I had a dream *)

Holographic Scenarios for HICs: EoS, viscosity, freeze-out, hadrons

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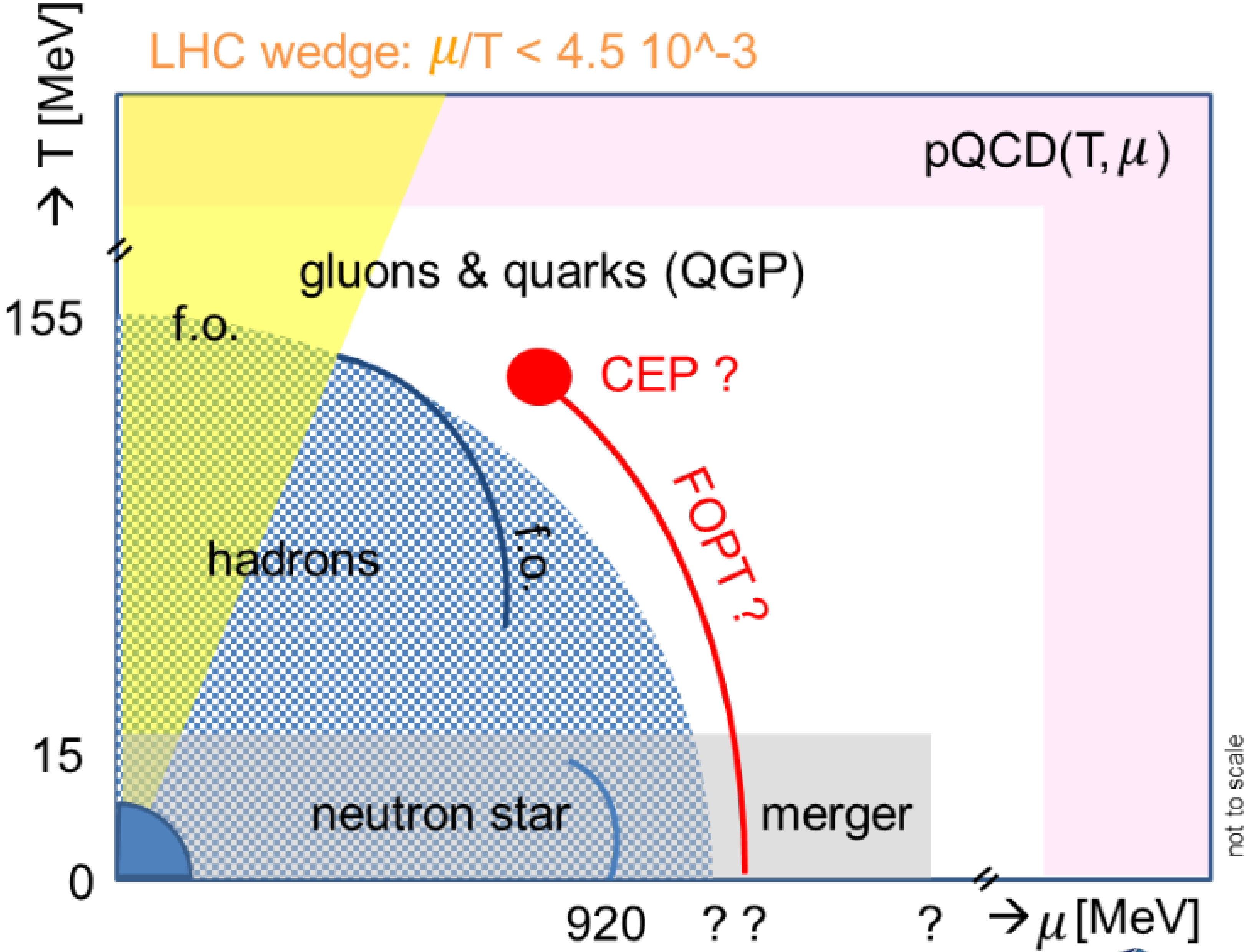
*) with J. Knaute, PLB (2018), PRD (2017)
R. Zollner, EPJA (2017), PRC (2016)
R. Yaresko, EPJC (2015), PLB (2015)



Gregor XV (1622): sacra congregatio de propaganda fide

questions: 1) CEP coordinates & HEE
input: p(T) & susceptibilities from lQCD

2) bulk viscosity
input: p(T) from lQCD

3) do hadrons exist at f.o. ?
input: hadrons in vacuum

tools: holography (AdS/CFT correspondence)
bottom-up engineering
(due to missing top-down from string theory
or QCD dual)

Einstein-dilaton-Maxwell in 5D (holographic QCD w/o gluons & quarks)

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(\underbrace{R-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)}_{\mu\ =\ 0} - \frac{f(\phi)}{4}F_{\mu\nu}^2\right) + \text{GH}$$

$\uparrow$ U(1) charge → μ

→ Einstein eqs. + EoM

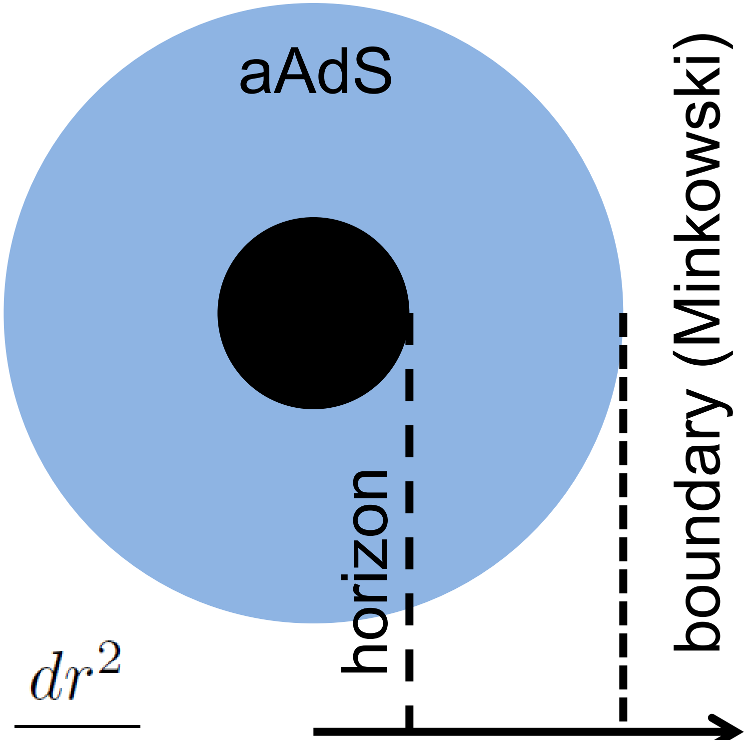
dictionary: 5D Riemann → 4D Minkowski

AdS + Schwarzschild BH

AdS: space-time with constant curvature & negative cosmolog. constant

Schwarzschild BH: black brane/horizon

mapping out the manifold
by various coordinates:



rH = 0	infinity
zH	0
phiH	0

$$ds^2 = \mathrm{e}^{2A(r)}\left(-h(r)dt^2 + d\vec{x}^2\right) + \frac{dr^2}{h(r)}$$

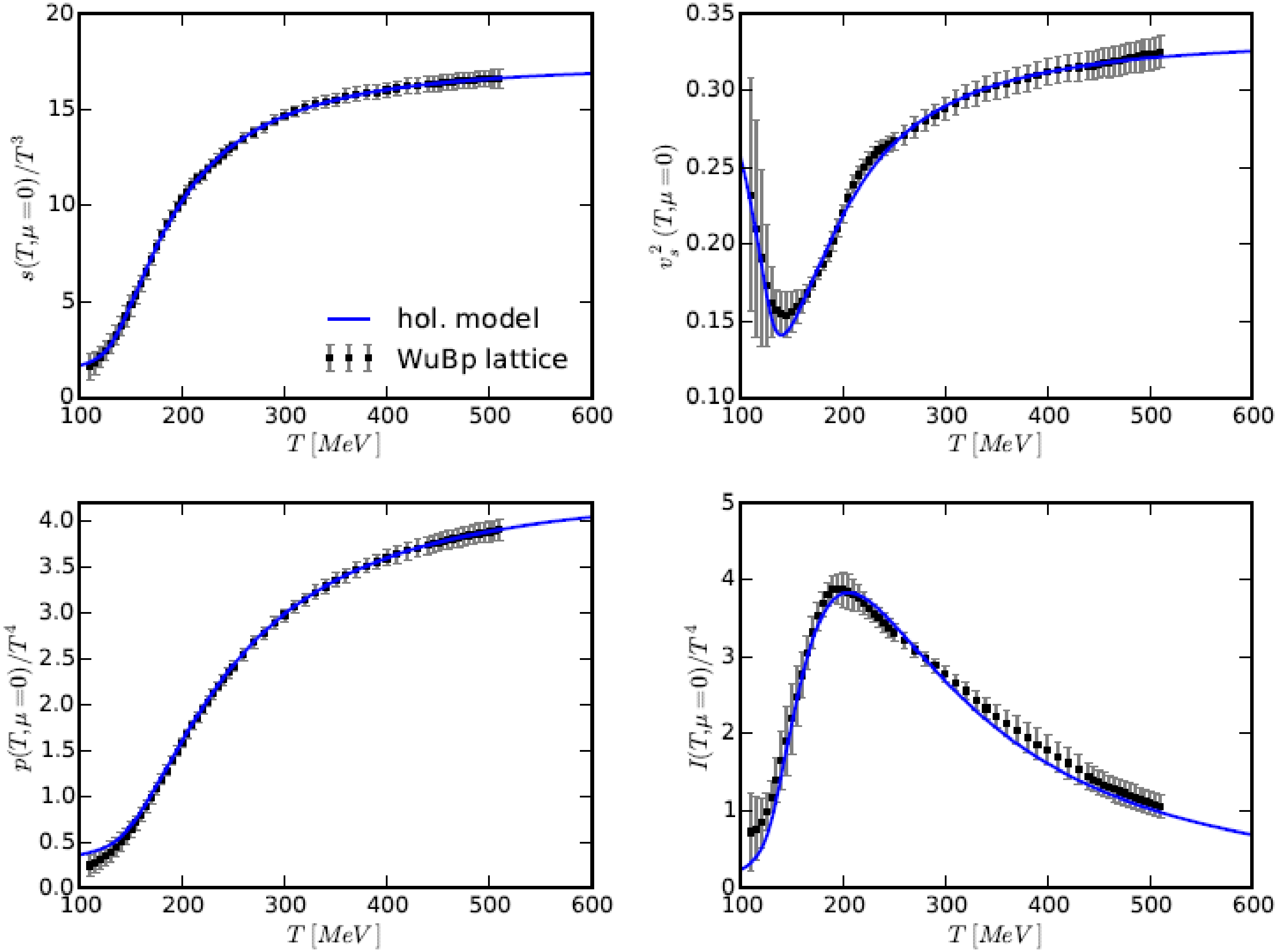
$$\mathrm{d}s^2 = e^{A(z)-\frac{2}{3}\Phi(z)}\left(f(z)\mathrm{d}t^2 - \mathrm{d}\vec{x}^{\,2} - \frac{\mathrm{d}z^2}{f(z)}\right)$$

$$ds^2 = e^{2A}(-hdt^2 + d\vec{x}^2) + e^{2B}\frac{d\phi^2}{h}$$

z, r … = holographic coordinate

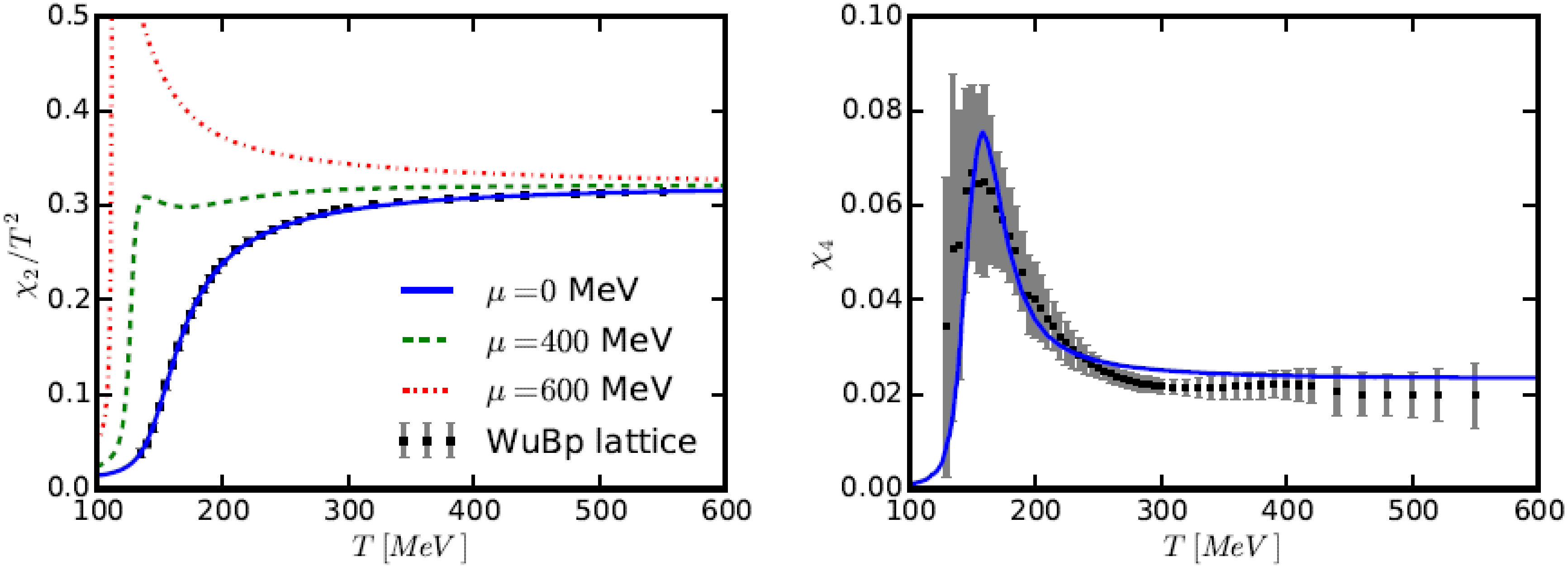
1) Phase Diagram: CEP, FOPT, HEE

strategy: encode 2+1 QCD gluons & quarks in V(phi), f(phi)



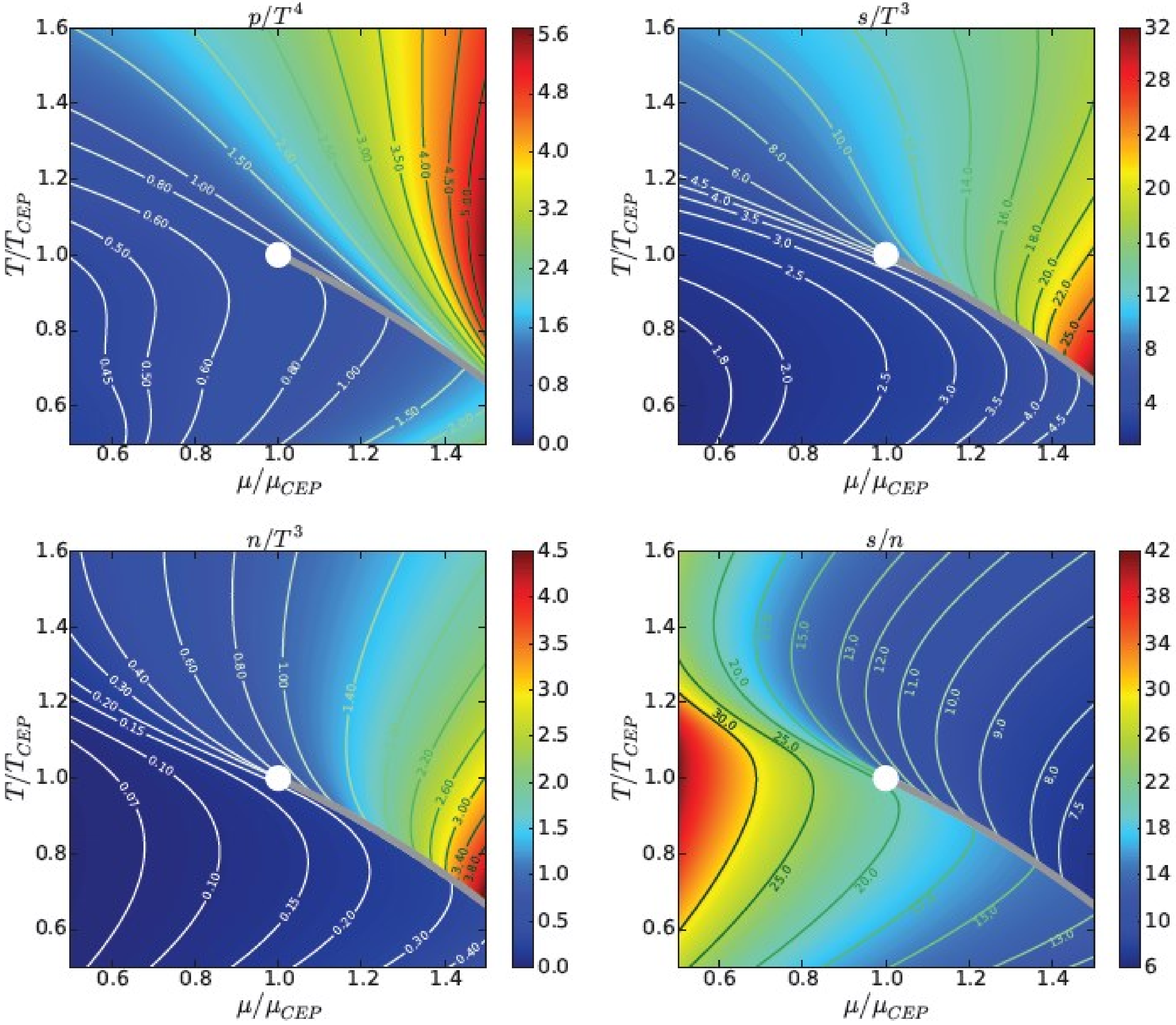
$$L^2V(\phi) = \begin{cases} -12\exp\left\{\frac{a_1}{2}\phi^2 + \frac{a_2}{4}\phi^4\right\} & : \phi < \phi_m \\ a_{10}\cosh\left[a_4(\phi - a_5)\right]^{a_3/a_4}\exp\left\{a_6\phi + \frac{a_7}{a_8}\tanh\left[a_8(\phi - a_9)\right]\right\} & : \phi \geq \phi_m \end{cases}$$

$$f(\phi) = c_0 + c_1\tanh\left[c_2(\phi - c_3)\right] + c_4\exp\left[-c_5\phi\right]$$

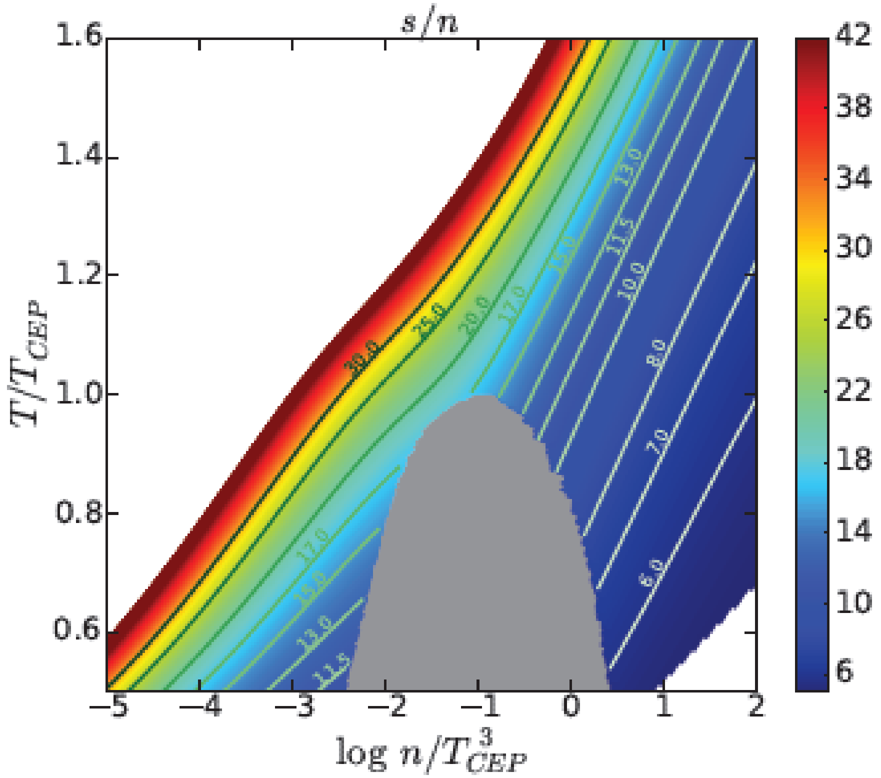


CEP (T, μ) = (112, 612) MeV

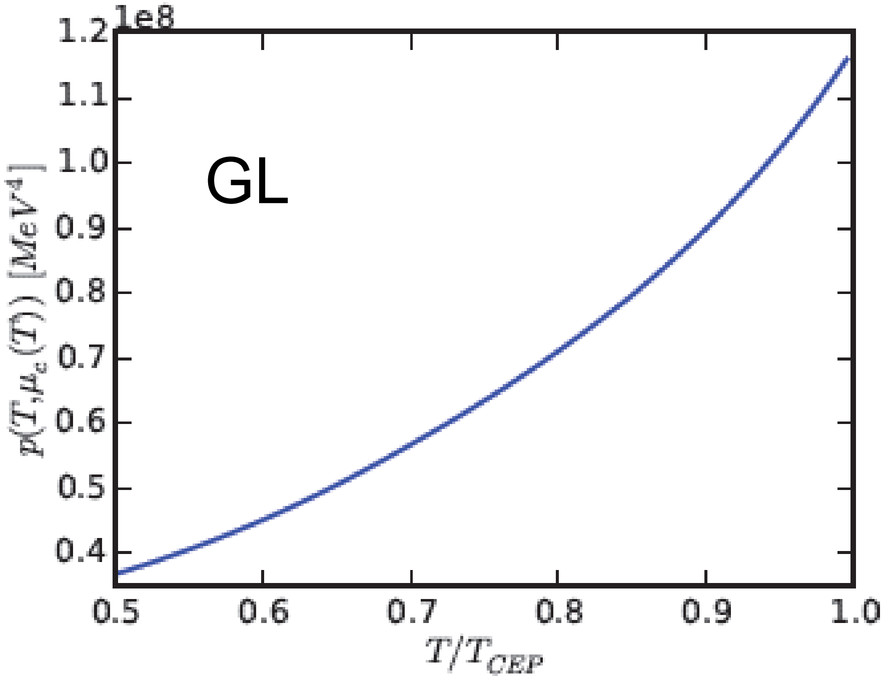
vs. (89, 723) MeV in 1706.00445


p/T^4
s/T^3
n/T^3
s/n
T/T_{CEP}
μ/μ_{CEP}
0.6
0.8
1.0
1.2
1.4
1.6
0.0
0.8
1.6
2.4
3.2
4.0
4.8
5.6
4
8
12
16
20
24
28
32
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.5
6
10
14
18
22
26
30
34
38
42

important: pattern of isentropic curves



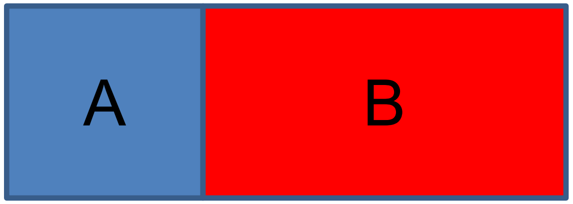
graceful exit



Clausius-Clapeyron

entanglement entropy: $S_{\mathrm{EE}} := -\operatorname{Tr}_{\mathcal{A}}\, \rho_{\mathcal{A}} \ln \rho_{\mathcal{A}}$

$$\rho_{\mathcal{A}} = \operatorname{Tr}_{\mathcal{B}}\, \rho_{tot}$$



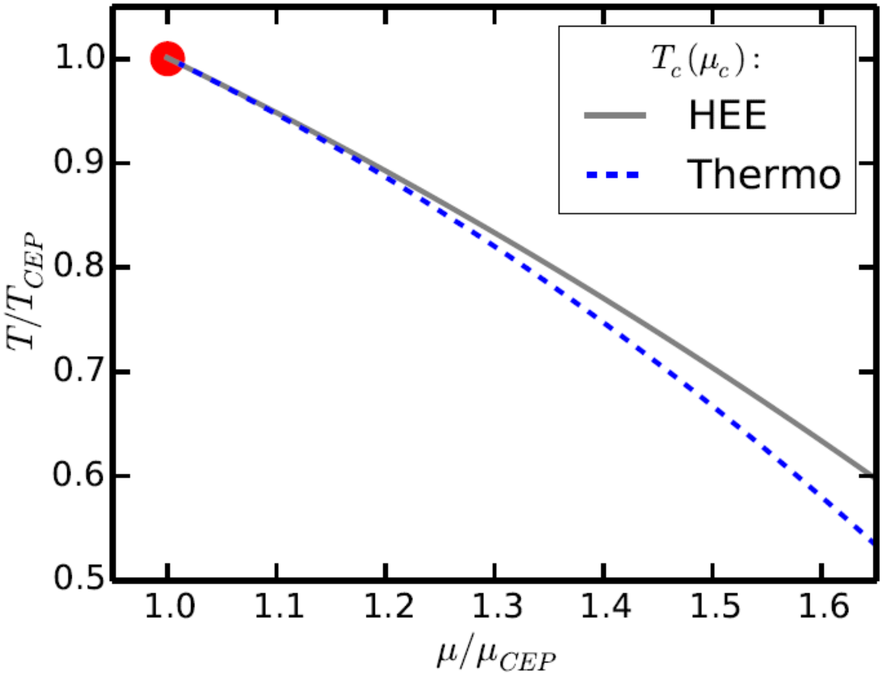
$$S_{\mathrm{HEE}} = \frac{\mathrm{Area}(\gamma_{\mathcal{A}})}{4G_N^{(d+1)}}$$

Ryu, Takayanagi (2006)

$$= \frac{1}{4}\int dx_1 dx_2 dx_3 \sqrt{\gamma}$$

$$= \frac{V_2}{2}\int_0^{l/2} dx_1 \mathrm{e}^{2A(r)}\sqrt{\mathrm{e}^{2A(r)} + \frac{r'^2}{h(r)}}$$

+ cut-off regularization



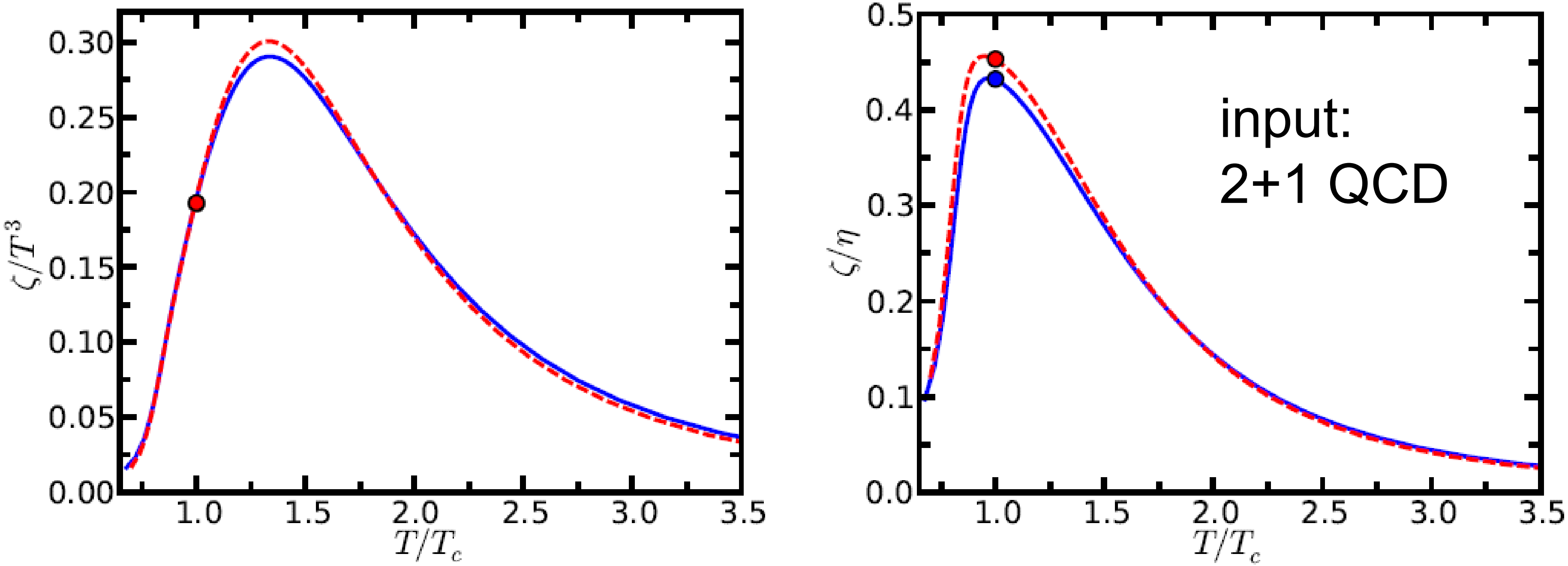
$$s^2_{\gamma_{\mathcal{A}}} = \left(\mathrm{e}^{2A} + \frac{r'^2}{h}\right) dx_1^2 + \mathrm{e}^{2A}\left(dx_2^2 + dx_3^2\right)$$

critical behavior/exponents

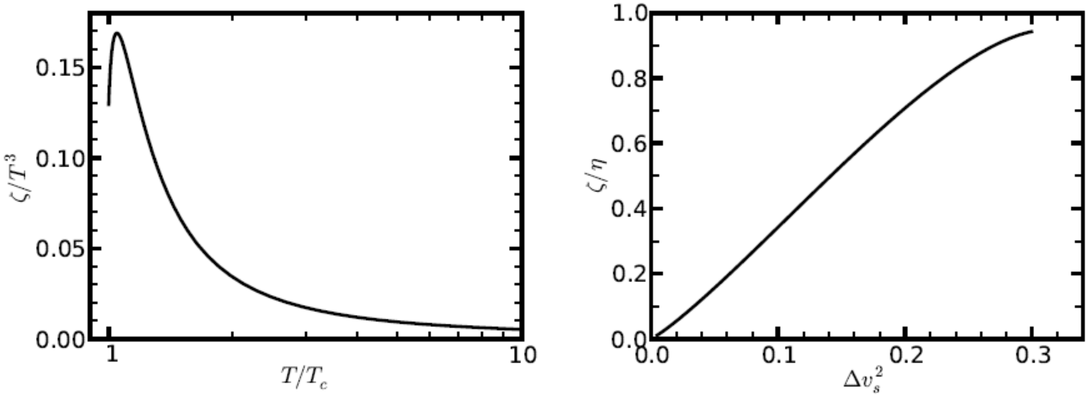
2) Bulk viscosity in LHC wedge

Eling-Oz formula:

$$\frac{\zeta}{\eta}\Big|_{\phi_H} = \left(\frac{d\log s}{d\phi_H}\right)^{-2} = \left(\frac{1}{v_s^2}\frac{d\log T}{d\phi_H}\right)^{-2};$$

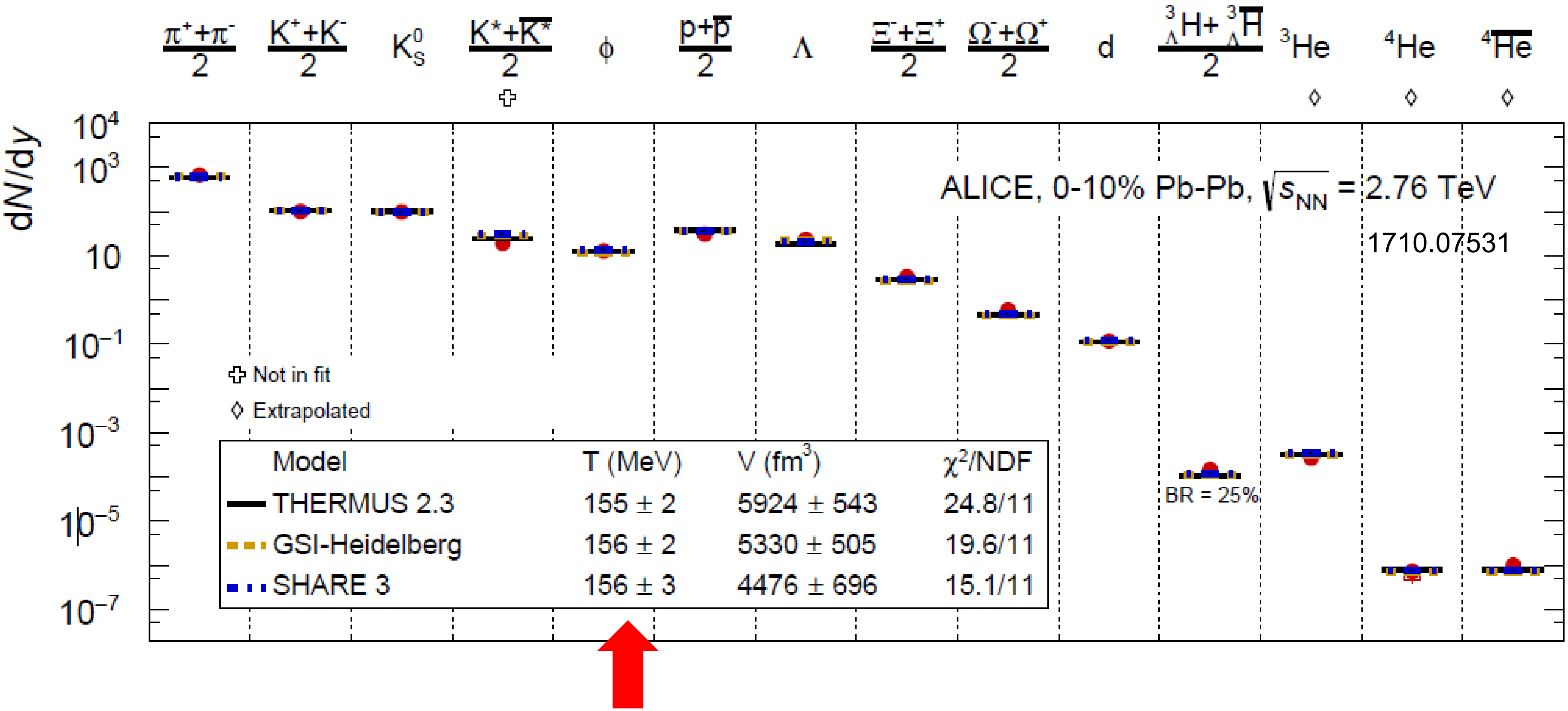


input:
2+1 QCD

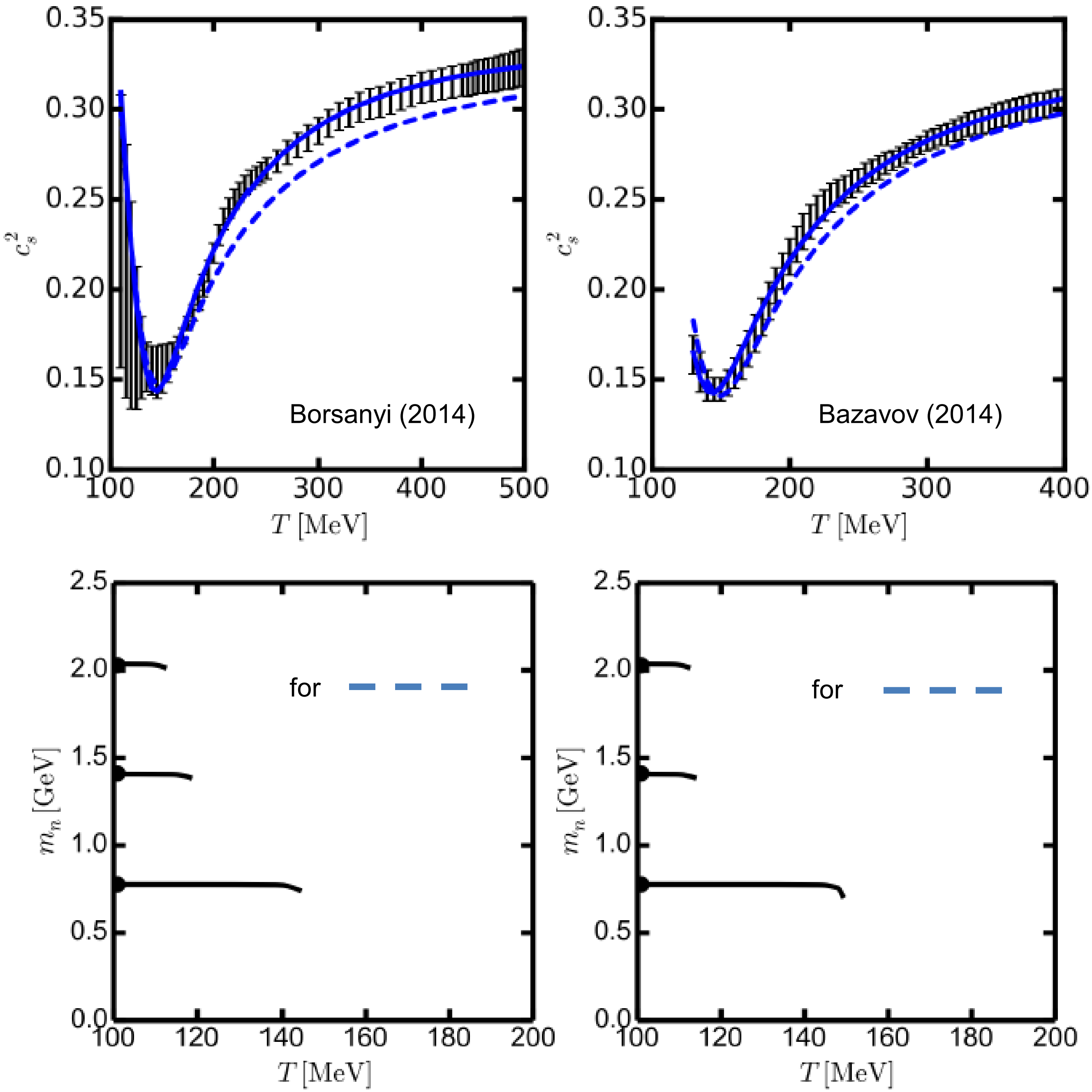


input:
SU(3) YM

3) Freeze-out



problem in given EdM model:
hadrons (vector mesons) in probe limit
melt at T(dis) << T(f.o.)



sequential melting

holographic vector mesons in probe limit

action: $$S_V = \frac{1}{k}\int \mathrm{d}^4x\,\mathrm{d}z\,\sqrt{g}e^{-\Phi(z)}F^2$$

EoM: $$\left(\partial_\xi^2 - (U_T - m_n^2)\right)\psi = 0$$

$$U_T = \left(\frac{1}{2}(\frac{1}{2}\partial_z^2 A - \partial_z^2\Phi) + \frac{1}{4}(\frac{1}{2}\partial_z A - \partial_z\Phi)^2\right) f^2 + \frac{1}{4}\underbrace{(\frac{1}{2}\partial_z A - \partial_z\Phi)}_{s'}\partial_z f^2.$$

popular requirement: U(T = 0) → Regge spectrum
(radial excitations n)

→ FOPT: 2+1 QCD in chiral limit (cf. Columbia plot)

2+1 QCD input (cross over) → no discrete hadron states

Gürsoy, Kiritsis, Mazzani, Nitti (2009), pure gluon sector:

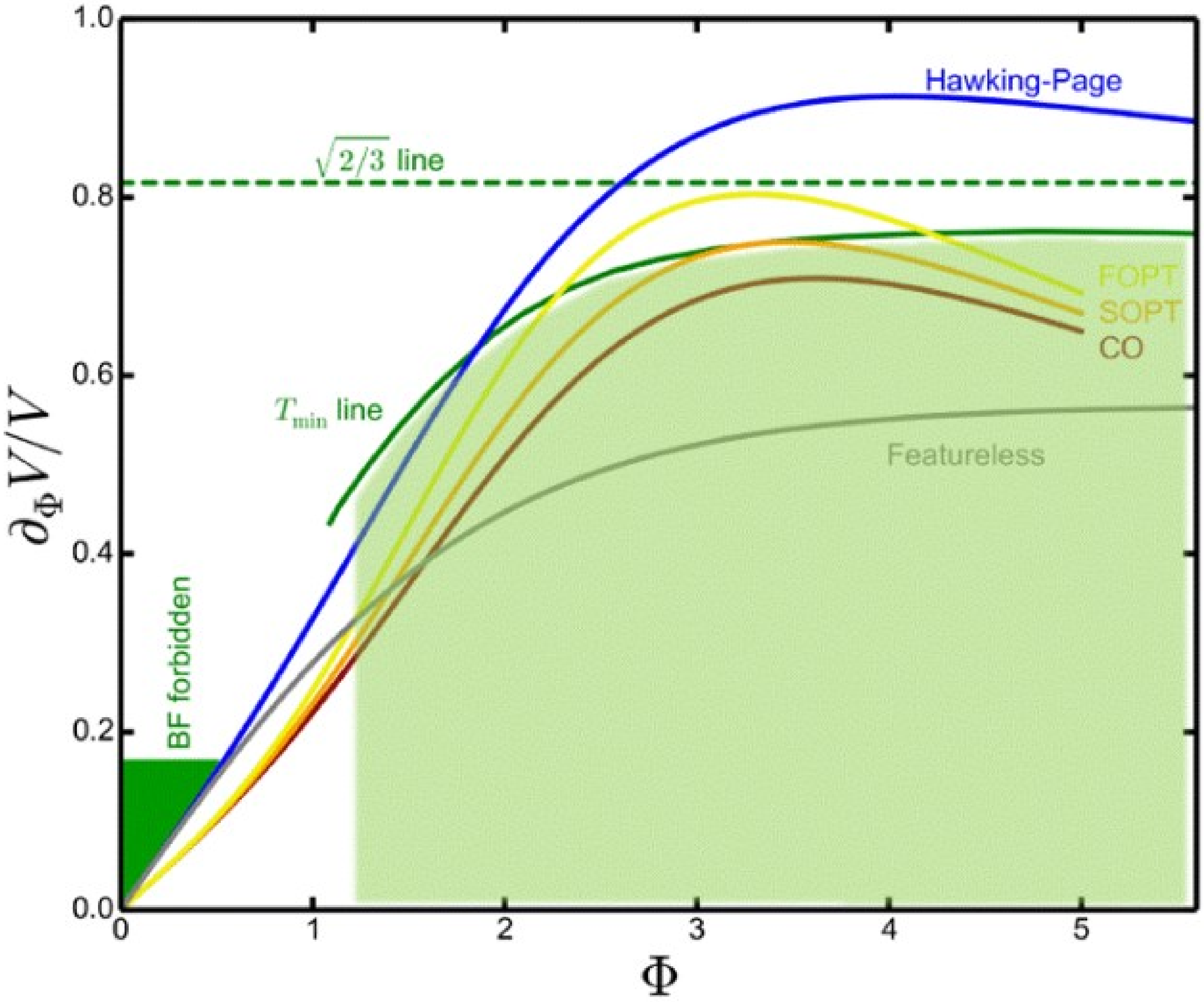
a gapped, discrete spectrum at $T = 0$
facilitates a FOPT at $T > 0$

no-go conjecture within EdM model & probe vector mesons:

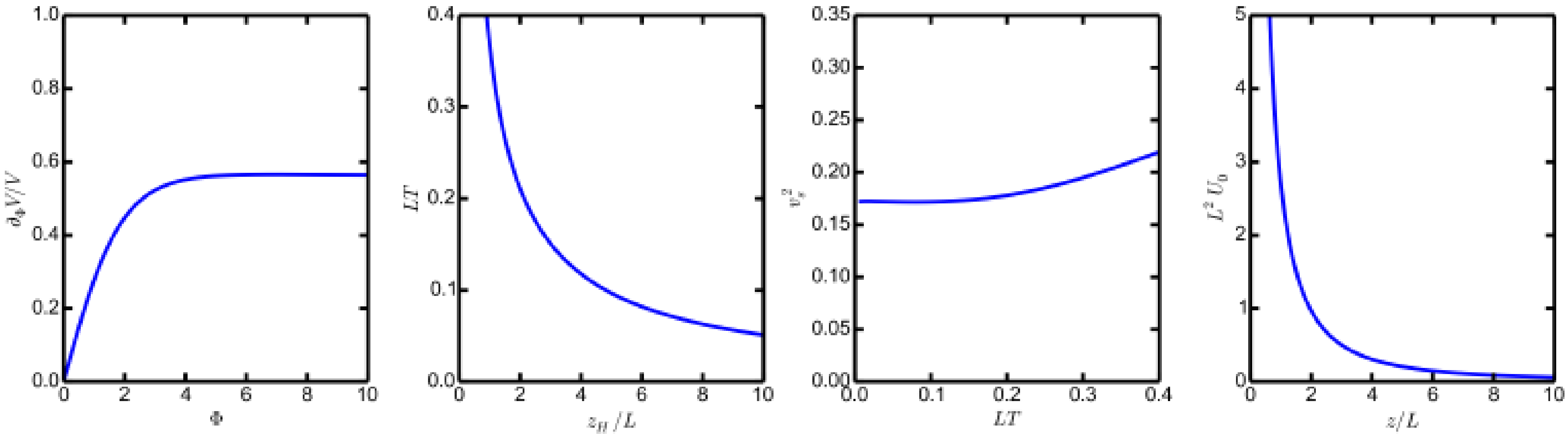
either FOPT & Regge spectrum
or cross over & meson melting sets in at $T = 0$

→ beyond probe limit (backreacted hadrons),
add systematically flavor to gluon dynamics,

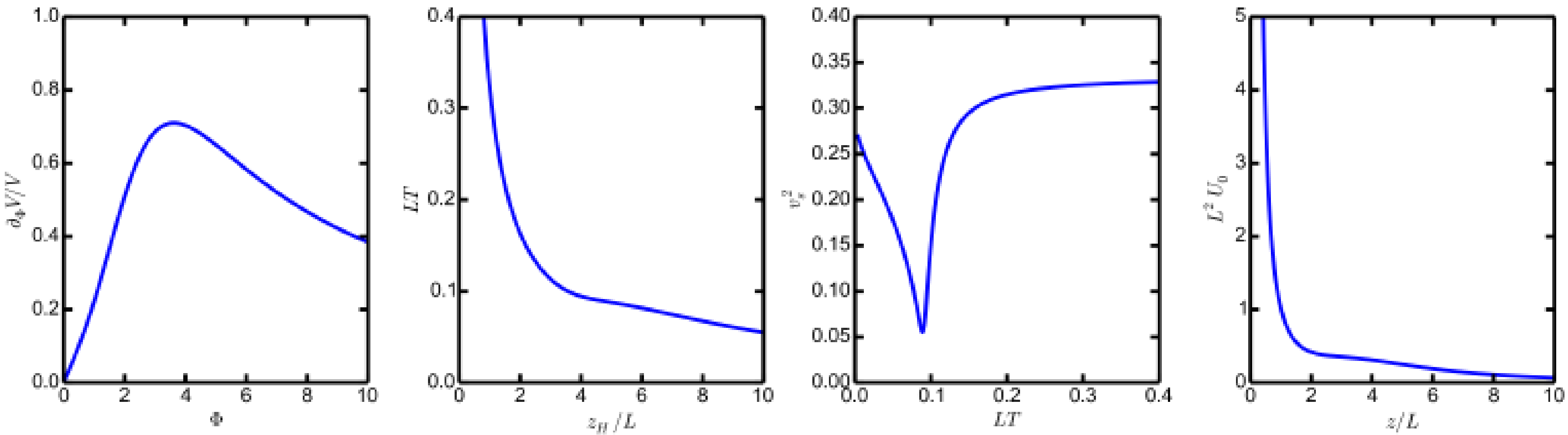
$$-L^2V_1(\Phi) = 12\exp(a\Phi^2 + b\Phi^4)$$
$$-L^2V_2(\Phi) = 12\cosh(\gamma\Phi) + a\Phi^2 + b\Phi^4$$



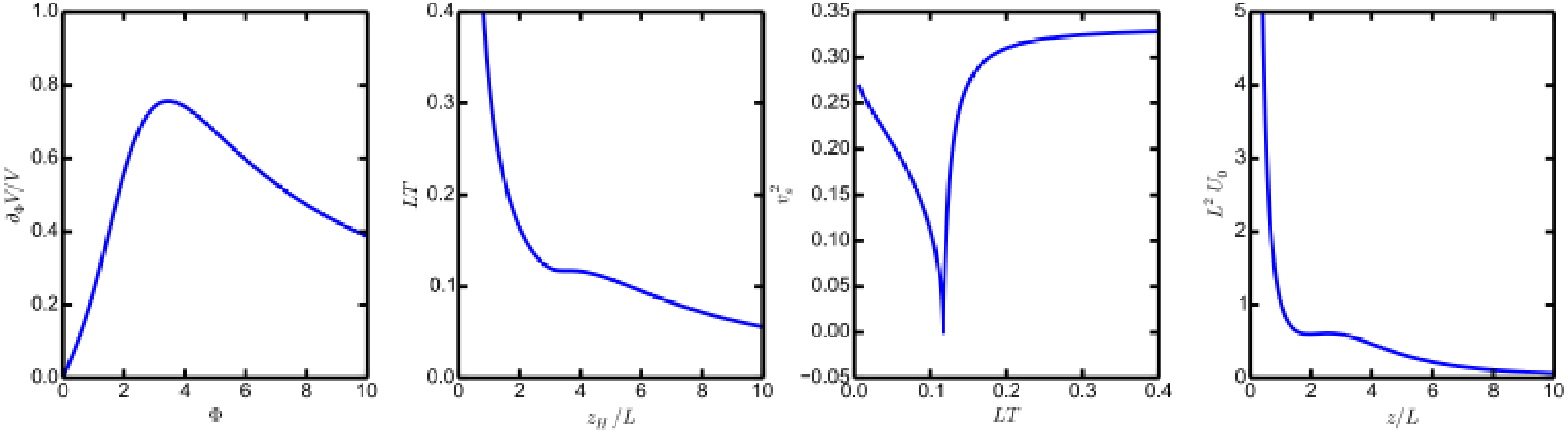
Featureless



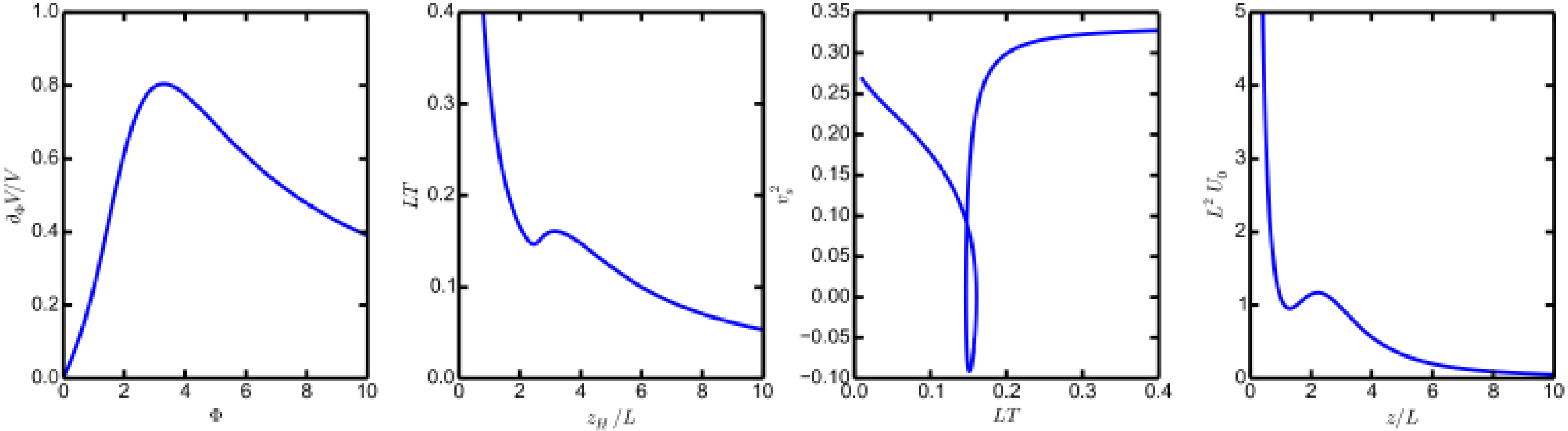
Cross-over



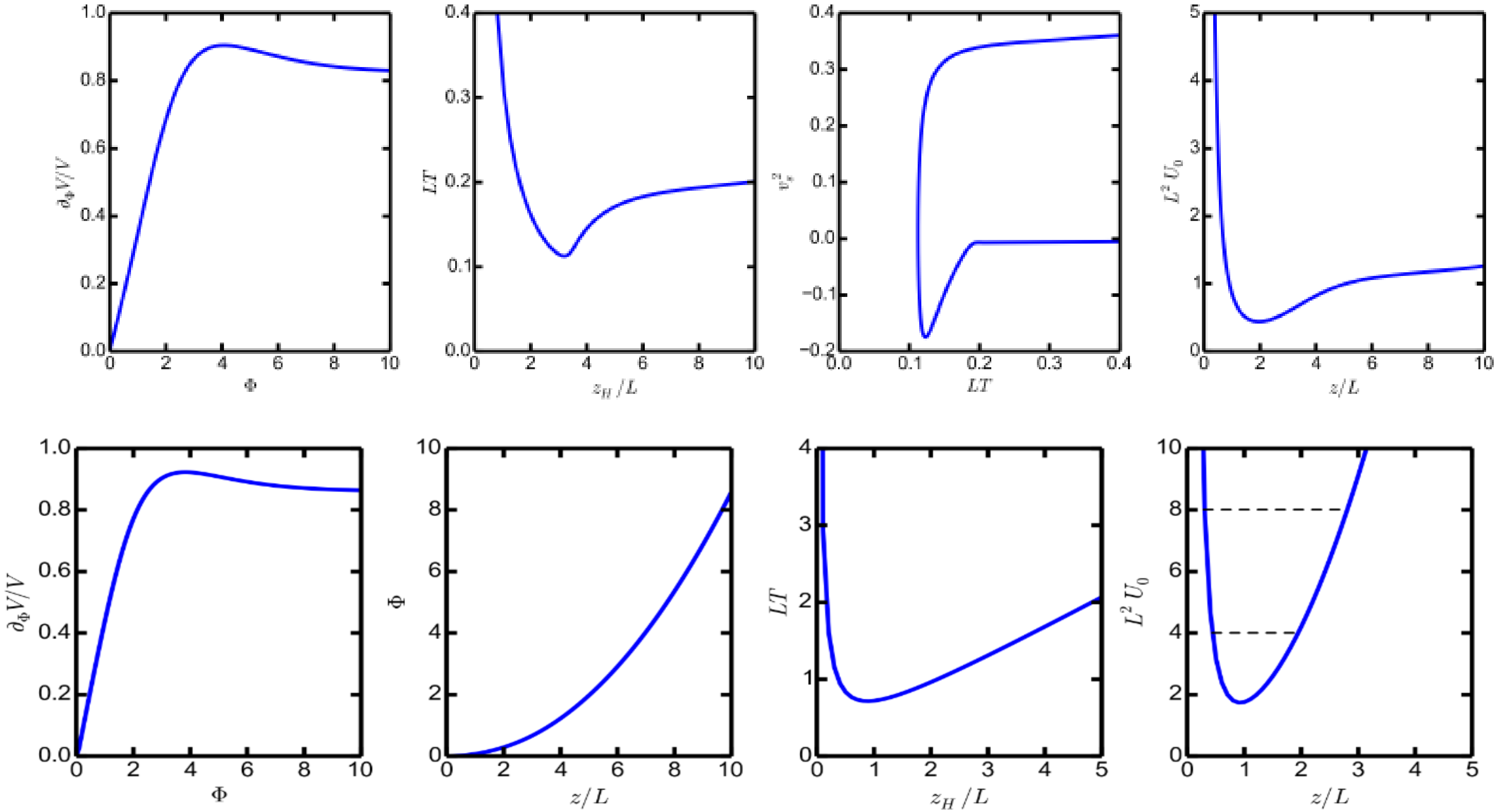
Second-order phase transition

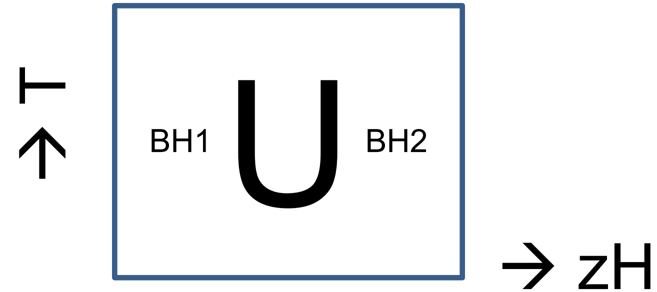


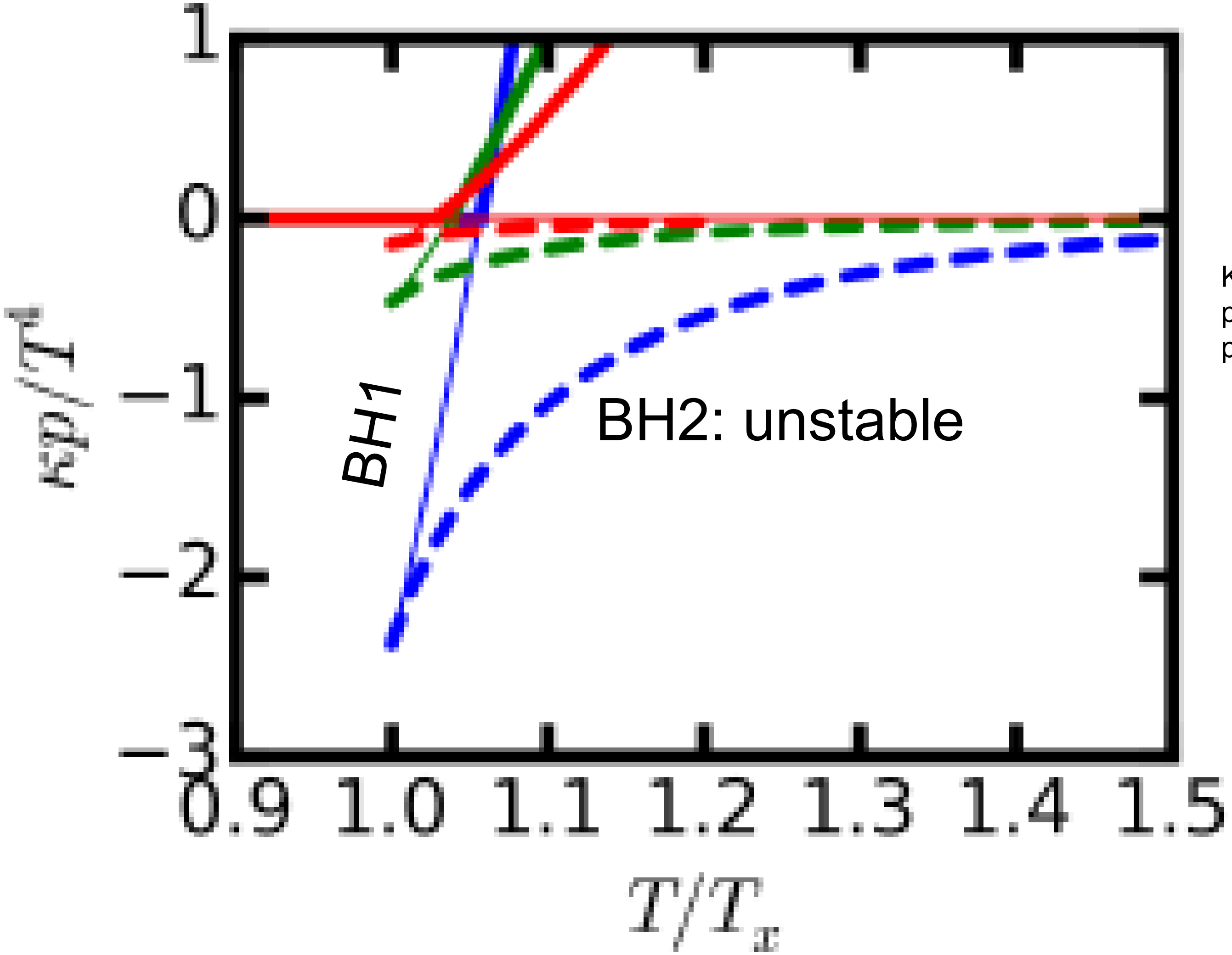
First-order phase transition



Hawking-Page



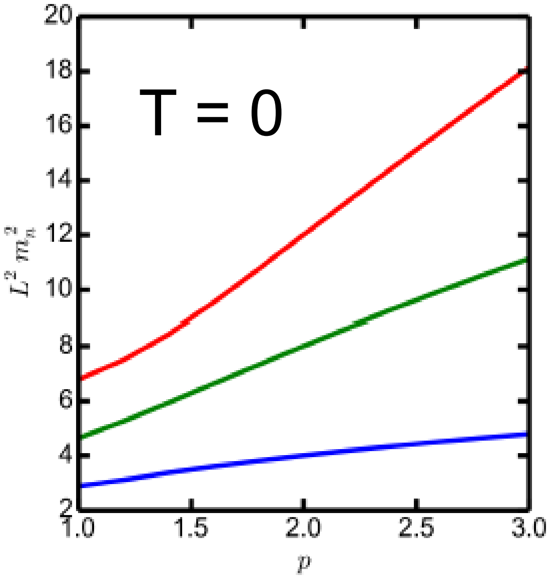




Kiritsis et al. (2008):
p(BH) ~ Nc^2
p(therm.gas) ~ O(Nc^0)

opposite way: start with

$$U_0 = \frac{3}{4z^2} + \left(\frac{z}{L}\right)^p$$



L. Die Dgl. $U_0 = \frac{1}{2}s'' + \frac{1}{4}s'^2$ besitzt die allgemeine Lösung

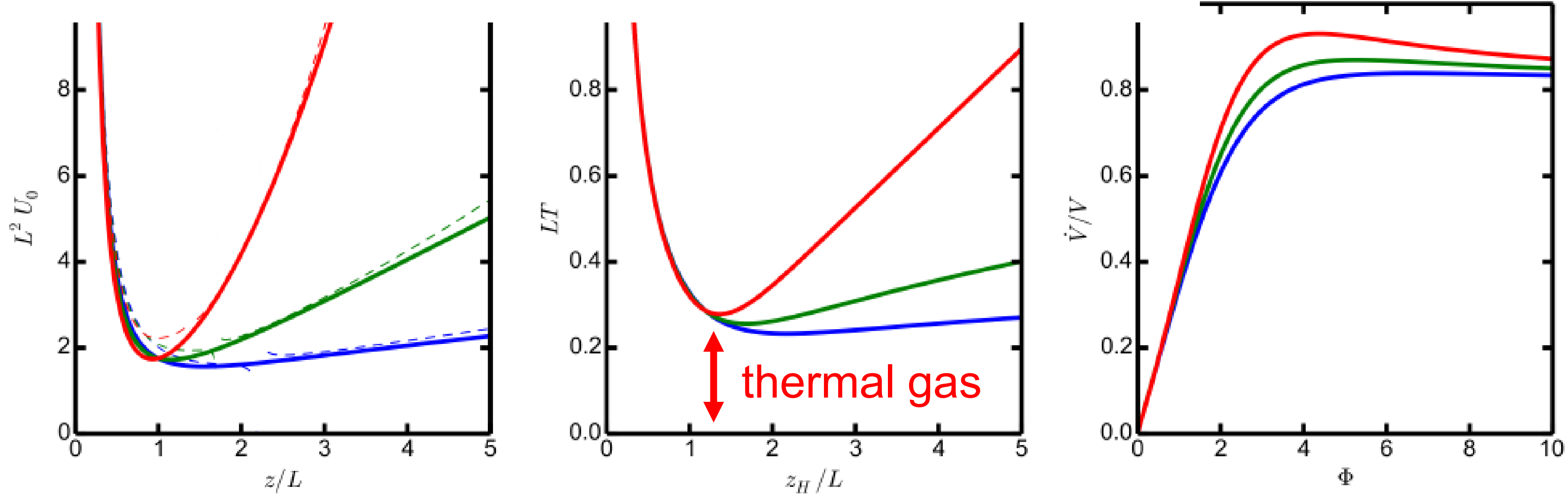
$$s = 2\ln\left(c_1 \hat{z}^{-\frac{1}{2}}\,{}_0F_1\left(\frac{p}{p+2}, \frac{\hat{z}^{p+2}}{(p+2)^2}\right) + c_2 \hat{z}^{\frac{3}{2}}\,{}_0F_1\left(\frac{p+4}{p+2}, \frac{\hat{z}^{p+2}}{(p+2)^2}\right)\right)$$

s = A/2 – 2 phi /3

Wenn man U_0 global annimmt, folgt daraus eindeutig c_2 als

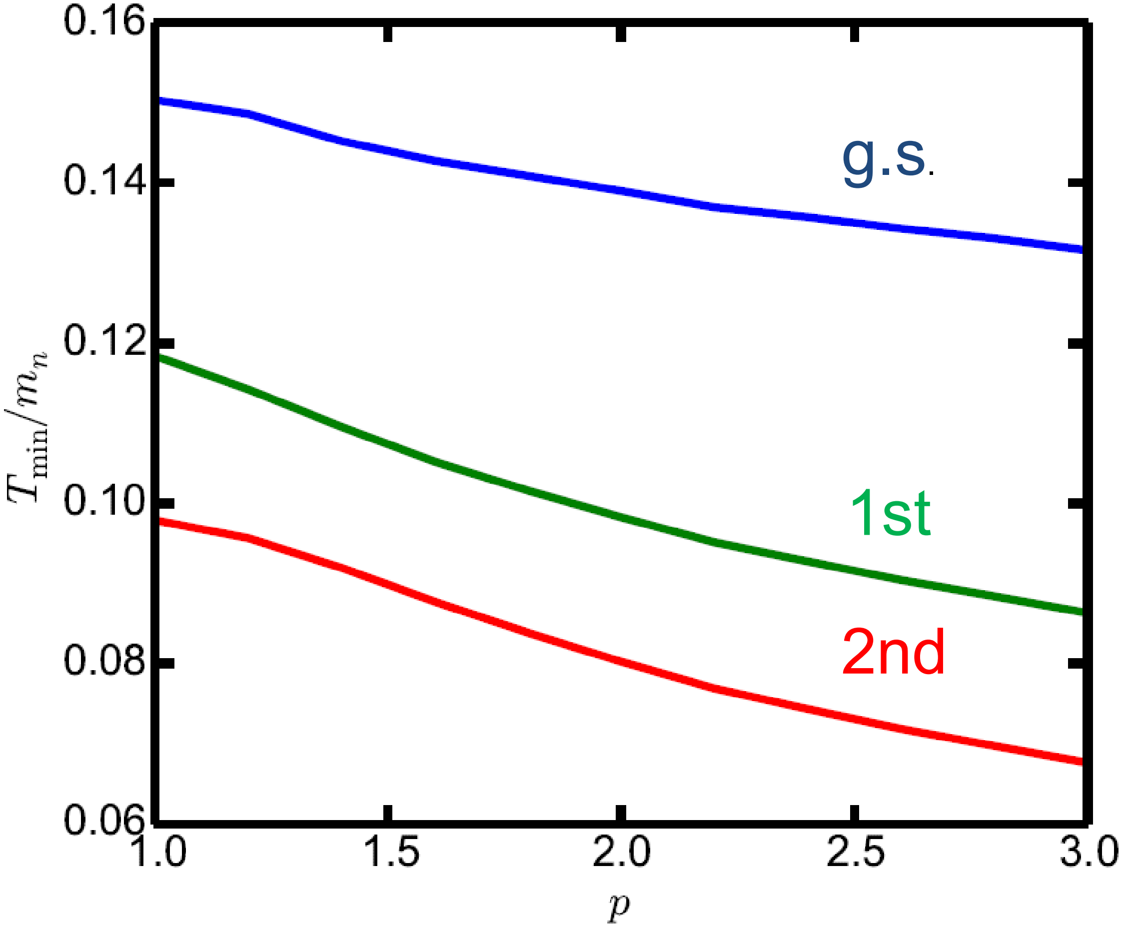
$$c_2 = -\frac{1}{2}\frac{\Gamma(\frac{p}{2+p})}{\Gamma(\frac{2}{p+2})}(p+2)^{\frac{p-2}{p+2}}$$

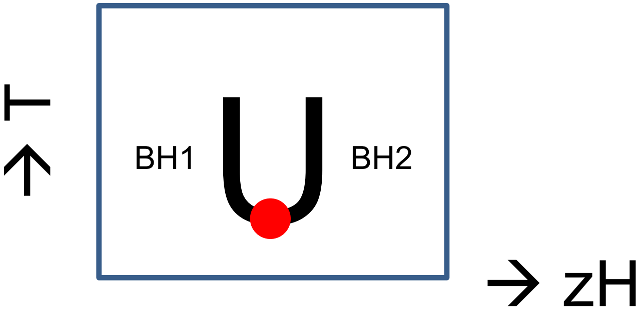
, c1 = 1

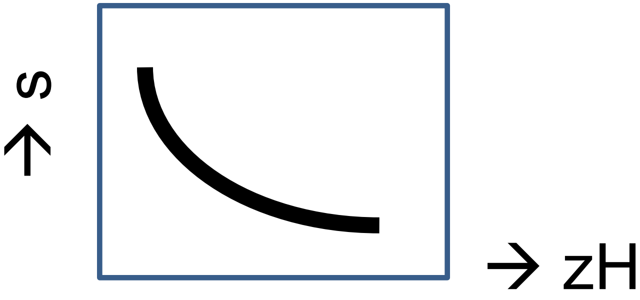


0.20

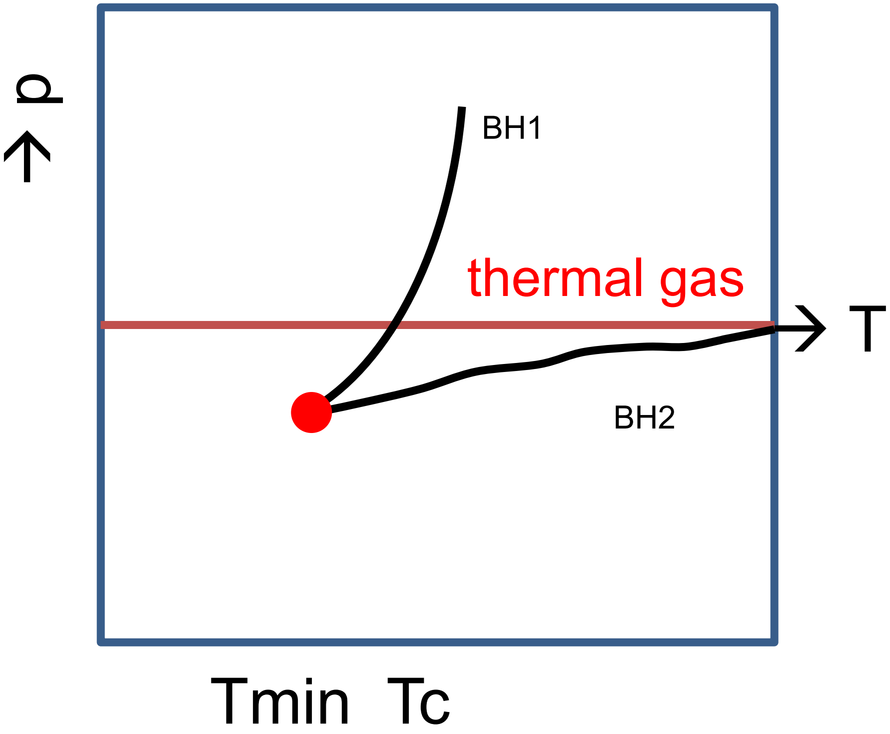
2 +1 QCD:
T_co = 155 MeV, m_rh0 = 777 MeV



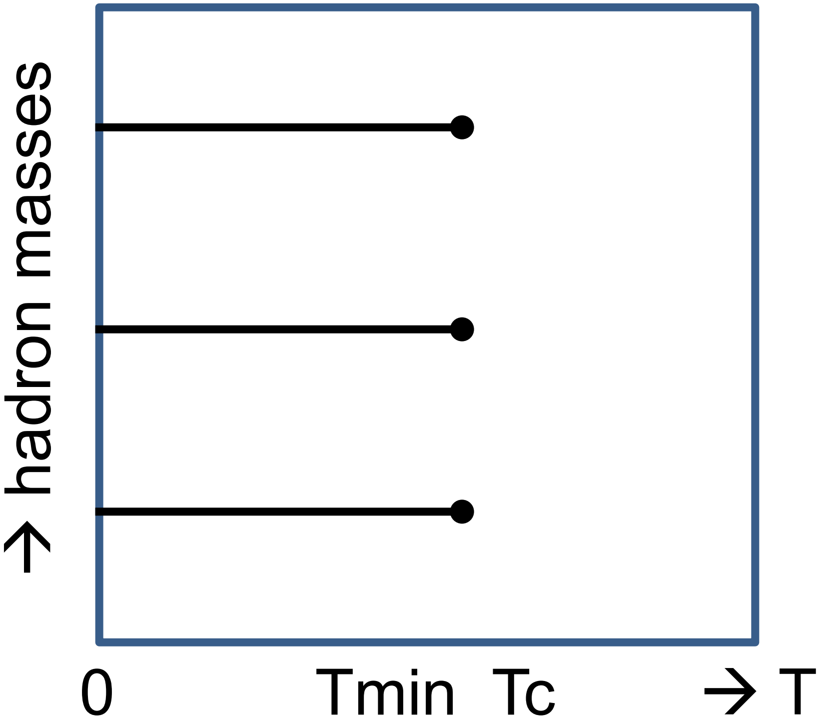




Hawking –Page as FOPT



instant. melting



Summary of holographic EdM model

1) phase structure & CEP coordinates
   HEE (cut-off) yields same information

prediction

2) bulk viscosity = 50% shear viscosity (2+1 QCD)
   100% (SU(3) YM)

predictions

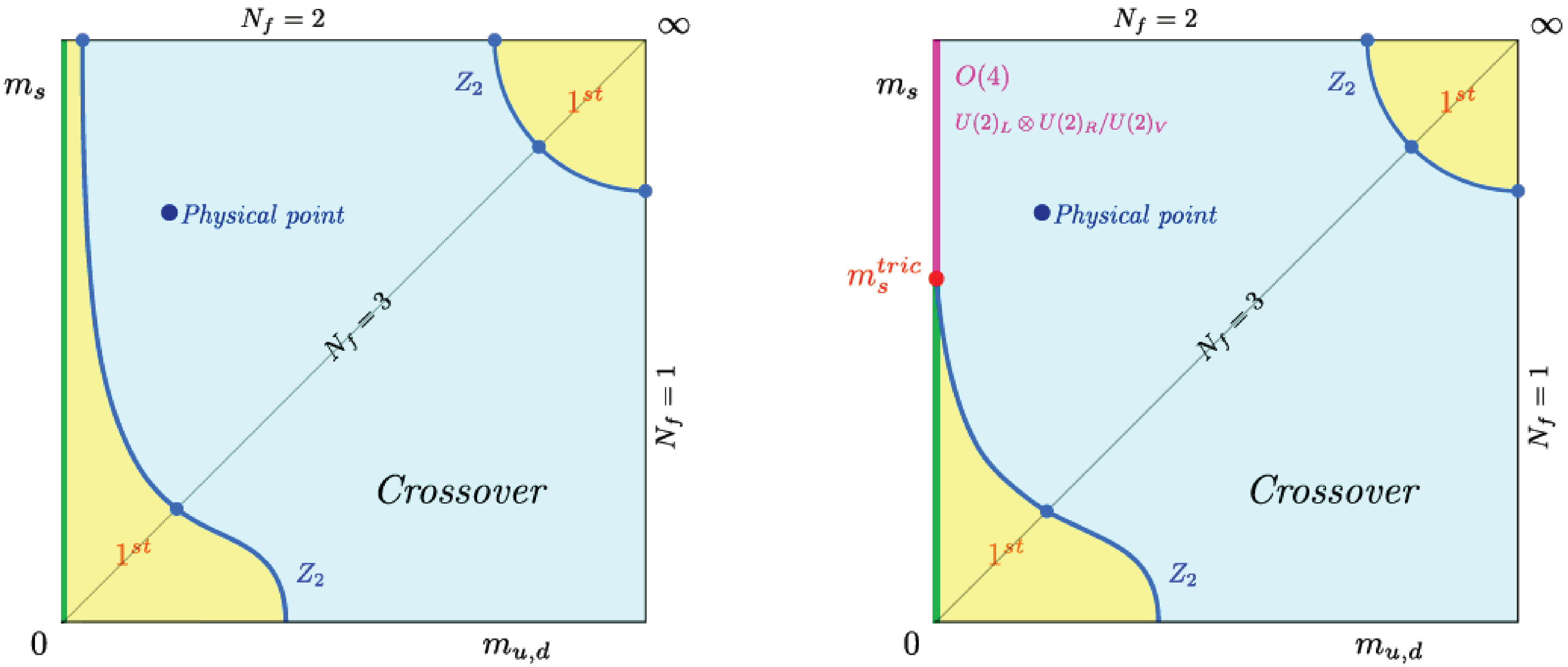
3) vector mesons in probe limit: no-go conjecture
   either
   dilaton potential to match QCD thermodynamics
   → no hadrons at/below CO Tc

desaster

or
   Schrodinger equivalent potential for Regge states
   → FOPT

desaster

Columbia Plot



(a) First order scenario in the $m_s - m_{u,d}$ plane

(b) Second order scenario in the $m_s - m_{u,d}$ plane.

EPJ Web Conf. 175 (2018) 07032, Philipsen et al.

$$S = \frac{1}{k} \int \sqrt{g} (R - \frac{1}{2} (\partial \Phi)^2 - V(\Phi)) \, \mathrm{d}^5 x$$

$$\mathrm{d}s^2 = e^A (f \mathrm{d}t^2 - \mathrm{d}\vec{x}^2 - \mathrm{d}z^2 / f)$$

$$\begin{aligned}
f'' + \frac{3}{2} A' f' &= 0 \\
A'' - \frac{1}{2} A'^2 + \frac{1}{3} \Phi'^2 &= 0 \\
(A'^2 - \frac{1}{6} \Phi'^2) f + \frac{1}{2} A' f' - \frac{1}{3} e^A V &= 0
\end{aligned}$$

$$\Phi'' + \left(\frac{3}{2} A' + \frac{f'}{f} \right) \Phi' + \frac{e^A}{f} \dot{V} = 0$$

Chesler & Yaffe

$$ds^2 = -A\,dv^2 + \Sigma^2\left[e^{B}dx_\perp^2 + e^{-2B}dz^2\right] + 2dv\,(dr + F\,dz)$$

$$0 = \Sigma'' + \tfrac{1}{2}(B')^2\,\Sigma\,, \tag{2a}$$

$$\begin{aligned} 0 &= \Sigma^2\left[F'' - 2(d_3B)' - 3B'd_3B\right] + 4\Sigma' d_3\Sigma\,,\\ &\quad - \Sigma\left[3\Sigma' F' + 4(d_3\Sigma)' + 6B'd_3\Sigma\right], \end{aligned} \tag{2b}$$

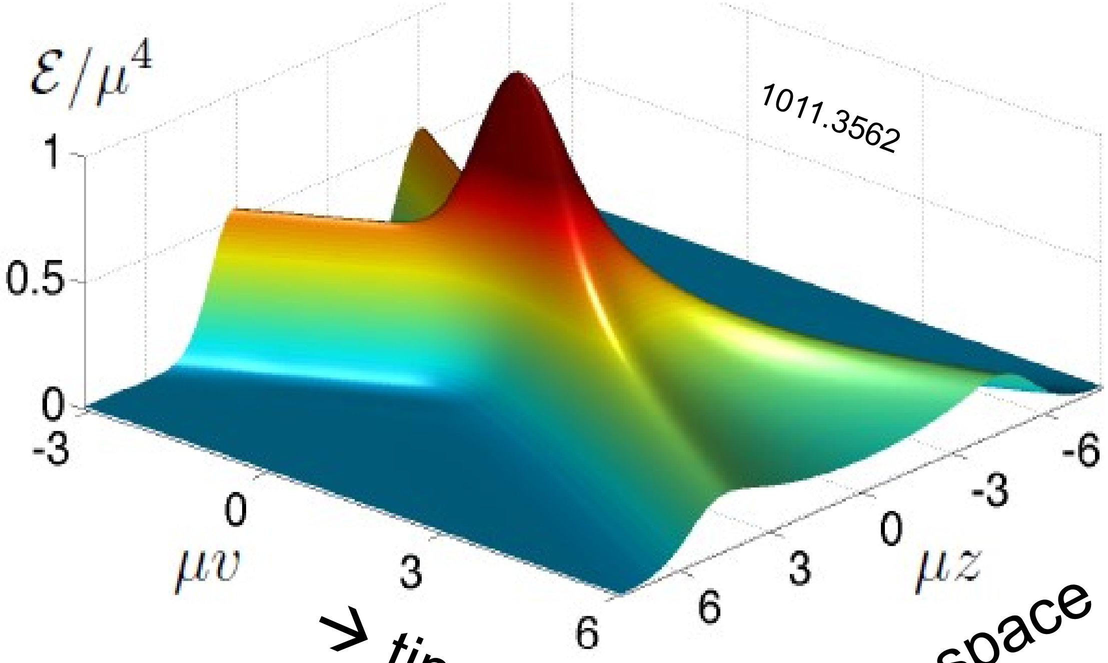
$$\begin{aligned} 0 &= \Sigma^4\left[A'' + 3B'd_+B + 4\right] - 12\Sigma^2\Sigma' d_+\Sigma\\ &\quad + e^{2B}\big\{\Sigma^2\left[\tfrac{1}{2}(F')^2 - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right]\\ &\qquad + 2(d_3\Sigma)^2 - 4\Sigma\left[2(d_3B)d_3\Sigma + d_3^2\Sigma\right]\big\}\,, \end{aligned} \tag{2c}$$

$$\begin{aligned} 0 &= 6\Sigma^3(d_+\Sigma)' + 12\Sigma^2(\Sigma' d_+\Sigma - \Sigma^2) - e^{2B}\big\{2(d_3\Sigma)^2\\ &\quad + \Sigma^2\left[\tfrac{1}{2}(F')^2 + (d_3F)' + 2F'd_3B - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right]\\ &\qquad + \Sigma\left[(F' - 8d_3B)\,d_3\Sigma - 4d_3^2\Sigma\right]\big\}\,. \end{aligned} \tag{2d}$$

$$\begin{aligned} 0 &= 6\Sigma^4(d_+B)' + 9\Sigma^3(\Sigma' d_+B + B'd_+\Sigma)\\ &\quad + e^{2B}\big\{\Sigma^2[(F')^2 + 2(d_3F)' + F'd_3B - (d_3B)^2 - d_3^2B]\\ &\qquad + 4(d_3\Sigma)^2 - \Sigma\left[(4F' + d_3B)\,d_3\Sigma + 2d_3^2\Sigma\right]\big\}\,, \end{aligned} \tag{2e}$$

$$\begin{aligned} 0 &= 6\Sigma^2 d_+^2\Sigma - 3\Sigma^2 A' d_+\Sigma + 3\Sigma^3(d_+B)^2\\ &\quad - e^{2B}\big\{(d_3\Sigma + 2\Sigma d_3B)(2d_+F + d_3A)\\ &\qquad + \Sigma\left[2d_3(d_+F) + d_3^2A\right]\big\}\,, \end{aligned} \tag{2f}$$

$$\begin{aligned} 0 &= \Sigma\left[2d_+(d_3\Sigma) + 2d_3(d_+\Sigma) + 3F'd_+\Sigma\right]\\ &\quad + \Sigma^2\left[d_+(F') + d_3(A') + 4d_3(d_+B) - 2d_+(d_3B)\right]\\ &\quad + 3\Sigma\,(\Sigma d_3B + 2d_3\Sigma)\,d_+B - 4(d_3\Sigma)d_+\Sigma\,, \end{aligned} \tag{2g}$$



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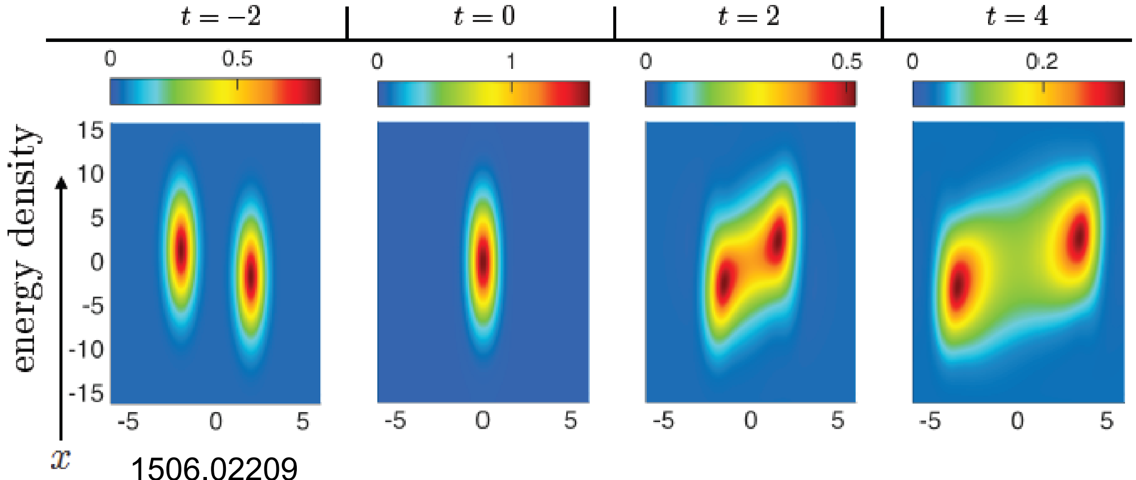
→ time

→ long. space



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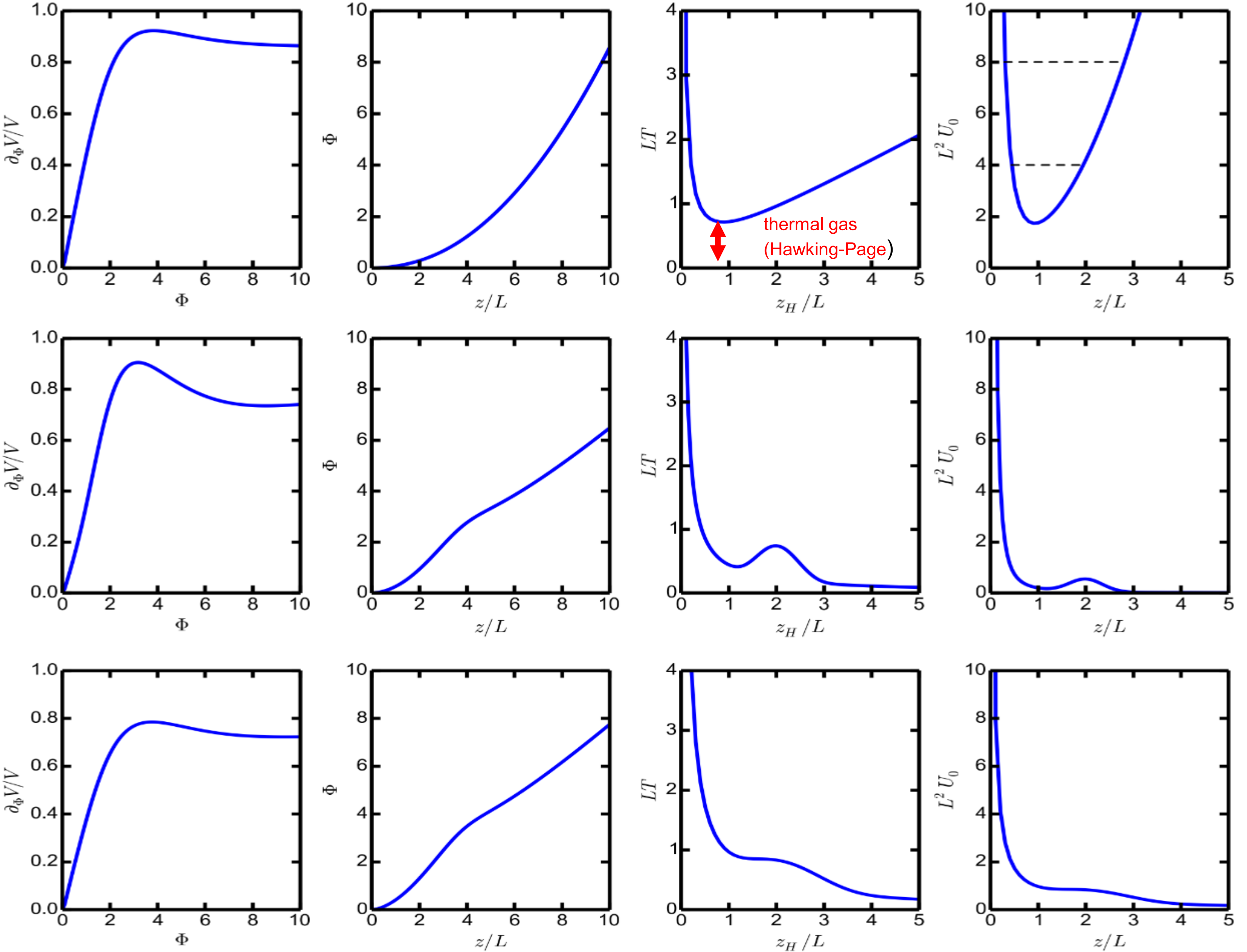
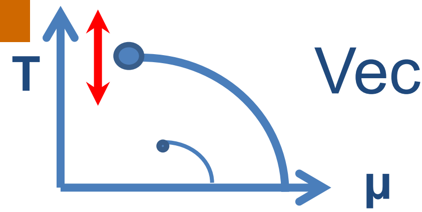


thermal gas
(Hawking-Page)

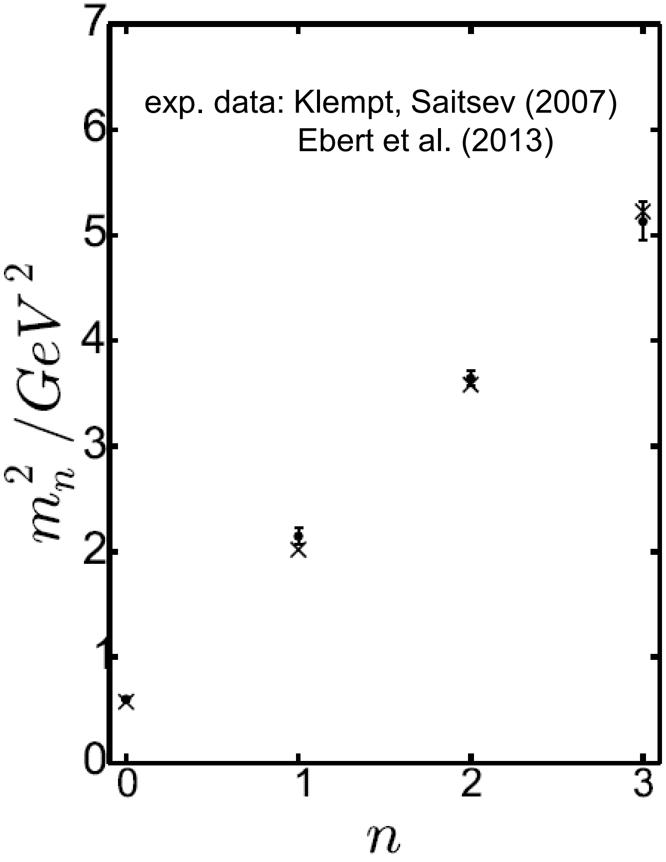


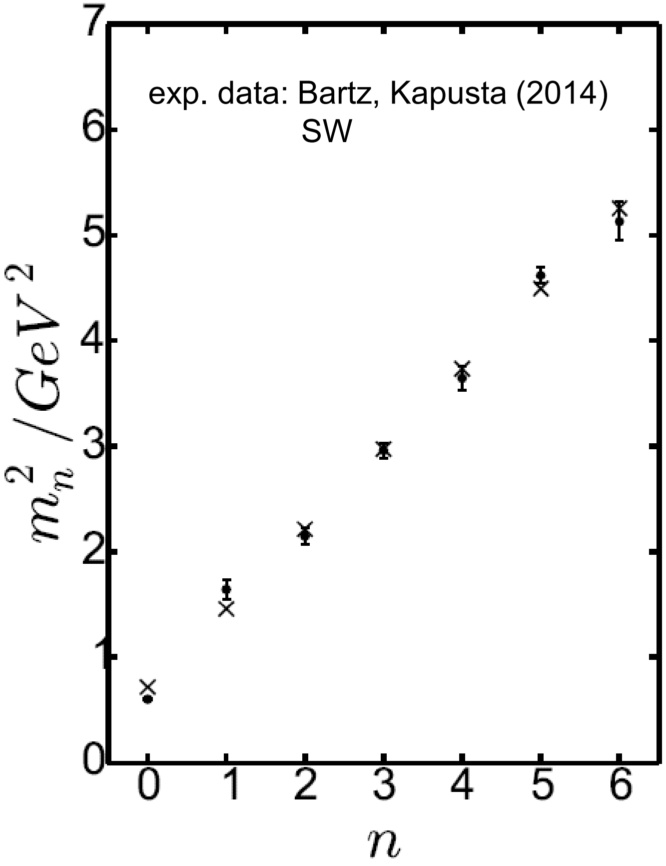
Vector mesons in AdS/CFT – extended soft wall model

$$S_V = F(\overbrace{\text{warp factor, blackening function}}^{\text{5D gravity}}, \overbrace{\text{dilaton}}^{\text{conf. symmetry breaker}}, \overbrace{\text{V wave function}}^{\text{sourced by } \bar{q}\gamma^\mu q})$$

soft wall (probe limit): $A(z) = \ln{(L/z)^2}$ $f(z) = 1 - (\frac{z}{H})^4$ $\Phi(z) = (cz)^2$

EoM of V→Schrödinger eq. in tortoise coordinate, T = 0 → Regge type spectrum



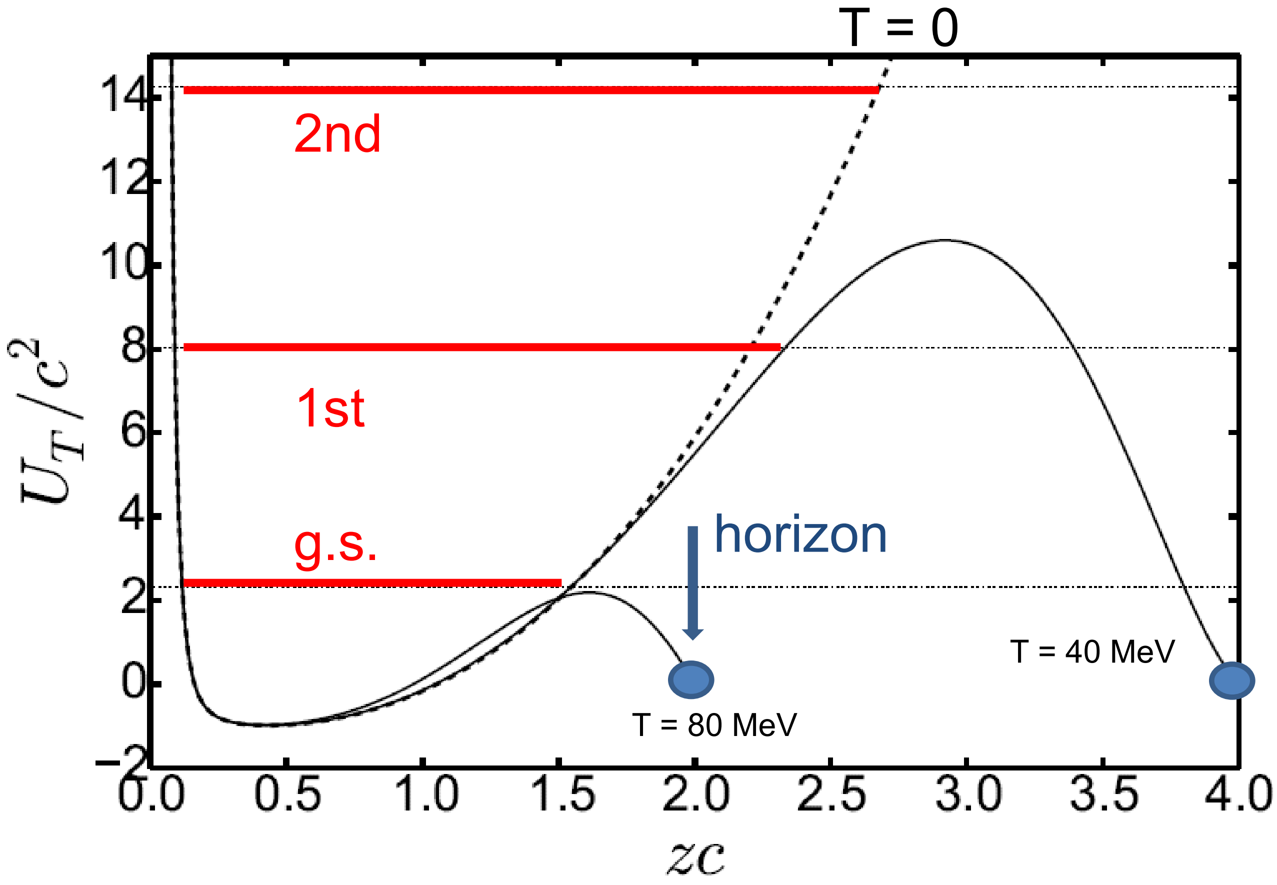


rho trajectory from mod. SW: $\tilde{A}, \tilde{f}, \tilde{\phi}$

SW & theor. reasoning:
Karch, Katz, Son, Stephanov
PRD (2006)

Schrödinger equivalent potential

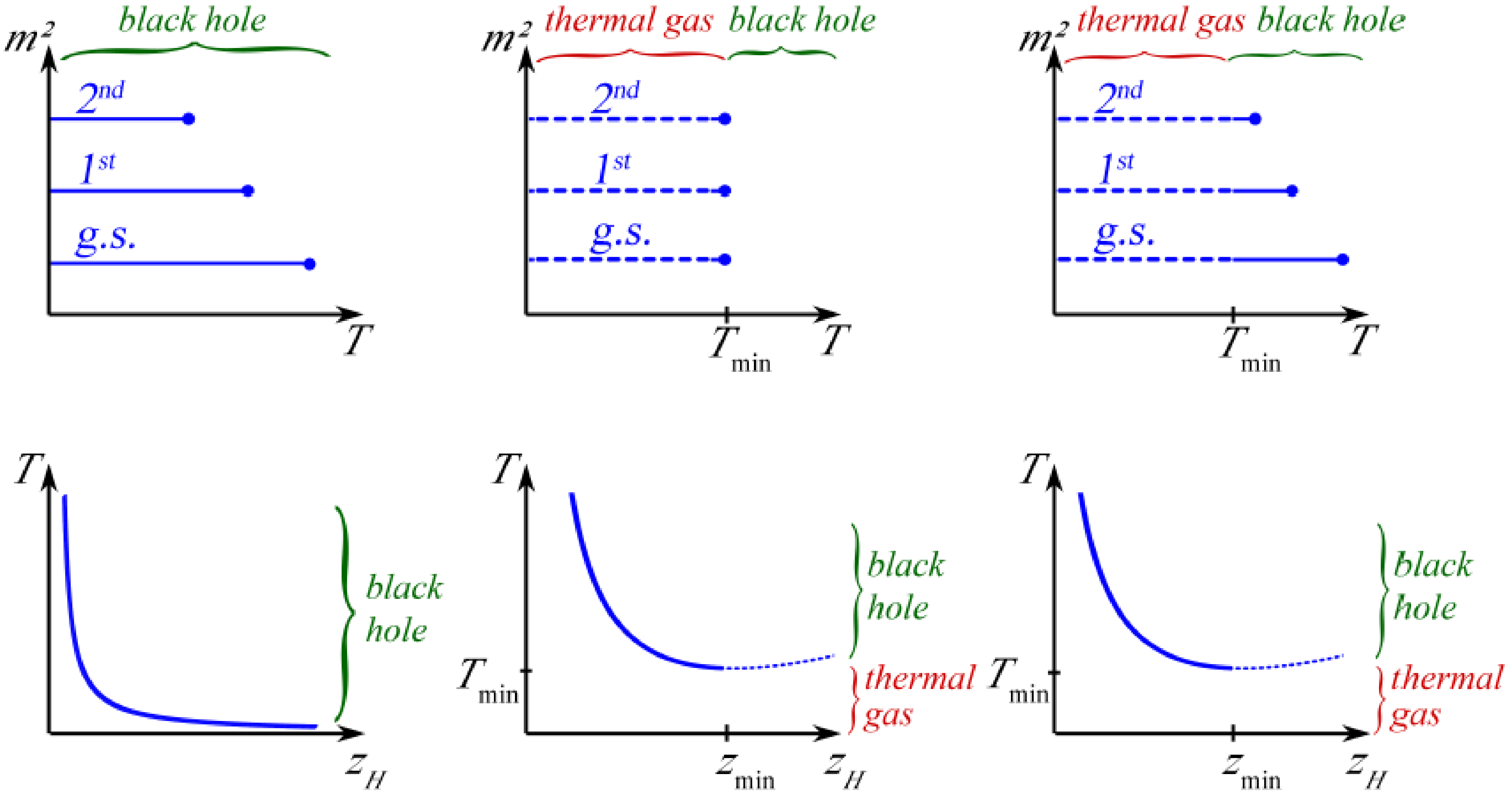
for modes in Klein-Kaluza decomposition of V in axial gauge



SW(T > 0):
Colangelo, Giannuzzi,
Nicotri, JHEP (2012)

H → T(H)
needs
fine tuning

sequential disappearance upon temperature increase



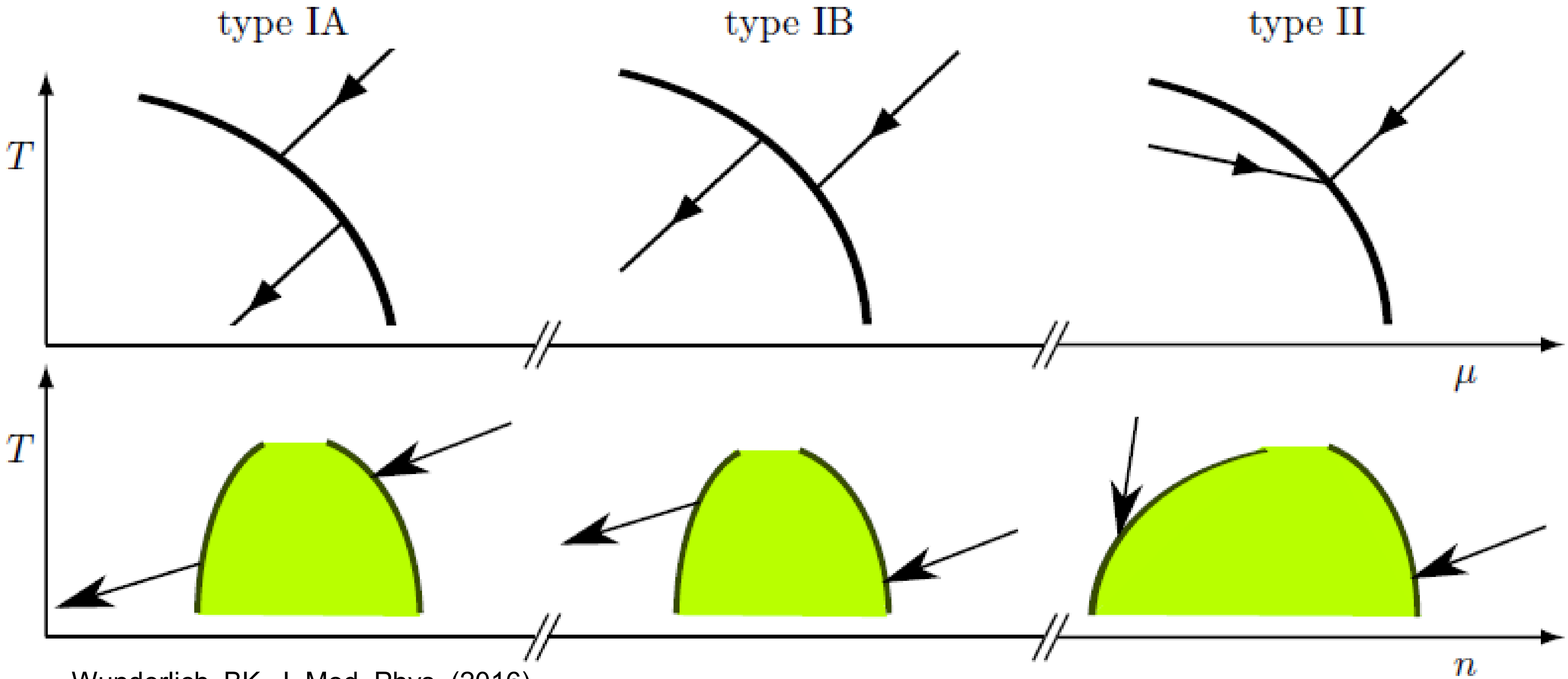
sequential vs. instantan. vs. mixed sequential

disappearance

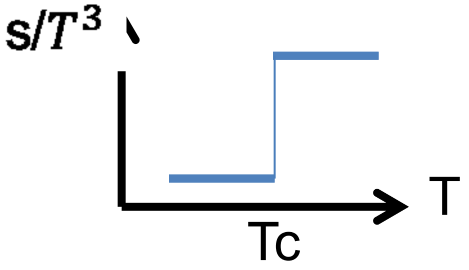
thermodyn. options:

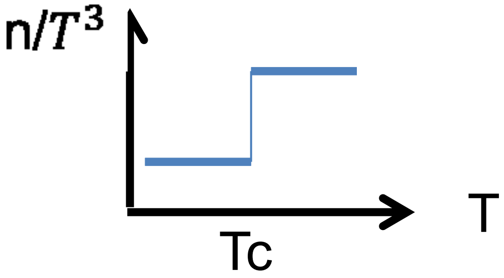
continuous – cross over – 2nd order – 1st order transitions

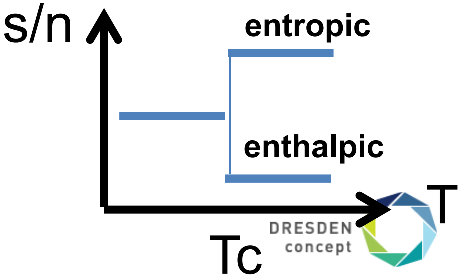
Crossing the phase border line



Wunderlich, BK, J. Mod. Phys. (2016)

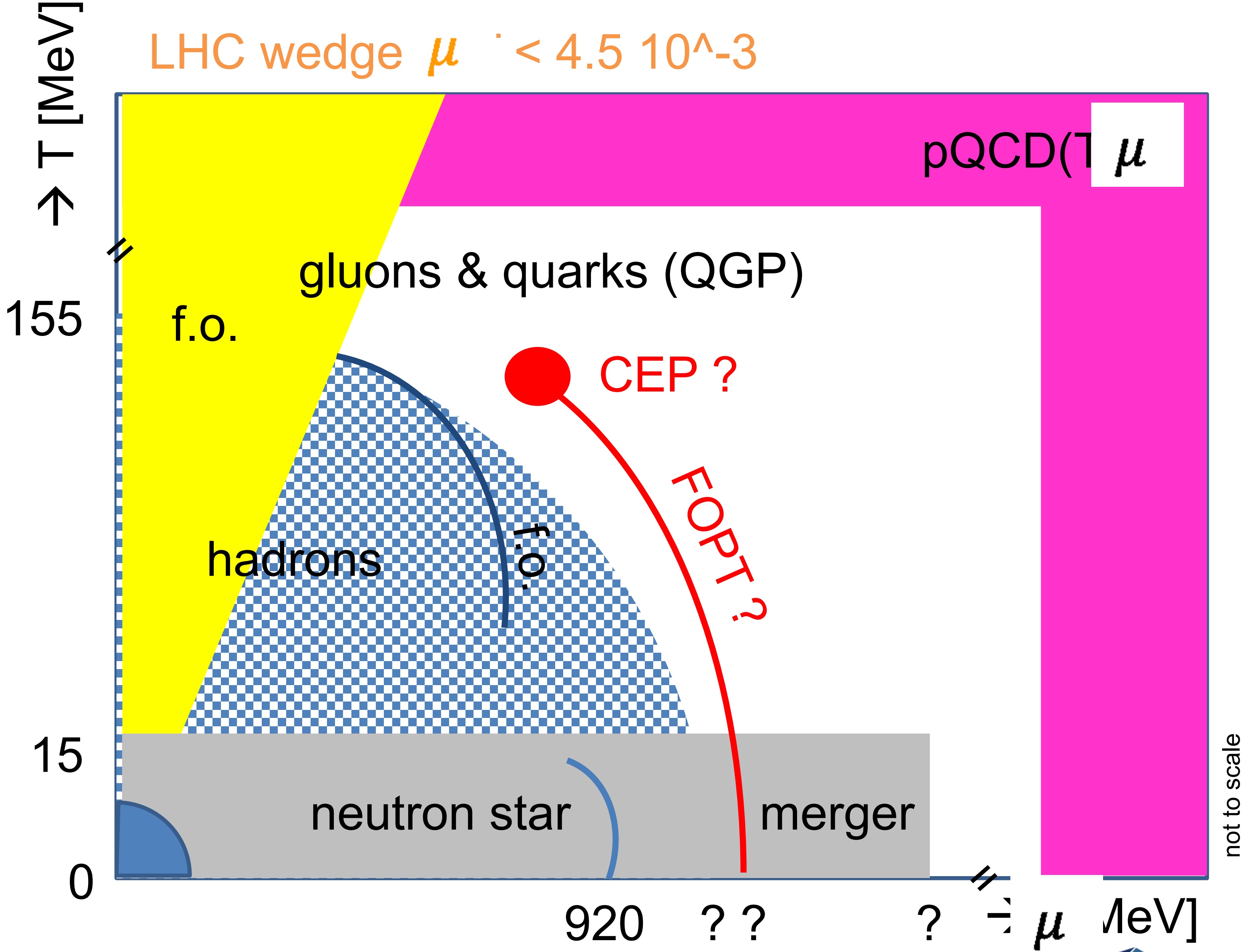






mu = const

LHC wedge μ < 4.5 10^-3



Gregor XV (1622): sacra congregatio de propaganda fide

$$-L^2 V_1(\Phi) = 12\exp(a\Phi^2 + b\Phi^4)$$

$$-L^2 V_2(\Phi) = 12\cosh(\gamma\Phi) + a\Phi^2 + b\Phi^4$$