Exact solutions of Einstein equations for anisotropic magnetic sources

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in collaboration with:

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Overview

Motivation

- Magnetized quantum systems exhibit anisotropic stress-energy tensor:

$$T^{\mu}_{\nu} = \mathrm{diag}(-\varrho, P_{\perp}, P_{\perp}, P_{\parallel})$$

$$\varrho = \Omega(B, \mu) + \mu N(B, \mu)$$
$$P_{\perp} = -\Omega(B, \mu) - B\mathcal{M}(B, \mu)$$
$$P_{\parallel} = -\Omega(B, \mu)$$

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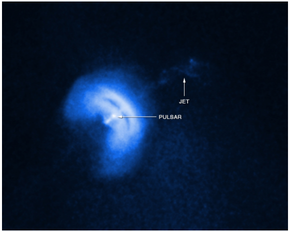
  The collapsed gas can be pushed towards the magnetic field axis while exerting a positive parallel pressure.
  Connected to *Astrophysical Jets*.



Motivation

The problem for the jets description is twofold:

1. Understand the production mechanism.
2. Look for GR solutions to study the stability.

Our aim:

Find exact solutions of Einstein's equations allowing the system to collapse in the direction of the magnetic field.

Plane-symmetric solutions

Considering:

- $T^{\mu}_{\nu} = \mathrm{diag}(-\varrho, P_1, P_2, P_{\parallel})$, with $P_1 = P_2 = P_{\perp}$
- Einstein's field equations exactly soluble
- Static Metric:

$$ds^2 = -f(z)dt^2 + g(z)(dx^2 + dy^2) + dz^2$$
$$f(z) = p(z)^2 q(z)^{-2/3}, \quad g(z) = q(z)^{4/3}$$

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Solutions imply:

$$T^{\mu}_{\nu} = \mathrm{diag}\left(-\varrho, -\frac{\varrho^2 + 3P_{\parallel}^2}{4\varrho}, -\frac{\varrho^2 + 3P_{\parallel}^2}{4\varrho}, P_{\parallel}\right).$$

$$P_{\perp} \neq 0$$

Plane-symmetric solutions: Cosmological constant

Including a cosmological constant, Λ, in the geometric sector of the field equations:

$$T^{\mu}_{\nu} = \mathrm{diag}\left(-\varrho, -\frac{\varrho^2 + 3P_{\parallel}^2 - 2\varrho\Lambda - 6P_{\parallel}\Lambda}{4\varrho + 4\Lambda}, -\frac{\varrho^2 + 3P_{\parallel}^2 - 2\varrho\Lambda - 6P_{\parallel}\Lambda}{4\varrho + 4\Lambda}, P_{\parallel}\right)$$

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Therefore,

$$\Lambda = \frac{1}{2}\frac{3P_{\parallel}^2 + \varrho^2 + 4P_{\perp}\varrho}{3P_{\parallel} + \varrho - 2P_{\perp}}$$

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Collapsed solution

$P_{\perp} = 0$. Strong magnetic field limit

$$\Lambda = \frac{1}{2}\frac{3P_{\parallel}^2 + \varrho^2}{3P_{\parallel} + \varrho}$$

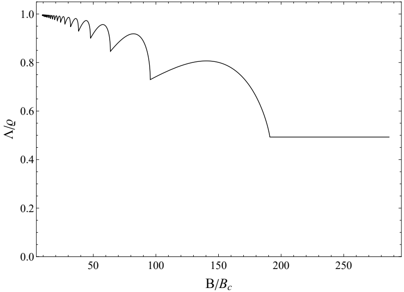
Isotropic case

$P_{\perp} \simeq P_{\parallel}$. Weak magnetic field limit

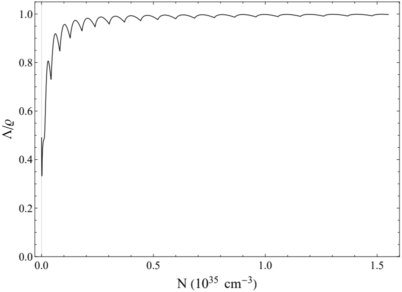
$$\Lambda \simeq \frac{1}{2}\left(3P_{\parallel} + \varrho\right)$$

Plane-symmetric solutions: Cosmological constant. Fermions gas results

- $0 < \Lambda/\varrho < 1$
- Strong magnetic field limit: gas collapsed, all particles in LLL and $\Lambda/\varrho = 1/2$.
- Weak magnetic field limit: higher Landau levels occupied, Haas van Alphen oscillations, $\varrho \to 3P_{\|}$ and $\Lambda/\varrho \to 1$.



Fixed N ($\mu = 19.6 m_e$).



Fixed $B = 100\ B_c$.

Conclusions

- Solutions obtained for plane–symmetric spacetime.
- With the inclusion of the cosmological constant, it is possible to recover the isotropic case from the anisotropic one. By tuning Λ, the system can be driven continuously from one case to the other.
- In the weak magnetic field regime, $\Lambda/\varrho \to 1$, while in the strong magnetic field limit, $\Lambda/\varrho \to 1/2$, which corresponds to the magnetic collapse situation that may be of importance for the description of jets.
- Cosmological constant has been used previously in the study of compact objects as a modification to structure equations (TOV). If taking the accepted value from the cosmological point of view, there is no visible effect on the mass and the radius.

Thank you!

Plane-symmetric solutions: Field equations & metric functions

Without Λ:

$$\frac{4q''}{3q} = -\varrho$$
$$\frac{p''}{p} + \frac{q''}{3q} = P_\perp$$
$$\frac{4p'q'}{3pq} = P_\parallel$$

$$q(z) = e^{-\frac{\sqrt{3A_1}}{2}z}\left(e^{\sqrt{3A_1}z}c_1 + c_2\right)$$
$$p(z) = c_3 e^{-\frac{\sqrt{3}A_2 z}{2\sqrt{A_1}}}\left(c_2 - c_1 e^{\sqrt{3A_1}z}\right)^{\frac{A_2}{A_1}}$$

With Λ:

$$\frac{4q''}{3q} + \Lambda = -\varrho$$
$$\frac{p''}{p} + \frac{q''}{3q} - \Lambda = P_\perp$$
$$\frac{4p'q'}{3pq} - \Lambda = P_\parallel$$

$$q(z) = e^{-\frac{\sqrt{3(A_1-\Lambda)}}{2}z}\left(e^{\sqrt{3(A_1-\Lambda)}z}c_1 + c_2\right)$$
$$p(z) = c_3 e^{\frac{(\Lambda - A_2)\left(3z\sqrt{A_1-\Lambda} - 2\sqrt{3}\log\left[c_2 - c_1 e^{\sqrt{3(A_1-\Lambda)}z}\right]\right)}{2\sqrt{3}(A_1-\Lambda)}}$$

c_1, c_2 and c_3 are integration constants, $A_1 = -\varrho$ and $A_2 = P_\parallel$.

EoS for a fermions gas in a constant magnetic field

$$\varrho = \frac{m^2}{4\pi^2}\frac{B}{B_c}\sum_{l=0}^{l_{max}} g_l \left(\mu\, p_F + \mathcal{E}_l{}^2 \ln\frac{\mu + p_F}{\mathcal{E}_l}\right)$$

$$P_\parallel = \frac{m^2}{4\pi^2}\frac{B}{B_c}\sum_{l=0}^{l_{max}} g_l \left[\mu\, p_F - \mathcal{E}_l{}^2 \ln\left(\frac{\mu + p_F}{\mathcal{E}_l}\right)\right]$$

$$P_\perp = \frac{m^4}{2\pi^2}\left(\frac{B}{B_c}\right)^2 \sum_{l=0}^{l_{max}} g_l\, l \ln\left(\frac{\mu + p_F}{\mathcal{E}_l}\right)$$

where:

- l runs for the Landau levels, $l_{max} = I[\frac{\mu^2 - m^2}{2|eB|}]$
- Fermi momenta: $p_F = \sqrt{\mu^2 - \mathcal{E}_l^2}$
- rest energy: $\mathcal{E}_l = \sqrt{2|eB|l + m^2}$
- $B_c = m^2/|e|$: Schwinger critical magnetic field