Lecture: (Far-from-equilibrium) dynamics in magnetic charged chiral plasma

Non-Equilibrium Dynamics - NED2018, April 19th, 2018, Varadero, Cuba

by Matthias Kaminski
(University of Alabama)
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Assume we have a hard problem that is difficult to solve in a given theory, for example \textbf{QCD}.

Methods: holography & hydrodynamics

- gravity dual to QCD or standard model?
- not known yet

\textbf{holography (gauge/gravity correspondence)}

\begin{itemize}
  \item \textbf{QFT}
  \item \textbf{Gravity}
\end{itemize}
Assume we have a hard problem that is difficult to solve in a given theory, for example \textbf{QCD}.

\textbf{Methods: holography \& hydrodynamics}

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- not known yet

\textit{Simple problem in a particular gravitational theory}

\textit{Methods: holography (gauge/gravity correspondence)}

\textbf{(Hard) problem in “similar” model theory}
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example, QCD

- gravity dual to QCD or standard model?
  - not known yet

Solve problems in effective field theory (EFT), e.g.:

- hydrodynamic approximation of original theory

Simple problem in a particular gravitational theory

Hard problem in “similar” model theory

QFT

Gravity

Hydrodynamics
Teaser: Good agreement of lattice QCD data with holography (N=4 SYM)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

\[ p_T = - \frac{L_T}{V} \frac{\partial F_{\text{QCD}}}{\partial L_T} \]

\[ p_L = - \frac{L_L}{V} \frac{\partial F_{\text{QCD}}}{\partial L_L} \]

\[ F_{\text{QCD}} \ldots \text{free energy} \]

\[ L_T \ldots \text{transverse system size} \]

\[ L_L \ldots \text{longitudinal system size} \]

\[ V \ldots \text{system volume} \]

conformal even here?

conformal regime

preliminary results
Teaser: Good agreement of lattice QCD data with holography (N=4 SYM)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

Preliminary results

conformal even here?

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Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: \( p_T = -\frac{L_T}{V} \frac{\partial F_{QCD}}{\partial L_T} \)

longitudinal pressure: \( p_L = -\frac{L_L}{V} \frac{\partial F_{QCD}}{\partial L_L} \)

\( F_{QCD} \) ... free energy

\( L_T \) ... transverse system size

\( L_L \) ... longitudinal system size

\( V \) ... system volume
Teaser: Good agreement of lattice QCD data with holography (N=4 SYM)
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pT / pL vs T / sqrt(B)

conformal even here?

holographic model with B approximates QCD very well

Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

\[ p_T = -\frac{L_T}{V} \frac{\partial F_{QCD}}{\partial L_T} \]

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\[ F_{QCD} \quad \text{free energy} \]

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Contents

1. Hydrodynamics 2.0
   (near equilibrium)

2. Holography
   (near equilibrium)

3. Results for charged chiral plasma

4. Far-From Equilibrium

5. Conclusions
Hydrodynamic variables

**Thermodynamics**

\[ T, \mu, u^\nu \]

**Hydrodynamics**

\[ T(t, \bar{x}), \mu(t, \bar{x}), u^\nu(t, \bar{x}) \]
Hydrodynamic variables

Thermodynamics

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Hydrodynamics

Universal effective field theory for microscopic QFTs, expansion in gradients of temperature, chemical potential and velocity

• fields $T(x), \mu(x), u^\nu(x)$

• conservation equations

• constitutive equations (Landau frame)
Hydrodynamics

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$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$\nabla_\nu j^\nu = 0$$

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  \[ \nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \]
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- constitutive equations (Landau frame)
  
  **Energy momentum tensor**
  \[ T^{\mu\nu} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu) + \tau^{\mu\nu} \]

  **Conserved current**
  \[ j^\mu = n u^\mu + \nu^\mu \]
Hydrodynamics

Universal effective field theory for microscopic QFTs, expansion in gradients of temperature, chemical potential and velocity

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Conserved current
\[ j^\mu = n u^\mu + \nu^\mu \]
An old idea

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group

Old example: \( \nabla_\nu u^\nu \)

New example: \( \omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho \) (vorticity)
Constructing hydrodynamic constitutive equations

An old idea

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group

   Old example: \[ \nabla_\nu u^\nu \]

   New example: \[ \omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho \text{ (vorticity)} \]

2. Restricted by conservation equations

   Example: \[ \nabla_\mu j^{\mu}_{(0)} = \nabla_\mu (n u^\mu) = 0 \]
Constructing hydrodynamic constitutive equations

An old idea

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group

   Old example: \( \nabla_\nu u^\nu \)

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2. Restricted by conservation equations

   Example: \( \nabla_\mu j_\nu^{(0)} = \nabla_\mu (n u^\mu) = 0 \)

3. Further restricted by positivity of local entropy production: \( \nabla_\mu J_\nu^\mu \geq 0 \)

Alternatively, use field theory restrictions (Onsager,...)

[Jensen, MK, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]

[Landau, Lifshitz]
Chiral hydrodynamics

Derived for any QFT with a chiral anomaly (e.g. QCD)

\[ \nabla \nu j^\nu = 0 \quad \text{classical theory} \]
Chiral hydrodynamics

Derived for any QFT with a chiral anomaly (e.g. QCD)

\[ \nabla_\mu j^\mu = C \varepsilon^{\nu\rho\sigma\lambda} F_{\nu\rho} F_{\sigma\lambda} \]

quantum theory
Chiral hydrodynamics

Derived for any QFT with a **chiral anomaly** (e.g. QCD)

\[ \nabla_\mu j^\mu = C \varepsilon^{\nu \rho \sigma \lambda} F_{\nu \rho} F_{\sigma \lambda} \]

Completed constitutive equation **with external fields**

\[ j^\mu = n u^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu + \ldots \]

**Vorticity** \( \omega^\mu = \frac{1}{2} \varepsilon^{\mu \nu \lambda \rho} u_\nu \nabla Lamda u_\rho \)

**Magnetic field** \( B^\mu = \frac{1}{2} \varepsilon^{\mu \nu \lambda \rho} u_\nu F_{\lambda \rho} \)

Def.: \( V^\mu = E^\mu - T \Delta^{\mu \nu} \nabla_\nu \left( \frac{\mu}{T} \right) \)

**Agrees with gauge/gravity prediction:**

[Loganayagam; arXiv (2011)]
[Jensen et al.; JHEP (2012)]
[Jensen et al.; PRL (2012)]
[Son, Surowka; PRL (2009)]

Chiral vortical effect

Chiral magnetic effect
Chiral hydrodynamics

Derived for any QFT with a chiral anomaly (e.g. QCD)

\[ \nabla_\mu j^\mu = C \varepsilon^{\nu\rho\sigma\lambda} F_{\nu\rho} F_{\sigma\lambda} \]

quantum theory

Completed constitutive equation with external fields

\[ j^\mu = n u^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu + \ldots \]

vorticity magnetic field

Agrees with gauge/gravity prediction:

\[ \omega^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho \]

\[ B^\mu = \frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} u_\nu F_{\lambda\rho} \]

\[ \xi = C \left( \mu^2 - \frac{2}{3} n \mu^3 + \frac{1}{2} \frac{n \mu^2}{\epsilon + P} \right), \]

anomaly-coefficient C

[Son, Surowka; PRL (2009)]
[Loganayagam; arXiv (2011)]
[Jensen et al.; JHEP (2012)]
[Jensen et al.; PRL (2012)]
Chiral hydrodynamics

Derived for any QFT with a chiral anomaly (e.g. QCD)

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Completed constitutive equation with external fields

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vorticity magnetic field

\[ \omega^\mu = \frac{1}{2} \epsilon^{\mu \nu \lambda \rho} u_\nu \nabla \lambda u_\rho \quad B^\mu = \frac{1}{2} \epsilon^{\mu \nu \lambda \rho} u_\nu F_{\lambda \rho} \]

Agrees with gauge/gravity prediction:

[Loganayagam; arXiv (2011)]

[Def.: \( V^\mu = E^\mu - T \Delta^{\mu \nu} \nabla_\nu \left( \frac{\mu}{T} \right) \)]

\[ \xi = C \left( \mu^2 - \frac{2}{3} \frac{n \mu^3}{\epsilon + P} \right) \]

\[ \xi_B = C \left( \mu - \frac{1}{2} \frac{n \mu^2}{\epsilon + P} \right) \]

Observable in:

heavy ion collisions?

[Kharzeev, Son.; PRL (2011)]

neutron stars?

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2014)]

condensed matter?

[Cortijo, Ferreiros, Landsteiner, Vozmediano; (2015)]

chiral vortical effect

chiral magnetic effect

condensed matter?
Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:

\[ j^\mu = n u^\mu + \sigma \left[ E^\mu - T \Delta^{\mu \nu} \partial_\nu \left( \frac{\mu}{T} \right) \right] \]

\[ \Delta^{\mu \nu} = g^{\mu \nu} + u^\mu u^\nu \]

\[ u^\mu = (1, 0, 0) \]
Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:

\[ j^\mu = nu^\mu + \sigma \left[ E^\mu - T \Delta^{\mu\nu} \partial_\nu \left( \frac{\mu}{T} \right) \right] \]

sources \[ A_t, A_x \propto e^{-i\omega t + ikx} \]

fluctuations \[ n = n(t, x, y) \propto e^{-i\omega t + ikx} \] (fix \( T \) and \( u \))

\[ \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \]

\[ u^\mu = (1, 0, 0) \]
Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:
\[ j^\mu = n u^\mu + \sigma \left[ E^\mu - T \Delta^{\mu\nu} \partial_\nu \left( \frac{\mu}{T} \right) \right] \]
\[ \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \]

sources
\[ A_t, A_x \propto e^{-i\omega t + ikx} \]
+ other sources

fluctuations
\[ n = n(t, x, y) \propto e^{-i\omega t + ikx} \] (fix \( T \) and \( u \))
+ fluctuations in \( T \) and \( u \)

one point functions
\[ \nabla_\mu j^\mu = 0 \]
\[ \langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + kA_t) \]
\[ \langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + kA_t) \]
\[ \langle j^y \rangle = 0 \]

\[ \Rightarrow \] two point functions
\[ \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2} \]
\[ \Rightarrow \] hydrodynamic poles in spectral function
Thermal Spectral Function

Thermal spectral function $\mathcal{A}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

$$\mathcal{A}(\omega, q) = -2 \operatorname{Im} G^\text{ret}(\omega, q)$$
Thermal Spectral Function

Thermal spectral function $\mathcal{R}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

$$\mathcal{R}(\omega, q) = -2 \text{Im} G^{\text{ret}}(\omega, q)$$
Thermal spectral function $\mathcal{R}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

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Thermal Spectral Function

Thermal spectral function $\mathcal{R}$ contains all information about diffusion and quasiparticle resonances in fluid/plasma.

$$\mathcal{R}(\omega, q) = -2 \text{Im} G^{\text{ret}}(\omega, q)$$

Transport coefficients using Kubo formulae, e.g.

**electric conductivity**

$$\sigma \sim \lim_{\omega \to 0} \frac{1}{\omega} \langle [J^t, J^t] \rangle$$
Exercise 1.b): hydrodynamic correlators

Simple (non-chiral) example in 2+1

two point function: \[ \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2\sigma}{\omega + iDk^2} \]

spectral function: \[ -\text{Im} G^R = -\text{Im} \langle j^x j^x \rangle = -\sigma \omega_R \frac{2Dk^2\omega_I + \omega_R^2 + \omega_I^2}{\omega_R^2 + (\omega_I + Dk^2)^2} \]
Exercise 1.b): hydrodynamic correlators

Simple (non-chiral) example in 2+1

two point function: \[ \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\sigma \omega^2}{\omega + iDk^2} \]
spectral function: \[ -\text{Im} G^R = -\text{Im} \langle j_x j_x \rangle = -\sigma \omega^R \frac{2Dk^2\omega_I + \omega^2_R + \omega^2_I}{\omega^2_R + (\omega_I + Dk^2)^2} \]

hydrodynamic pole (diffusion pole) in spectral function at decreasing momentum \( k \):

- pole goes to zero
- spectral function vanishes with \( k \)
Far beyond hydrodynamics

Example: 3+1-dimensional $N=4$ Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle (\omega, k) = G^R_{xy, xy}(\omega, k) = -i \int d^4 x \ e^{-i \omega t + i k z} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$

\[\text{hydrodynamics valid}\]
Far beyond hydrodynamics: holography

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle(\omega, k) = G_{xy, xy}^R(\omega, k) = -i \int d^4x \, e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$

hydrodynamics valid

[Starinets; JHEP (2002)]
2. Holography
Holography (gauge/gravity) concepts - I

Gauge/Gravity Correspondence based on holographic principle

\[ S_{max}(\text{volume}) \propto \text{surface area} \]

String theory gives one example (AdS/CFT).

\[ \begin{align*}
N=4 \text{ Super-Yang-Mills} & \quad \iff \quad \text{Typ II B Supergravity} \\
\text{in } 3+1 \text{ dimensions} & \quad \text{in } (4+1)\text{-dimensional} \\
(CFT) & \quad \text{Anti de Sitter space (AdS)} \\
\end{align*} \]

[t Hooft (1993)]

[Susskind (1995)]

[Maldacena (1997)]
Holography (gauge/gravity) concepts - II

- strongly coupled quantum field theory
- correspondence
- weakly curved gravity

[correspondence]

[Maldacena (1997)]

radial AdS coordinate

Anti-de Sitter space boundary

QFT
Holography (gauge/gravity) concepts - II

strongly coupled quantum field theory

\( \text{correspondence} \)

weakly curved gravity

[Maldaçena (1997)]

QFT temperature

Hawking temperature

radial AdS coordinate

Anti-de Sitter space boundary

black hole

QFT

Matthias Kaminski

Lecture: Far-from-equilibrium dynamics in magnetic charged chiral plasma
Holography (gauge/gravity) concepts - II

strongly coupled quantum field theory ↔ correspondence ↔ weakly curved gravity

[Maldacena (1997)]

QFT temperature ↔ Hawking temperature

conserved charge ↔ charged black hole

radial AdS coordinate

Anti-de Sitter space boundary

black hole

QFT

Matthias Kaminski
Lecture: Far-from-equilibrium dynamics in magnetic charged chiral plasma
How does this give us correlators/transport?
**Famous transport result:**

**low shear viscosity/entropy density**

<table>
<thead>
<tr>
<th>Theory/Model</th>
<th>$\eta/s$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice QCD</td>
<td>$0.134(33)$</td>
<td>[Meyer, 2007]</td>
</tr>
<tr>
<td>Hydro (Glauber)</td>
<td>0.19</td>
<td>[Drescher et al., 2007]</td>
</tr>
<tr>
<td>Hydro (CGC)</td>
<td>0.11</td>
<td>[Drescher et al., 2007]</td>
</tr>
<tr>
<td>Viscous Hydro (Glauber)</td>
<td>$0.08, 0.16, {0.03}$</td>
<td>[Romatschke et al., 2007]</td>
</tr>
</tbody>
</table>

Gauge/Gravity: $\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$

[Pollicastro, Son, Starinets, 2001]
[Kovtun, Son, Starinets, 2003]

✓ Correct prediction!
Correspondence by zooming in on boundary
Correspondence by zooming in on boundary

Anti-de Sitter space

gravity field

\[ \phi \]

\[ \mathcal{A} \]

source

\[ \langle \mathcal{O} \rangle \]

operator vev

QFT on boundary

radial AdS coordinate

black hole
\[ \phi = \phi(0) + \phi(1) \frac{1}{r} + \phi(2) \frac{1}{r^2} + \ldots \]

Correspondence by zooming in on boundary

Gravity field

QFT on boundary

\[ \langle \mathcal{O} \rangle \]

Source

Operator vev

AdS space

Radial coordinate

Black hole
Correspondence by zooming in on boundary

Anti-de Sitter space

\[ \phi = \phi(0) + \phi(1) \frac{1}{r} + \phi(2) \frac{1}{r^2} + \ldots \]

gravity field

mathematical map: gauge/gravity correspondence

QFT on boundary

A

source

\langle \mathcal{O} \rangle

operator vev

radial AdS coordinate

black hole

re: Far-from-equilibrium dynamics in magnetic charged chiral plasma
Anti-de Sitter space

\[ \phi = \phi(0) + \phi(1) \frac{1}{r} + \phi(2) \frac{1}{r^2} + \ldots \]

Correspondence by zooming in on boundary

\[ \langle \mathcal{O} \mathcal{O} \rangle = \left. \frac{\delta \mathcal{O}}{\delta A} \right|_{A=0} \sim \frac{\delta \phi(1)}{\delta \phi(0)} \]

gravity field

Correlation function

radiation AdS coordinate

black hole

mathematical map: gauge/gravity correspondence

source

operator vev

QFT on boundary

re: Far-from-equilibrium dynamics in magnetic charged chiral plasma
Holographic correlator calculation

• start with **gravitational background** (metric, matter content)

• choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed \( \phi(t) \propto e^{-i\omega t} \phi(\omega) \))

• impose **boundary conditions** that are
  in-falling at horizon:

  \[ \lim_{u \to u_{\text{bdy}}} \phi(u) = 0 \]

  (and for QNMs also vanishing at AdS-boundary: \( \lim_{u \to u_{\text{bdy}}} \phi(u) = 0 \))
Holographic correlator calculation

• start with **gravitational background** (metric, matter content)

  \[ds^2 = \frac{r^2}{L^2} (-f dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} dr^2\]
  \[f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}\]
  \[A_t = \mu - \frac{Q}{L r^2}\]

  **Example:** (charged) Reissner-Nordstrom black brane in 5-dim AdS

  \[\text{[Janiszewski, Kaminski; PRD (2015)]}\]

• choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed \( \phi(t) \propto e^{-i\omega t} \phi(\omega) \))

  **Example:** metric tensor fluctuation

  \[\phi := h_{x'y'} \quad 0 = \phi'' - \frac{f(u) - u f'(u)}{u f(u)} \phi' + \frac{\omega^2 - f(u) k^2}{4r_H^2 u f(u)^2} \phi \quad u = \left( \frac{r H}{r} \right)^2\]

• impose **boundary conditions** that are \textbf{in-falling} at horizon:  \[\phi = (1 - u)^{i\omega} \left[ \phi^{(0)}_H + \phi^{(1)}_H (1 - u) + \phi^{(2)}_H (1 - u)^2 + \ldots \right] \]

  (and for QNMs also \textbf{vanishing} at AdS-boundary:  \( \lim_{u \to u_{\text{bdy}}} \phi(u) = 0 \))
Holographic correlator calculation

• start with **gravitational background** (metric, matter content)

\[ ds^2 = \frac{r^2}{L^2} (-f dt^2 + dx^2) + \frac{L^2}{r^2 f} dr^2 \]

\[ f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6} \]

\[ A_t = \mu - \frac{Q}{L r^2} \]

*Example:* (charged) Reissner-Nordstrom black brane in 5-dim AdS

[Janiszewski, Kaminski; PRD (2015)]

• choose one or more **fields to fluctuate**

(obeying linearized Einstein equations; Fourier transformed \( \phi(t) \propto e^{-i\omega t} \phi(\omega) \))

*Example:* metric tensor fluctuation

\[ \phi := h_{x^y} \]

\[ 0 = \phi'' - \frac{f(u) - u f'(u)}{u f(u)} \phi' + \frac{\omega^2 - f(u)k^2}{4r_H^2 u f(u)^2} \phi \]

\[ u = \left( \frac{r_H}{r} \right)^2 \]

• impose **boundary conditions** that are

**in-falling** at horizon:

\[ \phi = (1 - u)^{\pm \frac{i\omega}{2(2-d)}} \left[ \phi_H^{(0)} + \phi_H^{(1)}(1 - u) + \phi_H^{(2)}(1 - u)^2 + \ldots \right] \]

(and for QNMs also **vanishing** at AdS-boundary: \( \lim_{u \to u_{bdy}} \phi(u) = 0 \))

\[ \Rightarrow \langle \mathcal{O}\mathcal{O} \rangle = \left. \frac{\delta \mathcal{O}}{\delta A} \right|_{A=0} \sim \frac{\delta \phi(1)}{\delta \phi(0)} \]

**holographic correlator**

**Exercise 2.**

a.) transform metric to FG/EF

b.) show scale invariance

cf. Casey Cartwright’s talk
What are quasi-normal modes?

• heuristically: the eigenmodes of black holes or black branes

\[ H \phi = -\partial_r^2 \phi + V_S \phi = E\phi \]

• the “ringing” of black holes

• quasi-eigensolutions to the linearized Einstein equations
What are quasi-normal modes?

• heuristically: the eigenmodes of black holes or black branes

\[ H \phi = -\partial_r^2 \phi + V_S \phi = E\phi \]

• the “ringing” of black holes

• quasi-eigensolutions to the linearized Einstein equations

• quasinormal modes (gravity) holographically correspond to poles of correlators

\[ \omega_{QNM} = \text{pole of } G^{ret}_{QFT} \]
✓ Hydrodynamics 2.0 (near equilibrium)

✓ Holography

3. Results for charged chiral plasma
   \[\text{Ammon, Kaminski, Koirala, Leiber, Wu; JHEP (2017)}\]

4. Far-From Equilibrium

5. Conclusions
Hydro result: hydrodynamic poles

Weak B hydrodynamics - poles of 2-point functions:

\[ \langle T^{\mu \nu} T^{\alpha \beta} \rangle, \langle T^{\mu \nu} J^\alpha \rangle, \langle J^\mu T^{\alpha \beta} \rangle, \langle J^\mu J^\alpha \rangle \]

[Ammon, Kaminski et al.; JHEP (2017)]

[Abbasi et al.; PLB (2016)]

[Kalaydzhyan, Murchikova; NPB (2016)]

\[
\omega = -ik^2 \frac{\eta}{\epsilon_0 + P_0} +
\]

former momentum diffusion modes

\[ s_0 = s_0/n_0 \]

\[ \tilde{c}_p = T_0 (\partial s/\partial T)_p \]
Hydro result: hydrodynamic poles

Weak B hydrodynamics - poles of 2-point functions

\[ \langle T^{\mu \nu} T^{\alpha \beta} \rangle, \langle T^{\mu \nu} J^{\alpha} \rangle, \langle J^{\mu} T^{\alpha \beta} \rangle, \langle J^{\mu} J^{\alpha} \rangle \]

[Ammon, Kaminski et al.; JHEP (2017)]

[Abbasi et al.; PLB (2016)]

[Kalaydzhyan, Murchikova; NPB (2016)]

vector modes under SO(2) rotations around \( B \)

\[
\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}
\]

former momentum diffusion modes

\[
\begin{align*}
\delta_0 &= s_0 / n_0 \\
\tilde{c}_P &= T_0 (\partial s / \partial T)_P
\end{align*}
\]
Hydro result: hydrodynamic poles

Weak B hydrodynamics - poles of 2-point functions

\[ \langle T^{\mu\nu} T^{\alpha\beta} \rangle, \langle T^{\mu\nu} J^\alpha \rangle, \langle J^\mu T^{\alpha\beta} \rangle, \langle J^\mu J^\alpha \rangle \]

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\text{former momentum diffusion modes}
\]

\[
\frac{s_0}{s_0/n_0} = c_p = T_0 (\partial s/\partial T)_P
\]

scalar modes under SO(2) rotations around \( B \)

\[
\omega_0 = v_0 k - i D_0 k^2 + O(\partial^3) \\
\text{former charge diffusion mode}
\]

\[
\omega_+ = v_+ k - i \Gamma_+ k^2 + O(\partial^3)
\]

\[
\omega_- = v_- k - i \Gamma_- k^2 + O(\partial^3) \\
\text{former sound modes}
\]
Hydro result: hydrodynamic poles

Weak B hydrodynamics - poles of 2-point functions:
\[ \langle T^{\mu\nu} T^{\alpha\beta} \rangle, \langle T^{\mu\nu} J^\alpha \rangle, \langle J^\mu T^{\alpha\beta} \rangle, \langle J^\mu J^\alpha \rangle \]

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
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Vector modes under SO(2) rotations around \( B \):

\[
\omega = \mp \frac{Bn_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + \frac{Bn_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}
\]

former momentum diffusion modes

\[
\text{s}_0 = s_0/n_0
\]
\[
\tilde{c}_P = T_0(\partial s/\partial T)_P
\]

Scalar modes under SO(2) rotations around \( B \):

\[
\omega_0 = v_0 k - i D_0 k^2 + O(\partial^3)
\]
former charge diffusion mode

\[
\omega_+ = v_+ k - i \Gamma_+ k^2 + O(\partial^3)
\]
former sound modes

\[
\omega_- = v_- k - i \Gamma_- k^2 + O(\partial^3)
\]

\[ \Rightarrow \text{a chiral magnetic wave} \]
[Kharzeev, Yee; PRD (2011)]

\[
v_0 = \frac{2BT_0}{\tilde{c}_p n_0} \left( \tilde{C} - 3C s_0^2 \right)
\]

\[
D_0 = \frac{w_0^2 \sigma}{\tilde{c}_p n_0^3 T_0}
\]

\[ \Rightarrow \text{dispersion relations of hydrodynamic modes are heavily modified by anomaly and } B \]
Fluctuations around charged magnetic black branes

- Weak $B$: holographic results are in “agreement” with hydrodynamics.

- Strong $B$: holographic result in agreement with thermodynamics, and numerical result indicates that chiral waves propagate at ... the speed of light and without attenuation

confirming conjectures and results in probe brane approach

[Ammon, Kaminski et al.; JHEP (2017)]

[Kharzeev, Yee; PRD (2011)]
Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes

- Weak $B$: holographic results are in “agreement” with hydrodynamics.
- Strong $B$: holographic result in agreement with thermodynamics, and numerical result indicates that chiral waves propagate at the speed of light and without attenuation

$\omega_0 = v_0 k - i D_0 k^2 + O(\partial^3)$

$\omega_+ = v_+ k - i \Gamma_+ k^2 + O(\partial^3)$

$\omega_- = v_- k - i \Gamma_- k^2 + O(\partial^3)$

RECALL: weak B hydrodynamic poles

confirming conjectures and results in probe brane approach

[Kharzeev, Yee; PRD (2011)]
Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes

- Weak $B$: **holographic results are in “agreement” with hydrodynamics.**

- Strong $B$: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at ...** the speed of light and without attenuation

**RECALL: weak B hydrodynamic poles**

\[
\omega_0 = v_0 k - i D_0 k^2 + \mathcal{O}(\partial^3) \\
\omega_+ = v_+ k - i \Gamma_+ k^2 + \mathcal{O}(\partial^3) \\
\omega_- = v_- k - i \Gamma_- k^2 + \mathcal{O}(\partial^3)
\]

confirming conjectures and results in probe brane approach

[Kharzeev, Yee; PRD (2011)]
Contents

✓ Hydrodynamics 2.0
   (near equilibrium)

✓ Holography

✓ Results for charged chiral plasma
   [Ammon, Kaminski, Koirala, Leiber, Wu; JHEP (2017)]

4. Far-From Equilibrium

5. Conclusions
Holographic thermalization

Thermalization:

T=0 particle “soup”

\[ \text{thermal QFT} \]

\[ \text{nonzero T plasma} \]
Holographic thermalization

Thermalization:

T=0 particle “soup”

correspondence

thermal QFT

nonzero T plasma

radial AdS coordinate

black hole

Anti-de Sitter space boundary

thermal QFT

cf. Casey Cartwright’s talk
Holographic thermalization

Thermalization:
- T=0 particle “soup”
- Thermal QFT
- Nonzero T plasma

Horizon formation:
- T=0 QFT
- Radial AdS coordinate
- Black hole
- Thermal QFT

Correspondence

cf. Casey Cartwright’s talk

Horizon: Anti-de Sitter space boundary
Holographic thermalization

Thermalization:

T=0 particle “soup”

thermal QFT

nonzero T plasma

correspondence

Horizon formation:

T=0 QFT

radial AdS coordinate

black hole

thermal QFT

Anti-de Sitter space boundary

cf. Casey Cartwright’s talk

Horizon formation:
Holographic thermalization

Thermalization:

- T=0 particle “soup”
- thermal QFT
- nonzero T plasma

Horizon formation:

- horizon formation: black hole
- T=0 QFT
- radial AdS coordinate
- black hole
- thermal QFT
- Anti-de Sitter space boundary

Correspondence:

- cf. Casey Cartwright’s talk
- cf. Duvier Suarez Fontanella’s talk
- T=0 particle “soup”
- thermal QFT
- nonzero T plasma

cf. Casey Cartwright’s talk

cf. Duvier Suarez Fontanella’s talk
Holographic thermalization

Thermalization:
- T=0 particle “soup”
- thermal QFT
- nonzero T plasma

Horizon formation:
- correspondence
- T=0 QFT
- radial AdS coordinate
- black hole
- thermal QFT

Anti-de Sitter space boundary

Thermalization:

Holographic thermalization

Horizon formation:

Thermalization:

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Holographic thermalization

Horizon formation:
Colliding shock waves in AdS

\[
\begin{align*}
0 &= \Sigma^2 \left[ F'' - 2(d_3 B)' - 3B'd_3 B + 4\Sigma'd_3 \Sigma, \\
&\quad - \Sigma \left[ 3\Sigma'F'' + 4(d_3 \Sigma)' + 6B'd_3 \Sigma \right],
\end{align*}
\]

\[
0 = \Sigma^4 \left[ A'' + 3B'd_3 B + 4 - 12\Sigma^2 \Sigma'd_3 \Sigma + e^{2B}\left\{ \Sigma^2 \left[ \frac{1}{2}(F')^2 - \frac{1}{2}(d_3 B)^2 - 2d_3^2 B \right] \\
&\quad + 2(d_3 \Sigma)^2 - 4\Sigma \left[ 2(d_3 B)d_3 \Sigma + d_3^2 \Sigma \right] \right\} \right.
\]

\[
0 = 6\Sigma^4 (d_3 \Sigma)' + 12\Sigma^2 (\Sigma'd_3 \Sigma - \Sigma^2) - e^{2B} \left\{ 2(d_3 \Sigma)^2 \\
&+ \Sigma^2 \left[ \frac{1}{2}(F')^2 + 2(d_3 F)' + 2F'd_3 B - \frac{1}{2}(d_3 B)^2 - 2d_3^2 B \right] \\
&+ \Sigma \left[ (F'' - 8d_3 B) d_3 \Sigma - 4d_3^2 \Sigma \right] \right\}.
\]

\[
0 = 6\Sigma^4 (d_3 B)' + 9\Sigma^3 (\Sigma'd_3 B + B'd_3 \Sigma) \\
&+ e^{2B} \left\{ \Sigma^2 \left[ (F')^2 + 2(d_3 F)' + 2F'd_3 B - (d_3 B)^2 - 2d_3^2 B \right] \\
&+ 4(d_3 \Sigma)^2 - 4\Sigma \left[ (4F' + d_3 B) d_3 \Sigma + 2d_3^2 \Sigma \right] \right\},
\]

\[
0 = 6\Sigma^2 (d_3 \Sigma)'.
\]

\[
0 = \Sigma \left\{ 2d_3 (d_3 \Sigma) + 2d_3 (d_3 \Sigma) + 3F'd_3 \Sigma \right\} \\
&+ \Sigma^2 \left[ d_3 (F') + d_3 (A') + 4d_3 (d_3 B) - 2d_3 (d_3 B) \right]
\]

cf. Burkhard Kämpfer’s talk

[Chesler, Yaffe; PRL (2011)]
[Janik; PRD (2006)]
[Fuini, Yaffe; (JHEP) 2015]
[Cartwright, Kaminski; work in progress]
Colliding shock waves in AdS

Method: numerical computation in gravity

holographic model

cf. Burkhard Kämpfer’s talk

[Chesler, Yaffe; PRL (2011)]
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Colliding shock waves in AdS

Method: numerical computation in gravity

hydro works too well too early!

2-pt functions? cf. Casey Cartwright’s talk

[Chesler, Yaffe; PRL (2011)]
[Janik; PRD (2006)]
[Fuini, Yaffe; (JHEP) 2015]
[Cartwright, Kaminski; work in progress]
Far-from equilibrium in hydrodynamics?

• hydrodynamics describes pressures much **earlier than expected** (lesson from holographic thermalization)

• similar effects in Bjorken flow **numerical hydro calculations**

• hydrodynamic expansion in gradients is asymptotic — resummation reveals analogies to QFT expansion in Planck’s constant, addressed by **resurgence**

• hydrodynamics may be rewritten with different fields, in order to describe far from equilibrium dynamics

[Romatschke; (2017)]
5. Conclusions
Things for which there was no time …

➡ transport coefficients and correlators
  [Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; work in progress]

➡ magnetohydrodynamics (dynamic B)
  [Hernandez, Kovtun; JHEP (2017)]
  [Grozdanov, Hofman, Iqbal; PRD (2017)]
  [Hattori, Hirono, Yee, Yin; (2017)]

➡ axial and vector current
  [Landsteiner, Megias, Pena Benitez; PRD (2014)]
  [Ammon, Grieninger, Jimenez-Alba, Macedo, Melgar; JHEP (2016)]
Holography: Fluid/gravity correspondence

see appendix

Perturbing the surface of a black hole.
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD.

Gravity dual to QCD or standard model? Not known yet.

Solve problems in effective field theory (EFT), e.g.:
- Hydrodynamic approximation of original theory
- Hydrodynamic approximation of model theory

(Hard) problem in “similar” model theory.

Simple problem in a particular gravitational theory.

QFT

Gravity

Hydrodynamics
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD

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Solve problems in effective field theory (EFT), e.g.:
- hydrodynamic approximation of original theory
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(Hard) problem in “similar” model theory

model

holography (gauge/gravity correspondence)

Simple problem in a particular gravitational theory
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example **QCD**

- gravity dual to QCD or standard model?
- not known yet

**model**

**holography** (gauge/gravity correspondence)

(Hard) problem in “similar” model theory

Simple problem in a particular gravitational theory

Holography is good at predictions that are **qualitative** or **universal**.

- **Compare** holographic result to hydrodynamics of model theory.
- **Compare** hydrodynamics of original theory to hydrodynamics of model.
- **Understand** holography as an **effective description**.
Methods: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example QCD

⇒ gravity dual to QCD or standard model?
⇒ not known yet

Holography is good at predictions that are qualitative or universal.

⇒ Compare holographic result to hydrodynamics of model theory.
⇒ Compare hydrodynamics of original theory to hydrodynamics of model.
⇒ Understand holography as an effective description.

REALITY CHECK: MODEL APPROPRIATE?

CONSISTENCY CHECK

Solve problems in effective field theory (EFT), e.g.:

hydrodynamic approximation of original theory
Thanks to collaborators

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Regensburg University, Germany

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University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA

Dr. Jackson Wu

Dr. Julian Leiber

Sebastian Grieninger

Prof. Dr. Martin Ammon

Roshan Koirala

Casey Cartwright

Matthias Kaminski

Lecture: Far-from-equilibrium dynamics in magnetic charged chiral plasma
APPENDIX
Fluid/gravity correspondence

\[\begin{align*}
\text{Einstein equations} & = \text{hydrodynamic conservation equations} + \text{dynamical equations of motion} \\
\text{ Constitutive equations from geometry near boundary.}
\end{align*}\]
Fluid/gravity correspondence

Conservation equations from gravity

5-dimensional Einstein-Maxwell-Chern-Simons equations of motion:

\[ R_{MN} + 4g_{MN} = \frac{1}{2} F_{MK} F^K_N - \frac{1}{12} g_{MN} F^2 \]

\[ \partial_N (\sqrt{-g} F^{NM}) = \frac{1}{4\sqrt{3}} \epsilon^{MNPQR} F_{NO} F_{PQ} \]

\( \xi_N = dr \)

Constraint equations arise from contraction with one-form \( dr \) (normal to boundary):

\((\text{contraints})_M = \xi^N \text{(Einstein equations)}_{MN}\)

\((\text{contraint}) = \xi^N \text{(Maxwell – Chern – Simons equations)}_N\)

Constitutive equations from gravity

Example: no matter content, vanishing gauge fields:

\[ \langle T_{\mu\nu} \rangle = \lim_{r \to \infty} \left[ \frac{r^{(D-3)}}{\kappa_D^2} (K_{\mu\nu} - K \gamma_{\mu\nu} - (D-2) \gamma_{\mu\nu}) \right] \]

with extrinsic curvature

\[ K_{\mu\nu} = -\frac{1}{2n} (\partial_r \gamma_{\mu\nu} - \nabla_\mu n_\nu - \nabla_\nu n_\mu) \]

\[ ds^2 = n^2 dr^2 + \gamma_{\mu\nu} (dx^\mu + n^\mu dr)(dx^\nu + n^\nu dr) \]
Example: $N=4$ Super-Yang-Mills with anomaly

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

Gravity dual: 5-dimensional Einstein-Maxwell-Chern-Simons action

\[
S = -\frac{1}{2\kappa_5^2} \int \left[ \sqrt{-g} \left( R + 12 - \frac{1}{4} F^2 \right) - \frac{1}{12\sqrt{3}} \epsilon^{MNOPQ} A_M F_{NO} F_{PQ} \right] d^4 x \, dr
\]
Example: $N=4$ Super-Yang-Mills with anomaly

[Ernmerger, Haack, Kaminski, Yarom; JHEP (2009)]

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CS-term dual to chiral anomaly
Example: $N=4$ Super-Yang-Mills with anomaly

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CS-term dual to chiral anomaly

Black hole with R-charge (in Eddington-Finkelstein coordinates):

$$ds^2 = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 \Delta_{\mu\nu} dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$

solution with constant parameters $Q, b, u^\mu$.

$$f(r) = 1 + \frac{Q^2}{r^6} - \frac{1}{b^4 r^4} \quad A_r = 0 \quad A_\mu = -\frac{\sqrt{3}Q}{2} u_\mu \quad \Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$
Example: \( N=4 \) Super-Yang-Mills with anomaly

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**CS-term dual to chiral anomaly**

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\]

\[\Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu\]

Make parameters boundary-coordinate-dependent: \( b \rightarrow b(x), \quad Q \rightarrow Q(x), \quad u^\mu \rightarrow u^\mu(x) \)
Example: \( N=4 \) Super-Yang-Mills with anomaly

\[ [\text{Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)}] \]

Gravity dual: 5-dimensional Einstein-Maxwell-Chern-Simons action

\[
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\]

**CS-term dual to chiral anomaly**

Black hole with R-charge (in Eddington-Finkelstein coordinates):

\[
ds^2 = -r^2 f(r) u_\mu u_\nu \, dx^\mu \, dx^\nu + r^2 \Delta_{\mu\nu} dx^\mu \, dx^\nu - 2 u_\mu dx^\mu \, dr
\]

solution with constant parameters \( Q, b, u^\mu \).

\[
f(r) = 1 + \frac{Q^2}{r^6} - \frac{1}{f^2 A r^4} \quad A_r = 0 \quad A_\mu = -\frac{\sqrt{3}Q}{r^2} u_\mu \quad \Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu
\]

Make parameters boundary-coordinate-dependent:

\[ b \rightarrow b(x), \quad Q \rightarrow Q(x), \quad u^\mu \rightarrow u^\mu(x) \]

\( \text{dual to hydrodynamic fields} \)

• expand in gradients of \( b, Q \) and \( u \)

**dual to hydrodynamic expansion in the field theory**

• new analytical solutions to Einstein equations

**give values of transport coefficients in field theory**
Proofs of Gauge/Gravity Correspondences
- Some examples

- Three-point functions of N=4 Super-Yang-Mills theory
- Conformal anomaly of the same theory
- RG flows away from most symmetric case
- ... many other symmetric instances of the correspondence
Evidence for Gauge/Gravity

-Reasonable example results from Gauge/Gravity!
Evidence for Gauge/Gravity

-Reasonable example results from Gauge/Gravity!

- Compute observables in strongly coupled QFTs
Evidence for Gauge/Gravity
-Reasonable example results from Gauge/Gravity!

- Compute observables in strongly coupled QFTs
- **Meson spectra/melting**, glueball spectra
Evidence for Gauge/Gravity
-Reasonable example results from Gauge/Gravity!

- Compute observables in strongly coupled QFTs
  - Meson spectra/melting, glueball spectra
  - Quark energy loss, Jets
Evidence for Gauge/Gravity

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- **Hydrodynamics** (beyond Muller-Israel-Stewart), **chiral effects**
Evidence for Gauge/Gravity
-Reasonable example results from Gauge/Gravity!

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- **Meson spectra/melting**, glueball spectra
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- **Thermodynamics/Phase diagrams**
- **Hydrodynamics** (beyond Muller-Israel-Stewart), chiral effects
- Transport coefficients (e.g. ‘universal’ viscosity bound)
Evidence for Gauge/Gravity

-Reasonable example results from Gauge/Gravity!

- Compute observables in strongly coupled QFTs
- **Meson spectra/melting**, glueball spectra
- Quark energy loss, Jets
- **Thermodynamics/Phase diagrams**
- **Hydrodynamics** (beyond Muller-Israel-Stewart), **chiral effects**
- Transport coefficients (e.g. ‘universal’ viscosity bound)
- **Deconfinement & Break**: Chiral, Conformal, SUSY
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- [AdS/QCD (bottom-up approach) distinct from string constr.]
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Gauge/Gravity is a Powerful Tool

- non-perturbative results, strong coupling
- treat many-body systems
- direct computations in real-time thermal QFT (transport)
- no sign-problem at finite charge densities
- methods often just require solving ODEs in classical gravity
- quick numerical computations (~few seconds on a laptop)
  (turn around: study strongly curved gravity)
Outline: Gauge/gravity correspondence

-Why does it work?

Two distinct ways to describe this stack:

Stack of Nc D3-branes (coincident) in 10 dimensions

4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side

AdS$_5 \times S^5$ near-horizon geometry (e.g. Supergravity)

gravity side
Outline: Gauge/gravity correspondence

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**Stack of Nc D3-branes** (coincident) in 10 dimensions

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- 4-dimensional worldvolume theory on the D3-branes (e.g. \( N = 4 \) Super-Yang-Mills)
  - **gauge side**

- **AdS\(_5\) x S\(^5\)** near-horizon geometry (e.g. Supergravity)
  - **gravity side**

-How does it work?

**Add/change geometric objects on ‘gravity side’:**

Geometric setup: Strings/Branes → Find solution configuration → Field Theory result

Example: Schwarzchild radius corresponds to temperature
Simple example: Eigenfrequencies / normal modes of the quantum mechanical harmonic oscillator (no damping)

\[ \omega_n = \frac{1}{2} + n \]
Quasi Normal Modes (QNMs)

Example: Poles of charge current correlator

- QNMs are the quasi-eigenmodes of gauge field
- Dual QFT: lowest QNM identified with hydrodynamic diffusion pole (not propagating)
- Higher QN modes: gravity field waves propagate through curved b.h. background while decaying (dual gauge currents analogously)

\[ G_{ret} \propto \frac{1}{i\omega - Dq^2} \]
Quasi Normal Modes (QNMss)

Complex frequency plane

Trajectories (dial k)

Instabilities
Chiral effects in vector and axial currents

Vector current (e.g. QCD $U(1)$)

\[
J^\mu_V = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu
\]

chiral magnetic effect

Axial current (e.g. QCD axial $U(1)$)

\[
J^\mu_A = \cdots + \xi_\omega \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu
\]

chiral vortical separation effect
Chiral effects in vector and axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD $U(1)$)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial $U(1)$)

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chiral vortical effect

chiral separation effect

Matthias Kaminski

Lecture: Far-from-equilibrium dynamics in magnetic charged chiral plasma
Chiral effects in vector and axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

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chiral vortical effect chiral separation effect
Full chiral vortical effect & gravity

\[ \xi = C \left( \mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right) + \ldots \]
More than one anomalous current

$$
\xi = C \left( \mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right) + \ldots
$$

$$\nabla_{\nu} J^\nu_a = \frac{1}{8} C_{abc} \epsilon^{\nu \rho \sigma \gamma} F^b_{\nu \rho} F^c_{\sigma \gamma}
$$

$$
\xi_a = C_{abc} \mu^b \mu^c + 2\beta_a T^2 - \frac{2n_a}{\epsilon + p} \left( \frac{1}{3} C_{bcd} \mu^b \mu^c \mu^d + 2\beta_b \mu^b T^2 + \gamma T^3 \right)
$$

[Neiman, Oz; JHEP (2010)]
Full chiral vortical effect & gravity

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various charges (e.g. axial, vector)
previously neglected

\[ \beta = -4\pi^2 c m \]

[Neiman, Oz; JHEP (2010)]

[Jensen, Loganayagam, Yarom; (2012)]
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Various charges (e.g. axial, vector) previously neglected

\[ \beta = -4\pi^2 c m \]

Gravitational anomalies

\[ \nabla_\nu T^\mu_\nu_{\text{cov}} = F^\mu_\nu J^\nu_{\text{cov}} + \frac{c_m}{2} \nabla_\nu \left[ \epsilon^{\rho \sigma \alpha \beta} F_{\rho \sigma} R^{\mu \nu}_{\alpha \beta} \right] \]

Formal approach guarantees completeness

full transport coefficient exactly known; first measurement of gravitational anomaly?