Thermodynamics and phase diagram of the Polyakov-Nambu-Jona-Lasinio model

in collaboration with Joerg Aichelin

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Outline

- Motivation: QCD Phase Diagram
- Introduction to Polyakov-Nambu-Jona-Lasinio Model

(P)NJL-like models contain much more than quarks (and static gluons)!

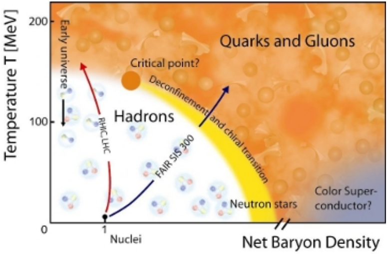
$\downarrow$

- Collective excitations: Mesons, Diquarks & Baryons

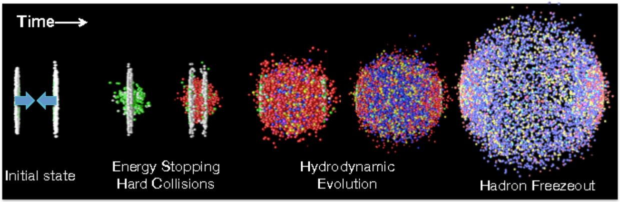
Equation of State

$\downarrow$

- Thermodynamics at $\mu = 0$: mean-field results & meson-like fluctuations
- Thermodynamics at $\mu \neq 0$: mean-field **prelim.** results
- Summary and Outlook

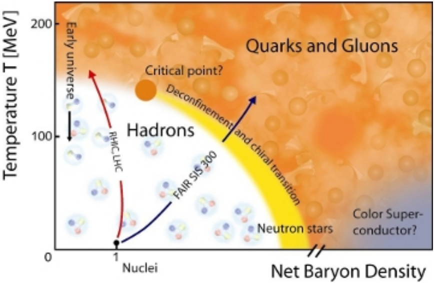


QCD phase diagram: Hadron and quark-gluon plasma phases



Heavy-ion collision to explore them (figure taken from T. Nayak, 2012).

We would like to be able to explore the confined and deconfined phases (and the phase transition itself) from a **single** framework.



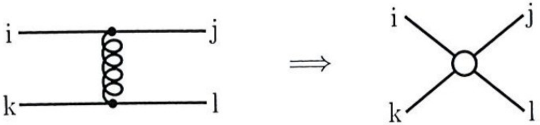
QCD phase diagram: Hadron and quark-gluon plasma phases

Simplest model to describe the properties of **both** phases (hadronization eventually?)

One good candidate is

(Polyakov)-Nambu-Jona–Lasinio Model

NJL Lagrangian



From QCD, the NJL model is inspired on the limit $t \to 0$ of the gluon exchange

Interaction Lagrangian: color current interaction

$$\mathcal{L}_{int} = -g \quad [\bar{q}_i \gamma^\mu T^a \delta_{ij} q_j] \quad [\bar{q}_k \gamma_\mu T^a \delta_{kl} q_l]$$

Flavor: $i, j = 1...N_f = 3$; T^a : color representations $a = 1...N_c^2 - 1 = 8$.

For reviews see Vogl and Weise (1991), Klevansky (1992), Ebert, Reinhardt and Volkov (1994), Hatsuda and Kunihiro (1994), Buballa (2004)...

NJL for mesons

Exchange Lagrangian (after Fierz transformation): accounts for color singlet vertex

Pseudoscalar sector

$$\mathcal{L}_{ex} = G \; (\bar{q}_i \; \tau^a_{ij} \; \mathbb{I}_c i\gamma_5 \; q_j) \; (\bar{q}_k \; \tau^a_{kl} \; \mathbb{I}_c i\gamma_5 \; q_l) \; ; \qquad G = (N_c^2 - 1)/N_c^2 g$$

$\mathbb{I}_c$: color singlet interaction
τ^a : flavor generators $a = 1...8$ ($N_f = 3$).

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The effective Lagrangian should share the global symmetries of (massless) QCD:

Symmetries of massless NJL model

$$SU_V(3) \otimes SU_A(3) \otimes U_V(1) \otimes U_A(1)$$

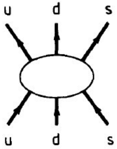
In our scheme, chiral symmetry is explicitly broken to $SU_V(3)$ by the bare quark masses. We keep an isospin $SU_V(2)$ symmetry. The $U_A(1)$ is broken by quantum effects...

$U_A(1)$ symmetry is broken by the **axial anomaly**.
The anomaly is the respondible for the mass difference between the η and η'.
(mixing between flavor octet and singlet).

't Hooft Lagrangian

$$\mathcal{L}_{'t\ Hooft} = H \det_{ij} \left[\bar{q}_i(1-\gamma_5)q_j\right] - H \det_{ij} \left[\bar{q}_i(1+\gamma_5)q_j\right]$$

For $N_f = 3$ it represents a six-quark contact interaction.



H is fixed by the $\eta - \eta'$ mass difference.

Gluon (static) properties are implemented in the model through an effective potential for the Polyakov loop

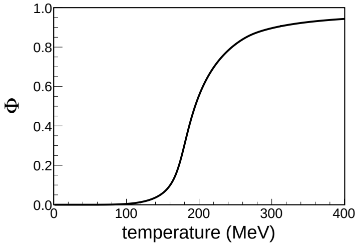
$$\frac{U(T,\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^3$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

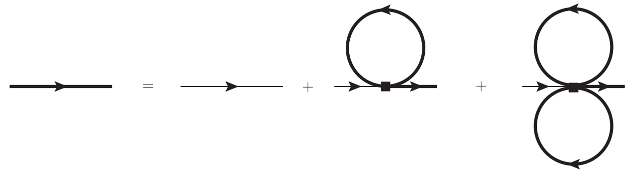
$a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, $b_4 = 7.5$ from fit to lattice-QCD results of Yang-Mills and $T_0 = 190$ MeV.

Φ is the order parameter of the deconfinement transition

$$\Phi = \frac{1}{N_c}\,\mathrm{Tr}_c\,\left\langle \mathcal{P}\exp\left(-\int_0^\beta d\tau A_0(\mathbf{x},\tau)\right)\right\rangle$$



The dressed quark masses are calculated through the **gap equation** in the mean-field approximation

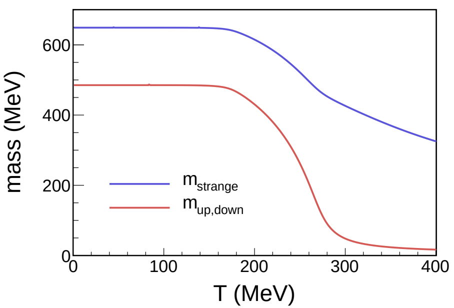


$$m_q = m_{q0} - 4G\langle\bar{q}q\rangle + 2H\langle\bar{q}'q'\rangle\langle\bar{q}''q''\rangle$$

Quark condensate

$$\langle\bar{q}q\rangle = -iN_c \mathrm{Tr}\, S_q \ , \quad S_q : \ \text{quark propagator}$$

Quark masses as a function of the temperature (zero chemical potential)



The limit $m_q \to m_{q0}$ at large T is a signal of the chiral symmetry restoration.

Hadrons can also be generated by the PNJL model!

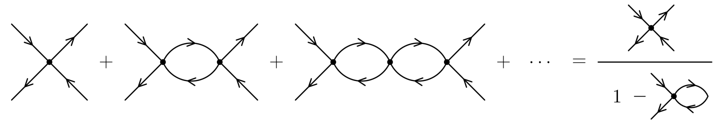
Key results of

Flavor dependence of baryon melting temperature
in effective models of QCD
Phys.Rev. C 91, Issue 6, 065206 (2015)
J.M. Torres-Rincon, J. Aichelin and B. Sintes

$\bar{q}q$ rule for mesons

The $\bar{q}q$ interaction is used as the kernel for the Bethe-Salpeter equation

$$T(p) = \mathcal{K} + i \int \frac{d^4 k}{(2\pi)^4} \mathcal{K} \, S\left(k + \frac{p}{2}\right) S\left(k - \frac{p}{2}\right) T(p)$$



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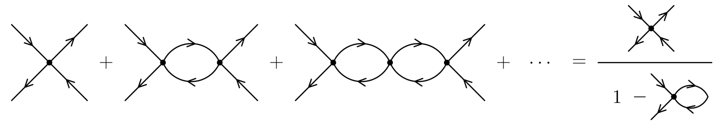
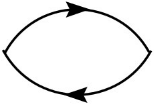
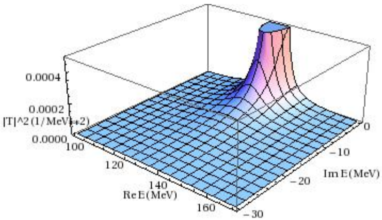


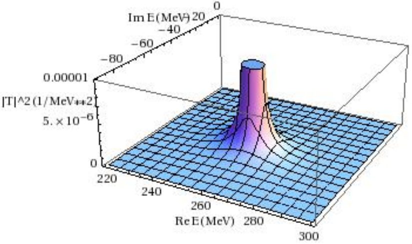
The solution of the $T-$amplitude at finite temperature

$$T(p) = \frac{\mathcal{K}}{1 - \mathcal{K}\Pi(p)} \,, \qquad \Pi(p_0, \mathbf{p}) = -T \sum_n \int \frac{d^3k}{(2\pi)^3} \, S\left(k + \frac{p}{2}\right) S\left(k - \frac{p}{2}\right)$$

 $\equiv \Pi$

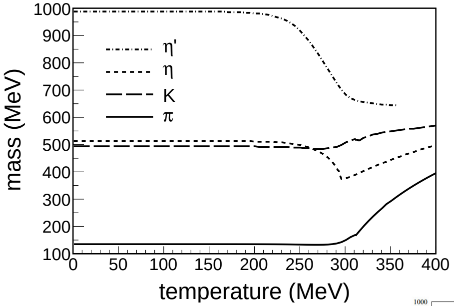
Mesons appear as bound states/resonances of $\bar{q}q$ scattering, i.e. **poles of the** $T-$**matrix**.



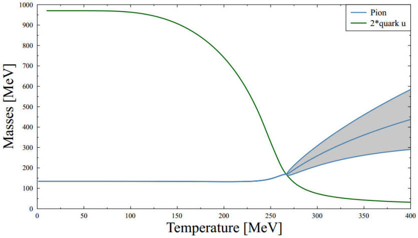


The position of the pole gives the mass and width of the meson as a function of the temperature.

$$1 - \mathcal{K}\,\Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, \mathbf{p} = 0) = 0$$

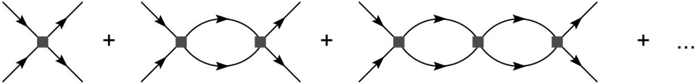


Pseudoscalar mesons
in the PNJL model at $\mu = 0$



Pion mass (blue line) and
decay width (grey band)

We can also combine two quarks to form a DIQUARK.



The procedure is analogous to mesons taking care of the different reps, spin structure and charge conjugations.

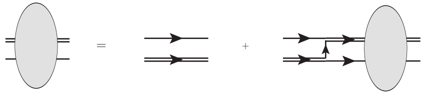
Solution of Bethe-Salpeter equation

$$T(p) = \frac{2G_{DIQ}}{1 - 2G_{DIQ}\Pi(p)}$$

$$\Pi(p) = i \int \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr} \, \left[\bar{\Omega} \; S\left(k + \frac{p}{2}\right) \Omega S^T \left(\frac{p}{2} - k\right) \right]$$

Two-body equation from (3-body) Fadeev equation (see Reinhardt 1990; Buck et al. 1992 for details)

$$G(P) \quad = \quad G_0 \quad + \quad G_0 Z G(P)$$



Solution

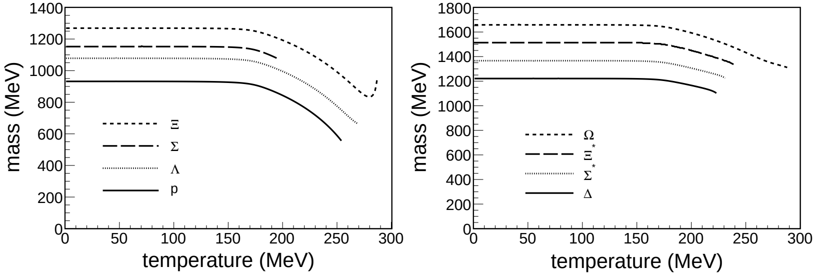
$$G(P) = \frac{G_0}{1 - G_0 Z}$$

Baryon mass

$$1 - G_0 Z(P_0 = M_{baryon}, \mathbf{P} = 0) = 0$$

Solution and Baryon mass

$$G(P) = \frac{G_0}{1 - G_0 Z} \quad \rightarrow \quad 1 - G_0 Z(P_0 = M_{baryon}, \mathbf{P} = 0) = 0$$



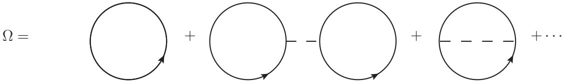
Baryon masses in $\mathbf{8}_f$ and $\mathbf{10}_f$ representations, as a function of temperature.

Thermodynamics and phase diagram of the PNJL model

We want to compute the thermodynamical properties of the PNJL model at finite T and μ_q

Grand-canonical potential of the model: $\Omega(T, \mu_q)$

The computation is usually oeganized within **large-N_c** limit considerations and Ω is expanded in powers of $1/N_c$ (see Klevansky (1992), Hüfner et al. (1993)).



- The LO term contains the non-interacting contribution which is $\mathcal{O}(N_c)$
- At $\mathcal{O}(N_c)$ we also get the Hartree diagram
- Fock diagram is topologically equivalent in the NJL limit but already $\mathcal{O}(1)$

At order $\mathcal{O}(N_c)$ the expression for the gran-canonical potential is

$\Omega_{PNJL}(\Phi, \bar{\Phi}, m_i, T, \mu_i)$

$$
\begin{aligned}
= \quad & 2G \sum_{i=u,d,s} \langle \bar{\psi}_i \psi_i \rangle^2 - 4H \prod_i \langle \bar{\psi}_i \psi_i \rangle - 2N_c \sum_i \int \frac{d^3p}{(2\pi)^3} E_i \\
- \quad & 2T \sum_i \int \frac{d^3p}{(2\pi)^3} \left[\mathrm{tr}_c \log(1 + L e^{-(E_i - \mu_i)/T}) + \mathrm{tr}_c \log(1 + L^\dagger e^{-(E_i + \mu_i)/T}) \right] \\
+ \quad & U(T, \Phi, \bar{\Phi})
\end{aligned}
$$

with L the Polyakov loop and $E_i = \sqrt{p^2 + m_i^2}$.

Note that $G \sim \mathcal{O}(1/N_c), H \sim \mathcal{O}(1/N_c^2), \langle \bar{\psi}_i \psi_i \rangle \sim \mathcal{O}(N_c)$

(see details e.g. in Klevansky (1992) for the NJL model
or Ratti, Thaler, Weise (2006) for PNJL model)

Equation of state at $\mu_q = 0$

The pressure of the system

Pressure

$$P(T) = -\,[\,\Omega(T) - \Omega(0)\,]$$

- We need to substract the vacuum contribution of Ω.
- We remove the cutoff ($\Lambda \to \infty$) to all UV-convergent integrals (SB limit).
- We use polynomial ansatz for the YM effective potential U (other choices are possible).
- For $N_f = 3$, only the parameter T_0 is readjusted (B.J. Schaefer, J.W. Pawlowski, J. Wambach, 2007).

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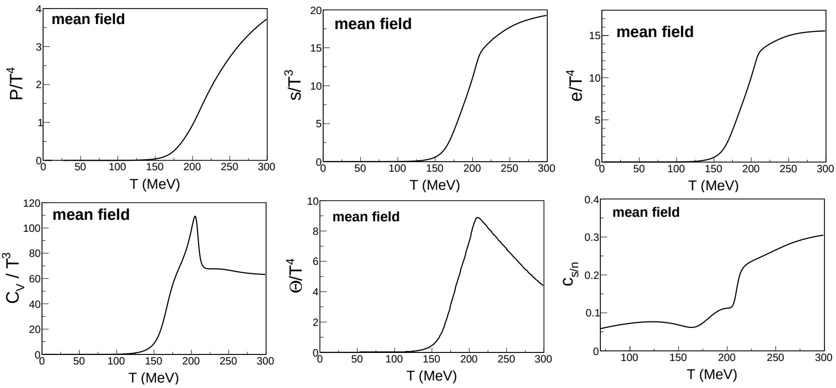
Other thermodynamical quantities at $\mu_q = 0$

$$s = \frac{dP}{dT} \; ; \quad e = sT - P \; ; \quad C_V = T \frac{ds}{dT} \; ; \quad \Theta = \epsilon - 3P \; ; \quad c_s = \frac{s}{C_V}$$

- s: entropy density; e: energy density; c_s: speed of sound
- C_V: specific heat; Θ: trace anomaly

Thermodynamics at $\mathcal{O}(N_c)$ in the mean-field approximation and $\mu_q = 0$.



How it compares with lattice-QCD calculations?

Comparison to Lattice-QCD calculations

PNJL model at MF did a really good job with "OLD" lattice-QCD data ...

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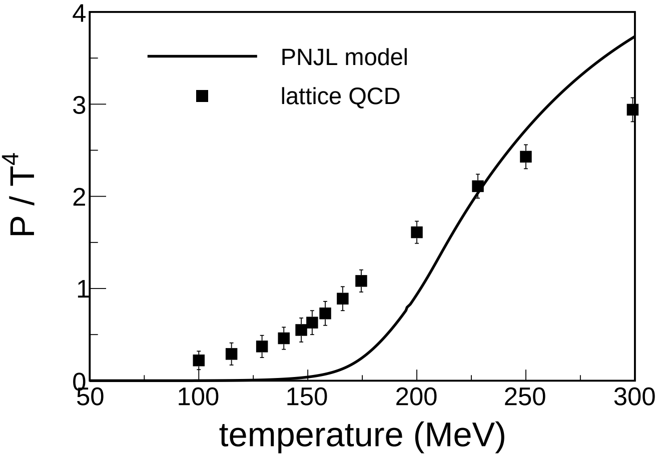
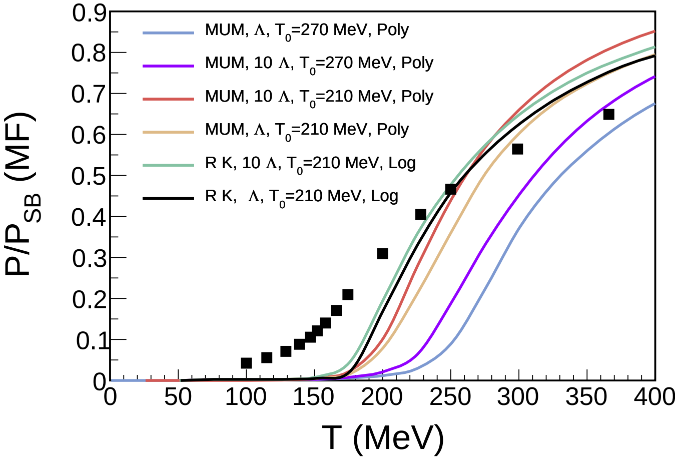
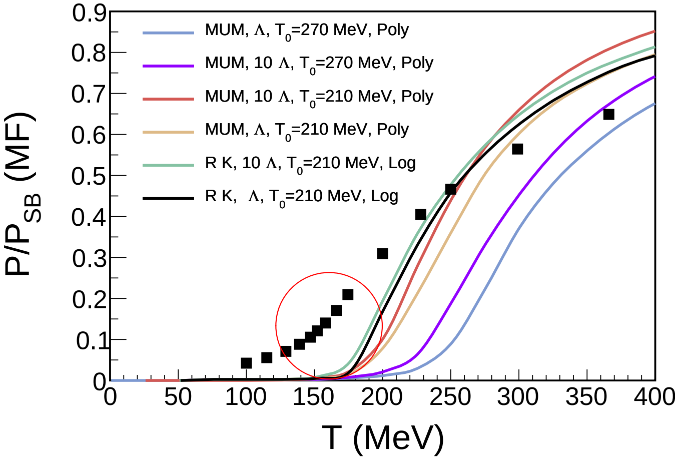


but it does not describe well the CURRENT lattice-QCD data
(S. Borsanyi et al. 2010).



Any parametrization underestimates the pressure at low temperature, and push it very quickly to the Stefan-Boltzmann limit.

Comparison to Lattice-QCD calculations



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Low-temperature range

- Hadronic states are missing at low temperature
- In fact, they should be dominant in this limit
- The first contributions come from π, K, η...
- ...but PNJL can account for these states!

- Hadronic states are missing at low temperature
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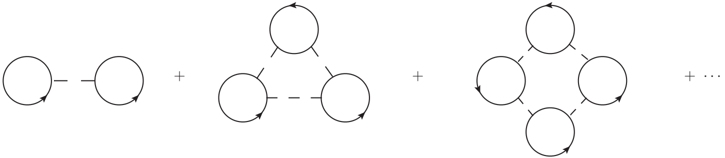
- The first contributions come from π, K, η...
- ...but PNJL can account for these states!

How can we take into account these states in $\Omega(T)$?

- Mesonic fluctuations on top of the mean-field pressure
- This means to include the diagrams at $\mathcal{O}(1)$ in the $1/N_c$ expansion: Ring summation



(see Hüfner, Klevansky, Zhuang (1994,1995), Blaschke et al. (2014)...)

The $\mathcal{O}(1)$ contribution to the thermodynamic potential:

Mesonic contribution

$$\Omega_{fluc}^{meson} = \sum_M \frac{N_M}{2} T \int \frac{d^3p}{(2\pi)^3} \sum_n \log\left(2G\ t_M^{-1}\right)$$

- The summation sum runs over all *M*esons
- N_M is the degeneracy of each state (e.g. $N_\pi = 3$ for pions)
- t_M is the meson propagator coming from the BS equation

$$t_M(i\omega_n, \mathbf{p}) = \frac{\mathcal{K}}{1 - \mathcal{K}\Pi_M(i\omega_n, \mathbf{p})}$$

Main message

The mesonic contribution to Ω is directly related to the resummed quark-antiquark **scattering amplitude** in the medium.

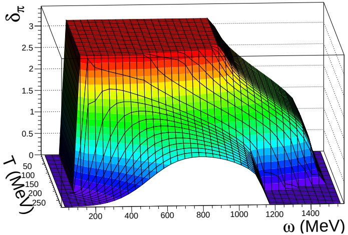
The contribution can be rewritten in terms of the **phase-shift**

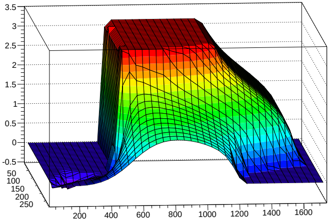
$$e^{2i\delta_M(\omega,\mathbf{p})} \equiv \frac{1 - \mathcal{K}\Pi_M(\omega + i\epsilon, \mathbf{p})}{1 - \mathcal{K}\Pi_M(\omega - i\epsilon, \mathbf{p})}$$

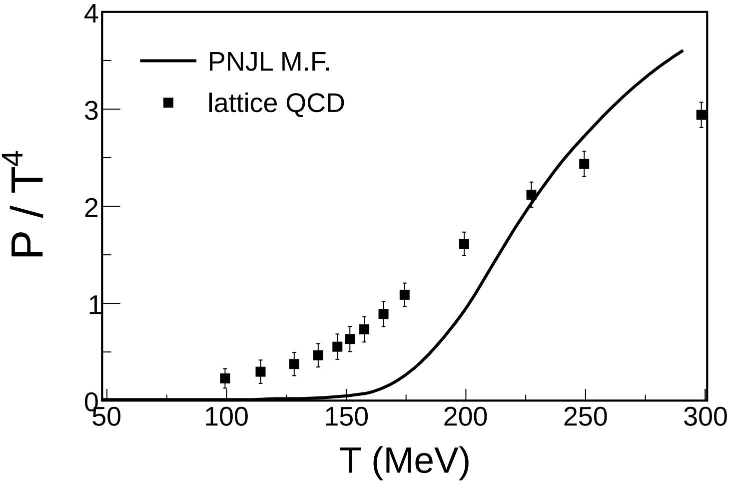
The thermodynamic potential

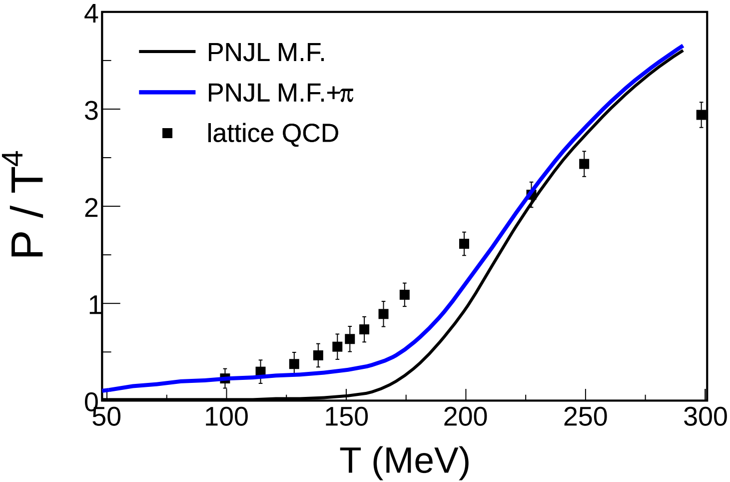
$$\Omega^{meson}_{fluc,M} = -N_M \int \frac{d^3p}{(2\pi)^3} \int_{-p^2}^{\infty} \frac{ds}{2\pi} \frac{1}{2\sqrt{s+p^2}} \left(1 + \frac{2}{e^{\beta\sqrt{s+p^2}} - 1}\right) \delta_M(s)$$

The analytic structure of the polarization function will determine the contribution to Ω^{meson}. In particular a nonzero phase-shift appears at any cut on the real axis (unitarity and Landau cuts)






PNJL M.F.
lattice QCD
P / T4
T (MeV)
0
1
2
3
4
50
100
150
200
250
300


PNJL M.F.
PNJL M.F.+π
lattice QCD
P / T4
T (MeV)
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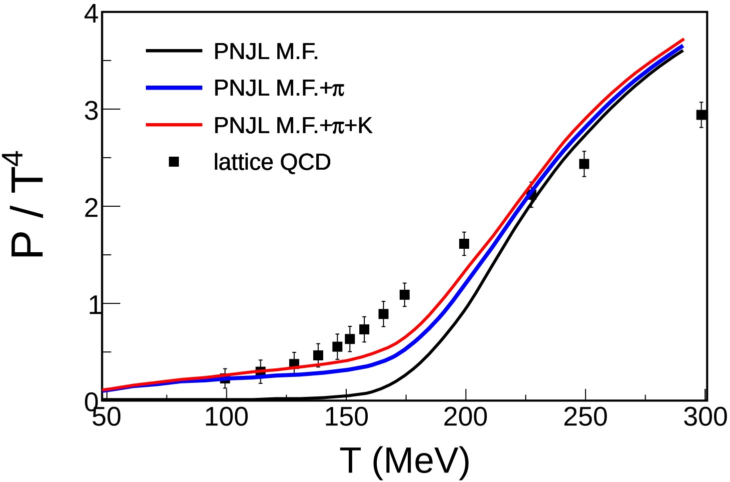
PNJL M.F.
PNJL M.F.+π
PNJL M.F.+π+K
lattice QCD
P / T⁴
T (MeV)
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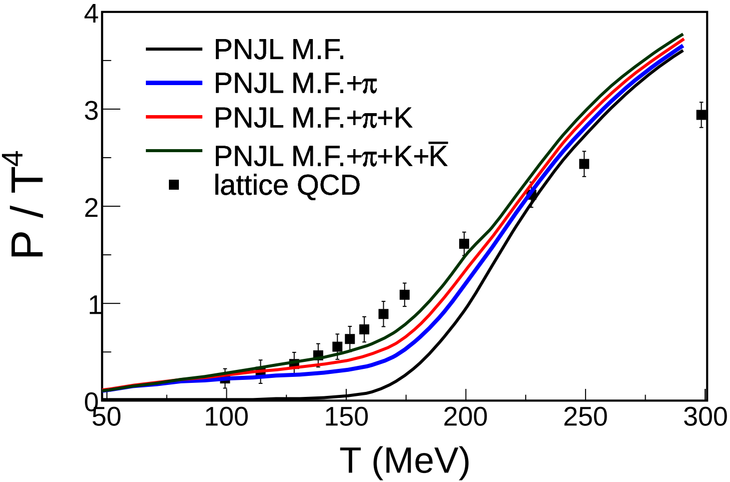
300


PNJL M.F.
PNJL M.F.+π
PNJL M.F.+π+K
PNJL M.F.+π+K+K̄
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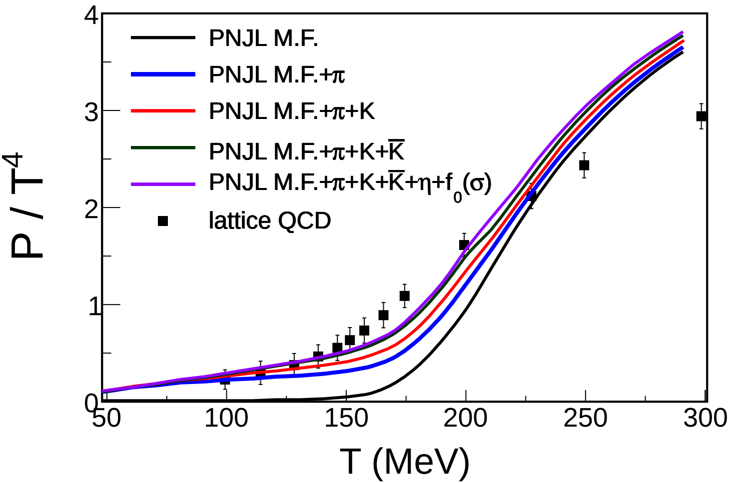
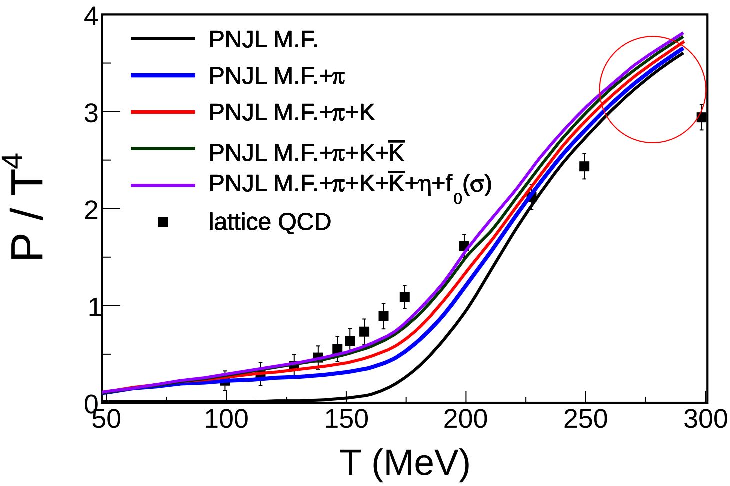


Note that more and more states are needed at intermediate temperatures.



Note that more and more states are needed at intermediate temperatures (eventually also baryon should be added).

High temperature limit

At large-T we have a gas of quasi-massless quarks (chiral limit) and the pressure gets very close to the SB limit.

Solutions we are considering

1. Include perturbative-QCD contribution from fermion Fourier modes $k \in (\Lambda, \infty)$ (Turko, Blaschke et al. 2013)
2. Consider backreaction of quarks into the gluonic Yang-Mills potential

$$\frac{U_{YM}}{T^4}(\Phi, \bar{\Phi}, T) \to \frac{U_{glue}}{T_{YM}^4}(\Phi, \bar{\Phi}, T_{YM}(T))$$

where the relation between *glue* and *YM* potentials comes an FRG study (Hass, Stiele et al. 2013)

WORK IN PROGRESS...

Finite chemical potential

Finally, we are working on the thermodynamics at finite chemical potential (very preliminary results!)

Scenario

$$\mu_q = \mu_u = \mu_d; \mu_s = 0 \quad \rightarrow \quad \mu_B = 3\mu_q, \mu_S = 0$$

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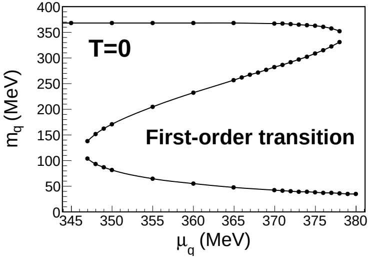
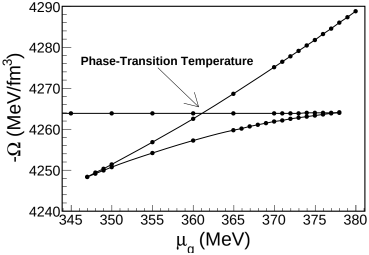
First we need to determine the phase boundary of the model, because for $\mu_q \neq 0$ one finds several solutions of the gap equation

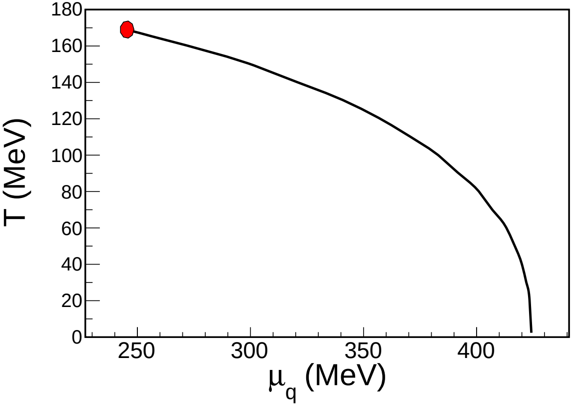




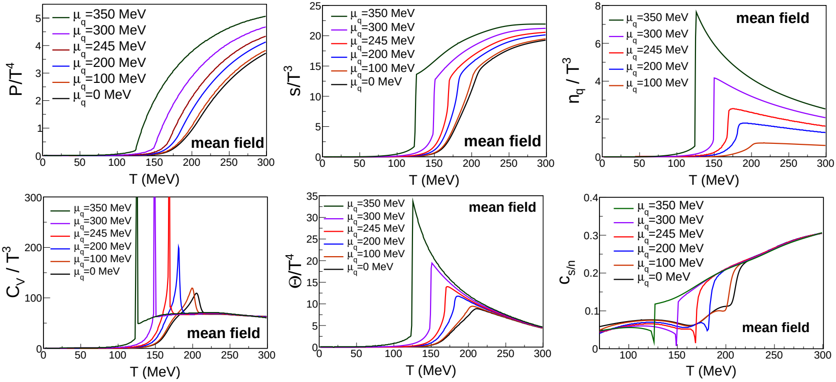
(be careful: this example is done with a different parametrization)

First order transition

Phase Boundary:



Critical point coordinates: $(T_c, \mu_{qc}) \simeq (169 \text{ MeV}, 245 \text{ MeV})$



Very preliminar results at finite μ_q in the mean-field approximation.

Summary

Conclusions:

- PNJL model provides quark, meson, diquark and baryon masses (and widths) at finite T and μ.
- Thermodynamics at $\mu_q = 0$ taking into account mesonic fluctuations (beyond mean-field).
- Recent lattice-QCD EoS is reasonable good described by the model.
- Straightforward extension to finite chemical potential: first-order boundary and critical point.

Outlook:

- Improve matching to lattice-QCD at $\mu_q = 0$ and predict for $\mu_q \neq 0$ (feedback to lattice-QCD).
- $\mu_q \neq 0$: Incorporate vectors mesons, diquark condensation, Polyakov eff. potential.
- Generalized susceptibilities (fluctuations of conserved quantities).
- Signatures of critical behavior around (T_c, μ_{qc}).
- Collision and hadronization cross sections (transport coefficients at finite T and μ).
- ...
- Full implementation in a heavy-ion collision evolution routine

THANKS FOR YOUR ATTENTION!

We use imaginary time formalism ($0 \leq it \leq \beta = 1/T$) with the prescription

Imaginary time formalism

$$k_0 \to i\omega_n, \qquad \int \frac{d^4 k}{(2\pi)^4} \to iT \sum_{n \in \mathcal{Z}} \int \frac{d^3 k}{(2\pi)^3}$$

with $i\omega_n = i\pi T(2n+1)$ the fermionic Matsubara frequencies.

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The quark chemical potential can be introduced by the Lagrangian

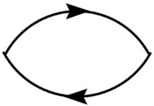
Quark chemical potential

$$\mathcal{L}_\mu = \sum_{ij} \bar{q}_i \mu_{ij} \gamma_0 q_j$$

where

$$\mu_{ij} = \text{diag}\,(\mu_u, \mu_d, \mu_s)$$

$$\Pi^P_{12}(p_0,\mathbf{p}) = -\frac{N_c}{8\pi^2}\left\{A(m_1)+A(m_2)+\left[(m_1-m_2)^2-p_0^2+\mathbf{p}^2\right]B_0(\mathbf{p},m_1,m_2,p_0)\right\}$$



$$\equiv \Pi$$

with

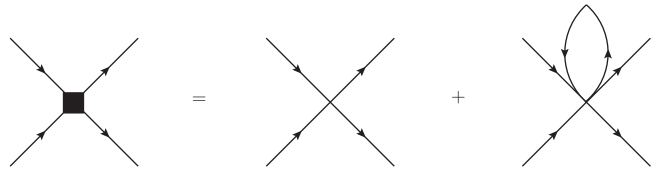
$$A(m_1) = 16\pi^2 T\sum_n \int \frac{d^3k}{(2\pi)^3}\frac{1}{(i\omega_n)^2-\mathbf{k}^2-m_1^2}$$

and

$$B_0(\mathbf{p},m_1,m_2,i\nu_m\to p_0+i\epsilon) = 16\pi^2 T\sum_n \int \frac{d^3k}{(2\pi)^3}\frac{1}{(i\omega_n)^2-\mathbf{k}^2-m_1^2} \times \frac{1}{(i\omega_n-i\nu_m)^2-(\mathbf{p}-\mathbf{k})^2-m_2^2}$$

Combine G and K at mean-field level into an effective coupling, e.g. for pion:

$$G_{eff} = G + \frac{1}{2} H \langle \bar{s}s \rangle$$



The complete Feynmann rule for the 4-point function

$$\mathcal{K} = \Omega \; 2 G_{eff} \; \bar{\Omega}$$

where Ω encodes color, flavor and spin factors

$$\Omega = \mathbb{I}_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$

We take the inverse of the T-matrix

$$T^{-1}(p^2) = \frac{1 - \mathcal{K}\Pi(p^2)}{\mathcal{K}}$$

and make a Taylor expansion around the pole mass of the meson/diquark $p^2 = m^2$

$$T^{-1}(p^2) = \left.\frac{\partial T^{-1}(p^2)}{\partial p^2}\right|_{p^2=m^2} (p^2 - m^2) + \cdots$$

$$T^{-1}(p^2) = -\left.\frac{\partial \Pi(p^2)}{\partial p^2}\right|_{p^2=m^2} (p^2 - m^2) + \cdots$$

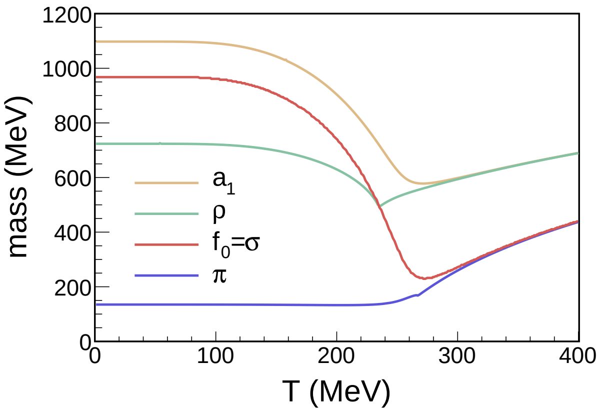
Finally, let us define and effective coupling constant

$$g^2_{eff} \equiv \frac{1}{\left.\frac{\partial \Pi(p^2)}{\partial p^2}\right|_{p^2=m^2}} \tag{1}$$

to finally obtain the meson propagator from the $T-$matrix

$$t(p^2) \simeq \frac{-g^2_{eff}}{p^2 - m^2}$$

Restoration of chiral symmetry at large temperatures



We can also combine two quarks to form a DIQUARK.

However, it cannot be color singlet $\not{\mathbb{1}}_c$
$\rightarrow$ they are not interesting as observable states

Allowed combinations

Color	Flavor	J^P	Denomination
$\mathbf{6}_S$	Not considered here		
$\bar{\mathbf{3}}_A$	$\mathbf{6}_S$	1^+_S	Axial
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^+_A	Scalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	0^-_A	Pseudoscalar
$\bar{\mathbf{3}}_A$	$\bar{\mathbf{3}}_A$	1^-_A	Vector

We focus on SCALAR and AXIAL diquarks to build baryons.
(the other diquarks have masses much higher and not suitable to form baryons)

Auxiliar slides: Diquarks

We repeat the calculation with diquarks $\langle q^T \Omega q \rangle$.
There are many types of diquarks according to $\Omega = \Omega_c \otimes \Omega_f \otimes \Omega_{Dirac}$:

$$\Omega_c \in \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$\Omega_f \in \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

$$\Omega_{Dirac} \in 1/2 \otimes 1/2 = 0 \oplus 1$$

With the constraint of Pauli principle: total antisymmetry

$$\Omega^T = -\Omega$$

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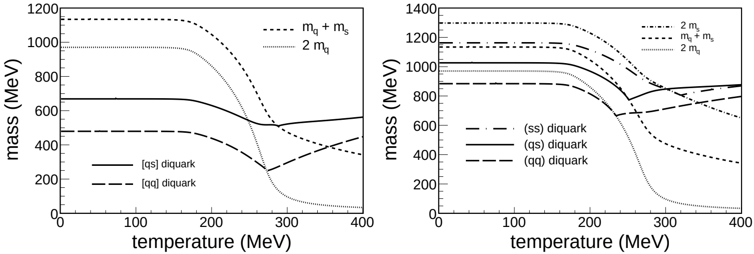
Result for [Scalar] and (Axial) diquarks (q denotes either u or d quark)

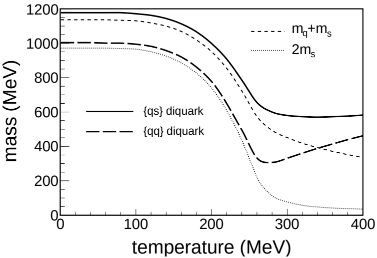


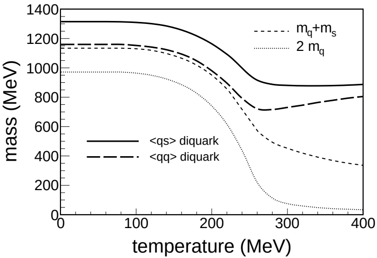
SCALAR AXIAL

They will be used to contruct the members of the baryon octet and decuplet, respectively.

$$\mathbf{3} \otimes (\bar{\mathbf{3}} \oplus \mathbf{6}) = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

Pseudoscalar and vector diquarks have **too large masses** and they present decay widths even at $T = 0$





Therefore, we are not assume these states to take part of baryons.

Auxiliar slides: Modelization of baryons

The formal equation for the baryon wavefunction $\mathcal{X}$ (flavor octet) reads (Reinhardt 1990; Buck et al. 1992)

Two-body equation

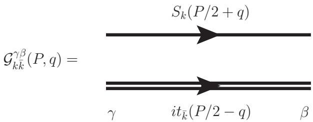
$$\mathcal{X}_j^{\bar{j}}(P,q) - \int \frac{d^4k}{(2\pi)^4} Z_{jk}^{\bar{k}\bar{j}}(q,k)\ G_{0,k\bar{k}}(P,q)\ \mathcal{X}_k^{\bar{k}}(P,k)\ |_{P^2=M^2_{Baryon}} = 0$$

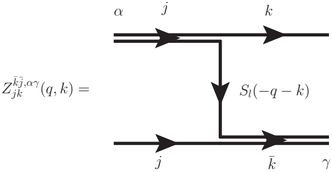
Quark index j, diquark index $\bar{j}$. The two-body quark-diquark propagator reads

$$G_{0,k\bar{k}}(P,q) = S_k(P/2+q)\ it_{\bar{k}}(P/2-q)$$

And the interaction subdiagram with two diquark-quark-quark vertices

$$Z_{jk}^{\bar{k}\bar{j}}(q,k) = \Omega_{jl}^{\bar{k}}\ S_l(-q-k)\ \Omega_{lk}^{\bar{j}}$$





It is convenient to use normalizated projectors on to the physical baryons.

$$(\mathcal{P}^{A}_{j\bar{i}})^\dagger \mathcal{P}^{A'}_{\bar{i}j} = \delta^{*,AA'}$$

In the octet representation of $SU(3)$ we can construct them from the Gell-Mann matrices

$$\mathcal{P}^{p}_{\bar{i}j} = \frac{1}{2}\left(\lambda^4 - i\lambda^5\right)_{\bar{i}j} \; ; \qquad \mathcal{P}^{n}_{\bar{i}j} = \frac{1}{2}\left(\lambda^6 - i\lambda^7\right)_{\bar{i}j}$$

$$\mathcal{P}^{\Lambda}_{\bar{i}j} = \mathcal{P}^{8}_{\bar{i}j} = \sqrt{\frac{1}{2}}\lambda^8_{\bar{i}j} \; ; \qquad \mathcal{P}^{\Sigma^0}_{\bar{i}j} = \mathcal{P}^{3}_{\bar{i}j} = \sqrt{\frac{1}{2}}\lambda^3_{\bar{i}j}$$

$$\mathcal{P}^{\Sigma^\pm}_{\bar{i}j} = \frac{1}{2}\left(\lambda^1 \mp i\lambda^2\right)_{\bar{i}j} \; , \qquad \mathcal{P}^{\Xi^0}_{\bar{i}j} = \frac{1}{2}\left(\lambda^6 + i\lambda^7\right)_{\bar{i}j}$$

$$\mathcal{P}^{\Xi^-}_{\bar{i}j} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)_{\bar{i}j} \; , \qquad \mathcal{P}^{0}_{\bar{i}j} = \sqrt{\frac{1}{3}}\mathbb{I}_{\bar{i}j}$$

Where the singlet flavor mixes with the $\Lambda - \Sigma^0$, producing a coupled channel problem (note that the Λ, Σ and singlet contains the same quark content).
Because of this fact, the projectors are not totally orthogonal (*).
Similar for the decuplet representation of $SU(3)$ to obtain the $\Delta, \Sigma^*, \Xi^*, \Omega$.