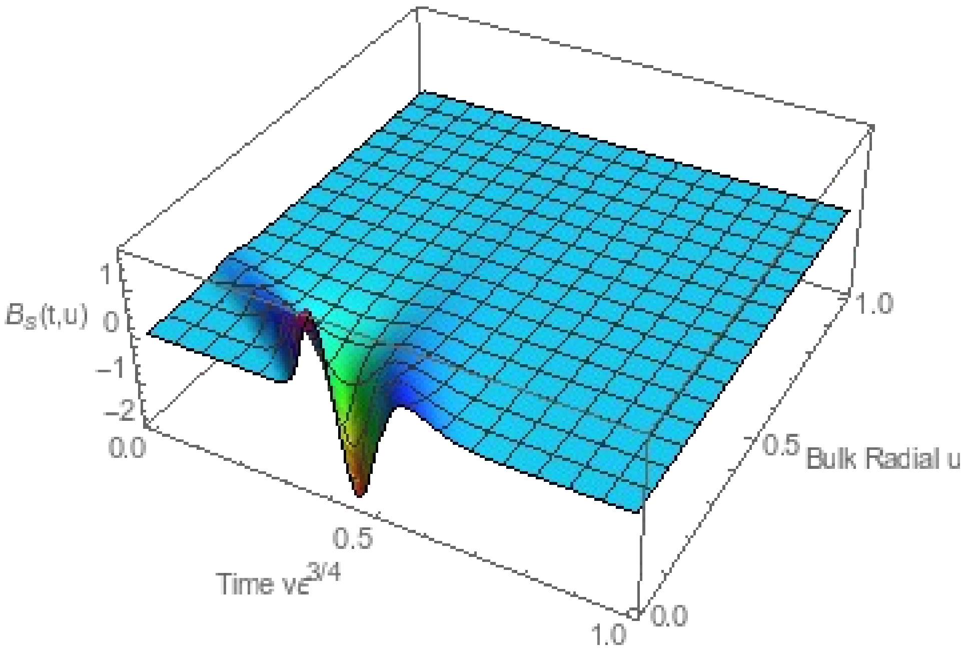
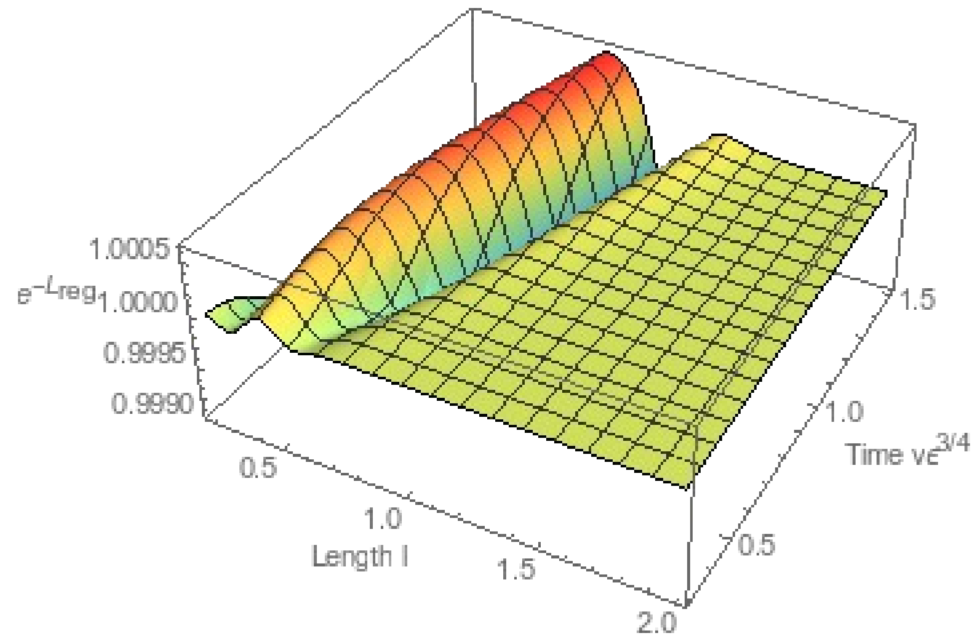
Correlations in strongly coupled plasmas via the gauge/gravity duality

Non-Equilibrium Dynamics (NeD) – Varadero, Cuba April 2018





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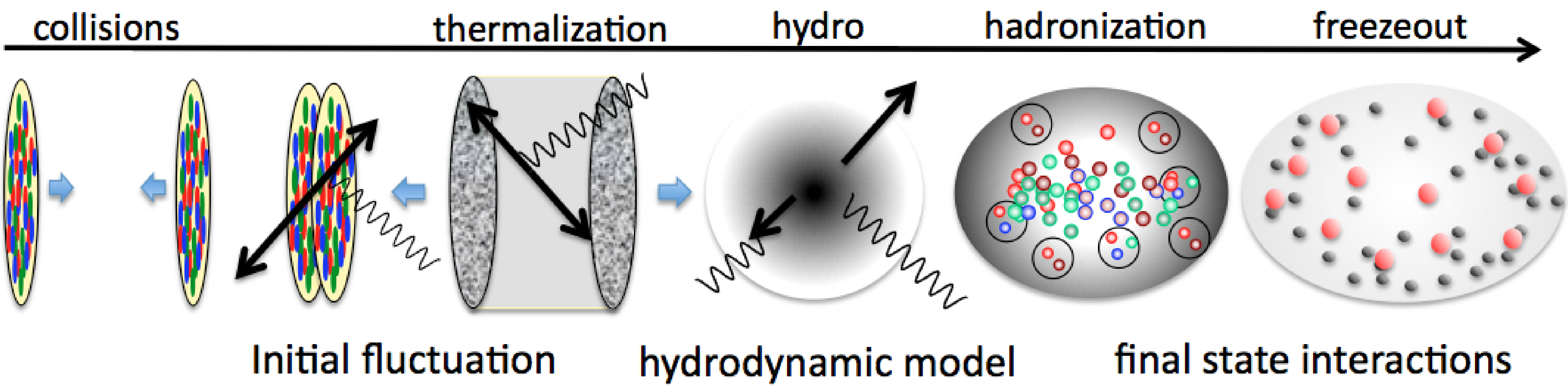
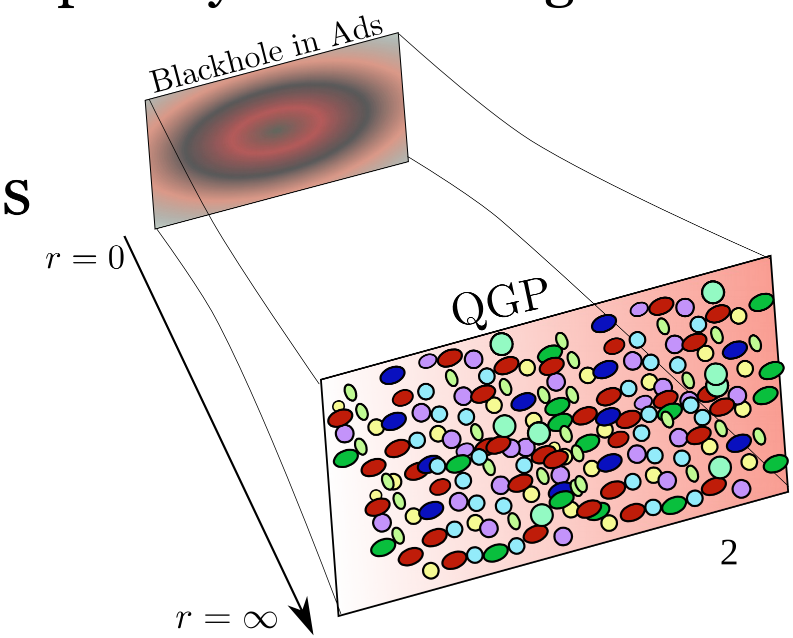
Heavy Ion Collision Schematic [Nonaka,Asakawa; PTEP (2012)]



- Process of thermalization (isotropization)
  - Heavy ion collisions appear to be strongly coupled systems during this phase
- Want model to understand this process
  - AdS/CFT correspondence provides models
  - Our focus: External Magnetic Field effects on 2 pt functions



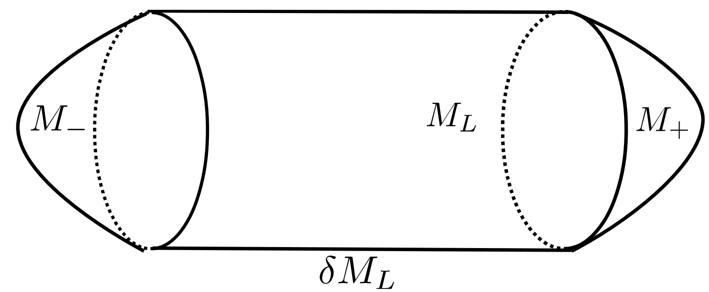
The Duality

- AdS/CFT
  - Form of correspondence for Minkowski signature [Skenderis,Ress; JHEP (2009)]

$$\left\langle 0 \middle| T e^{-i\int_{\delta M_L} \mathrm{d}^d x \sqrt{-g} O \phi_{(0)}} \middle| 0 \right\rangle = e^{(iS_L[\phi_{(0)},\phi_-,\phi_+] - S_E[0,\phi_-] - S_E[0,\phi_+])}$$

Metric in bulk $g_{\mu\nu}$ dual to $T^{CFT}_{\mu\nu}$

Gauge field in bulk A_μ dual to j^{CFT}_μ



- Our focus: $\mathrm{AdS}_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ SYM

  More from the dictionary:

- Approximate Wightman Functions [Balasubramanian, Ross; PRD (2000)]

$$\langle O(t,\vec{x}_1) O(t,\vec{x}_2) \rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L},$$

Geodesic Approximation

Geometry

- Bulk Action:

$$\frac{1}{16\pi G}\int \mathrm{d}^5x\sqrt{-g}(R-2\Lambda-F_{\mu\nu}F^{\mu\nu})$$

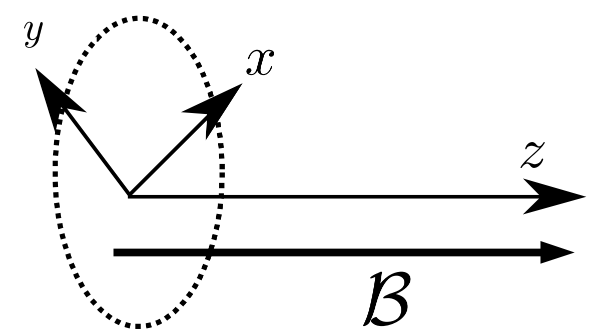
- Metric Ansatz [Chesler, Yaffe; JHEP,2014]

$$\mathrm{d}s^2 = G_{ij}\mathrm{d}x^i\mathrm{d}x^j + 2\mathrm{d}t(\mathrm{d}r - \frac{1}{2}A\mathrm{d}t) \quad G_{ij} = S^2\ \mathrm{Diag}(e^B, e^B, e^{-2B})$$

- Local $\mathrm{U}(1)$ gauge field A_μ in the gravity theory

- Radial Gauge $A_r = 0$

$$A_\mu = \frac{\mathcal{B}}{2}(\delta^2_\mu x_1 - \delta^1_\mu x_2)$$



- $\mathcal{B}$ In z direction in Bulk and Boundary

- Metric Ansatz allows for Characteristic Formulation [Fuini, Yaffe; JHEP,2015]
  - Evolve Einsteins equations in bulk via spectral methods

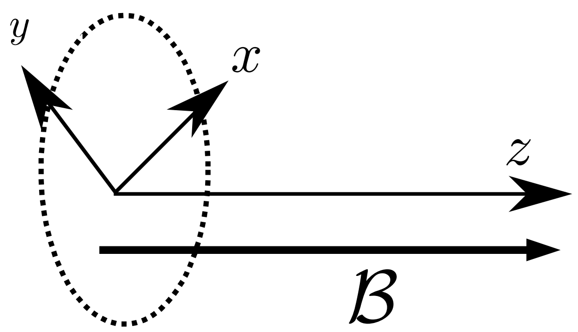
Geometry: Geodesics

- Proper length of a charged particle

$$L = m\int \mathrm{d}\lambda\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} + \frac{q}{m}A_\mu\dot{x}^\mu$$

- Variation yields geodesic equations

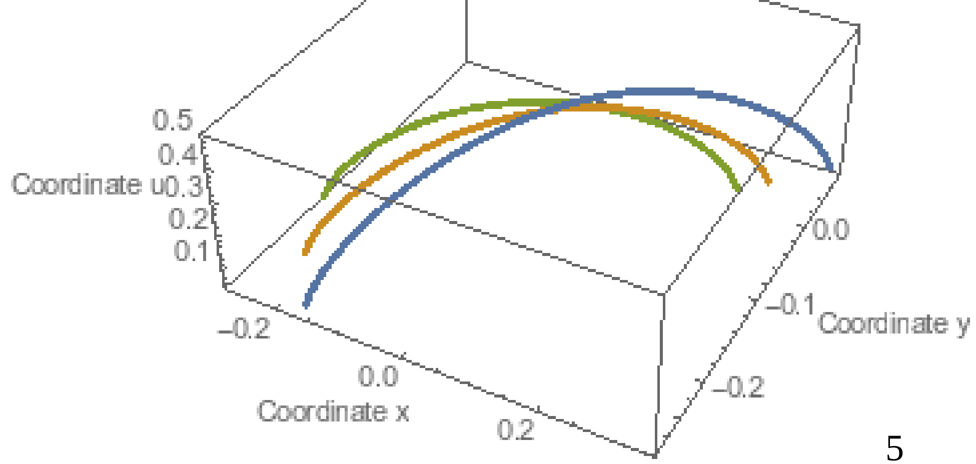
$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} = \frac{q}{m}F^{\mu\nu}g_{\nu\alpha}\dot{x}^\alpha$$



- 2 steps [Ecker et. al.; JHEP 2015]
  - solve geodesic equations via relaxation
  - Compute length via Riemann sum

Example set of geodesics

$l\epsilon^{1/4} = .6$ $B = 2$ $q/m = 10$

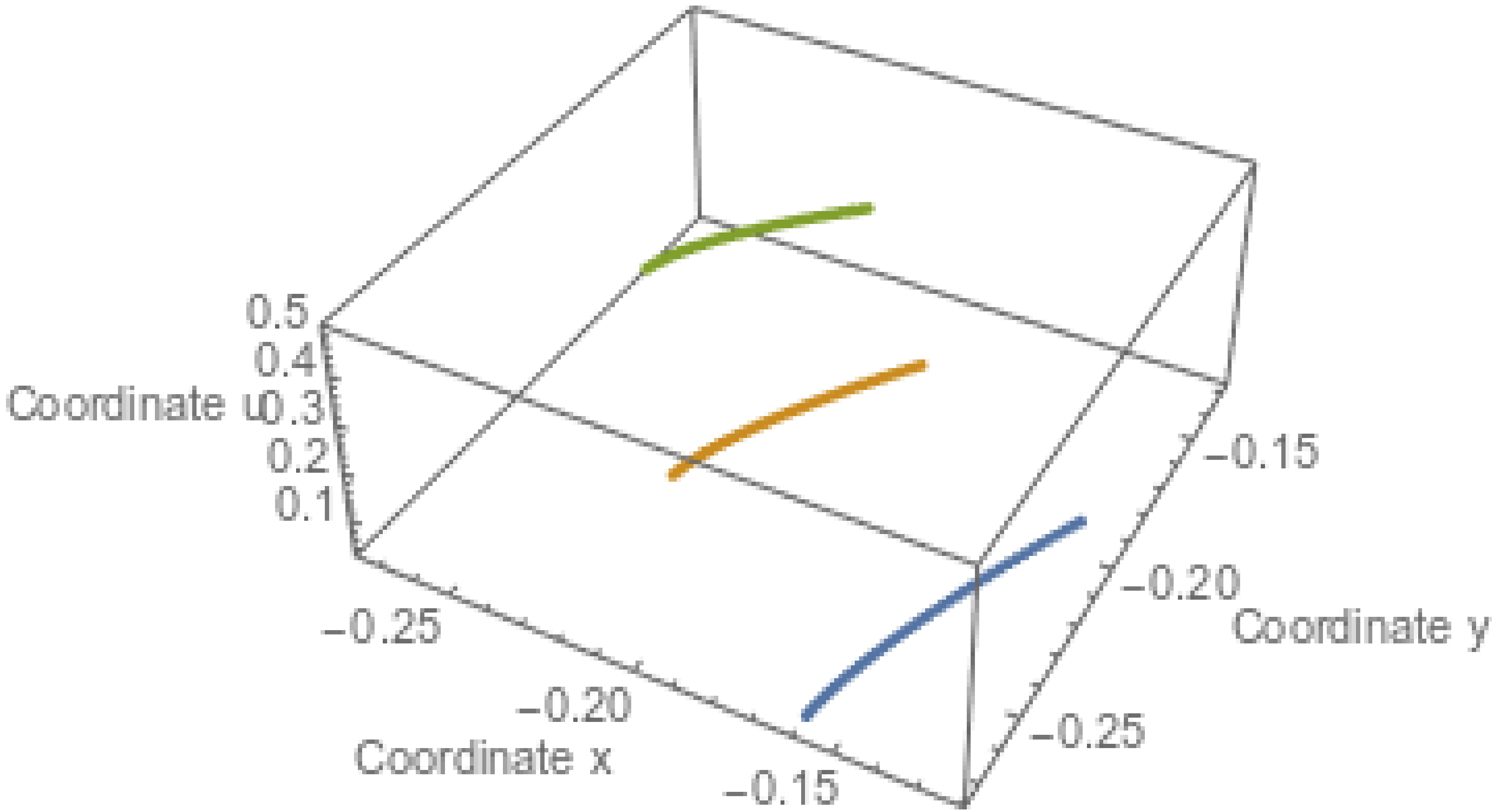


- Use empty AdS_4 geodesics as relaxation guess
- By symmetry choose AdS_4 slice

$$ds^2 = \frac{1}{u^2}(-\mathrm{d}\nu^2 - 2\mathrm{d}\nu\mathrm{d}u + \mathrm{d}x^2 + \mathrm{d}y^2)$$

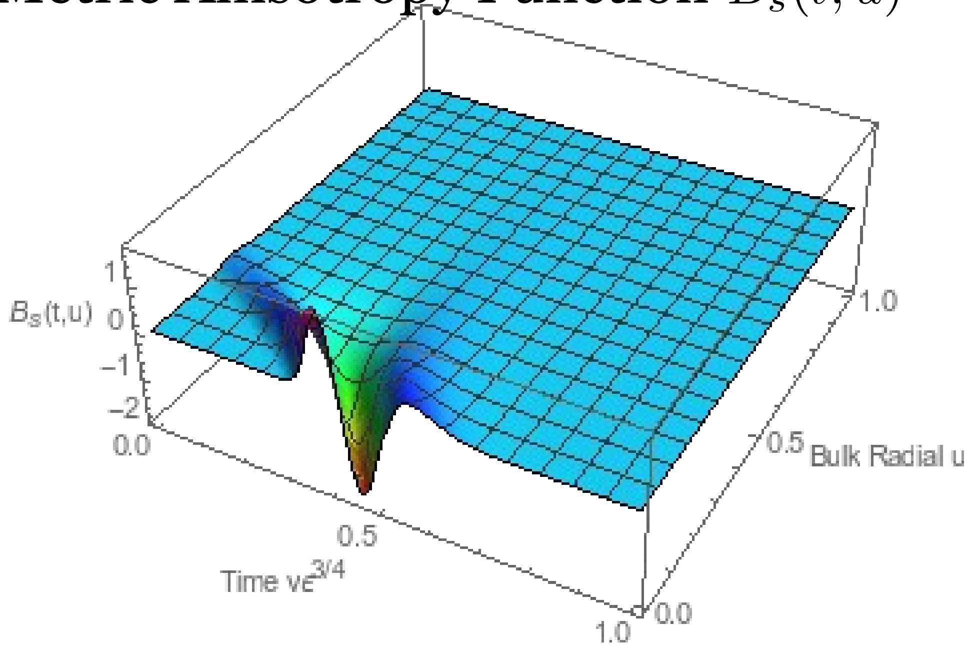
Geometry: Real time trajectory

$$B = 2 \qquad q/m = 10 \qquad l = .6$$



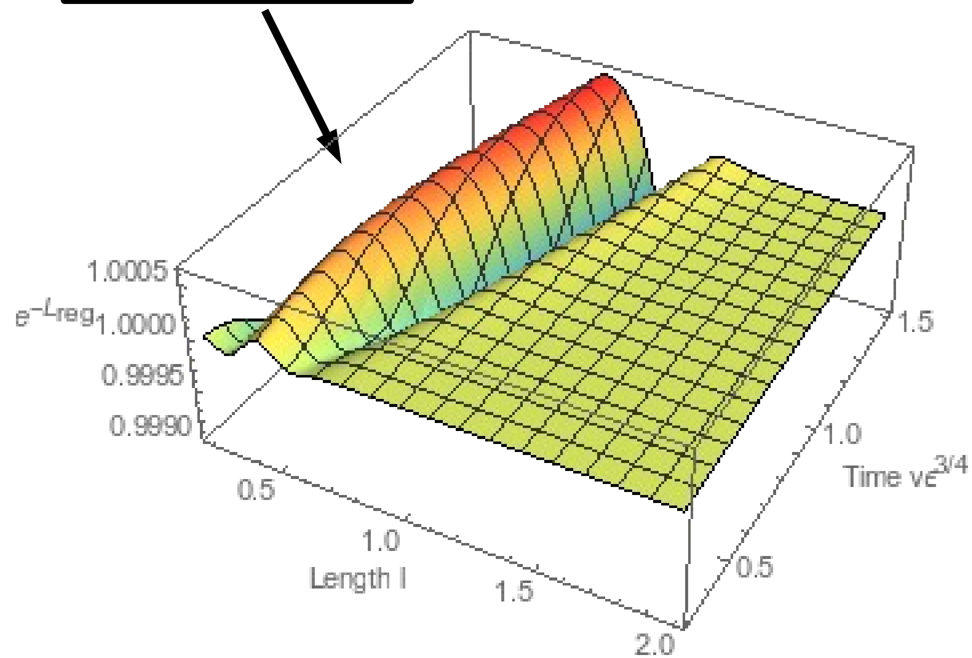
1 and 2 pt. Functions: No Magnetic Field

Metric Anisotropy Function $B_s(t,u)$



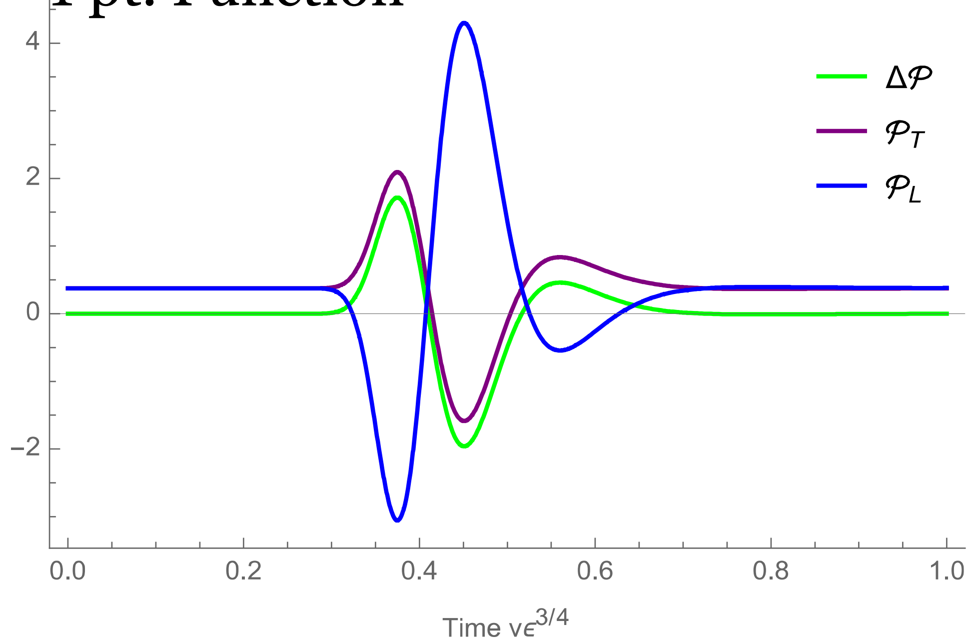
2 pt. Function

Preliminary

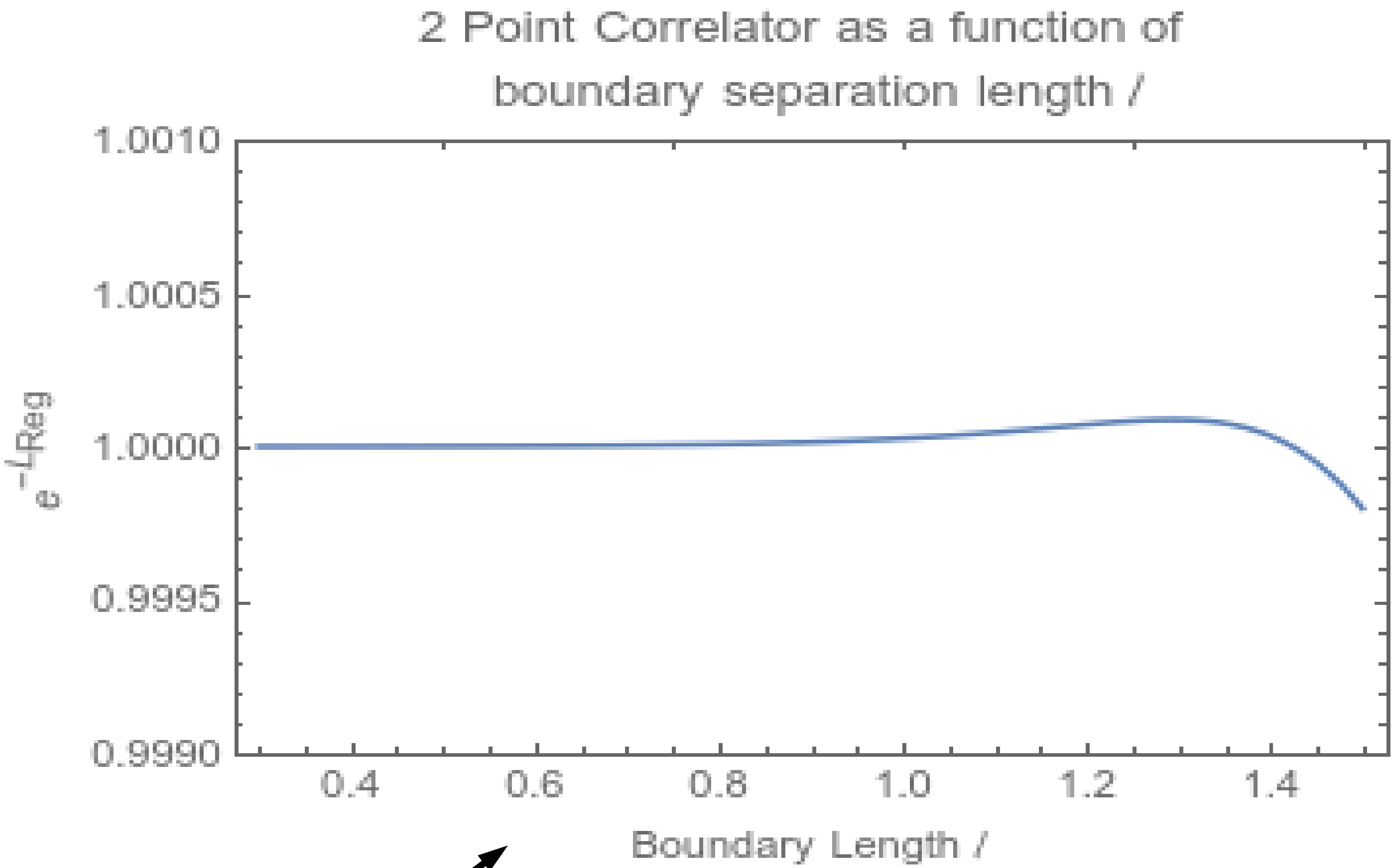


$$< O(x_2,t_2)O(x_1,t_1) >= f(x_2,x_1;t_2,t_1)$$

1 pt. Function



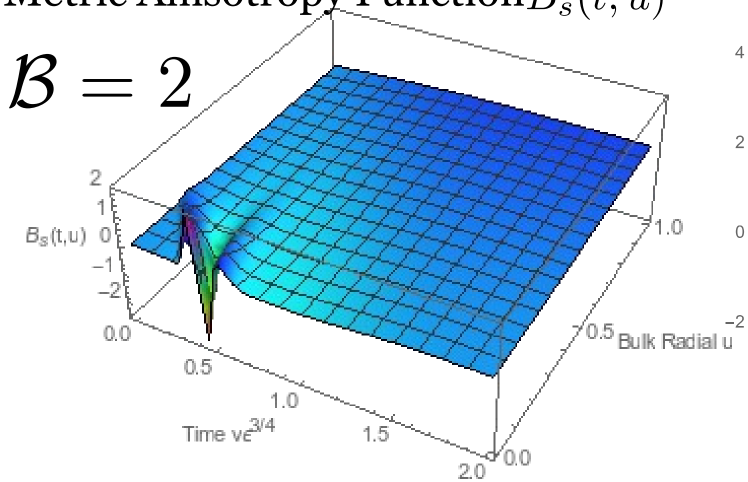
1 and 2 pt. Functions: No Magnetic Field

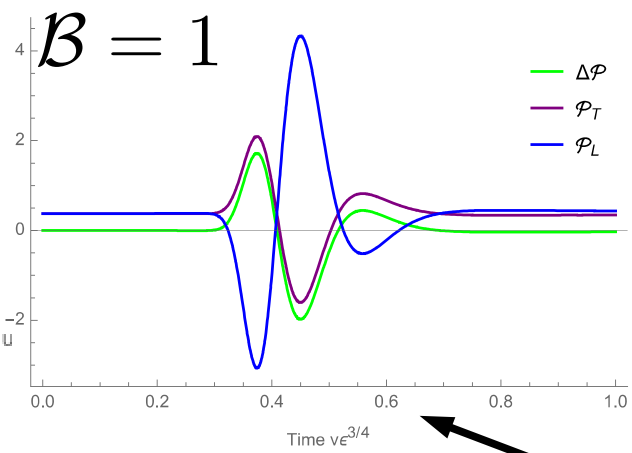


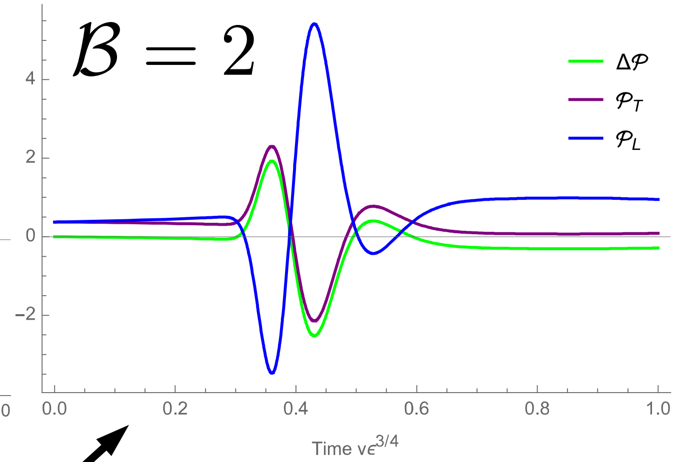
Preliminary Results

1 and 2 pt. Functions: Magnetic Field

Metric Anisotropy Function $B_s(t,u)$

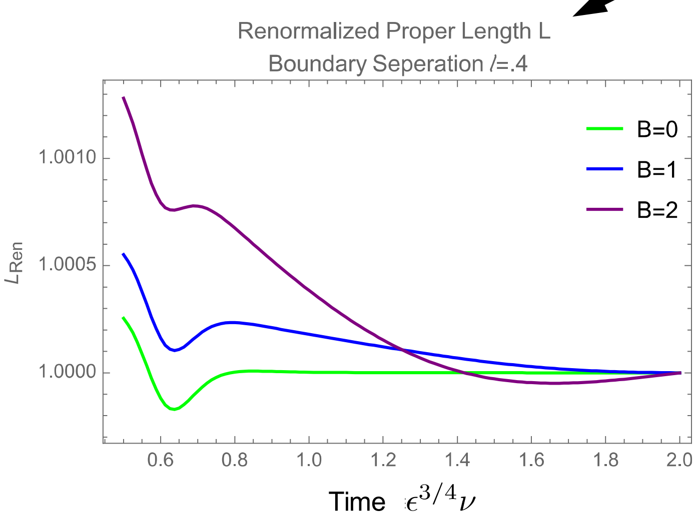


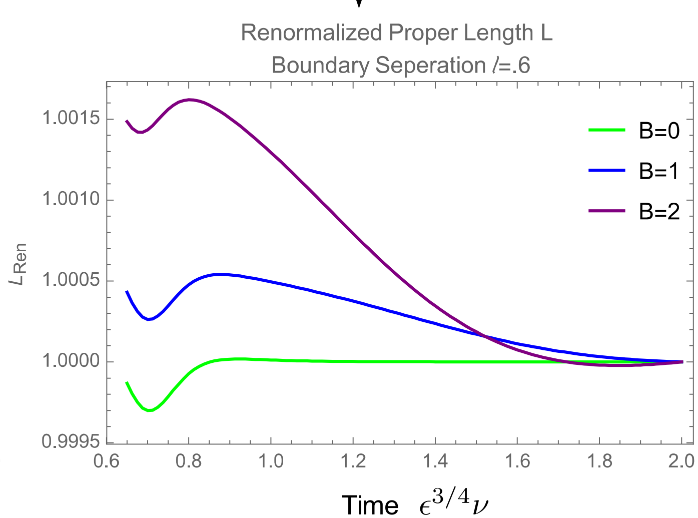


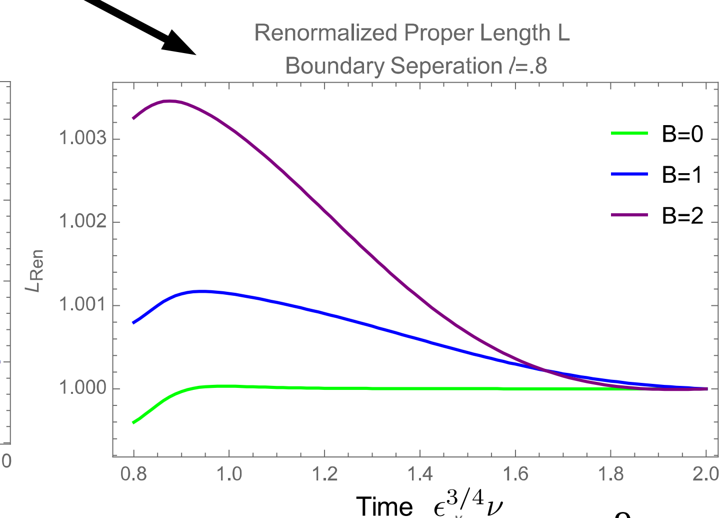


1 pt. Functions

Preliminary Results:
2 pt functions



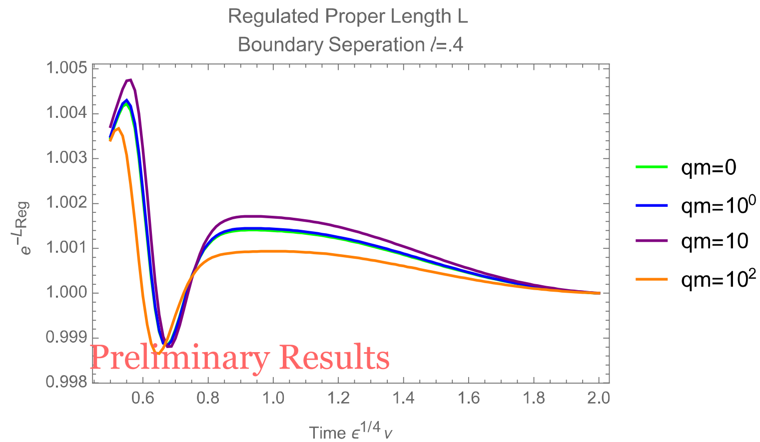


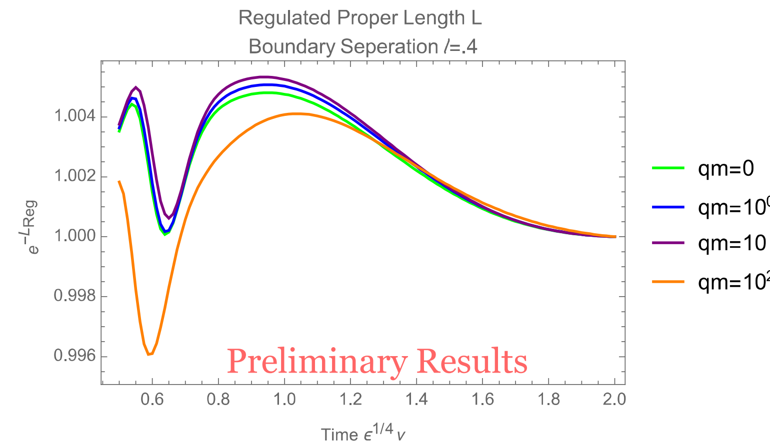


2 pt. Functions: Magnetic Field Charged Particles

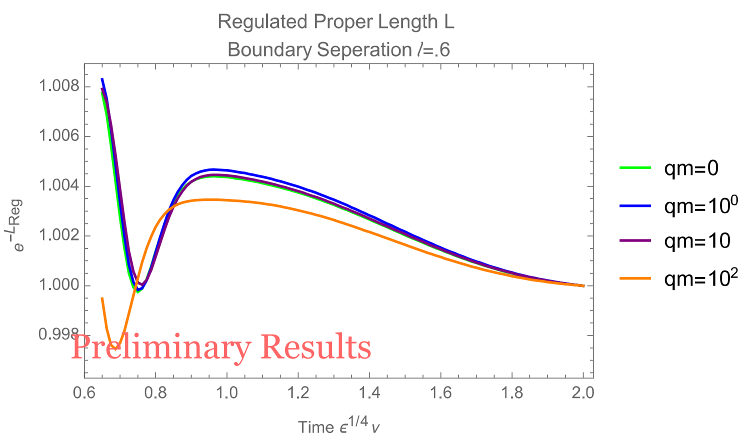
Boundary Separation	$\mathcal{B} = 1$	$\mathcal{B} = 2$

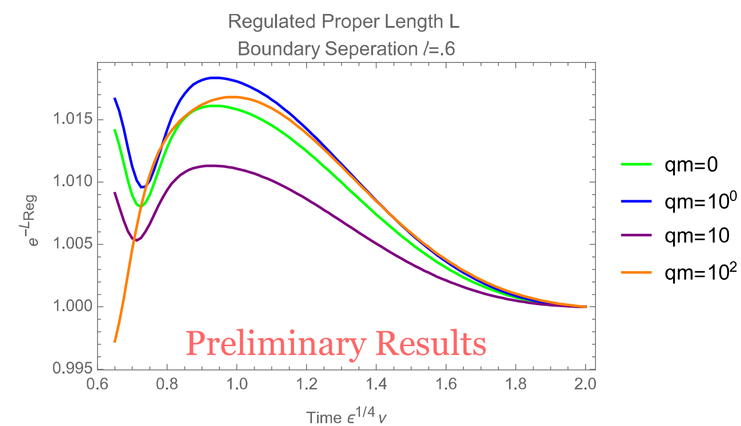
$l\epsilon^{1/4} = .4$



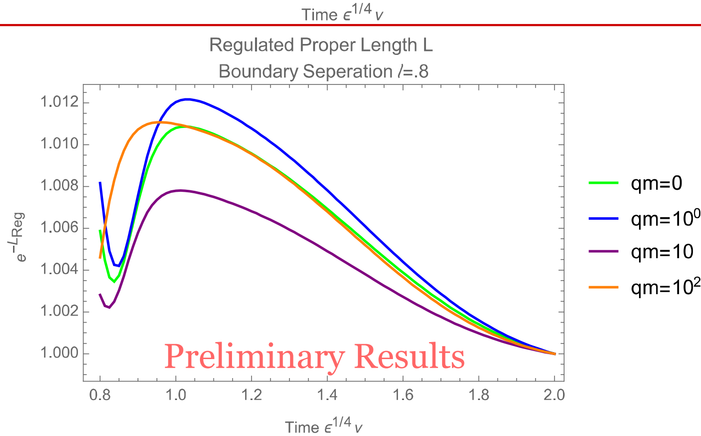


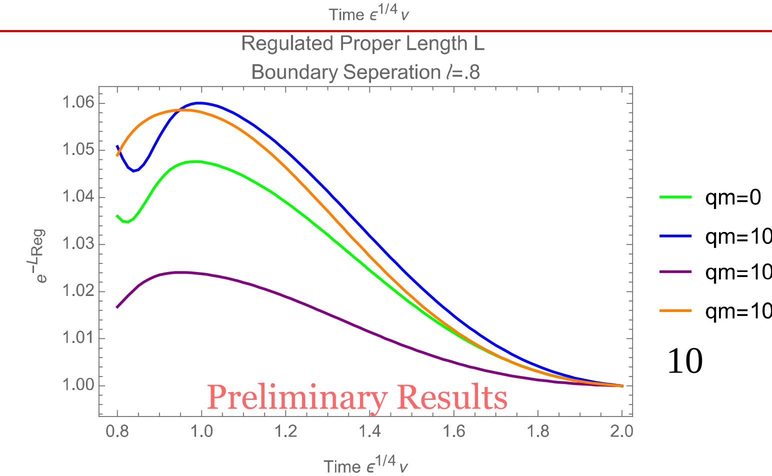
$l\epsilon^{1/4} = .6$





$l\epsilon^{1/4} = .8$

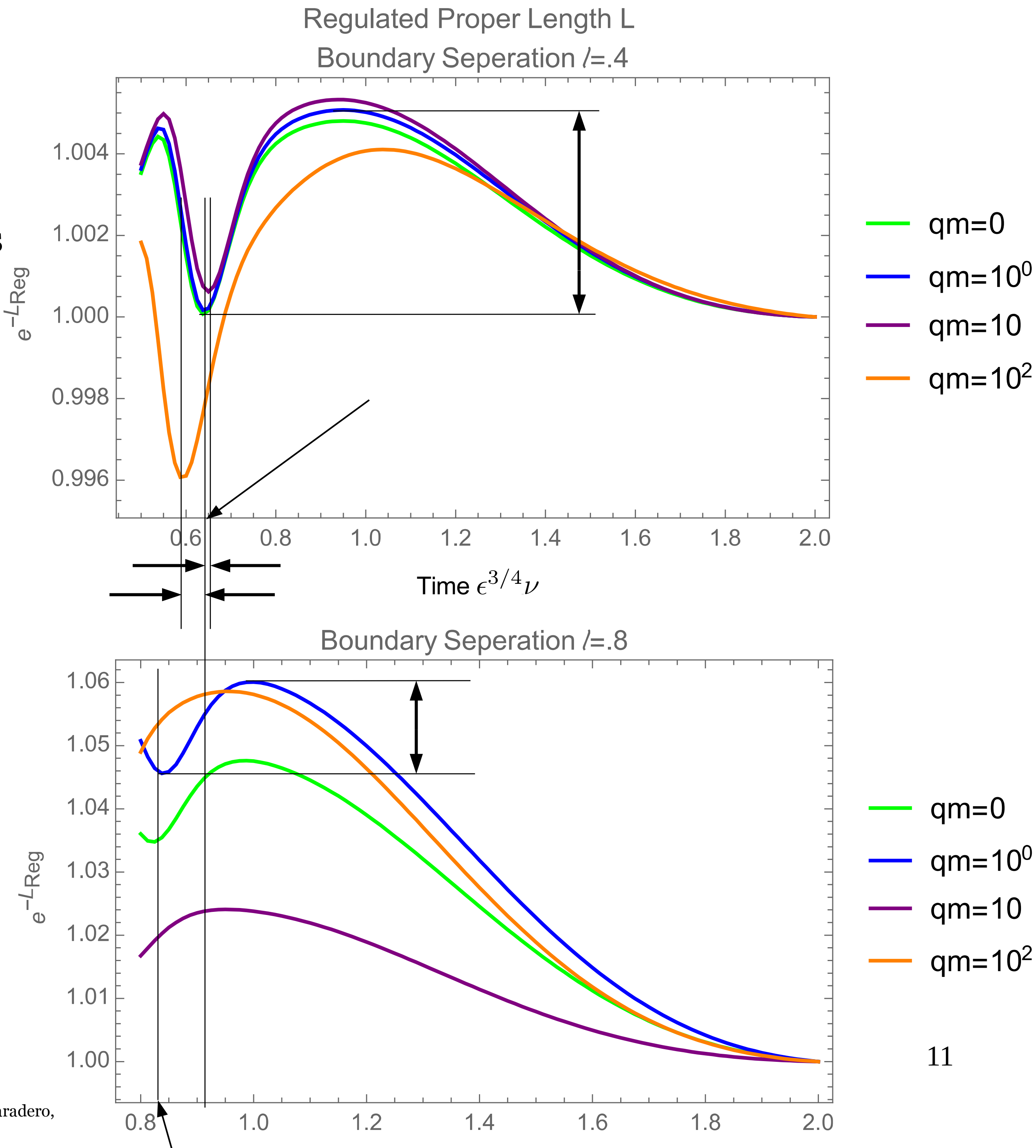




2 pt. Functions: Magnetic Field Charged Particles

Features:

- Longer boundary lengths lead to time delay in early time dynamics
- Larger q/m operators experience early time dynamics faster
- Longer lengths correlations become dominated by magnetic field
- Mid range q/m reveal competition between magnetic and medium dynamics

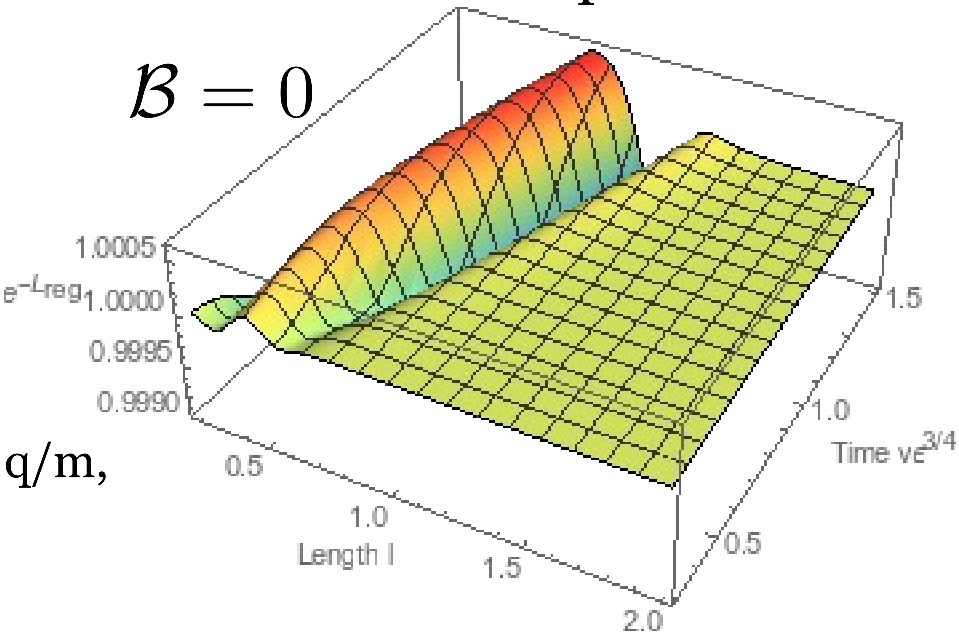


Conclusions

So Far:

- Numerically computed 2 pt of anisotropic plasma as a function of length and time
- Numerically computed 2 pt functions in presence of external magnetic field as a function of time at fixed lengths
- Found interesting features due to magnetic field
  - Early time dynamics and amplitude depends on q/m, length separation and B

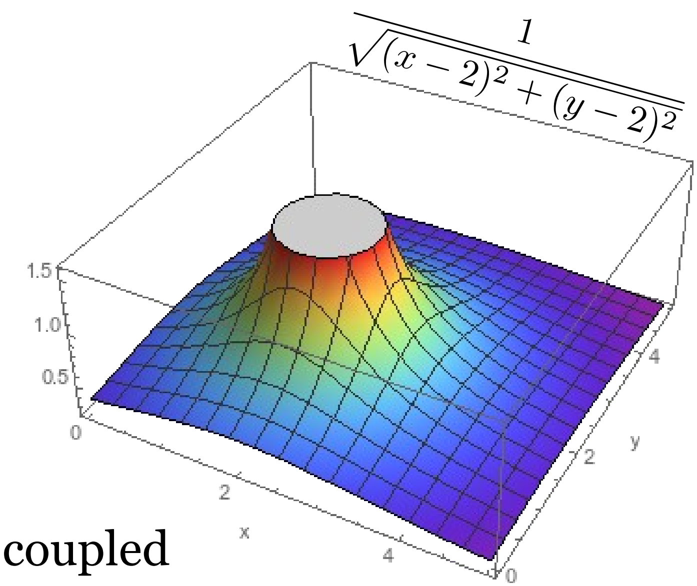
2 pt. Function



To be done:

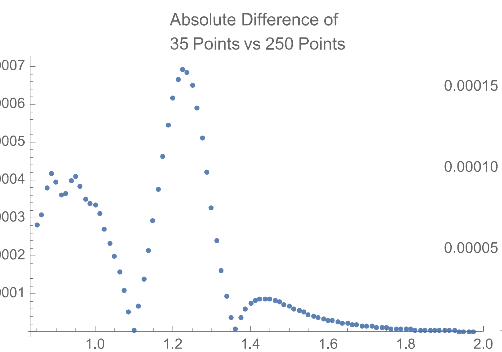
- Determine functional form $\langle O(t,\vec{x}_1)O(t,\vec{x}_2)\rangle$ as a function of boundary separation in non-magnetic case
- Begun numerical calculation of $\langle O(t,\vec{x}_1)O(t,\vec{x}_2)\rangle$ as a function of boundary separation in magnetic case
- Electromagnetic and conformal correlation functions serve as guide/comparison

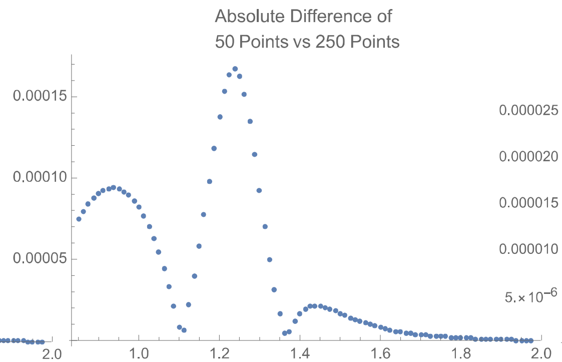
$$G(x_1,x_2) = \frac{1}{|\vec{x_1} - \vec{x_2}|} \qquad G(x_1,x_2) = \frac{1}{|\vec{x_1} - \vec{x_2}|^{2\Delta}}$$

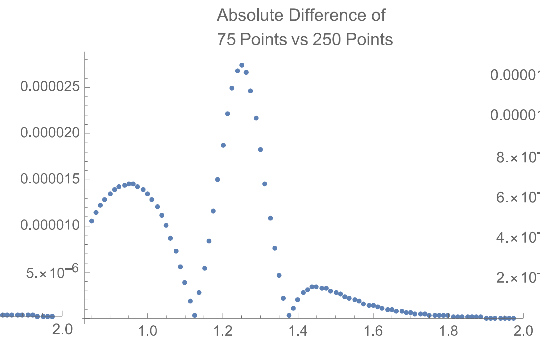


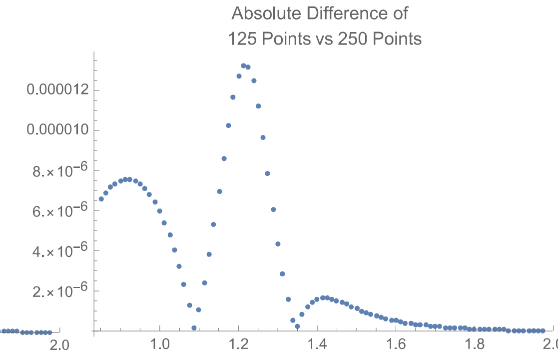
- Goal: Identify medium effects as a function of time in a strongly coupled plasma with and without external magnetic field
  - Enhancement? Reduction?

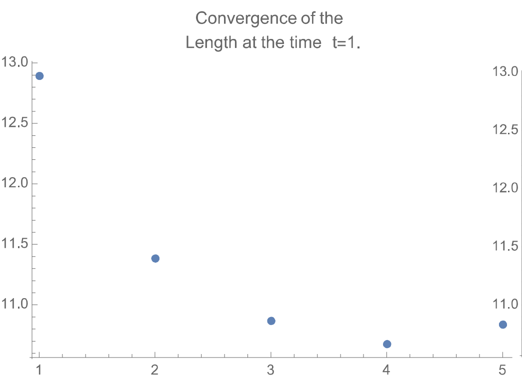
Convergence

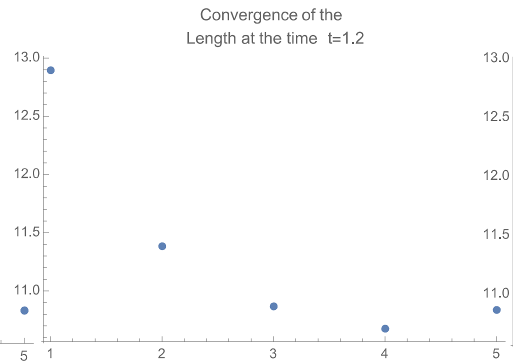


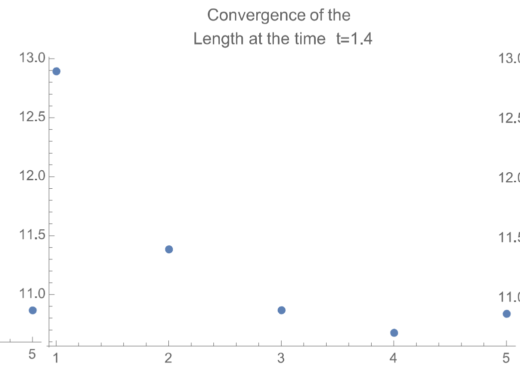


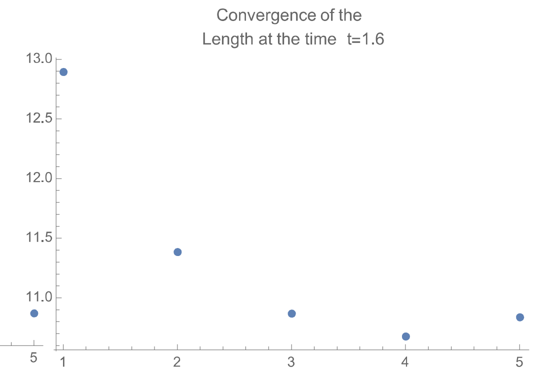












1-35 grid points; 2-50 grid points; 3-75 grid points; 4-125 grid points; 5-250 grid points

$$< \mathcal{O}(x)\mathcal{O}(x) > \approx e^{-L_{reg}}$$

Cutoff Dependence

zuv	v=0.525	v=0.7625	v=1.0125	v=1.2625	v=1.5125	v=1.7625
0.01	1.00029	1.00039	0.999741	1.00003	1.00001	1.
0.02	1.00032	1.00034	0.999804	1.00003	1.	1.
0.03	1.00035	1.0003	0.999859	1.00003	1.	1.
0.04	1.00037	1.00024	0.999906	1.00002	1.	1.
0.05	1.00039	1.00019	0.999945	1.00002	1.	1.
0.06	1.00041	1.00013	0.999976	1.00002	1.	1.
0.07	1.00043	1.00007	1.	1.00002	1.	1.
0.08	1.00044	1.	1.00002	1.00002	1.	1.
0.09	1.00045	0.99993	1.00003	1.00001	1.	1.
0.1	1.00046	0.999857	1.00004	1.00001	1.	1.