Energy Dependence and Fluctuations of Anisotropic Flow

Raimond Snellings
Heavy-Ion Collisions at the LHC

- What happens to matter when you heat and compress it to extreme magnitudes which existed in the primordial universe?
- Lattice QCD predicts a phase transition to a quark-gluon-plasma at an energy density of about 1 GeV/fm$^3$ and at a temperature of about $10^{12}$ K
- Temperatures $10^5$ larger than the core of the sun
- Magnetic fields of order $10^{18}$ Gauss (strongest magnetic fields known of order $10^8$ in the lab and $10^{15}$ in nature (magnetar))
Anisotropic Flow

- QCD in the strong coupling regime is complicated to calculate from first principles, pQCD only useful or small fraction of observables
- Measure properties of this hot and dense matter with respect to simpler systems p-p and p-A
- Use geometry as a control parameter
  - Anisotropic flow
- Can we constrain the transport parameters?
- Can we constrain the initial conditions?
A Single Collision
A Single Collision (coordinate space)

Simple Glauber Model Monte Carlo
Many Collisions versus the Reaction Plane

Simple Glauber Model Monte Carlo

![Graph showing the distribution of spectators and wounded nucleons with RMS values]

- **Spectators**
  - RMS x: 7.362 fm
  - RMS y: 3.318 fm

- **Wounded Nucleons**
  - RMS x: 2.42 fm
  - RMS y: 2.76 fm
Plane(s) of Symmetry (coordinate space)

Simple Glauber Model Monte Carlo
Plane(s) of Symmetry (coordinate space)

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Plane(s) of Symmetry (coordinate space)

Simple Glauber Model Monte Carlo

![Diagram showing planes of symmetry and corresponding mean and RMS values for x and y coordinates.](image-url)
Anisotropic Flow (momentum space)

\[
\frac{dN}{d\varphi} \propto 1 + 2 \sum_{n=1}^{+\infty} v_n \cos [n(\varphi - \Psi_n)],
\]

- Use geometry as a control parameter
- If the constituents interact they convert the coordinate space asymmetries into momentum space asymmetries
- The \(v_n\) coefficients provide information about the initial state anisotropies, the transport parameters and the EoS, and can be used to constrain them
  - Many parameters to constrain, need many different observables
- The energy dependence can be used to constrain the temperature dependence of these parameters on which the \(v_n\) depend the most
- Detailed measurements can constrain the p.d.f. of the \(v_n\)
Anisotropic Flow

The difference between $v_2\{2\}$ and $v_2\{4\}$ depends on the $v_2$ event-by-event fluctuations (later in this talk).

A small increase between 2-10\% for the $v_n$ is observed from 2.76 to 5.02 TeV.

The two parameterisations of $\eta/s$ which describe the data might indicate no or a small dependence on temperature.
Anisotropic Flow

- The dependence of $v_n$ on transverse momentum provides more differential information.
- At low transverse momentum the data can be interpreted in a "hydrodynamical" picture while at high-$p_t$ the dominant mechanism is thought to be path length dependent energy loss of high energetic partons.
- The $v_2$ coefficients dominate over all transverse momenta except for the most central collisions.
- The $v_2$ is significant up to the highest transverse momenta.
Anisotropic Flow

- The ratios between $v_n$ at 5.02 and 2.76 TeV are consistent with unity
- The increase in integrated $v_n$ due to increase in $<p_t>$ (due to radial flow in hydro picture)
- Also consistent with almost no change of eta/s between the two beam energies
Anisotropic Flow; compared to models

- Models use IP-Glasma, AMPT-IC or TRENTo initial conditions and all use UrQMD for the hadronic phase
- All models qualitatively describe the low pt data while AMPT-EC + iEBE-VISHNU does best in details (artefact?)
- At large $p_t$ the azimuthal asymmetries are thought to be due to path length dependent parton energy loss
- The model compared to the data uses an event-by-event hydro description ($v$-USPhydro) and jet quenching model (BBMG)
- Tested is a linear $dE/dx \sim L$ and quadratic energy loss
  - The $v_2$ at large $p_t$ is compatible with linear energy loss
Anisotropic Flow; the shape of $v_n$

- The dependence of $v_n$ on transverse momentum follows the previous observed power law scaling of $v_n^{1/n} \sim v_m^{1/m}$
- The scaling works over a surprisingly large momentum range particularly for $v_3$ compared to $v_2$
- The scaling does not exist in ideal hydro however seems to hold reasonably well for viscous hydro calculations shown
Anisotropic Flow Fluctuations

\[ v_n \{2\} = \sqrt{\langle v_n^2 \rangle}, \]
\[ v_n \{4\} = \sqrt{2 \langle v_n^2 \rangle^2 - \langle v_n^4 \rangle}, \]
\[ v_n \{6\} = \sqrt{3 \langle v_n^6 \rangle - 9 \langle v_n^2 \rangle \langle v_n^4 \rangle + 12 \langle v_n^2 \rangle^3}, \]
\[ v_n \{8\} = \sqrt{4 \langle v_n^8 \rangle - 16 \langle v_n^6 \rangle \langle v_n^2 \rangle - 18 \langle v_n^4 \rangle^2 + 144 \langle v_n^2 \rangle^2 \langle v_n^4 \rangle - 144 \langle v_n^2 \rangle^4}. \]

The different estimates of \( v_2 \) are sensitive to the moments of the \( v_2 \) distribution, if \( v_2 \{4\} = v_2 \{6\} = v_2 \{8\} \) then the distribution is a Bessel-Gaussian p.d.f.
A fine splitting is observed which is centrality dependent showing the non Bessel Gaussian contribution.

The splitting does not depend on the $p_t$ range used and collision energy.

The results agree well with model calculations as well as with ATLAS results based on a different technique.
Anisotropic Flow Fluctuations

\[ \gamma_1 = \frac{\langle (v_n \{RP\} - \langle v_n \{RP\}\rangle)^3 \rangle}{\langle (v_n \{RP\} - \langle v_n \{RP\}\rangle)^2 \rangle^{3/2}}, \]

\[ \gamma_1^{\text{exp}} = -6\sqrt{2}v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}. \]

\[ v_2\{6\} - v_2\{8\} = \frac{1}{11}(v_2\{4\} - v_2\{6\}). \]

The standardised skewness

The standardised skewness can estimated using the multi-particle cumulants

The experimental estimate depends on the fact that the higher order moments, e.g. kurtosis are small
Anisotropic Flow Fluctuations

$$v_2\{6\} - v_2\{8\} = \frac{1}{11}(v_2\{4\} - v_2\{6\}).$$
Anisotropic Flow Fluctuations

\[ \gamma_1^{\text{exp}} = -6\sqrt{2}v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}. \]

- A negative skewness is observed as expected due to the constraint on \( \epsilon_2 \) between 0-1.
- The skewness agrees well with model calculations and increases towards peripheral collisions due to the constraint of 1.
Anisotropic Flow Fluctuations

\[ P(\varepsilon_2) = \frac{1}{k_2^2} 2 \alpha \varepsilon_2 (1 - \varepsilon_2^2)^{\alpha - 1} (1 - \varepsilon_0^2)^{\alpha + 1/2} \frac{1}{\pi} \int_0^\pi (1 - \varepsilon_2 \varepsilon_0 \cos \varphi)^{-2\alpha - 1} d\varphi, \]

The elliptic power distribution can be used to describe the underlying p.d.f. of \( \varepsilon_2 \)

The parameter \( \alpha \) qualifies the magnitude of the flow fluctuations, \( \varepsilon_0 \) the mean eccentricity in the reaction plane and \( k_2 \) the proportionality between \( \varepsilon_2 \) and \( v_2 \); \( v_2 = k_2 \varepsilon_2 \)
Anisotropic Flow p.d.f.

\[ P(\varepsilon_2) = \frac{1}{k_2} 2 \alpha \varepsilon_2 (1 - \varepsilon_2^2)^{\alpha - 1} \left( 1 - \varepsilon_0^2 \right)^{1/2} \frac{1}{\pi} \int_0^\pi (1 - \varepsilon_2 \varepsilon_0 \cos \varphi)^{-2\alpha - 1} d\varphi, \]

**Fluctuations of elliptic flow ALICE Collaboration**

**Ratios of elliptic flow coefficients**

\[ \frac{(2n+1)}{2} \frac{F_{n+1}}{F_n} \]

**Elliptic Power parameters** is given by [27] between initial-state eccentricity and flow fluctuations, with this coefficient: 

\[ P(\varepsilon_2) = \frac{1}{k_2} 2 \alpha \varepsilon_2 (1 - \varepsilon_2^2)^{\alpha - 1} \left( 1 - \varepsilon_0^2 \right)^{1/2} \frac{1}{\pi} \int_0^\pi (1 - \varepsilon_2 \varepsilon_0 \cos \varphi)^{-2\alpha - 1} d\varphi, \]

**multi-particle cumulant methods, as a function of centrality. Measurements at**
Summary

• Anisotropic flow is precisely measured at the LHC as function of collision energy using multi-particle cumulants

• The measurements show that the system created behaves as an almost perfect liquid and constrain the path length dependence of parton energy loss

• The underlying p.d.f. of $v_2$ can be determined and used to constrain the initial conditions

![Graph showing the comparison between ideal hydro, viscous hydro, and AdS/CFT limits over time.](image)
Gracias!