ABOUT A DYNAMICAL GRAVASTAR

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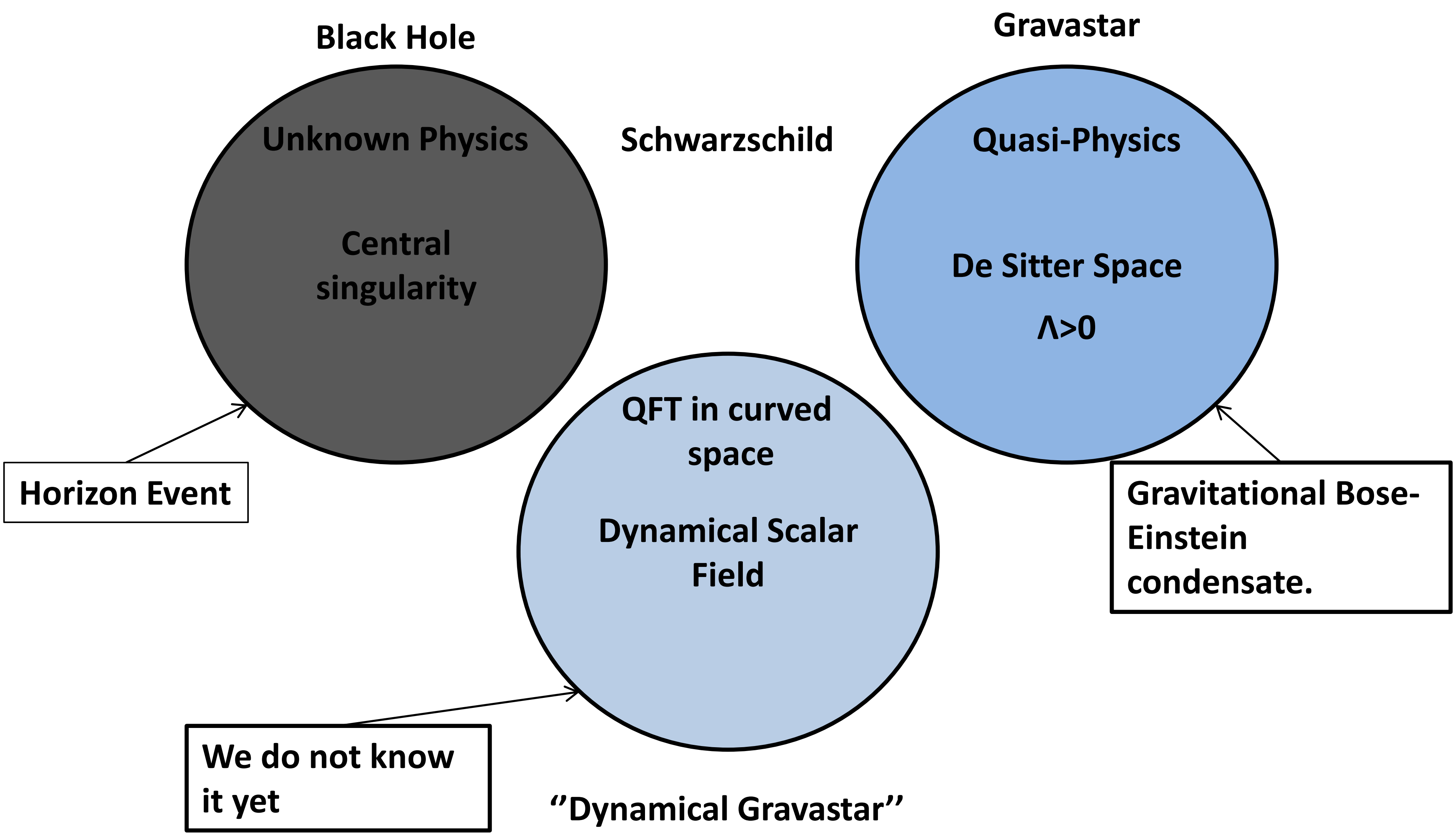
Outline

- Black Hole, Gravastar and Dynamical Gravastar
- A scalar solution without central singularity.
- Weak equality. A short review of the Columbeau-Egorov (C-E) theory.
- The design of the sequence of infinitely differentiable set of fields to construct the solutions of the Einstein-Klein-Gordon (EKG) in the sense of the C-E method.
- Behavior of the field in the boundary zone.
- The fulfillment of the EKG equations.
- Summary.

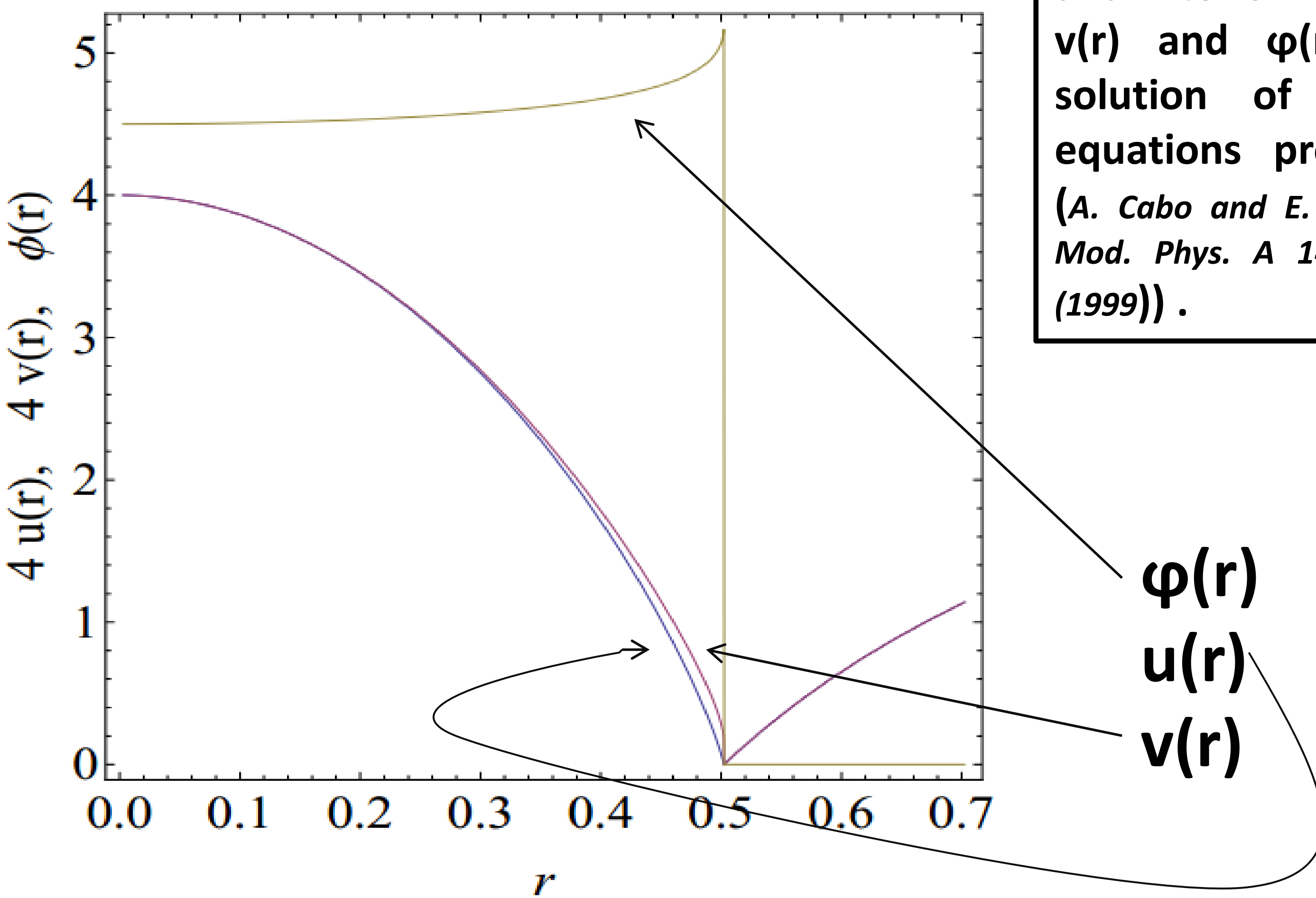
Black Hole and Gravastar



The previous model: The interior and external solutions

Scalar Field

Gravitational field



The plot of the exterior and interior fields u(r), v(r) and φ(r) of the solution of the EKG equations proposed in (*A. Cabo and E. Ayon, Int. J. Mod. Phys. A 14, 2013-2022 (1999)*).

A SHORT REVIEW TO THE COLOMBEAU-EGOROV THEORY

A new definition of generalized functions is currently under consideration (*J. F. Colombeau, ¨New Generalized Functions and Multiplication of Distributions¨*) .
The space of distributions can be defined as the set of all sequences of infinitely differentiable functions {*f*j} such that the integral ∫fj(x)φ(x)dx has a finite limit as j → ∞.

The distribution in the C-E sense is defined as a sequence {*f*j} for which this limit exists

$$\lim \int f_j(x)\varphi(x)\,dx,$$

φ is an arbitrary and infinitely differentiable function.

The weak equality between two sequences {*f*j} and {*g*j} is defined as:

$$\lim_{j\to\infty} \int [f_j - g_j]\varphi(x)\,dx = 0,$$

Defining the fields in the boundary region

$$u_\epsilon(r) = u_i(r)s_l(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_1(\epsilon)) + u_e(r)s_r(x|\ r^* + \frac{\epsilon}{2} - \eta_2(\epsilon),\ r^* + \frac{\epsilon}{2})$$

$$\nu_\epsilon(r) = v_i(r)s_l(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_6(\epsilon)) + v_e(r)s_r(x|\ r^* + \frac{\epsilon}{2} - \eta_7(\epsilon),\ r^* + \frac{\epsilon}{2}) +$$
$$v_1^{(l)}(r)s_r(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_6(\epsilon))s_l(x|\ r^* + \frac{\epsilon}{2} - \eta_5(\epsilon),\ r^* + \frac{\epsilon}{2}) +$$
$$v_2^{(l)}(r)s_r(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_3(\epsilon))s_l(x|\ r^* + \frac{\epsilon}{2} - \eta_5(\epsilon),\ r^* + \frac{\epsilon}{2}) + \ldots,$$

$$v_1^{(l)}(r) = -\frac{v_i(r^* - \frac{\epsilon}{2}) - v_2^{(l)}(r^* + \frac{\epsilon}{2} - \eta_2)}{\epsilon - \eta_2}(r - (\ r^* + \frac{\epsilon}{2} - \eta_2)) + v_2^{(l)}(r^* + \frac{\epsilon}{2} - \eta_2),$$

$$v_2^{(l)}(r) = \gamma\ v_e'(r^* + \frac{\epsilon}{2})(r - (\ r^* + \frac{\epsilon}{2})) + v_e(r^* + \frac{\epsilon}{2}),$$

$$\phi_\epsilon(r) = \phi_i(r)s_l(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_5(\epsilon)) + \phi_e(r)s_l(x|\ r^* + \frac{\epsilon}{2} - \eta_6(\epsilon),\ r^* + \frac{\epsilon}{2}) +$$
$$\phi^{(l)}(r)s_r(x|\ r^* - \frac{\epsilon}{2},\ r^* - \frac{\epsilon}{2} + \eta_5(\epsilon))s_l(x|\ r^* + \frac{\epsilon}{2} - \eta_6(\epsilon),\ r^* + \frac{\epsilon}{2}),$$

$$\phi^{(l)}(r) = -\frac{\phi_i(r^* - \frac{\epsilon}{2}) - \phi_e(r^* + \frac{\epsilon}{2})}{\epsilon}(r - (\ r^* + \frac{\epsilon}{2})) + \phi_e(r^* + \frac{\epsilon}{2}),$$

The family of parameters η depends on ϵ (the size of the boundary interval). In the limit ϵ→0, η is chosen so that the EKG equations are satisfied.

The parameter r* is the midpoint of the regularization boundary interval Bϵ=(r*$- \frac{\epsilon}{2}$, r* $+ \frac{\epsilon}{2}$) and is a function of ϵ and differs from the singularity radius r_0

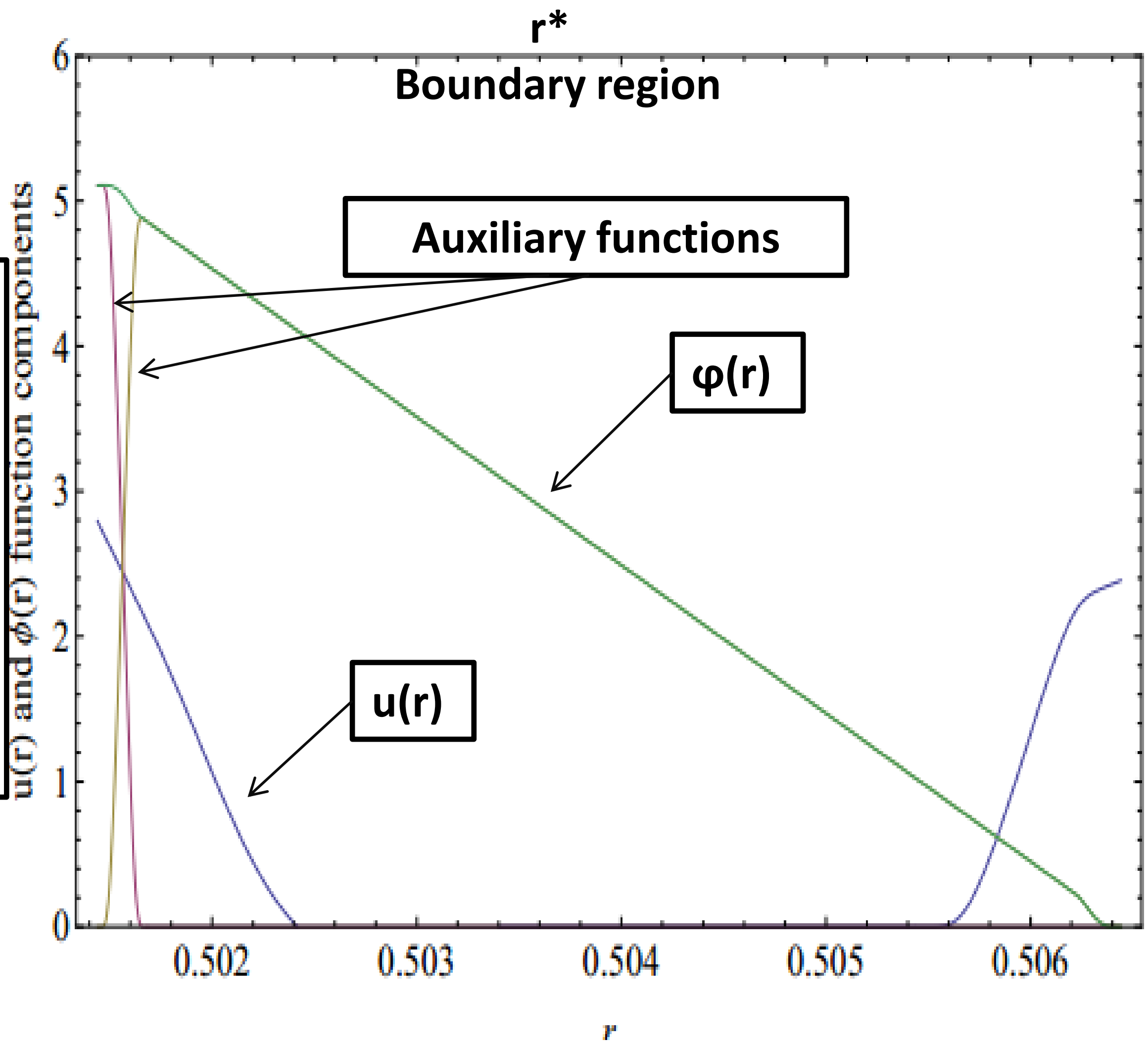
Si and Sr are auxiliary functions to interpolate functions in a C^∞ form

The field values in the extremes of interval are given by the numerical solutions obtained in previous work at the left side and the Schwarzschild solution at the right one.

Defining fields in the boundary region

Scalar plus gravitational field

The graphs correspond to a width of ϵ = 0.0005 of the boundary for certain chosen values of the other parameters.



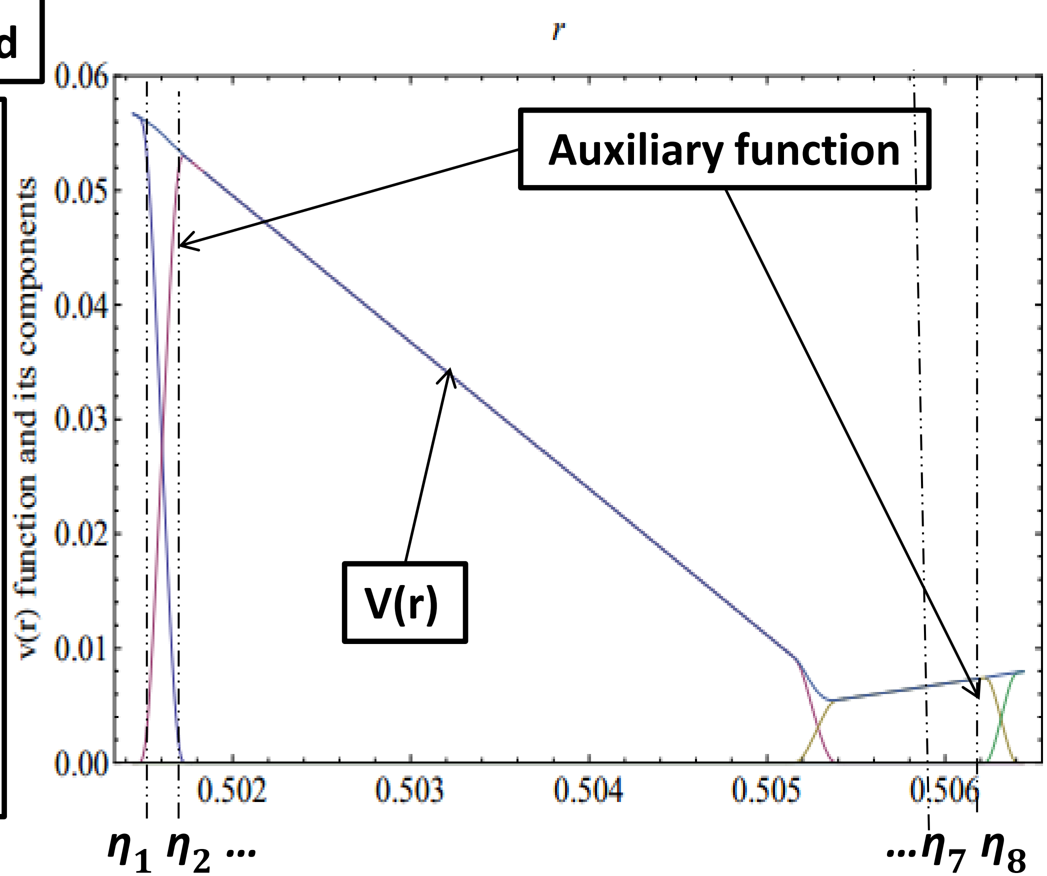
Only gravitational field

By construction, the curves match the interior and exterior fields at the left and right borders of the interval in all their derivatives.

Defining fields in the boundary region

Scalar field plus gravitational field

The "rising" and "falling" transition regions of their infinitely differentiable function components can be separated by different subinterval η



Only gravitational field

We will see that the fulfillment of the weak equality will depend on the appropriate form of fixing this parameter

The equality in the C-E sense

The integrations reduce to the boundary interval $B_{\epsilon k}$ of width ϵ and centered in the radius r*. The index k defines the sequence of widths. For simplicity, $\epsilon_k = 1/k$; k = N, N + 1,..,∞

$$\int_{R^+} dr E^{(l)}(r,\epsilon_k)\Phi(r) = \int_{B_{\epsilon_k}} dr\ E^{(l)}(r,\epsilon_k)\Phi(r)$$

$$= \int_{r^*(\epsilon_k)-\frac{\epsilon}{2}}^{r^*(\epsilon_k)+\frac{\epsilon}{2}} dr\ E^{(l)}(r,\epsilon_k)\Phi(r)$$

$E^{(l)}$ is the particular EKG equation for: l= u, v, θ,Φ

$$E^{(u)}(r,\epsilon) \equiv -\frac{u'_\epsilon(r)}{r} + \frac{1-u_\epsilon(r)}{r^2} - \frac{u_\epsilon(r)\,\phi'_\epsilon(r)^2}{2} - \frac{\phi_\epsilon(r)^2}{2},$$

$$E^{(v)}(r,\epsilon) \equiv -\frac{u_\epsilon(r)}{r}\frac{v'_\epsilon(r)}{v_\epsilon(r)} + \frac{1-u_\epsilon(r)}{r^2} + \frac{u_\epsilon(r)\,\phi'_\epsilon(r)^2}{2} - \frac{\phi_\epsilon(r)^2}{2},$$

Fulfillment of EKG equation

Finally the fulfillment of this expression resulted in the following set of conditions for the functions η_k...

$$\lim_{k\to\infty}\left(\frac{\eta_4(\epsilon_k)}{\eta_1(\epsilon_k)}\right)=0,\quad \lim_{k\to\infty}\left(\frac{\eta_1(\epsilon_k))}{\epsilon_k}\right)=0\ ,\quad \lim_{k\to\infty}\left(\frac{\eta_4(\epsilon_k)}{\epsilon_k}\right)=0\ ,$$

$$\lim_{k\to\infty}\left(\frac{\eta_5(\epsilon_k)}{\eta_2(\epsilon_k)}\right)=0,\quad \lim_{k\to\infty}\left(\frac{\eta_2(\epsilon_k)}{\epsilon_k}\right)=0,\quad \lim_{k\to\infty}\left(\frac{\eta_5(\epsilon_k)}{\epsilon_k}\right)=0,$$

...and the relation between δ_k and the width of the transition zone.
Where δ_k is distance of the r* (center of the interval) y r_o (point of singularity)

$$\lim_{k\to\infty}\left(\frac{\delta_k}{\epsilon_k}\right)=\frac{r_0^2\ \phi_{r_0}^2}{6(r_0^2\phi_{r_0}^2-1)},$$

$$1-\lim_{k\to\infty}\left(\frac{\delta_k}{\epsilon_k}\right)=\frac{5\ r_0^2\ \phi_{r_0}^2}{6(r_0^2\phi_{r_0}^2-1)}.$$

Summary

1. Physical and mathematical arguments giving further support to the existence of black holes with non-trapping interior are given. The internal part of this solution consisted of a scalar field interacting with gravity, tending to a finite value at the boundary. On the contrary, the derivative of the scalar field diverges at the border.

2. The external solution was chosen as the Schwarzschild space-time. We regularize the fields in a neighborhood of the transition zone between the two space-times in an infinitely differentiable form. After choosing the regularization in an appropriate way, as a function of the width of the transition zone, the equations are satisfied in the sense of the modern theory of generalized functions constructed by Colombeau and Egorov after removing the regularization.

3. One can expect that the proposed solution of the EKG equations might be the final outcome of collapsed scalar matter. Further support for this idea is under current investigation by attempting to show that the inclusion of one loop quantum corrections for the scalar (or gravity) fields leads to a regular and stable solution.