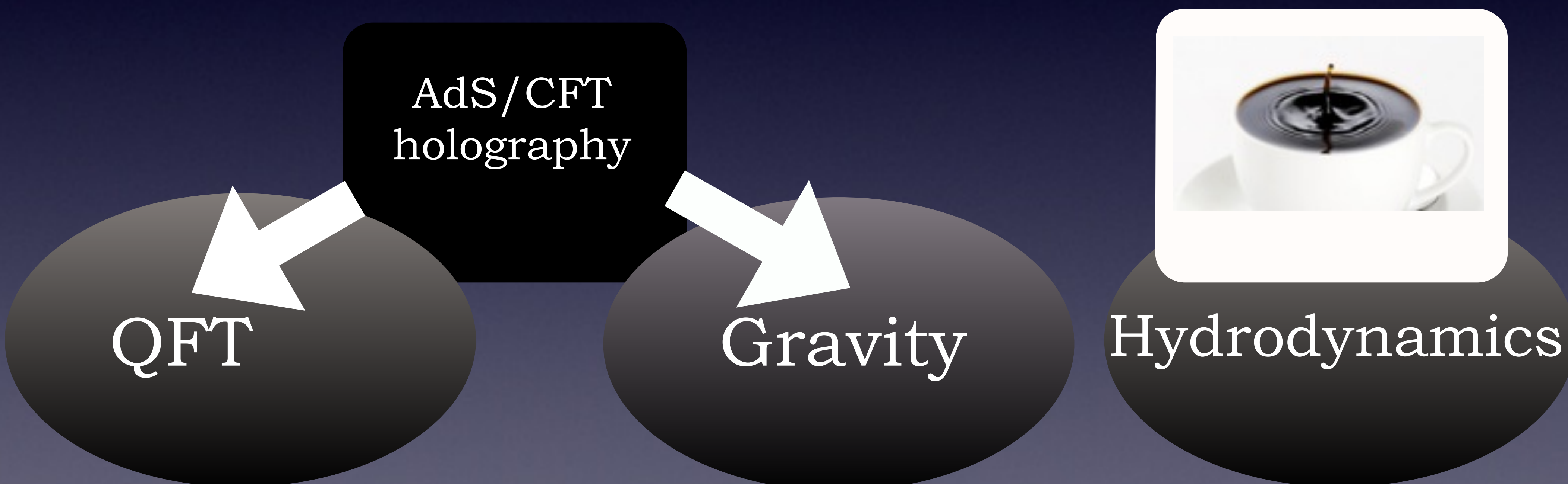
Lecture: (Far-from-equilibrium) dynamics in magnetic charged chiral plasma

Non-Equilibrium Dynamics - NED2018, April 19th, 2018, Varadero, Cuba



by Matthias Kaminski
(University of Alabama)

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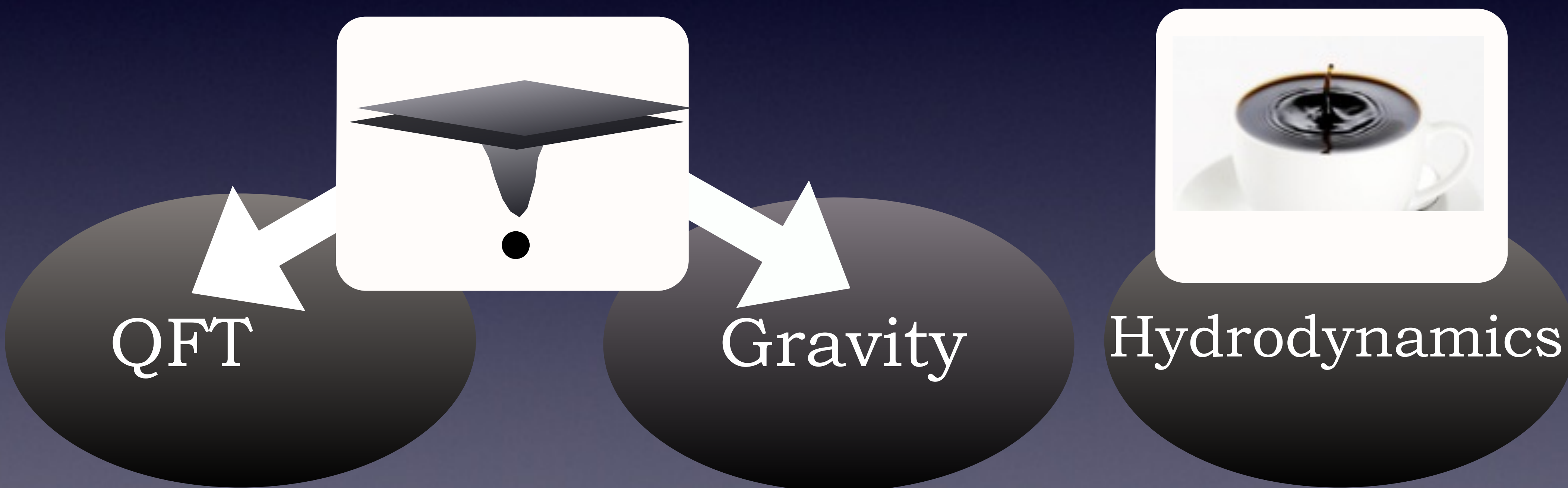

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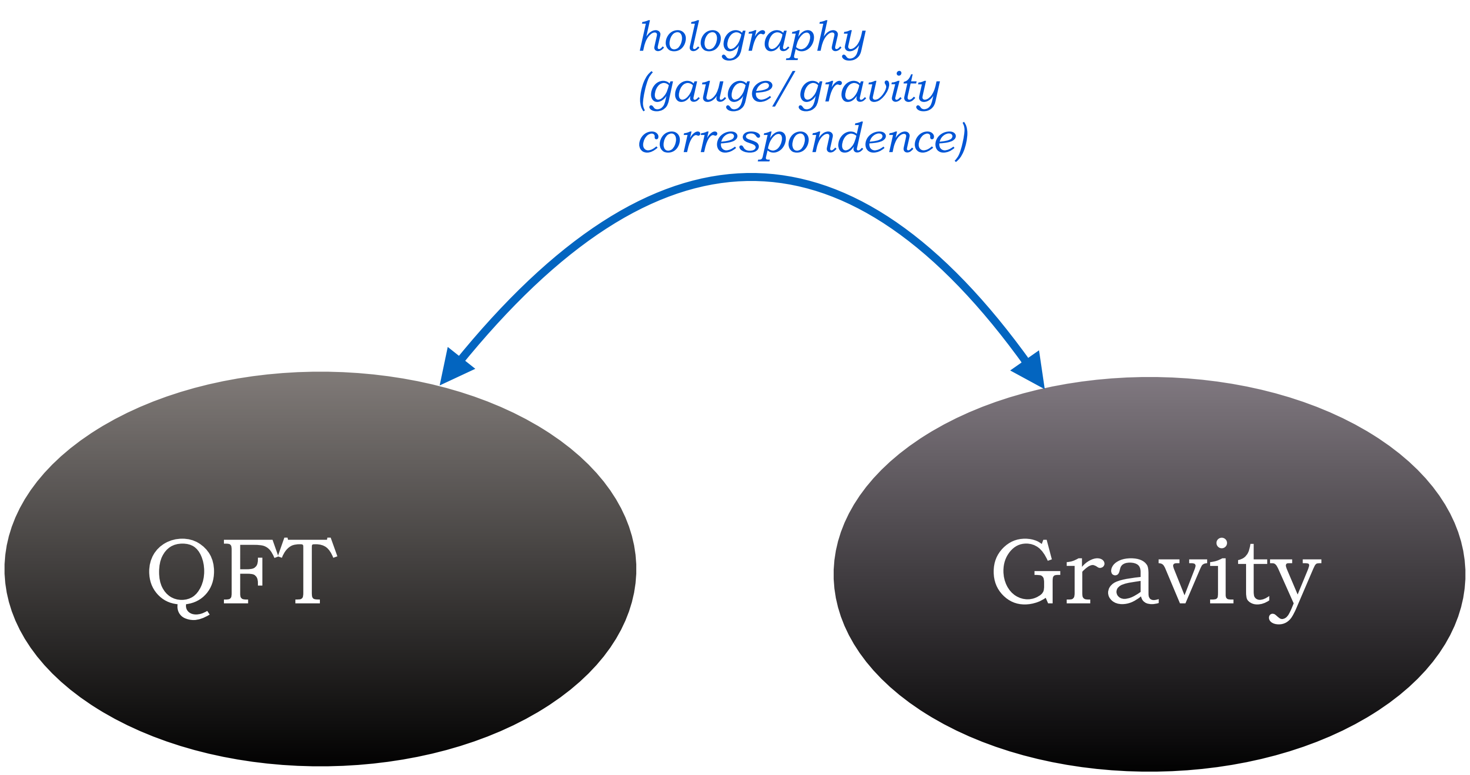
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory,
for example **QCD**

- gravity dual to QCD or standard model?
- not known yet



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model

holography (gauge/gravity correspondence)

(Hard) problem in "similar" model theory

Simple problem in a particular gravitational theory

QFT

Gravity

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Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory
- hydrodynamic approximation of model theory

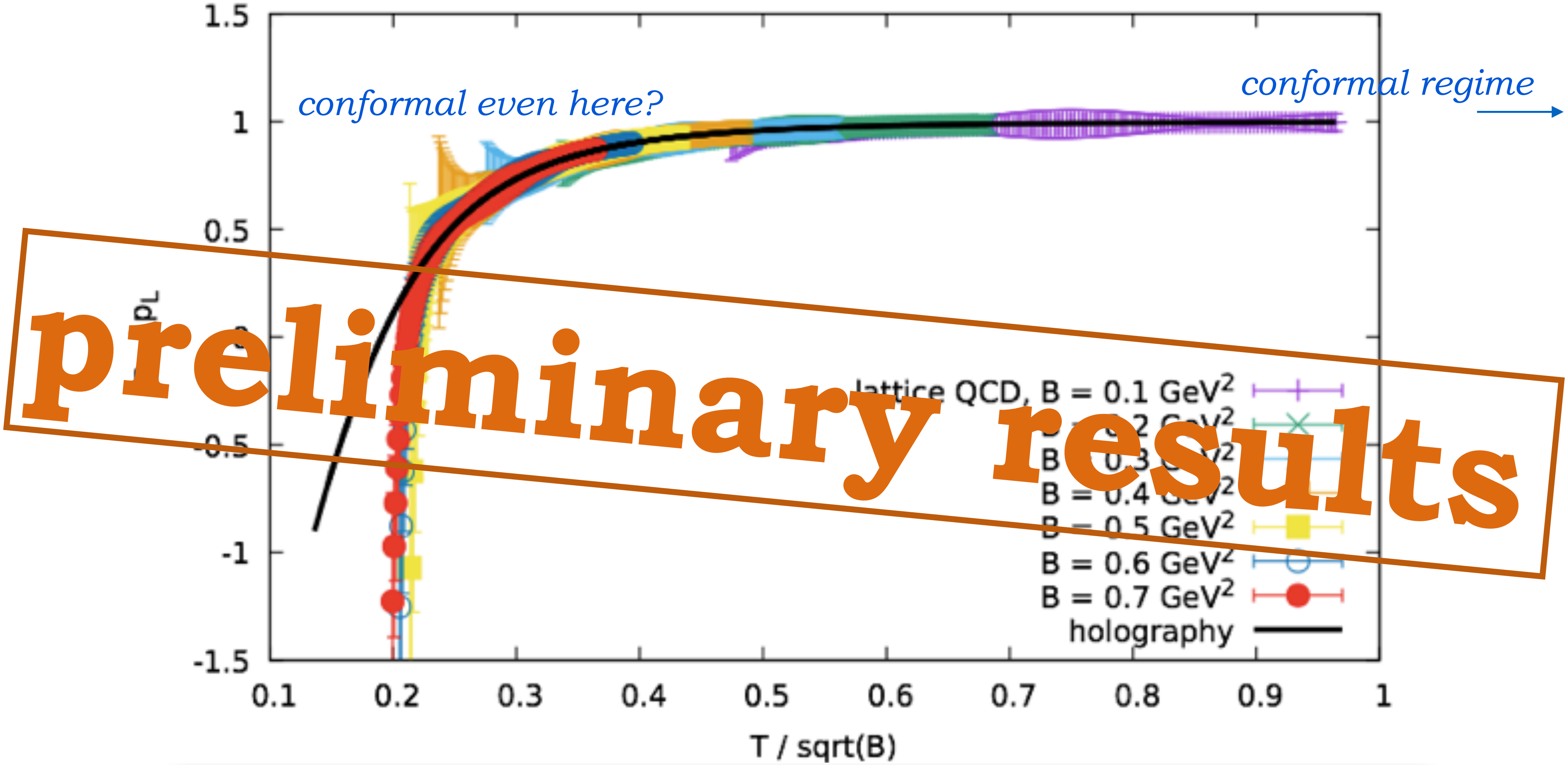
QFT

Gravity

Hydrodynamics

Teaser: Good agreement of lattice QCD data with holography (N=4 SYM)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_T = -\frac{L_T}{V}\frac{\partial F_{QCD}}{\partial L_T}$

longitudinal pressure: $p_L = -\frac{L_L}{V}\frac{\partial F_{QCD}}{\partial L_L}$

F_{QCD} ... *free energy*

L_T ... *transverse system size*

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V ... *system volume*

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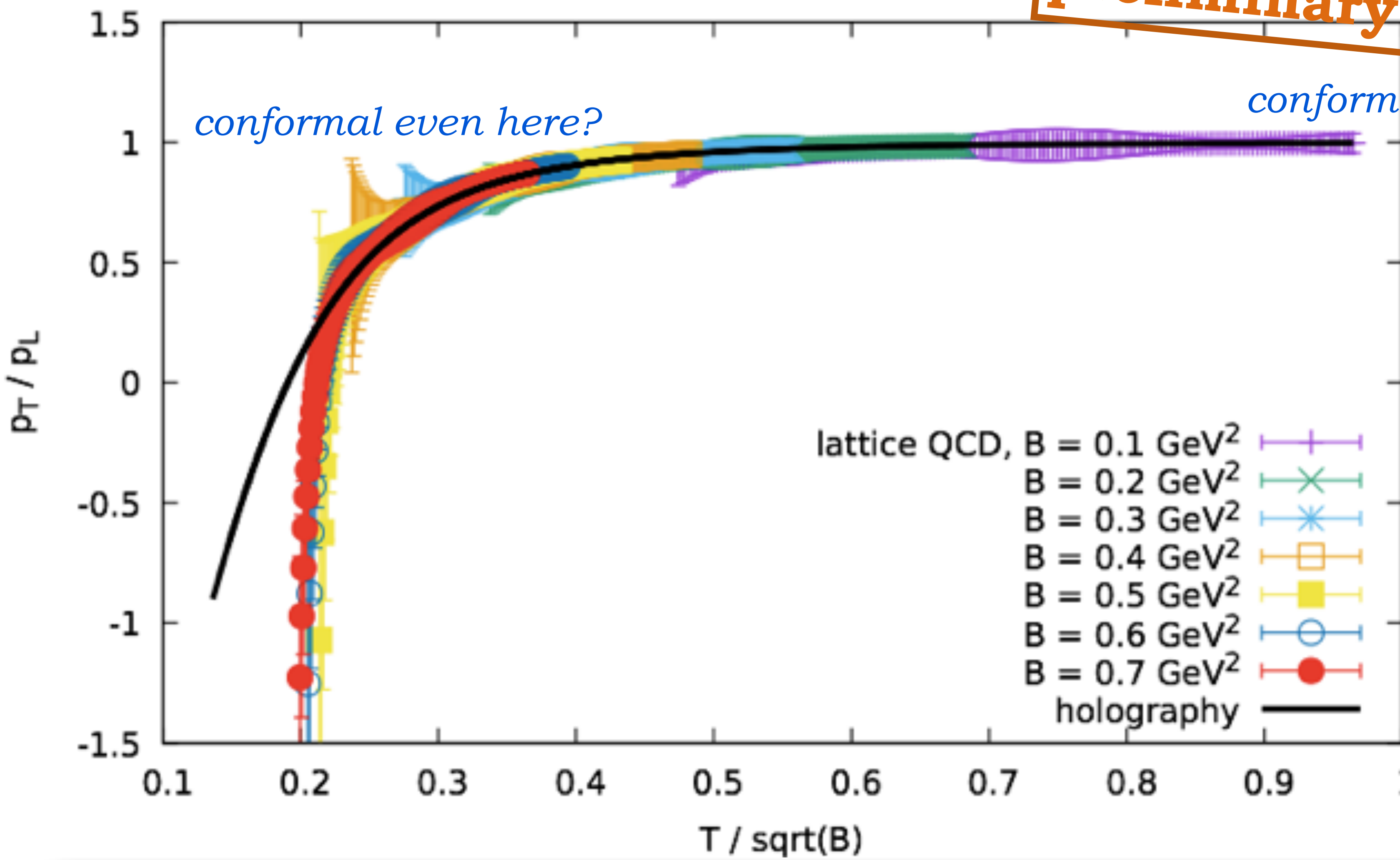
preliminary results



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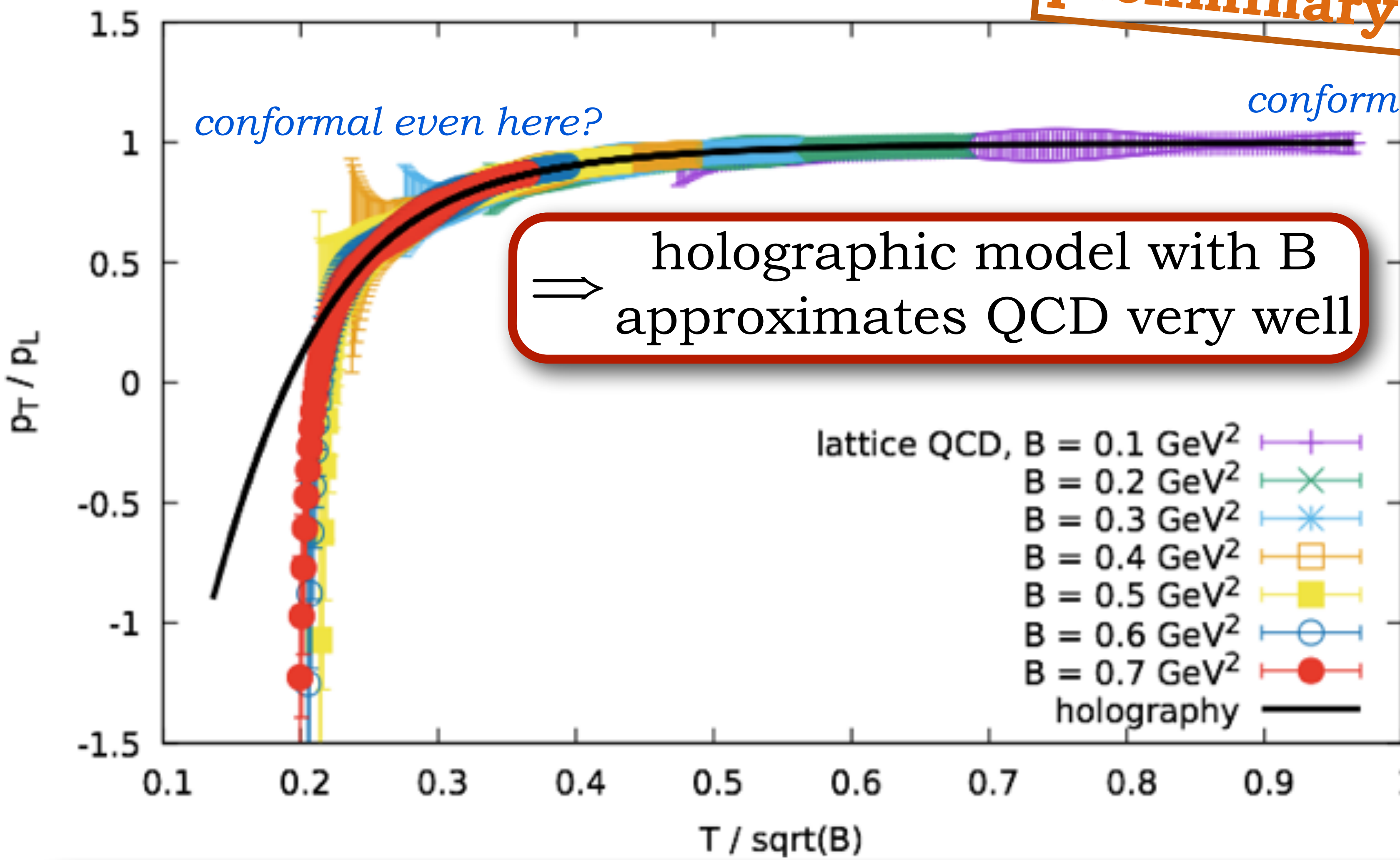

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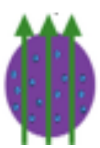
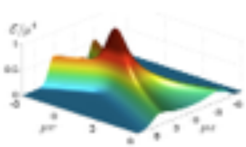
Contents

1. Hydrodynamics 2.0
   (near equilibrium) → *correlation functions (transport coefficients)*

2. Holography
   (near equilibrium) → *correlation functions (transport coefficients)*



3. Results for charged chiral plasma



4. Far-From Equilibrium

5. Conclusions

Hydrodynamic variables

Thermodynamics

$$T,\, \mu,\, u^\nu$$

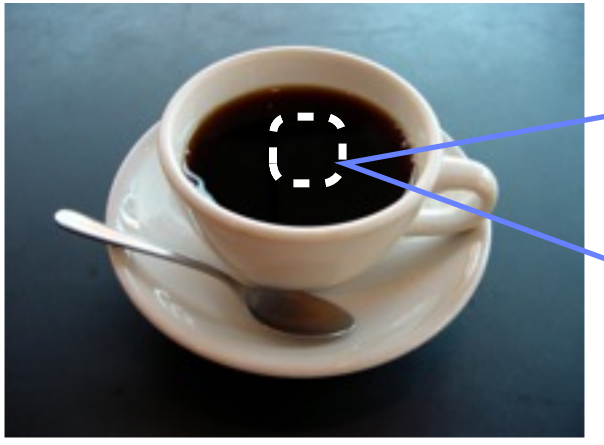
Hydrodynamics

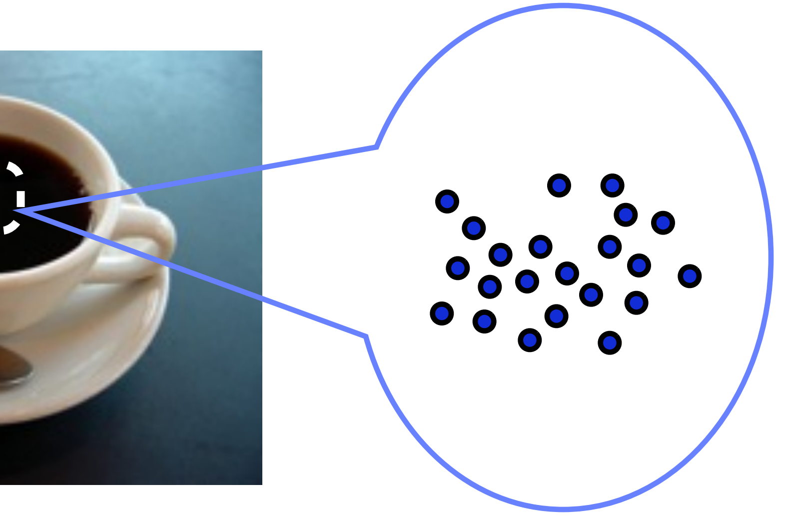
$$T(t,\vec{x}),\, \mu(t,\vec{x}),\, u^\nu(t,\vec{x})$$

Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$





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$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

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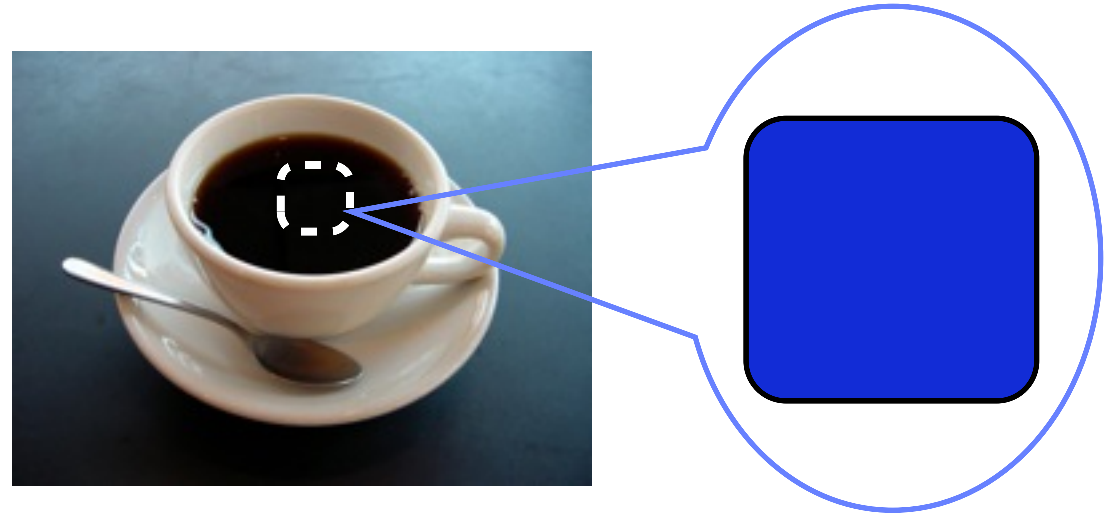

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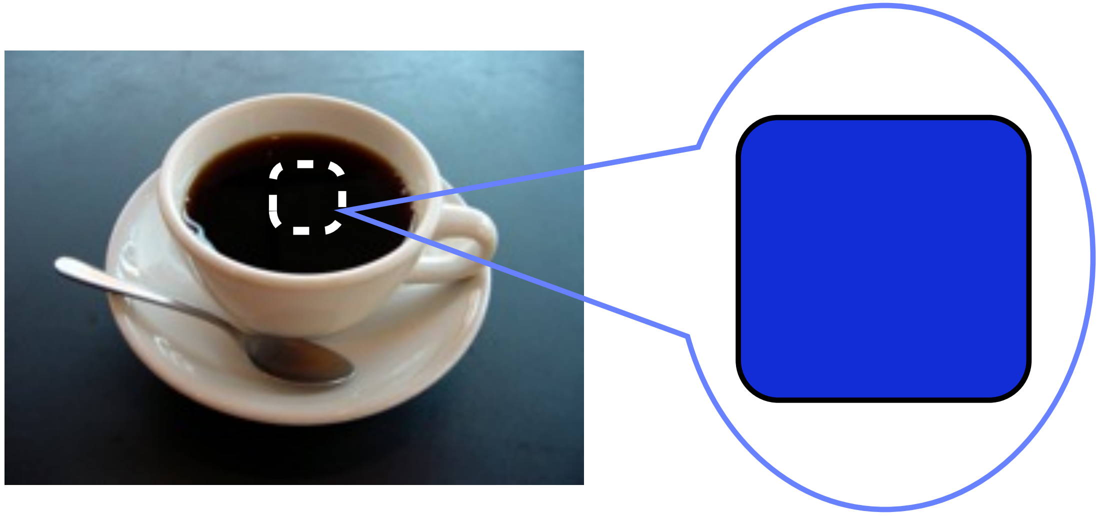

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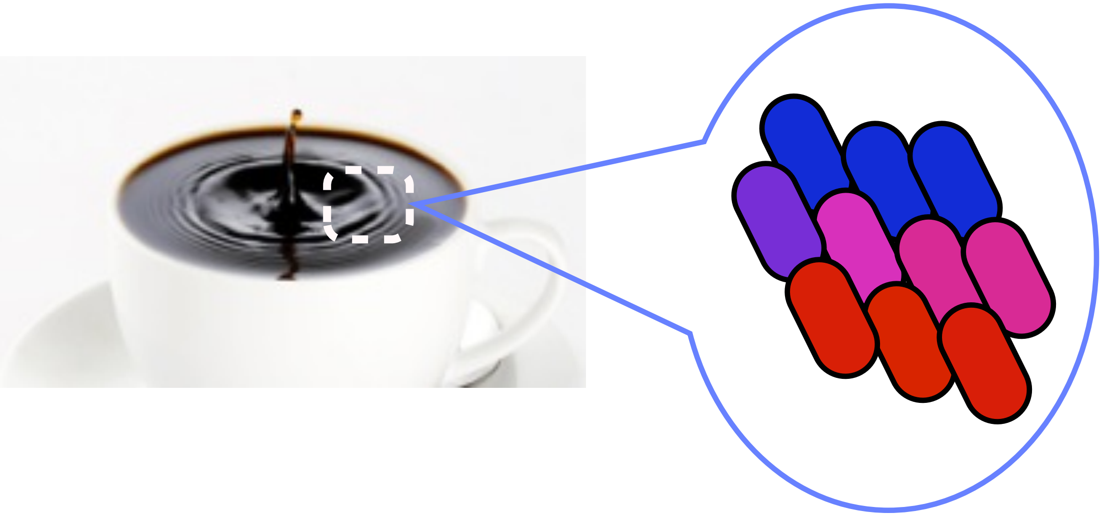

Hydrodynamics

Universal effective field theory for microscopic QFTs, expansion in gradients of temperature, chemical potential and velocity

- fields $T(x),\ \mu(x),\ u^\nu(x)$
- conservation equations
- constitutive equations (Landau frame)

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Energy momentum tensor $$T^{\mu\nu} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu) + \tau^{\mu\nu}$$

Conserved current $$j^\mu = n u^\mu + \nu^\mu$$

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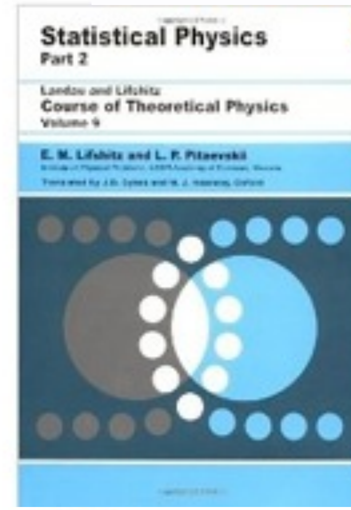
Constructing hydrodynamic constitutive equations

An old idea

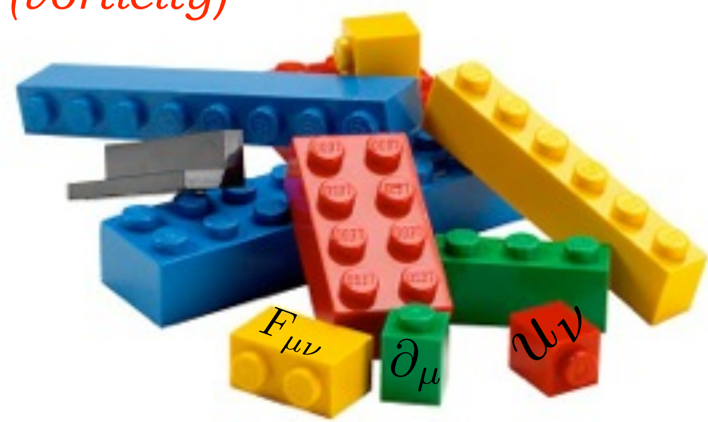
[Landau, Lifshitz]



1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group

Old example: $\nabla_\nu u^\nu$

New example: $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\nabla_\lambda u_\rho$ *(vorticity)*



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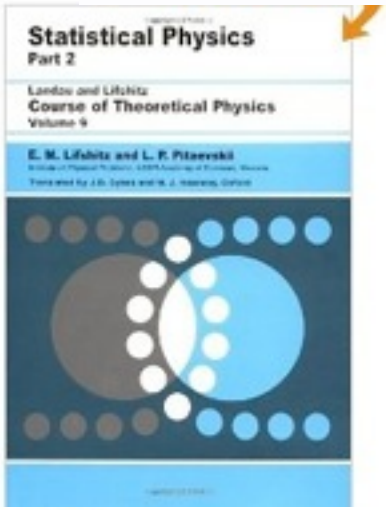

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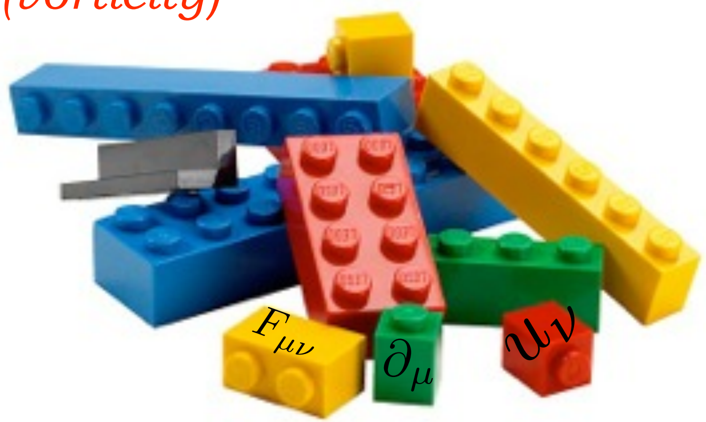

2. Restricted by conservation equations

   Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

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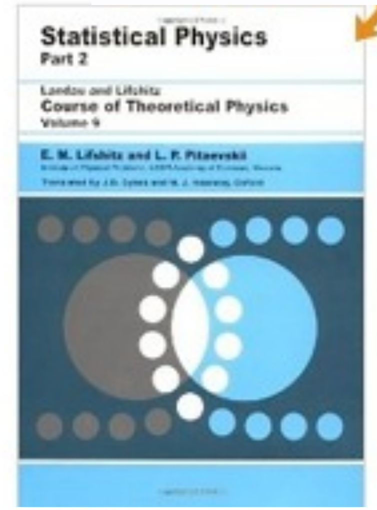

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3. Further restricted by positivity of local entropy production: $\nabla_\mu J^\mu_s \geq 0$

Alternatively, use field theory restrictions (Onsager,...)

[Jensen, MK, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]
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Derived for any QFT with a chiral anomaly (e.g. QCD)

$\nabla_\nu j^\nu = 0$ classical theory

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$$\nabla_\mu j^\mu = C\,\epsilon^{\nu\rho\sigma\lambda} F_{\nu\rho} F_{\sigma\lambda}$$

quantum theory

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Def.: $V^\mu = E^\mu - T\Delta^{\mu\nu}\nabla_\nu \left(\frac{\mu}{T}\right)$

Completed constitutive equation with external fields

$$j^\mu = n u^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu + \dots$$

vorticity *magnetic field*

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Agrees with gauge/gravity prediction:

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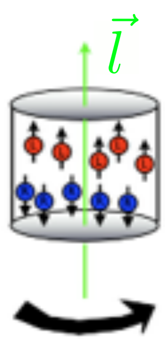
chiral vortical effect



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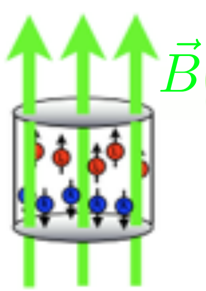

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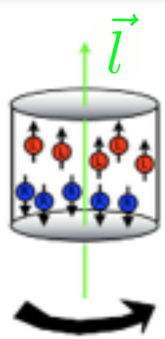
anomaly-coefficient C

chiral vortical effect



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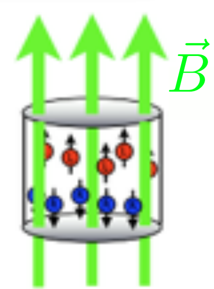

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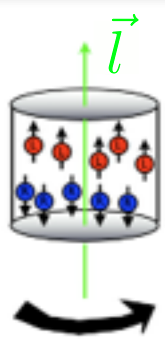

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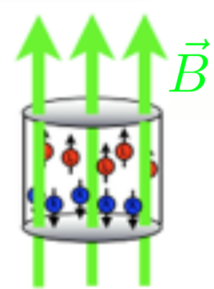

Observable in:

heavy ion collisions?

[Kharzeev, Son.; PRL (2011)]

neutron stars?

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2014)]

condensed matter?

[Li et al; (2014)]

[Cortijo, Ferreiros, Landsteiner, Vozmediano; (2015)]

Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

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sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \quad u^\mu = (1,0,0)$

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx} \quad$ (fix T and u)

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sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1,0,0)$

+other sources

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx}$ (fix T and u)

+ fluctuations in T and u

one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

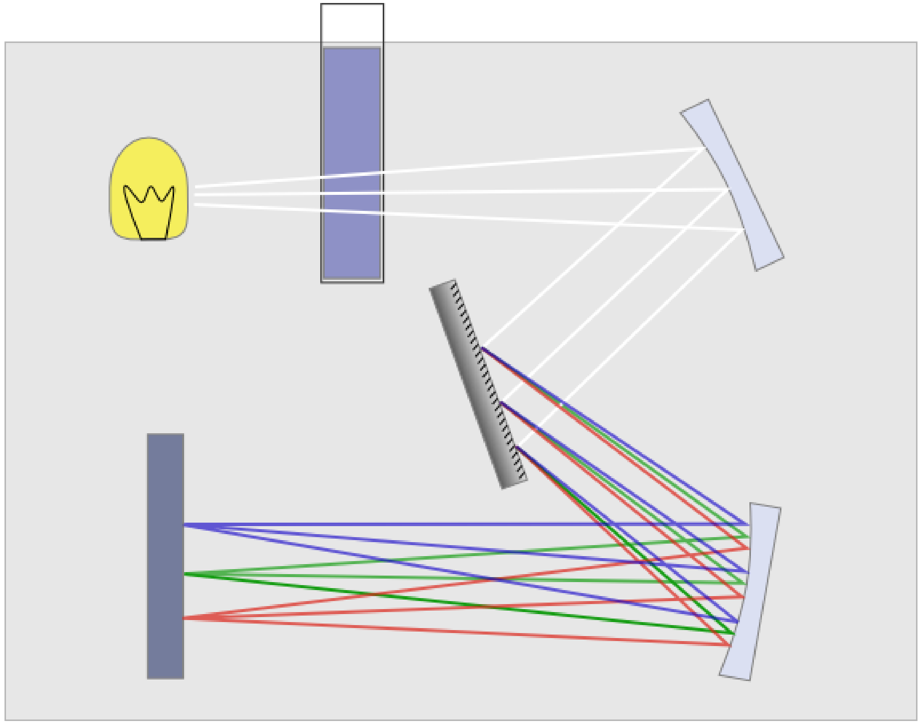
susceptibility: $\quad \chi = \frac{\partial n}{\partial \mu}$

Einstein relation: $\quad D = \frac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

Thermal Spectral Function



Thermal spectral function $\mathfrak{R}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

$$\mathfrak{R}(\omega,\, \mathbf{q}) = -2\,\mathrm{Im}G^{\mathrm{ret}}(\omega,\, \mathbf{q})$$

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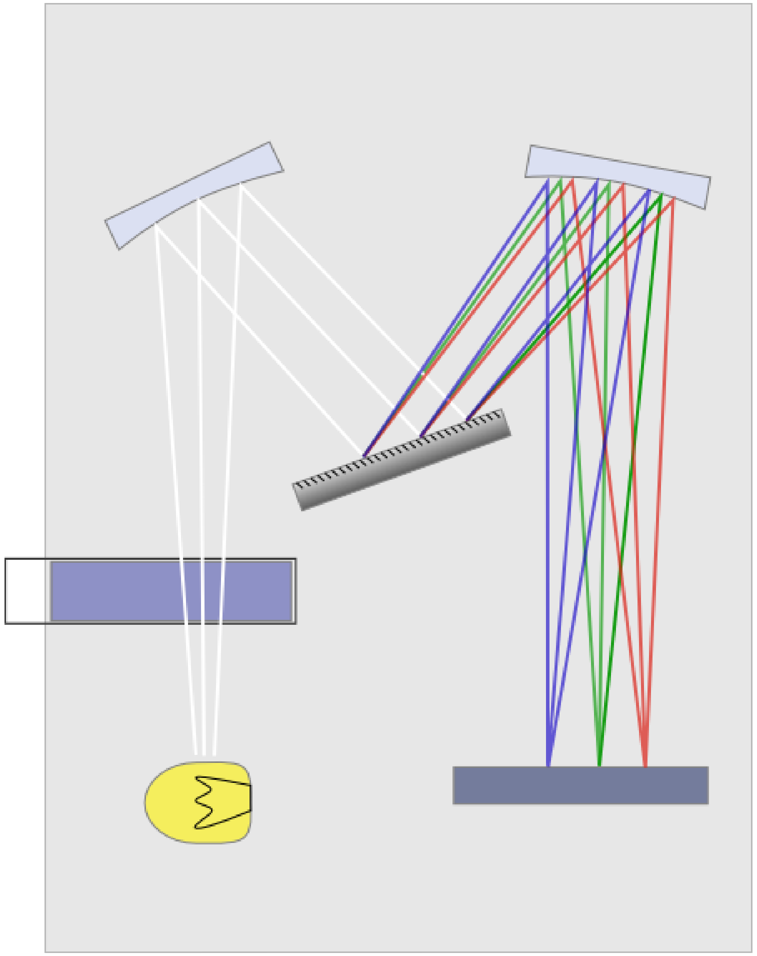

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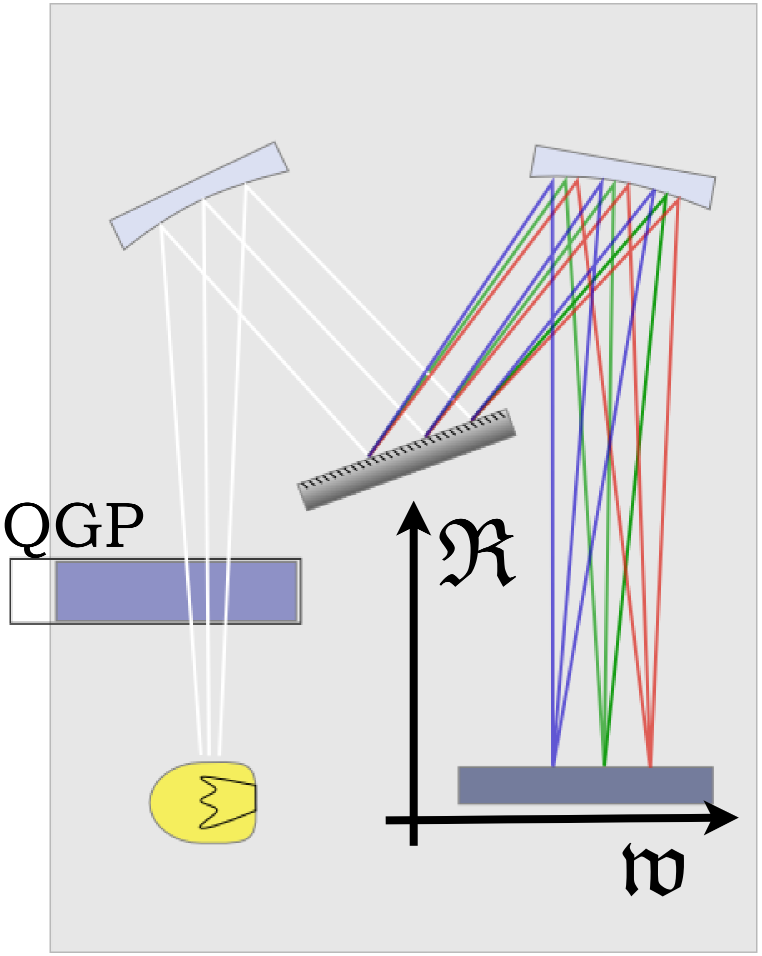

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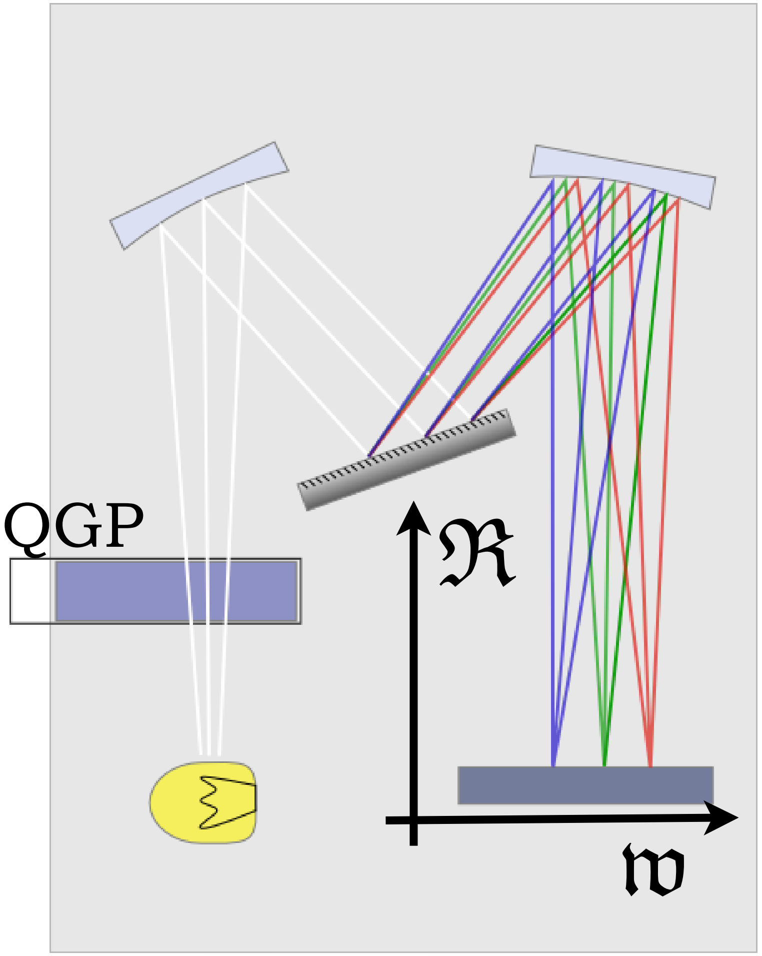

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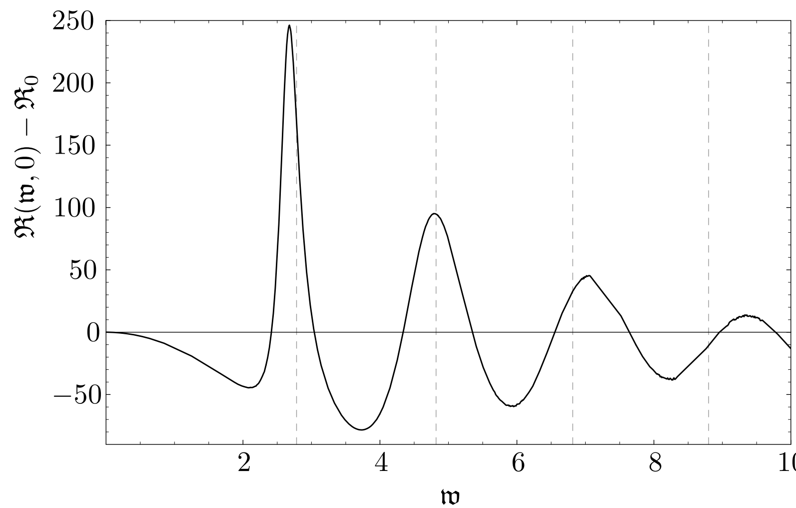

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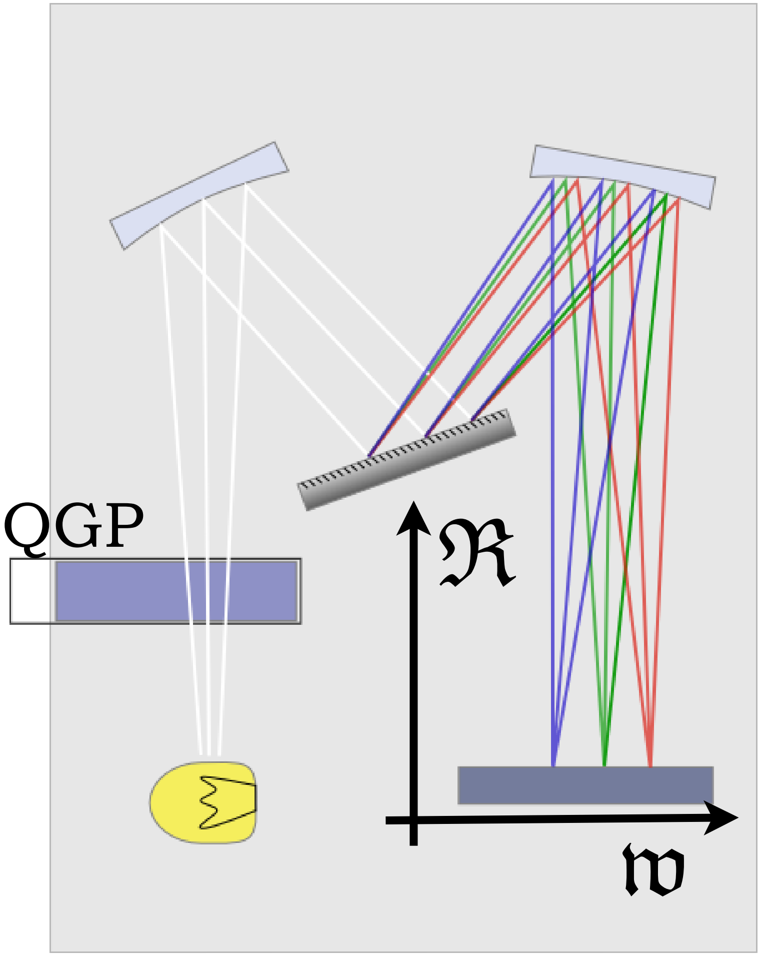

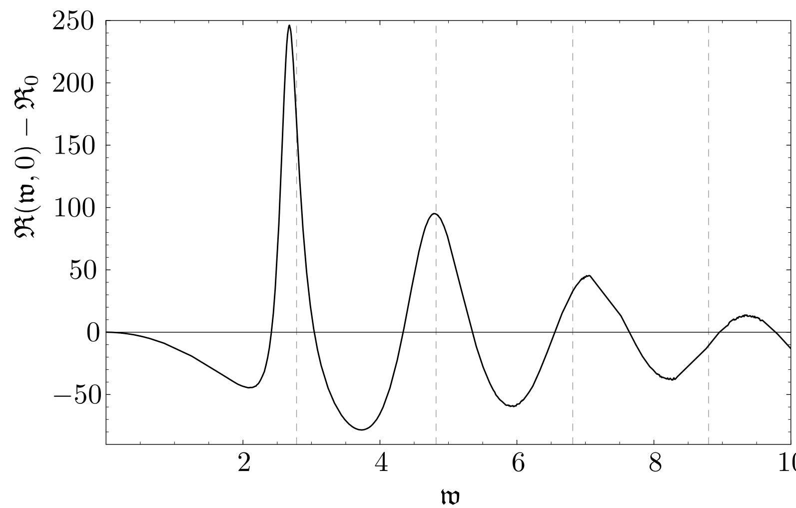

Thermal spectral function $\mathfrak{R}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

$$\mathfrak{R}(\omega,\, \mathbf{q}) = -2\, \mathrm{Im} G^{\mathrm{ret}}(\omega,\, \mathbf{q})$$

Transport coefficients using Kubo formulae, e.g.

electric conductivity $$\sigma \sim \lim_{\omega\to 0} \frac{1}{\omega} \langle [J^t, J^t] \rangle$$

Exercise 1.b): hydrodynamic correlators

Simple (non-chiral) example in 2+1

two point function: $\langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

spectral function: $-\mathrm{Im}\, G^R = -\mathrm{Im}\, \langle j_x j_x \rangle = -\sigma\, \omega_R \frac{2Dk^2\omega_I + \omega_R^2 + \omega_I^2}{\omega_R^2 + (\omega_I + Dk^2)^2}$

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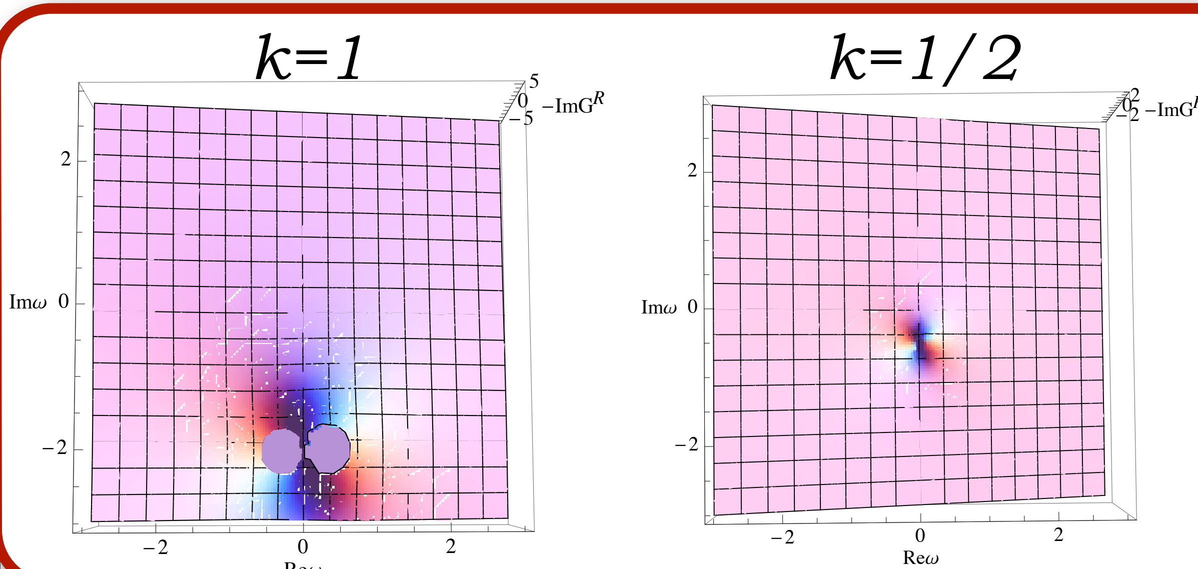
Simple (non-chiral) example in 2+1

two point function: $$\langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$$

spectral function: $$-\mathrm{Im}\, G^R = -\mathrm{Im}\, \langle j_x j_x \rangle = -\sigma\, \omega_R \frac{2Dk^2\omega_I + \omega_R^2 + \omega_I^2}{\omega_R^2 + (\omega_I + Dk^2)^2}$$

hydrodynamic pole (diffusion pole) in spectral function at decreasing momentum *k*:

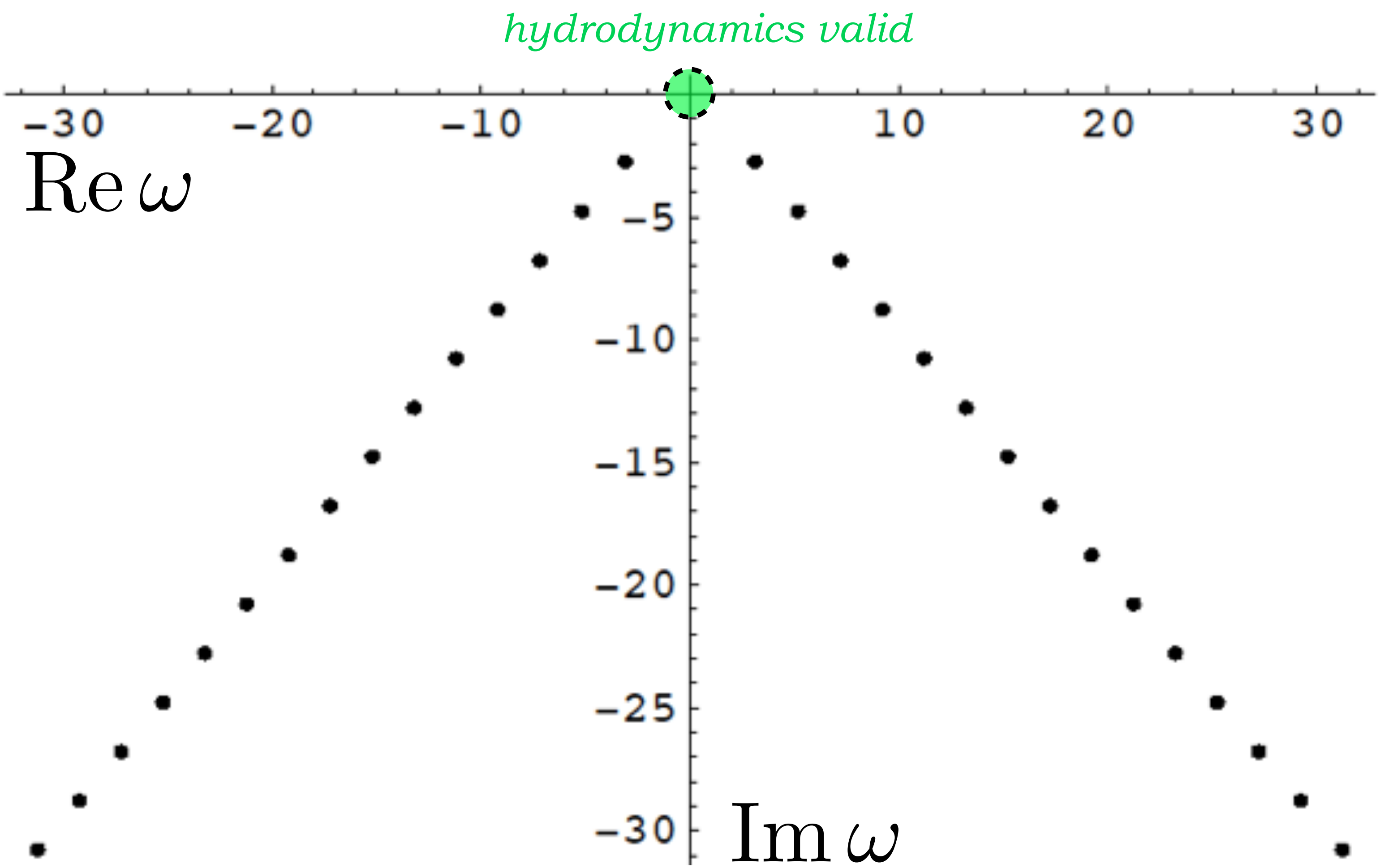


- pole goes to zero
- spectral function vanishes with *k*

Far beyond hydrodynamics

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle(\omega, k) = G^R_{xy,xy}(\omega, k) = -i \int d^4x \; e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$



Far beyond hydrodynamics: holography

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

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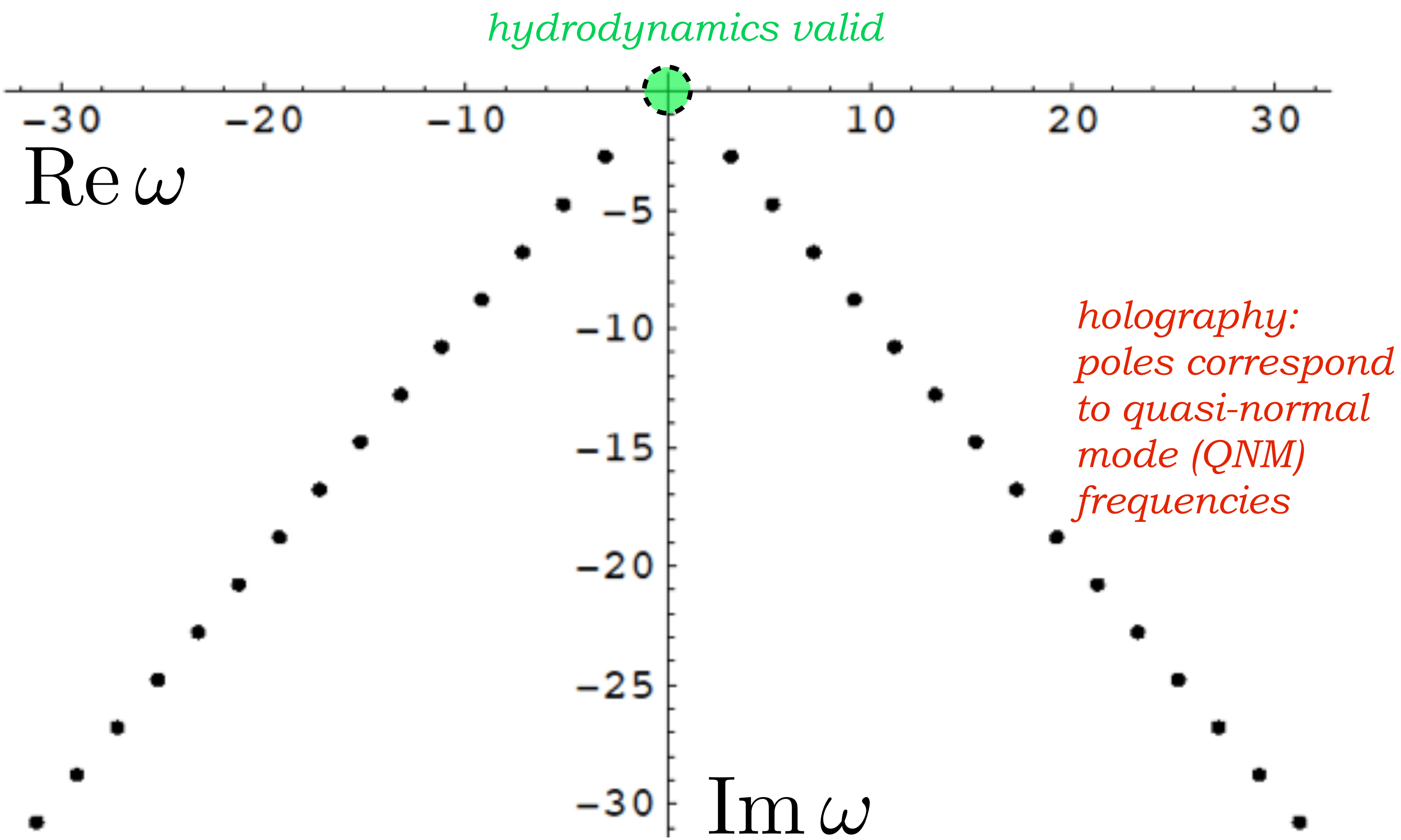


[Starinets; JHEP (2002)]

2. Holography

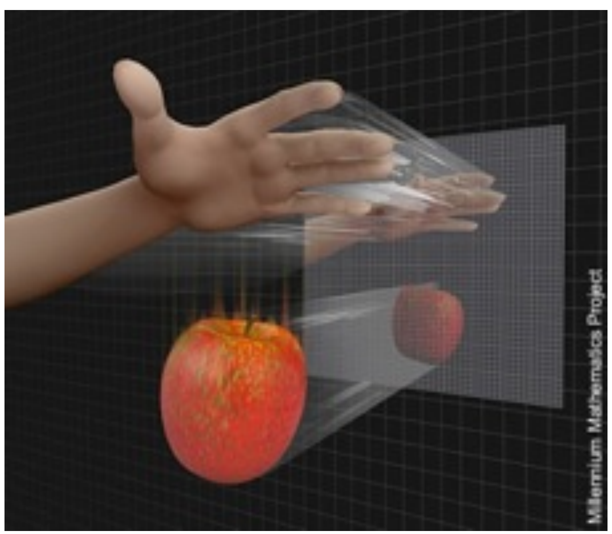
Holography (gauge/gravity) concepts - I

Gauge/Gravity Correspondence based on
holographic principle *['t Hooft (1993)]*

$$S_{max}(\text{volume}) \propto \text{surface area}$$

String theory gives one example (AdS/CFT).

N=4 Super-Yang-Mills in 3+1 dimensions (CFT)	$\Leftrightarrow$	Typ II B Supergravity in (4+1)-dimensional Anti de Sitter space (AdS)	*[Susskind (1995)]* *[Maldacena (1997)]*

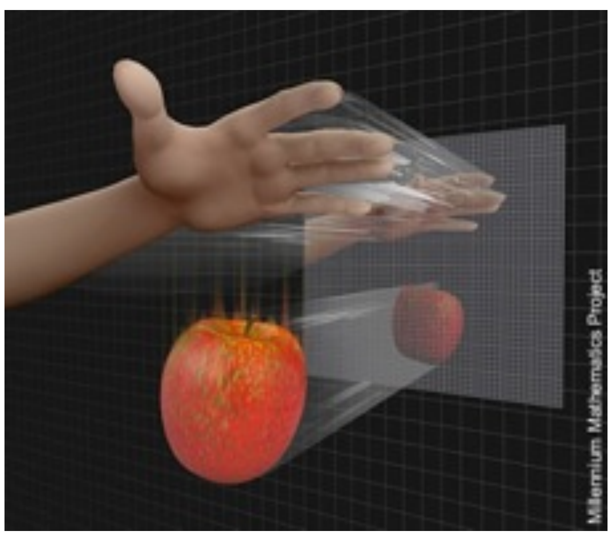


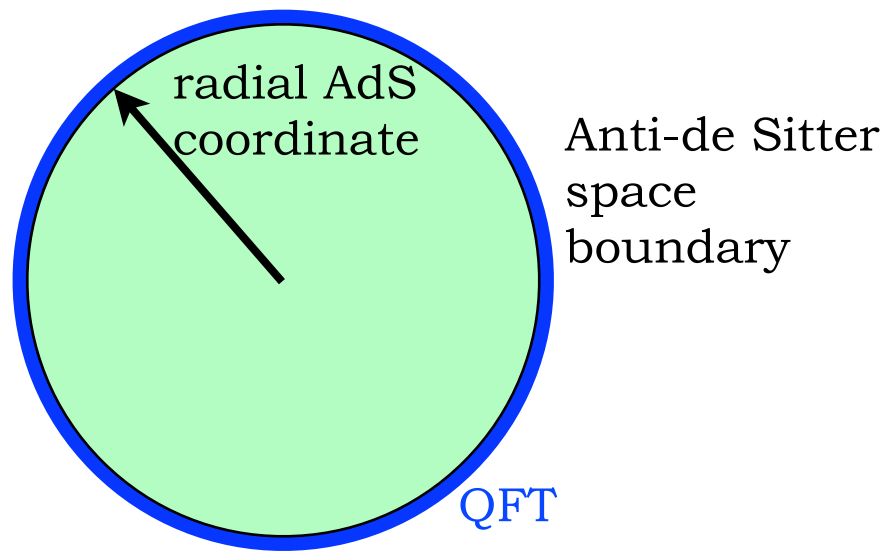
Holography (gauge/gravity) concepts - II

strongly coupled quantum field theory ⟷ weakly curved gravity

correspondence

[Maldacena (1997)]





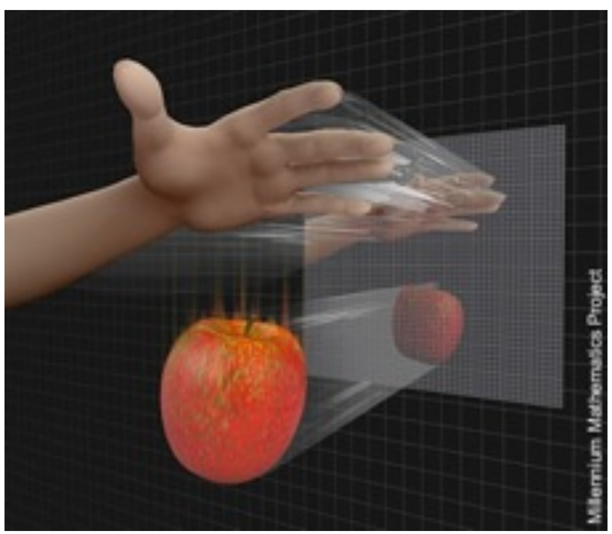
Holography (gauge/gravity) concepts - II

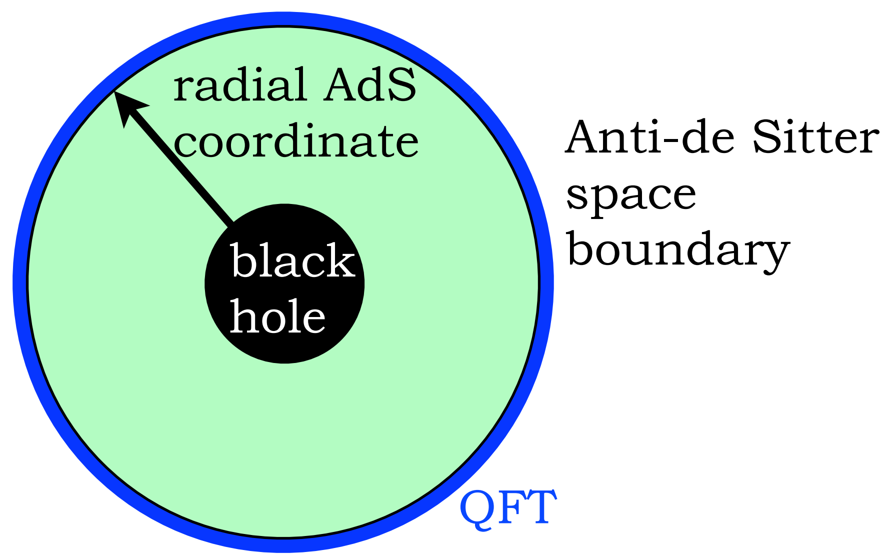
strongly coupled quantum field theory

correspondence ⟷ *[Maldacena (1997)]*

weakly curved gravity

QFT temperature ⟷ Hawking temperature





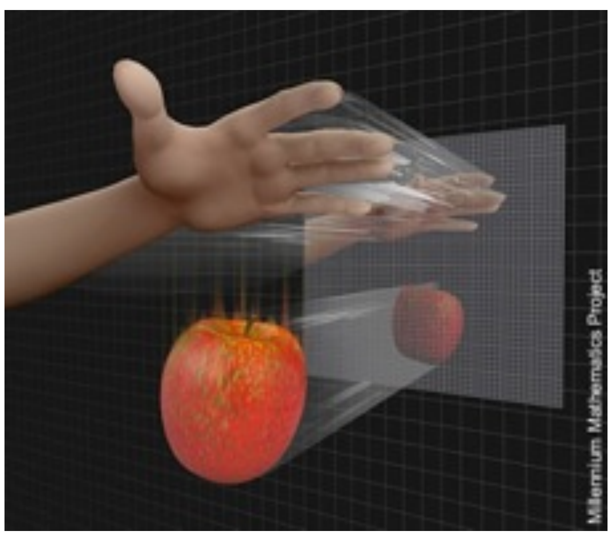
Holography (gauge/gravity) concepts - II

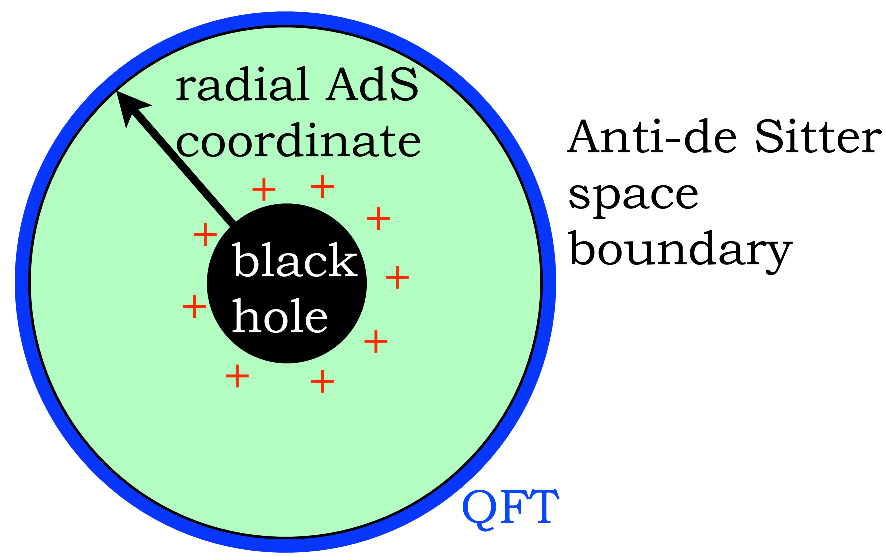
strongly coupled quantum field theory ⟷ *correspondence* ⟷ weakly curved gravity

[Maldacena (1997)]

QFT temperature ⟷ Hawking temperature

conserved charge ⟷ charged black hole





How does this give us correlators/transport?

Famous transport result: low shear viscosity/entropy density

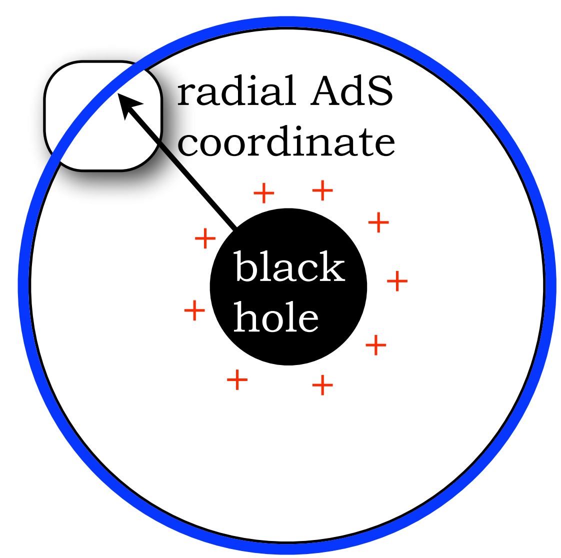
Theory/Model	η/s	Reference
Lattice QCD	$0.134(33)$	*[Meyer, 2007]*
Hydro (Glauber)	0.19	*[Drescher et al., 2007]*
Hydro (CGC)	0.11	*[Drescher et al., 2007]*
Viscous Hydro (Glauber)	$0.08\,, 0.16,\ \{0.03\}$	*[Romatschke et al.,2007]*

Gauge/Gravity: $$\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$$

[Policastro, Son, Starinets, 2001]
[Kovtun, Son, Starinets, 2003]

✓ Correct prediction!

Correspondence by zooming in on boundary



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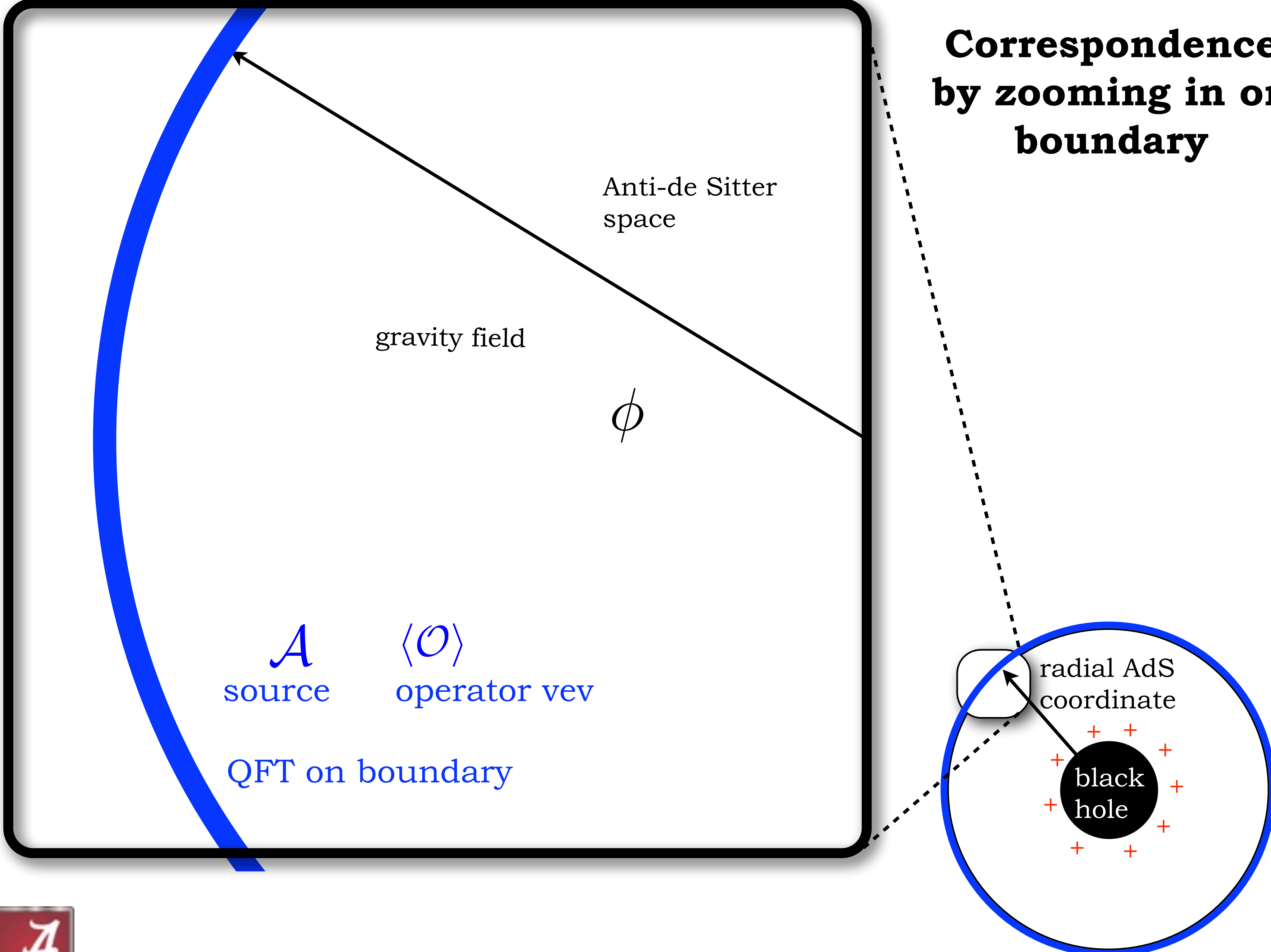

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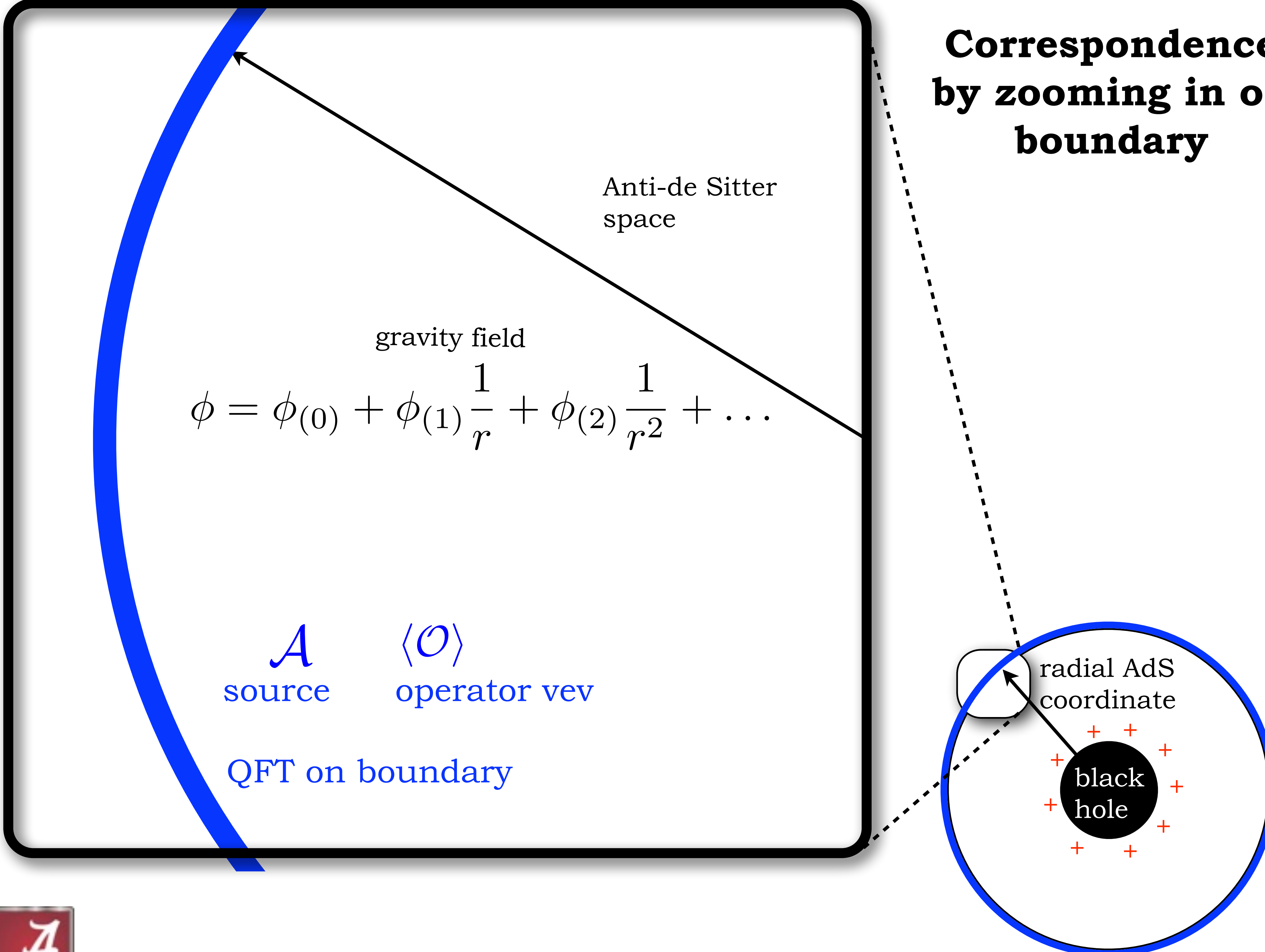

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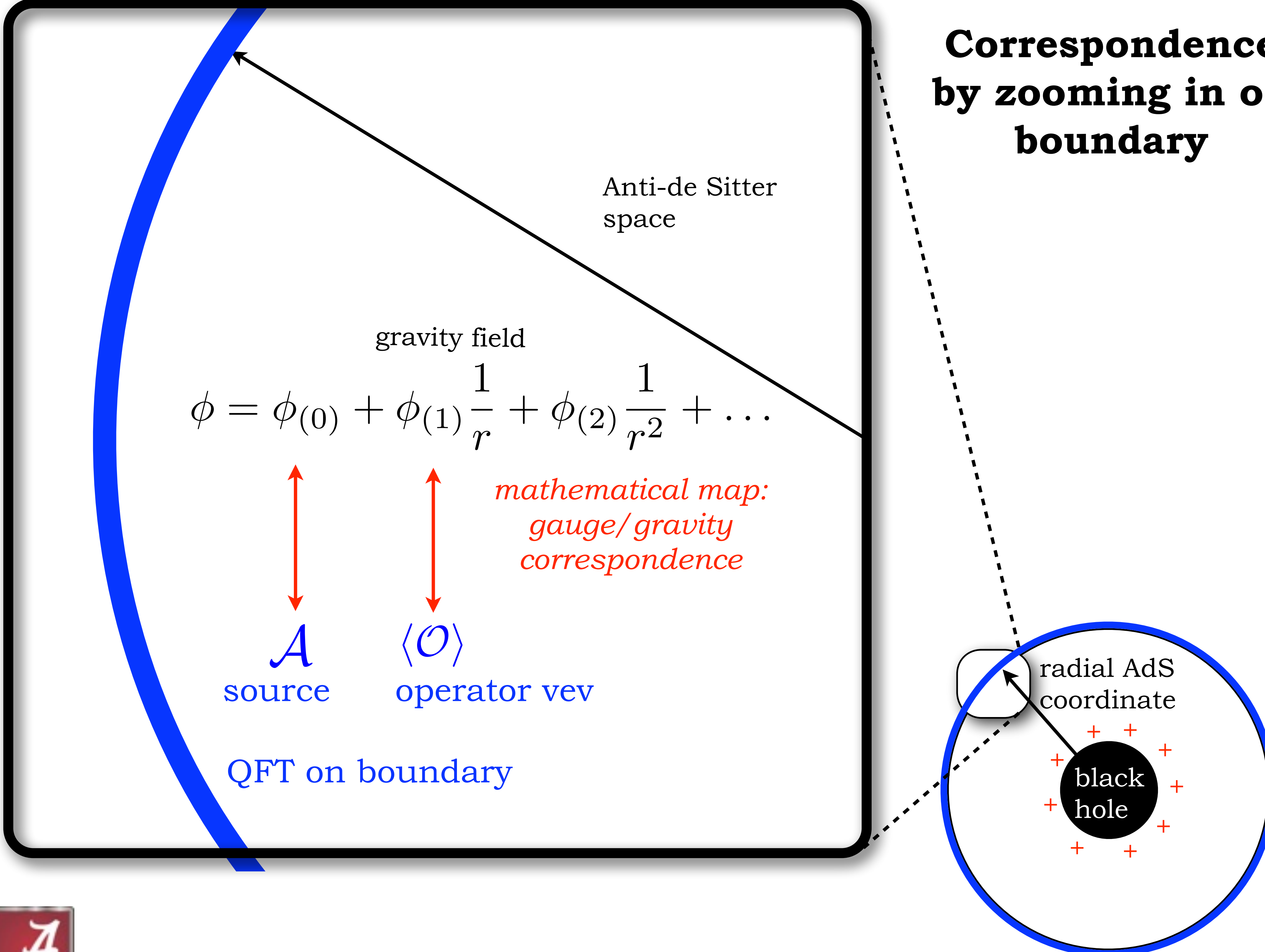

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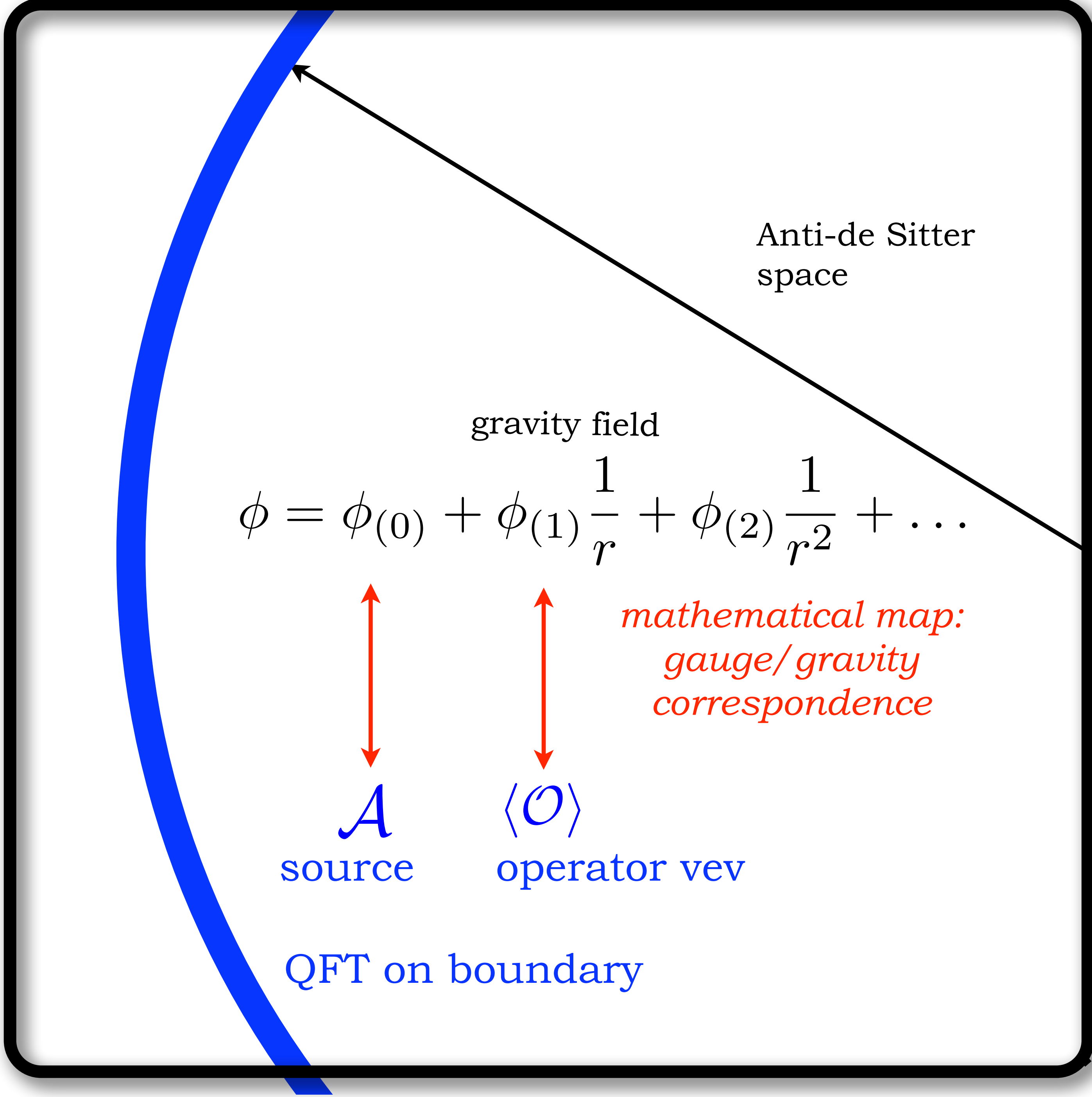
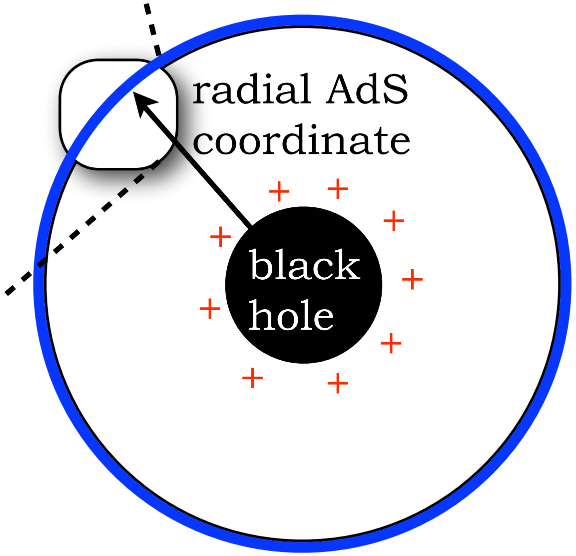


Correlation function

$$\langle \mathcal{O}\mathcal{O} \rangle = \left.\frac{\delta \mathcal{O}}{\delta \mathcal{A}}\right|_{\mathcal{A}=0}$$

$$\sim \frac{\delta \phi_{(1)}}{\delta \phi_{(0)}}$$



Holographic correlator calculation

- start with **gravitational background** (metric, matter content)

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

- impose **boundary conditions** that are
  in-falling at horizon:

(and for QNMs also vanishing at AdS-boundary: $\lim_{u \to u_{bdy}} \phi(u) = 0$)

Holographic correlator calculation

- start with **gravitational background** (metric, matter content)

Example: (charged) Reissner-Nordstrom black brane in 5-dim AdS

[Janiszewski, Kaminski; PRD (2015)]

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2 \qquad f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

$$A_t = \mu - \frac{Q}{L r^2}$$

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Example: metric tensor fluctuation

$$\phi := h_x{}^y \qquad 0 = \phi'' - \frac{f(u) - u f'(u)}{u f(u)}\phi' + \frac{\omega^2 - f(u)k^2}{4 r_H^2 u f(u)^2}\phi \qquad u = \left(\frac{r_H}{r}\right)^2$$

- impose **boundary conditions** that are
in-falling at horizon: $\phi = (1-u)^{\pm\frac{i\tilde{\omega}}{2(2-\tilde{q}^2)}}\left[\phi_H^{(0)} + \phi_H^{(1)}(1-u) + \phi_H^{(2)}(1-u)^2 + \dots\right]$

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Exercise 2.
a.) transform metric to FG/EF
b.) show scale invariance

cf. Casey Cartwright's talk

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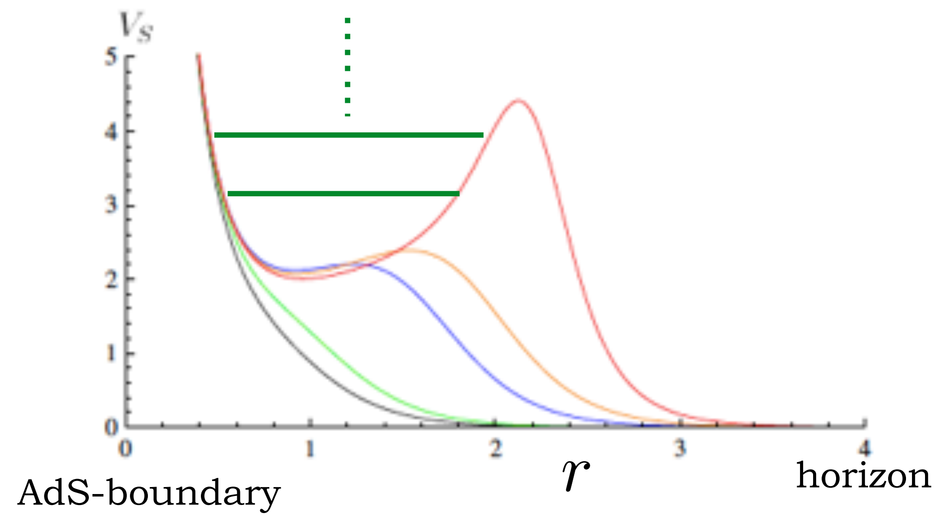
$$\Rightarrow \quad \langle \mathcal{O}\mathcal{O}\rangle = \left.\frac{\delta \mathcal{O}}{\delta \mathcal{A}}\right|_{\mathcal{A}=0} \sim \frac{\delta\phi_{(1)}}{\delta\phi_{(0)}}$$

holographic correlator

What are quasi-normal modes?

- heuristically: the eigenmodes of black holes or black branes

$$H\,\phi = -{\partial_r}^2\phi + V_S\phi = E\phi$$



- the "ringing" of black holes

- quasi-eigensolutions to the linearized Einstein equations

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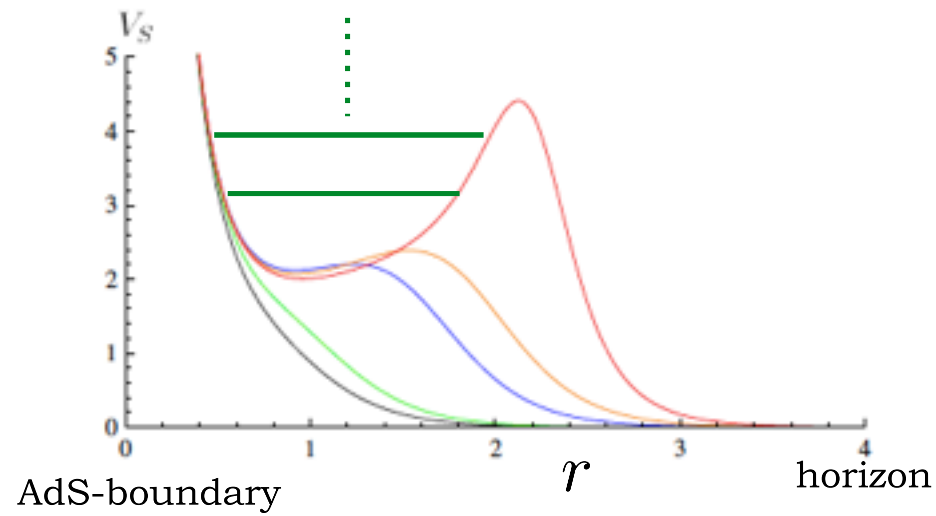

- the “ringing” of black holes

- quasi-eigensolutions to the linearized Einstein equations

- quasinormal modes (gravity) holographically correspond to poles of correlators

$$\omega_{QNM} = \text{pole of } G^{ret}_{QFT}$$

Contents

Hydro result: hydrodynamic poles

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions
: $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle, \langle T^{\mu\nu}\, J^{\alpha}\rangle, \langle J^{\mu}\, T^{\alpha\beta}\rangle, \langle J^{\mu}\, J^{\alpha}\rangle$

vector modes under SO(2) rotations around *B*

$$\omega = \quad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$\mathfrak{s}_0 = s_0/n_0$
$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$

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former sound modes

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former sound modes

➡ **a chiral magnetic wave**
[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3 C \mathfrak{s}_0^2\right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

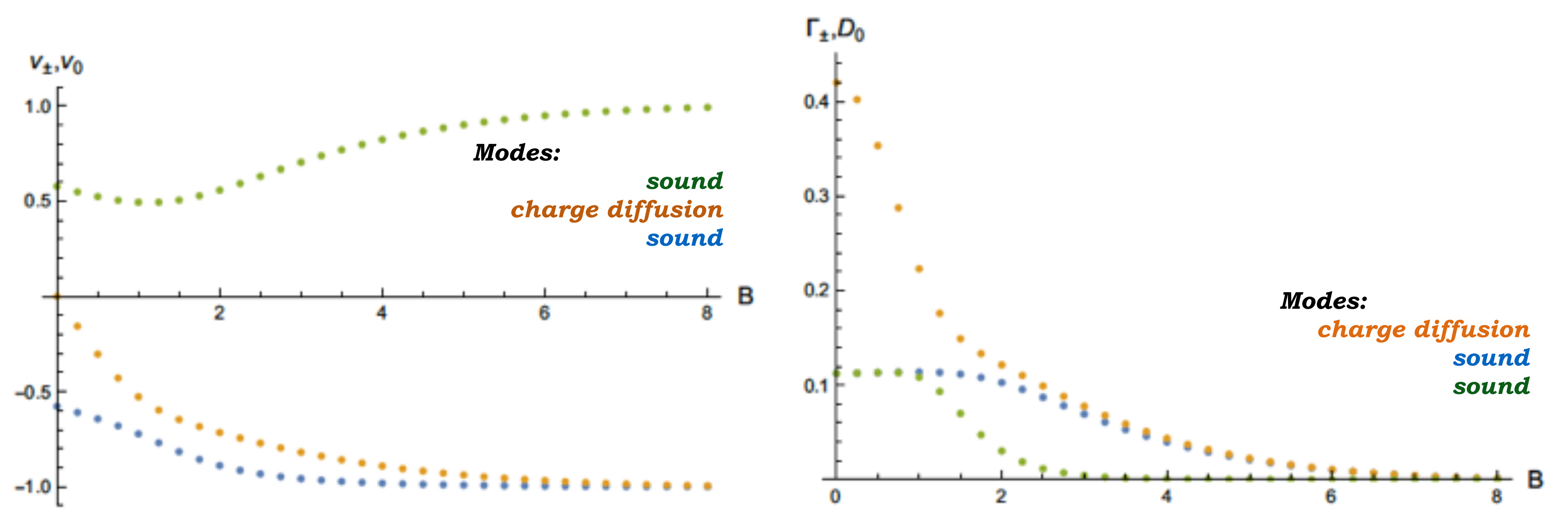
Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes

[Ammon, Kaminski et al.; JHEP (2017)]

- Weak B: **holographic results are in "agreement" with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

the speed of light **and without attenuation**



confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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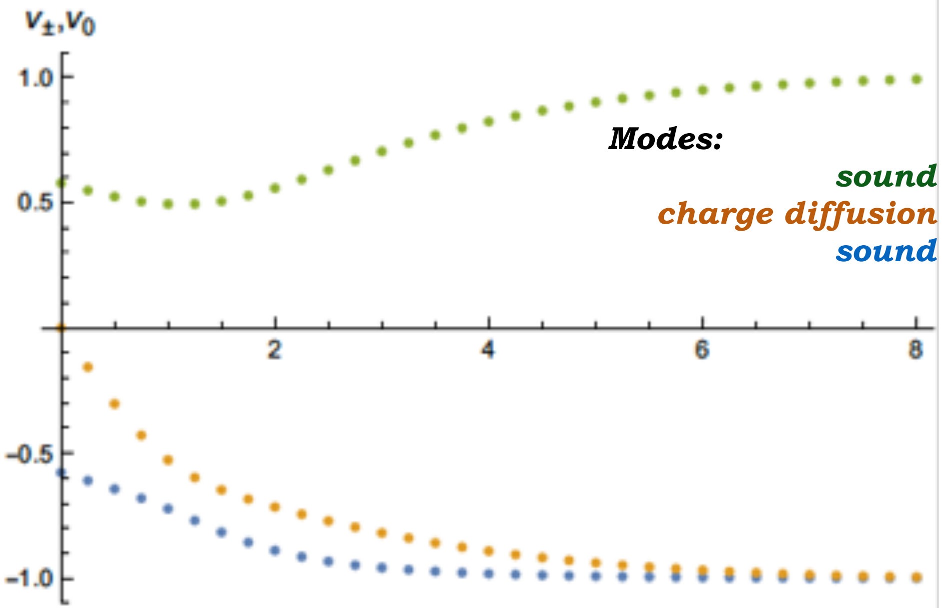

RECALL: weak B hydrodynamic poles

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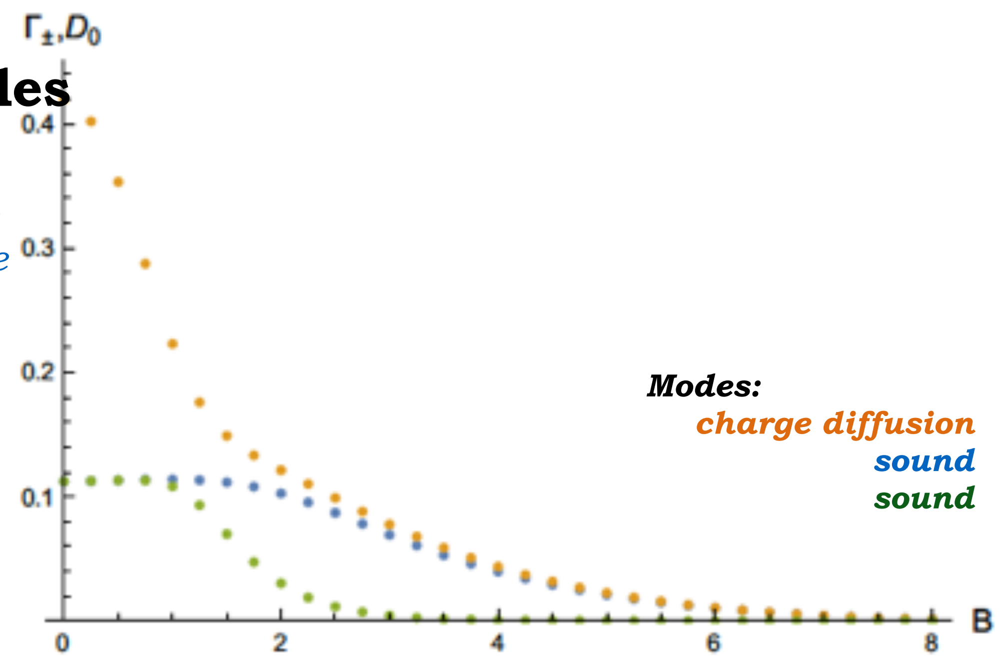

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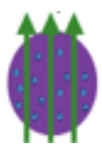
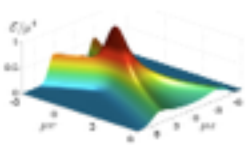
Contents

✓ Hydrodynamics 2.0 (near equilibrium) → *correlation functions (transport coefficients)*

✓ Holography → *correlation functions (transport coefficients)*



✓ Results for charged chiral plasma
[Ammon, Kaminski, Koirala, Leiber, Wu; JHEP (2017)]



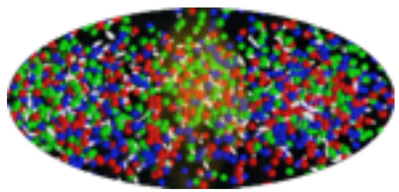
4. Far-From Equilibrium

5. Conclusions

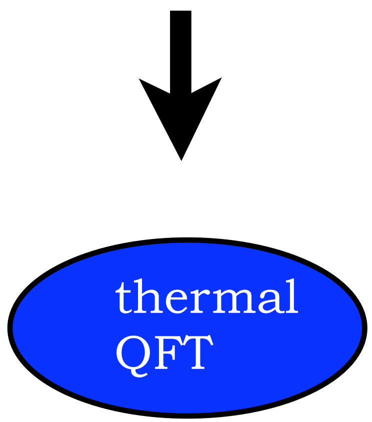
Holographic thermalization

cf. Casey Cartwright's talk

Thermalization:



T=0 particle "soup"



nonzero T plasma

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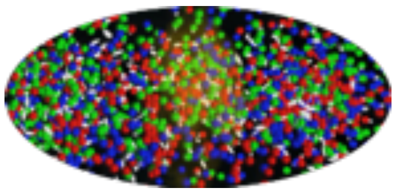

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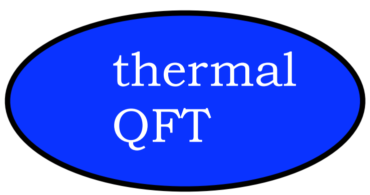

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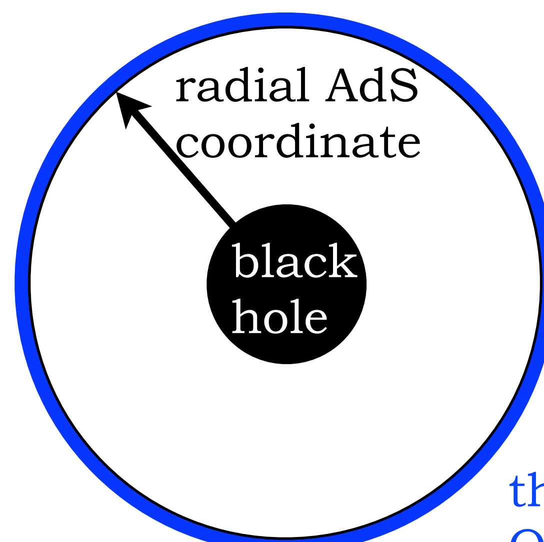

Anti-de Sitter space boundary

thermal QFT

Holographic thermalization

cf. Casey Cartwright's talk

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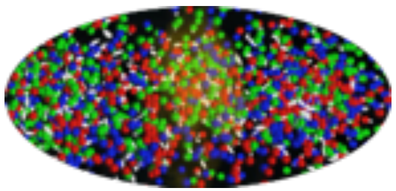

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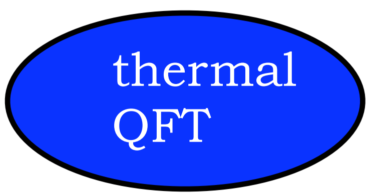
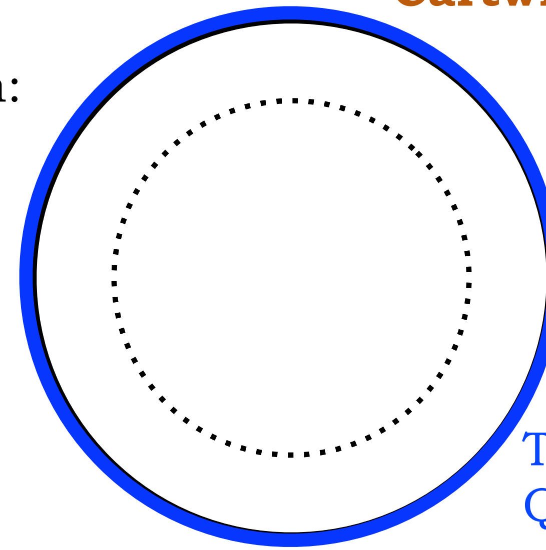


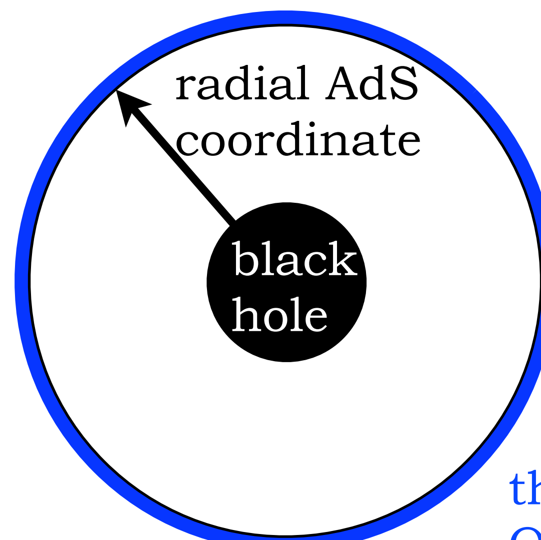
nonzero T plasma

correspondence

Horizon formation:



T=0
QFT



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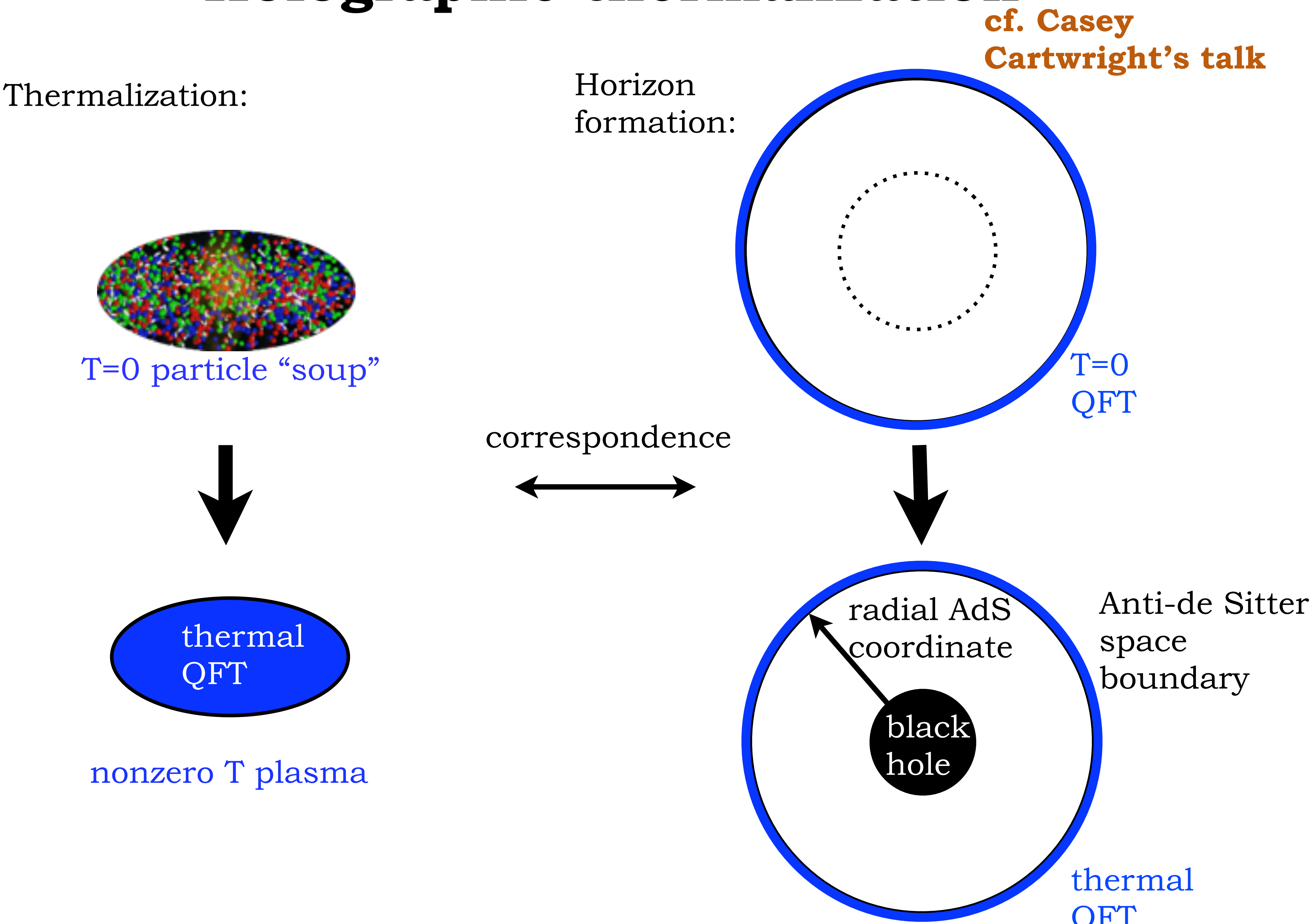

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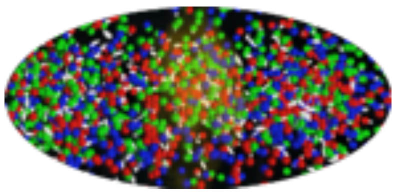
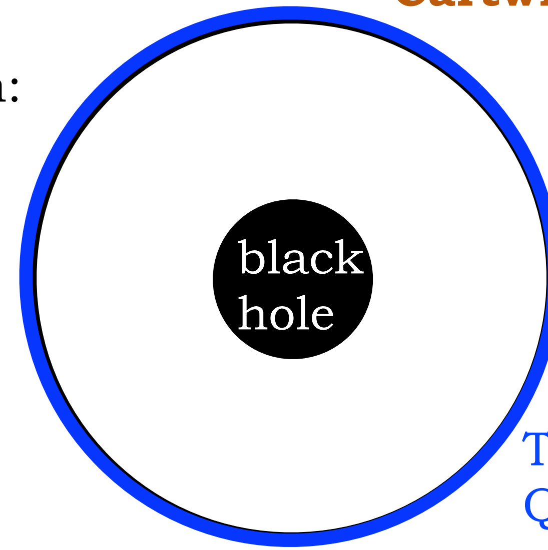


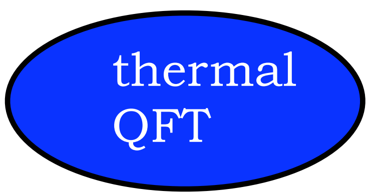
T=0 particle "soup"



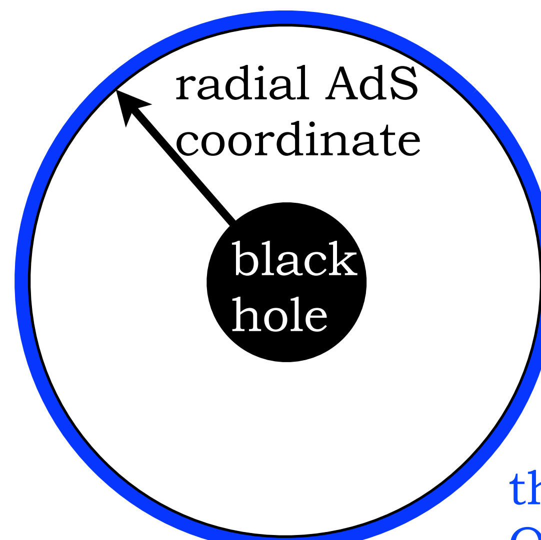
cf. Duvier Suarez Fontanella's talk

T=0 QFT

correspondence



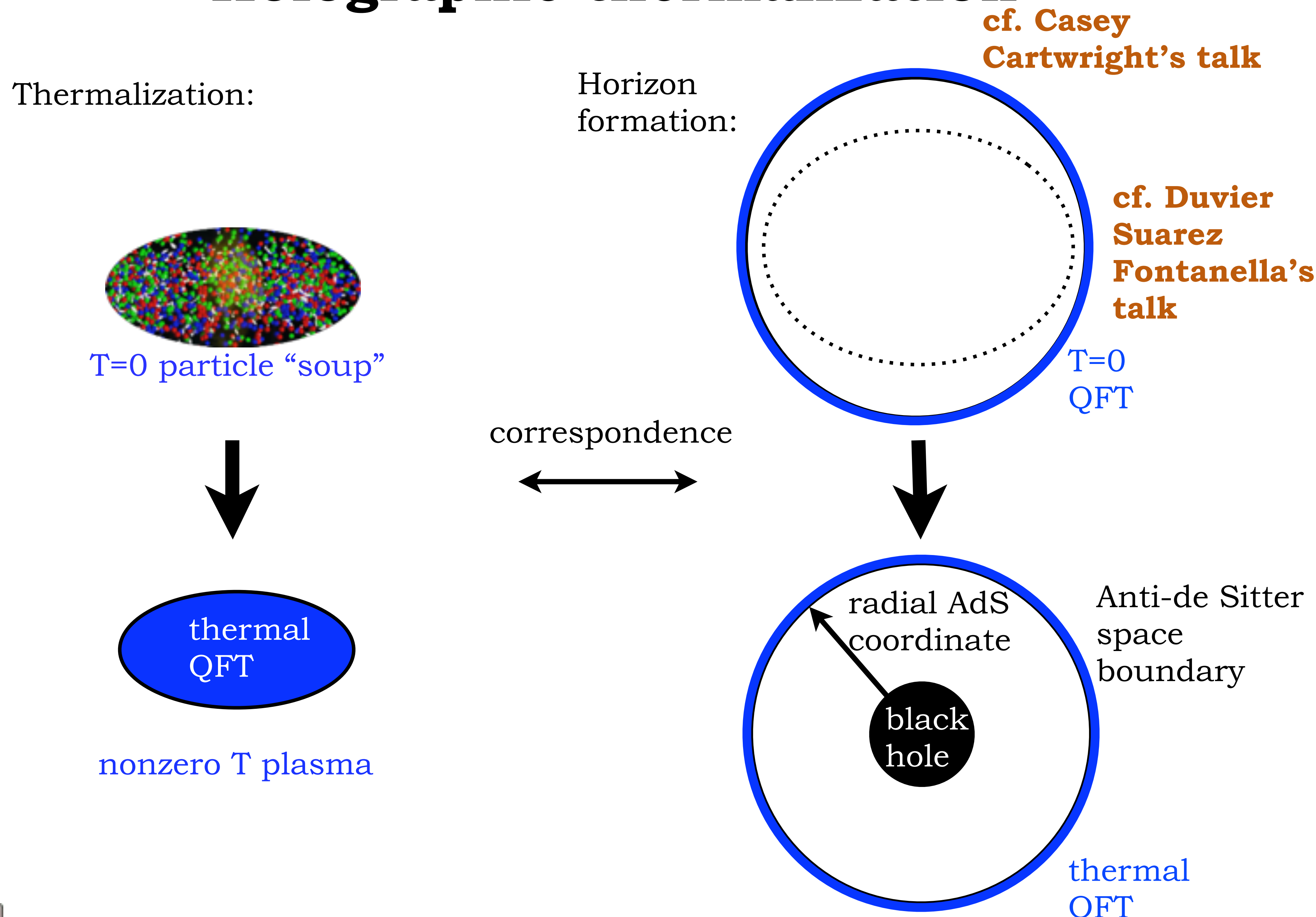
nonzero T plasma



Anti-de Sitter space boundary

thermal QFT

Holographic thermalization



Colliding shock waves in AdS

$$0 = \Sigma^2\left[F'' - 2(d_3B)' - 3B'd_3B\right] + 4\Sigma' d_3\Sigma\,, \quad - \Sigma\left[3\Sigma'F' + 4(d_3\Sigma)' + 6B'd_3\Sigma\right],$$

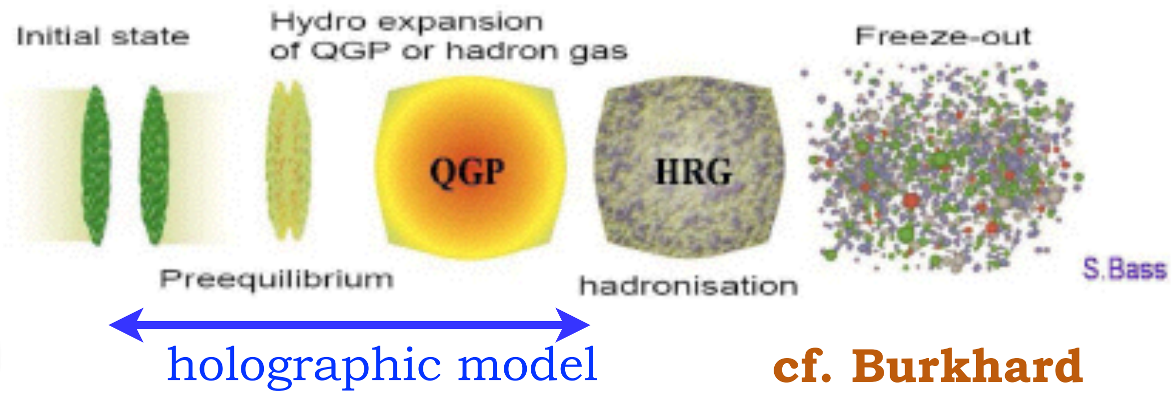
$$0 = \Sigma^4\left[A'' + 3B'd_+B + 4\right] - 12\Sigma^2\Sigma'd_+\Sigma + e^{2B}\Big\{\Sigma^2\left[\tfrac{1}{2}(F')^2 - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right] + 2(d_3\Sigma)^2 - 4\Sigma\left[2(d_3B)d_3\Sigma + d_3^2\Sigma\right]\Big\}\,,$$

$$0 = 6\Sigma^3(d_+\Sigma)' + 12\Sigma^2(\Sigma'd_+\Sigma - \Sigma^2) - e^{2B}\Big\{2(d_3\Sigma)^2 + \Sigma^2\left[\tfrac{1}{2}(F')^2 + (d_3F)' + 2F'd_3B - \tfrac{7}{2}(d_3B)^2 - 2d_3^2B\right] + \Sigma\left[(F' - 8d_3B)\,d_3\Sigma - 4d_3^2\Sigma\right]\Big\}\,.$$

$$0 = 6\Sigma^4(d_+B)' + 9\Sigma^3(\Sigma'd_+B + B'd_+\Sigma) + e^{2B}\Big\{\Sigma^2\left[(F')^2 + 2(d_3F)' + F'd_3B - (d_3B)^2 - d_3^2B\right] + 4(d_3\Sigma)^2 - \Sigma\left[(4F' + d_3B)\,d_3\Sigma + 2d_3^2\Sigma\right]\Big\}\,,$$

$$0 = 6\Sigma^2 d_+^2\Sigma - 3\Sigma^2A'd_+\Sigma + 3\Sigma^3(d_+B)^2 - e^{2B}\Big\{(d_3\Sigma + 2\Sigma d_3B)(2d_+F + d_3A) + \Sigma\left[2d_3(d_+F) + d_3^2A\right]\Big\}\,,$$

$$0 = \Sigma\left[2d_+(d_3\Sigma) + 2d_3(d_+\Sigma) + 3F'd_+\Sigma\right] + \Sigma^2\left[d_+(F') + d_3(A') + 4d_3(d_+B) - 2d_+(d_3B)\right]$$



cf. Burkhard Kämpfer's talk

[Chesler, Yaffe; PRL (2011)]

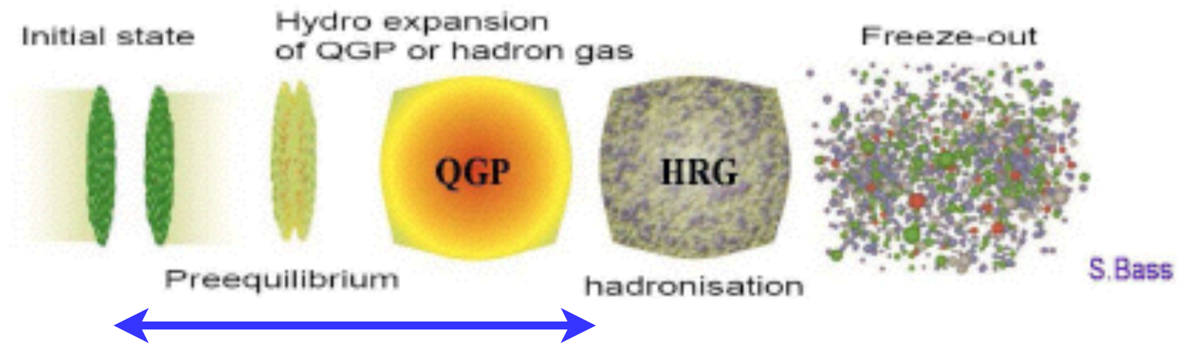
[Janik; PRD (2006)]

[Fuini, Yaffe; (JHEP) 2015)]

[Cartwright, Kaminski; work in progress]

Colliding shock waves in AdS

Method: numerical computation in gravity



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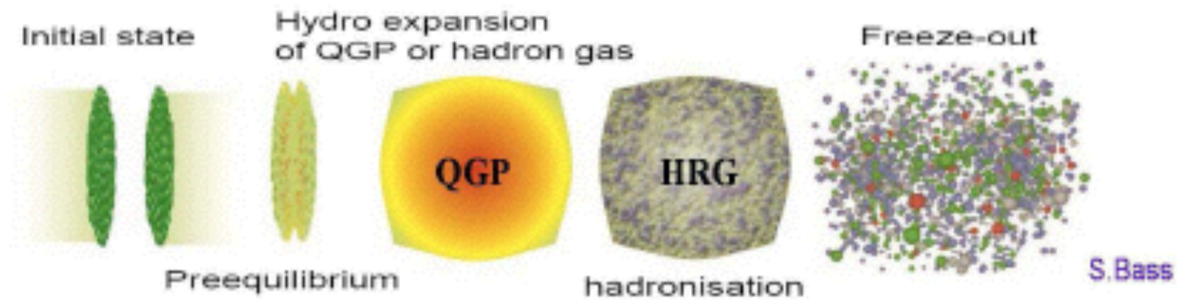
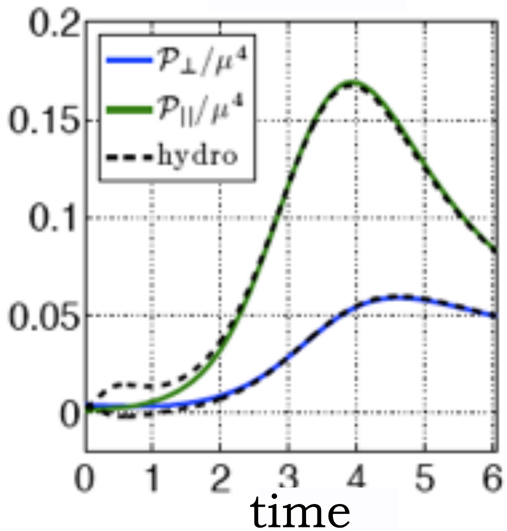
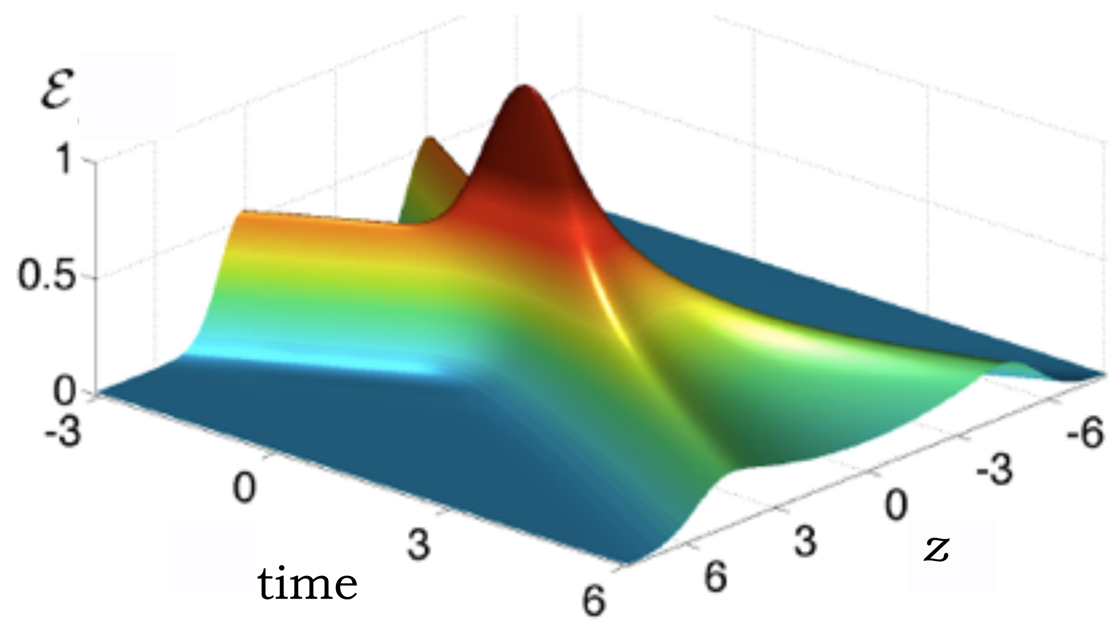


holographic model

cf. Burkhard Kämpfer's talk





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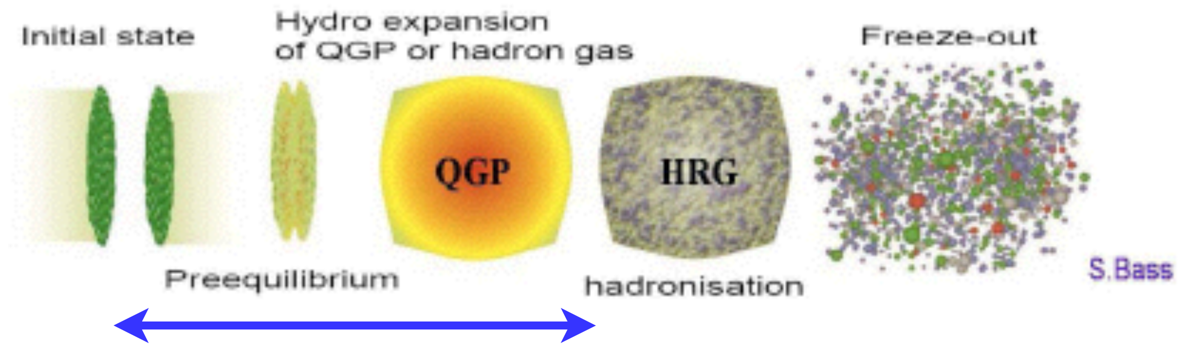

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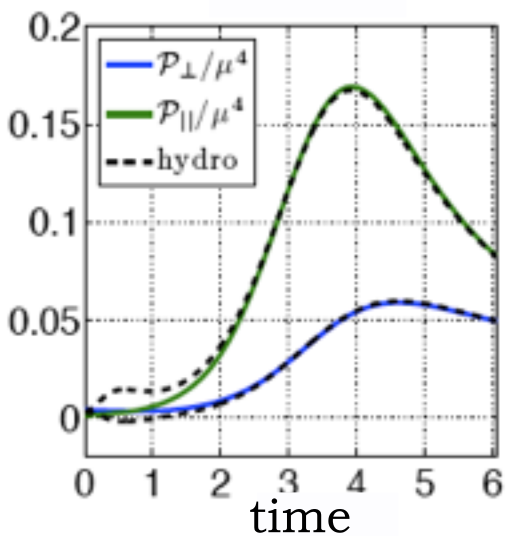

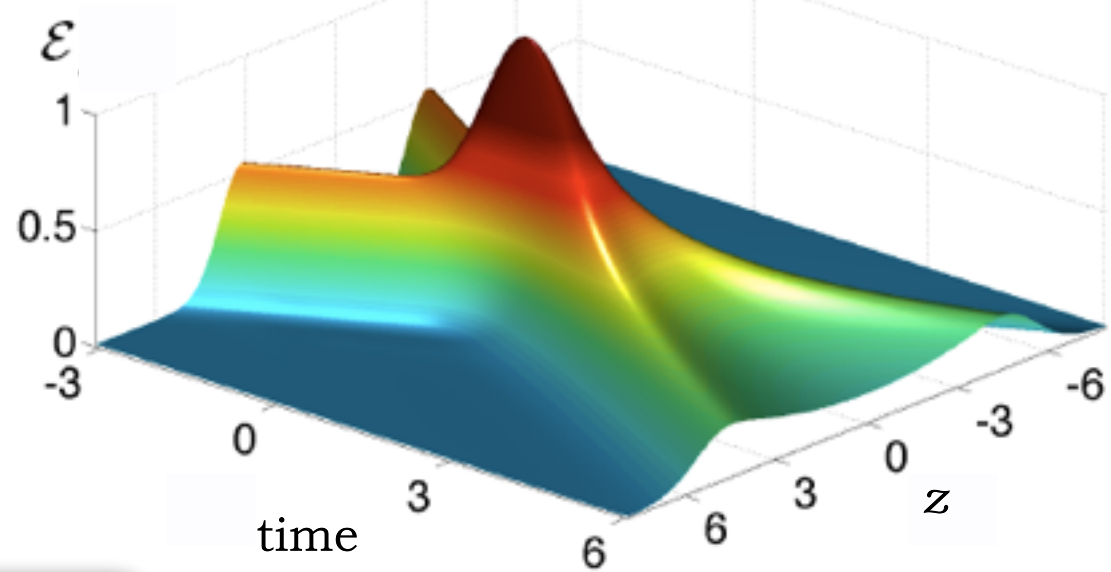

hydro works too well too early!

[Chesler, Yaffe; PRL (2011)]

[Janik; PRD (2006)]

[Fuini, Yaffe; (JHEP) 2015)]

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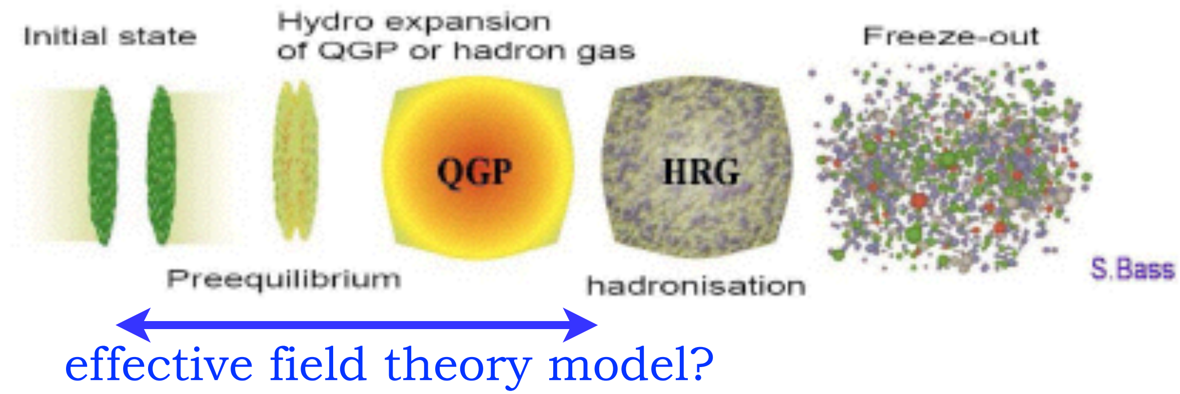
2-pt functions? **cf. Casey Cartwright's talk**

Far-from equilibrium in hydrodynamics?



- hydrodynamics describes pressures much **earlier than expected** (lesson from holographic thermalization)
- similar effects in Bjorken flow **numerical hydro calculations**
- hydrodynamic expansion in gradients is asymptotic — resummation reveals analogies to QFT expansion in Planck's constant, addressed by **resurgence**
- hydrodynamics may be rewritten with different fields, in order to describe far from equilibrium dynamics

[Romatschke; (2017)]

5. Conclusions

Things for which there was no time ...

➡ transport coefficients and correlators

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; work in progress]

➡ magnetohydrodynamics (dynamic B)

[Hernandez, Kovtun; JHEP (2017)]

[Grozdanov, Hofman, Iqbal; PRD (2017)]

[Hattori, Hirono, Yee, Yin; (2017)]

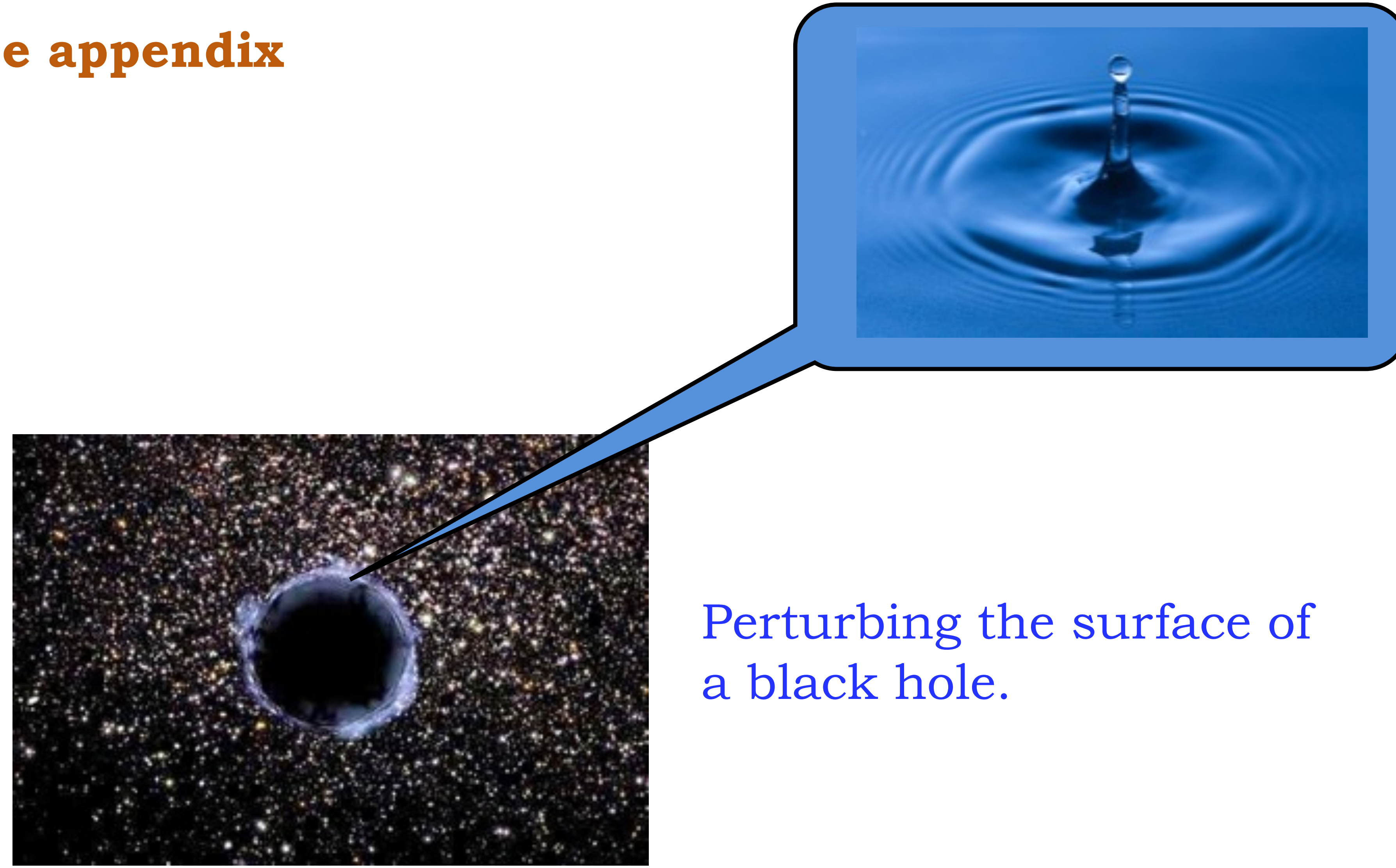
➡ axial *and* vector current

[Landsteiner, Megias, Pena Benitez; PRD (2014)]

[Ammon, Grieninger, Jimenez-Alba, Macedo, Melgar; JHEP (2016)]

Holography: Fluid/gravity correspondence

see appendix



Perturbing the surface of a black hole.

Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory,
for example **QCD**

- gravity dual to QCD or standard model?
- not known yet

model

holography (gauge/gravity correspondence)

(Hard) problem in "similar" model theory

Simple problem in a particular gravitational theory

Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory
- hydrodynamic approximation of model theory

QFT

Gravity

Hydrodynamics

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➡ Holography is good at predictions that are **qualitative** or **universal**.
➡ **Compare** holographic result to hydrodynamics of model theory.
➡ **Compare** hydrodynamics of original theory to hydrodynamics of model.
➡ **Understand holography as an effective description.**

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Simple problem i

CONSISTENCY CHECK

gravitational theo

Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory

REALITY CHECK: MODEL APPROPRIATE?

- hydrodynamic approximation of model theory

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Thanks to collaborators

Friedrich-Schiller University of Jena, Germany

Prof. Dr. Martin Ammon

Dr. Julian Leiber

Sebastian Grieninger

Regensburg University, Germany

Prof. Dr. Andreas Schäfer

Goethe University, Frankfurt, Germany

Dr. Gergely Endrödi

University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA

Dr. Jackson Wu

Roshan Koirala

Casey Cartwright

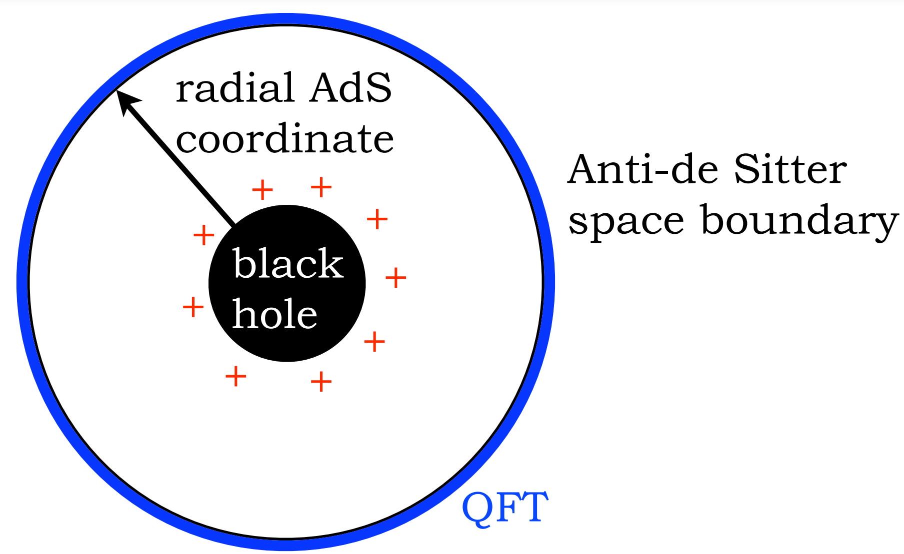
APPENDIX

Fluid/gravity correspondence

[Bhattacharyya et al.; JHEP (2008)]

Einstein equations = hydrodynamic conservation equations + dynamical equations of motion

Constitutive equations from geometry near boundary.



Fluid/gravity correspondence

Conservation equations from gravity

5-dimensional Einstein-Maxwell-Chern-Simons equations of motion :

$$R_{MN} + 4g_{MN} = \frac{1}{2}F_{MK}F_N{}^K - \frac{1}{12}g_{MN}F^2$$

$$\partial_N(\sqrt{-g}F^{NM}) = \frac{1}{4\sqrt{3}}\epsilon^{MNOPQ}F_{NO}F_{PQ}$$

dual to anomaly

$$\xi_N = dr$$

Constraint equations arise from contraction with one-form dr (normal to boundary) :

$$(\mathrm{contraints})_M = \xi^N (\mathrm{Einstein\,equations})_{MN} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$(\mathrm{contraint}) = \xi^N (\mathrm{Maxwell - Chern - Simons\,equations})_N \quad \Rightarrow \quad \nabla_\mu j^\mu = C E^\mu B_\mu$$

Constitutive equations from gravity

Example: no matter content, vanishing gauge fields :

$$\langle T_{\mu\nu}\rangle = \lim_{r\to\infty}\left[\frac{r^{(D-3)}}{\kappa_D^2}\left(K_{\mu\nu} - K\gamma_{\mu\nu} - (D-2)\gamma_{\mu\nu}\right)\right]$$

with extrinsic curvature $K_{\mu\nu} = -\frac{1}{2n}(\partial_r\gamma_{\mu\nu} - \nabla_\mu n_\nu - \nabla_\nu n_\mu)$

$$ds^2 = n^2 dr^2 + \gamma_{\mu\nu}(dx^\mu + n^\mu dr)(dx^\nu + n^\nu dr)$$

Example: $\mathcal{N}$=4 Super-Yang-Mills with anomaly

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

Gravity dual: 5-dimensional Einstein-Maxwell-Chern-Simons action

$$S = -\frac{1}{2{\kappa_5}^2}\int\left[\sqrt{-g}\left(R+12-\frac{1}{4}F^2\right)-\frac{1}{12\sqrt{3}}\epsilon^{MNOPQ}A_M F_{NO} F_{PQ}\right] d^4x\, dr$$

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CS-term dual to chiral anomaly

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CS-term dual to chiral anomaly

Black hole with R-charge (in Eddington-Finkelstein coordinates):

$$ds^2 = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 \Delta_{\mu\nu} dx^\mu dx^\nu - 2 u_\mu dx^\mu dr$$

solution with constant parameters $Q,\, b,\, u^\mu$.

$$f(r) = 1 + \frac{Q^2}{r^6} - \frac{1}{b^4 r^4} \qquad A_r = 0\,, \quad A_\mu = -\frac{\sqrt{3}Q}{r^2} u_\mu \qquad \Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$

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Make parameters boundary-coordinate-dependent: *dual to hydrodynamic fields*

$$b \to b(x)\,, \quad Q \to Q(x), \quad u^\mu \to u^\mu(x)$$

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Make parameters boundary-coordinate-dependent: *dual to hydrodynamic fields*

$$b \to b(x)\,, \quad Q \to Q(x), \quad u^\mu \to u^\mu(x)$$

- expand in gradients of b, Q and u
  dual to hydrodynamic expansion in the field theory
- new analytical solutions to Einstein equations
  give values of transport coefficients in field theory

Proofs of Gauge/Gravity Correspondences

-Some examples

- Three-point functions of N=4 Super-Yang-Mills theory
- Conformal anomaly of the same theory
- RG flows away from most symmetric case
- … many other symmetric instances of the correspondence

Evidence for Gauge/Gravity

-Reasonable example results from Gauge/Gravity!

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- Compute observables in strongly coupled QFTs

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- [AdS/QCD (bottom-up approach) distinct from string constr.]

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Gauge/Gravity is a Powerful Tool

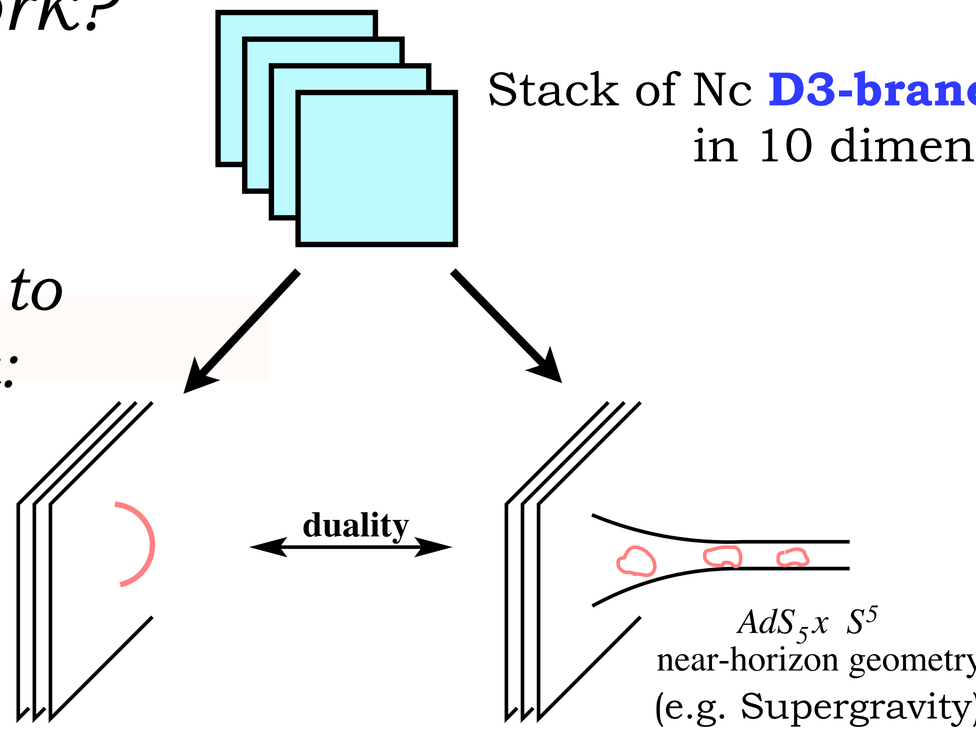
- non-perturbative results, strong coupling
- treat many-body systems
- direct computations in real-time thermal QFT (transport)
- no sign-problem at finite charge densities
- methods often just require solving ODEs in classical gravity
- quick numerical computations (~few seconds on a laptop)
- (turn around: study strongly curved gravity)

Outline: Gauge/gravity correspondence

-Why does it work?

Stack of Nc **D3-branes** (coincident)
in 10 dimensions

Two distinct ways to describe this stack:



4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side

(e.g. Supergravity)

gravity side

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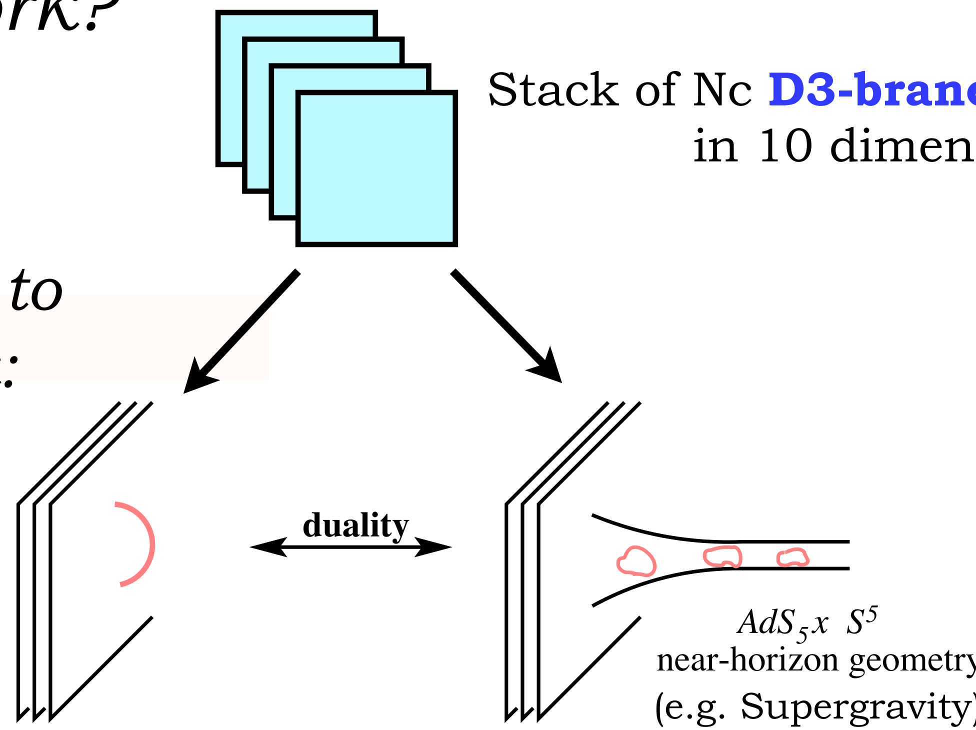

4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side

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gravity side

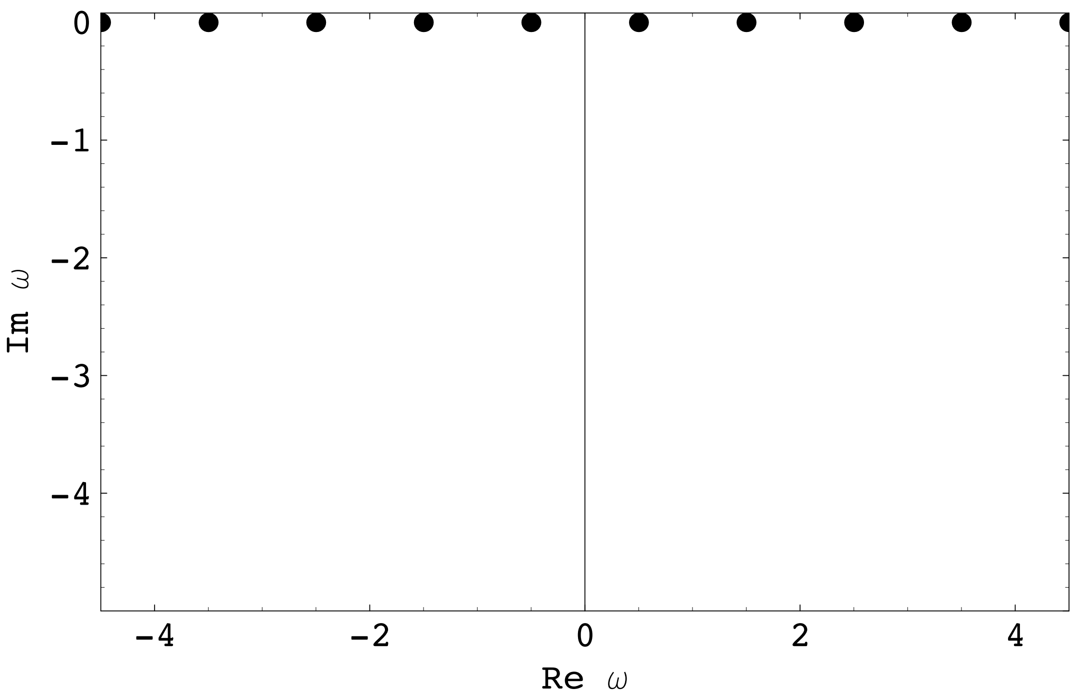
-How does it work?

Add/change geometric objects on 'gravity side':

Geometric setup: Strings/Branes → Find solution configuration → Field Theory result

Example: Schwarzschild radius corresponds to temperature

Quasi Normal Modes (QNMs)

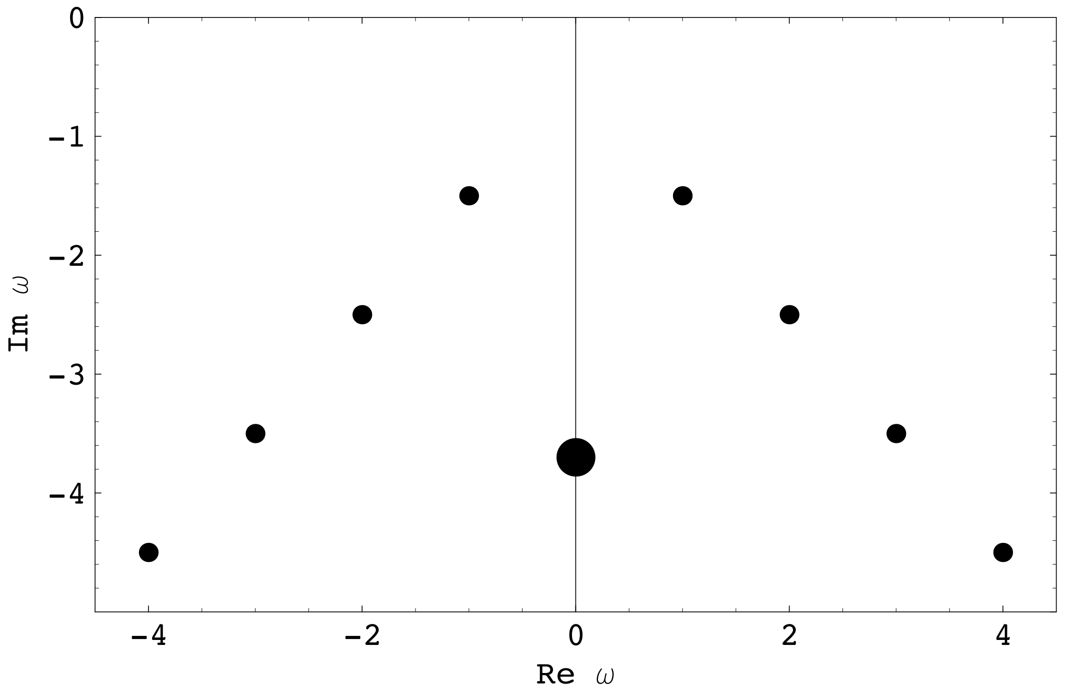


Simple example:
Eigenfrequencies / normal modes
of the quantum mechanical harmonic oscillator
(no damping)

$$\omega_n = \frac{1}{2} + n$$

quasinormal frequencies

Quasi Normal Modes (QNMs)



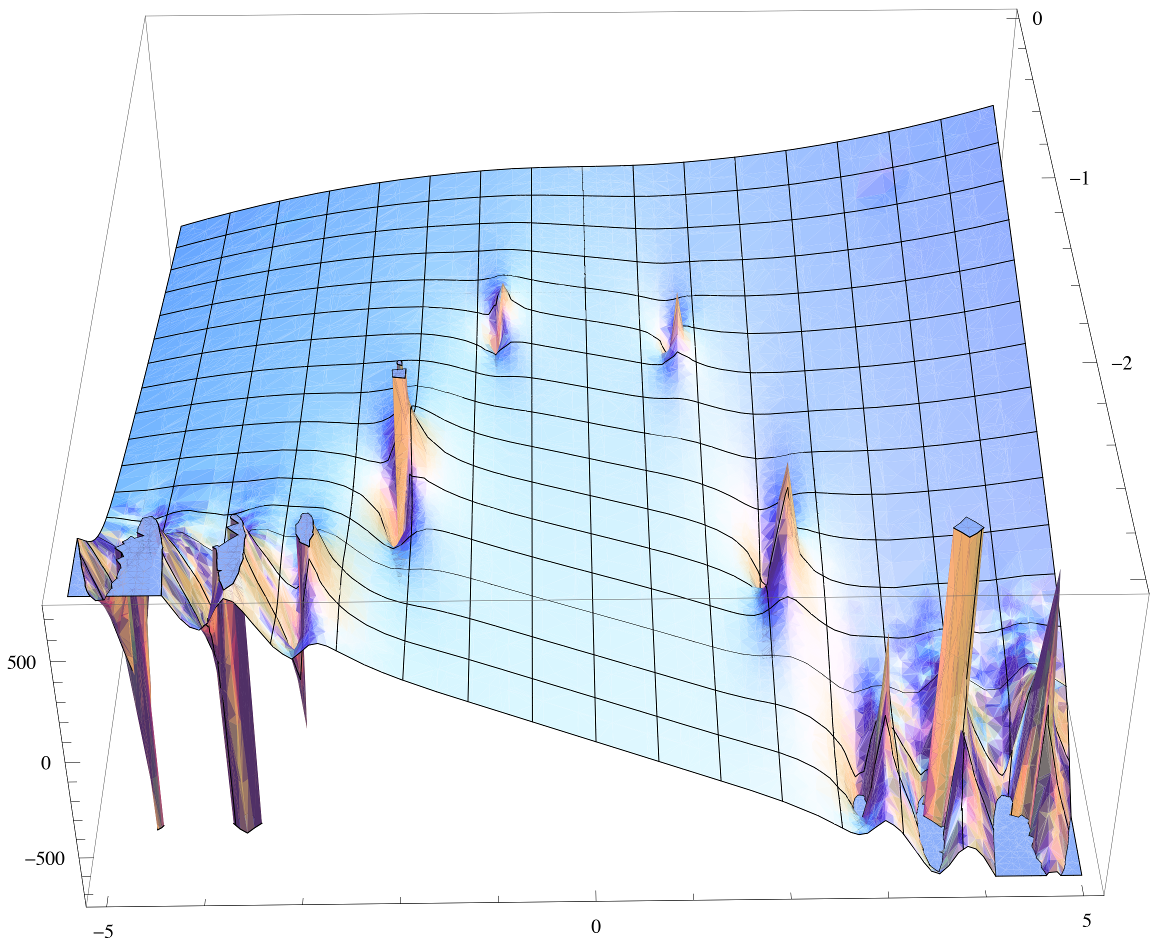
$$G_{ret} \propto \frac{1}{i\omega - Dq^2}$$

Example: Poles of charge current correlator

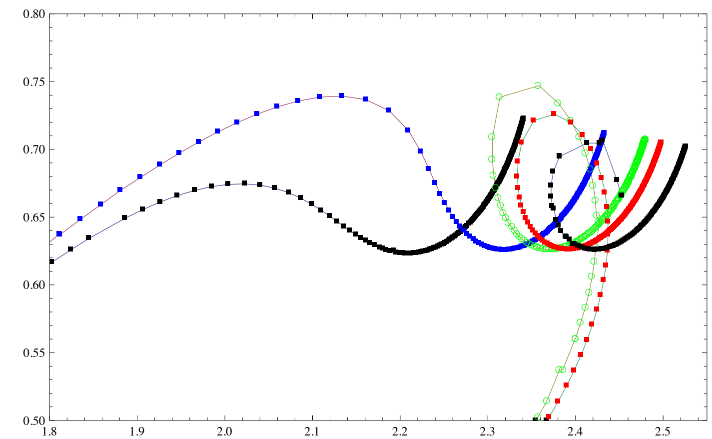
- QNMs are the quasi-eigenmodes of gauge field
- Dual QFT: lowest QNM identified with hydrodynamic diffusion pole (not propagating)
- Higher QN modes: gravity field waves propagate through curved b.h. background while decaying (dual gauge currents analogously)

Quasi Normal Modes (QNMs)

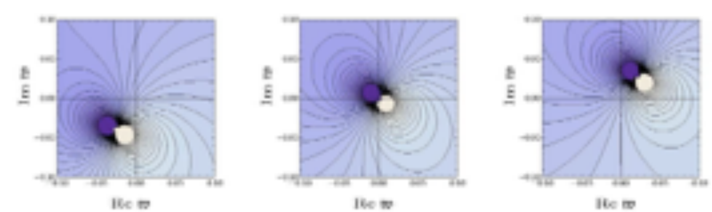
Complex frequency plane



Trajectories (dial k)



Instabilities



Chiral effects in vector and axial currents

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral vortical effect

chiral separation effect

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see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

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Full chiral vortical effect & gravity

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right) + \dots$$

formal approach guarantees completeness

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More than one anomalous current

$$\nabla_\nu J^\nu_a = \frac{1}{8} C_{abc}\epsilon^{\nu\rho\sigma\gamma} F^b_{\nu\rho} F^c_{\sigma\gamma}$$

$$\xi_a = C_{abc}\mu^b\mu^c + 2\beta_a T^2 - \frac{2n_a}{\epsilon + p}\left(\frac{1}{3}C_{bcd}\mu^b\mu^c\mu^d + 2\beta_b\mu^b T^2 + \gamma T^3\right)$$

[Neiman, Oz; JHEP (2010)]

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various charges (e.g. axial, vector)

previously neglected

$$\beta = -4\pi^2 c_m$$

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Gravitational anomalies

$$\nabla_\nu T^{\mu\nu}_{cov} = F^\mu{}_\nu J^\nu_{cov} + \frac{c_m}{2}\nabla_\nu\left[\epsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^{\mu\nu}{}_{\alpha\beta}\right]$$

full transport coefficient exactly known; first measurement of gravitational anomaly?