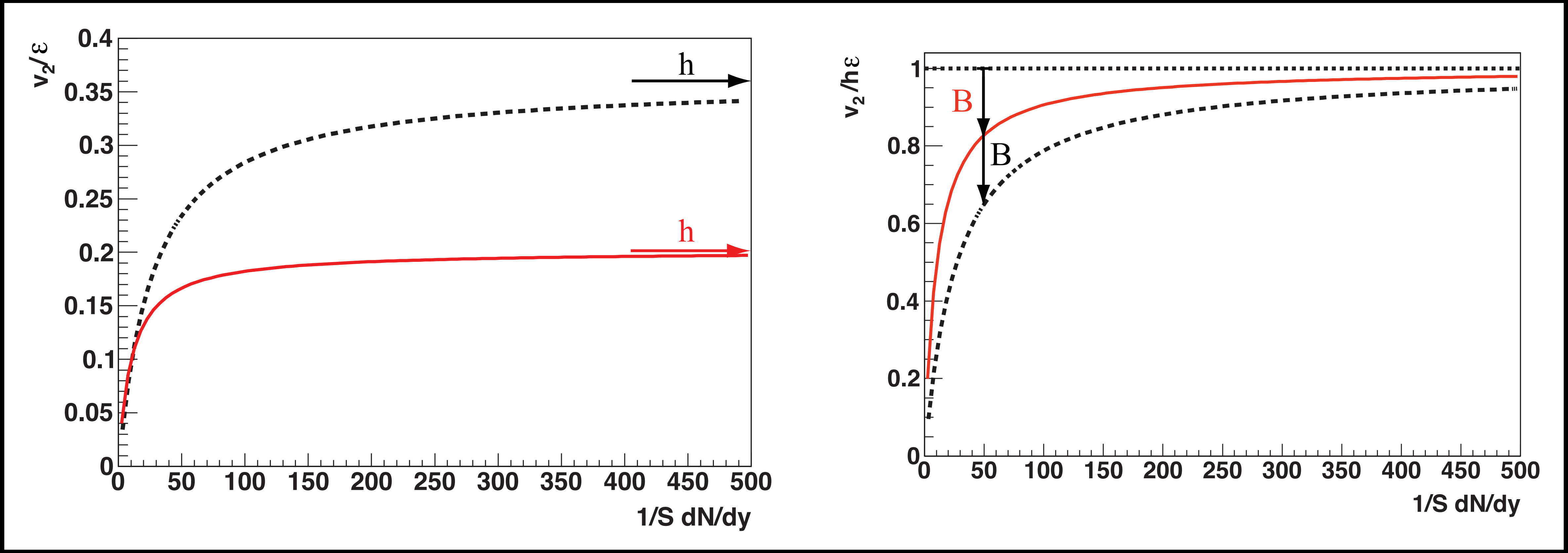




$$v_n \propto \varepsilon_n$$

sensitive to the EoS and transport parameters

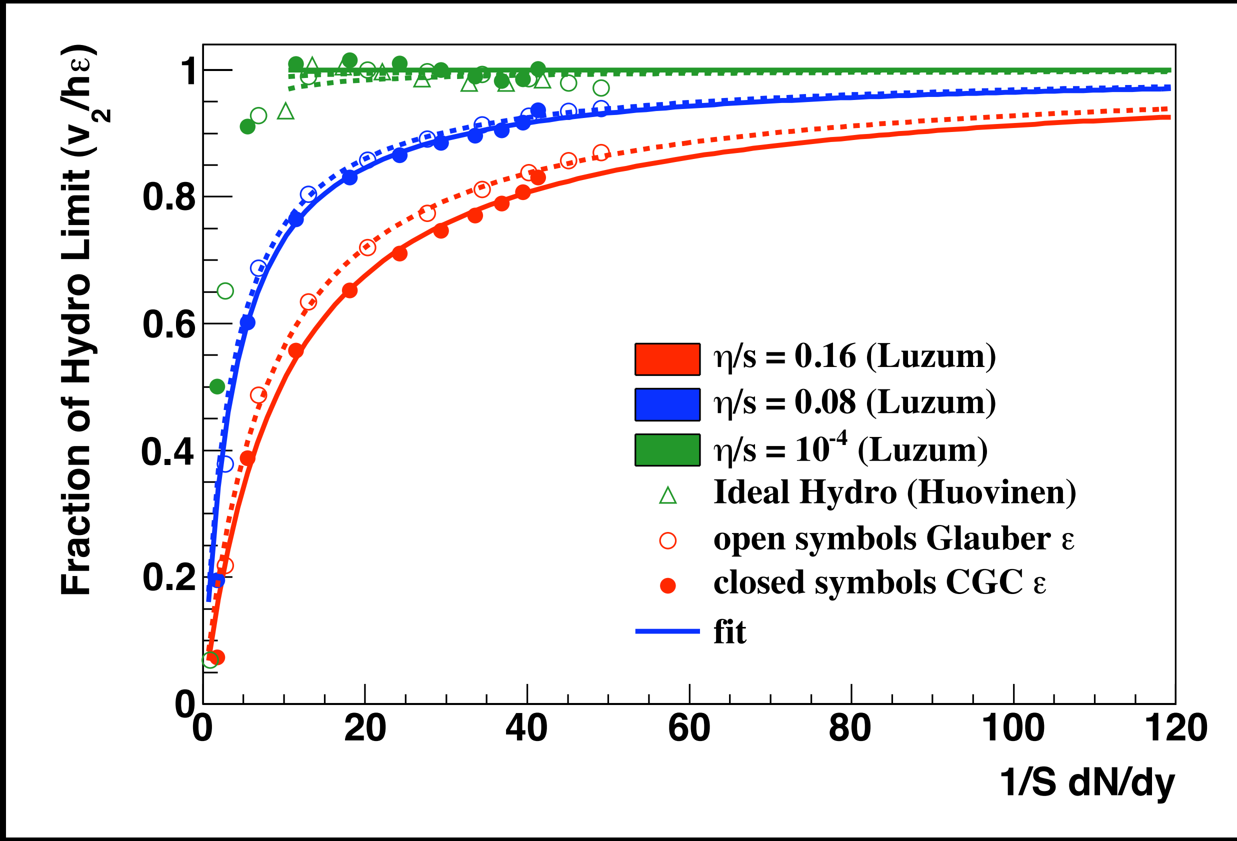
Integrated v_2 and the eccentricity

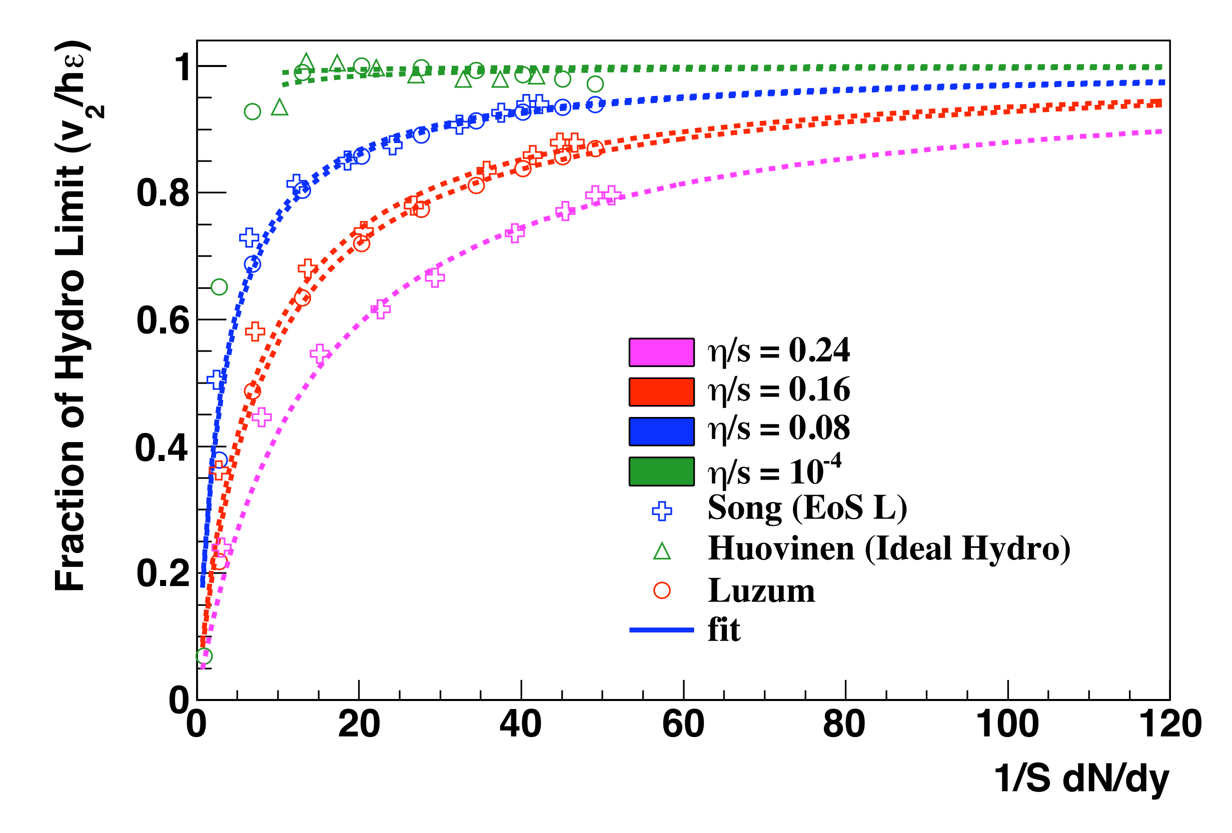


$$v_n \propto \varepsilon_n$$

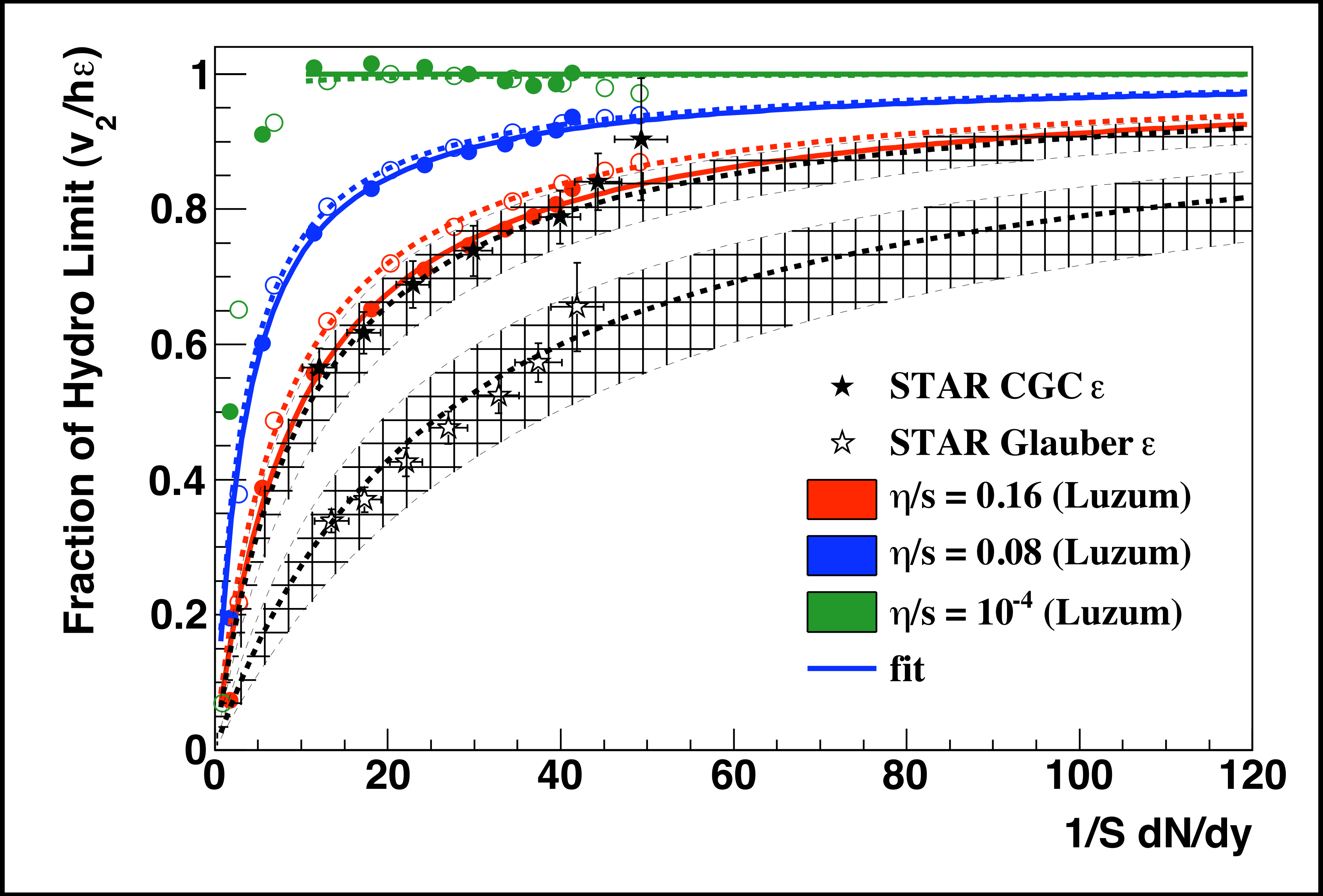
sensitive to the EoS and transport parameters

Integrated v_2 and the eccentricity



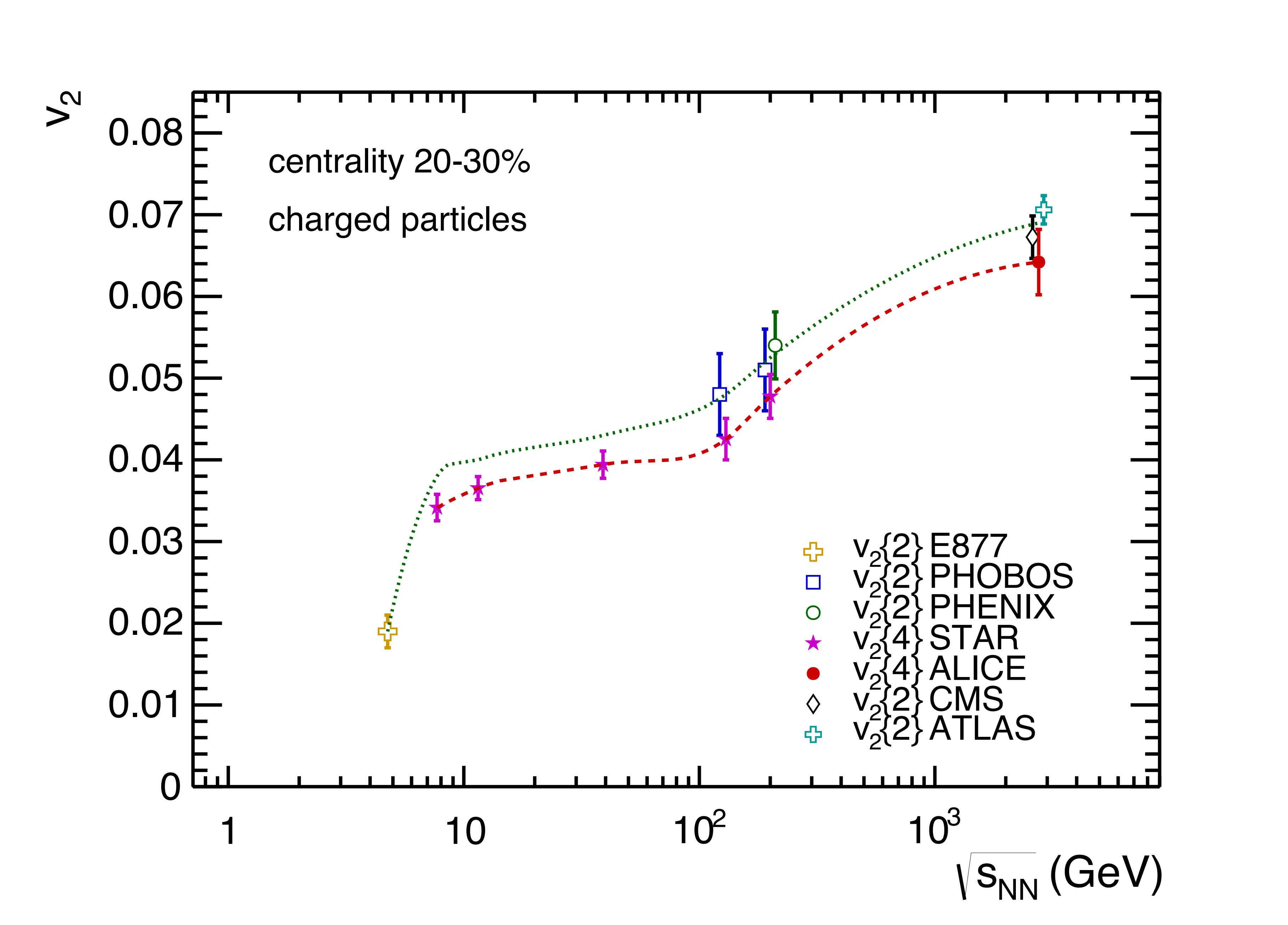


Integrated v_2 and the eccentricity



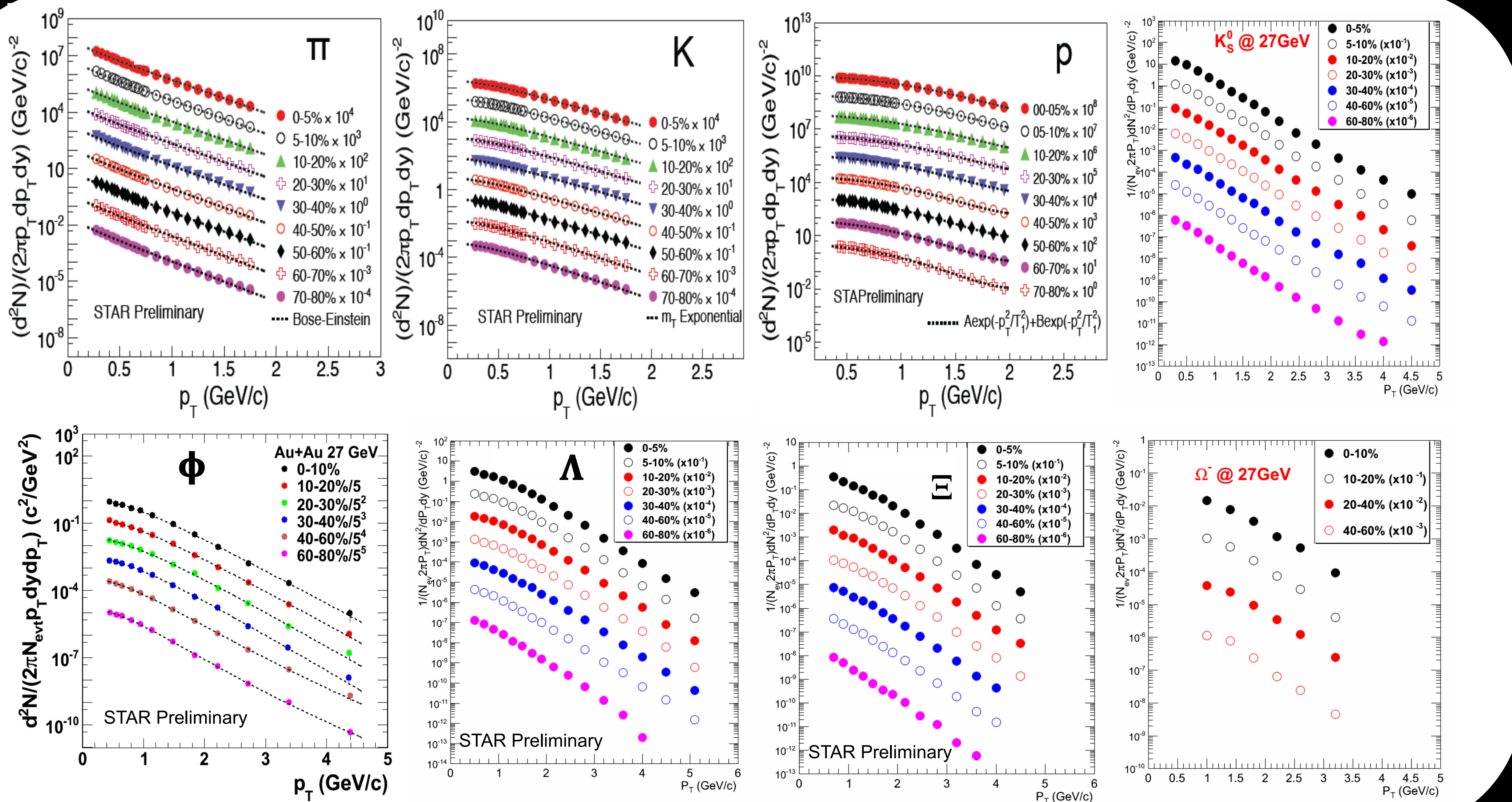
If the models match the data depends strongly on what the true eccentricity is (still an open question)

Integrated v_2



collision energy dependence of the elliptic flow
shows indication of changing slope

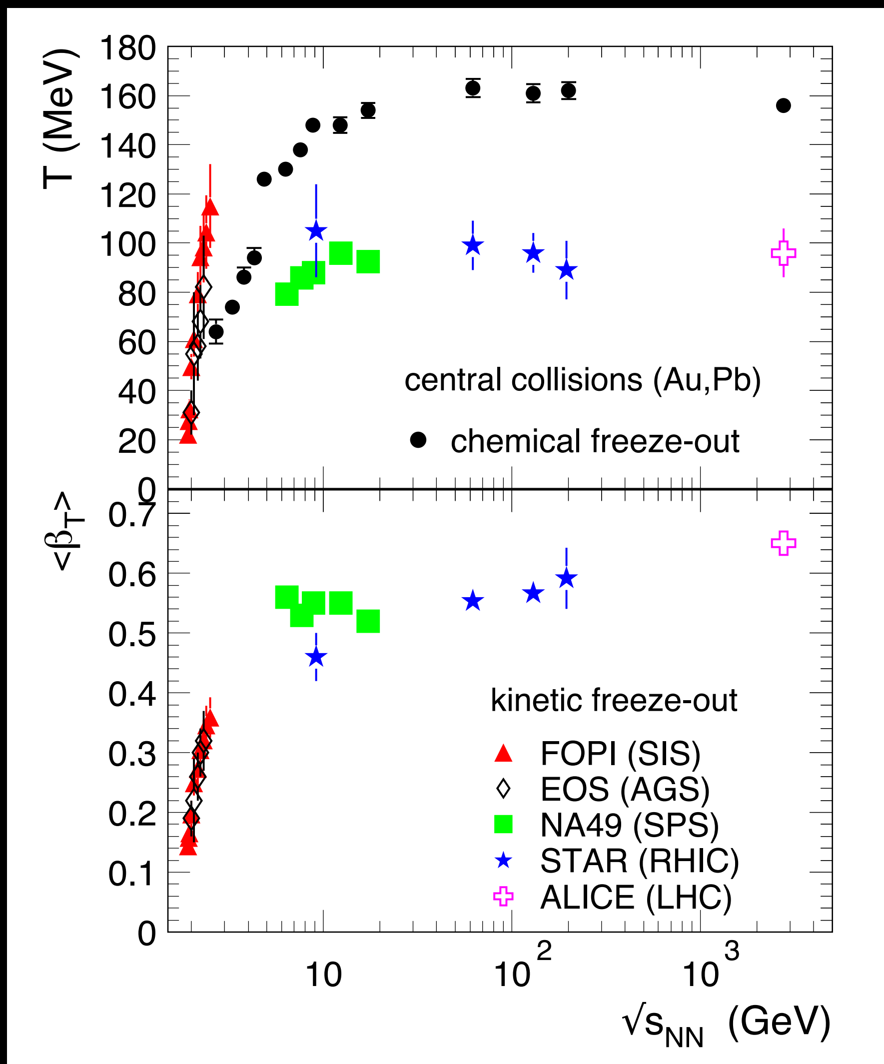
kinetic freeze-out

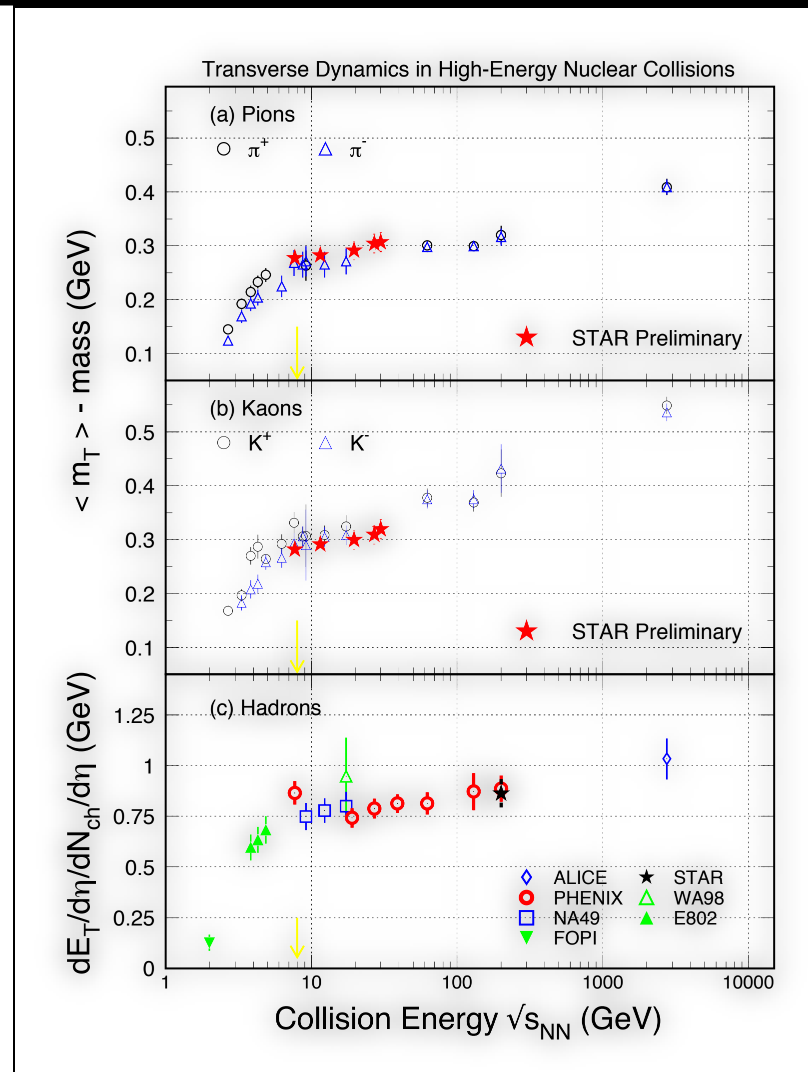


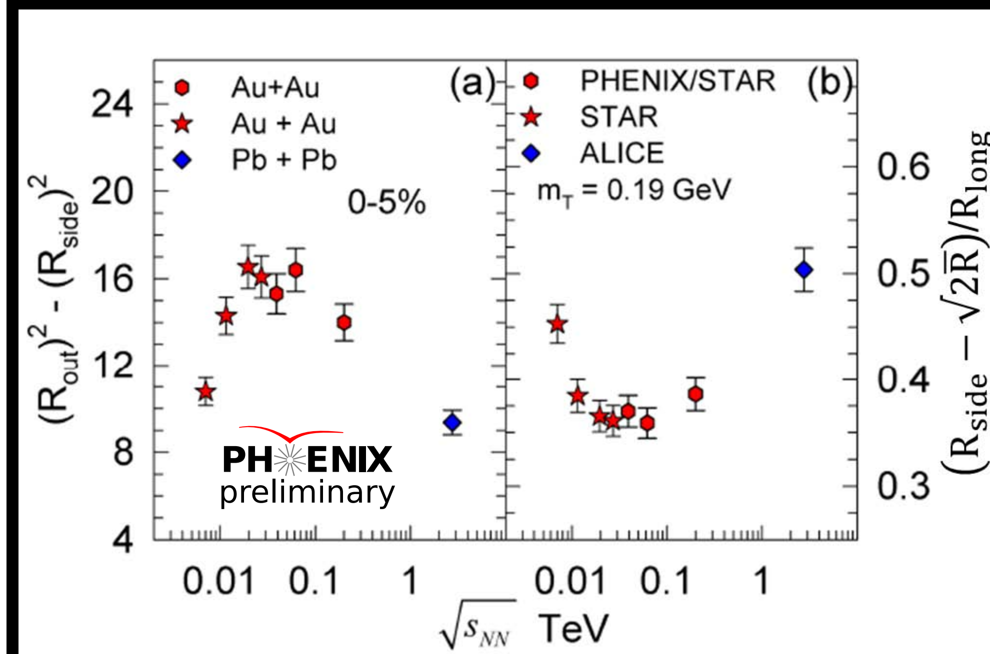
spectra a various particles follow trend expected
from a boosted "thermal" system

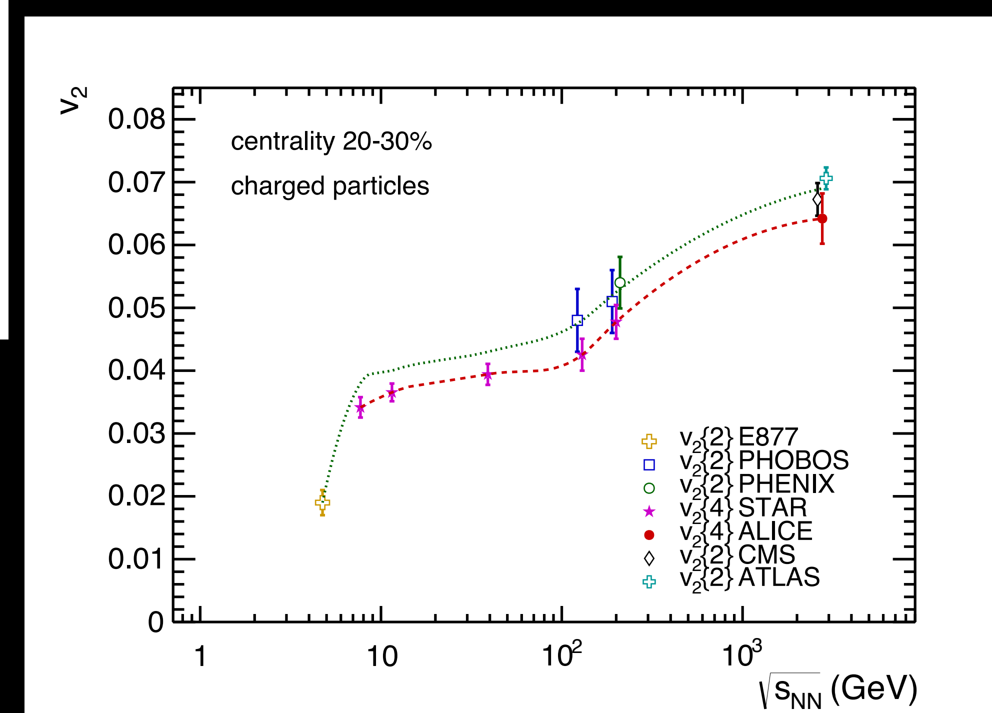
kinetic freeze-out

Anton Andronic, Int. J. Mod. Phys. A29 (2014) 1430047



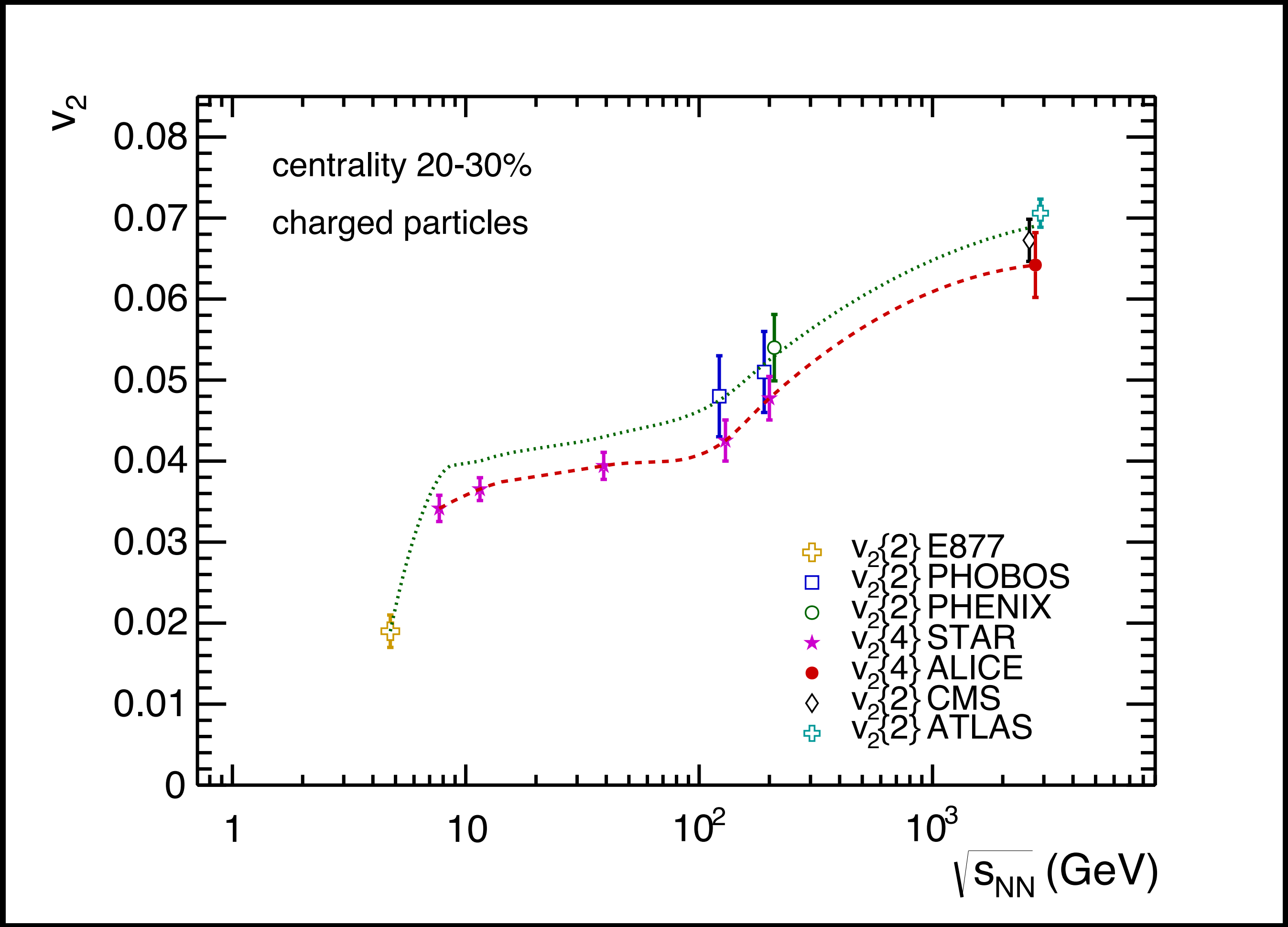


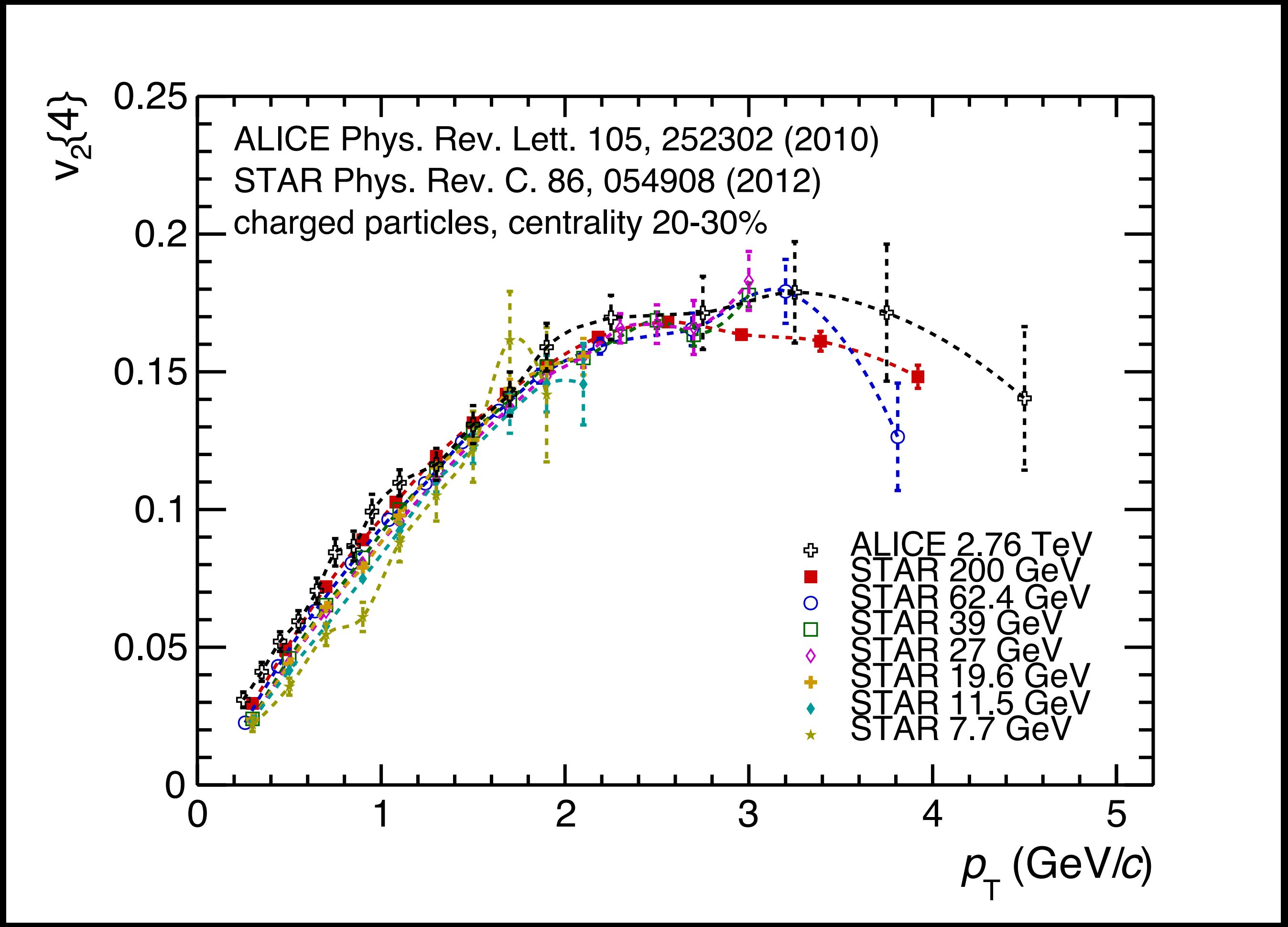




collision energy dependence shows nice
indications of changing slope in many observables

The increase of collective flow with collision energy?

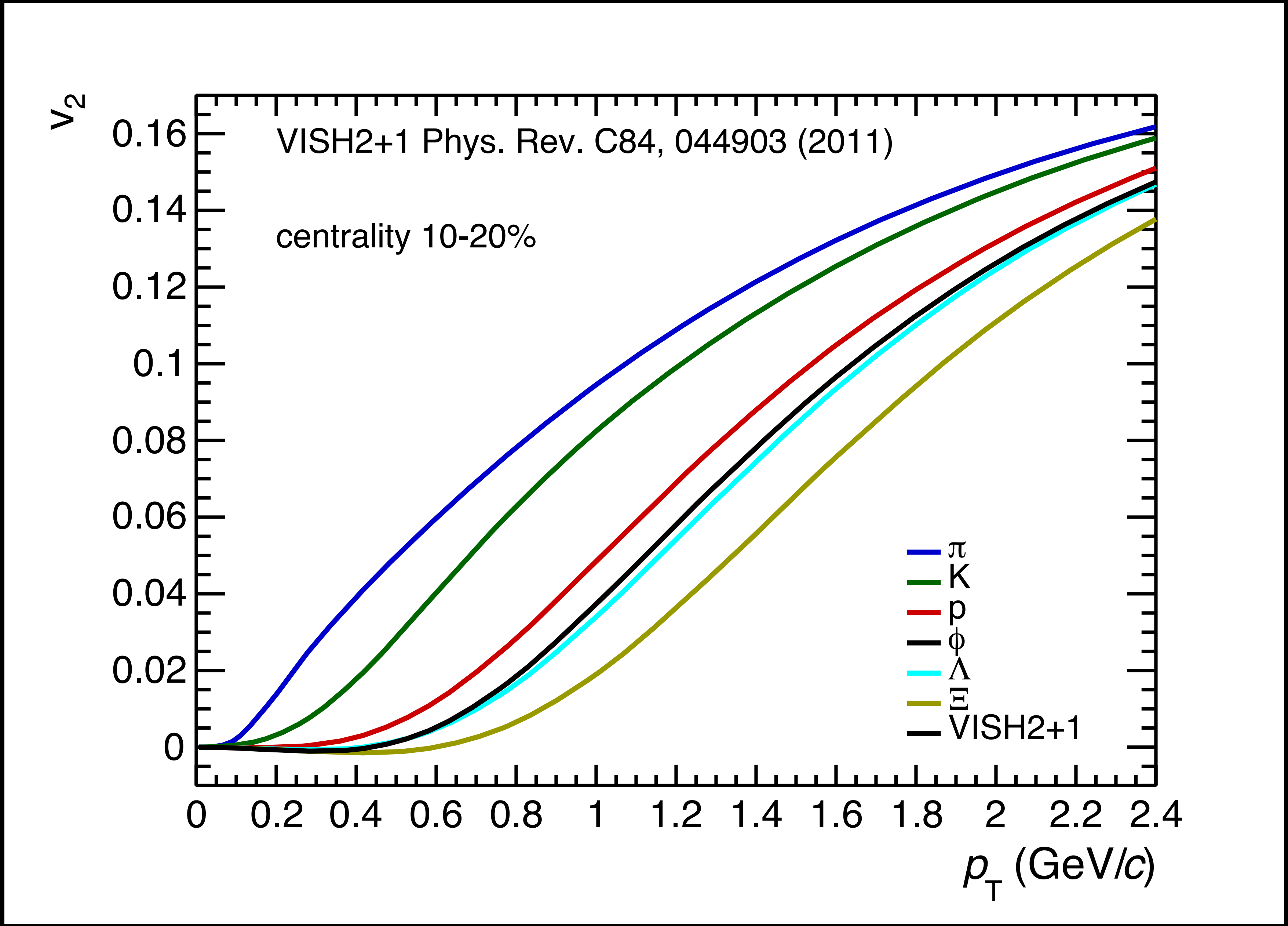




Elliptic flow increases from RHIC to LHC collision energies about 30%
Detailed measurements of $v_2\{4\}$ at RHIC in the beam energy scan combined with the LHC measurements show tantalising evidence for a change in slope.

The p_T-differential elliptic flow also increases with collision energy but difference is small over two orders of magnitude
Is this expected/understood?

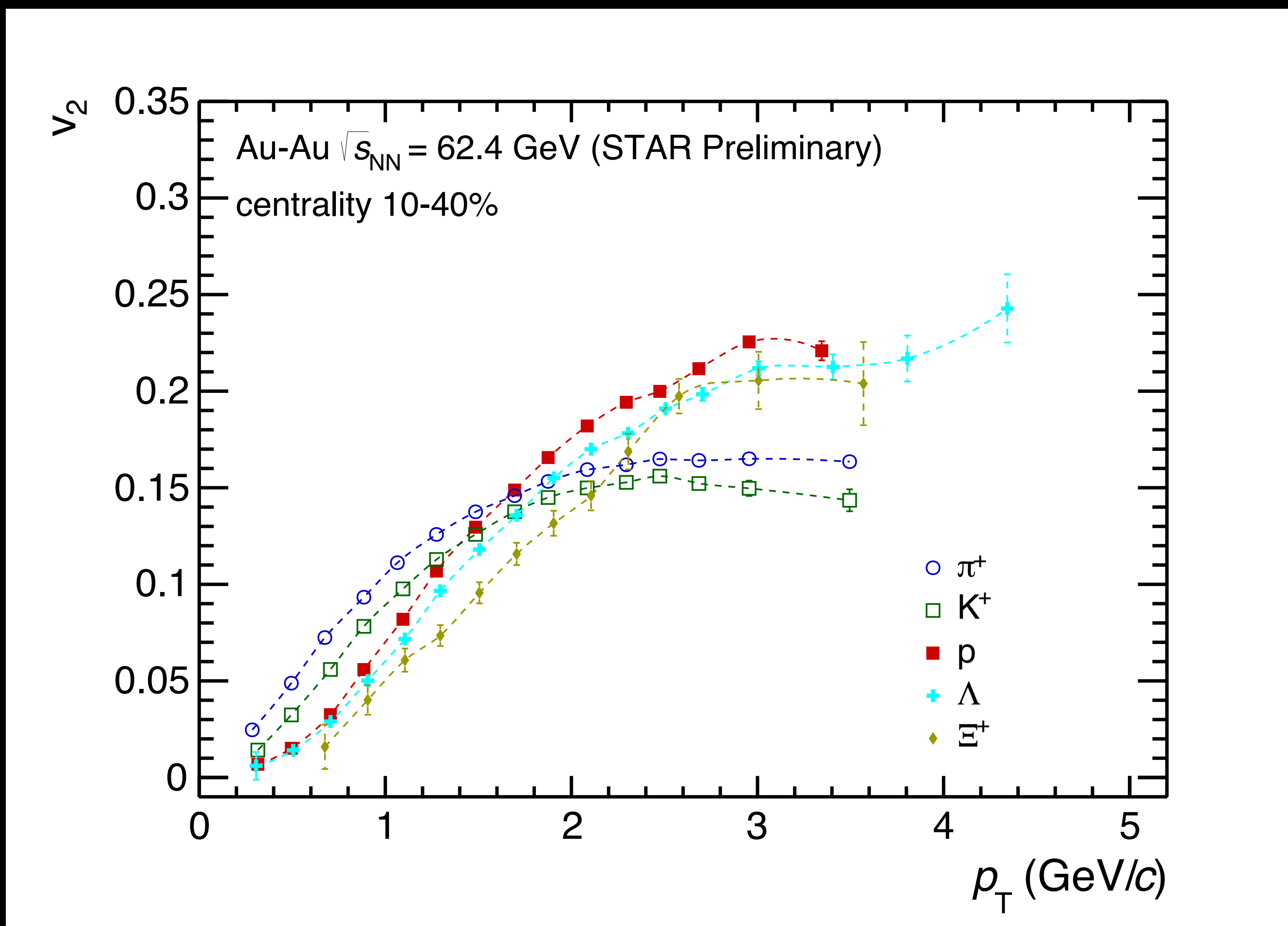
Collective behaviour



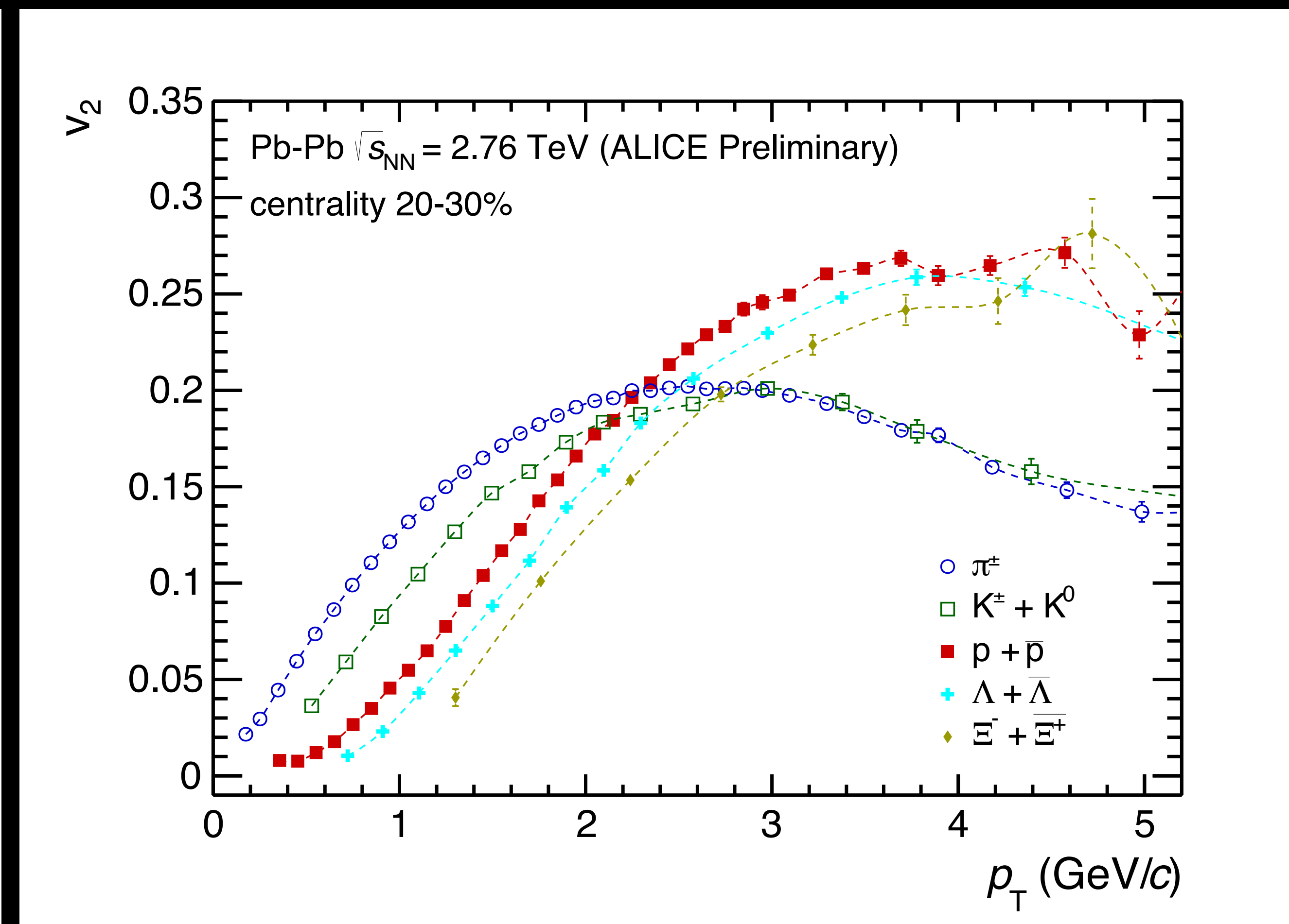
In the hydro and blast-wave picture
particles have a common temperature and flow velocity at freeze-out.
The difference in p_T-differential elliptic flow depends mainly on one parameter: the mass of the particle

Collision energy dependence of elliptic flow for particles with different masses

STAR QM2014

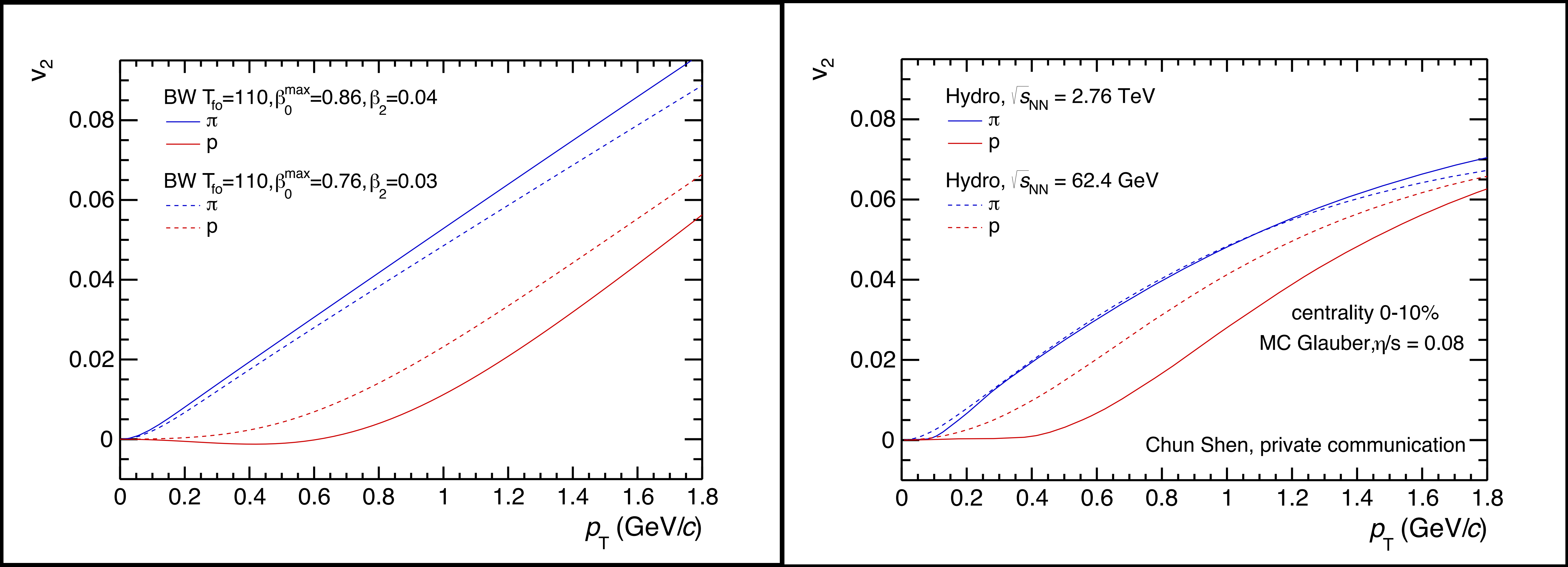


ALICE arXiv:1405.4632



mass hierarchy follows hydrodynamic and blast-wave picture at low p_T!

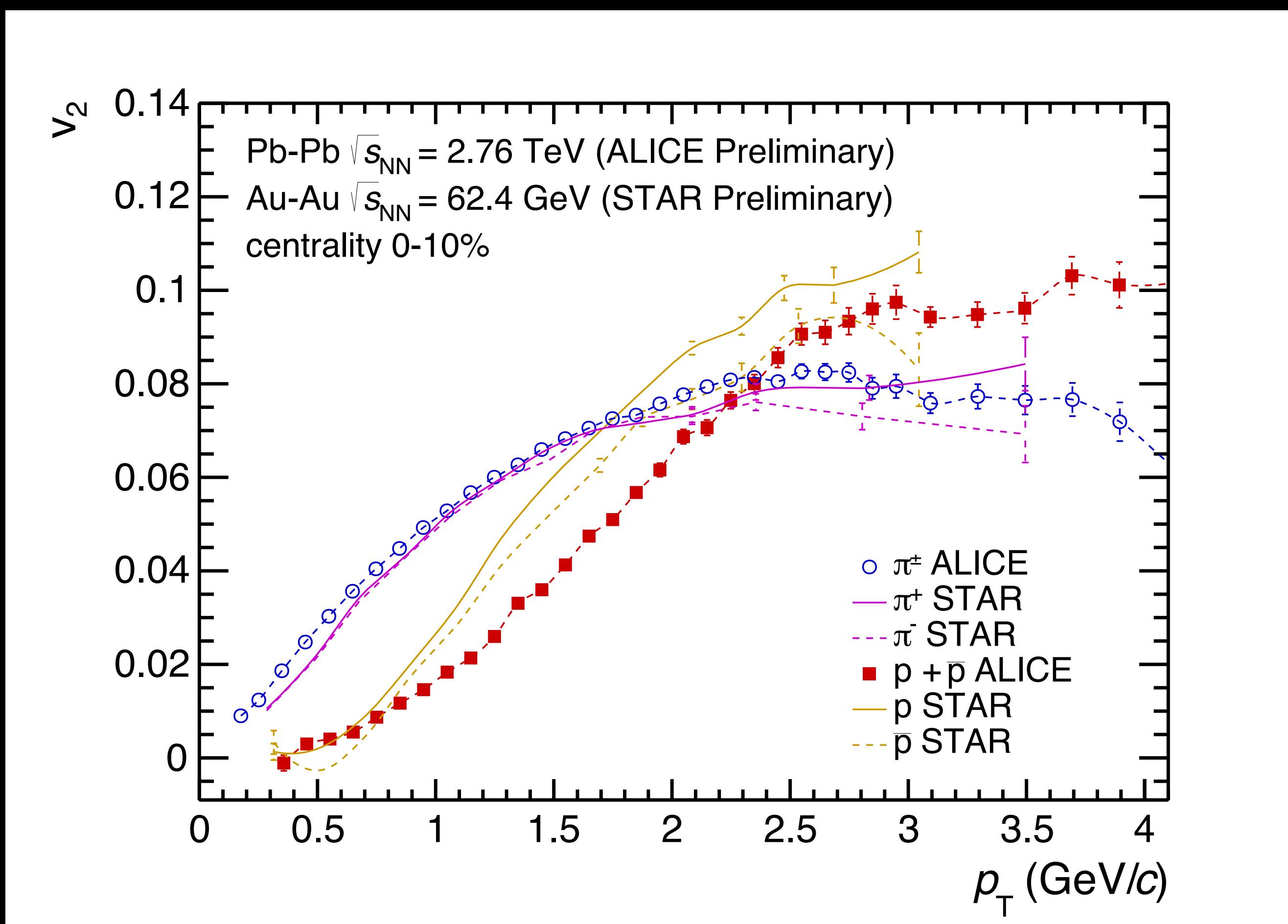
Hydrodynamic behaviour



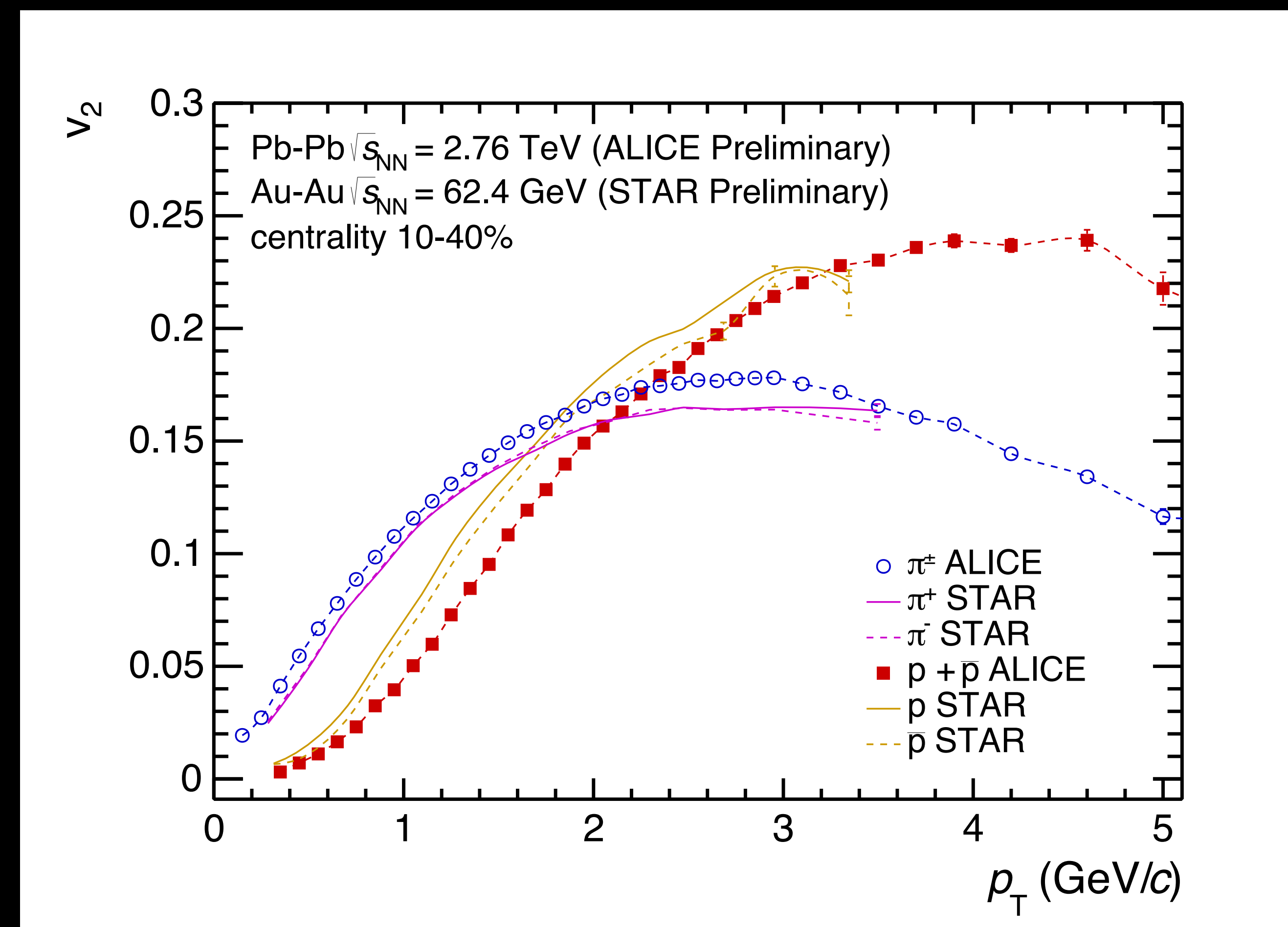
hydro and blast-wave picture
particles have a common temperature and flow velocity
larger radial flow increases mass splitting

Collision energy dependence of elliptic flow as function of transverse momentum

ALICE arXiv:1405.4632 STAR QM2014



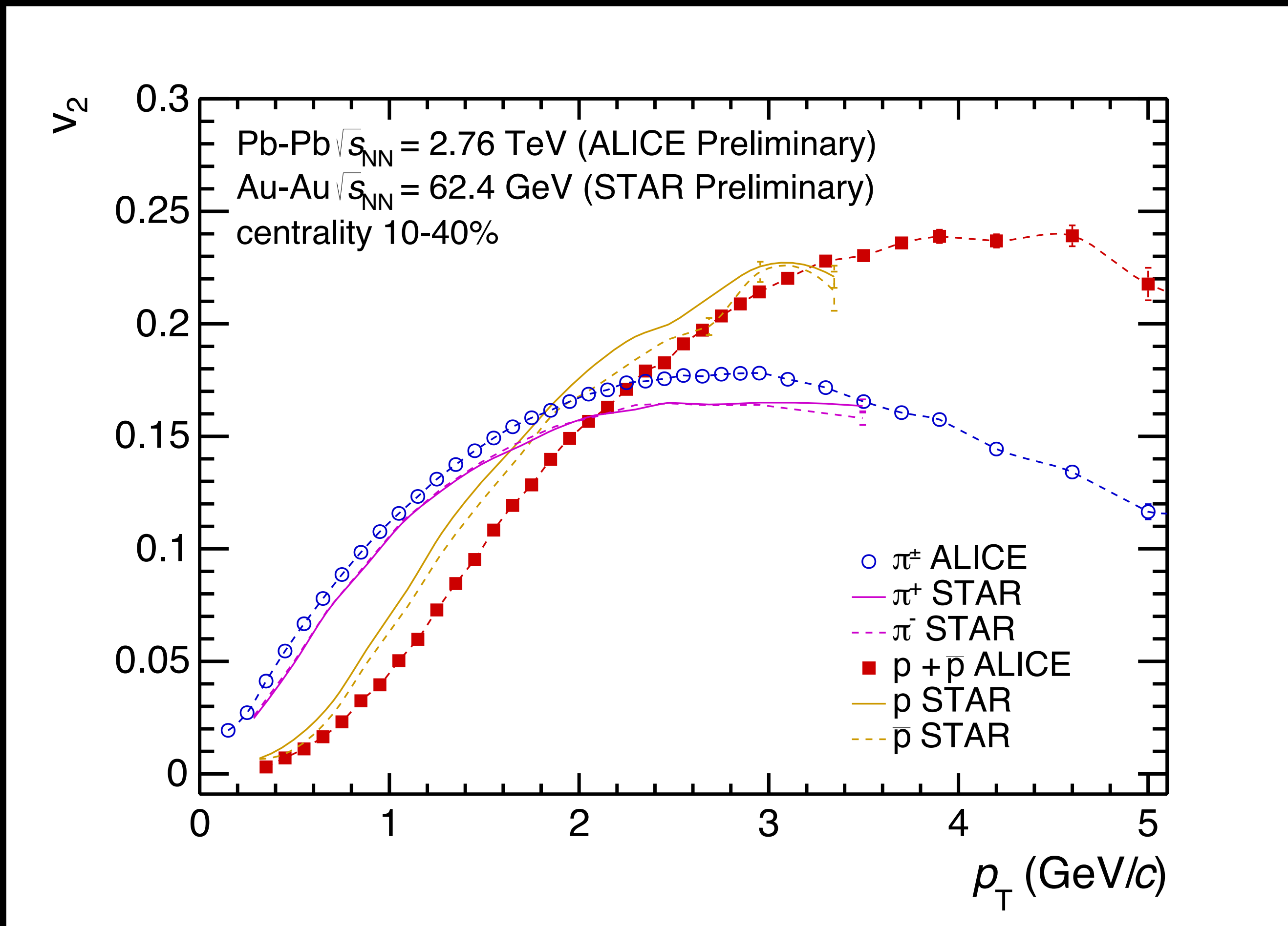
ALICE arXiv:1405.4632 STAR QM2014

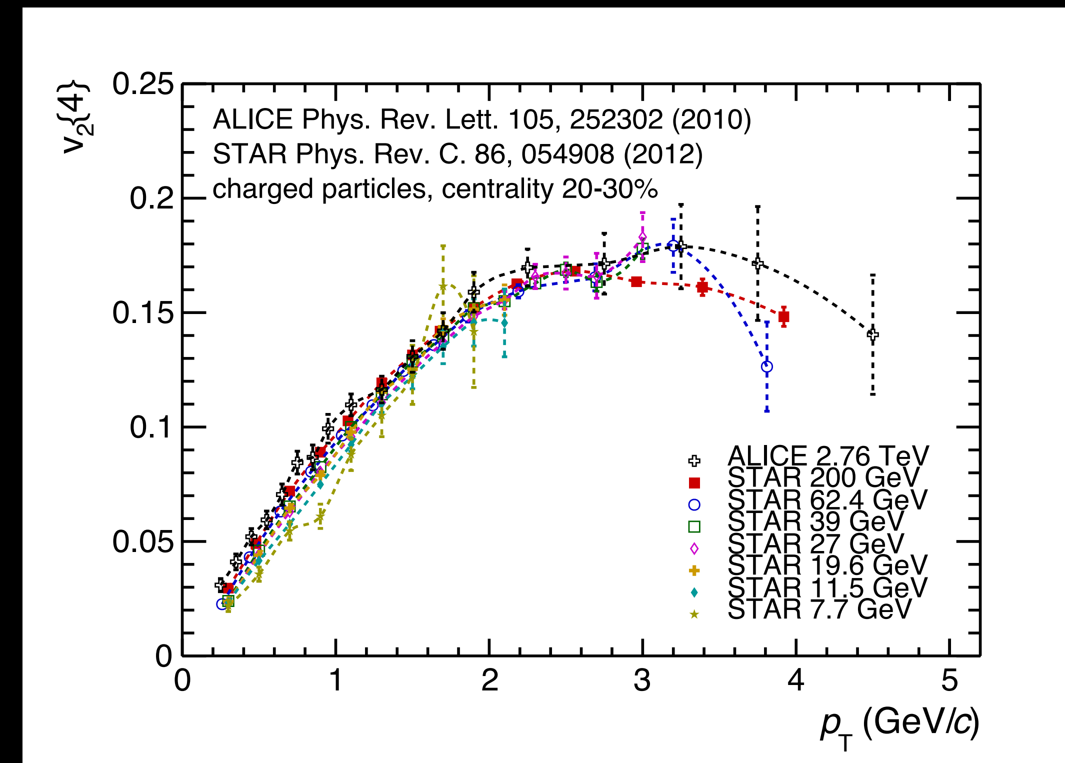


while the p_T-differential charged particle v_2 changes very little over two orders of magnitude the v_2 of heavier particles clearly shows the effect of the larger collective flow at higher collision energies

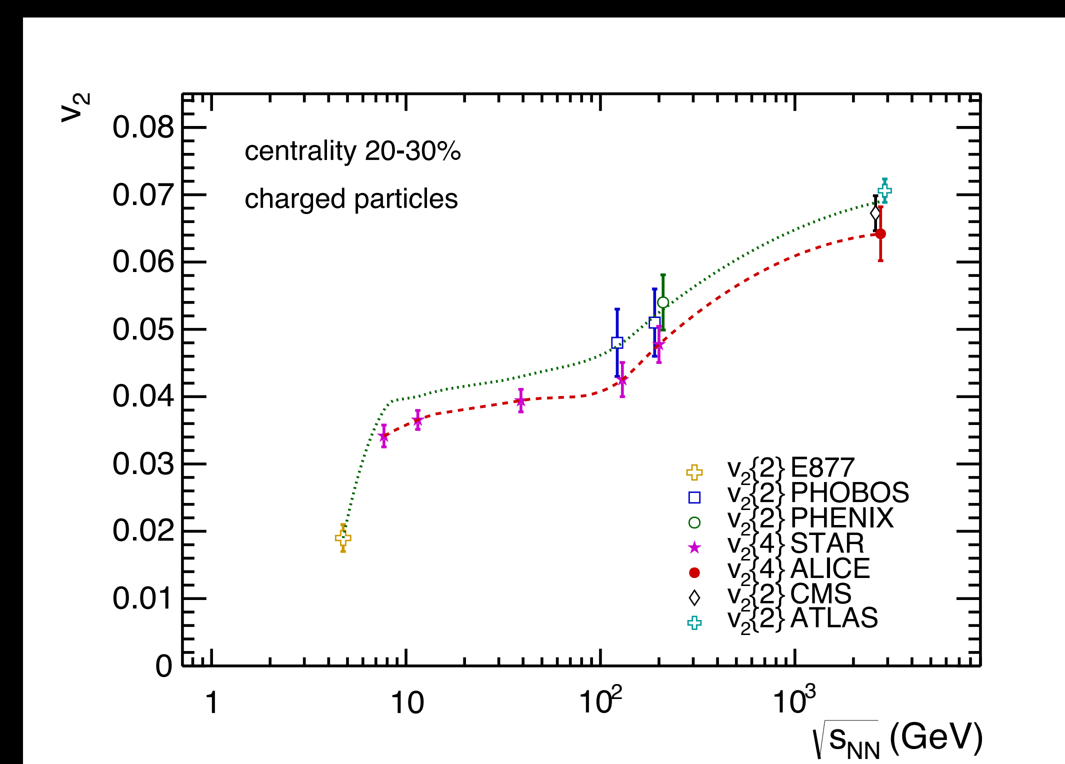
Collision energy dependence of elliptic flow as function of transverse momentum

ALICE arXiv:1405.4632 STAR QM2014



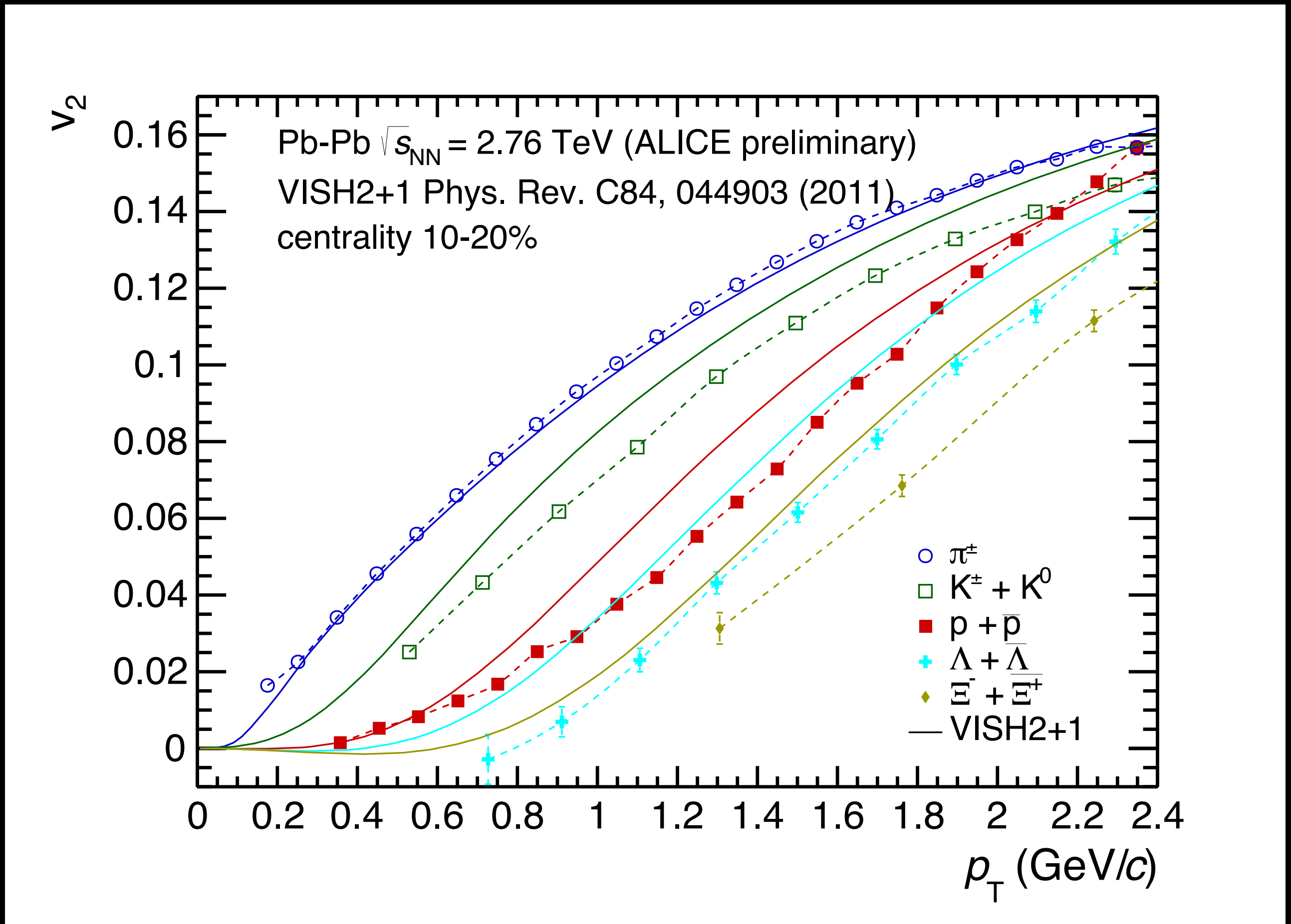


Elliptic flow as function of collision energy can be qualitatively understood in terms of a boosted thermal system



Compared to viscous hydrodynamics

ALICE arXiv:1405.4632



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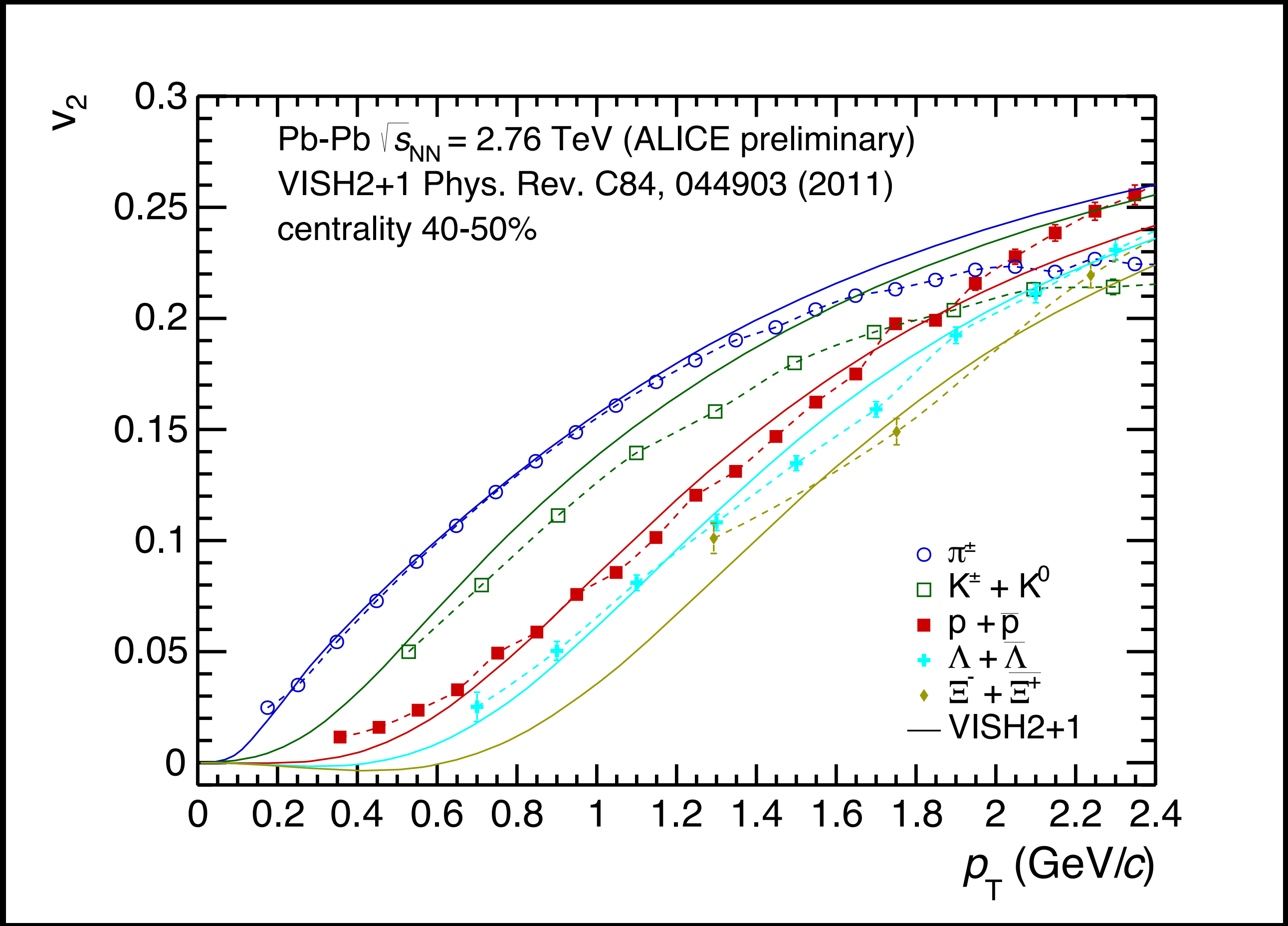


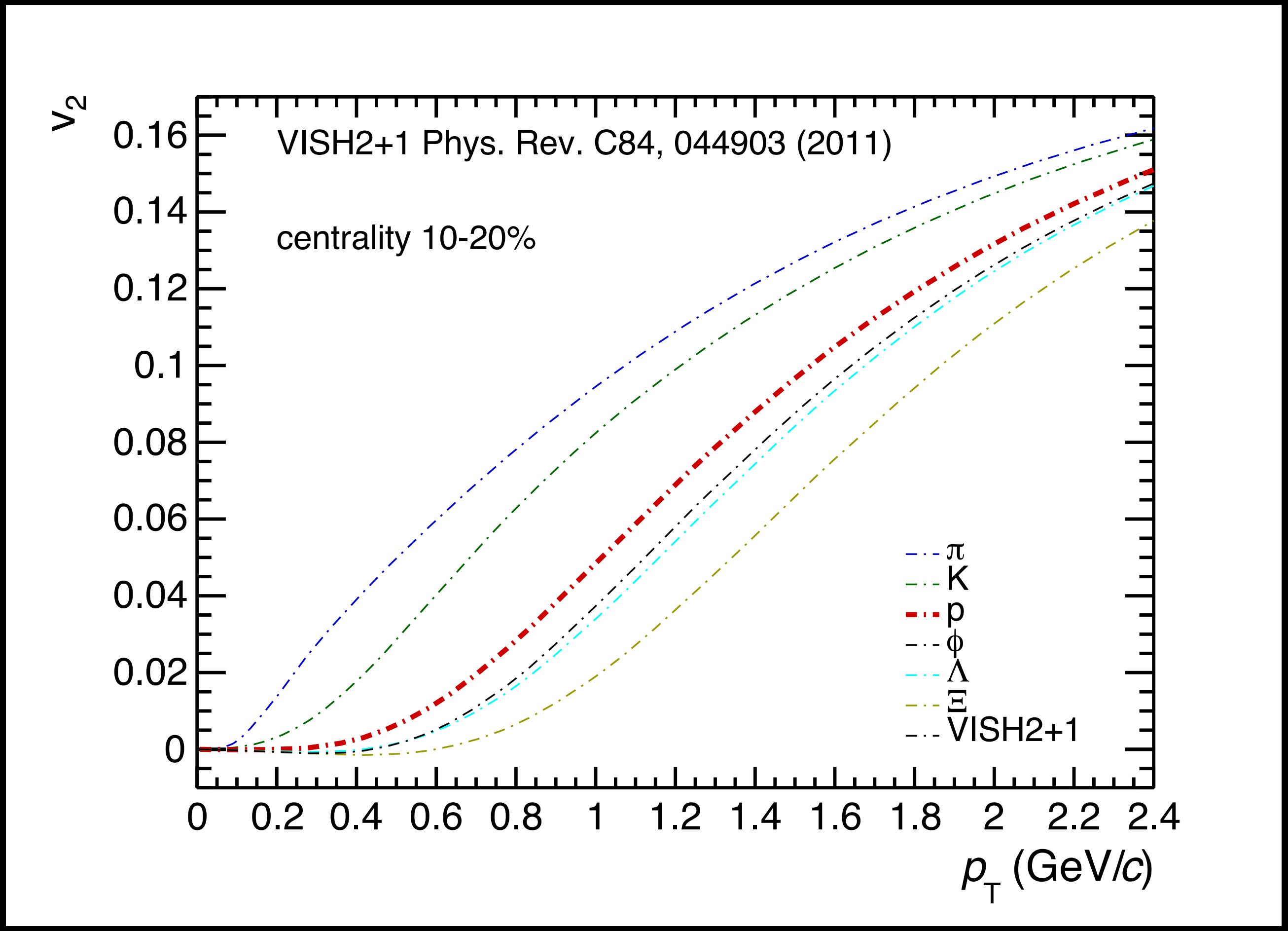
pure viscous hydrodynamics VISH2+1, status at QM2011

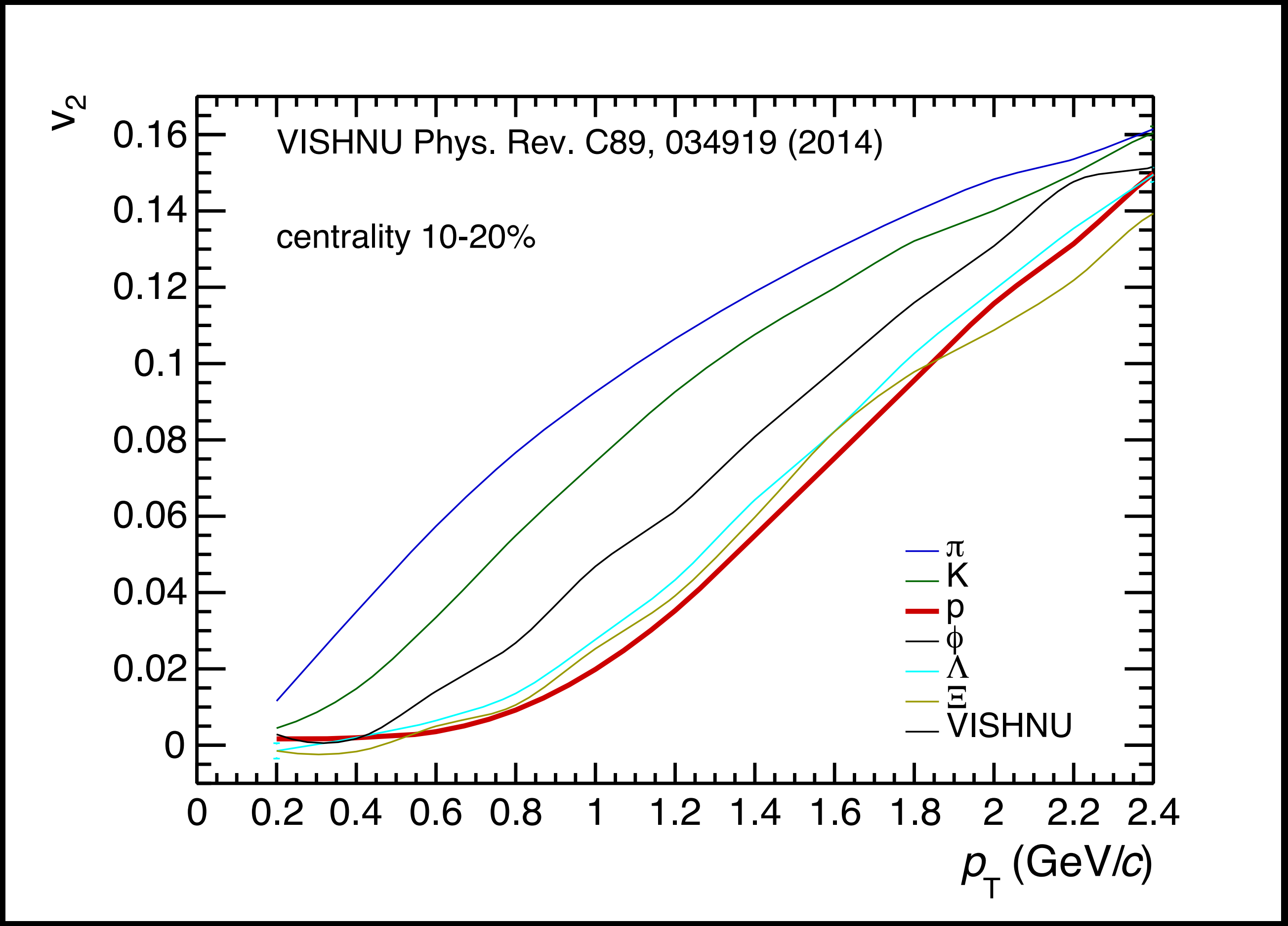
Viscous hydrodynamics predictions worked reasonably well for more peripheral collisions 40-50%

For more central collisions, 10-20%, the radial flow seems to be under-predicted as the protons deviate a lot and this was part of the proton puzzle (the new data plotted here shows this is not just for protons but all heavy particles)

can this be understood by a more dissipative hadronic phase (model with a hadron cascade)?

Viscous hydrodynamics and the effect of the hadronic cascade

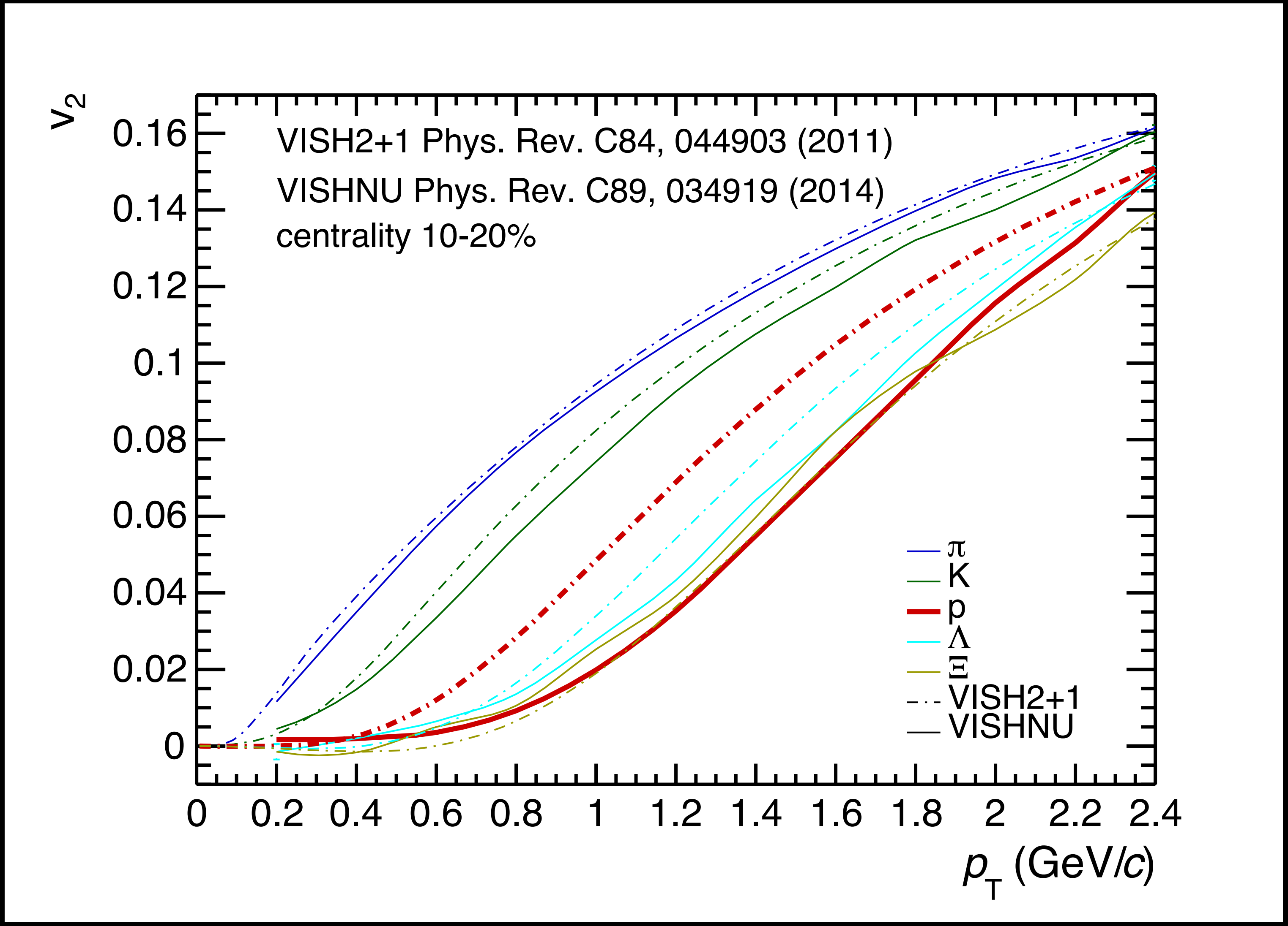




VISH2+1 viscous hydrodynamics
"standard" mass scaling

VISHNU viscous hydrodynamics + hadron cascade
mass scaling broken,
depending on individual hadronic re-interaction cross sections (pion wind pushing the protons)

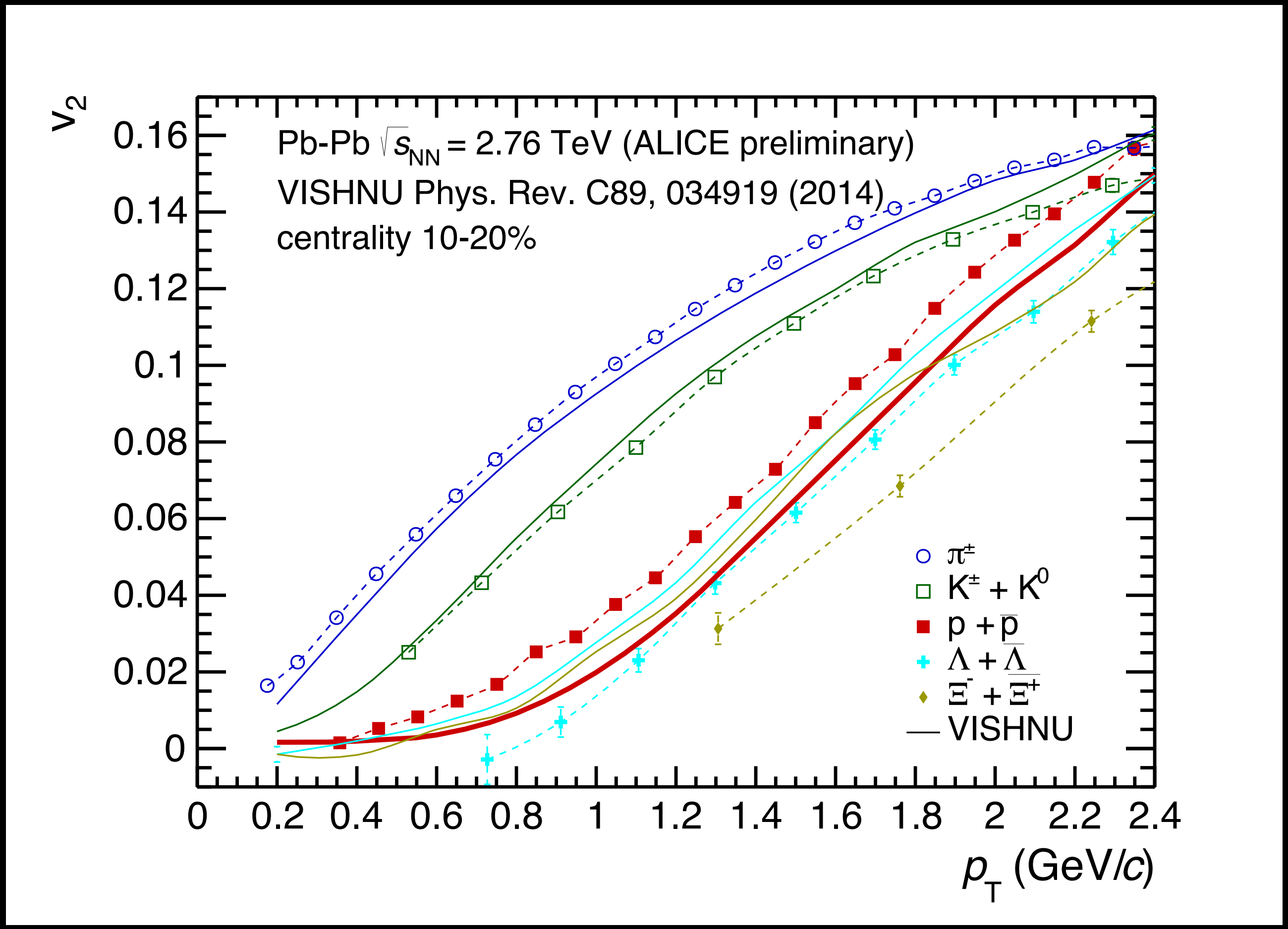
Viscous hydrodynamics and the effect of the hadronic phase



VISHNU viscous hydrodynamics + hadron cascade
big effect for the protons!
mass scaling broken,
depending on individual hadron-hadron re-interaction cross sections

Viscous hydrodynamics + hadron cascade

ALICE arXiv:1405.4632



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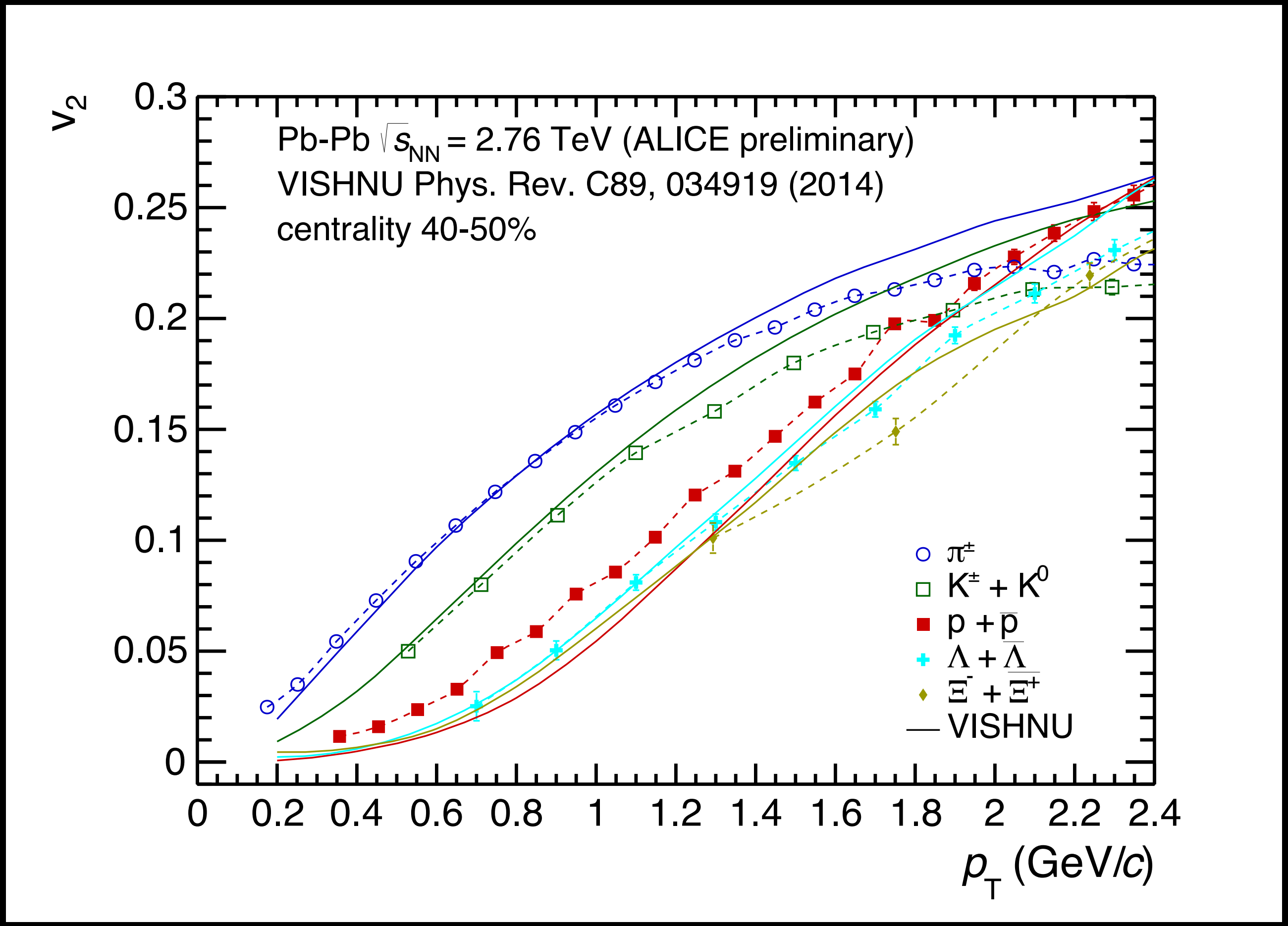
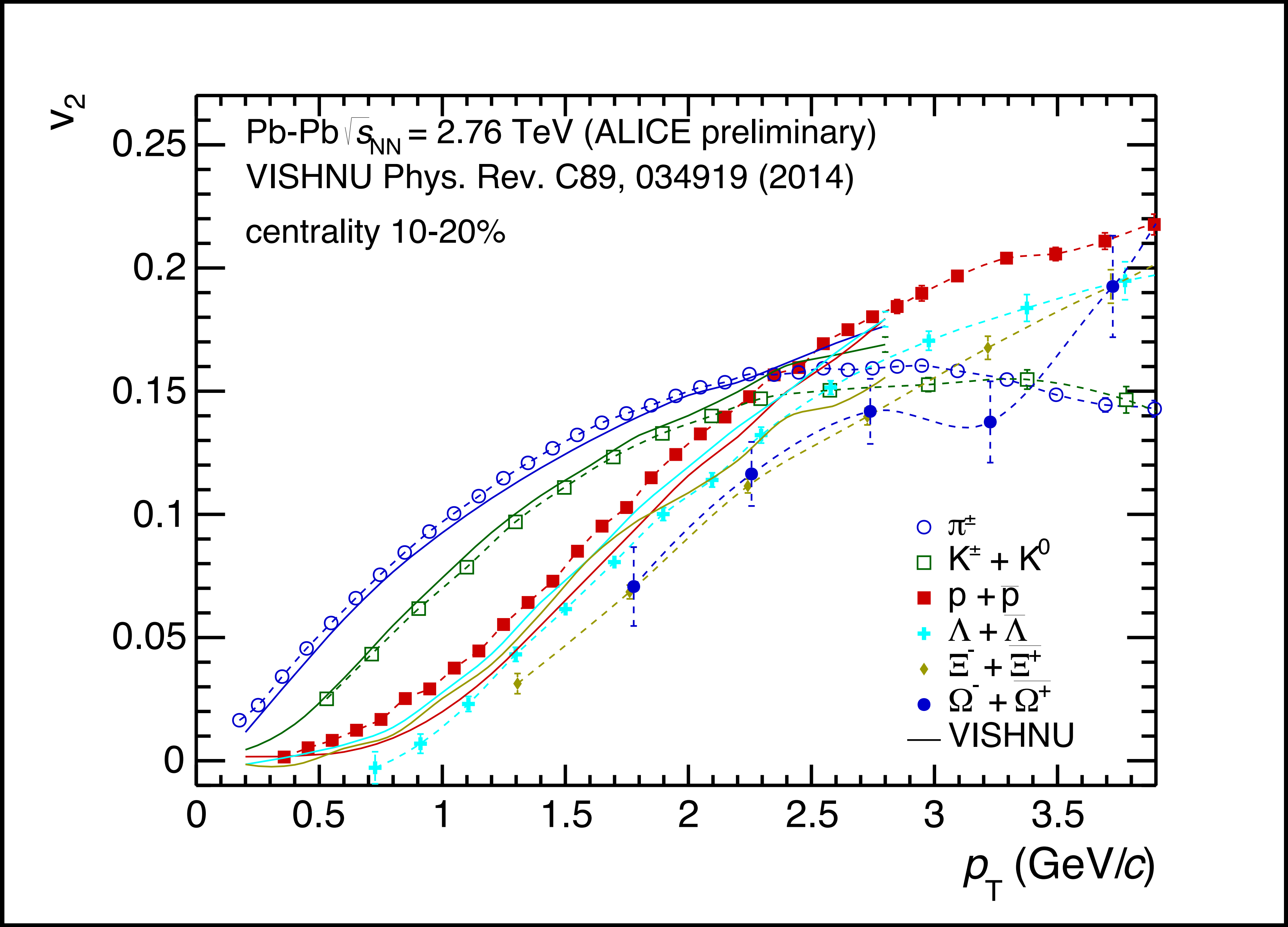


Viscous hydro +hadron cascade improves the Kaon v_2
It increases the push for the protons but actually over does it
It breaks the mass scaling and is incompatible with the data
It does a worse job than “simple” viscous hydrodynamics!!

over estimating effect of hadronic cascade?
or is the model lacking pre-equilibrium flow (AdS/CFT, CGC, …..)?

Viscous hydrodynamics + hadron cascade

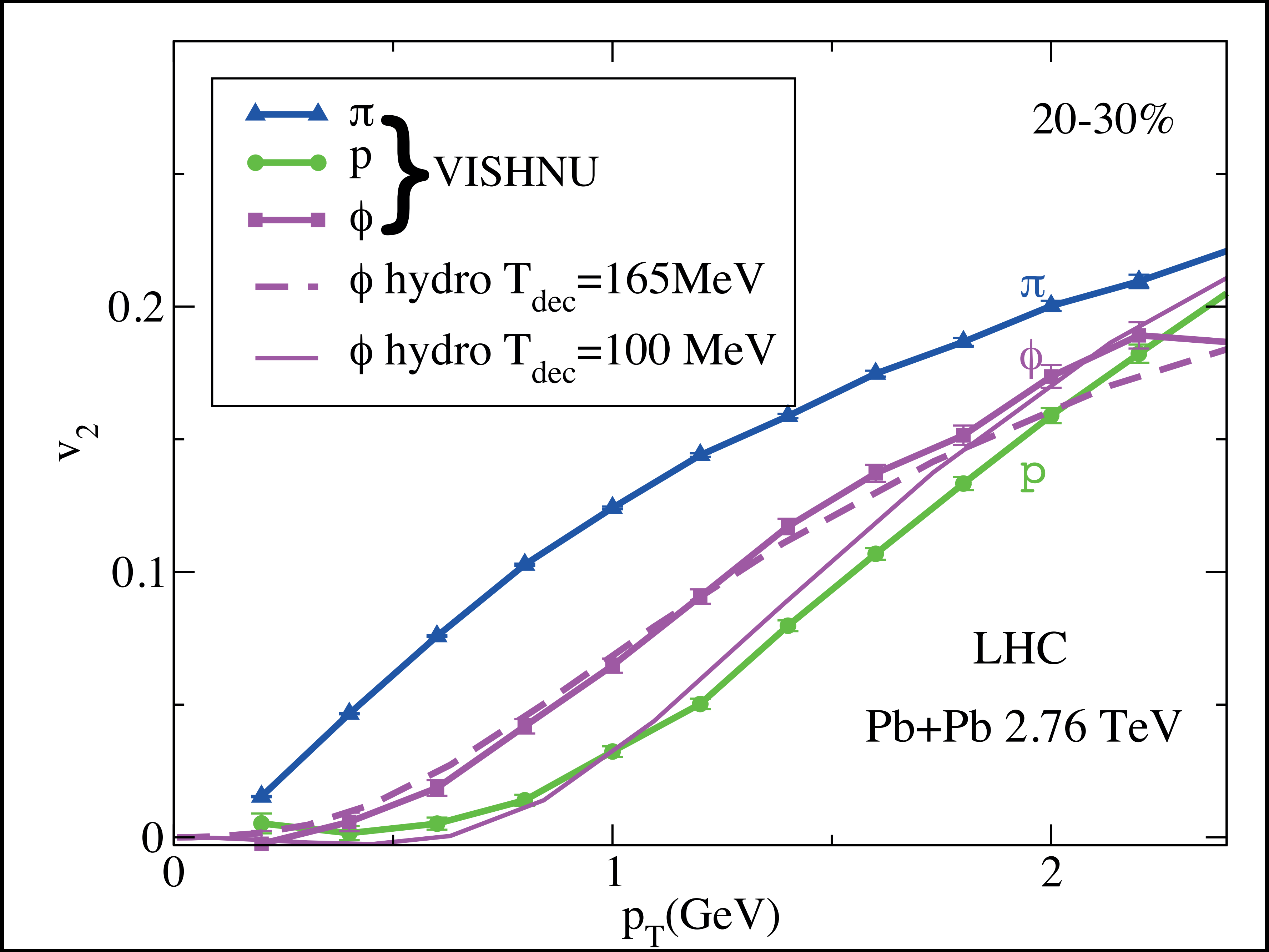
ALICE arXiv:1405.4632



VISHNU does not match with any of the baryons including the Ω's

ϕ-meson elliptic flow

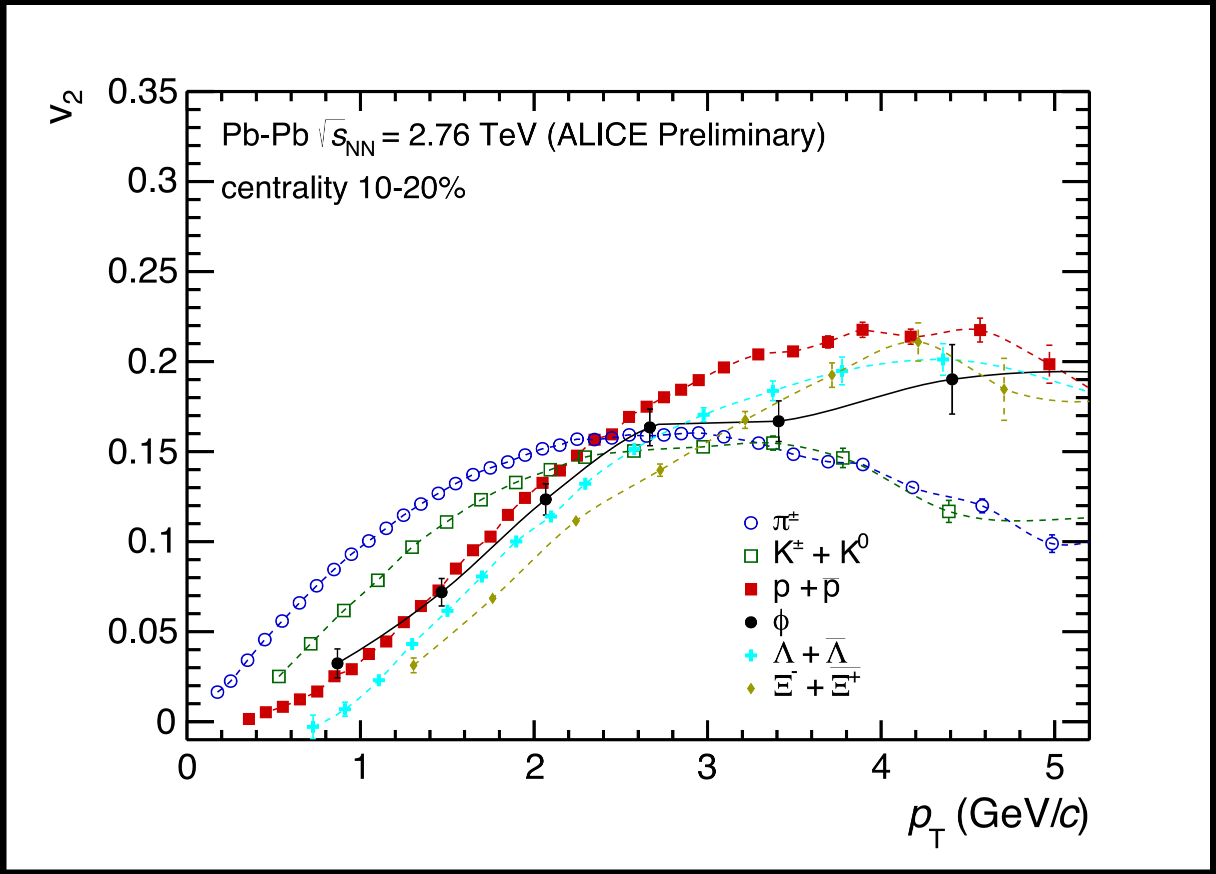
H. Song, S. Bass, U.W. Heinz, Phys. Rev. C89 034919 (2014)

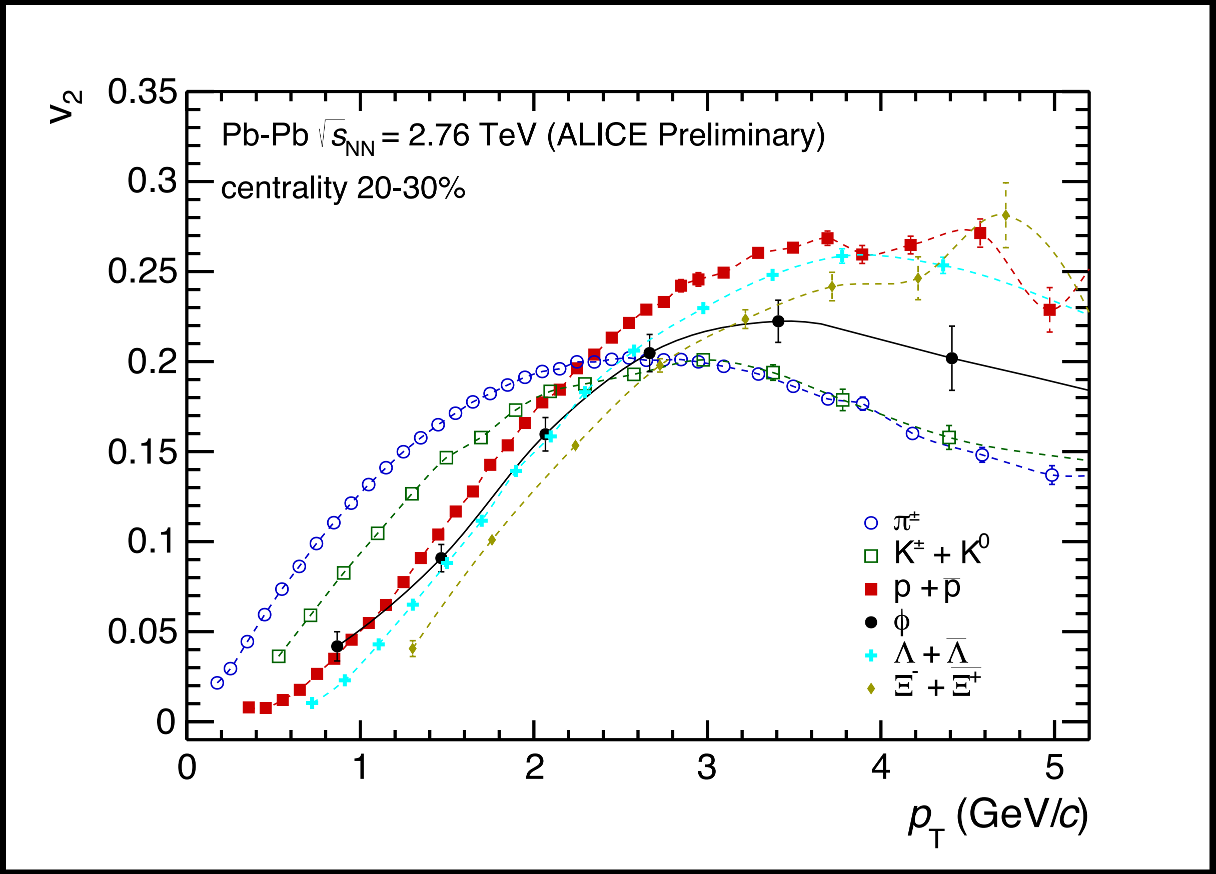


Individual hadronic cross sections matter

The ϕ-meson calculations show clear differences (expected because this is put in the model, not a priory clear if this has anything to do with reality)!

ϕ-meson elliptic flow





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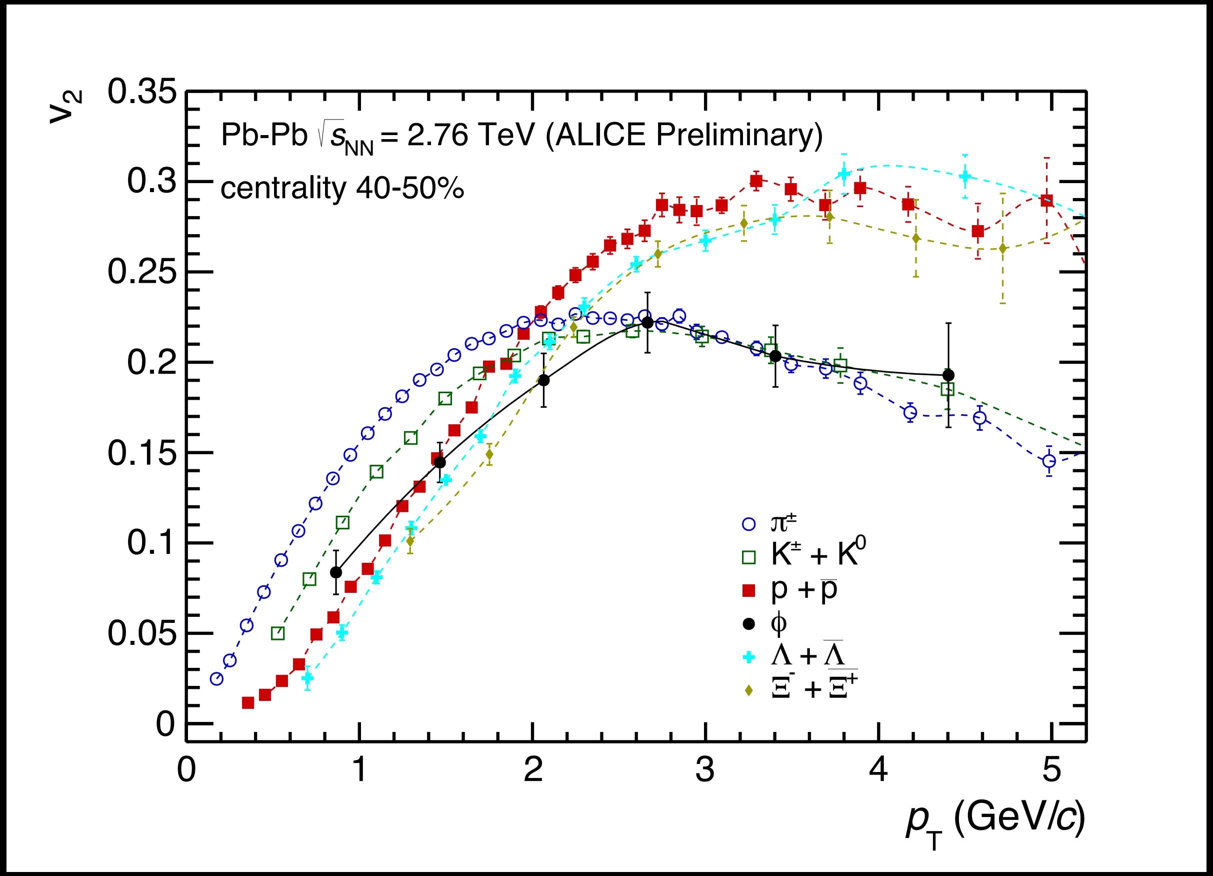
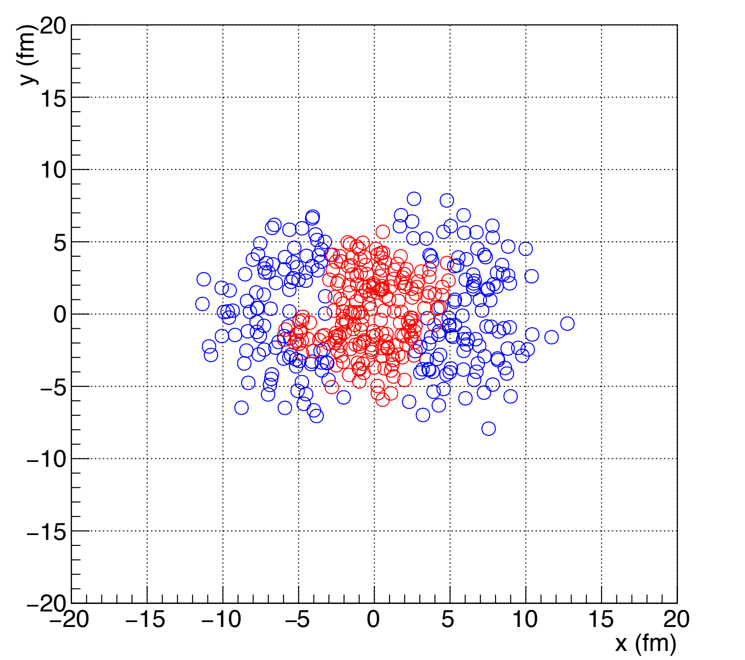
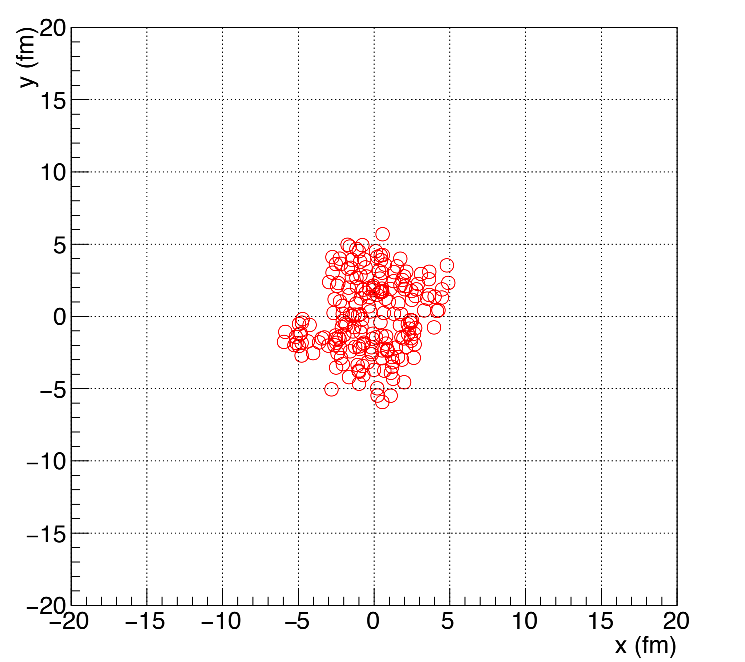
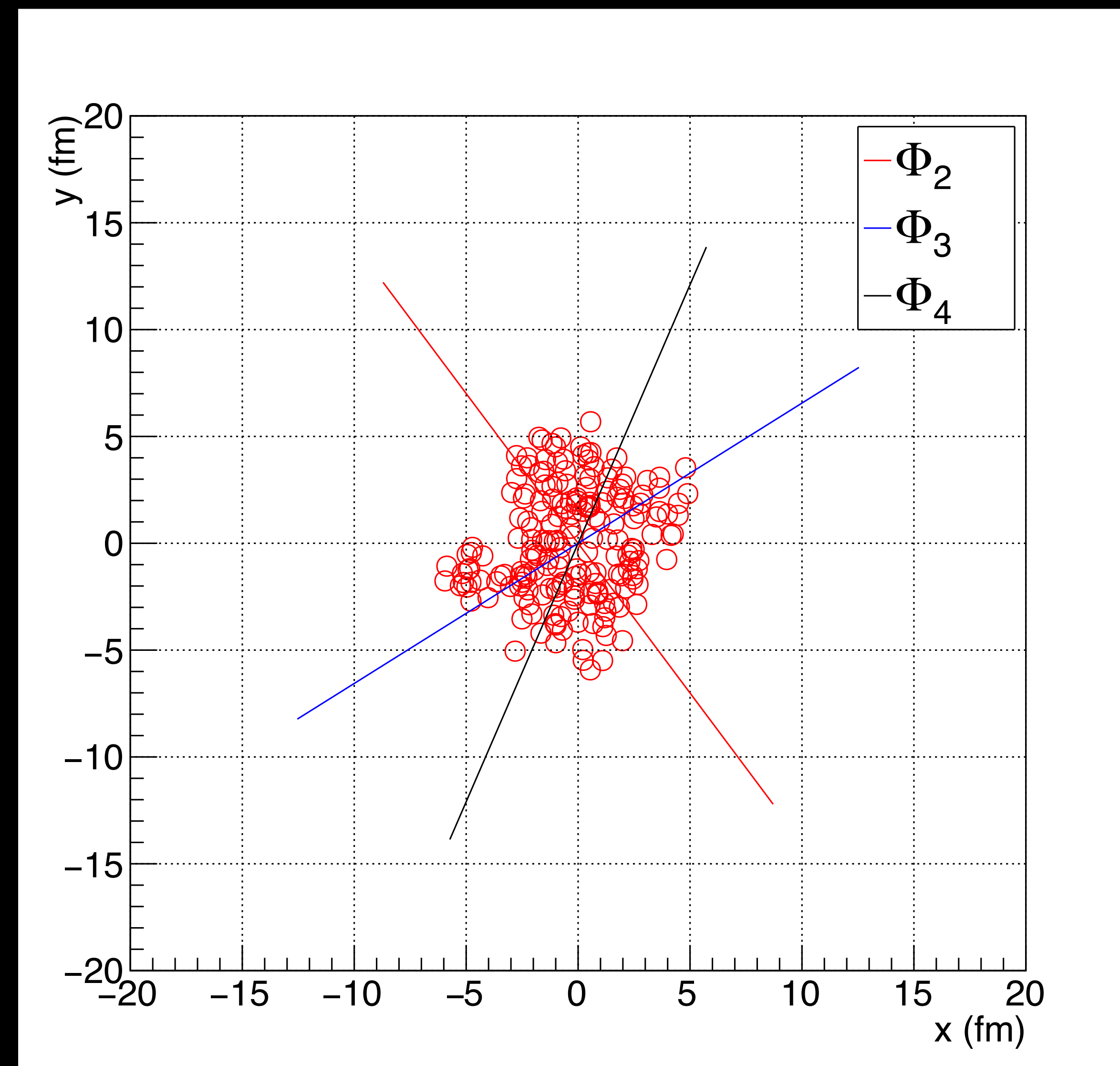


The ϕ-meson might behave differently also in data!

Symmetry Planes



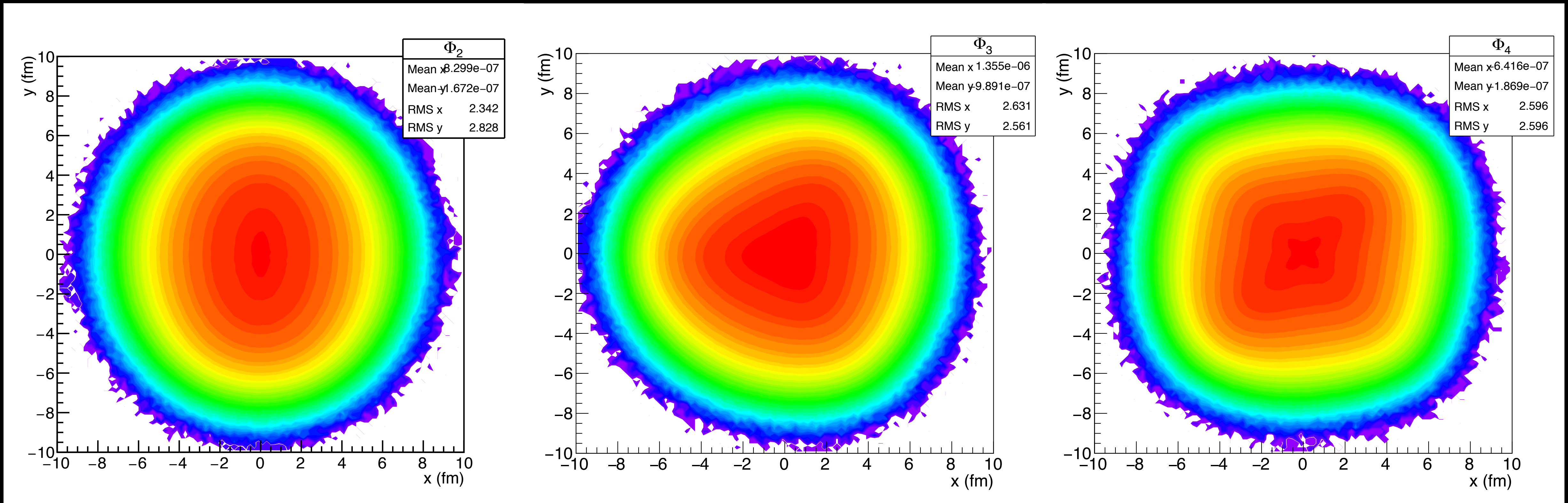




There are many more symmetry planes

$$v_n \propto \varepsilon_n$$

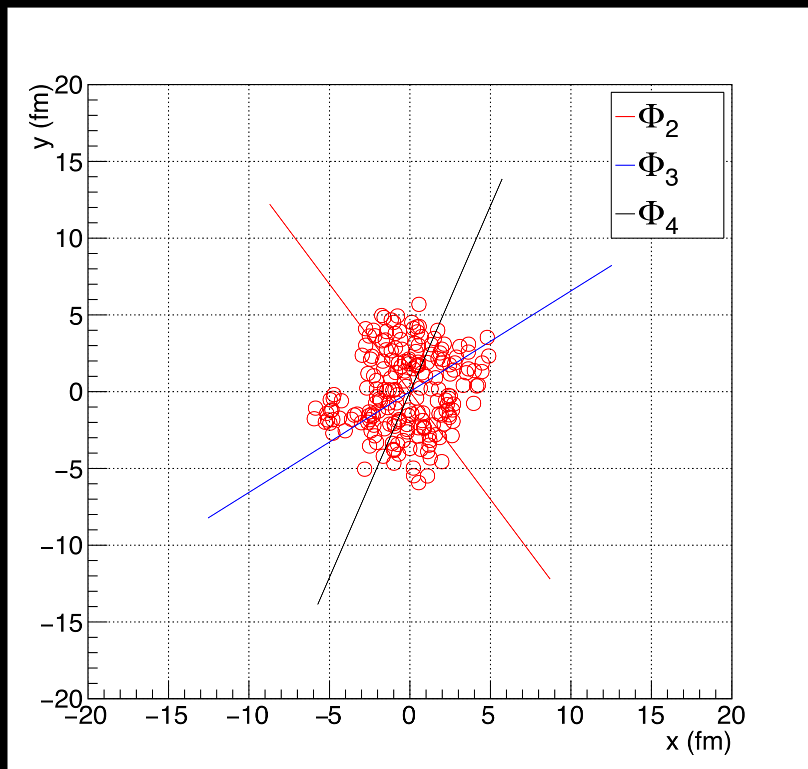
Symmetry Planes



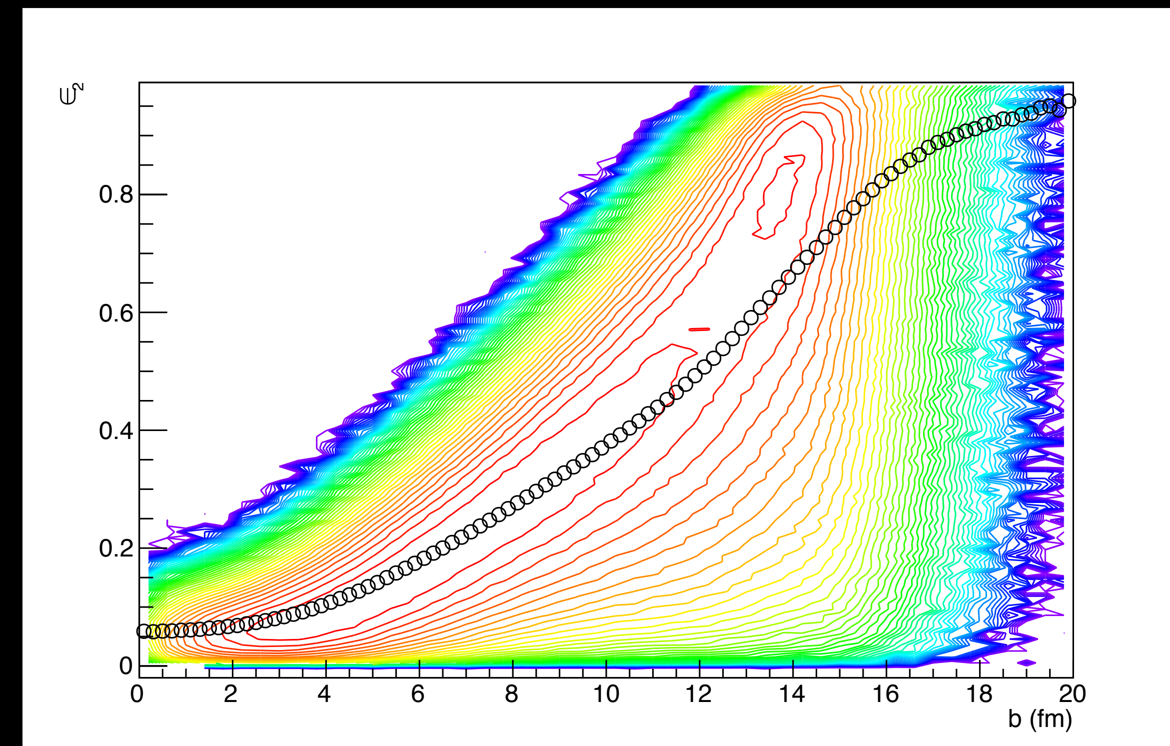
rotated to the planes of symmetry we clearly see the different harmonics

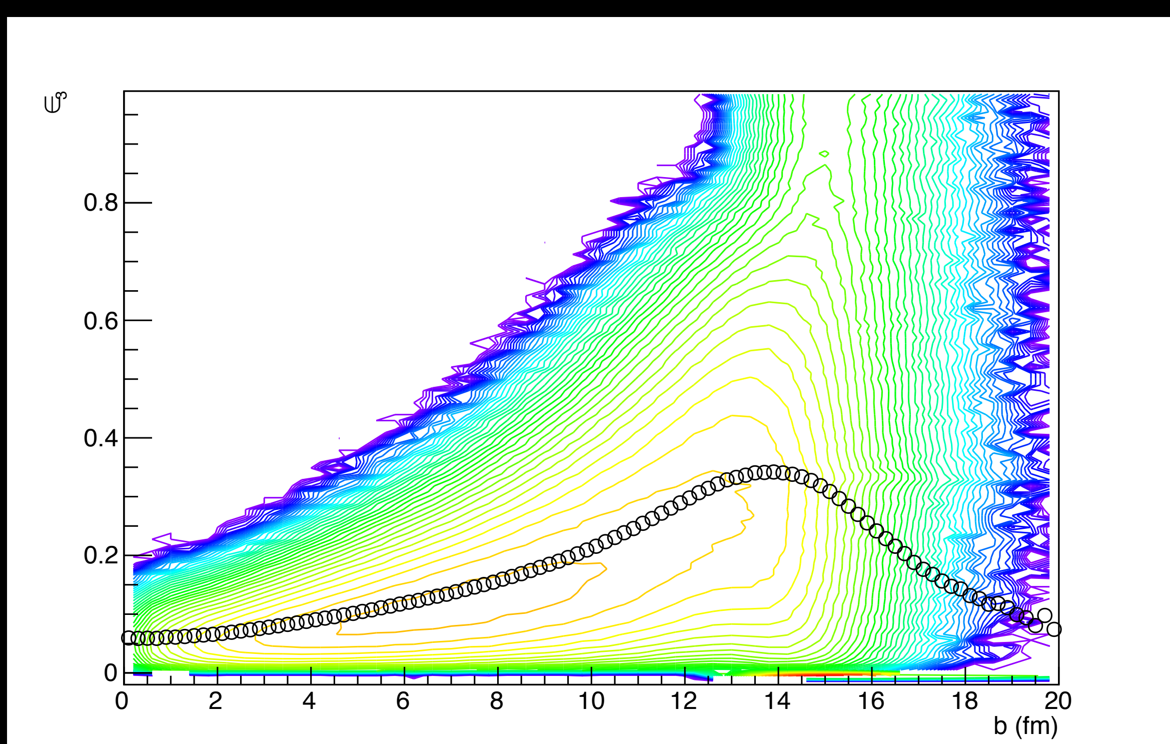
$$v_n \propto \varepsilon_n$$

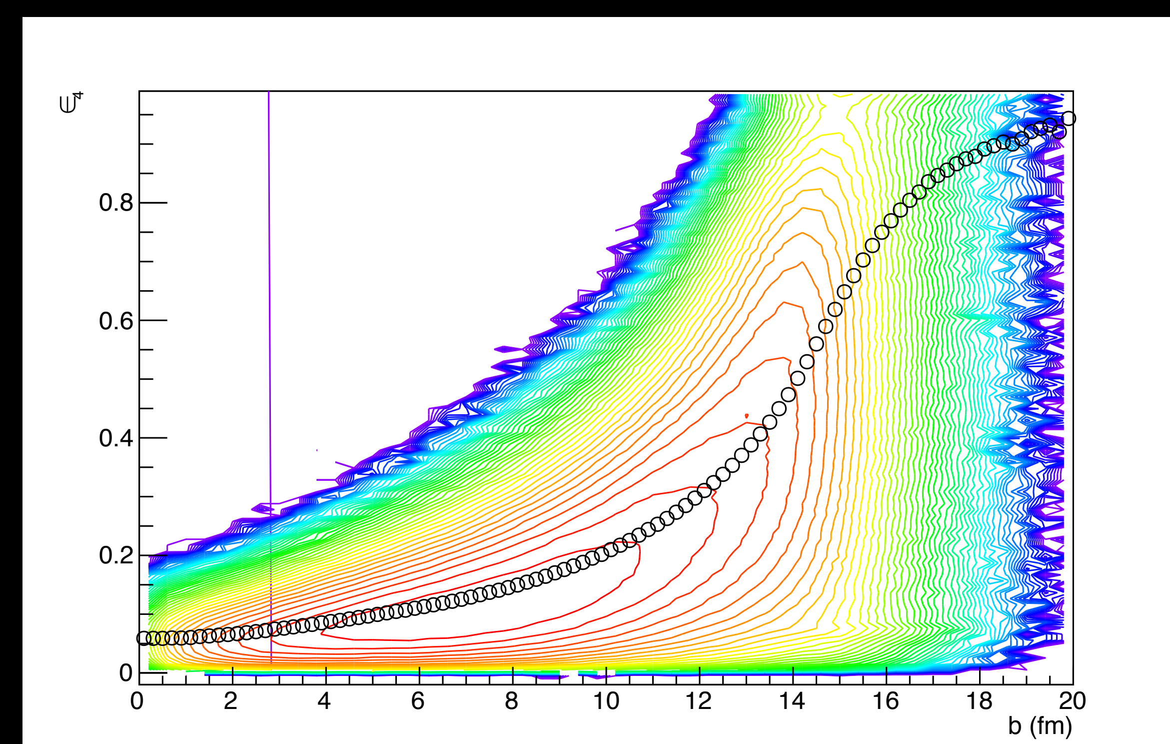
Eccentricities



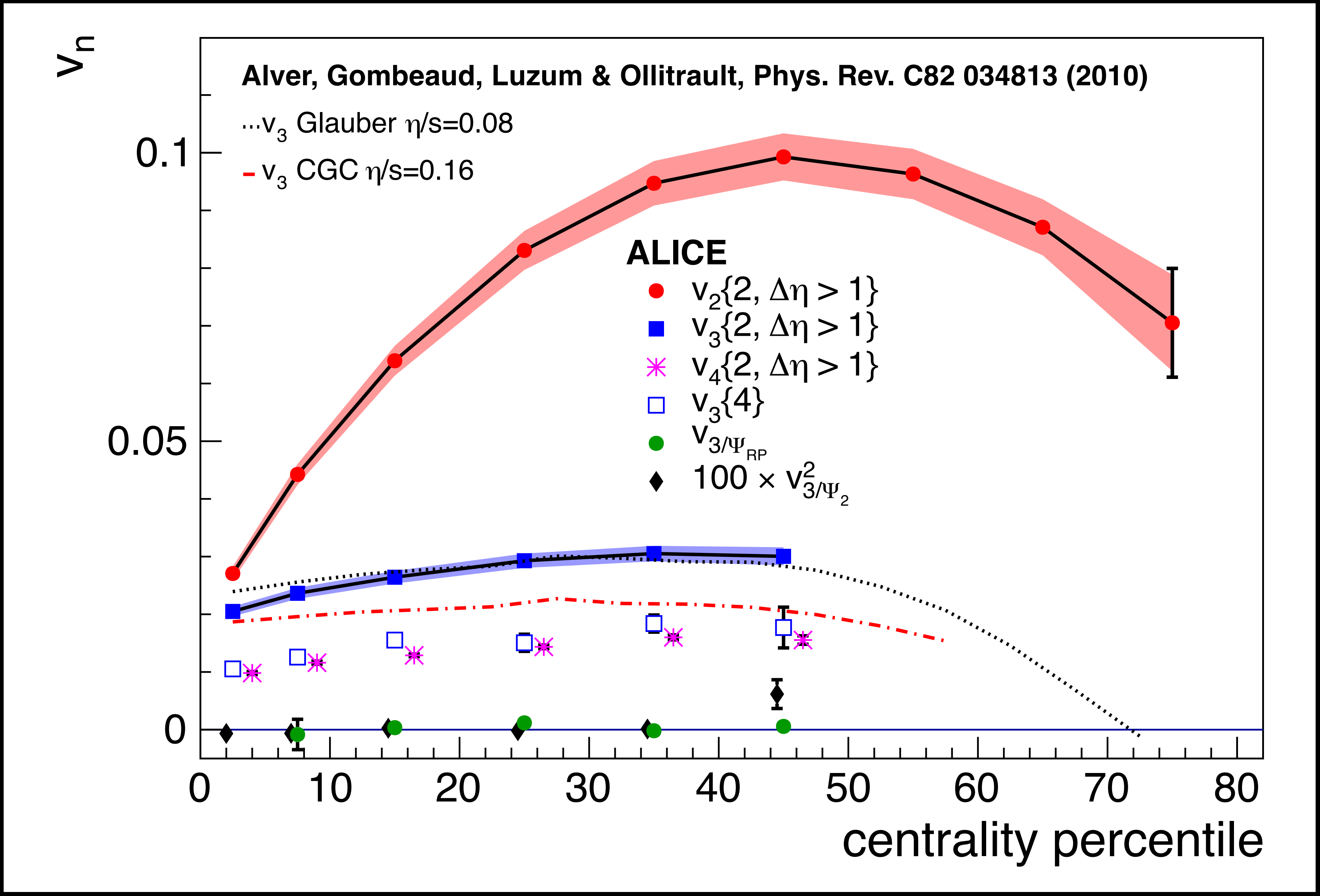
$v_n \propto \varepsilon_n$



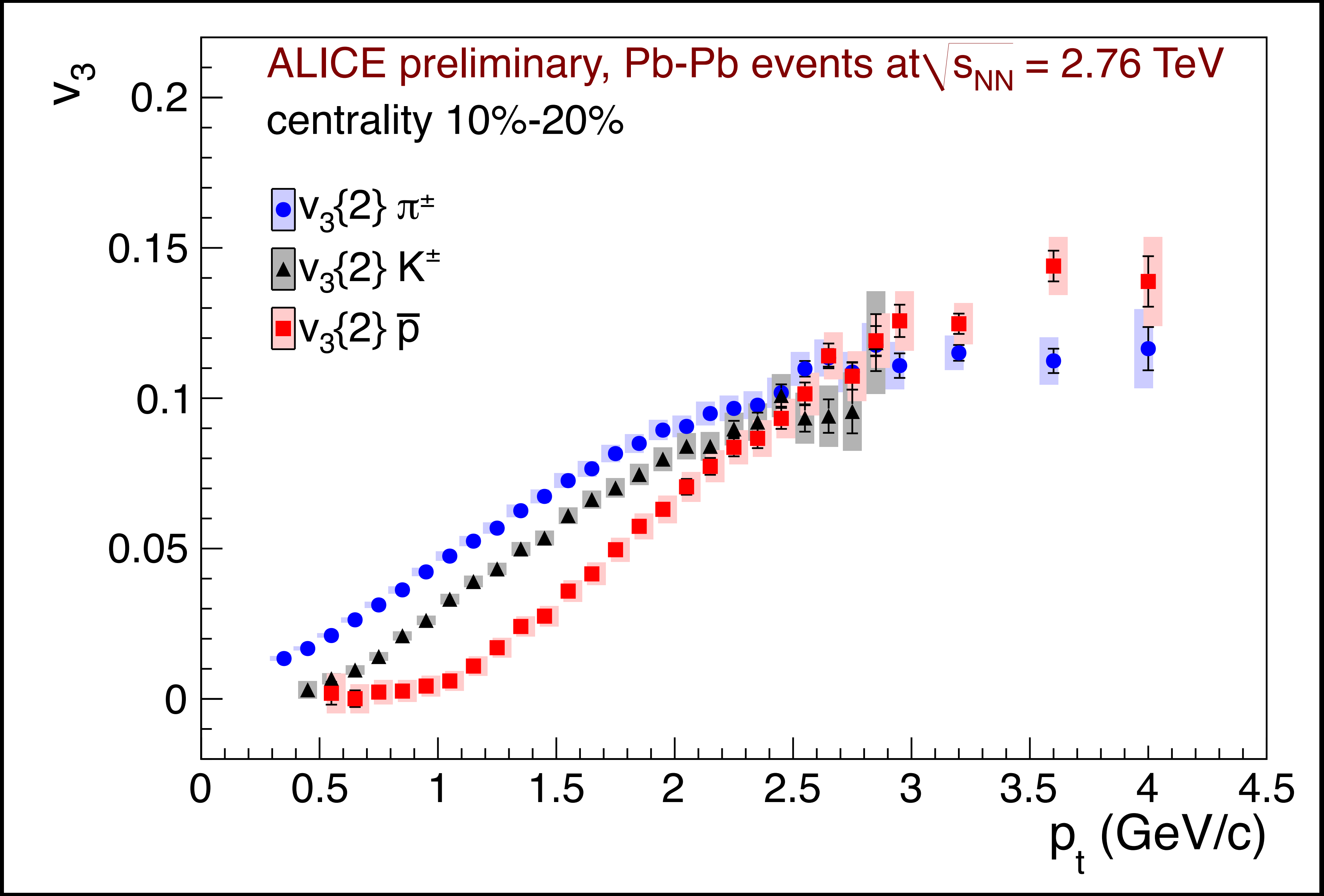




Higher harmonics



Higher harmonics



the mass ordering is also observed for higher harmonics

What do we measure?

We do not know the reaction plane Ψ_R or in more general Ψ_n

$$v_n \equiv \langle e^{in(\varphi - \Psi_n)} \rangle$$

We can calculate these observables only using correlations

$$\langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle = \langle\langle e^{in(\varphi_1)} \rangle\rangle \langle\langle e^{in(\varphi_2)} \rangle\rangle + \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle_c \rangle$$

zero for symmetric detector when averaged over many events

$$\begin{aligned} \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle &= \langle\langle e^{in(\varphi_1 - \Psi_n - (\varphi_2 - \Psi_n))} \rangle\rangle \\ &= \langle\langle e^{in(\varphi_1 - \Psi_n)} \rangle \langle e^{-in(\varphi_2 - \Psi_n)} \rangle\rangle \\ &= \langle v_n^2 \rangle \end{aligned}$$

when only Ψ_n correlations are present

What do we measure?

Build cumulants with multi-particle correlations (Ollitrault and Borghini, 2000)

$$
\begin{aligned}
c_n\{2\} &\equiv \left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = v_n^2 + \delta_2 \\
c_n\{4\} &\equiv \left\langle\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\right\rangle - 2\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle^2 \\
&= v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 - 2(v_n^2+\delta_2)^2 \\
&= -v_n^4
\end{aligned}
$$

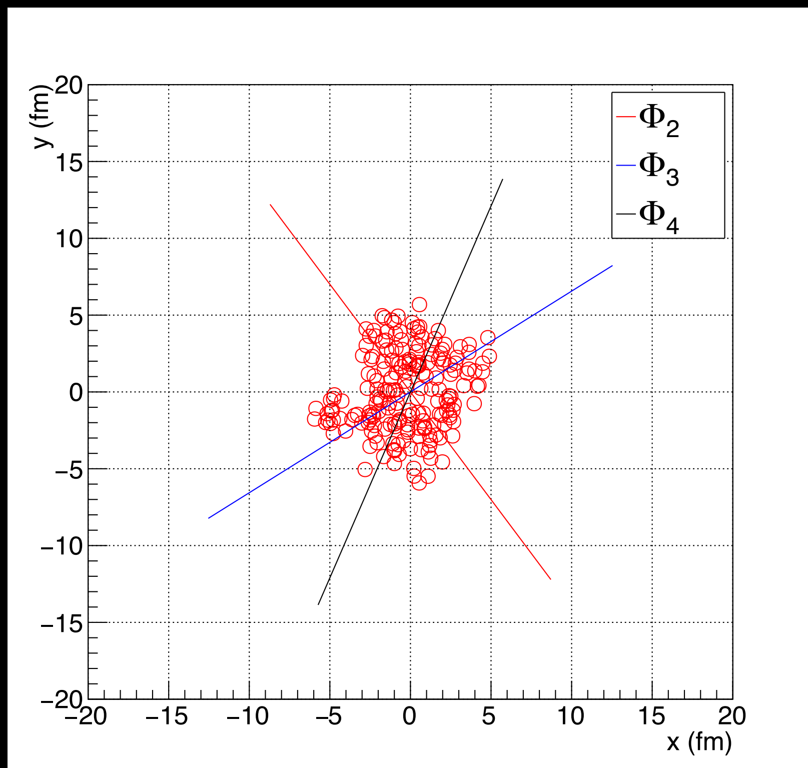
got rid of 2-particle correlations not related to collective flow however now we measure higher moment moments of the distribution

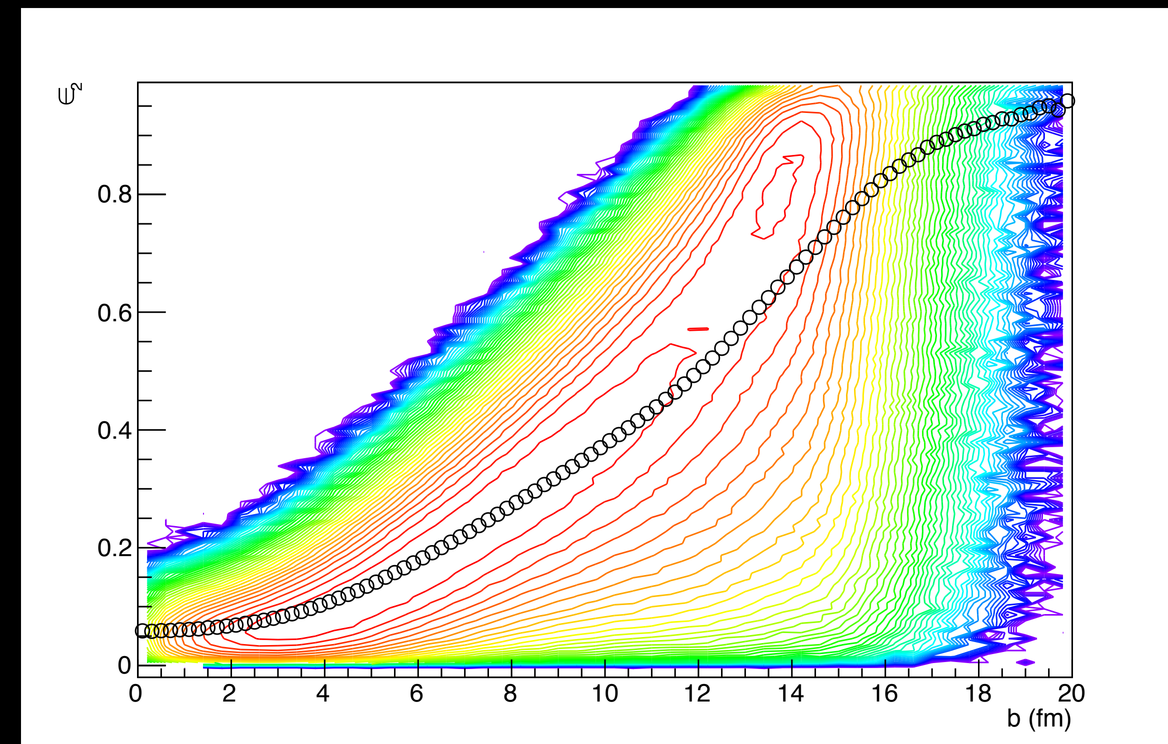
What do we measure?

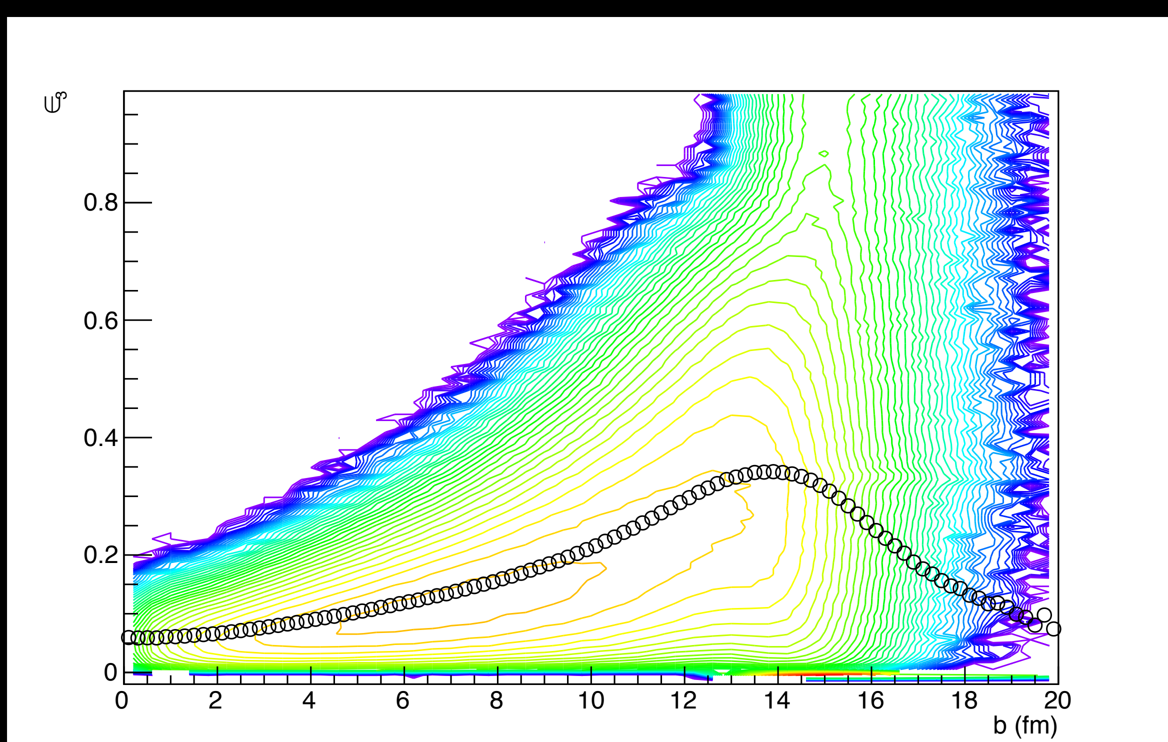
if the fluctuations are small we can say for any distributions that the various flow estimates follow:

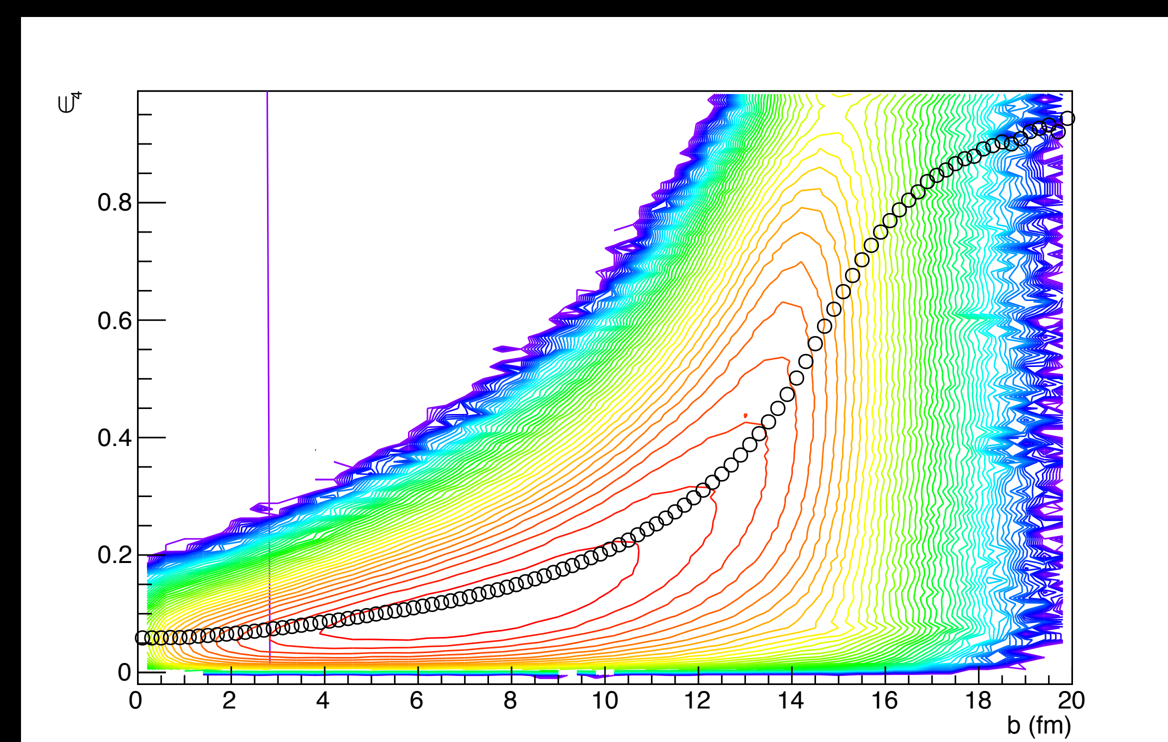
$$
\begin{aligned}
v\{2\} &= \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\
v\{4\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\
v\{6\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\
v\{8\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}
\end{aligned}
$$

Fluctuations

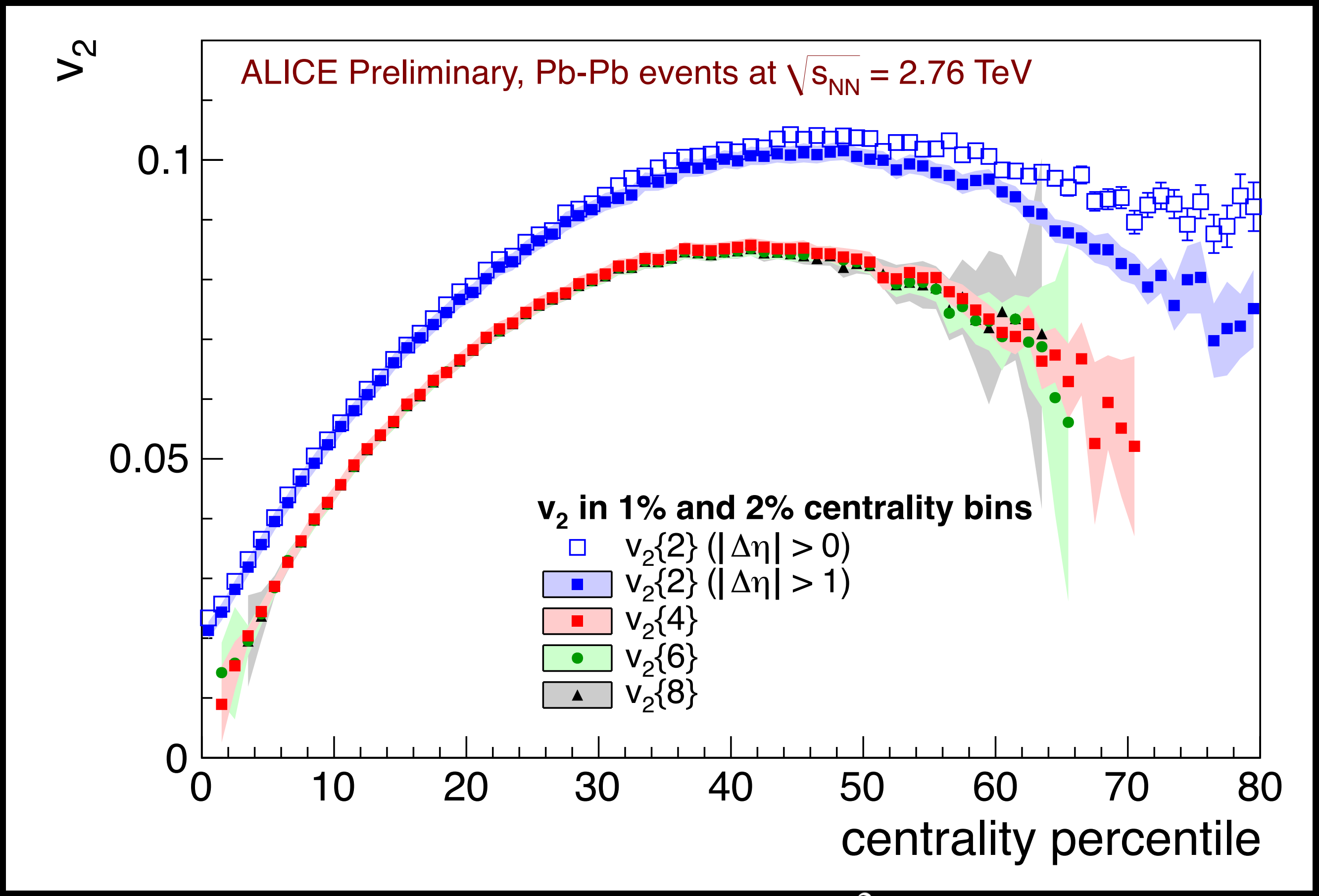


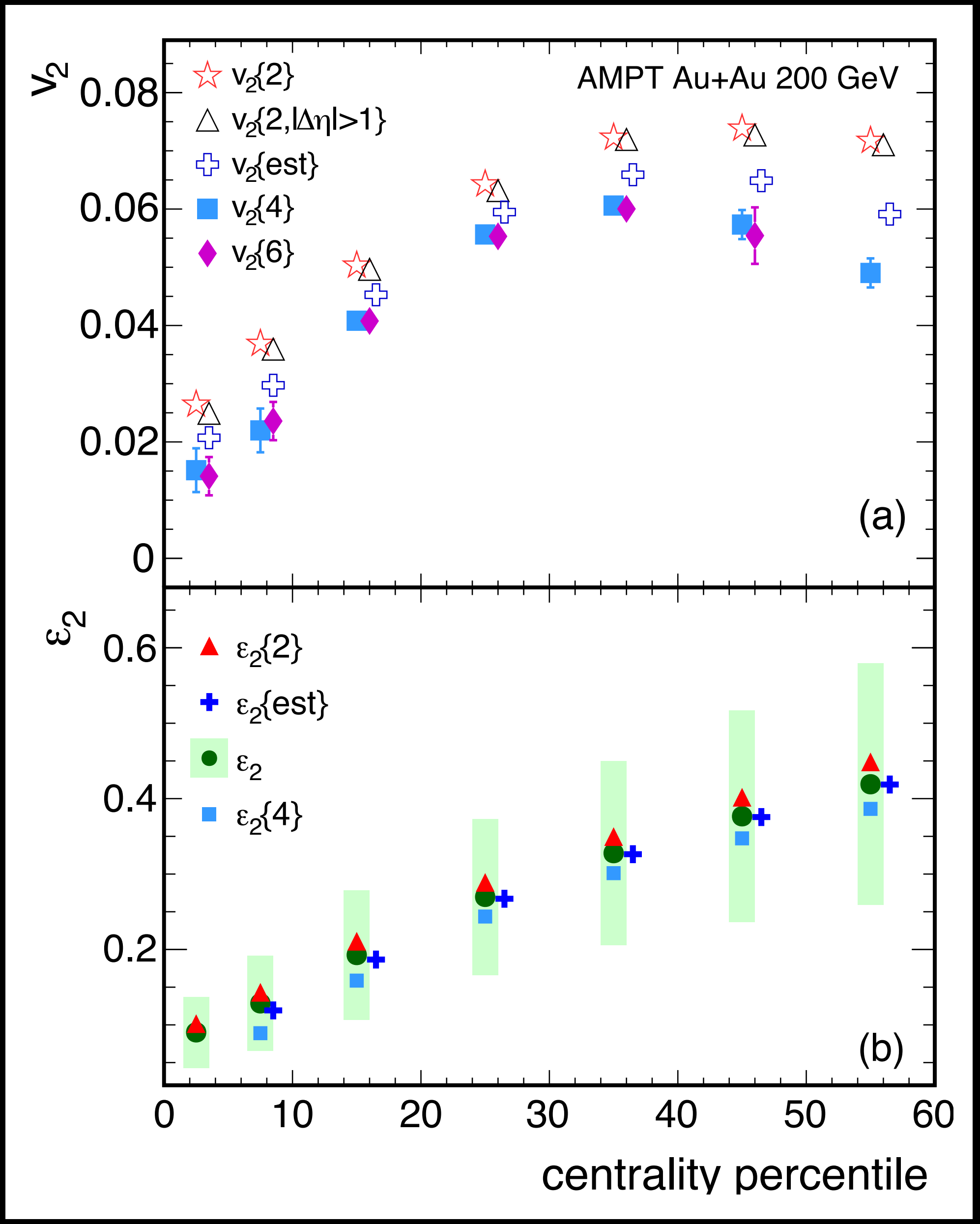






Fluctuations





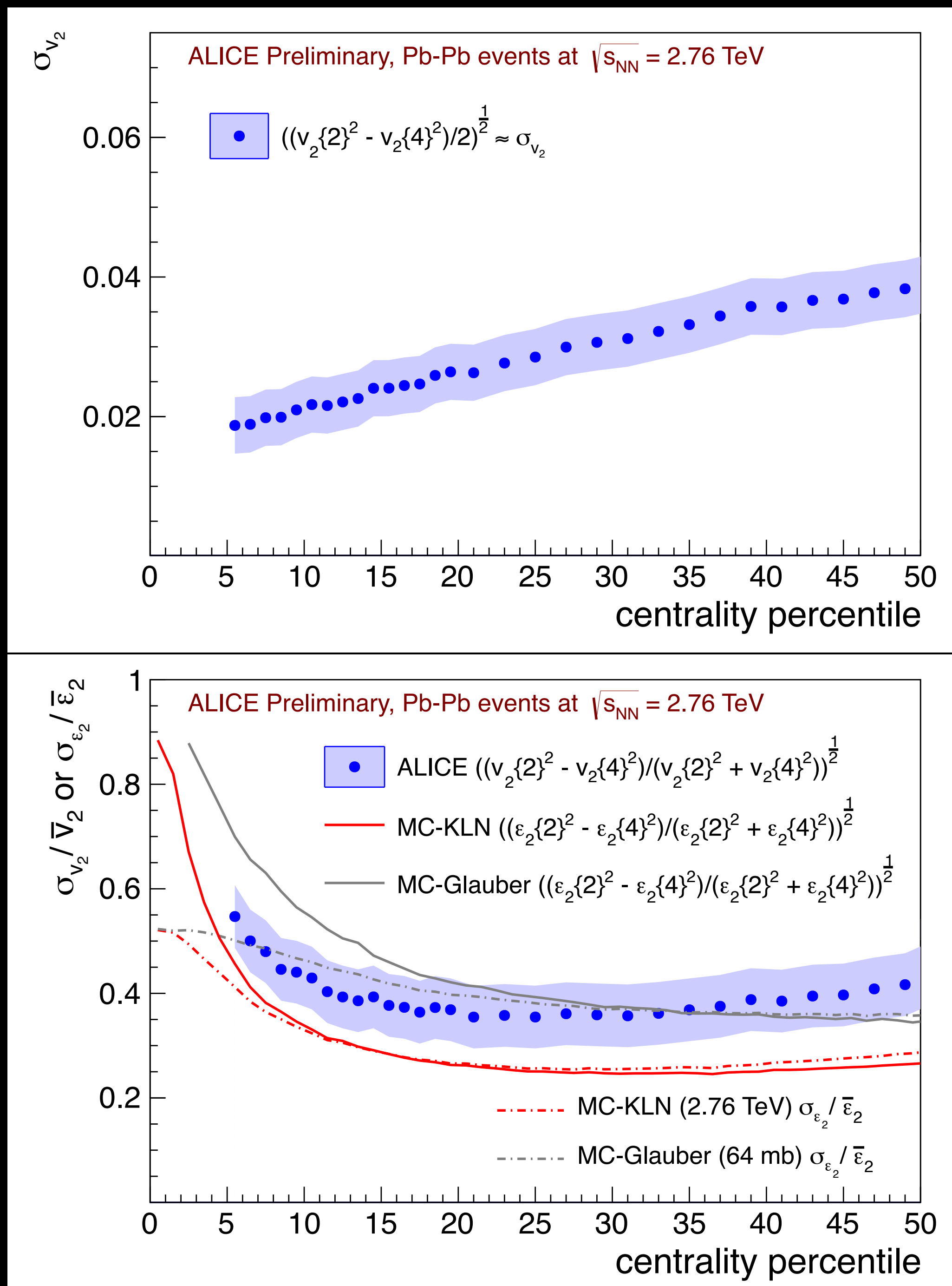
$$v\{2\} = \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

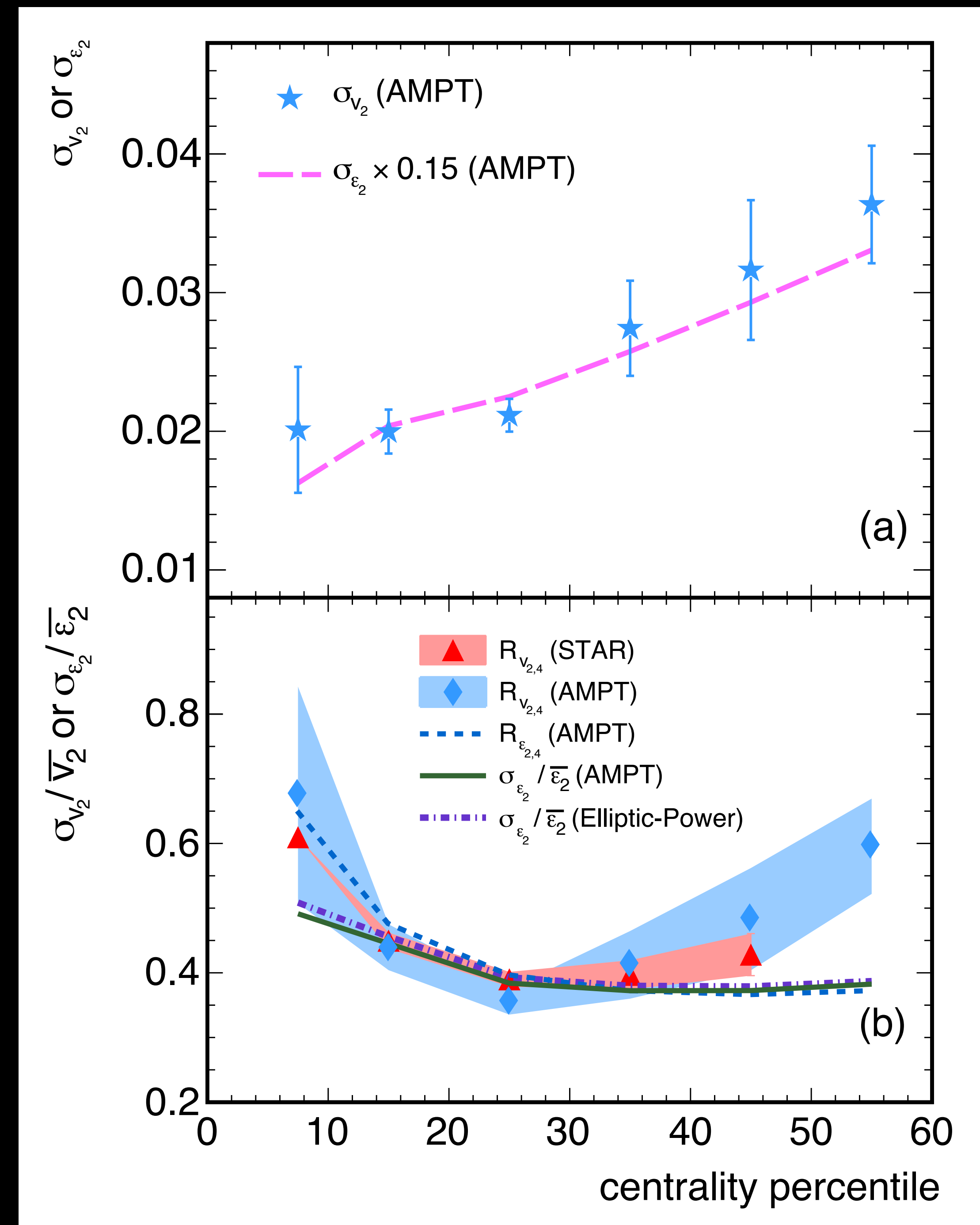
$$v\{4\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

$$v\{6\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

$$v\{8\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

Fluctuations





however the fluctuations are rather large

What do we measure?

- small fluctuations easy but not much info
- need the underlying pdf!

Fluctuations

Bessel-Gaussian

$$p(\varepsilon_n) = \frac{\varepsilon_n}{\sigma^2} I_0\left(\frac{\varepsilon_n \varepsilon_n}{\sigma^2}\right) \exp\left(-\frac{\varepsilon_0^2 + \varepsilon_n^2}{2\sigma^2}\right)$$

ε_0 is the anisotropy versus the reaction plane and σ the fluctuations.
Works for mid-central collisions, not expected to work for peripheral collisions because not constraint to 1
this distribution predict that $v_3\{4\}=0$

Power-law distribution

$$p(\varepsilon_n) = 2\alpha\, \varepsilon_n \left(1 - \varepsilon_n^2\right)^{\alpha - 1}$$

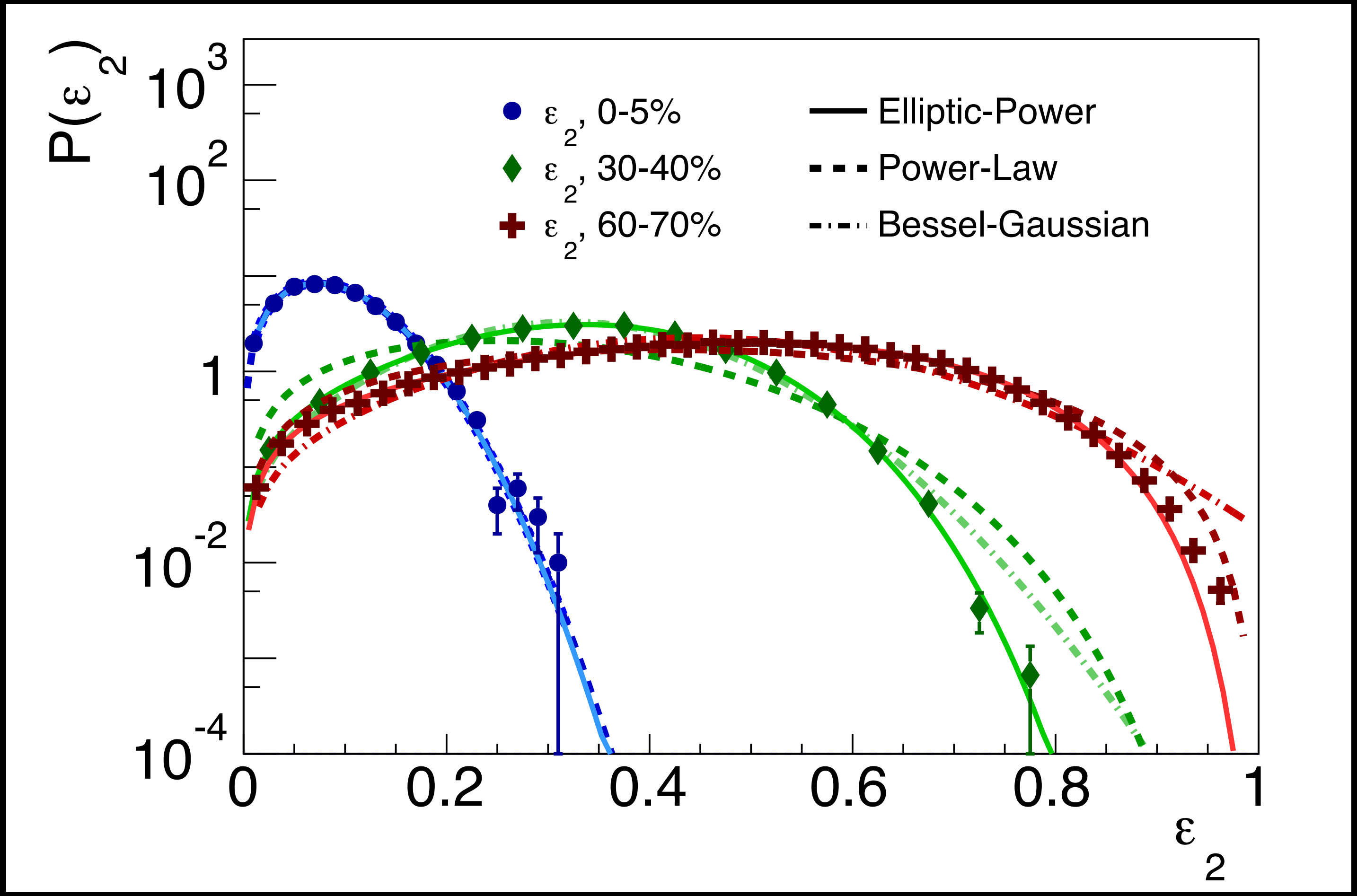
α quantifies the fluctuations, this function has no ε_0 therefore only describes the response due to fluctuations

Elliptic Power distribution

$$p(\varepsilon_n) = \frac{\alpha\, \varepsilon_n}{\pi} \left(1 - \varepsilon_0^2\right)^{\alpha + \frac{1}{2}} \int_0^{2\pi} \frac{\left(1 - \varepsilon_n^2\right)^{\alpha - 1} \mathrm{d}\phi}{\left(1 - \varepsilon_0\, \varepsilon_n \cos\phi\right)^{2\alpha + 1}}$$

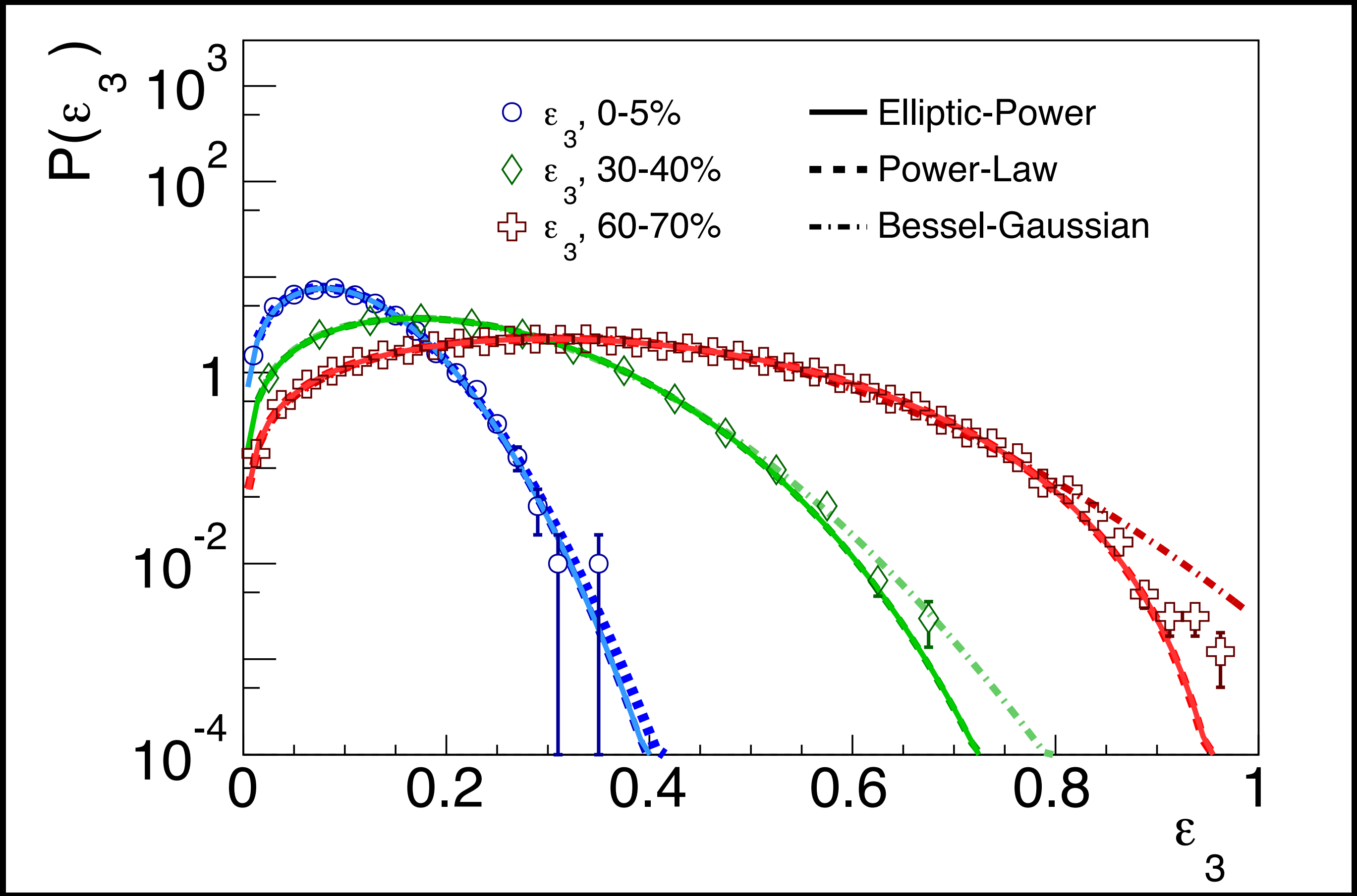
α and ε_0 are the ingredients with same definition as in previous distributions

Fluctuations



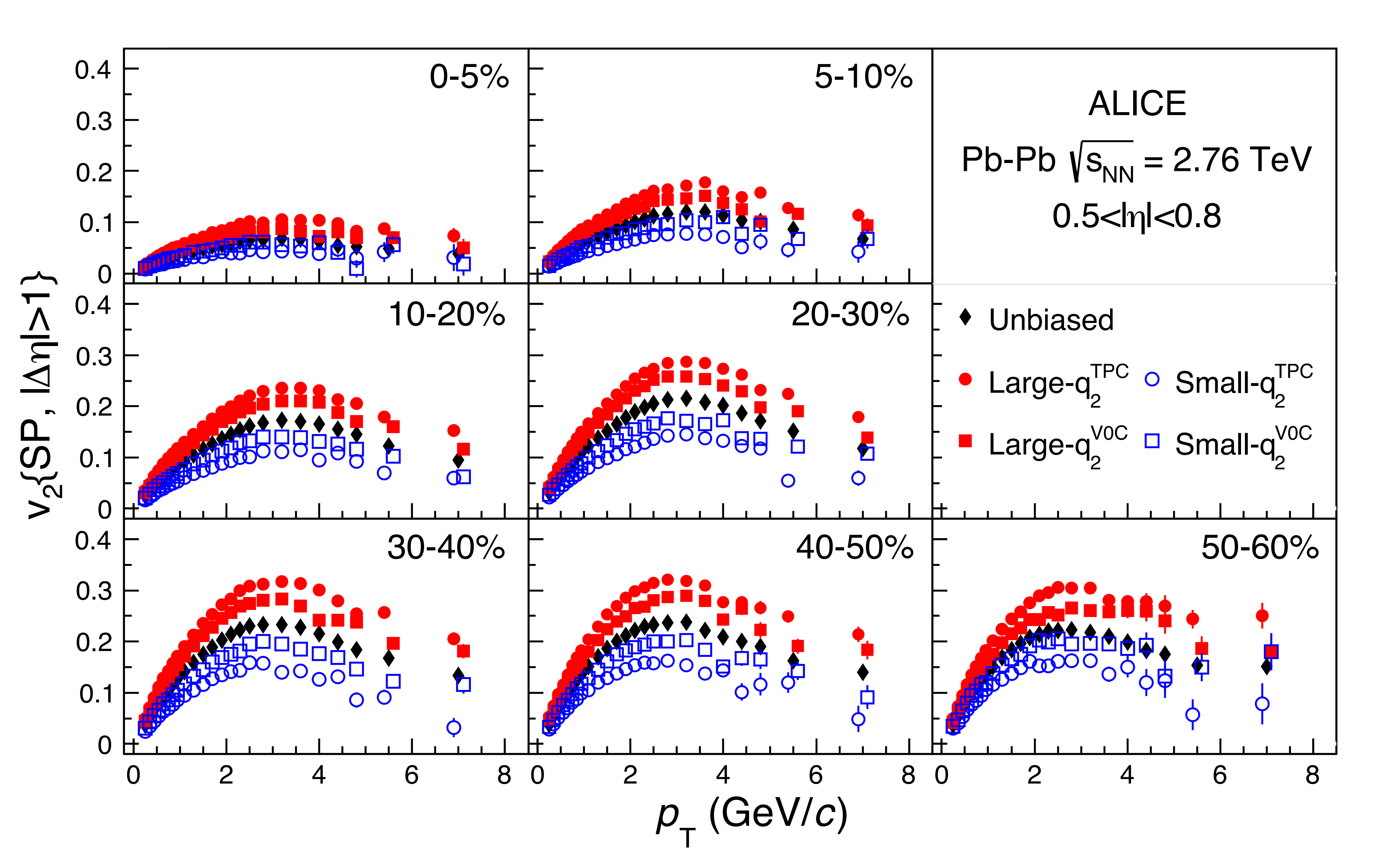
In 0-5% all three functions work rather well. This is understood, ε_0 is small and α is large. Elliptic Power turns into a Bessel Gaussian and with ε_0 small the anisotropy versus the reaction plane and power law also works. For more peripheral collisions the Elliptic Power is the only distributions which works well

Fluctuations



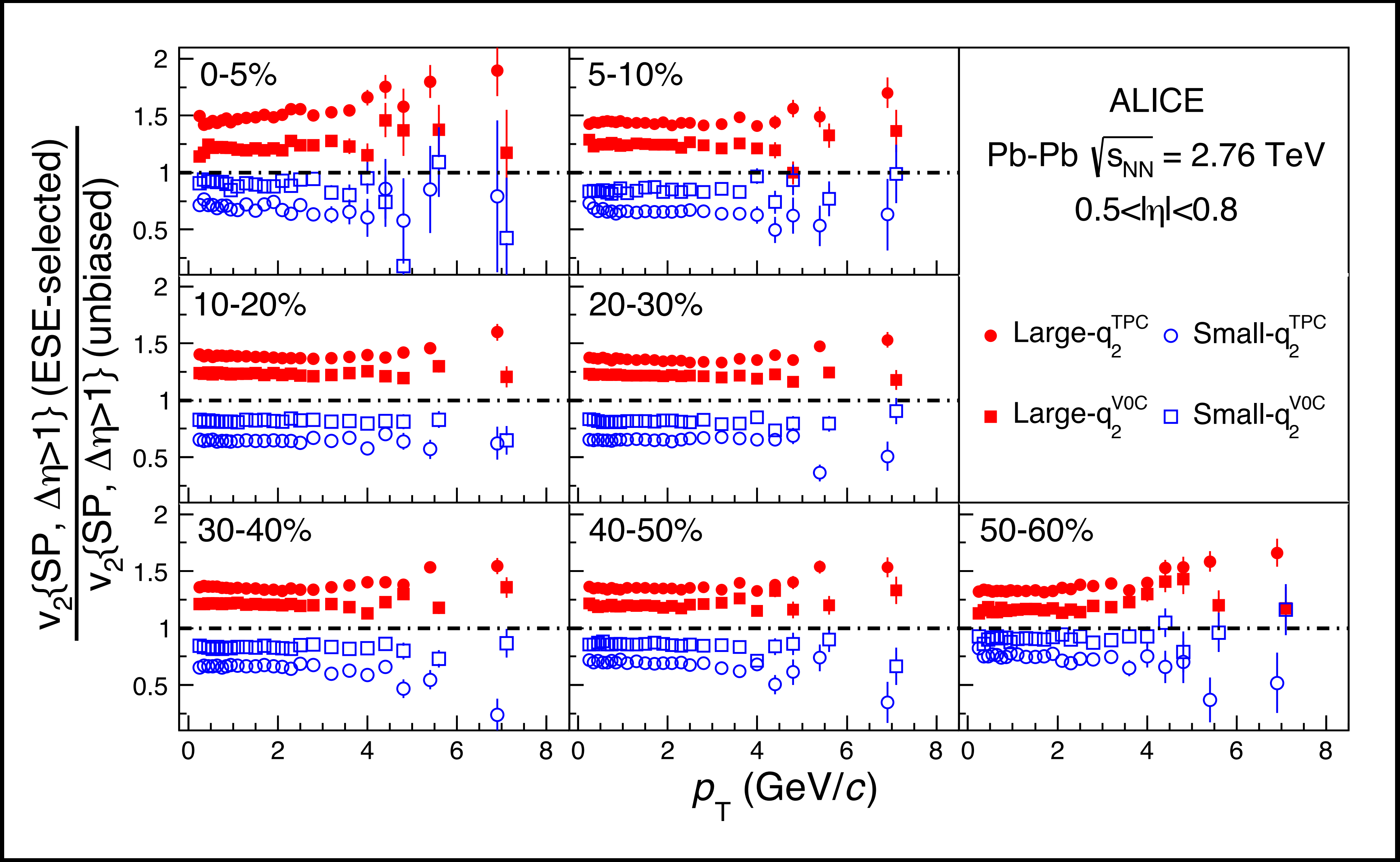
ε_3, v_3 dominated by fluctuations. For more central collisions all three functions work rather well. Again this is understood Bessel Gaussian fails for more peripheral due to lack of constraint < 1. The fact that $\varepsilon_3\{4\}$ and $v_3\{4\}$ are non-zero completely excluded the Bessel Gaussian

Event shape engineering

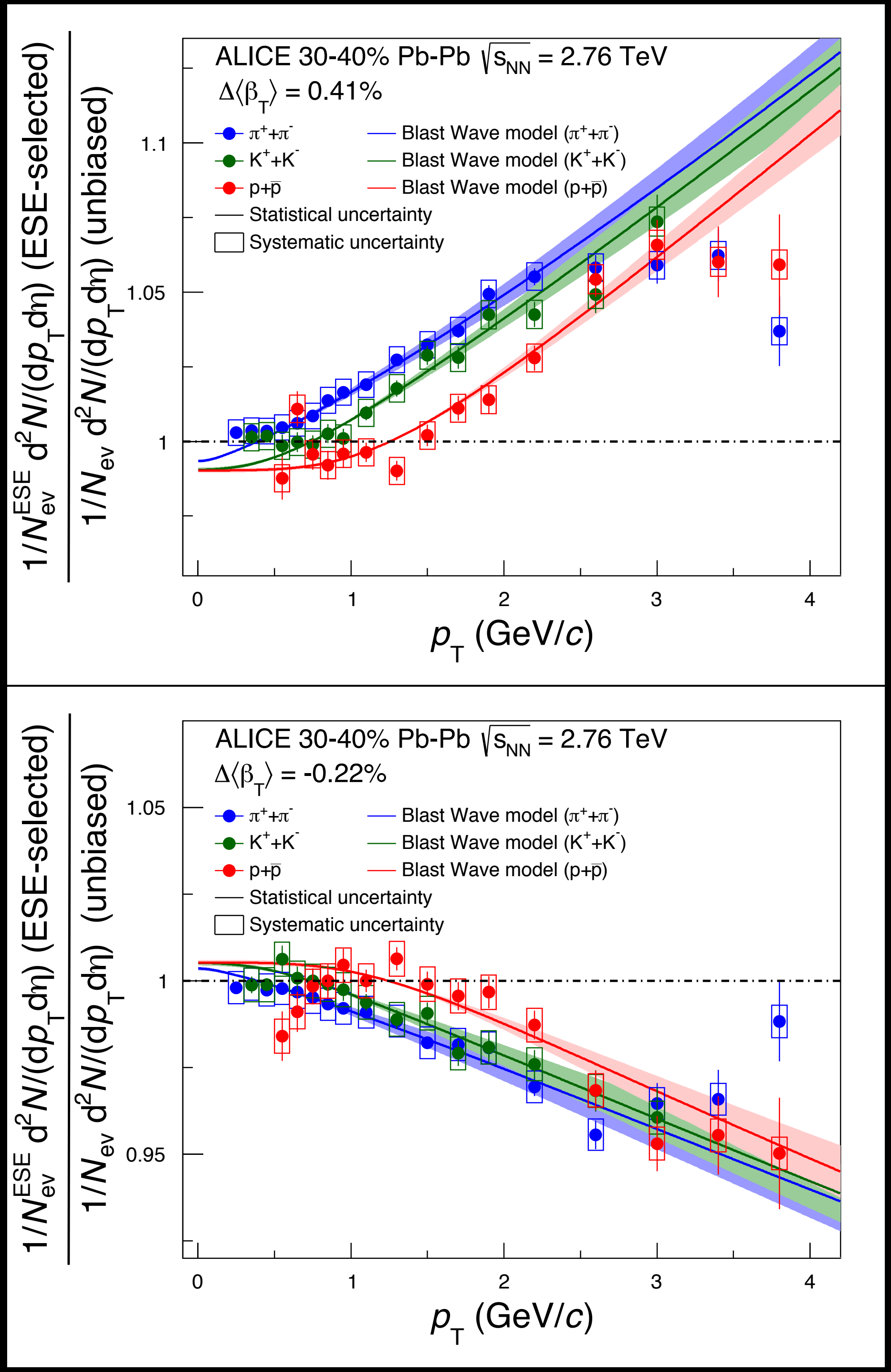


Can use the fluctuations to select events with different shapes at fixed centralities
Perfect tool to test the response of the system outside of our normal selection on centrality and/or collision system

Event shape engineering



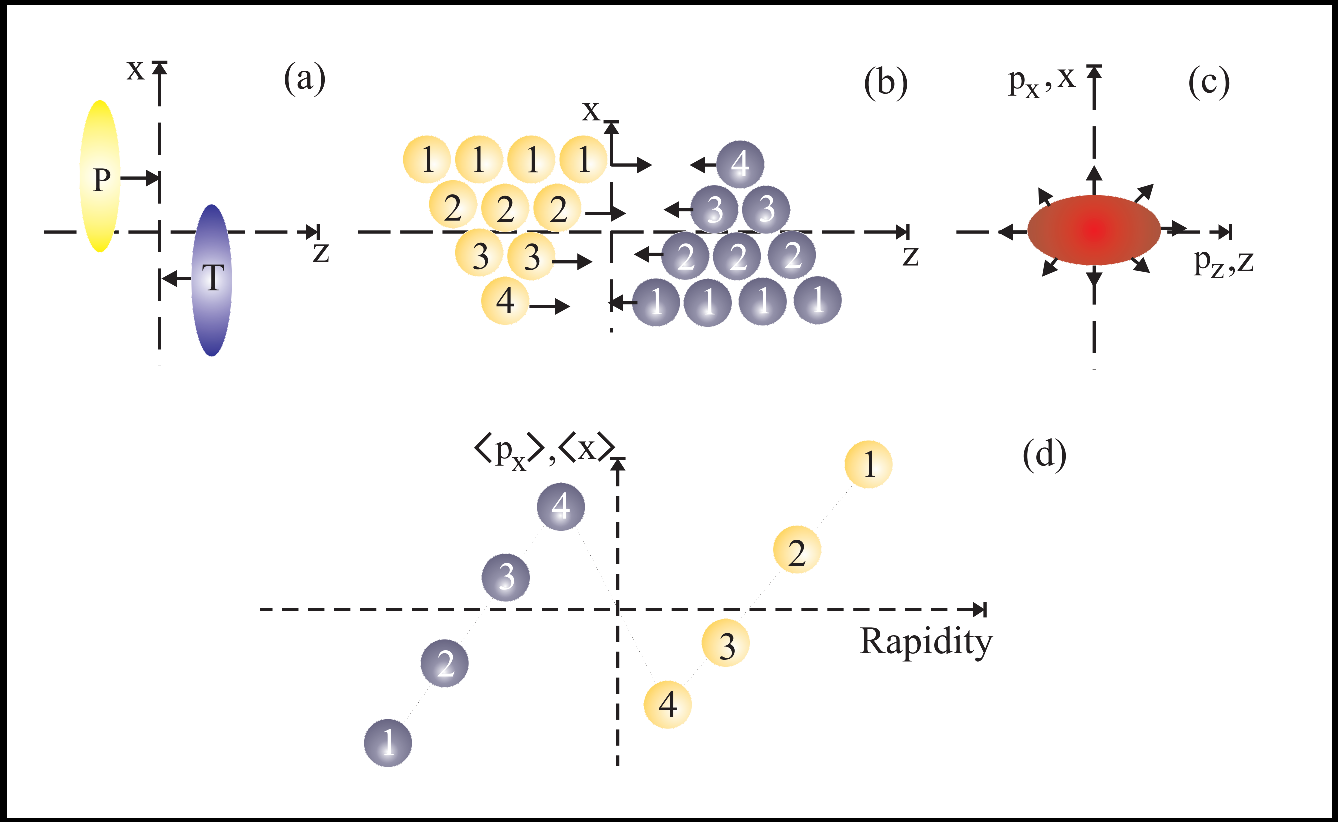
Event shape engineering

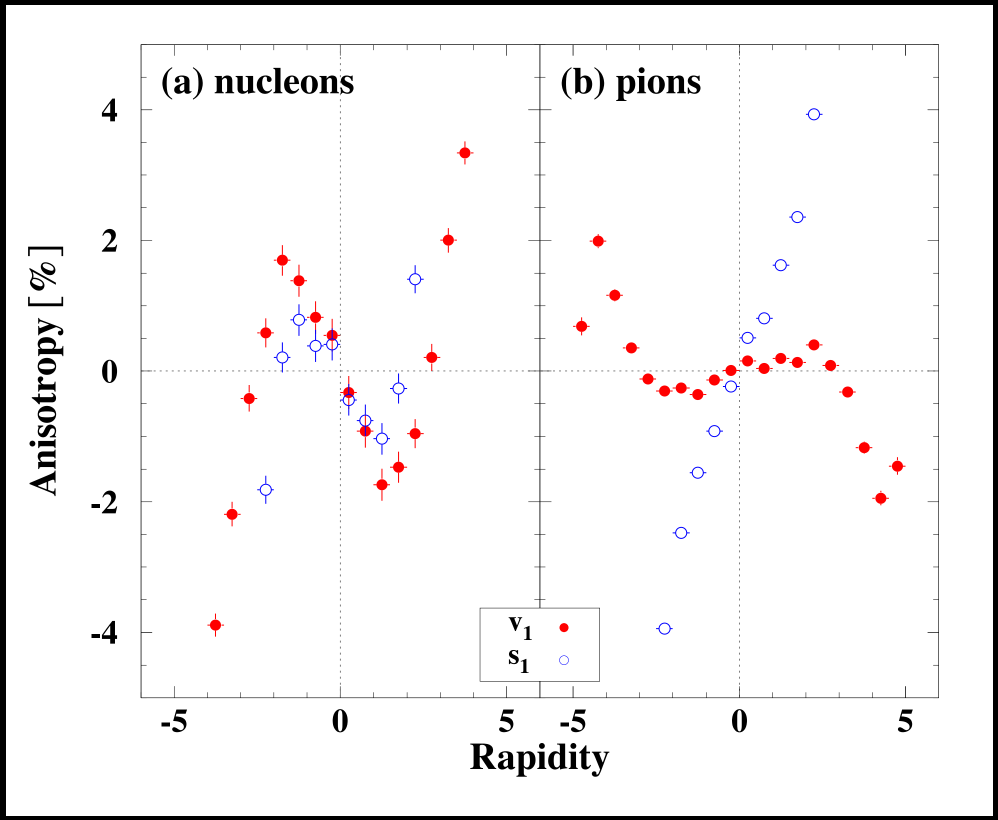


The radial flow indeed also changes for “identical” collisions which only differ in shape
new and starting field!

Rapidity Dependence

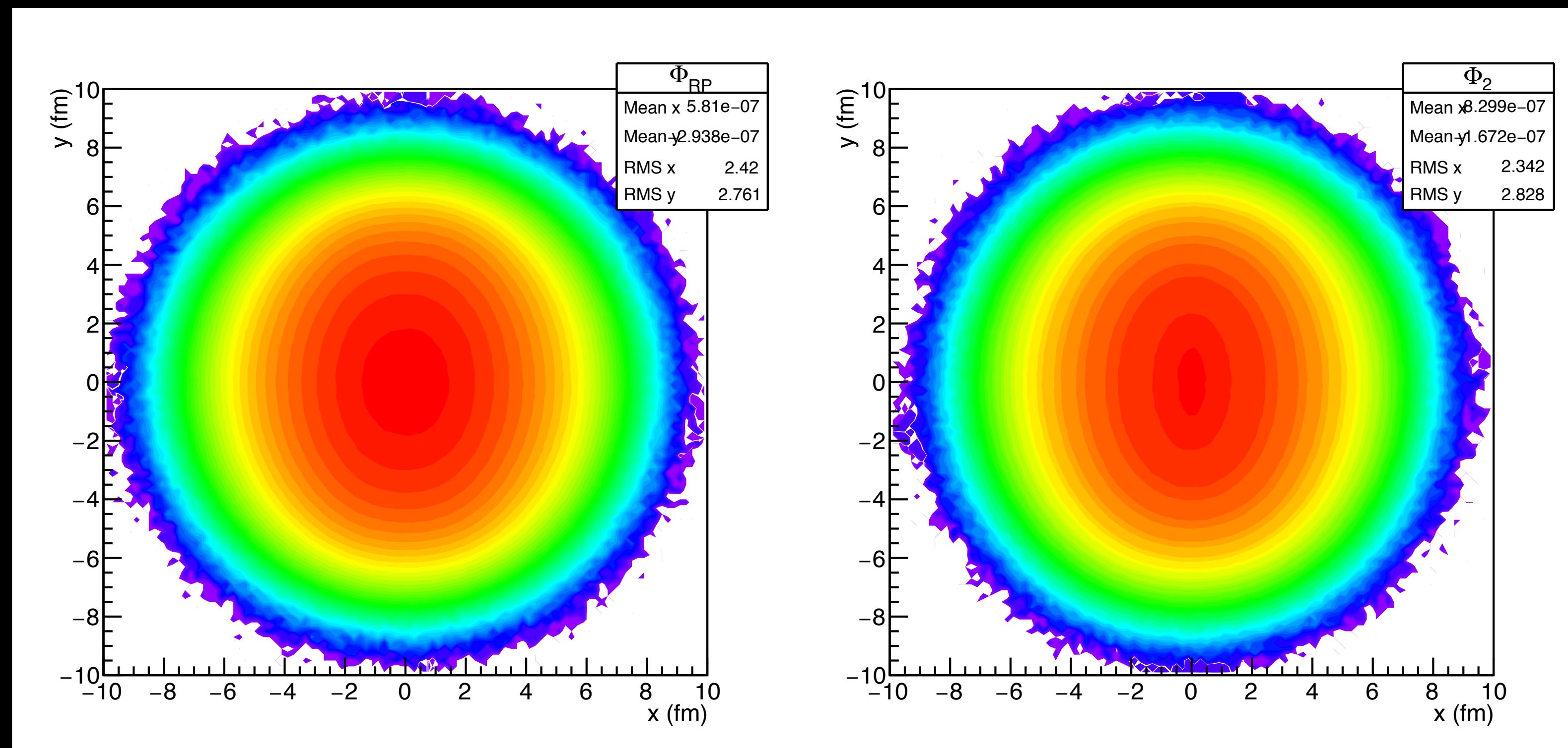
R.S, H. Sorge, S.A. Voloshin, F.Q. Wang, N. XU; PRL 84 (2000)

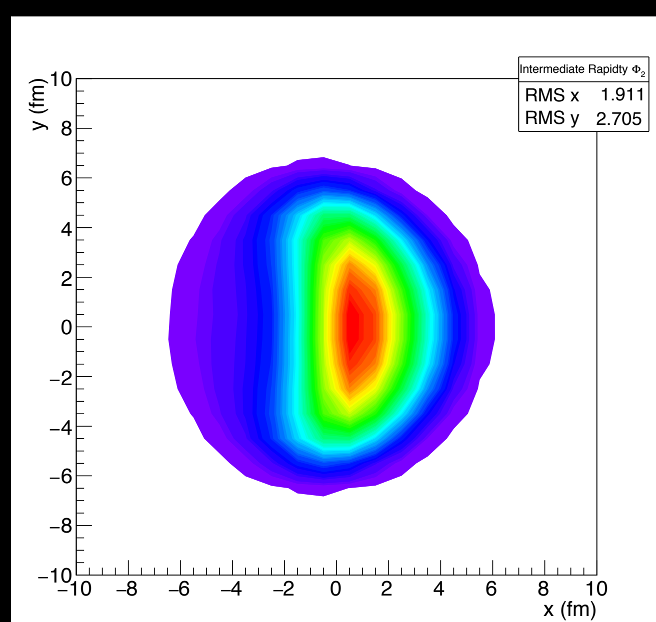


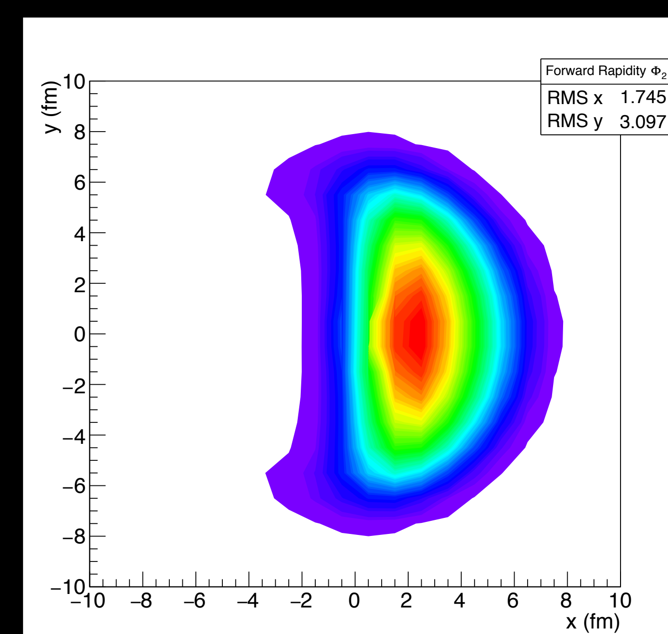


stopping can be important for how the initial spatial distributions looks like

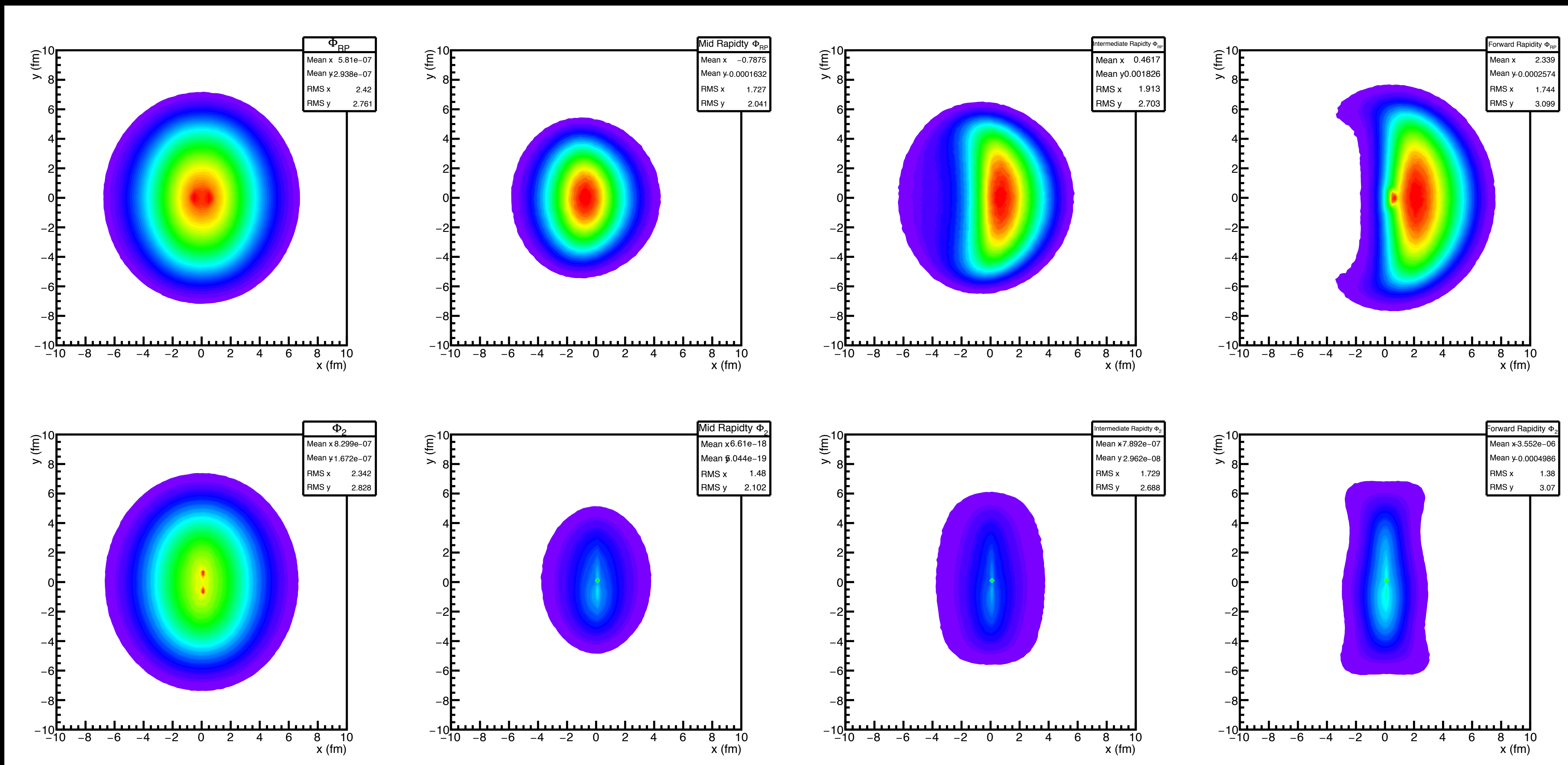
Rapidity Dependence







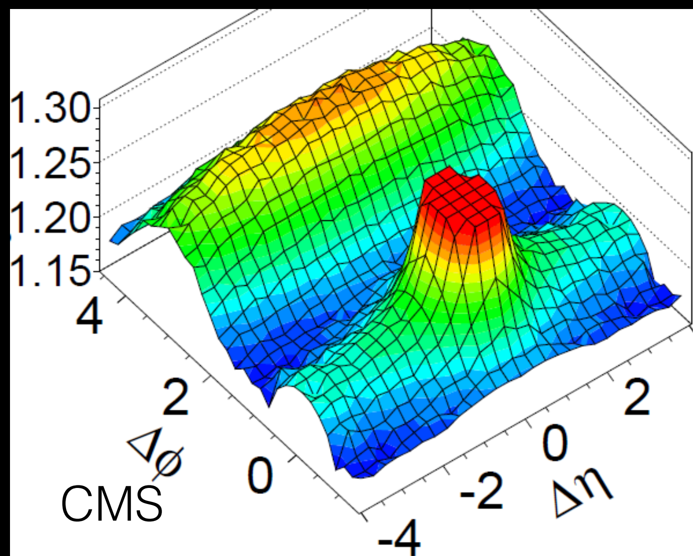
Rapidity Dependence

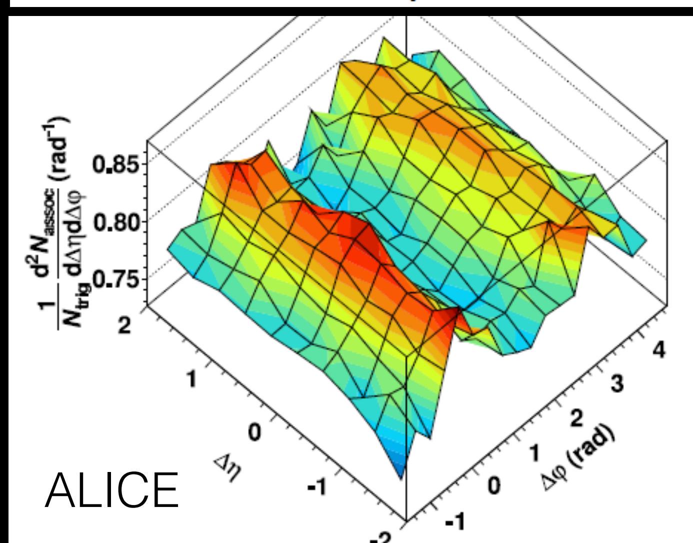


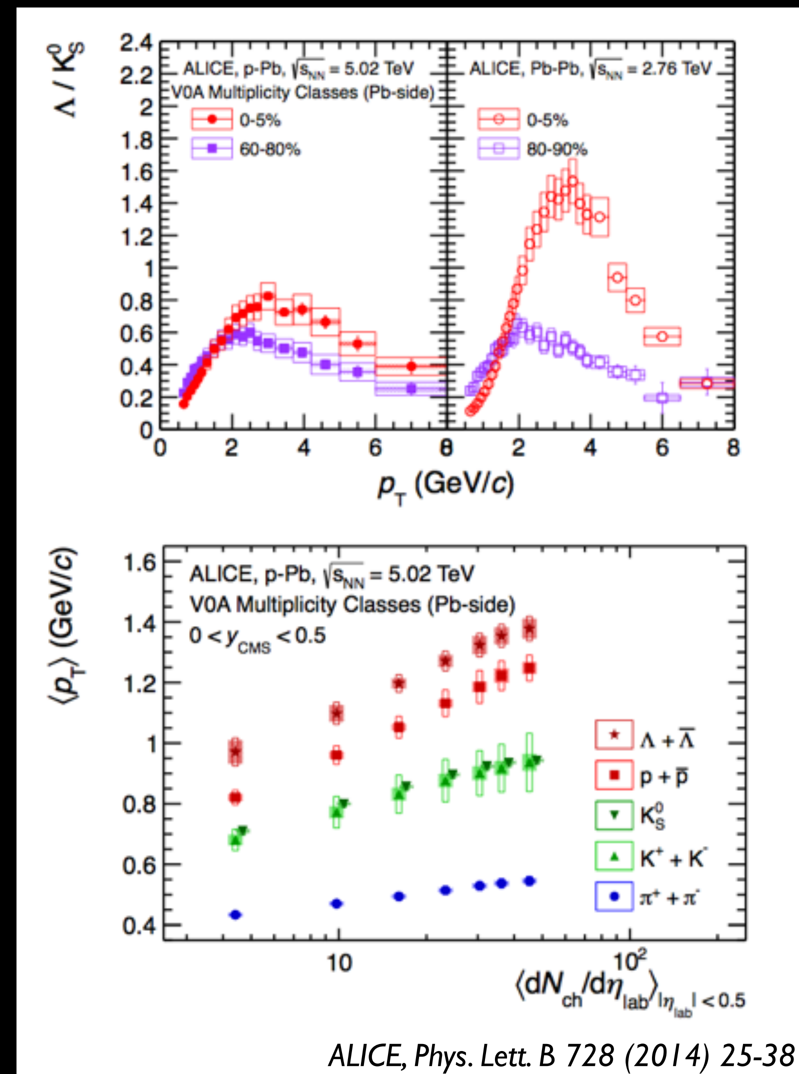
Small systems; pA collisions

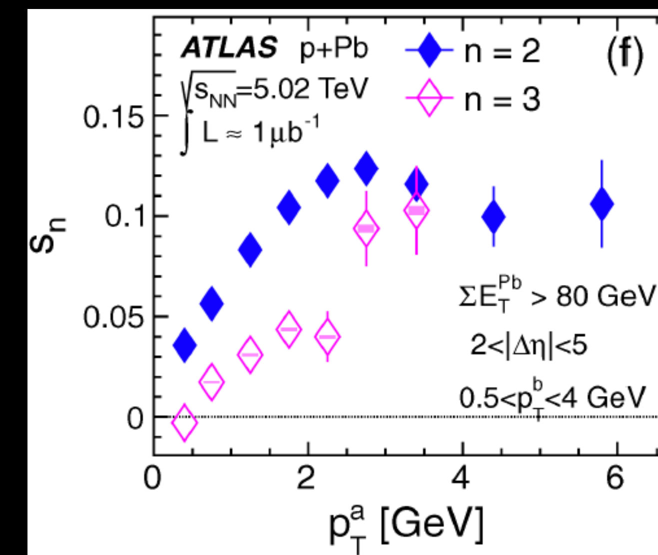
- a reference for AA (cold nuclear matter effects)
- a ideal system for CGC studies

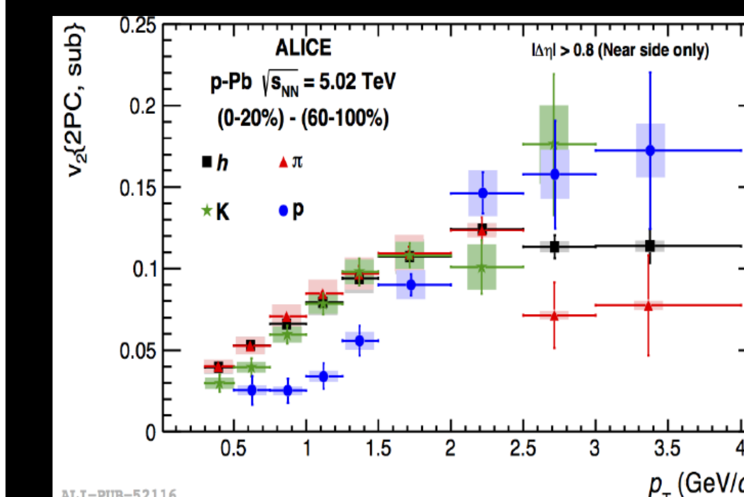
pA collisions

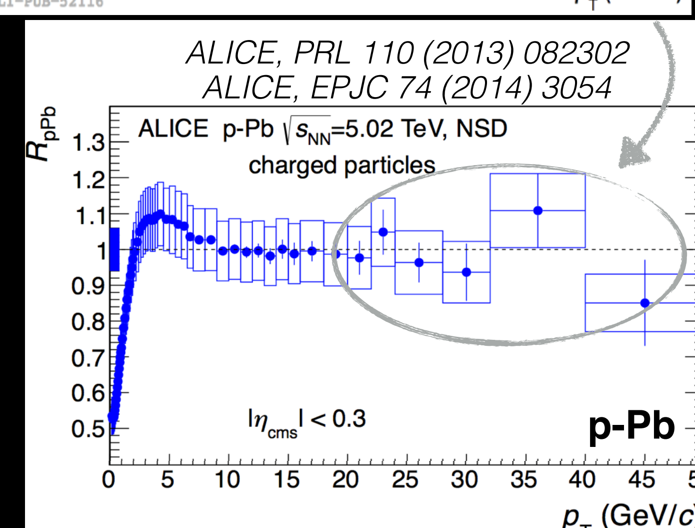






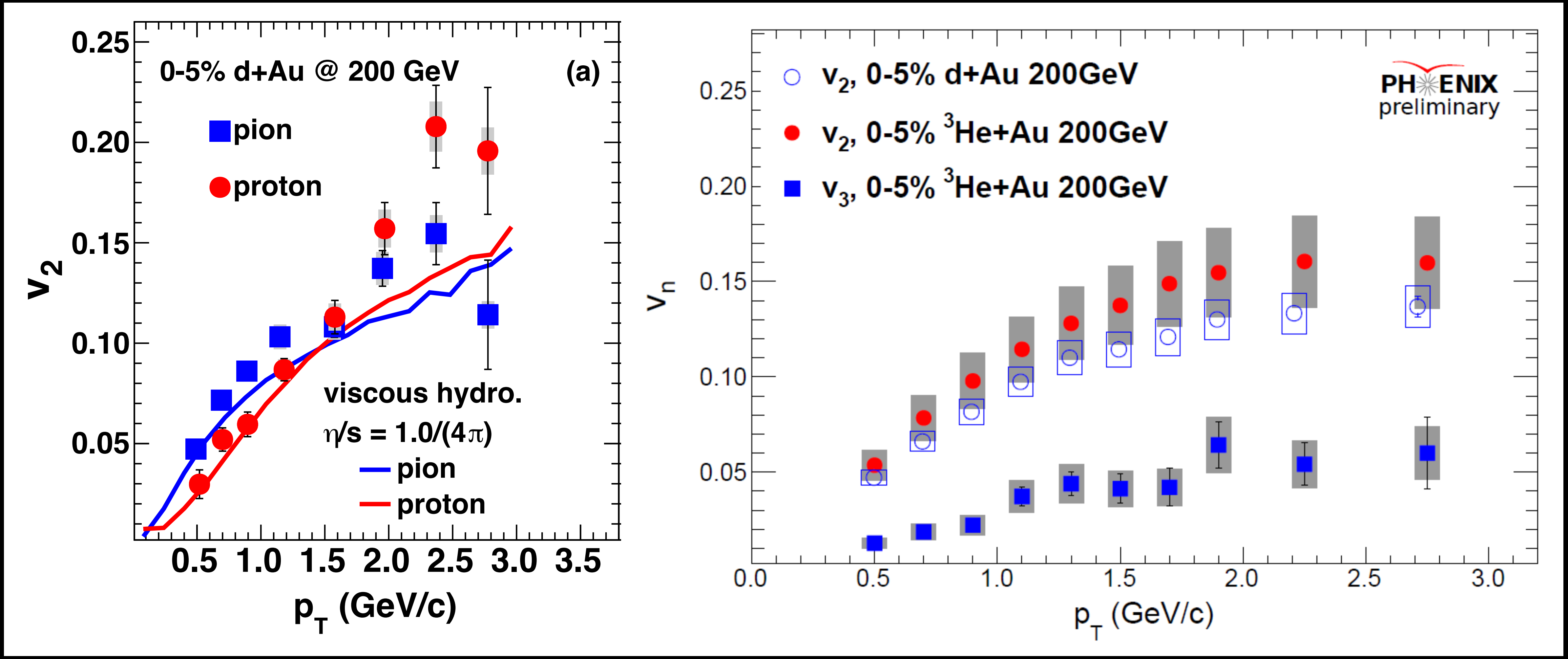






- collective effects in pA?
    - who ordered that?

d+A and ^{3}He+A collisions



- collective effects in dA and ^{3}He+A at RHIC?
  - who ordered that?

a rose?

- it smells like a rose
- it pricks like a rose
- .....

Collective motion

JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Vol. 17, No. 7, JULY 1962

Generalized Cumulant Expansion Method*

Ryogo KUBO
Department of Physics, University of Tokyo

(Received April 11, 1962)

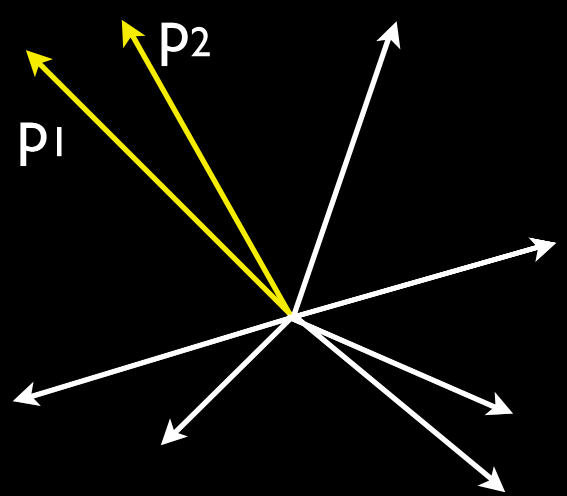
The moment generating function of a set of stochastic variables defines the cumulants or the semi-invariants and the cumulant function. It is possible, simply by formal properties of exponential functions, to generalize to a great extent the concepts of cumulants and cumulant function. The stochastic variables to be considered need not be ordinary c-numbers but they may be q-numbers such as used in quantum mechanics. The exponential function which defines a moment generating function may be any kind of generalized exponential, for example an ordered exponential with a certain prescription for ordering q-number variables. The definition of average may be greatly generalized as far as the condition is fulfilled that the average of unity is unity. After statements of a few basic theorems these generalizations are discussed here with certain examples of application. This generalized cumulant expansion provides us with a point of view from which many existent methods in quantum mechanics and statistical mechanics can be unified.

$$
\begin{aligned}
\langle X_j\rangle_c &= \langle X_j\rangle \\
\langle X_j^2\rangle_c &= \langle X_j^2\rangle - \langle X_j\rangle^2 \\
\langle X_jX_l\rangle_c &= \langle X_jX_l\rangle - \langle X_j\rangle\langle X_l\rangle \\
\langle X_jX_kX_l\rangle_c &= \langle X_jX_kX_l\rangle \\
&\quad -\{\langle X_j\rangle\langle X_kX_l\rangle+\langle X_k\rangle\langle X_lX_j\rangle+\langle X_l\rangle\langle X_jX_k\rangle\} \\
&\quad +2\langle X_j\rangle\langle X_k\rangle\langle X_l\rangle \\
\langle X_jX_kX_lX_m\rangle_c &= \langle X_jX_kX_lX_m\rangle \\
&\quad -\{\langle X_j\rangle\langle X_kX_lX_m\rangle+\langle X_k\rangle\langle X_jX_lX_m\rangle+\langle X_l\rangle\langle X_jX_kX_m\rangle+\langle X_m\rangle\langle X_jX_kX_l\rangle\} \\
&\quad -\{\langle X_jX_k\rangle\langle X_lX_m\rangle+\langle X_jX_l\rangle\langle X_kX_m\rangle+\langle X_jX_m\rangle\langle X_kX_l\rangle\} \\
&\quad +2\{\langle X_j\rangle\langle X_k\rangle\langle X_lX_m\rangle+\langle X_j\rangle\langle X_l\rangle\langle X_kX_m\rangle+\langle X_j\rangle\langle X_m\rangle\langle X_kX_l\rangle \\
&\qquad +\langle X_jX_k\rangle\langle X_l\rangle\langle X_m\rangle+\langle X_jX_l\rangle\langle X_k\rangle\langle X_m\rangle+\langle X_jX_m\rangle\langle X_k\rangle\langle X_l\rangle\} \\
&\quad -6\langle X_j\rangle\langle X_k\rangle\langle X_l\rangle\langle X_m\rangle
\end{aligned}
\tag{2.8}
$$

cumulants allow us to see if there are multi-particle correlations in the system (cumulants nonzero only mathematical proof)

Collective motion

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = \langle v_n^2 \rangle + \delta_2$$



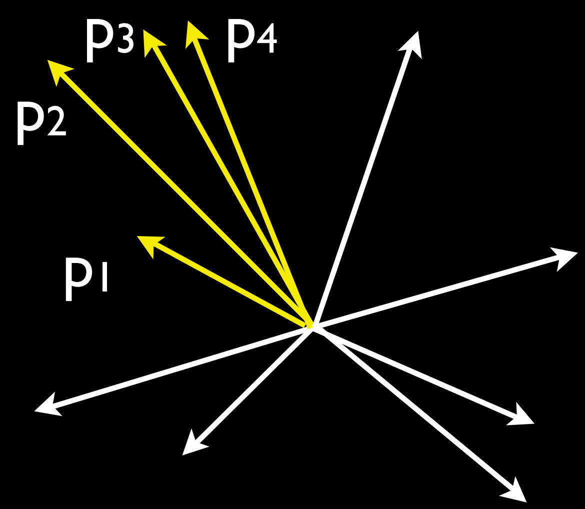
particle 1 coming from the resonance. Out of remaining M-1 particles there is only one which is coming from the same resonance, particle 2. Hence a probability that out of M particles we will select two coming from the same resonance is ~ 1/(M-1). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$

- not easily satisfied: M=200 v_n >> 0.07

Collective motion



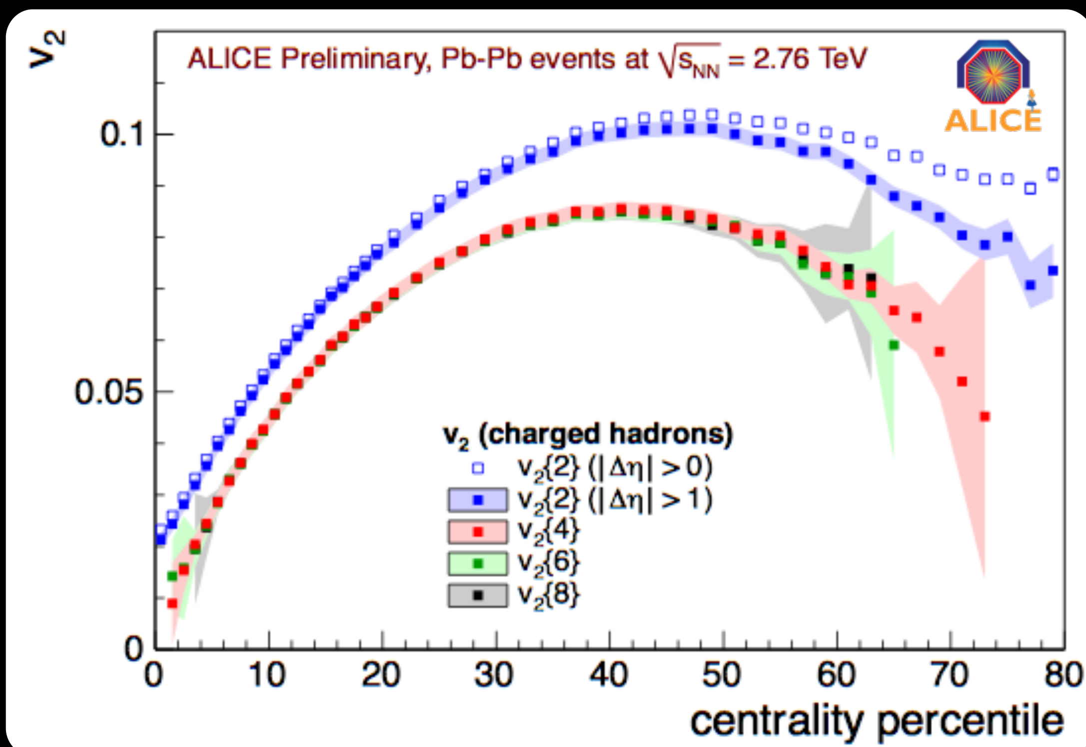
Particle 1 coming from the mini-jet. To select particle 2 we can make a choice out of remaining M-1 particles; once particle 2 is selected we can select particle 3 out of remaining M-2 particles and finally we can select particle 4 out of remaining M-3 particles. Hence the probability that we will select randomly four particles coming from the same resonance is 1/(M-1)(M-2)(M-3). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$

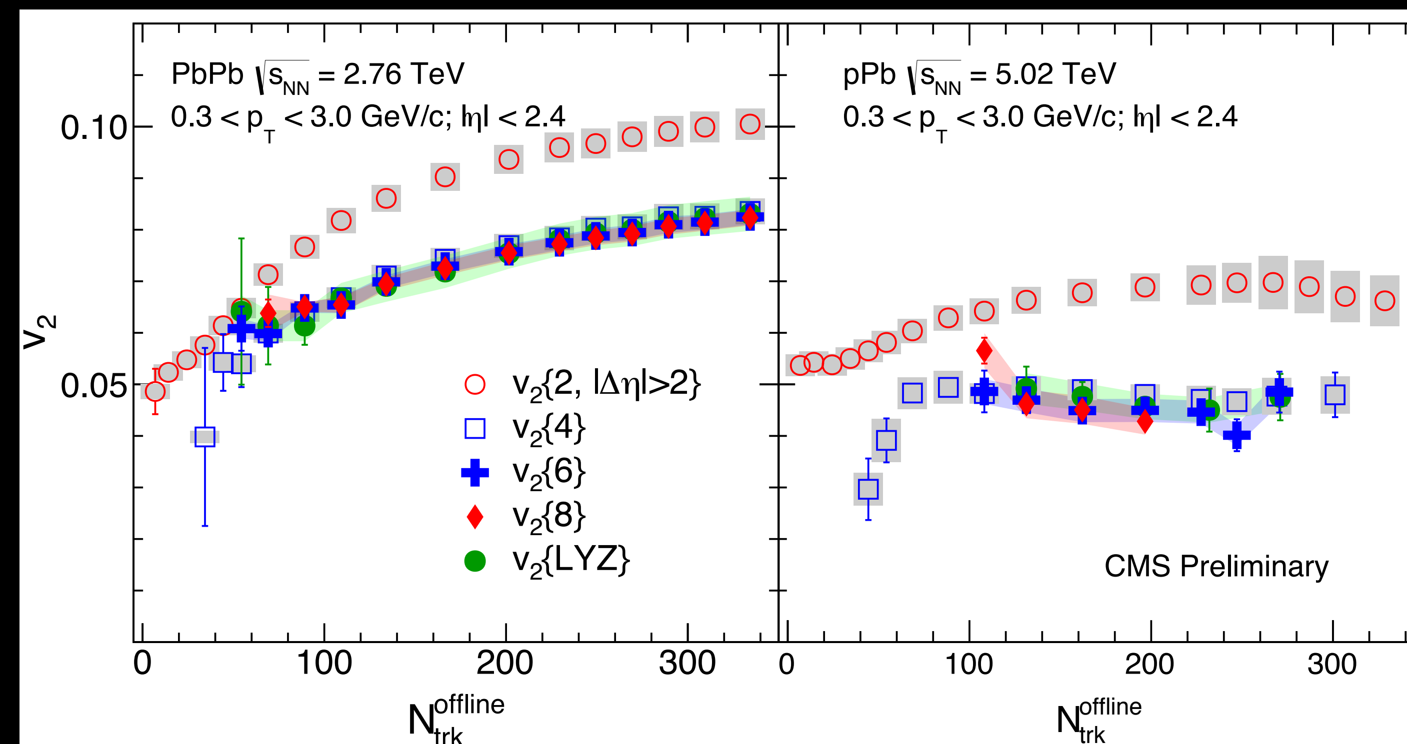
- therefore to reliably measure flow:

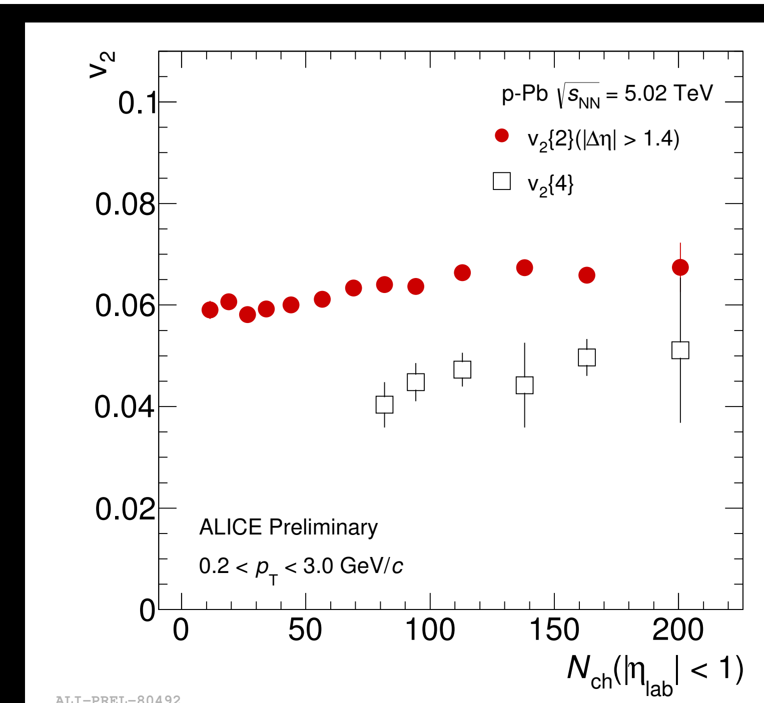
$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4}$$

Collective motion

Stefan Bass







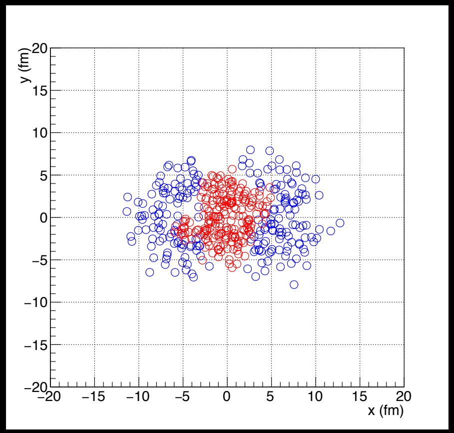
ALICE, Phys.Rev. C90 (2014) 5, 054901

- collective behaviour in pA!
- not necessarily hydrodynamics!
    - quantitative question
- initial state or final state?

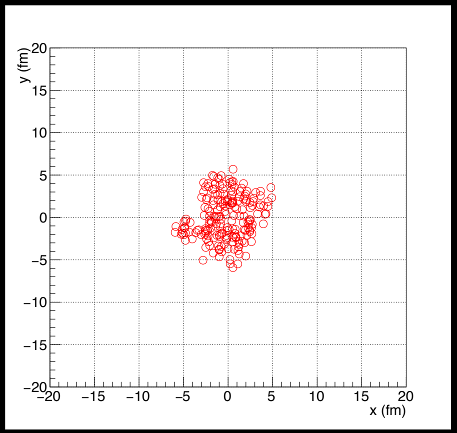
Roberto Preghenella

Who ordered that?

- What can we learn from pA?
  - better theoretical understanding initial state pA compared to AA
  - better experimental constraints on the initial *geometry* in AA compared to pA
  - if both initial state and the final state interactions are important in pA is there still a clear preference compared to peripheral AA?
    - new questions to answer
  - d+A and ^{3}He+A are very important to disentangle initial or final state origin!
  - (multi-particle) azimuthal correlations differentially as function of p_T and η should allow us to also test if there are different regimes where initial state or final state effects dominate



Summary



- clear evidence of the importance of the initial spatial distribution (in all gory details) in all the correlations
- naturally explained if the constituents have strong final state interactions
  - some depend non-trivially on the evolution (which is well captures in models with final state interactions)
- very rich playground for theorist and experimentalist!

Anisotropic Flow; ad infinitum

Raimond Snellings

Universiteit Utrecht

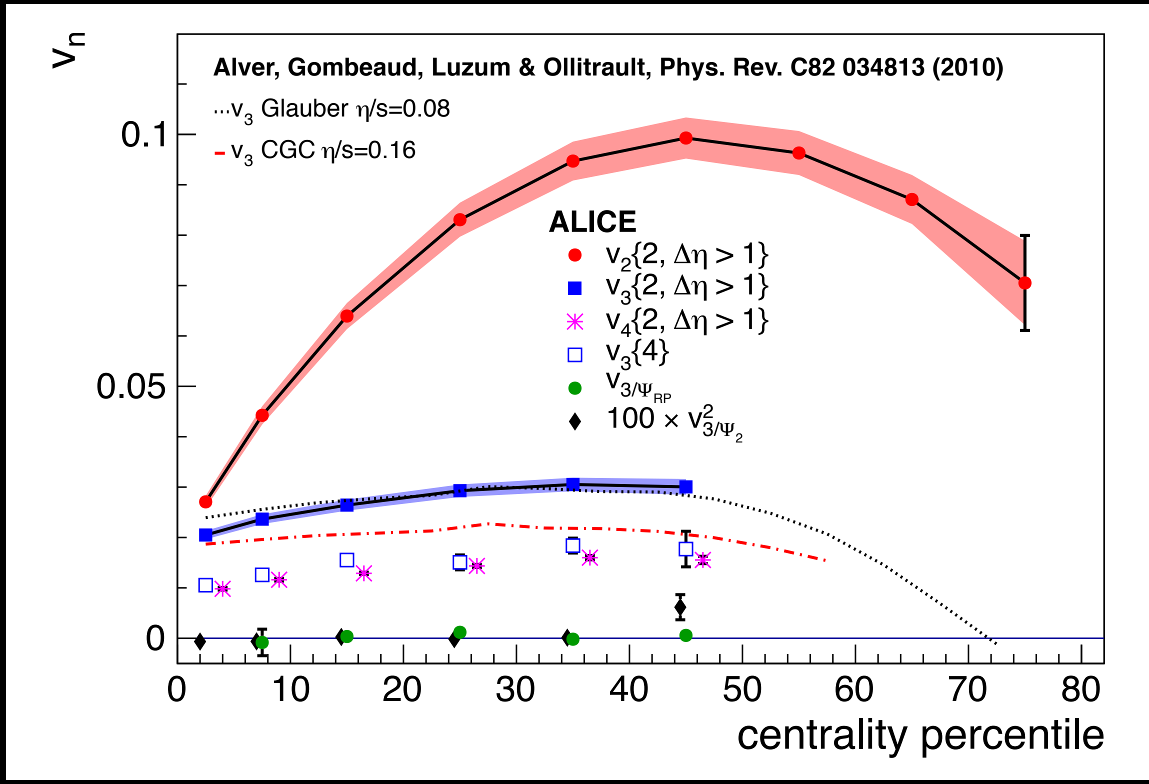
NIKHEF

4^{th} International Symposium on Non-equilibrium Dynamics
30-08 — 05-09-2015
Giardini Naxos, Sicily,Italy

Mixed Harmonics and Standard Candles

$$v_3\{\Psi_{RP}\} = \langle\langle \cos 3(\varphi - \Psi_{RP})\rangle\rangle$$
$$= \langle\langle \cos 3(\varphi - \Psi_3)\ \cos 3(\Psi_3 - \Psi_{RP})\rangle\rangle$$
$$= \langle v_3 \langle \cos 3(\Psi_3 - \Psi_{RP})\rangle\rangle$$

$$v_3^2\{\Psi_2\} = \frac{\langle \cos(2\varphi_1 + 2\varphi_2 + 2\varphi_3 - 3\varphi_4 - 3\varphi_5)\rangle}{v_2^3}$$



Mixed Harmonics and Standard Candles

$$\begin{aligned} v_3\{\Psi_{\mathrm{RP}}\} &= \langle\langle \cos 3(\varphi - \Psi_{\mathrm{RP}})\rangle\rangle \\ &= \langle\langle \cos 3(\varphi - \Psi_3)\ \cos 3(\Psi_3 - \Psi_{\mathrm{RP}})\rangle\rangle \\ &= \langle v_3 \, \langle \cos 3(\Psi_3 - \Psi_{\mathrm{RP}})\rangle\rangle \end{aligned}$$

$$v_3^2\{\Psi_2\} = \frac{\langle \cos(2\varphi_1 + 2\varphi_2 + 2\varphi_3 - 3\varphi_4 - 3\varphi_5)\rangle}{v_2^3}$$

$$\begin{aligned} &\langle \cos(n_1\varphi_1 + ... + n_k\varphi_k)\rangle \\ &= \langle v_{n_1} ... v_{n_k} \cos(n_1\Psi_{n_1} + ... + n_k\Psi_{n_k})\rangle \end{aligned}$$

$$\begin{aligned} \langle \cos(4\varphi_1 - 2\varphi_2 - 2\varphi_3)\rangle &= \langle v_4 v_2^2 \cos(4\Psi_4 - 4\Psi_2)\rangle, \\ \langle \cos(6\varphi_1 - 3\varphi_2 - 3\varphi_3)\rangle &= \langle v_6 v_3^2 \langle \cos(6\Psi_6 - 6\Psi_3)\rangle \end{aligned}$$