Towards a proper modeling of anisotropic magnetized compact objects

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Outline

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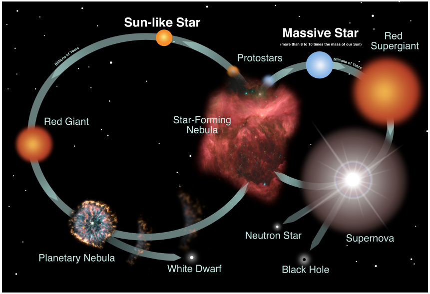
- Precedents
- Anistropic model for Compact Objects
- Preliminary results for White Dwarfs
- Summary and Future Work

Motivation

Compact Objects

- Final stage in the evolution of ordinary stars.
- Formed when thermonuclear fusion cannot compensate the gravitational collapse.

White Dwarfs	Neutron Stars
$M \sim M_\odot$	$M \sim 2M_\odot$
$R \sim 10^3 km$	$R \sim 10 km$
$B < 10^{13} G$	$B \sim 10^{18} G$
$M \sim R^{-3}$	$M \sim R^{-3}$
$\rho < 10^{11} \frac{g}{cm^3}$	$\rho < 10^{15} \frac{g}{cm^3}$



Motivation

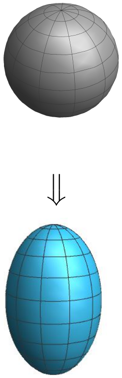
Constant magnetic field effects on a fermion gas:

- Anisotropic energy-momentum tensor

$$T_{\mu\nu} = \mathrm{diag}(E, P_{\perp}, P_{\perp}, P_{\parallel})$$

- Anisotropic equation of state.

$$E = \Omega + \mu N$$
$$P_{\parallel} = -\Omega$$
$$P_{\perp} = -\Omega - B\mathcal{M}$$



Spherical symmetry is broken.

Precedents

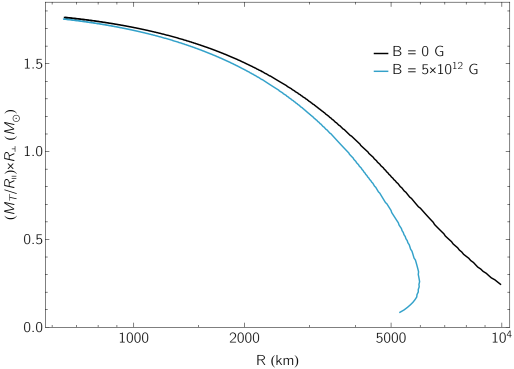
- Metric in cylindrical coordinates

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\phi^2 + e^{2\Psi(r)}dz^2$$

- Mass by Tolman definition

$$\frac{M_T}{R_\parallel} = 4\pi \int_0^{R_\perp} r e^{\Phi(r)+\Lambda(r)+\Psi(r)}(E - 2P_\perp - P_\parallel)dr$$

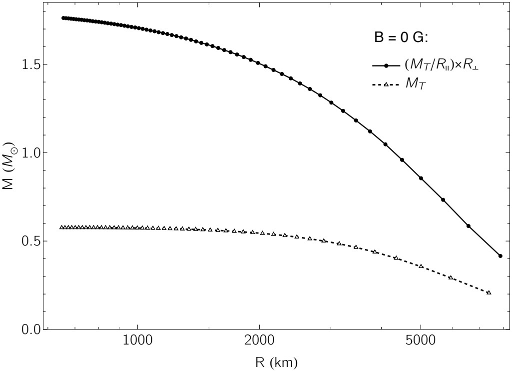
D. Manreza Paret et.al., RAA **15**, 2015.



Precedents

- Metric in cylindrical coordinates
- Ansatz: $\frac{R_\parallel}{R_\perp} = \frac{P_\parallel}{P_\perp}$
- Mass by Tolman definition

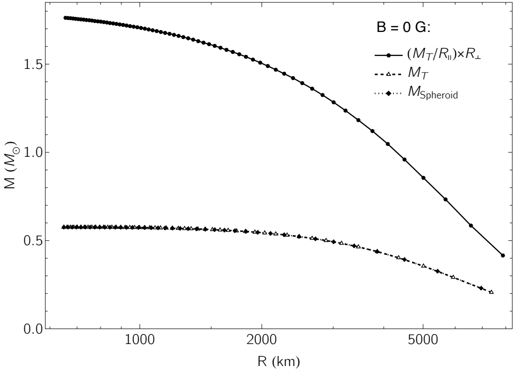
$$M_T = 4\pi \int_0^{R_\perp} \int_0^{r\frac{P_\parallel}{P_\perp}} r e^{\Phi(r)+\Lambda(r)+\Psi(r)} (E - 2P_\perp - P_\parallel)\, dzdr$$



Precedents

- Metric in cylindrical coordinates
- Ansatz: $\gamma = \frac{P_{\|_0}}{P_{\perp_0}} = \frac{z}{r}$
- Spheroid mass

$$\frac{dM}{dr} = 4\pi r^2 E\gamma$$



Anisotropic model for Compact Objects

- γ Metric in spherical coordinates (arXiv:gr-qc/9810079v1; Omair Zubairi et al 2015 J. Phys.: Conf. Ser. 615 012003)

$$ds^2 = -\left(1 - \frac{2M(r)}{r}\right)^{\gamma} dt^2 + \left(1 - \frac{2M(r)}{r}\right)^{-\gamma} dr^2 + r^2 \sin\theta d\phi^2 + r^2 d\theta^2$$

- Ansatz: $\gamma = \frac{P_{\parallel 0}}{P_{\perp 0}} = \frac{z}{r}$
- Structure equations:

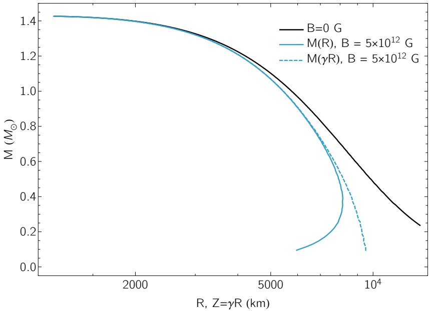
$$\frac{dM}{dr} = r^2 E \gamma$$

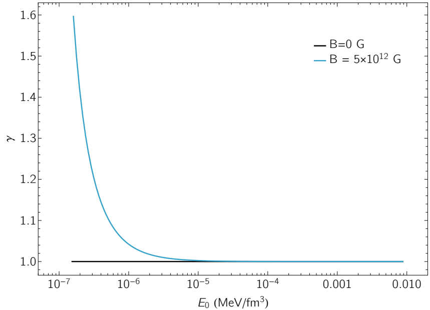
$$\frac{dP_{\parallel}}{dr} = -\frac{(E + P_{\parallel})[\frac{r}{2} + r^3 P_{\parallel} - \frac{r}{2}(1 - \frac{2M}{r})^{\gamma}]}{r^2(1 - \frac{2M}{r})^{\gamma}}$$

$$\frac{dP_{\perp}}{dz} = \frac{1}{\gamma}\frac{dP_{\perp}}{dr} = -\frac{(E + P_{\perp})[\frac{r}{2} + r^3 P_{\perp} - \frac{r}{2}(1 - \frac{2M}{r})^{\gamma}]}{\gamma r^2(1 - \frac{2M}{r})^{\gamma}}$$

Reduces to TOV equations at $P_{\parallel} \equiv P_{\perp}$ ($B = 0$ case)

Preliminary results for White Dwarfs





- Magnetic fields deform the star.
- The effects become important in the low density regime.

Summary and Future Work

- We have given the first steps towards a generalized structure equations that can account for the anisotropies present in magnetized compact stars and reduce to the standard TOV equations when $B = 0$.
- Magnetic fields seem to affect the shape of the star, and therefore, the radii, but not the maximum mass.

The near future:

- Study stability in more detail.
- Obtain solutions for different types of compact stars.
- Set $\gamma = P_{\parallel}/P_{\perp}$ varying throughout the star.

Structure equations in cylindrical coordinates

- Metric in cylindrical coordinates

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\phi^2 + e^{2\Psi(r)}dz^2$$

- Mass by Tolman definition

$$M_T = \int \sqrt{-g}(T_0^0 - T_1^1 - T_2^2 - T_3^3)dV$$

- Einstein equations

$$P'_\perp = -\Phi'(E + P_\perp) - \Psi'(P_\perp - P_\parallel),$$

$$4\pi e^{2\Lambda}(E + P_\parallel + 2P_\perp) = \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r},$$

$$4\pi e^{2\Lambda}(E + P_\parallel - 2P_\perp) = -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r},$$

$$4\pi e^{2\Lambda}(P_\parallel - E) = \frac{1}{r}(\Psi' + \Phi' - \Lambda').$$