Dynamical models for heavy-ion collisions and why we need a new one

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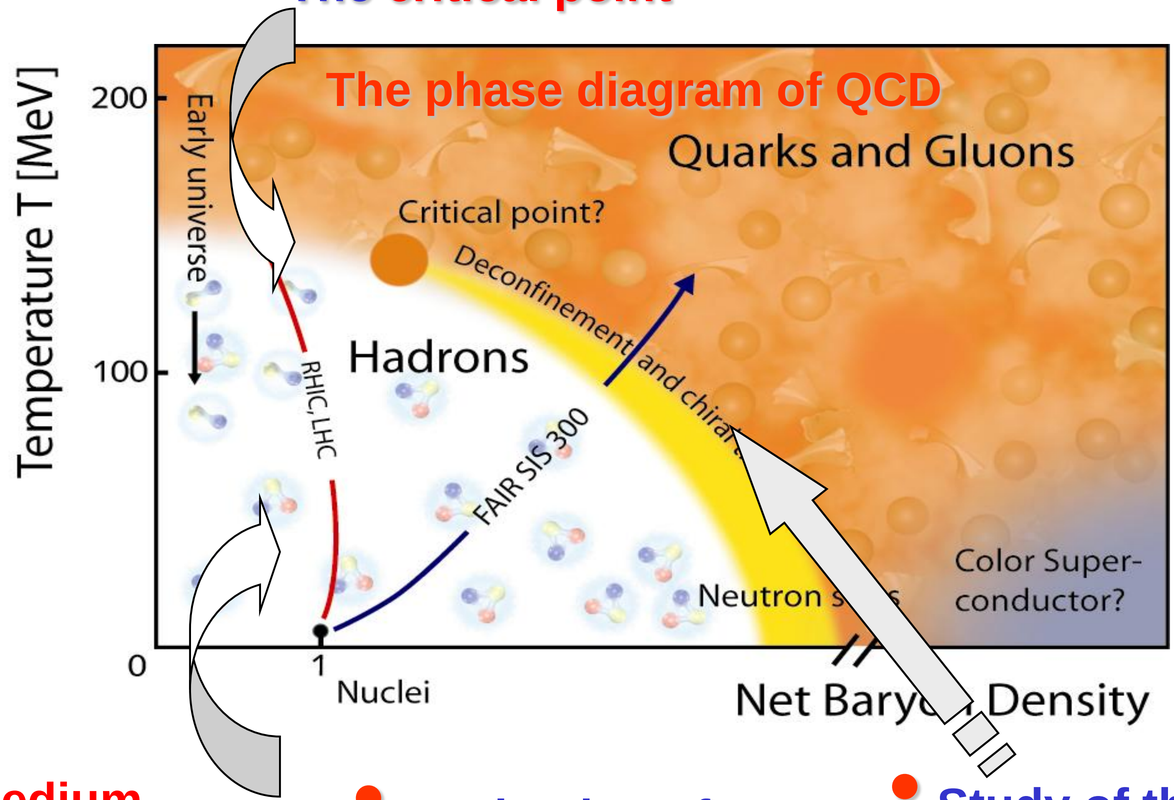
- ❑ Why do we need transport approaches in HI physics?
- ❑ What kind of transport approaches are available?
- ❑ Why do we need a new one?

Workshop on Non-Equilibrium Dynamics,
Valadero Cuba April 16-22 2018

The challenges of heavy-ion physics I:

What we want to know

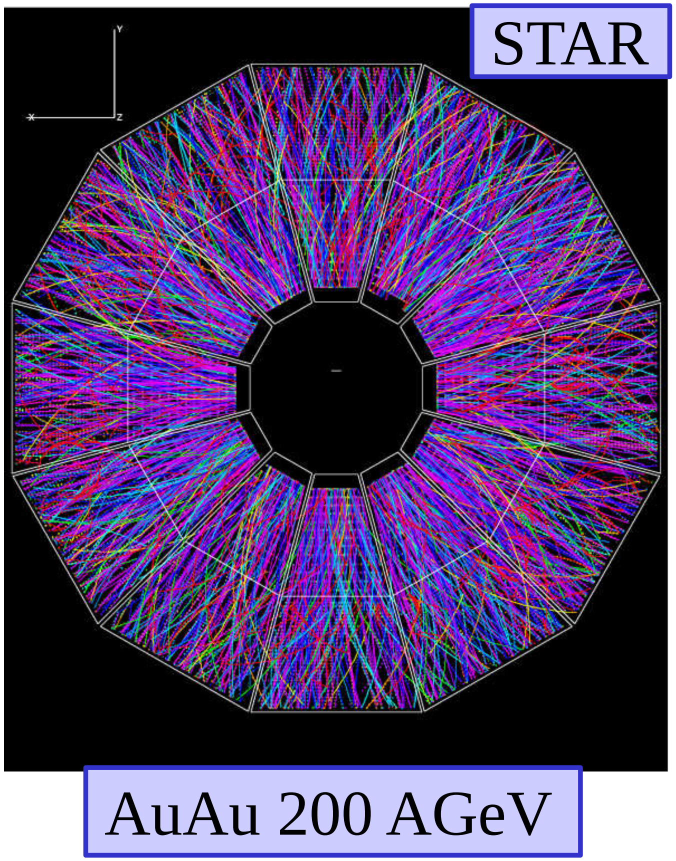
- The critical point



- in-medium properties of hadrons, and their production channels
- production of (hyper)fragments and fragments at midrapidity
- Study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma

The challenges of heavy-ion physics II:

What the experiments can provide:



Experiments deliver the momenta of thousands of particles of different masses

but

these particles contain only very indirectly the information on the interesting physics

To extract the physical quantities from the experimental results is the task of transport theories

Dynamical models for HIC

All approaches have their advantages and drawbacks.

Ideal hydrodynamics: only input: Equation of state (lQCD)
but to compare with data: initial condition + hadronization needed

The more sophisticated the approach the more input is needed

Microscopic models:
Elementary cross sections, (effective) masses, degrees of freedom ← (theory or exp.)
Theory of some of the ingredients needs improvement

Strategy to explore the physics:

- use the results of many body theory, elementary particle theory, lQCD, different experiments

treat the unknown as parameter to be determined by comparison with data.

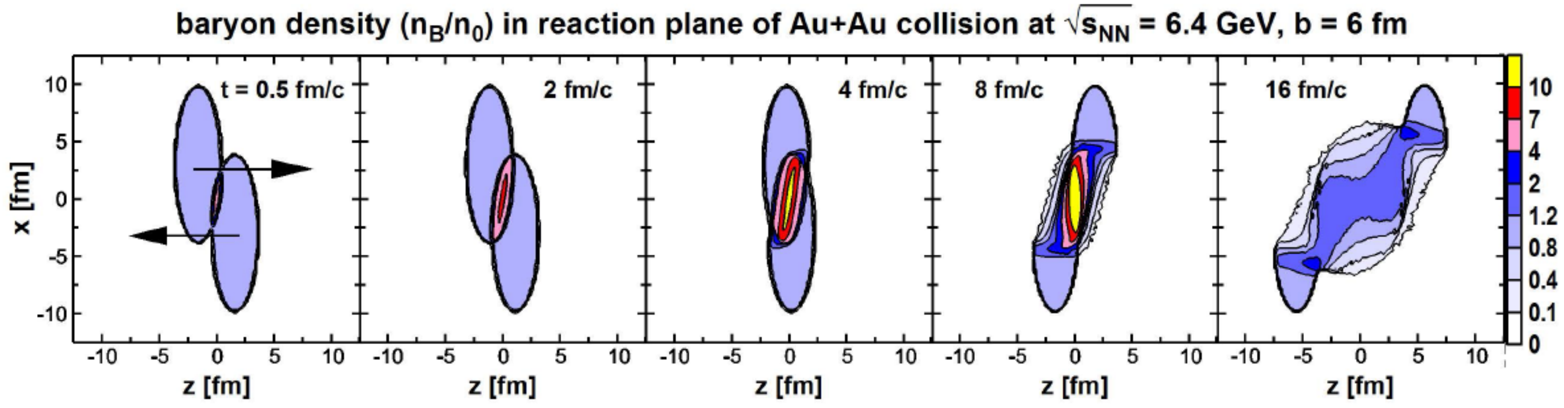
- Cross check with predictions of other observables

goal and necessity:
comprehensive understanding of all observables

Models suited for BM@N, NICA FAIR energies

Hydrodynamics

++: only input: equation of state (test of different EOS)



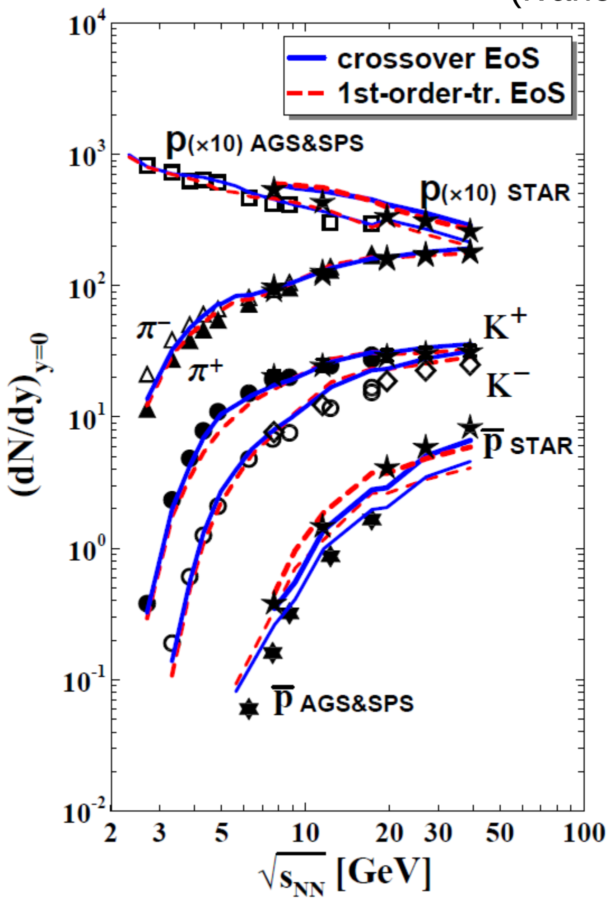
Ivanov et al. 1801.01764

to compare with data more input needed:

- -: initial condition
- -: hadronisation: according to grand canonical distribution functions
- -: eventually hadronic rescattering
  (importance seen by too low multiplicities of resonances)

BM@N/NICA/CBM physics needs more sophistication:

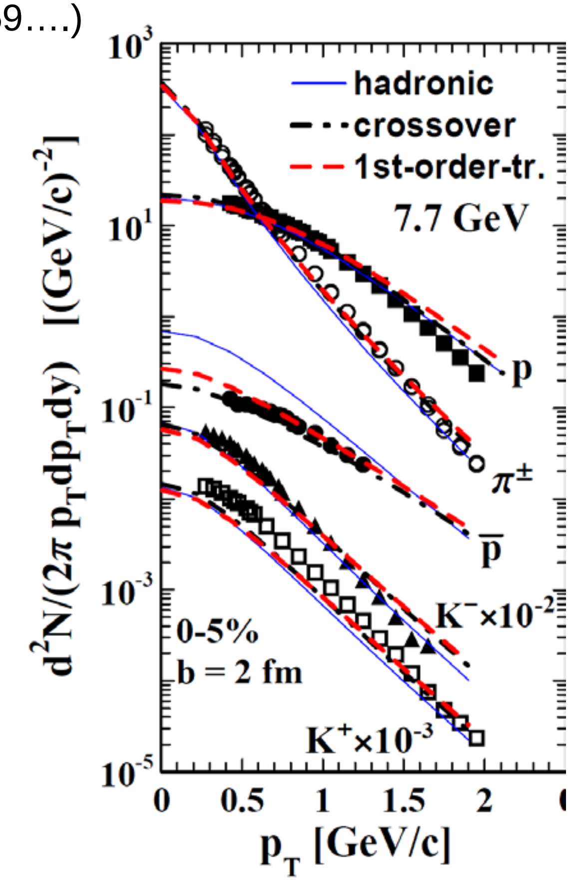
Three fluid hydrodynamics (proj, target, midrapidity source)
with phenomenomogical interaction between the fluids
(Ivanov et al. 1801.01764, 1711.07959….)



very useful results
for the bulk parts

Low p_T particle spectra →

← Excitation function



but fails in details by construction (using grand canonical particle production)
- if energy conservation becomes important (high p_T , multistrange baryons)
- if non equilibrium effects become important (details of spectra)
- if the conservation of quantum numbers becomes important

BUU (VUU, QGSM, AMPT, SMASH) equation

To say that we solve the BUU equation is quite misleading (and gives often rise to questions)

Why?

Boltzmann eq. for classical particles under the influence of an external potential U (for example an electrial field)

$$\frac{d}{dt} f(\vec{r},\vec{p},t) \equiv \frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}}\ f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\ \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Thus the particle described by f has two different types of interactions

Long range >> average distance described by U

Shart range << average distance described by collisions

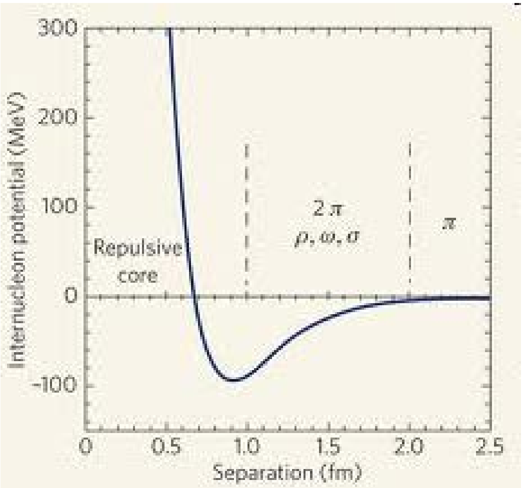
Interaction nucleons have just one interaction, V_{NN}

How two terms can appear?
Because we have cheated!! (in QMD as in BUU)

The Hamiltonian contains V = NN potential

The NN potential has a hard core

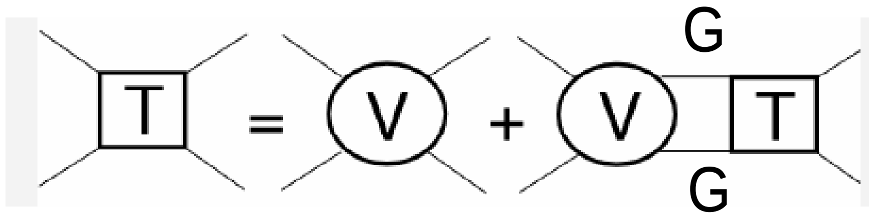
- makes TDHF calculations impossible
- makes also transport calculations impossible (Bodmer 75)
- In complete disagreement with experiment



Remember: hard core -> hard scattering
does not appear in low energy collisions
(Pauli blocked), therefore a completely different kinematics

Solution (taken over from TDHF):

Replace the NN potential V_{NN} by the solution of the Bethe-Salpeter eq. in T(G)-matrix approach (Brueckner)



$$T_\alpha(E;q,q') = V_\alpha(q,q') + \int k^2 dk \, V_\alpha(q,k) \, G^0_{Q\bar{Q}}(E,k) \, T_\alpha(E;k,q')$$

Consequences:

V_{NN} is real → T is complex = ReT + i Im T

Replaces V_{NN} In Hamiltonian (Skyrme)

σ_{elast} collisions done identically BUU (testp.) and QMD (part)

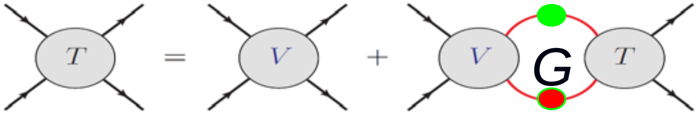
To this one adds inelastic collisions (BUU and QMD same way) !

BUU (VUU, QGSM, AMPT, SMASH) equation

Boltzmann (Vlasov)-Uehling-Uhlenbeck equation (in non-relativistic form!)
- propagation of particles in the self-generated HF mean-field potential with an
- on-shell collision term:

$$\frac{d}{dt} f(\vec{r},\vec{p},t) \equiv \frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}} f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

$$U_j = <\phi_i|T_{ij}|\phi_i> \quad ; \quad T_{ij} = V_{ij} + V_{il} G(\rho, T) T_{lj}$$



already a quantal approach: self generated mean field ~ Re T(Brückner G-matrix) NOT V_{NN}
collision term ~ Im T

Probability including Pauli blocking of fermions

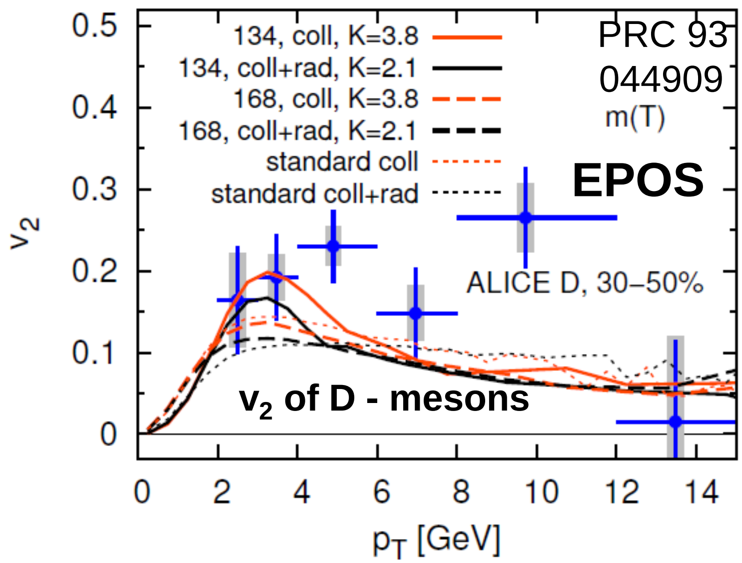
$$\left(\frac{\partial f}{\partial t}\right)_{coll} \Rightarrow \frac{1}{((2\pi)^3)^3} \int d^3p_2\, d^3p_3\, d^3p_4 \cdot w(1+2 \rightarrow 3+4) \cdot P$$

$$\times (2\pi)^3 \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\, (2\pi) \delta\left(\frac{\vec{p}_1}{2m_1} + \frac{\vec{p}_2}{2m_2} - \frac{\vec{p}_3}{2m_3} - \frac{\vec{p}_4}{2m_4}\right)$$

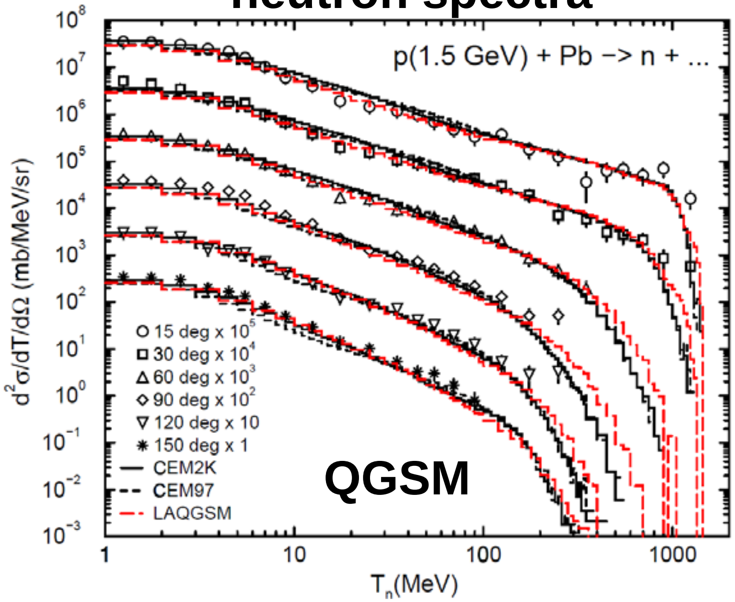
Transition probability for *1+2→3+4:* $w(1+2 \rightarrow 3+4) \Rightarrow \upsilon_{12} \cdot \frac{d^3\sigma}{d^3q}$

Collision integral can easily be extended to inelastic collisions

These transport approaches have been used for numerous studies

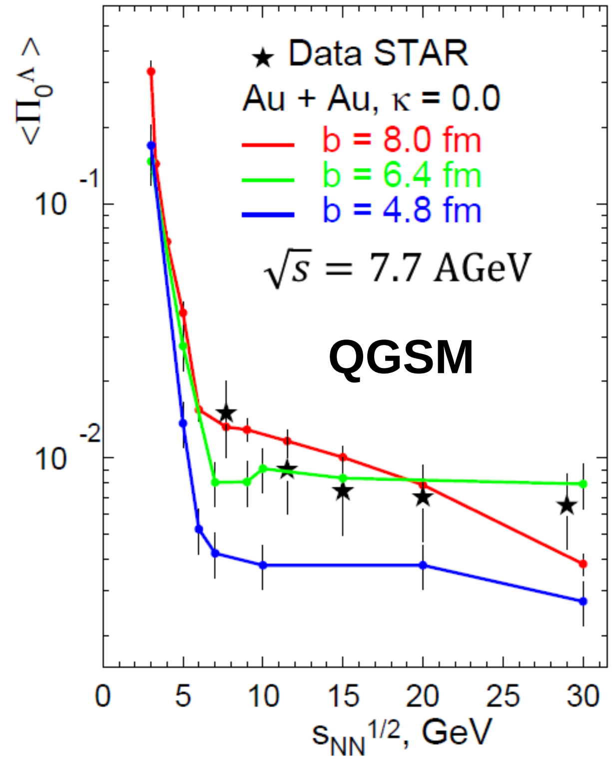


neutron spectra



K. Gudima, S. Mashnk, A. Sierk, nucl-th/001164

Λ polarisation



M. Baznat, K. Gudima, G. Prokhorov, A. Sorin, O. Teryaev and V. Zakharo, J.Phys.Conf.Ser.938,012063

Medium affects particle properties

In a dense and hot environment

- ❑ hadrons (partons) are not „on shell" ($E^2 = p^2 + m^2$) but develop a spectral function
- ❑ resonances modify their properties (width, life time)

➔ broad spectral function ➔ particles cannot be treated as quasi-particles
but are quantum objects
need resummation of the in-medium scattering matrix

❑ semi-classical BUU

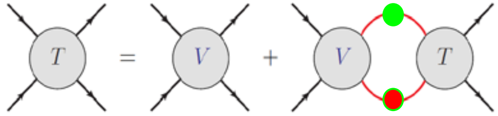
resummed 2 particle scattering matrix

❑ Kadanoff Baym equation

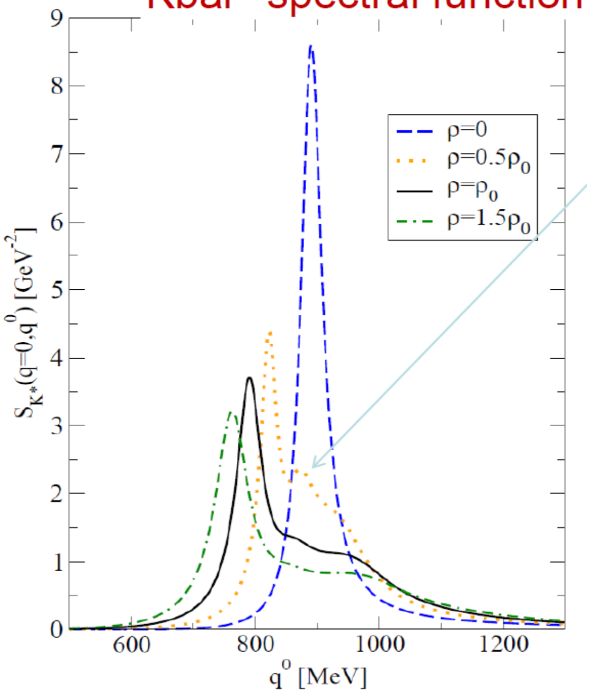
first order gradient expansion

Numerical realization: PHSD

(Parton Hadron String Dynamics)



Kbar* spectral function



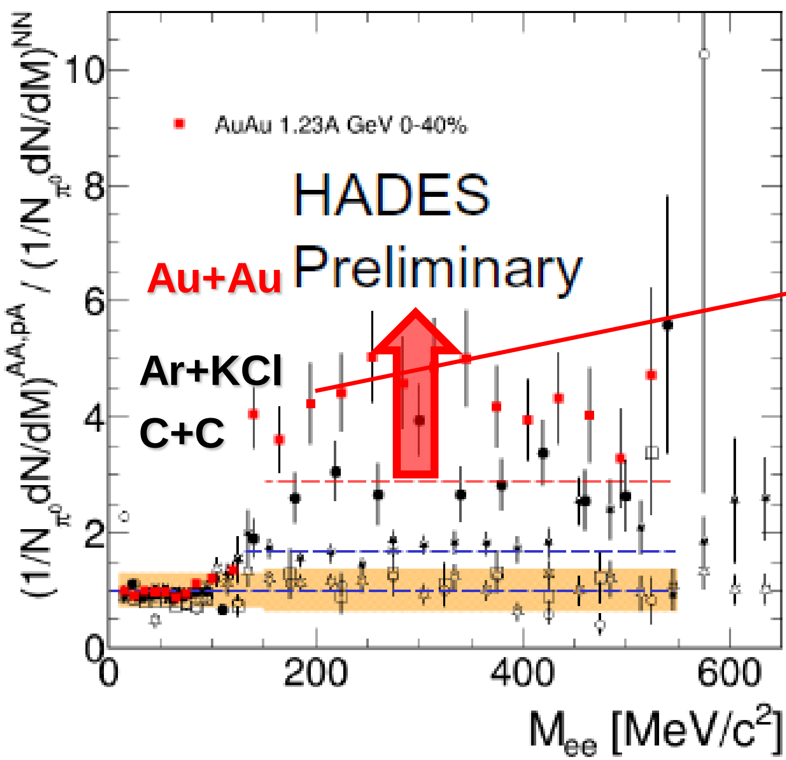
Λ(1783)N^{-1} and Σ(1830)N^{-1} exitations

Barcelona / Valencia group

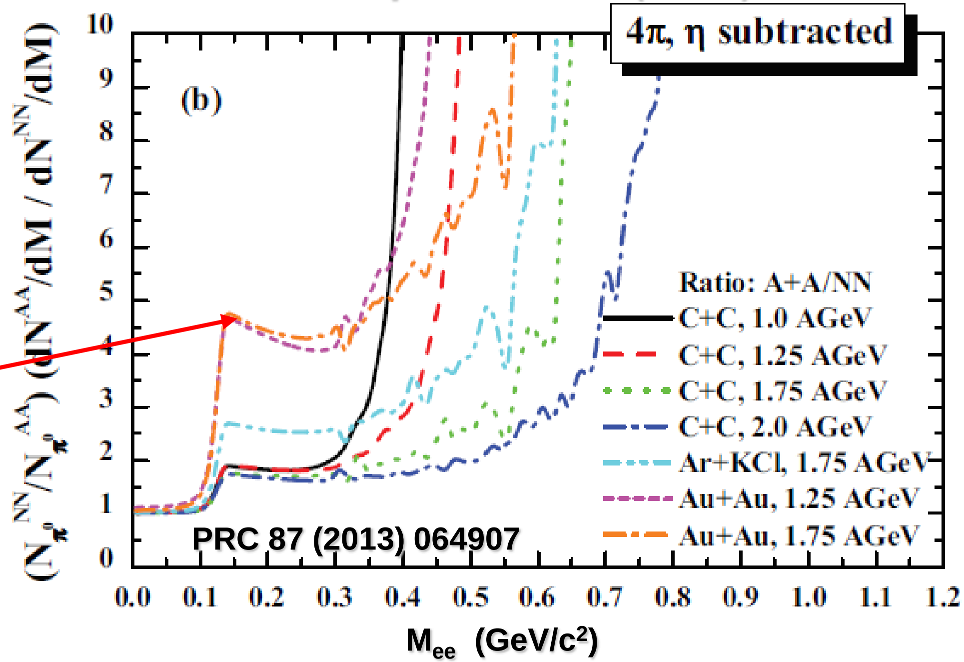
(P)HSD – transport approach based on Kadanoff Baym eqs.

Dileptons at SIS (HADES): Au+Au

CPOD-2016:
HADES preliminary: Au+Au, 1.23 A GeV



▪ **HSD predictions (2013)**



❑ **Strong in-medium enhancement** of dilepton yield in Au+Au vs. NN :

1) the **multiple Δ regeneration** – dilepton emission from intermediate Δ's which are part of the reaction cycles Δ→πN; πN →Δ and NN→NΔ; NΔ→NN

2) the **pN bremsstrahlung** which scales with N_{bin} and not with N_{part} , i.e. pions;

Physics explored transport approaches

Transport approaches presented so far
allow to investigate

Observables:

- Multiplicity of hadrons ($\sqrt{s}$) → (in medium) cross section
- Particle ratios → resonance suppression
- In-plane flow → interaction potential between hadrons
- Elliptic flow(light had) → spatial geometry of the interaction zone
- Elliptic flow (heavy had) → interaction of heavy quarks with QGP
- Dileptons→ production of resonances, heavy mesons
- Suppression of multi strange baryons → limited phase space
- Photons → more than bremsstrahlung?
- Vorticity → Λ polarization

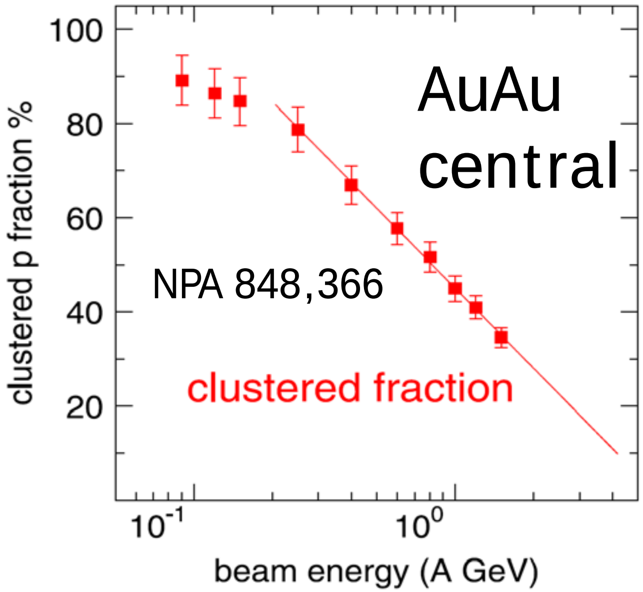
So why do we need more sophisticated model?

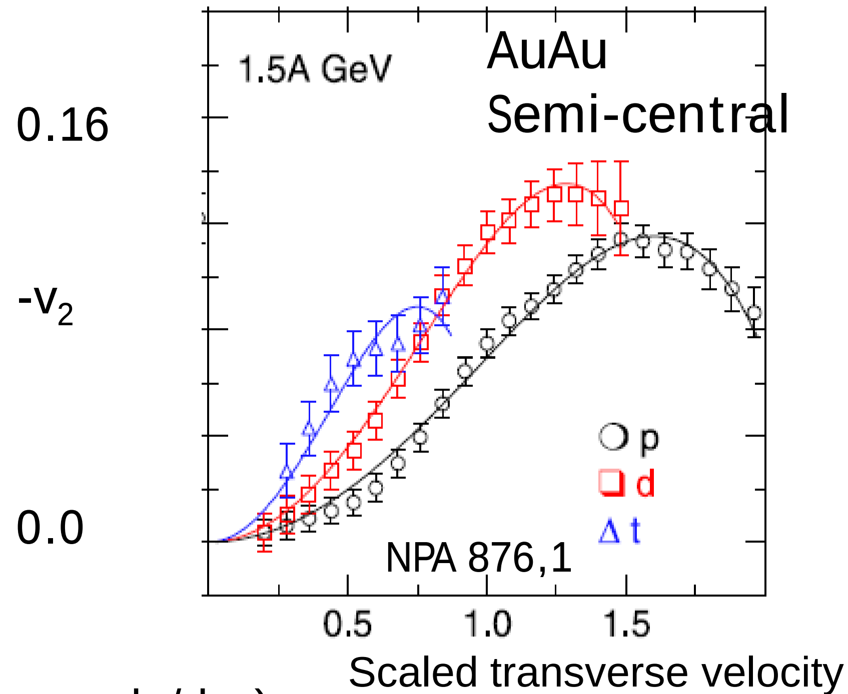
Many nucleons are in clusters

At 3 AGeV, even in central collisions:

20% of the baryons are in clusters

… and baryons in clusters have quite different properties





Without dynamical formation of fragments

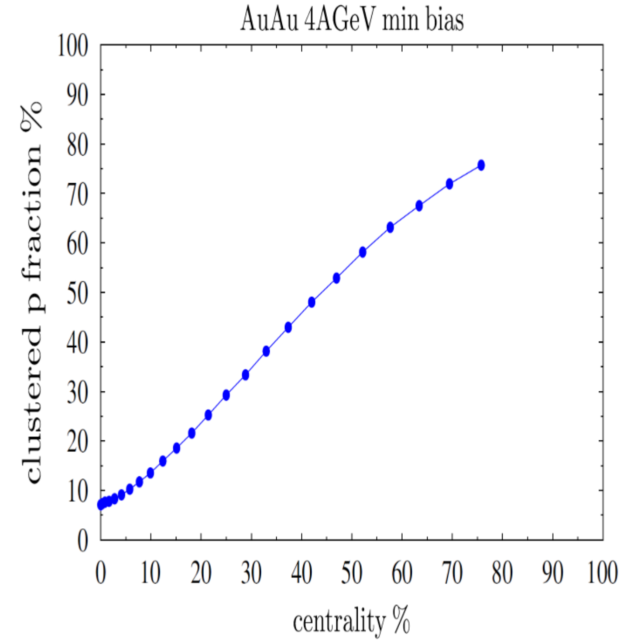
- we cannot describe the nucleon observables (v_1, v_2, dn/dp_T)

- we cannot explore the new physics opportunities like
  - hyper-nucleus formation
  - 1[st] order phase transition
  - fragment formation at midrapidity (RHIC, LHC)

If we do not describe the dynamical formation of fragments

- we cannot describe the nucl. observables (v_1, v_2, dn/dp_T)

- we cannot explore the new physics opportunities like
  - hyper-nucleus formation
  - 1st order phase transition
  - fragment formation at midrapidity (RHIC, LHC)



Present microscopic approaches fail to describe fragments at NICA/FAIR (and higher) energies

VUU(1983), BUU(1983), (P)HSD(96), SMASH(2016) solve the time evolution of the one-body phase space density → fragments only by coalescence

UrQMD is a n-body theory but has no (noy yet) potential
→ nucleons cannot be bound to fragments

(I)QMD is a n-body theory but is limited to energies < 1.5 AGeV
→ describes nicely fragments at SIS energies,
but conceptually not adapted for NICA/FAIR

N-body theory: Describe the exact time evolution of a system of N particles. All correlations of the system are correctly described and fluctuations correctly propagated.

Roots in classical physics:
A look into textbooks on classical mechanics:
If one has a given Hamiltonian

$$H(\mathbf{r}_1, .., \mathbf{r}_N, .., \mathbf{p}_1, .., \mathbf{p}_N, t)$$

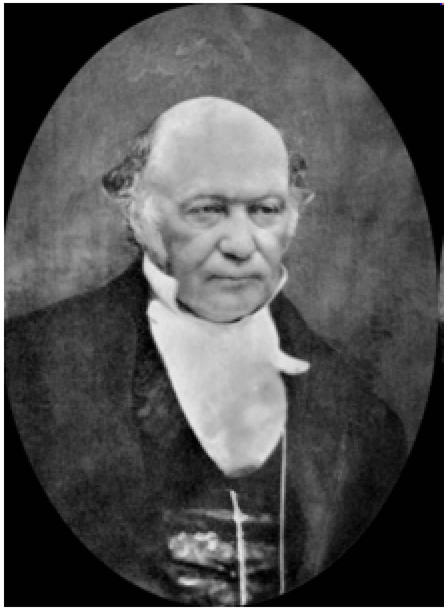
$$\frac{d\mathbf{r}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}; \quad \frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{r}_i}$$

For a given initial condition

$$\mathbf{r}_1(t=0), ..., \mathbf{r}_N(t=0), \mathbf{p}_1(t=0), .., \mathbf{p}_N(t=0)$$

the positions and momenta of all particles are predictible for all times.

fully relativistic version (PRC 87, 034912)
too time consuming (but also not necessary)



William Hamilton

Roots in Quantum Mechanics

Remember QM cours when you faced the problem

- we have a Hamiltonian $\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V$
- the Schrödinger eq. $\hat{H}|\psi_j> = E_j|\psi_j>$

  has no analytical solution
- we look for the ground state energy E_0

Walther Ritz

Ritz variational principle:

Assume a trial function $\psi(q, \alpha)$ which contains one (or more) adjustable parameter α, which is varied to find a lowest energy configuration.

$$\frac{d}{d\alpha} < \psi|\hat{H}|\psi >= 0$$

determines α for which $\psi(q, \alpha)$ is closest to the true ground state wfct and

$$< \psi|\hat{H}|\psi > > E_0$$

Quantal N-body dynamics is alsobased on a variational principle (Koonin, TDHF)

Take trial wavefct with time dependent parameters and solve

$$\frac{< \psi_N | i\frac{d}{dt}\hat{H} | \psi_N >}{< \psi_N | \psi_N >} = 0 \qquad (1)$$

OMD trial wavefct for one particle (Gaussian):

$$\psi_i(q_i, q_{0i}, p_{0i}) = Cexp[-(q_i - q_{0i} - \frac{p_{0i}}{m}t)^2/4L] \cdot exp[ip_{0i}(q_i - q_{0i}) - i\frac{p_{oi}^2}{2m}t]$$

For N particles: $\psi_N = \prod_{i=1}^{N} \psi_i(q_i, q_{0i}, p_{0i})$ QMD

For this QMD trial wavefct eq. (1) yields

$$\frac{dq}{dt} = \frac{\partial < H >}{\partial p} \quad ; \quad \frac{dp}{dt} = -\frac{\partial < H >}{\partial q}$$

For Gaussian wavefct eq. of motion very similar to Hamilton's eqs.

Potential: density dependent two body potential adjusted to nuclear EOS

All elastic and inelastic collisions are treated as in PHSD - therefore the spectra of produced particles are similar to PHSD results

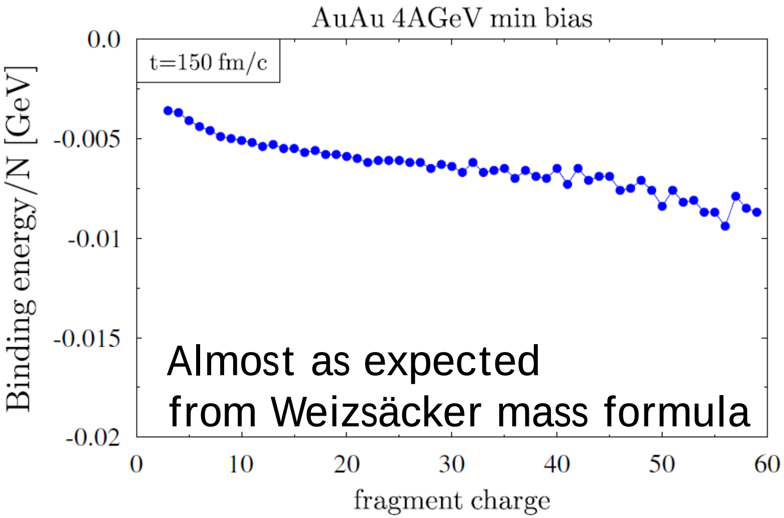
→ PHQMD : Parton Hadron Quantum Molecular Dynamics

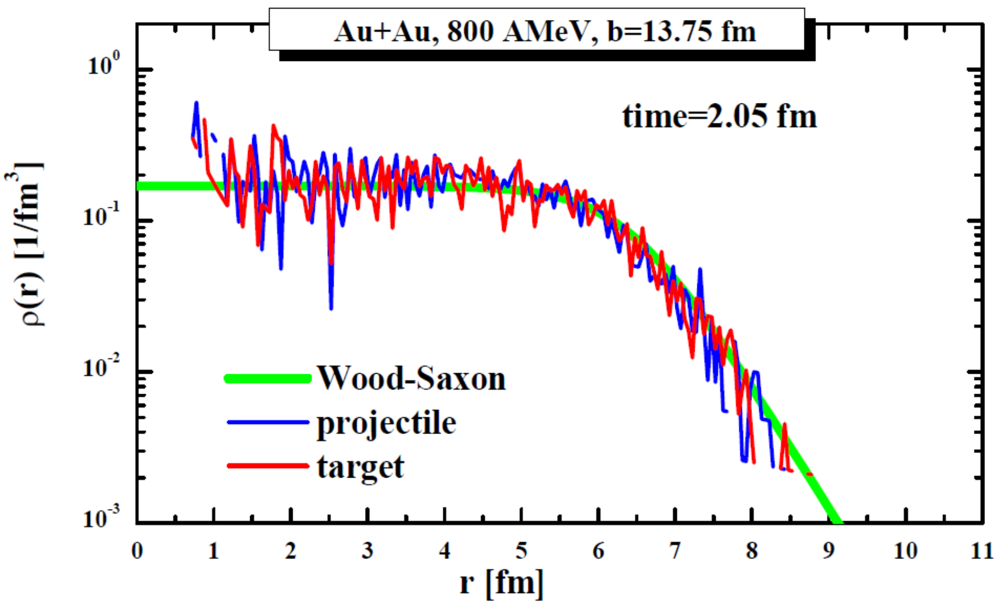
Initial condition in PHQMD

to describe fragment formation and
to guaranty the stability of nuclei

The initial distributions of nucleons in proj and targ has to be carefully modelled:

- Right density distribution
- Right binding energy





local Fermi gas model for the momentum distribution

Potential in PHQMD

Relativistic molecular dynamics (PRC 87, 034912) too time consuming

The potential interaction is most important in two rapidity intervals:

- at beam and target rapidity where the fragments are initial – final state correlations and created from spectator matter
- at midrapidity where – at a late stage - the phase space density is sufficiently high that small fragments are formed

In both situations we profit from the fact that the relative momentum between neighboring nucleons is small and therefore nonrelativistic kinematics can be applied. Potential interaction between nucleons

$$
\begin{aligned}
U_{ij}(\mathbf{r},\mathbf{r}') &= U_{\mathrm{Skyrme}} + U_{\mathrm{Coul}} \\
&= \frac{1}{2} t_1 \delta(\mathbf{r}-\mathbf{r}') + \frac{1}{\gamma+1} t_2 \delta(\mathbf{r}-\mathbf{r}') \rho^{\gamma-1}(\mathbf{r}) \\
&\quad + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}-\mathbf{r}|}.
\end{aligned}
$$

t_1 , t_2 and γ adjusted to reproduce a given nuclear equation of state

$$< U(\mathbf{r_i}) > \; = \sum_j \int d^3r d^3r' d^3p d^3p' \; U_{ij}(\mathbf{r},\mathbf{r}') f_i(\mathbf{r},\mathbf{p},t) f_j(\mathbf{r}',\mathbf{p}',t)$$

$$< U_i(\mathbf{r_i},t) > = \alpha \left(\frac{\rho_{int}}{\rho_0}\right) + \beta \left(\frac{\rho_{int}}{\rho_0}\right)^{\gamma}$$

To describe the potential interactions in the spectator matter we transfer the Lorentz-contracted nuclei back into the projectile and target rest frame, neglecting the small time differences

$$\rho_{int}(\mathbf{r_i},t) \;\rightarrow\; C \sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r_i^T}(t)-\mathbf{r_j^T}(t))^2} \cdot \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r_i^L}(t)-\mathbf{r_j^L}(t))^2} .$$

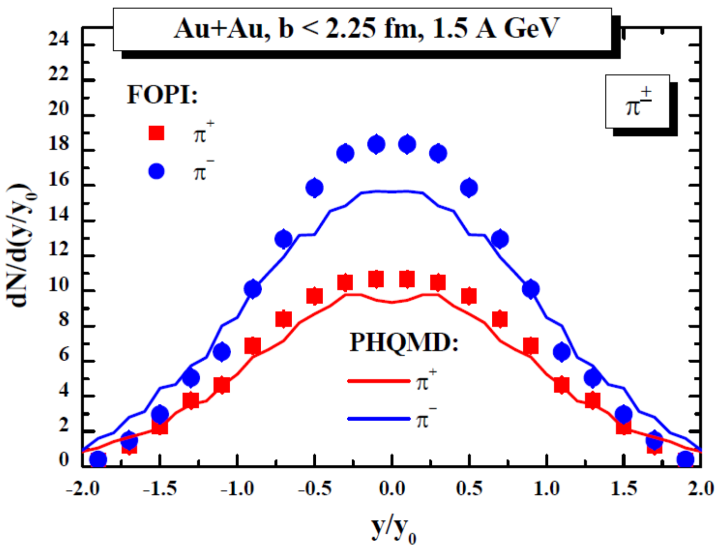
For the midrapidity region γ → 1. and we can apply nonrelativisitic kinematics as well

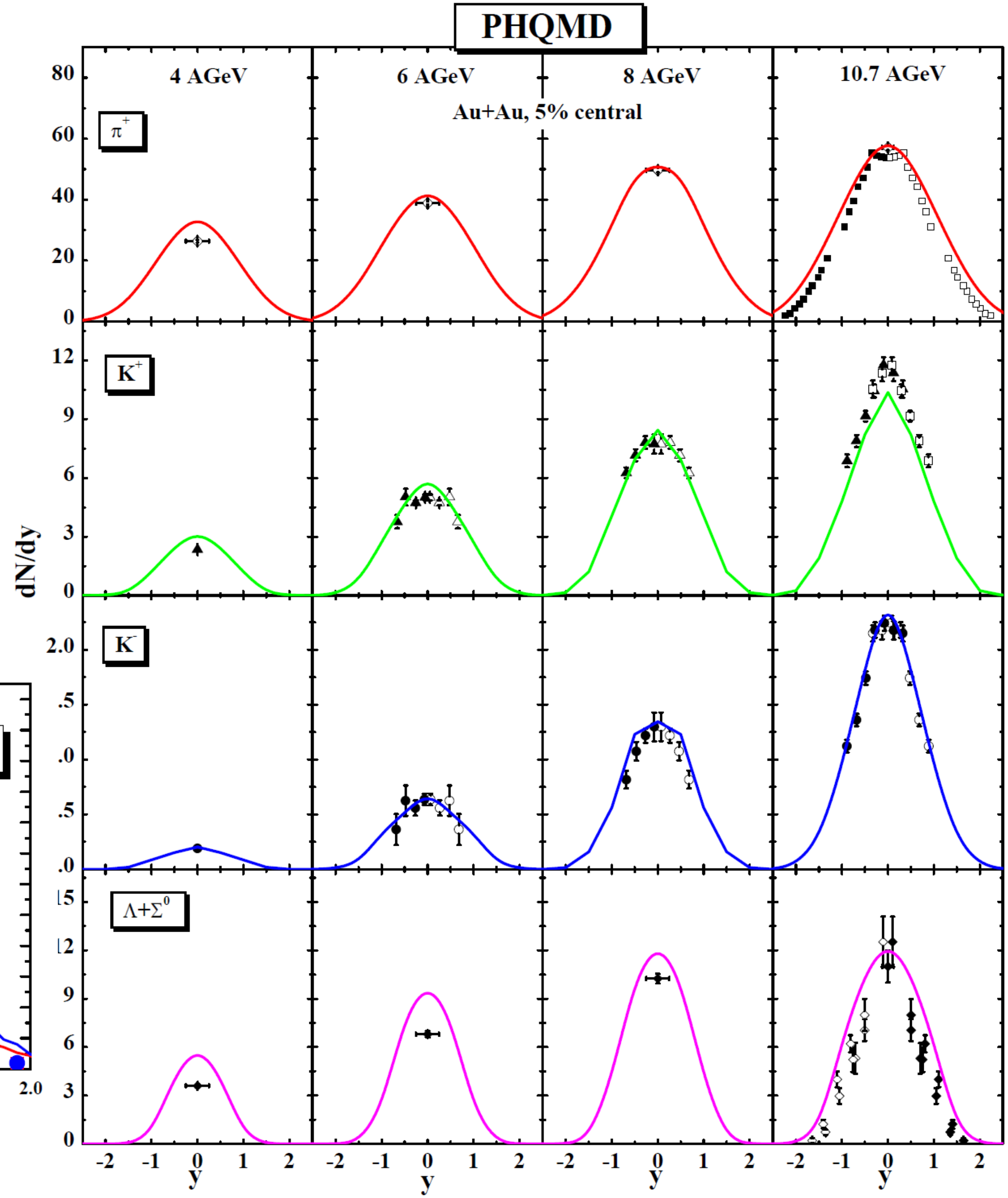
All elastic and inelastic collisions are treated as in PHSD - therefore the spectra of produced particles are similar to PHSD results

First Results of PHQMD

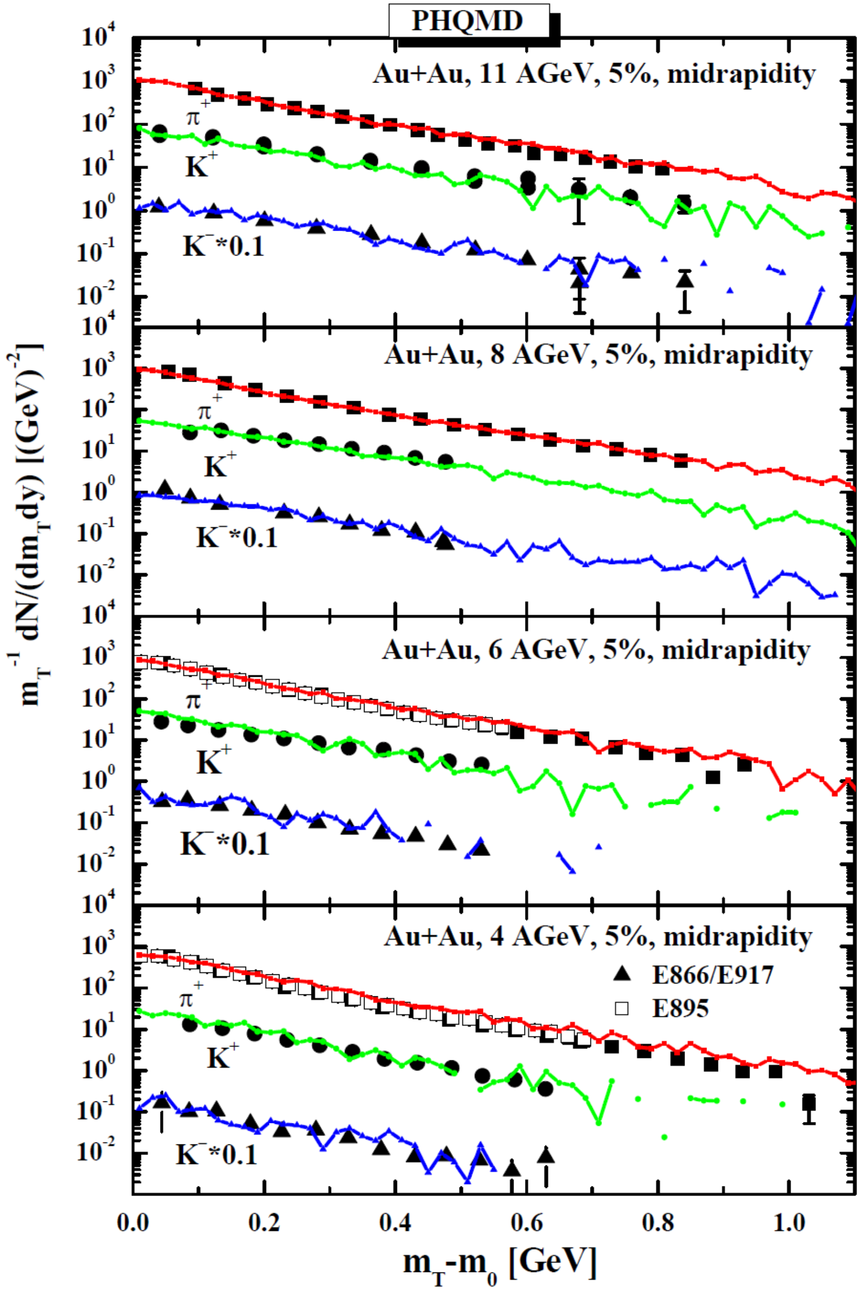
Produced particles
(dominated by collisions)

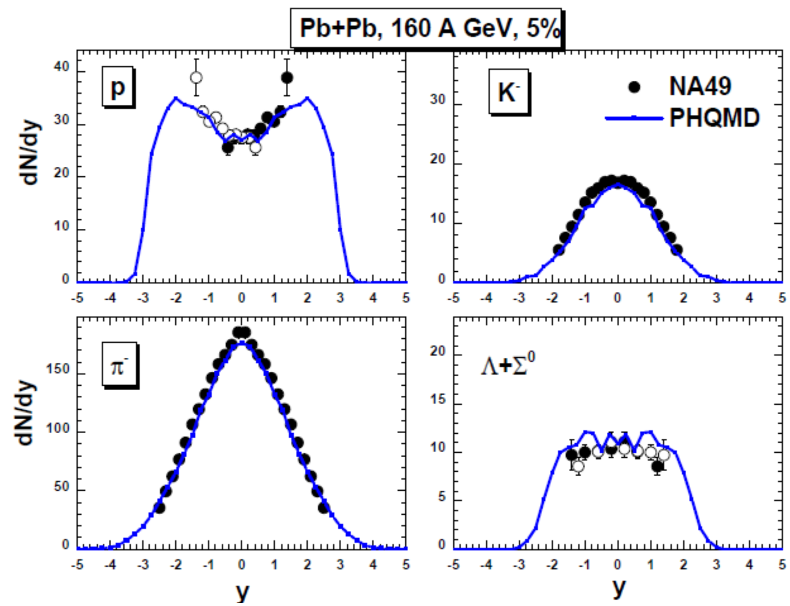
are in agreement with experiment
at SIS/AGS/NICA/FAIR energies

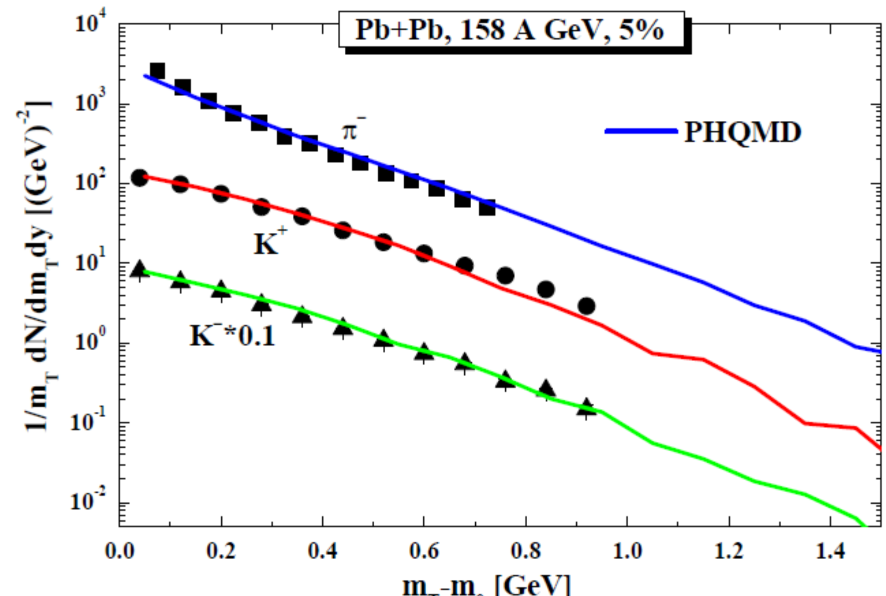




AGS and SPS energies







How to define fragments in transport theories which propagate nucleons ?

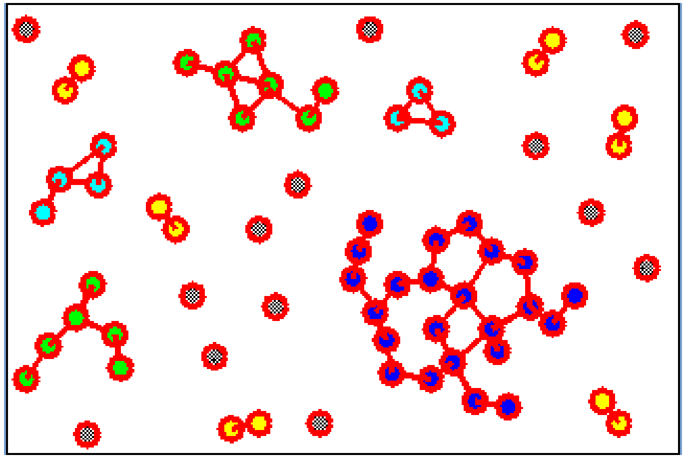
I. **Minimum Spanning Tree** (MST) is a cluster recognition method applicable for the **(asymptotic t → ∞) final state** where coordinate space correlations only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are bound if their distance in coordinate space is

$$|r_i - r_j| \leq 2.5\, fm$$

2. A particle is bound to a cluster if it is bound with at least one particle of the cluster.



Particles with large relative momentum are finally not at the same position
→ Additional momentum cuts (coalescence) change little:

Drawback: Does not allow to study HOW the fragment are formed

Early Fragment identification (SACA/FRIGA)

PLB301:328,1993

Nuovo Cim. C39 (2017) 399

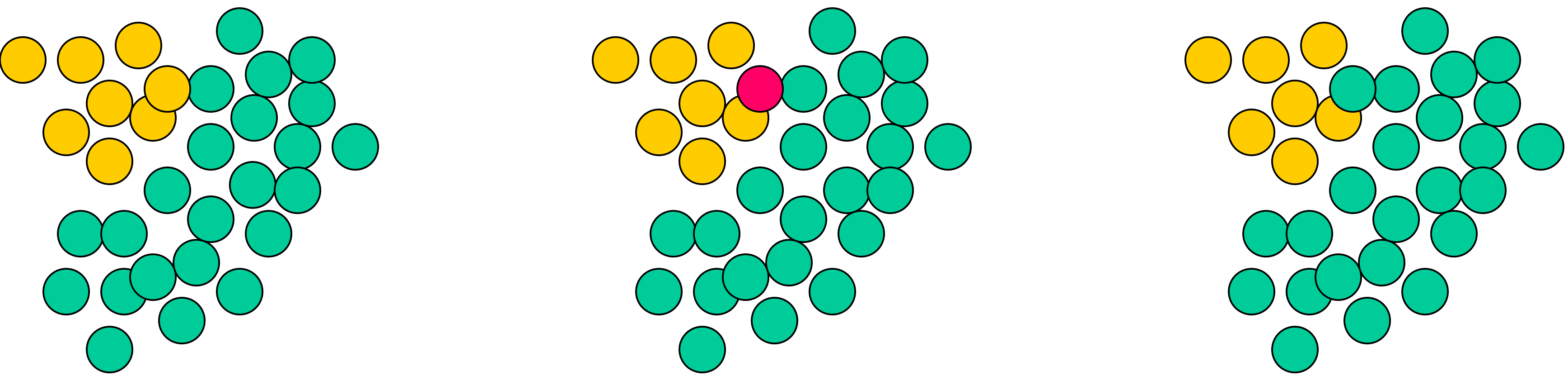
a) Take the positions and momenta of all nucleons at time t.

b) Combine them in all possible ways into all kinds of fragments or leave them as single nucleons

c) Neglect the interaction among clusters

d) Choose that configuration which has the highest binding energy

This configuration has a large overlap with the final fragment distribution



Take randomly 1 nucleon out of a fragment

Add it randomly to another Fragment

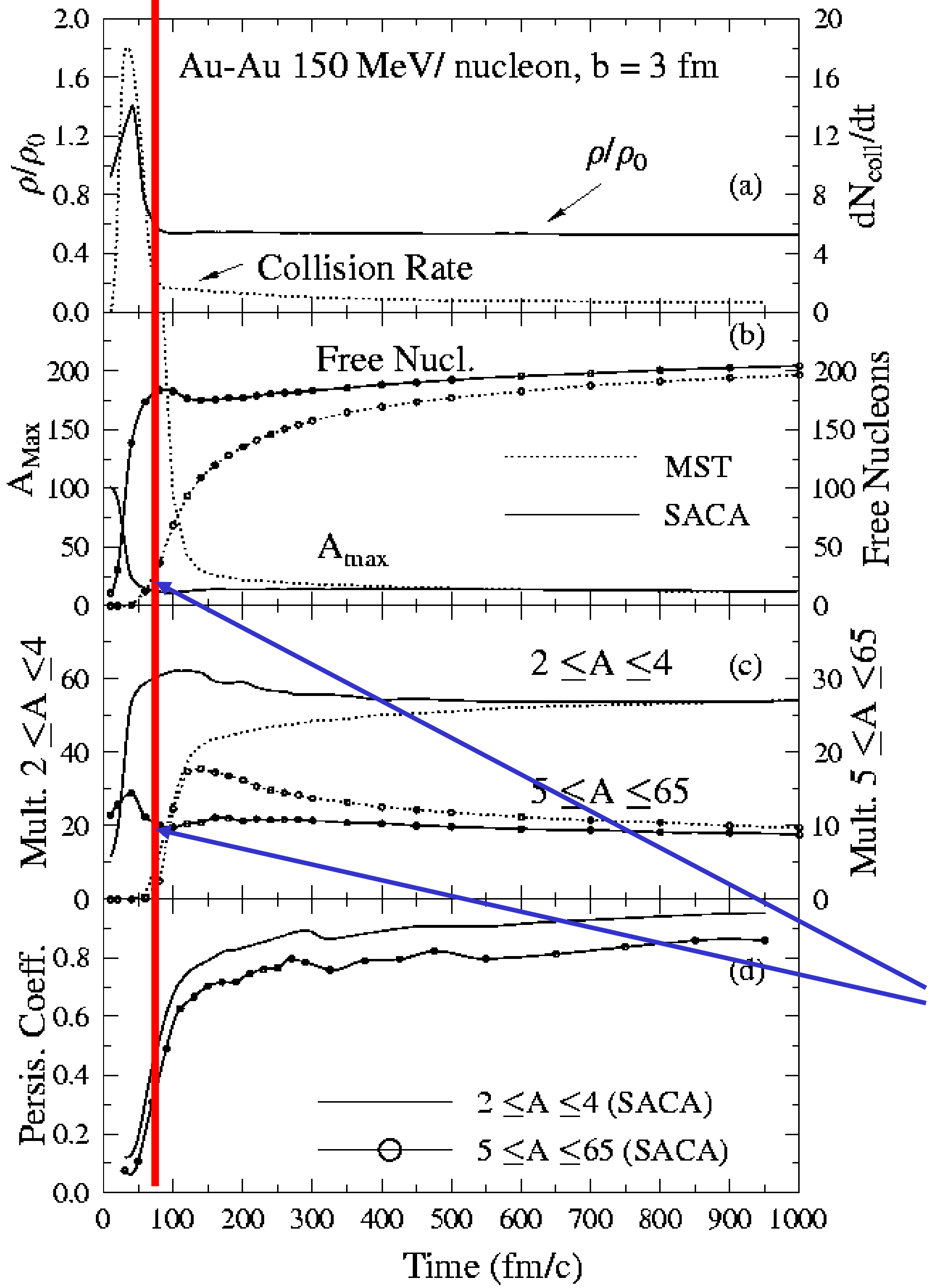
$$E=E^1_{kin} + E^2_{kin} + V^1 + V^2$$

$$E'=E^{1'}_{kin} + E^{2'}_{kin} + V^{1'} + V^{2'}$$

If E’ < E take the new configuration

If E’ > E take the old with a probability depending on E’-E

Repeat this procedure many times → Leads automatically to the most bound configuration

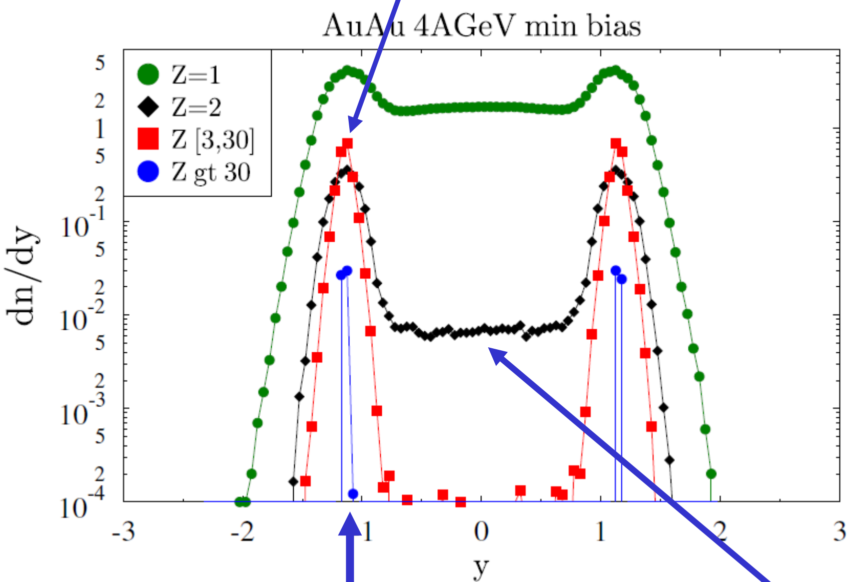


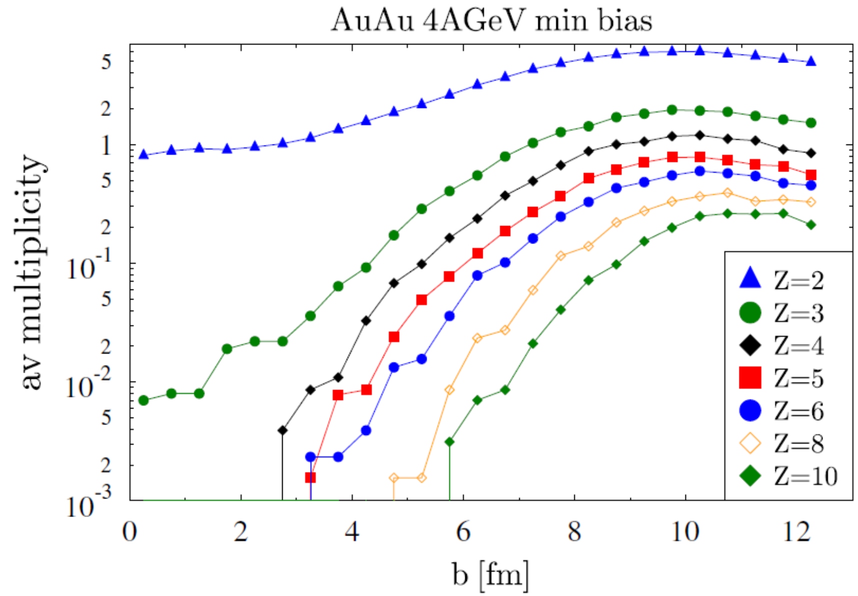
SACA/FRIGA can identify the fragment pattern very early as compared to the Minimum Spanning Tree (MST) which requires a minimal distance in coordinate space between two nucleons to form a fragment

After $t=1.5t_{pass}$ Amax and multiplicities of intermediate mass fragments do not change anymore

First Results of PHQMD

- ❑ Only for 10% most central events fragments do not play a role
- ❑ Heavy fragments appear only in the residue rapidity range
- ❑ Complicated fragment pattern for larger impact parameters (acceptance??)
- ❑ M_Z (b) is different for each fragment charge





b dependence non-trivial

There are two kinds of fragments

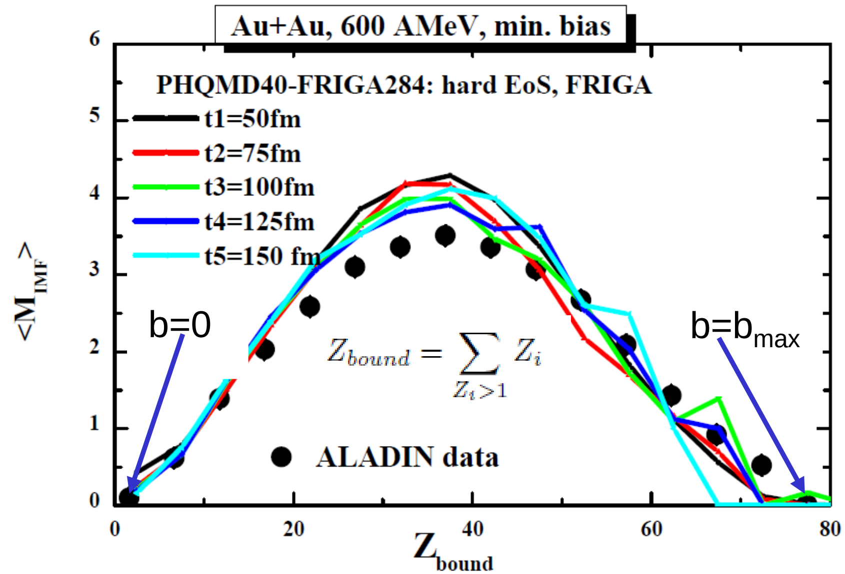
- ❑ formed from spectator matter
  close to beam and target rapidity
  initial-final state correlations
  HI reaction makes spectator matter unstable

- ❑ formed from participant matter
  created during the expansion of fireball
  “ice” (E_{bind} ≈8 MeV/N) in “fire”(T≥ 100MeV)
  origin not known yet
  seen from SIS to RHIC
  (quantum effects are important)

First Results of PHQMD

Spectator Fragments

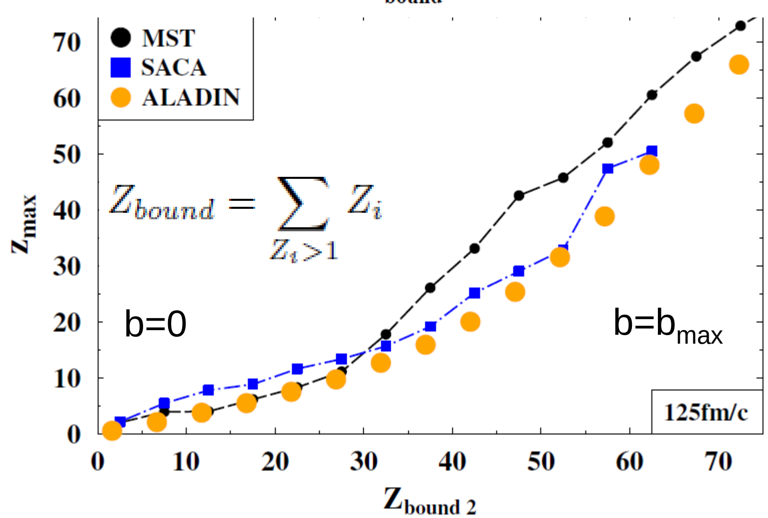
exp. measured up to E_{beam} =1 AGeV (ALADIN)

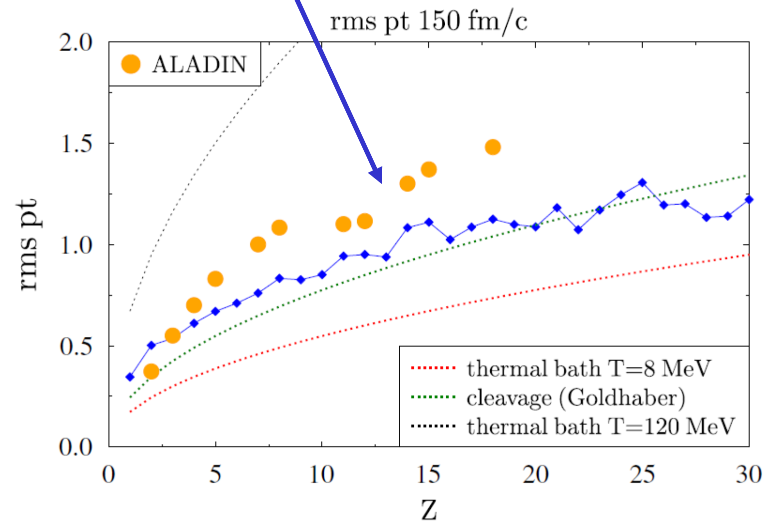


agreement for very complex fragment observables like the

- ❑ Energy independence of "rise and fall"
- ❑ Largest fragment (Z_{bound})

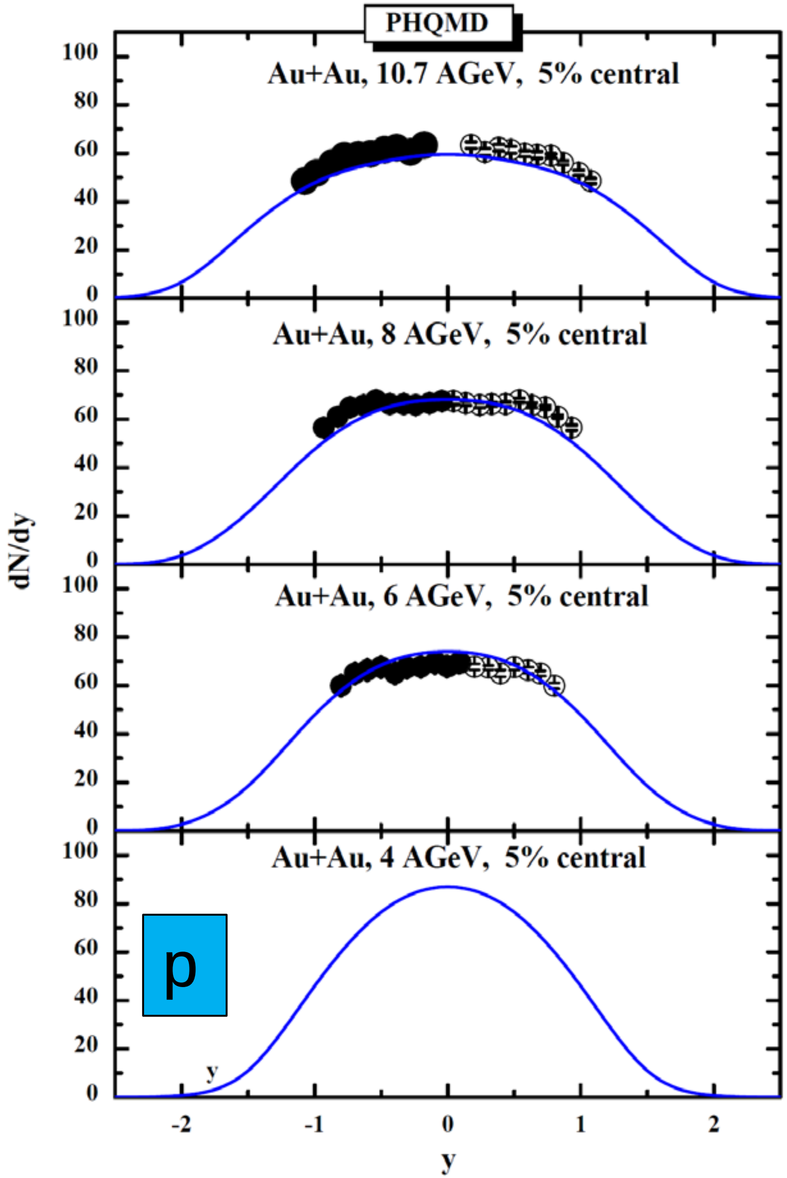
rms(p_t) shows $\sqrt{Z}$ dependence

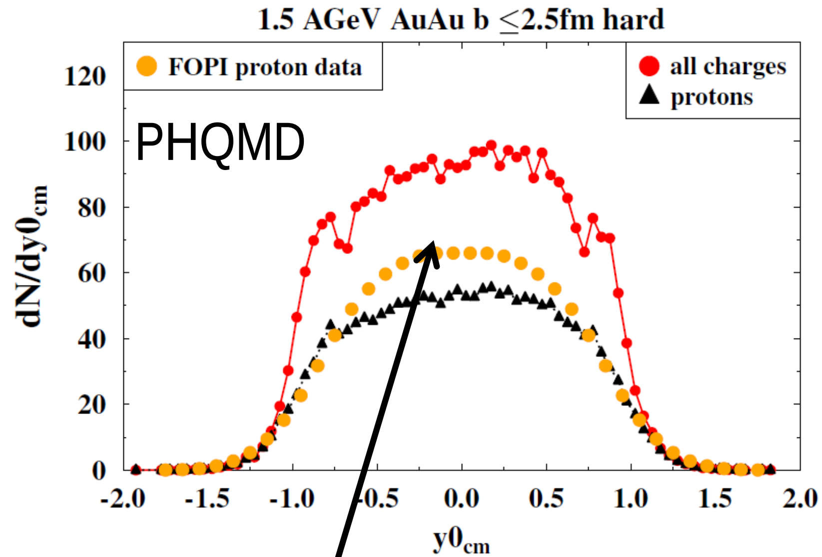




First Results of PHQMD

Protons at midrapidity well described

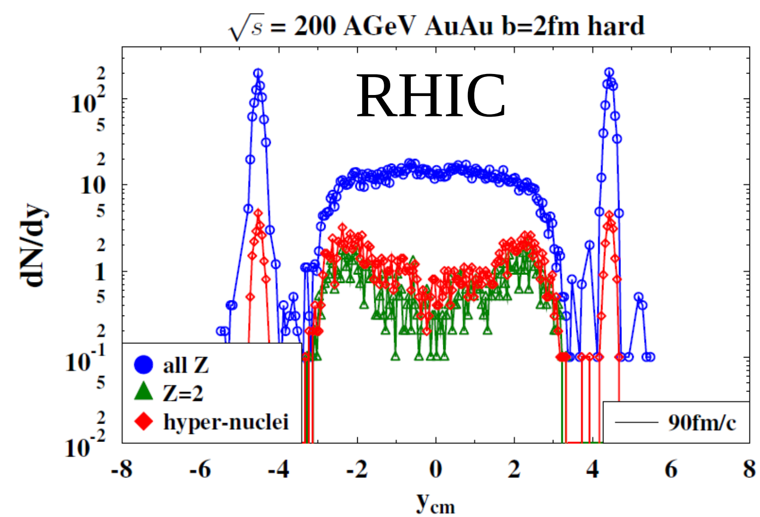




1.5 AGeV central

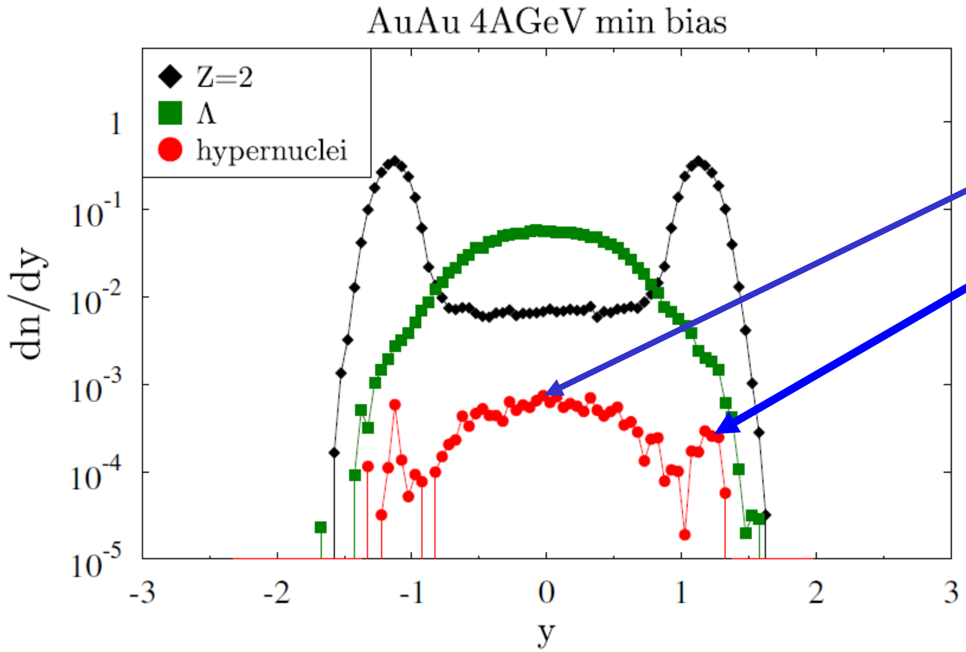
- 30% of protons bound in cluster

To improve: better potential for small clusters



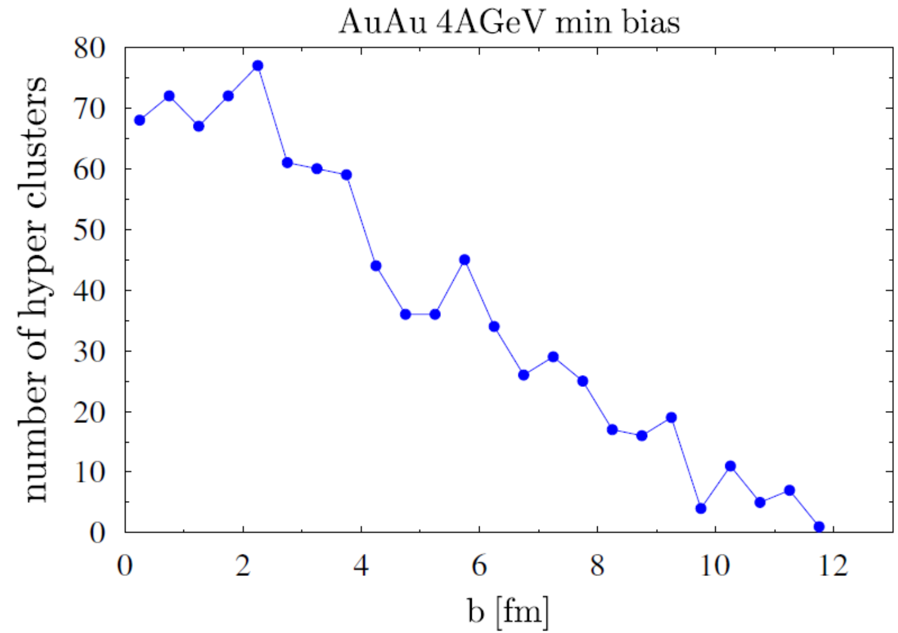
.. and what about hyper-nuclei ?

First Results of PHQMD



There are hyper-nuclei
- at midrapidity (small)
- at beam rapidity (large) few in number but more than in other reactions to create hyper-nuclei

Central collisions → light hyper-nuclei
Peripheral collisions → heavy hyper-nuclei



Conclusions and Perspectives

The number of transport theories available for HI studies is impressive

Hydrodynamics (coarse features)

BUU type (detailed 1-body dynamics)

PHSD (+ Medium modifications of hadrons and partons)

QMD (n-body dynamics)

To this we added a new one

PHQMD (n-body dynamics) which allows to study fragment and hyper-fragment production at all beam energies

First results show

- a good agreement with available experimental data
- the possibility to produce hyper-nuclei in quantity
- the dynamical formation of midrapidity fragments

and more will come

THANK YOU!!