Anisotropic Equations of State of a vector boson gas in a constant magnetic field: Astrophysical implications

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Motivation

- Nucleon superfluidity in Neutron Stars (NS) core and crust (1960's).
- Observational evidence of superfluidity in the interior of the NS at Cassiopeia A (Page, Shternin, 2011).
- Experimental observation of BCS-BEC crossover (1999).
- Bose-Einstein condensate stars (Chavanis 2012, Latifah 2014):

Stars formed totally by a an interacting Bose gas of mass $2m_n$:

  - Masses an radii in the order of those observed for NS.
  - Do not take into account the interaction of the bosons with the NS magnetic field.

Neutron star

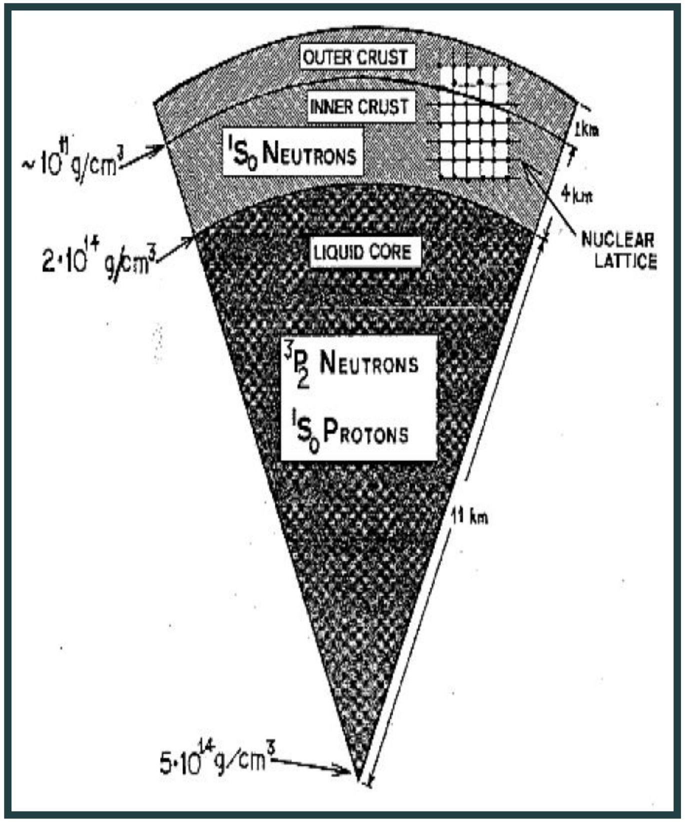
$M \sim 1 - 3M_{\odot}$

$R \sim 10\ km$

$N \sim 10^{30-38} cm^{-3}$

$T \sim 10^{9-11} K$

$B \sim 10^{12-18}$G



Motivation

Having a magnetic boson introduces new phenomelogy to the problem, in particular:

- **Bose-Einstein ferromagnetism (BEF)**: the appearance of an spontaneous magnetization below the BEC critical temperature.

Might be related to magnetic field generation.

- **Anisotropic Equations of State (EoS):** $P_{\parallel} = -\Omega$

$$P_{\perp} = -\Omega - MB$$

Influence the shape of astrophysical objects.

- **Quantum magnetic collapse (QMC)**: $P_{\perp} = 0$, i.e. $\Omega = -MB$ for certains values of temperature, magnetic field and particle density.

Might be related to matter ejection from the star and jets.

Our aim is to study BEF and QMC for a gas of neutral vector bosons (form by two paired neutrons) in astrophisical conditions

(i.e for particle densities and magnetic fields in the order of those of NS).

A vector boson gas in a constant magnetic field

The thermodynamic potential for a magnetized neutral vector boson gas in the low temperature limit is

$$\Omega = \Omega_{st}(\boldsymbol{N}, \boldsymbol{T}, \boldsymbol{b}) + \Omega_{vac}(\boldsymbol{b})$$

with

$$\Omega_{st}(N,T,b) \cong -\sqrt{\frac{\left(m\sqrt{1-b}\right)^3 T^5}{2(2-b)^2\pi^5}}\, L_{5/2}\big(z(N)\big),$$

$$\Omega_{vac}(b) = -\frac{m^4}{288\,\pi}\{b^2(66-5b^2) - 3(6-2b-b^2)(1-b)^2 ln(1-b)$$
$$-3(6+2b-b^2)(1+b)^2\, ln(1+b)\}.$$

- $L_n(x) = \sum_{i=1} \frac{x^i}{i^n}$,
- $z = e^{(\mu-m)/T}$
- $\boldsymbol{T \ll m\ (T \ll 10^{13}\mathrm{K})}$

$\boldsymbol{m = 2m_n, m_n}$ **the neutron mass**

$\boldsymbol{\kappa = 2\,\mu_n, \mu_n}$ **the neutron magnetic moment**

$\boldsymbol{b = B/B_c}$

$\boldsymbol{B_c = m/2\,\kappa = 2.98 \times 10^{20}\mathrm{G}}$

G. Quintero Angulo, A. Pérez Martínez and H. Pérez Rojas. Phys. Rev. C 96, 045810 (2017)

Bose-Einstein Ferromagnetism

Magnetization can be computed as

$$M = -\frac{\partial \Omega}{\partial B}$$

For low temperatures and high particle densities:

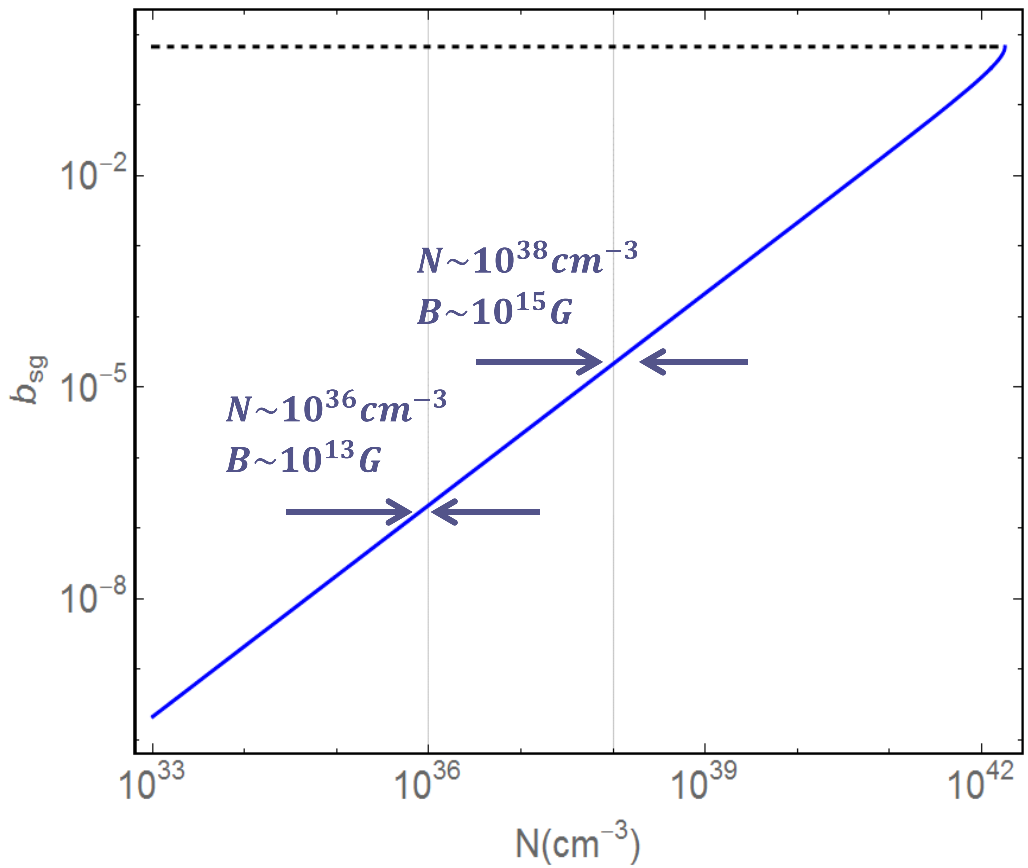
$$M = \frac{\kappa}{\sqrt{1-b}} N$$

Self-magnetization:

$$H = B - 4\pi M, \qquad H = 0$$

$$b = b_{sg}: \; b_{sg}\sqrt{1-b_{sg}} = 4\pi\kappa N/B_c$$

$$N_{max} = 1.81 \times 10^{42} cm^{-3}$$
$$B_{max} = {}^{2}/_{3}\, B_c \sim 10^{20} G$$



G. Quintero Angulo, A. Pérez Martínez and H. Pérez Rojas. Phys. Rev. C 96, 045810 (2017)

Anisotropic EoS

$$E = m\sqrt{1-b}\,N - \frac{3}{2}\Omega_{st}(N,T,b) + \Omega_{vac}(b)$$

$$P_{\parallel} = -\Omega_{st}(N,T,b) - \Omega_{vac}(b)$$

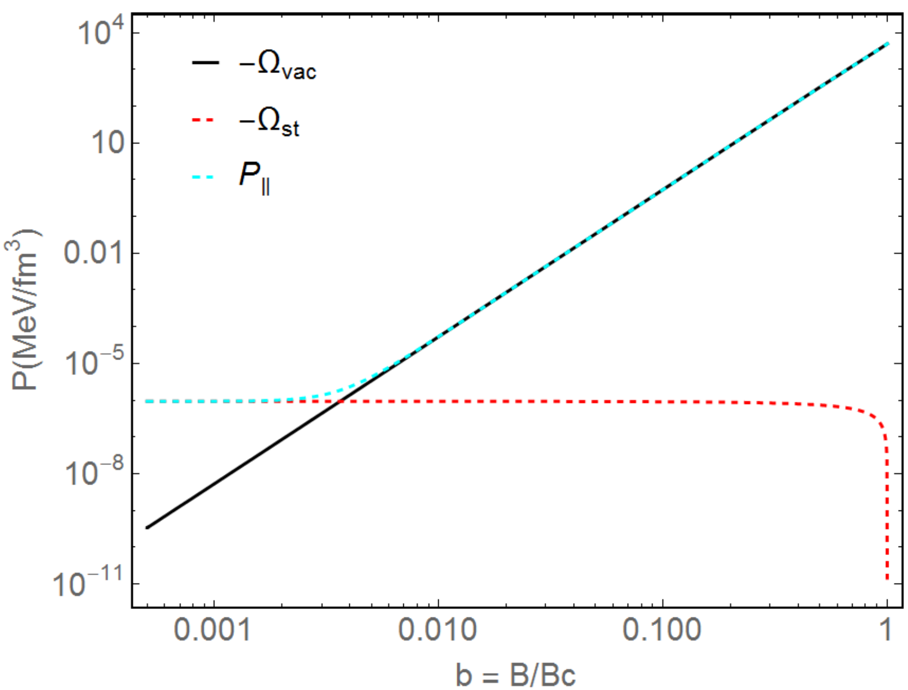
$$P_{\perp} = -\Omega_{st}(N,T,b) - \Omega_{vac}(b) - MB$$

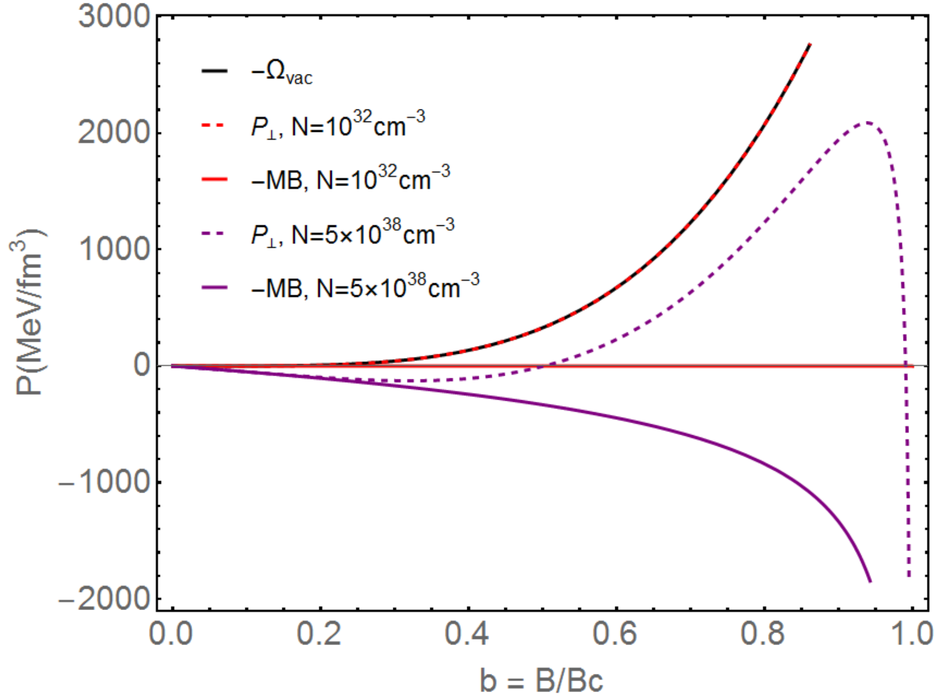
$$b = B/B_c$$

$$B_c = m/2\,\kappa = 2.98 \times 10^{20}\text{G}$$

$$T = 8 \times 10^{8} K$$

$$N = 10^{32} cm^{-3}$$

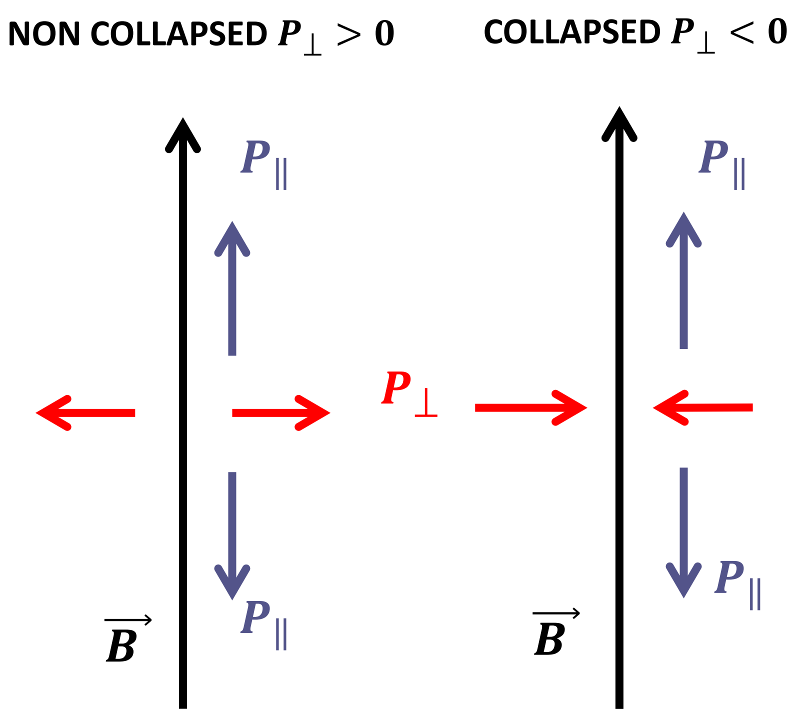


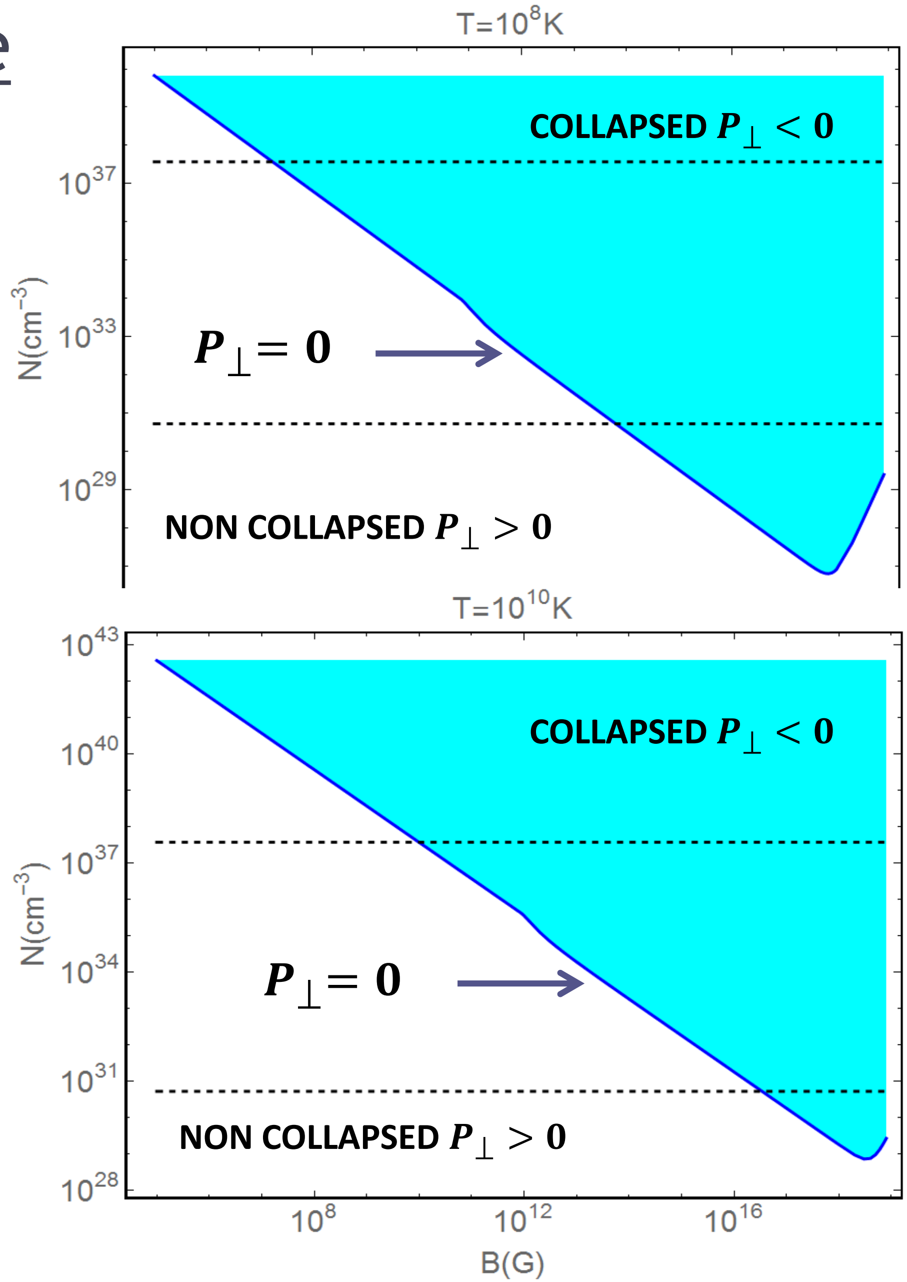


G. Quintero Angulo, A. Perez Martinez and H. Perez Rojas, Astron. Nachr. (2017) doi:10.1002/asna.201713448.

Quantum magnetic collapse

$N, B\ :\ P_\perp = 0$ for a fixed temperature





Non collapsed regime: Magnetized boson stars

- The star is composed by a non interacting gas of spin-1 bosons with mass $\boldsymbol{2m_N}$.
- The magnetic field is self-generated.
- Gravity is counterbalanced by the vacuum pressure $-\Omega_{vac}$.
- Mass-radius relation is obtained through the TOV + EoS scheme: $P_{\parallel} \cong P_{\perp}$

$$E = m\sqrt{1-b_{sg}}\,N - \frac{3}{2}\Omega_{st}\left(N,T,b_{sg}\right) + \Omega_{vac}\left(b_{sg}\right)$$

$$P = -\Omega = -\Omega_{st}\left(N,T,b_{sg}\right) - \Omega_{vac}\left(b_{sg}\right)$$

$$b_{sg}\sqrt{1-b_{sg}} = 4\pi\kappa N/B_c$$

$$+$$

TOV

$$\frac{dM(r)}{dr} = 4\pi r^2 E(r)$$

$$\frac{dP}{dr} = -G\frac{(E(r)+P(r))(M(r)+4\pi r^3 P(r))}{r^2 - 2rGM(r)}$$

- With respect to previous boson star models, we expect it will provide a natural way to include the magnetic field in the star description.

Magnetized boson stars: Preliminar results

$$E = m\sqrt{1-b_{sg}}\,N - \frac{3}{2}\Omega_{st}\left(N,T,b_{sg}\right) + \Omega_{vac}\left(b_{sg}\right)$$

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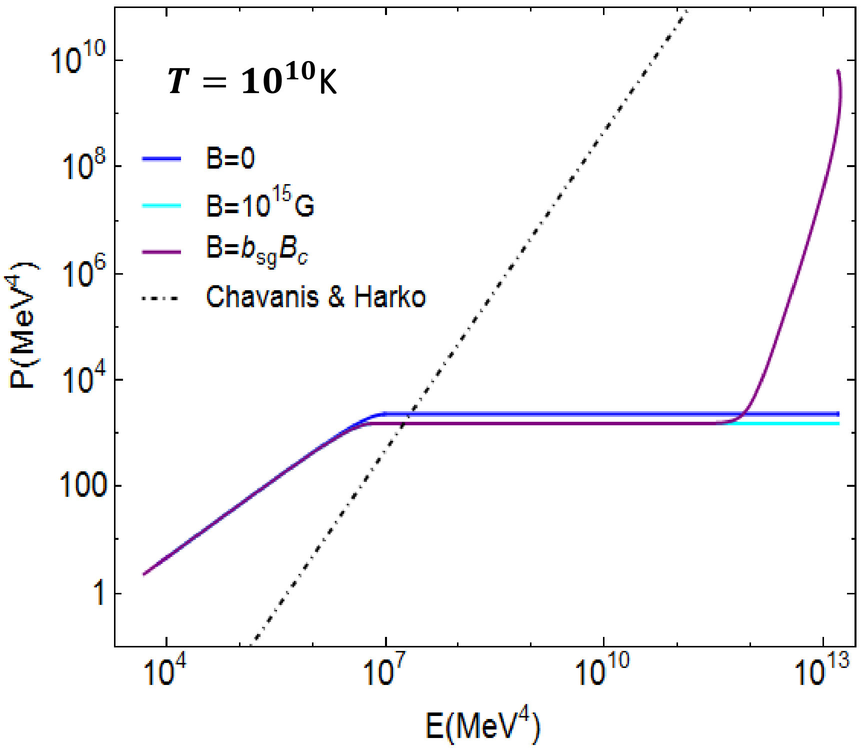
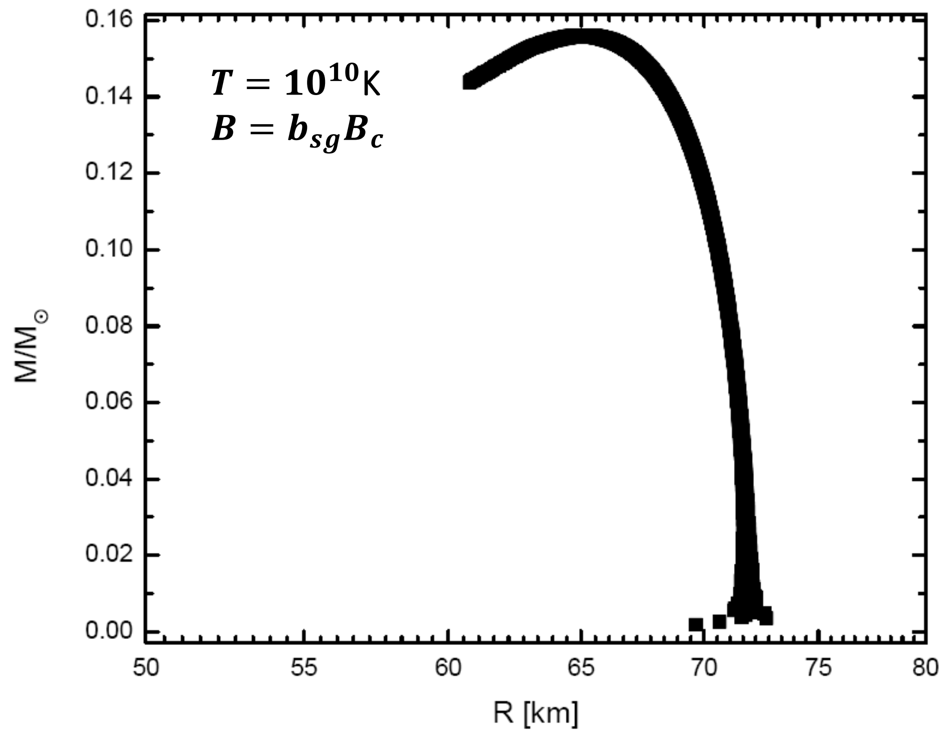




Collapsed regime: Matter ejection and jets

A *Jet* is an extended linear astronomical structure of matter that can be exerted by several astrophysical objects, such as:

star, stars forming regions,

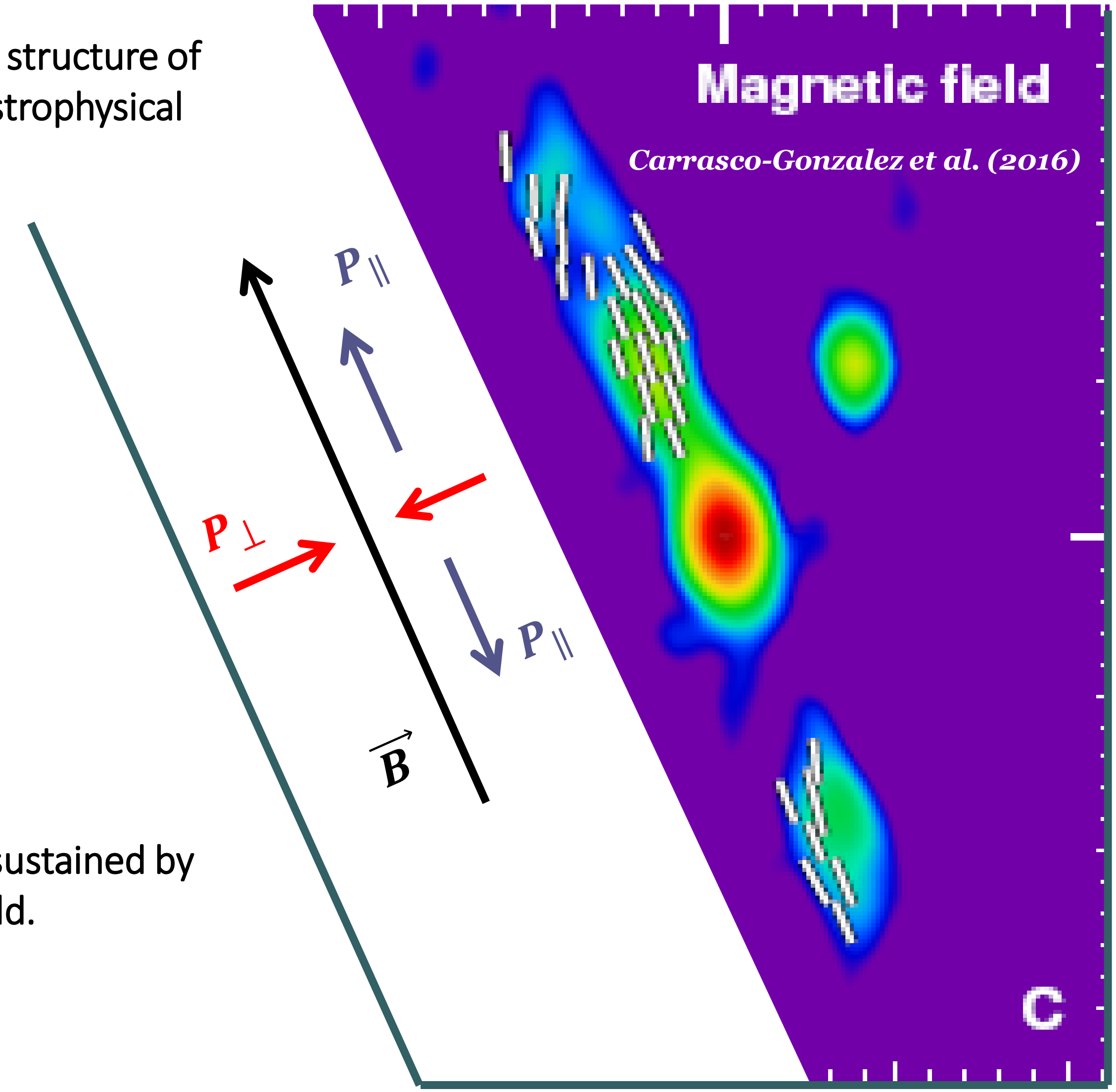
compact objects,

active galactic nuclei,

quasars,

galaxy clusters, etc.

Their one-dimensionality seems to be sustained by a self-generated magnetic field.



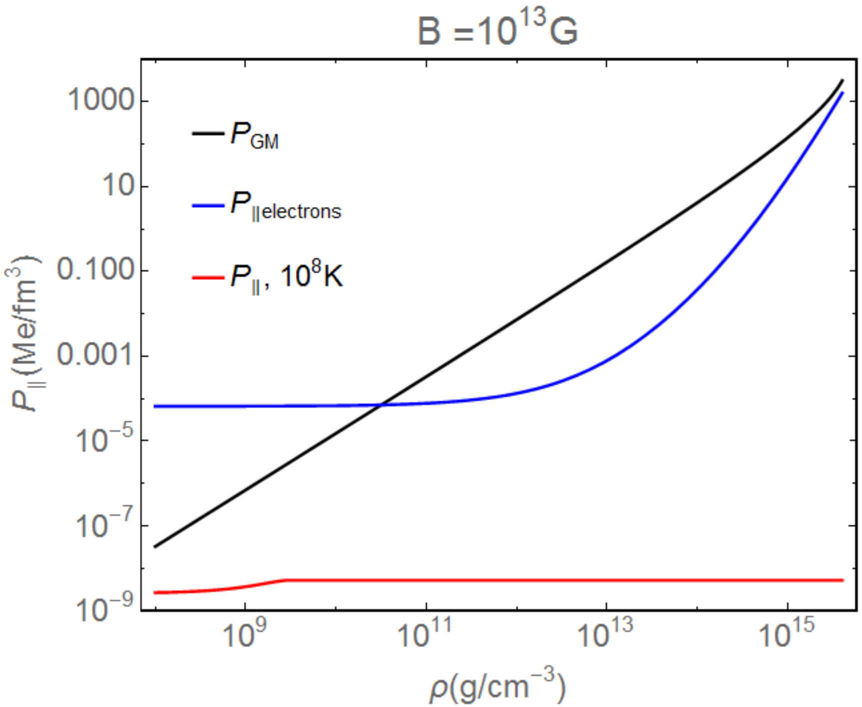
Matter ejection: Preliminar results

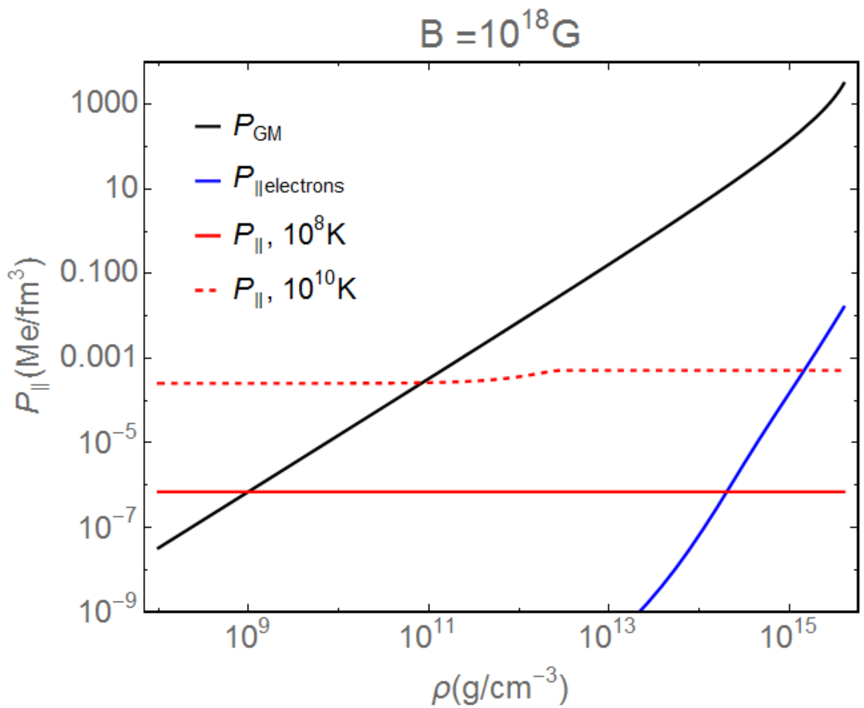
$P_{\|}$ vs. the gravitational pressure P_G

- $P_G < P_{GM} = \rho \frac{1-\sqrt{1-2M/R}}{3\sqrt{1-2M/R}-1}$

P_{GM} is the central pressure –the maximum pressure – of a star with mass M, radius R and constant density ρ.

$M = 1.5\ M_{\odot},\ R = 10km$





Concluding Remarks

We studied BEF and QMC for a gas of neutral vector bosons in astrophysical conditions and found that:

- For particle densities under $\boldsymbol{N_{max} = 1.81 \times 10^{42} cm^{-3}}$ the gas can maintain a self-generated magnetic field whose maximum value is $\boldsymbol{B_{max} = {}^{2}/_{3}\, B_c \sim 10^{20} G}$

  A mechanism to explain the strong magnetic fields of some astrophysical objects.

- In dependence on $\boldsymbol{N}$, $\boldsymbol{B}$ and $\boldsymbol{T}$, the perpendicular pressure might be negative and the system is susceptible to suffer a transversal magnetic collapse.
  - In the non collapsed regime the possibility of having a (self)magnetized boson star is confirmed.

    Next Steps: To construct the magnetic field profile of the star and to take into account the anisotropy in the EoS.

  - In the collapsed regime the parallel pressure might be enough to overcome the NS gravity and account for matter ejection.

    Next steps: To study in detail the possibles compositions of a jet model and to take into account general relativity effects.