Magnetized vacuum pressures

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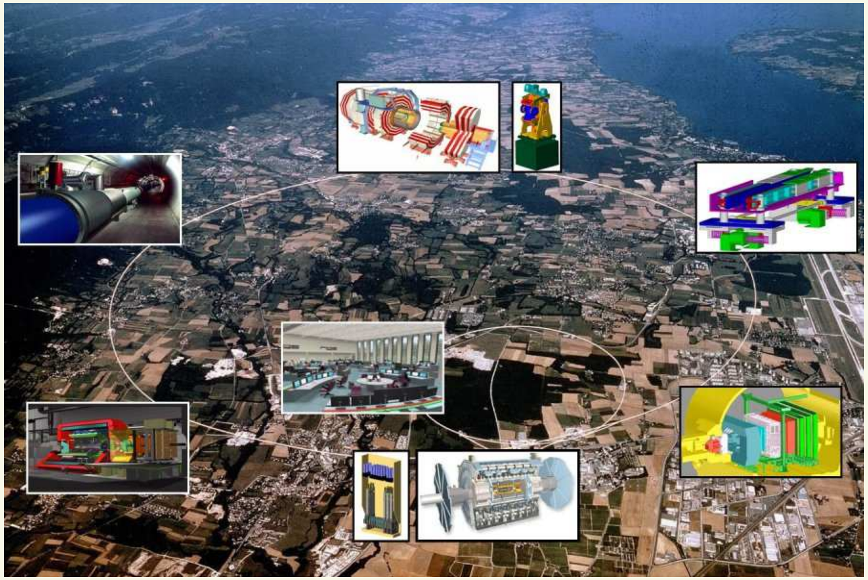
Outline

- Introduction
- Magnetized $e^- - e^+$ vacuum pressure
- Photon propagation and withdraw of momentum orthogonal to **B**
- The effect on the total pressure
- The unstable vacuum

Motivation

Heavy Ion Colliders

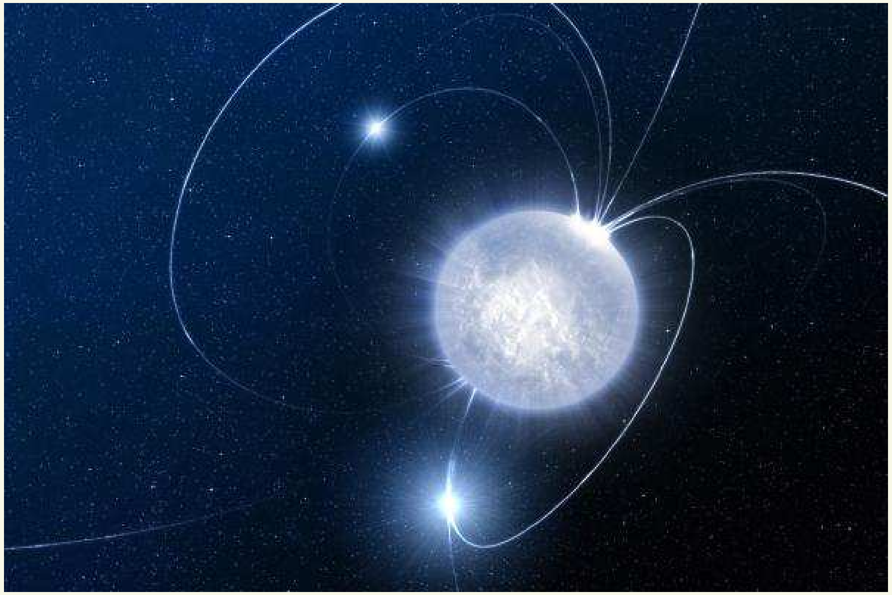
$B \sim 10^{19}$ G



A Large Ion Collider Experiment (ALICE) at LHC

Magnetars

$B \sim 10^{13}$ G at the surface



Introduction

- There is an analogy among certain boundary conditions and the effect of external fields. Some boundary conditions lead to new physical effects as it is for instance the well-known Casimir effect [H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948)].

- We may understand it phenomenologically as photons colliding elastically with the boundary plates, getting negative momentum in each collision, which we may understand as a manifestation of a sort of negative pressure, similarly as a gas contained in a rigid box, which acts on a plane.

- In a previous work, by starting from the Heisenberg-Euler expression for the vacuum energy in a constant magnetic field **B** [W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936)], we obtained the arising of negative pressure orthogonal to it. In this case, for a homogeneous field the negative pressure acts at each point with rotational symmetry around any straight line parallel to **B** passing through it.

Magnetized $e^- - e^+$ vacuum pressure

- There is a negative pressure in magnetized vacuum in the direction perpendicular to the field $\mathbf{B}$ [HPR, IJMPA 21, 3761 (2006)].

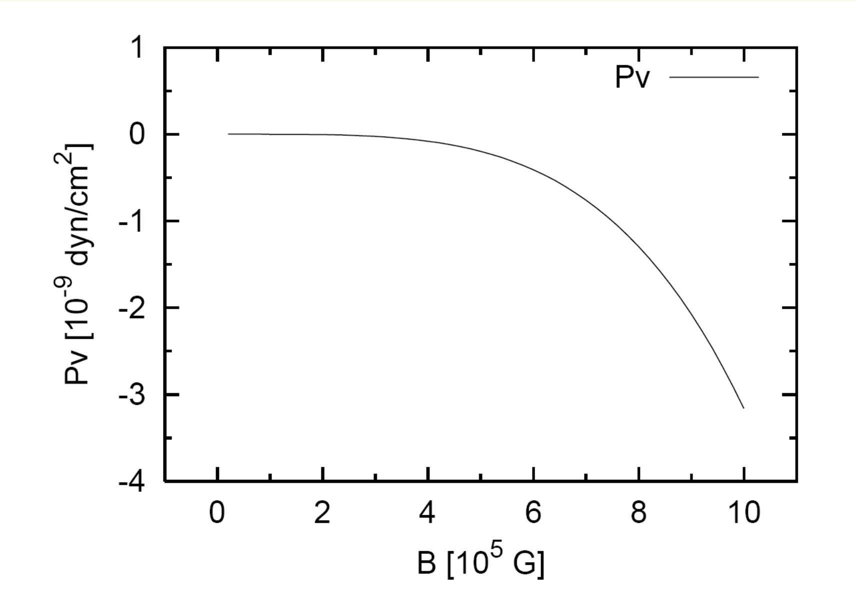


Figure 1: $P_\perp(B)$

- $B \ll B_c = m^2c^3/e\hbar \sim 4.41 \times 10^{13}$ G: $P_\perp \approx -\frac{\alpha B^4}{120\pi^2 B_c^2}$, $P_\| \approx \frac{\alpha B^4}{360\pi^2 B_c^2}$,
- For fields $B \sim 4.5 \times 10^5$ G: $P_\perp - 1.35 \times 10^{-9} dyn\ cm^{-2}$, $P_\| \simeq 4.6 \times 10^{-10} dyn\ cm^{-2}$, $P_\perp$ is larger than Casimir pressure few orders of magnitude. For instance, for a distance between plates $d = 0.1cm$, it gives $P_C \sim -10^{-14} dyn\ cm^{-2}$.

Magnetized $e^- - e^+$ vacuum pressure

- Physically the negative pressure can be understood since the quantity corresponding to the classical orbit radius $r_0 = \sqrt{\hbar c/eB}$ decreases with increasing B, thus the increase in B leads to a contraction of the orbit radius.
- We conclude that pressure exerted on a body by magnetized quantum vacuum stretches it along the field and contracts it perpendicularly as B increases.
- Contribution from heavier particles than the electron to quantum vacuum pressure are obviously less intense, since their masses lead to critical fields greater than B_c.

Photon propagation and withdraw of momentum orthogonal to B

The withdraw of momentum orthogonal to the magnetic field is easily seen from the dispersion equation for a photon propagating in magnetized vacuum.

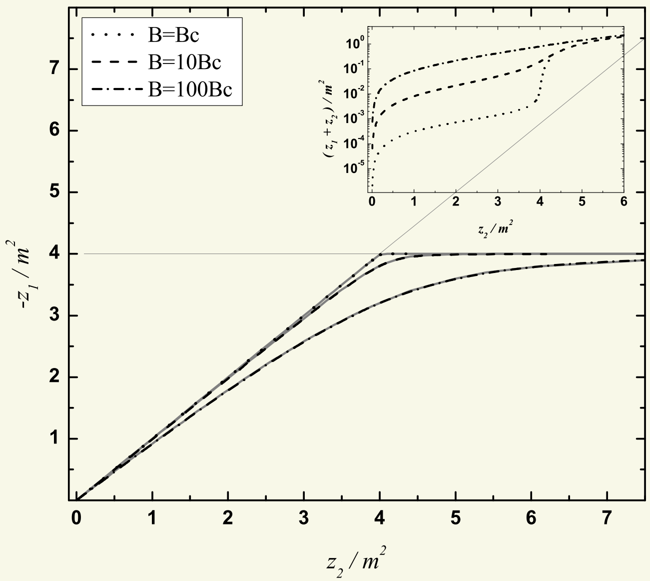


Figure 2: Dispersion Eq. for the second mode ($z_1 = k_{\parallel}^2 - \omega^2$, $z_2 = k_{\perp}^2$)

$B \ll B_c$:

- In the previously considered limit $B \ll B_c$,

$$\omega^{i2} - k_{\parallel}^2 = k_{\perp}^2 \left(1 - \frac{C^i \alpha b^2}{45\pi}\right), \; b = B/B_c, \tag{1}$$

  $C^i = 7, 4$ for $i = 2, 3$, corresponding respectively to photon plane polarization along and orthogonal to B.

- The last expression is the dispersion equation in presence of the magnetic field for an incoming photon which far from the magnetized region, satisfies the usual light cone equation $\omega_0^2 = k_{\parallel}^2 + k_{\perp}^2$.

- In other words, the effect of the magnetic field is to decrease the incoming transverse momentum $k_{\perp}$ to a value $k'_{\perp} = (1 - C^i \alpha b^2/45\pi)k_{\perp} < k_{\perp}$, and in consequence, the initial photon squared energy (frequency) is also decreased.

Photon propagation and withdraw of momentum orthogonal to B

- If the photon moves perpendicular to B,

$$k_{||} = 0, \qquad \text{and} \qquad \omega^{2\prime} = k_{\perp}^{2\prime}. \tag{2}$$

- Interestingly, if it is reflected perpendicularly by a half-silvered mirror, keeping its plane polarization, its dispersion equation is the light cone one $\omega' = k'_{\perp}$.

- We have, by calling $\Delta\omega^i = \omega^{i\prime} - \omega^i$

$$\Delta\omega^i = -\frac{C^i \alpha b^2}{90\pi}\omega^i \tag{3}$$

- Assuming $B \sim 4.5 \times 10^5$ G, we get for $i = 2$, $\Delta\omega^i \sim -10^{-20}\omega^i$.

The pendant task is to design an appropriate experiment to detect this effect. We consider that interferometry could be used successfully to test it.

The effect on the total pressure

Let us assume laser light propagating perpendicular to **B**, with intensity $I = E_l^2 = B_l^2$.

- This intensity is equal to the beam pressure orthogonal to **B**, $I = B_l^2$.

- The total pressure exerted on the surface of a body placed orthogonal to the beam is

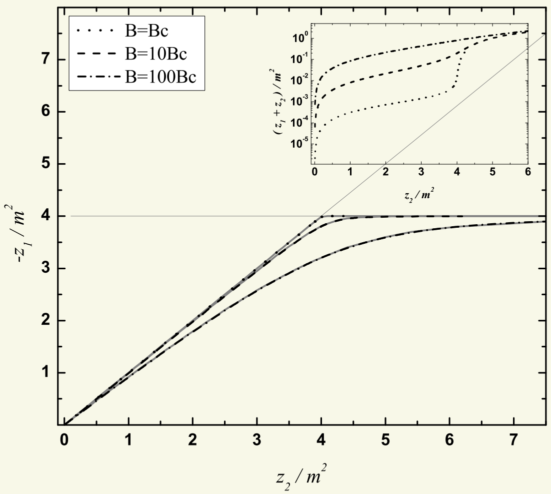
$$P = 2B_l^2 - \frac{\alpha B^2 b^2}{120\pi^2} = B_l^2[1 - \frac{\alpha b'^2 b^2}{120\pi^2}] \tag{4}$$

  where $b' = B/B_l$.

- This effect is expected to be observable on a solid body, or more properly, on the lattice formed by ions, atoms or molecules composing it. The vacuum in which it is immerse contracts orthogonal to the magnetic field for growing B_I. The parameter b' could be used (by varying B_I for instance), to produce a varying pressure. Its effect could be measured, for instance, by means of an appropriately sensitive piezoelectric device.

$B \sim B_c, \quad \omega \simeq 2m, \quad k_\parallel < \omega$:

$$z_1 = (\mathbf{k} \cdot \mathbf{B})^2/\mathbf{B}^2 - \omega^2 = k_\parallel^2 - \omega^2,$$
$$z_2 = (\mathbf{B} \times \mathbf{k})^2/\mathbf{B}^2 = k_\perp^2, \quad z_1 + z_2 = k_\mu k^\mu = k^2,$$



The dispersion equation for the second mode (having plane polarization parallel to B) may be written[a] as [Shabad, Ann. Phys. 90, 166 (1975)]

$$z_1 + z_2 = \frac{2\alpha eBme^{-z_2/2eB}}{\sqrt{z_1 + 4m^2}}. \tag{5}$$

This Eq. is valid in a neighborhood of $z_1 \lesssim -4m^2$.

[a] if the polarization operator is expressed as a sum over Landau levels n, n' of the virtual electron-positron pairs, in terms of the dominant term $n = n' = 0$

- Its limit for $\mathbf{k} \to \mathbf{0}$ is $\omega \neq 0$. Actually, it describes a massive vector boson particle closely related to the electron-positron pair (see below). This is not in contradiction with the gauge invariance property of the photon self energy.
- This dispersion equation has solutions found by Shabad as those of a cubic equation.
- One can estimate its behavior very near $z_1 = -4m^2$, by assuming $z_1 = -4m^2 + \epsilon$ and $z_2 = 4m^2 - \epsilon$, where ϵ is a small quantity. One can obtain the solution approximately as $(z_1 + 4m^2)^{3/2} = 2\alpha eBme^{-z_2/2eB}$, from which,

$$\omega^2 = \sqrt{k_\parallel^2 + 4m^2 - (2\alpha eBme^{-2m^2/eB})^{2/3}}. \tag{6}$$

- Thus, the transverse momentum of the original photon is trapped by the magnetized medium, the resulting quasi-particle being deviated to move along the field as a longitudinally polarized vector boson of mass

$$\omega_t = \sqrt{4m^2 - m^2(2\alpha be^{-2/b})^{2/3}} \tag{7}$$

- Notice that from the dispersion equation we observe that it and its solutions) become complex for $z_1 < -4m^2$, or equivalently $\omega^2 > k^2 + 4m^2$.

- Our approach is approximate. A more complete discussion would be made by following the method of [Shabad, Ann. Phys. 90, 166 (1975)], and a more exact one would require to consider the contribution from electromagnetic interaction terms among the electron-positron pairs, leading for instance to virtual positronium.

- However, the term depending on B substractive from the term $4m^2$ indicates the existence of a magnetic field intensity which reduces the effective mass to zero.

- It must be remarked that a transverse polarized wave propagating initially parallel to $\mathbf{B}$, continues moving in the same direction located on the light cone $\omega = k_{||}$. But the massive longitudinal mode, coexisting with an electron-positron pair, which is obviously paramagnetic differs totally from the transverse photons. For slightly larger energies such that $z_1 = -4m^2 - \epsilon$, and b of order unity, that is $B \sim B_c$, they decay in observable electron-positron pairs, and the polarized vacuum becomes absorptive.

- Thus, near the critical field B_c our problem bears some analogy to light passing near a black hole: for $r \simeq r_G$, the light is deviated enough to be absorbed by the black hole. Among other differences in both cases, it must be remarked that the gravitational field in black holes is usually centrally symmetric, whereas our magnetic field is axially symmetric. In the magnetic field case the resulting object is a massive longitudinally polarized vector boson.

- One can find an approximate expression for the energy of vacuum for fields around B_c, including radiative corrections, by starting from the general expression

$$\Omega = \frac{V}{2\pi^4}\int d^4k \int \frac{de}{e}\Pi_{\mu\nu}D_{\mu\nu}, \qquad (8)$$

which in our case may be reduced to a sum over the three eigenvalues of the polarization operator κ_i, $i = 1, 2, 3$, and more specifically, related to the case $i = 2$,

$$\begin{aligned} \Omega &= \frac{V}{2\pi^4}\int d^4k \int_0^e \frac{de'}{e'}\frac{\kappa_2}{k^2-\kappa_2} \qquad (9) \\ &= \frac{V}{2\pi^4}\int d^4k \ln(k^2-\kappa_2). \end{aligned}$$

- One would get for $D = k^2 - \kappa_2 = 0$, a logarithmic divergence. Mathematically , complex solutions appear from the solution of the dispersion equation which are related to the creation of electron positron pairs out of an incoming photon. But we have seen that for fields greater enough, the effective mass of the vector boson tends to zero, and this suggests the possibility of vacuum decay out of virtual photons of arbitrary small energy, in other words, of vacuum instability.

[HPR, Int. J. Mod. Phys. D 19, 1711 (2010)]

- As pointed out in [G. Quintero Angulo, A. Perez Martínez, H. Perez Rojas, Phys. Rev. C 96, 045810 (2017)] this instability is avoided in a magnetized neutral vector boson gas by self-magnetization at field intensities lower than the critical field B_c [M. Chaichian, S. Masood, C. Montonen, A. Perez Martinez, H. Perez Rojas, Phys. Rev. Lett. 84, 5261 (2000)].

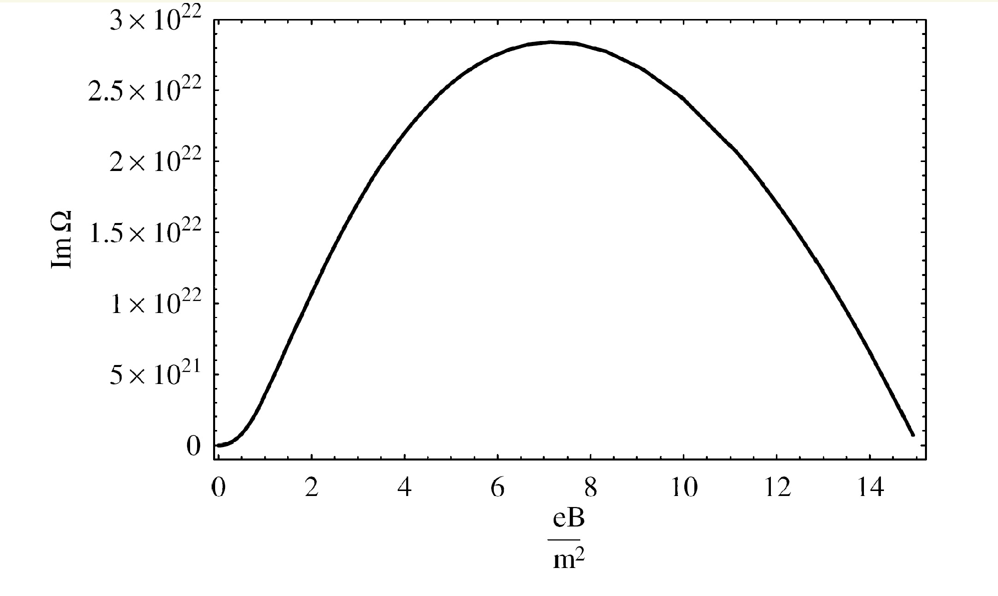


Figure 3: Imaginary part of vacuum energy