

Nucleosynthesis in heavy-ion collisions at the LHC via the Saha equation

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Non-equilibrium Dynamics, Castiglione della Pescaia, June 2019

in collaboration with:

V. Vovchenko, **K. Gallmeister** and **J. Schaffner-Bielich**

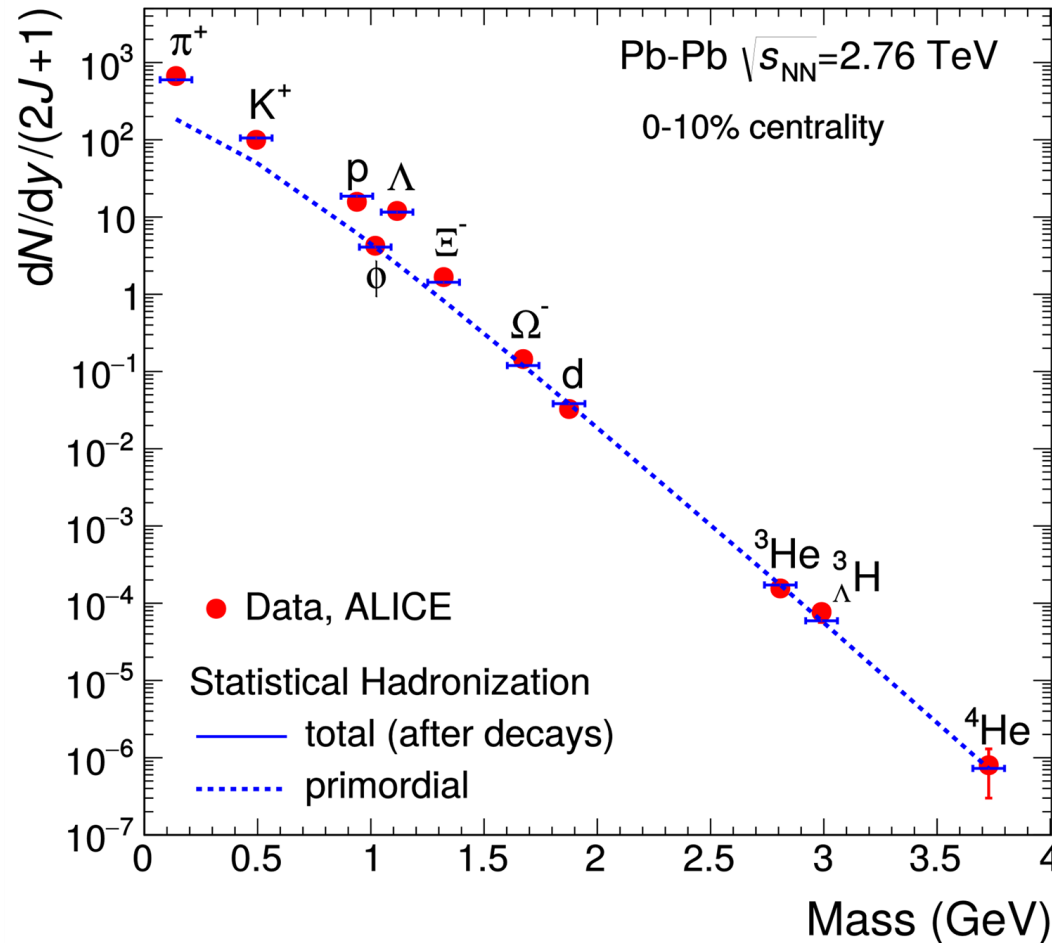
Big-Bang nucleosynthesis

Estimates

Numerical solution

arXiv:1903.10024

ALICE data



$$T_{\text{ch}} = 156.5 \text{ MeV}$$

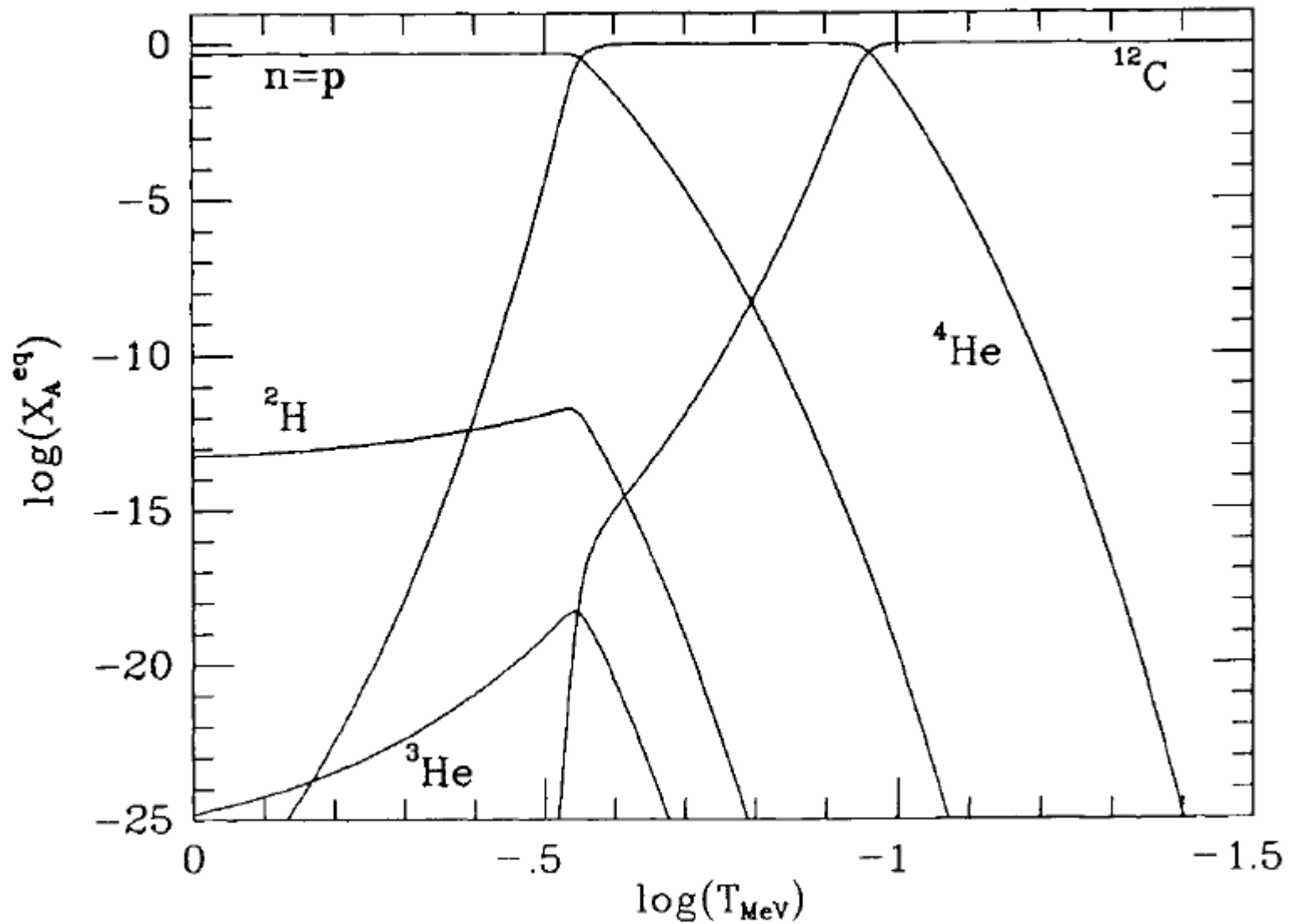
$$T_{\text{kin}} \sim 115 \text{ MeV}$$

Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

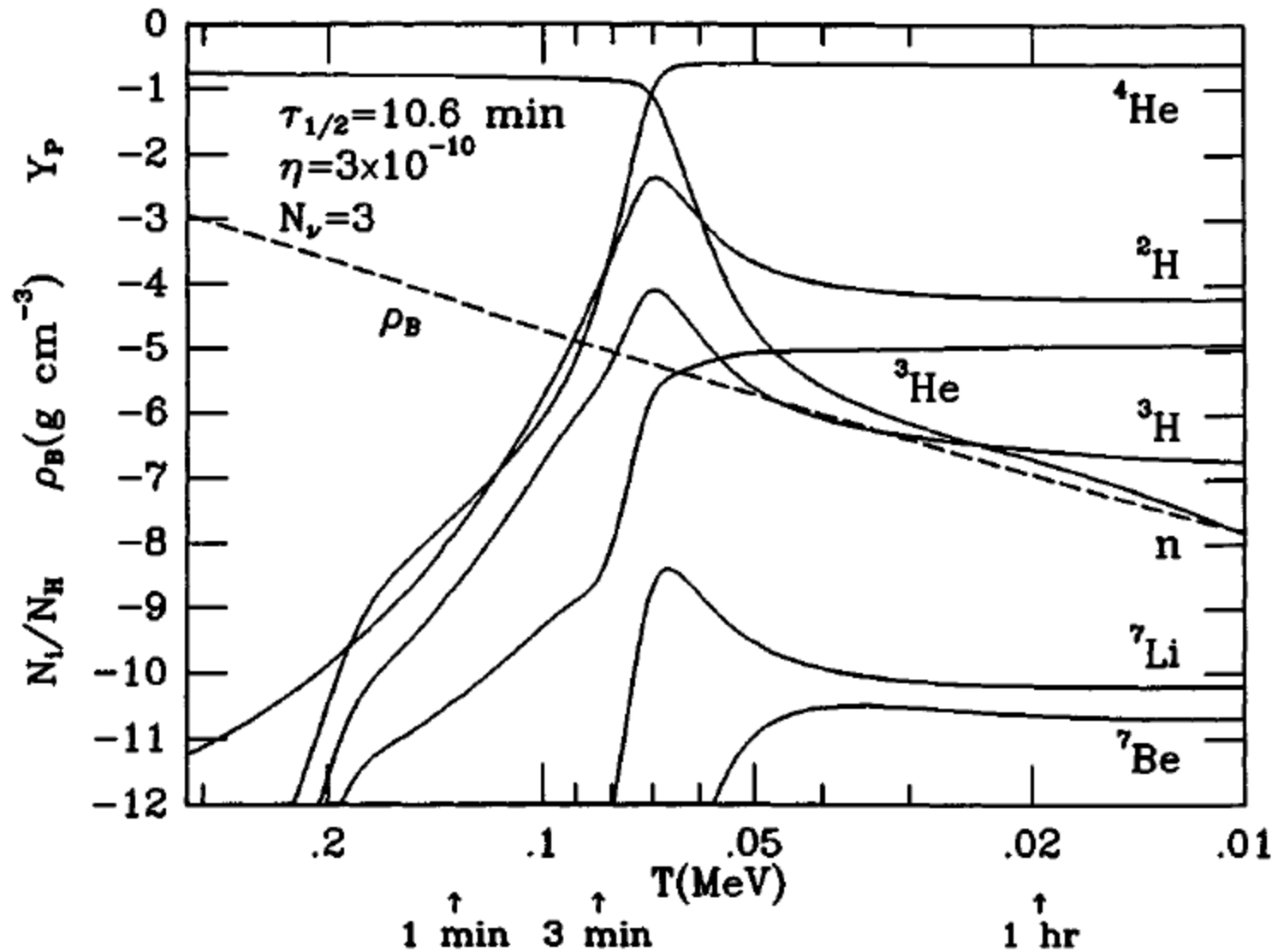
binding energies:

$${}^2\text{H}, {}^3\text{He}, {}^4\text{He} : 2.22, 7.72, 28.3 \text{ MeV} \quad {}^3_{\Lambda}\text{H} : 130 \text{ keV}$$

primordial nucleosynthesis – nuclear statistical eq.



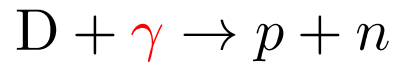
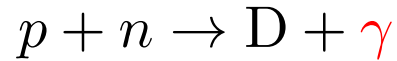
primordial (big bang) nucleosynthesis



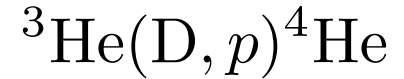
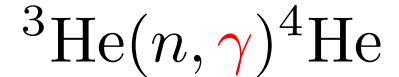
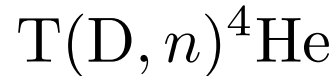
primordial nucleosynthesis: network

■ Deuterium

$$p(n, \gamma)D$$



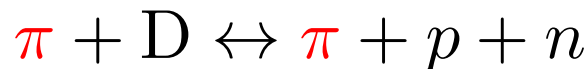
■ Helium



■ here now:

$$\pi + A \leftrightarrow \pi + A' + m_1 p + n_1 n$$

e.g.



“Bevalac” nucleosynthesis

VOLUME 43, NUMBER 20

PHYSICAL REVIEW LETTERS

12 NOVEMBER 1979

Evidence for a Soft Nuclear-Matter Equation of State

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(Received 3 August 1979)

The entropy of the fireball formed in central collisions of heavy nuclei at center-of-mass kinetic energies of a few hundred MeV per nucleon is estimated from the ratio of deuterons to protons at large transverse momentum. The observed paucity of deuterons suggests that strong attractive forces are present in hot, dense nuclear matter, or that degrees of freedom beyond the nucleon and pion may already be realized at an excitation energy of 100 MeV per baryon.

Because of the reaction $d + N \leftrightarrow p + n + N$, where N is a spectator nucleon or cluster, deuterons will be constantly breaking up and reforming. If collisions are frequent enough, the deuterons will quickly reach an equilibrium concentration determined by detailed balancing⁴:

$$\exp(-\mu_d/T) d_d(\vec{R}, \vec{P}, s_z) = \sum_{s_z} d_p(\vec{R}, \vec{P}/2, s_z) d_n(\vec{R}, \vec{P}/2, s_z - s_z) \exp[-(\mu_n + \mu_p)/T],$$

“Bevalac” nucleosynthesis

Nuclear Physics **A476** (1988) 718-772
North-Holland, Amsterdam

THE QUANTUM STATISTICAL MODEL OF FRAGMENT FORMATION: Entropy and temperature extraction in heavy ion collisions

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Received 8 October 1986

(Revised 5 October 1987)

Abstract: Quantum statistical model (QSM) calculations of nuclear fragment formation are presented. Various independent methods for extracting the temperature, T , and entropy, S/A , from fragment- and pion yields in heavy-ion collisions are analysed. It is emphasized that stable and unstable medium mass fragments play an important role in determining T and S/A : They alter the relation $S/A(R_{dp})$ dramatically and distort via feeding simple temperature measurements. However, these fragments allow by their very abundance for a variety of new, alternative methods to determine S/A from data on multifragmentation (ratios of complex fragment yields, mass yield curves, and charged-particle multiplicities).

Entropy values deduced from 4π plastic ball data exhibit a strong multiplicity dependence. For large multiplicities the entropy residing in nuclear fragments appears to be independent of the bombarding energy and low in absolute value, $S/A \approx 3.5$.

The corresponding break-up temperatures of the fragment conglomerate are $T = 12, 16$, and 20 MeV at $E_{lab} = 400, 650$, and 1050 MeV/n, respectively. These values are much smaller (a factor $\frac{1}{3}$) than the temperatures extracted from pion yields. This result can be understood only if the pions are created in the early, hot stage of the collision, while the fragments are formed after an isentropic expansion of the system at small densities where the temperature is low. This would imply that in the late stage of the reaction a large fraction ($\approx 80\%$) of the available center-of-mass energy resides in (possibly isotropic) flow.

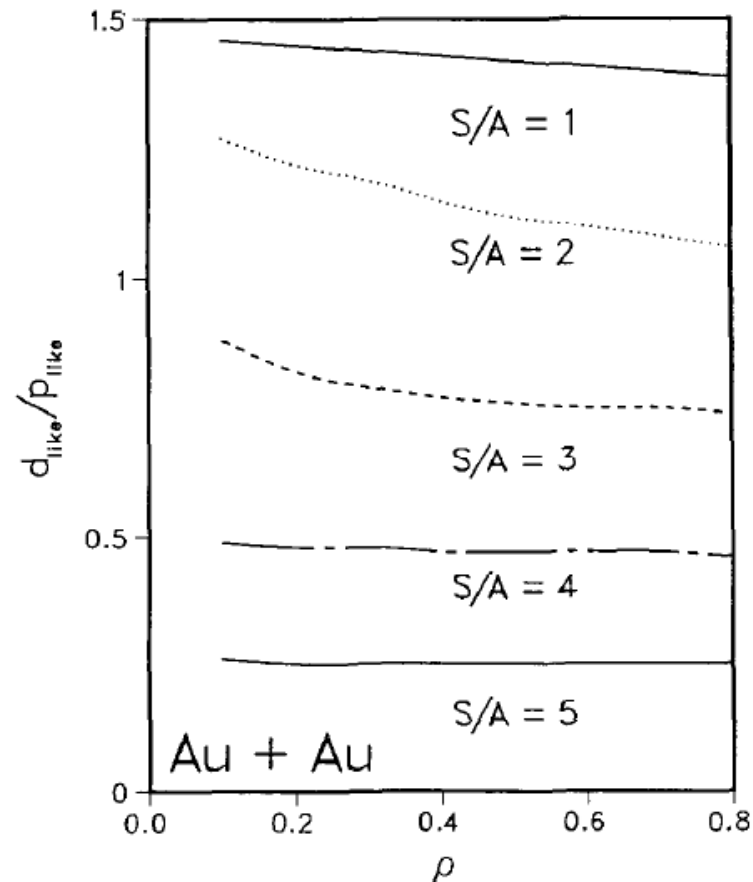


Fig. 23. Connection between d-like/p-like and the break-up density for constant entropy. The calculation has been done for the Au + Au system.

Saha equation

■ ionization of a gas



$$\frac{n_e^2}{n_0} = \frac{2}{\lambda_e^3} \frac{g_1}{g_0} \exp(-\epsilon/T) \quad n_1 = n_e \quad \lambda_e : \text{deBroglie}$$

Megh Nad Saha, Phil. Mag. Series 6 40:238 (1920) 472

■ equivalently: partition functions

$$\frac{Z_0}{N_0} = \frac{Z_1 Z_e}{N_1 N_e}$$

■ equivalently: chemical potentials

$$\mu_0 = \mu_1 + \mu_e$$

Saha equation
= detailed balance
= law of mass action

Saha equation

■ nuclear equivalent



■ 'Nuclear Statistical Equilibrium'

■ mass fraction of nucleus A:

$$X_A = g_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}} \right] A^{\frac{5}{2}} \left(\frac{T}{m_N} \right)^{\frac{3}{2}(A-1)} \\ \times \eta^{A-1} X_p^Z X_n^{A-Z} \exp \left(\frac{B_A}{T} \right)$$

$$\eta = \frac{n_B}{n_\gamma} \sim 6 \cdot 10^{-10}$$

Kolb, Turner, The Early Universe, 1990

■ this work:

$$\frac{N_A(T)}{N_p} = \frac{g_A}{g_M^{1-A}} \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{1+A}{2}} \right] A^{\frac{3}{2}} \left(\frac{T}{m_N} \right)^{\frac{3}{2}(A-1)} \\ \times \eta_B^{A-1} \exp \left(\frac{B_A}{T} \right)$$

$$\eta_B = \frac{N_N}{N_M} \sim 0.03$$

$$(N_M = g_M [\zeta(3)/\pi^2] V T^3, \quad g_M \sim 11 - 13)$$

Heavy ion collisions

- chemical freeze-out =
number of (anti-)protons/neutrons etc. constant below T_{ch}

$$n_i^{(0)} = \frac{g}{2\pi^2} m_i^2 T_{\text{ch}} K_2(m_i/T_{\text{ch}}) , \quad n_i = n_i^{(0)} e^{\mu_i/T}$$

- Saha equation/detailed balance: $X + A \longleftrightarrow X + \sum_i A_i$

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{(0)}}{\prod_i n_{A_i}^{(0)}} , \quad \mu_A = \sum_i \mu_{A_i}$$

e.g. π

$$\mu_{^2\text{H}} = \mu_p + \mu_n , \quad \mu_{^3\text{He}} = 2\mu_p + \mu_n , \quad \dots$$

$$N_A = V \frac{g_A}{2\pi^2} m_A^2 T K_2(m_A/T) e^{\mu_A/T}$$

$$\mu_A(T) ? \quad V(T) ?$$

Heavy ion collisions

■ isentropic expansion:

$$V/V_{\text{ch}} = (T/T_{\text{ch}})^{-3}$$

■ non-relativistic approximation:

$$N_i(T) \simeq g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} e^{\mu_i/T} V_{\text{ch}} \left(\frac{T_{\text{ch}}}{T} \right)^3$$
$$\Rightarrow \mu_i \simeq 3/2 T \ln (T/T_{\text{ch}}) + m_i (1 - T/T_{\text{ch}}).$$

$$\frac{N_A(T)}{N_A(T_{\text{ch}})} \simeq \left(\frac{T}{T_{\text{ch}}} \right)^{\frac{3}{2}(A-1)} \exp \left[B_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right]$$

$$\left[\frac{N_A(T)}{N_A(T_{\text{ch}})} \right]_{\text{eq.}} \simeq \left(\frac{T}{T_{\text{ch}}} \right)^{\frac{3}{2}} \exp \left[-m_A \left(\frac{1}{T} - \frac{1}{T_{\text{ch}}} \right) \right]$$


Full calculation (Thermal-FIST)

■ particle decays:

$i \in \text{stable} , j \in \text{HRG}$

$\langle n_i \rangle_j$ mean number of hadron i from decays of hadron j

■ effective chemical potentials:

$$\tilde{\mu}_j = \sum_i \langle n_i \rangle_j \mu_i$$

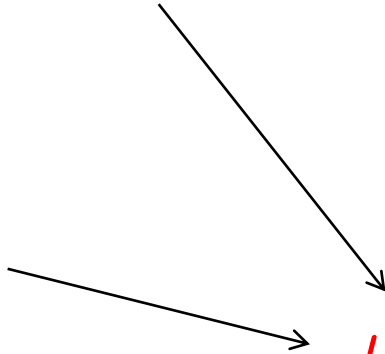
■ conservation of yields of stable hadrons:

$$V \sum_i \langle n_i \rangle_j n_j(T, \tilde{\mu}_j) \stackrel{!}{=} N_i(T_{\text{ch}})$$

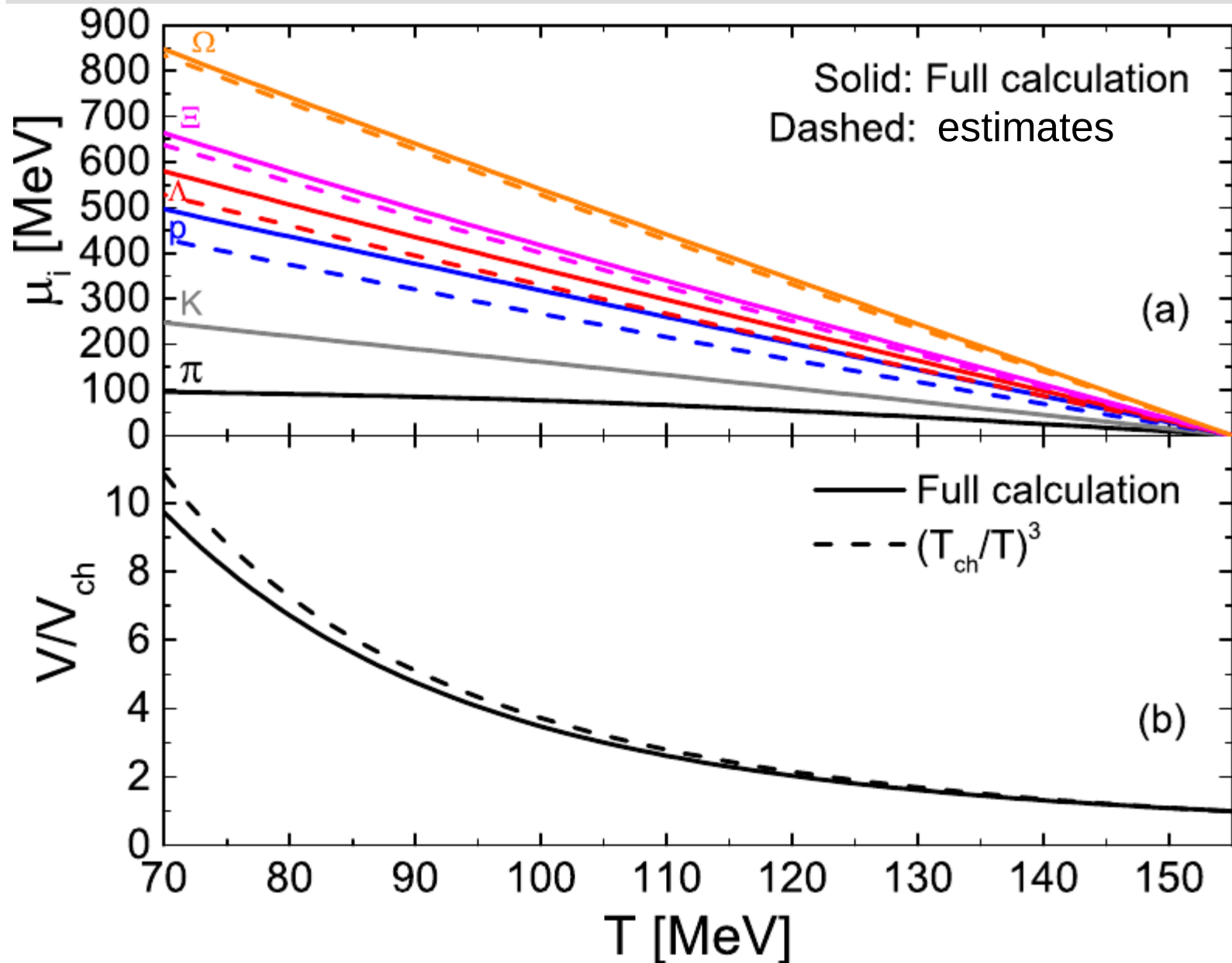
■ isentropic expansion:

$$V \sum_i s_j(T, \tilde{\mu}_j) \stackrel{!}{=} S(T_{\text{ch}})$$

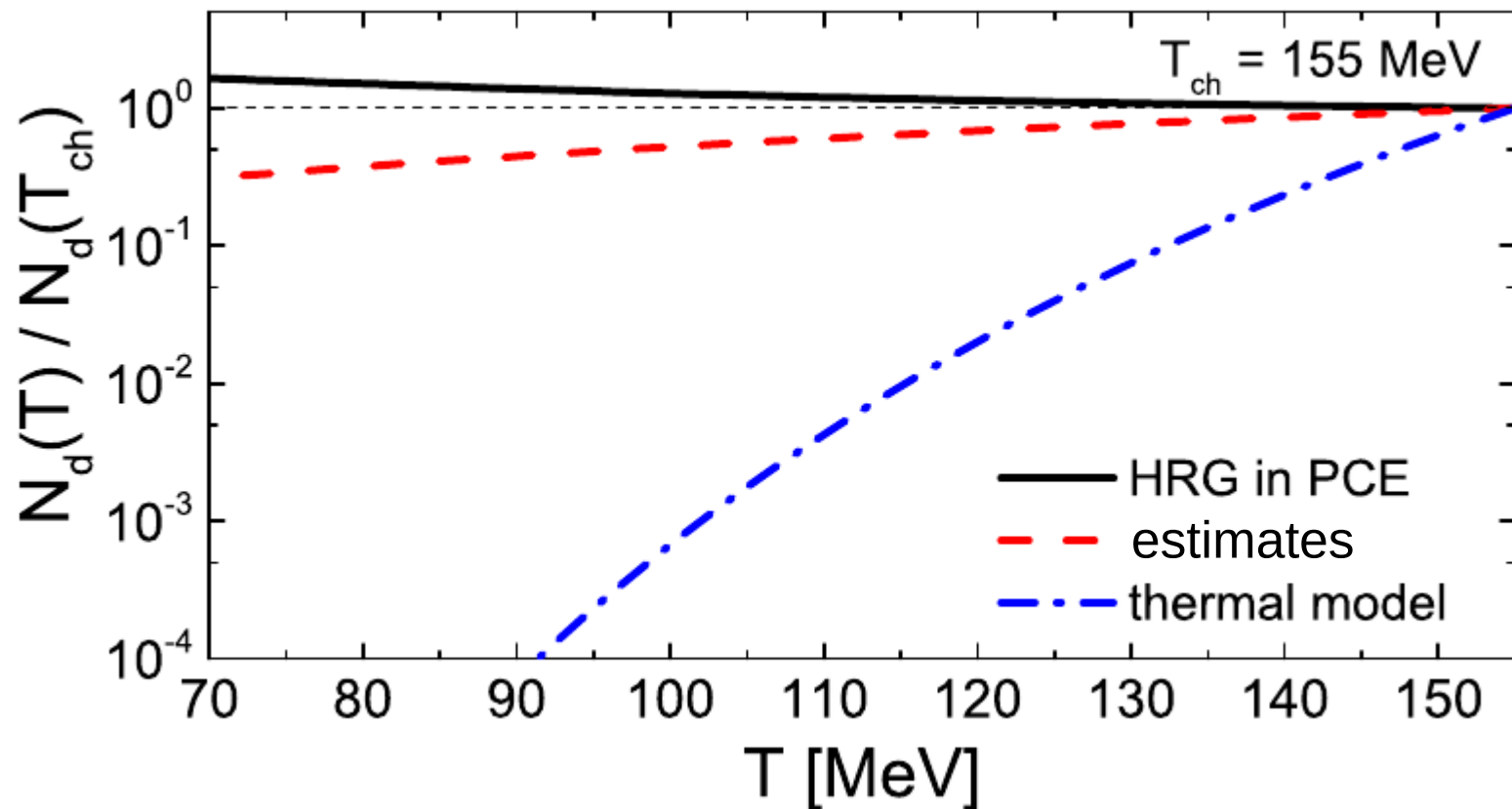
$\mu_i(T) , V(T)$

Two arrows originate from the equations above. One arrow points from the equation $V \sum_i \langle n_i \rangle_j n_j(T, \tilde{\mu}_j) \stackrel{!}{=} N_i(T_{\text{ch}})$ down and to the right towards the text $\mu_i(T) , V(T)$. The other arrow points from the equation $V \sum_i s_j(T, \tilde{\mu}_j) \stackrel{!}{=} S(T_{\text{ch}})$ down and to the right, also towards the text $\mu_i(T) , V(T)$.

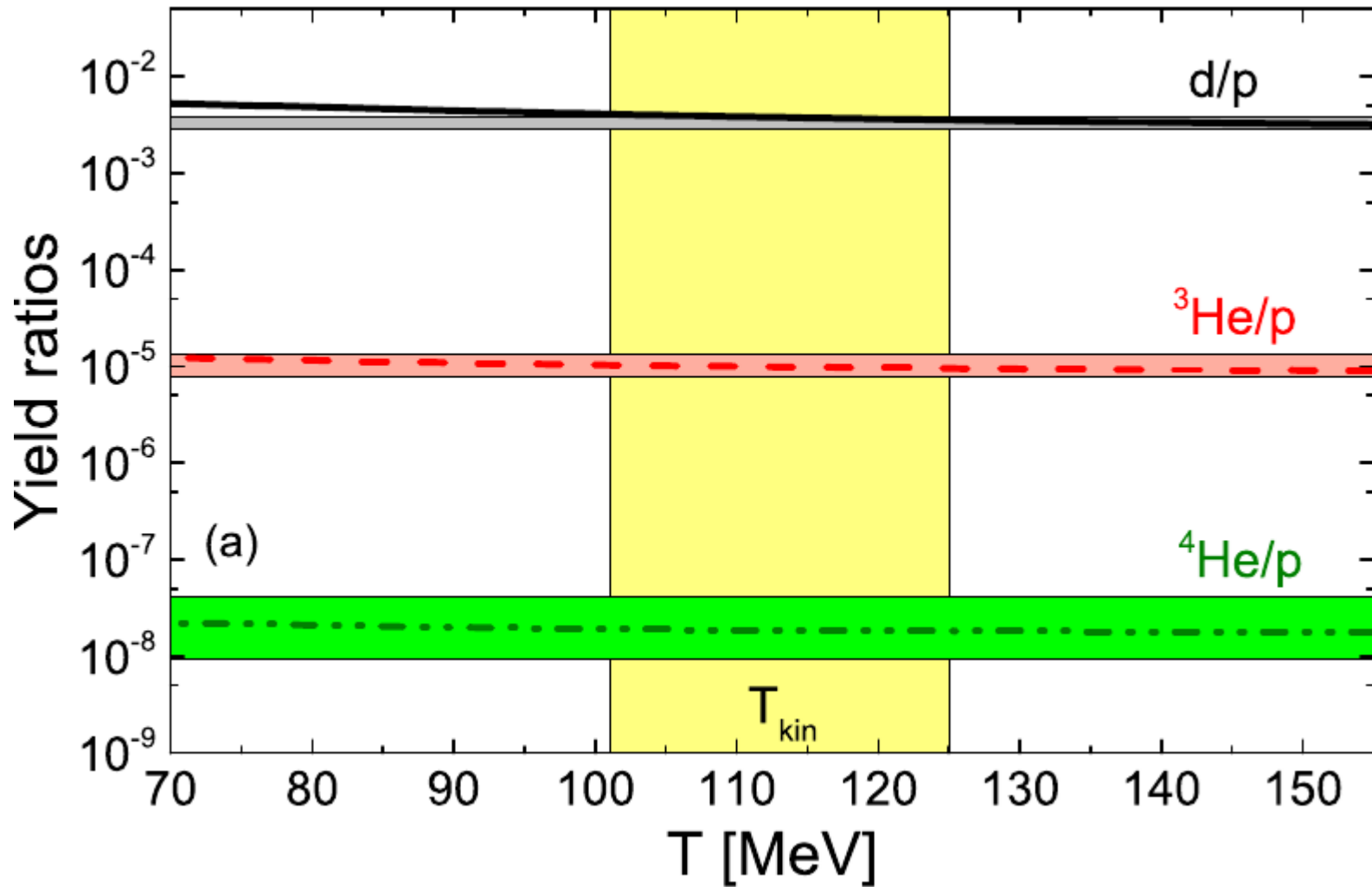
Full calculation: parameters



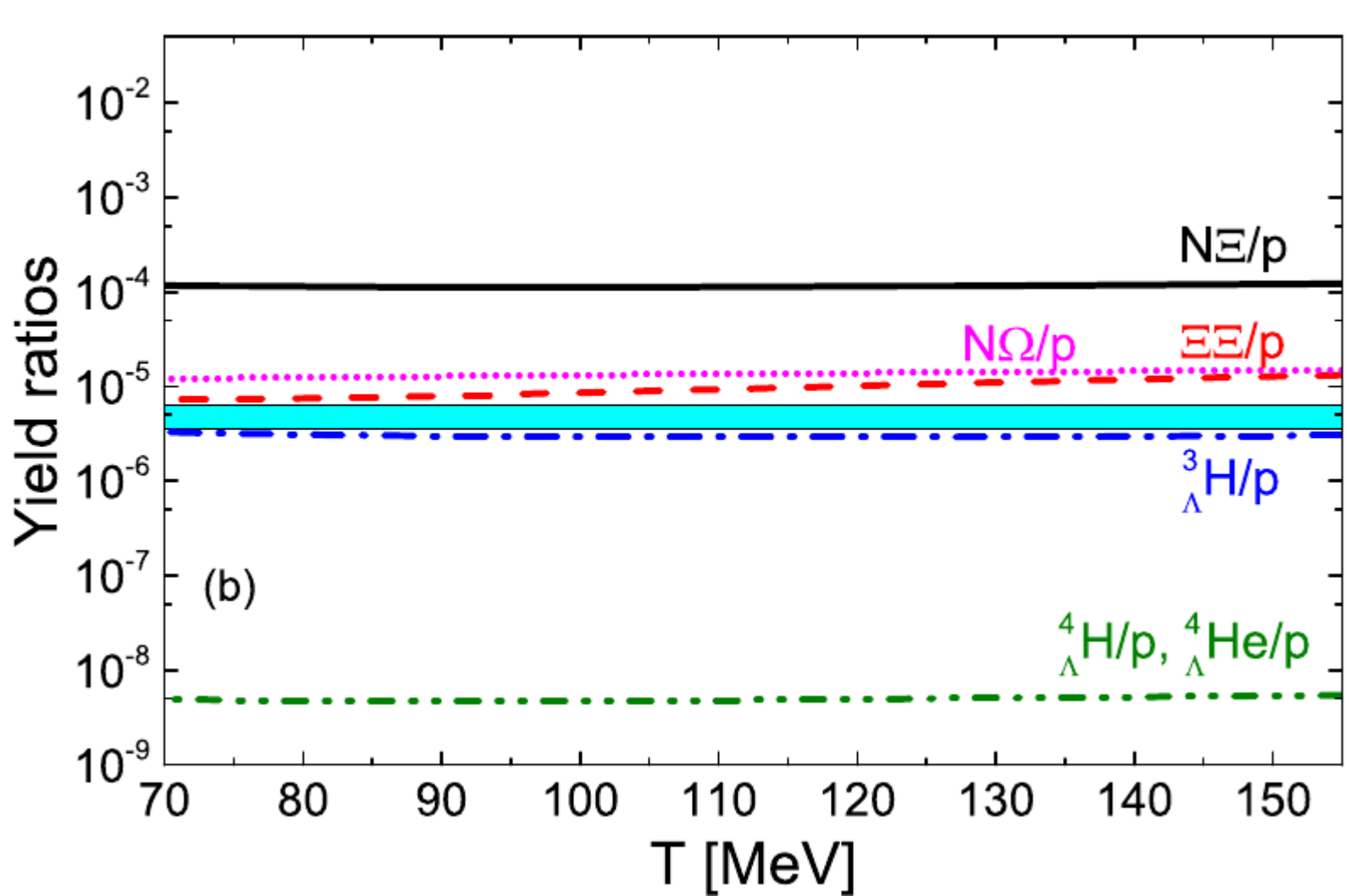
Full calculation: results for d



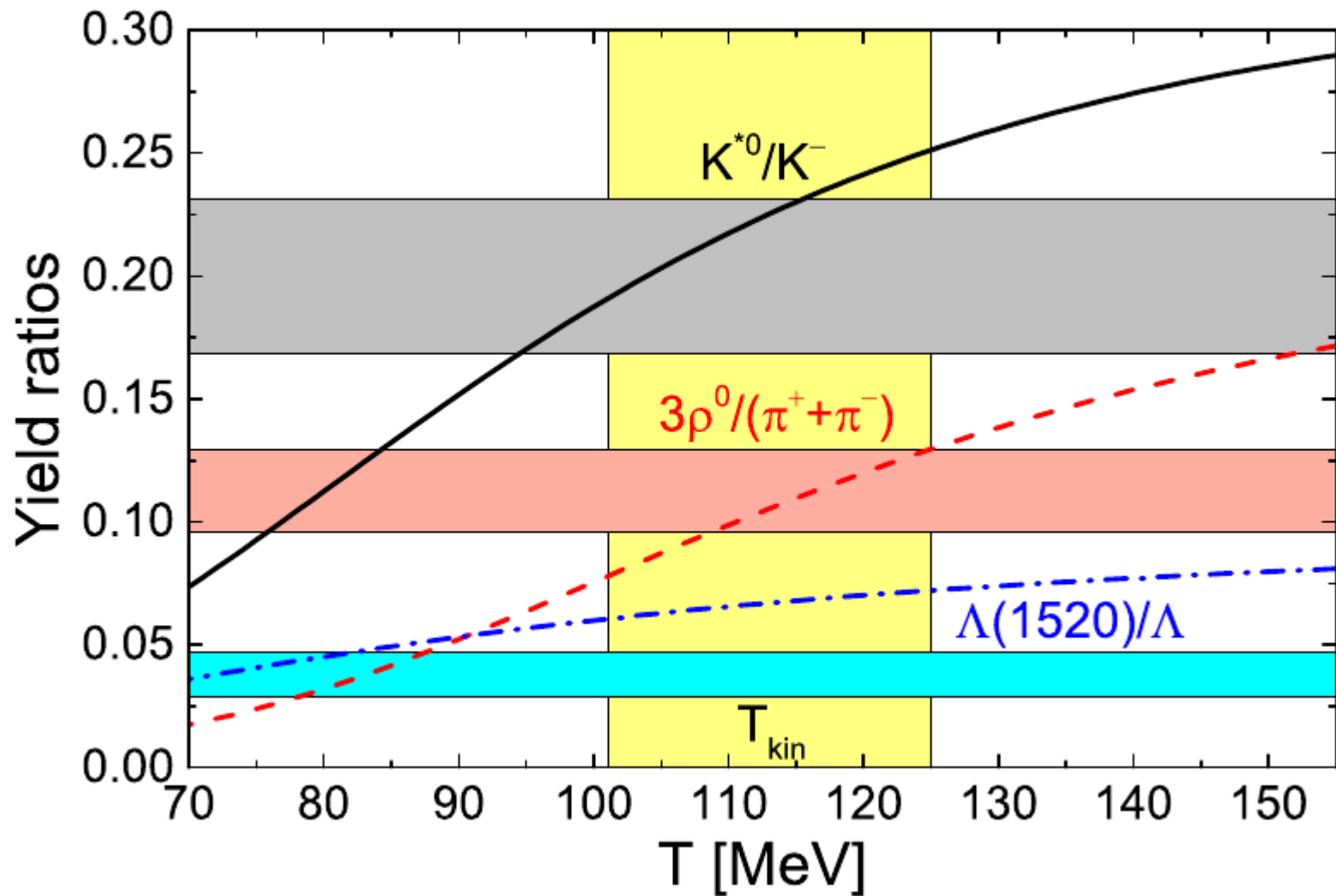
Full calculation: results (a)



Full calculation: results (b)



Full calculation: results for resonances



LHC nucleosynthesis

PHYSICAL REVIEW C **99**, 044907 (2019)

Editors' Suggestion

Featured in Physics

Microscopic study of deuteron production in PbPb collisions at $\sqrt{s} = 2.76$ TeV via hydrodynamics and a hadronic afterburner

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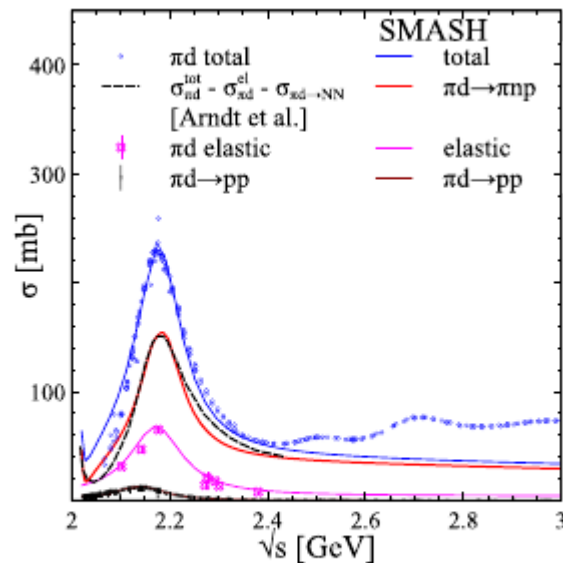


FIG. 1. Deuteron-pion interaction cross sections from SAID database [40] and partial wave analysis [41] are compared to our parametrizations (Tables II and III in the Appendix). Inelastic $d\pi \leftrightarrow$

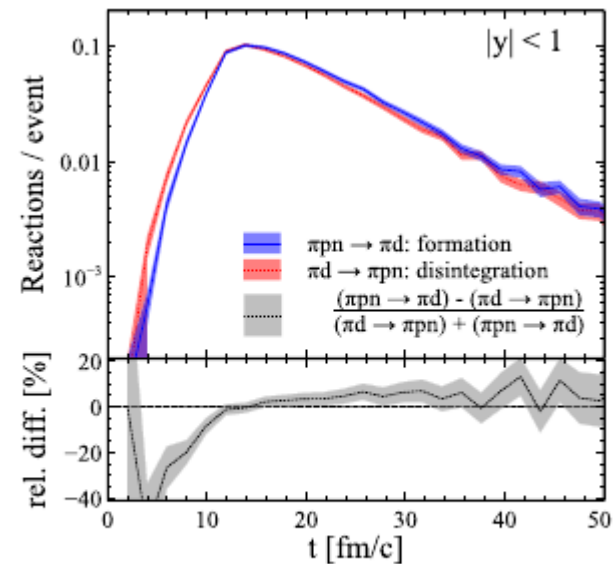
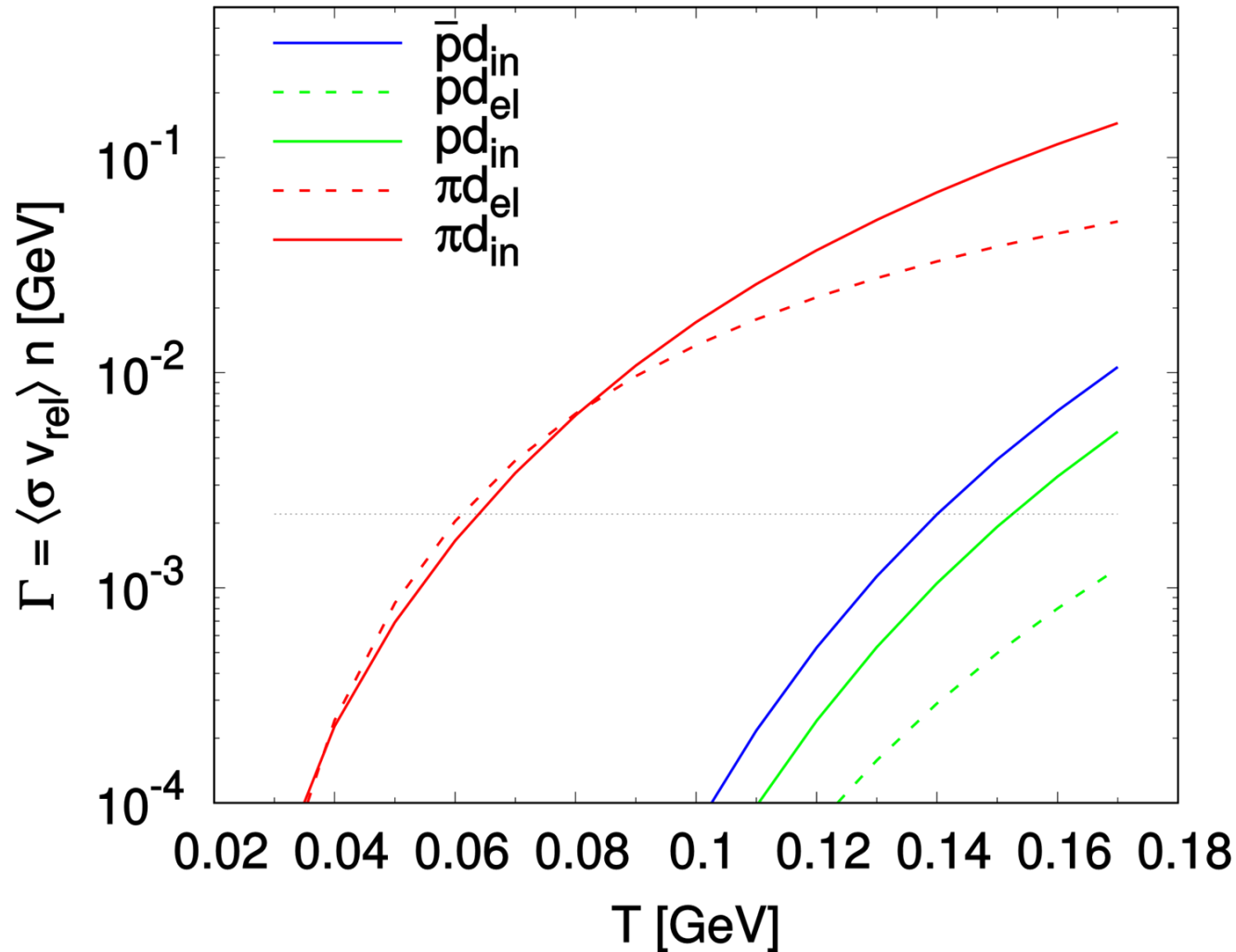


FIG. 5. Reaction rates of the most important $\pi d \leftrightarrow \pi pn$ reaction in forward and reverse direction.

■ law of mass action at work

Rate



$$\Delta E_{\text{Heisenberg}} \simeq \Gamma$$

$$\Leftrightarrow \Delta E_{\text{bound}}$$

$$\Leftrightarrow \Gamma_{\text{expansion}}$$

("kinetic freezeout")

Conclusions

■ Saha equation gives natural explanation of agreement of thermal model predictions and experimental observations

■ light nuclei may be formed at any $T < T_{\text{ch}}$!

■ who can give the answer?

■ building of clusters (Hagedorn states?)`

■ coalescence

■ rate equations

■ transport simulations (cf. D.Oliinychenko et al.)

■ ...

■ quantum mechanical treatment of creation/decreation of bound systems in medium needed