Correlations far from equilibrium

NED2019, 16-22 June 2019, Castiglione della Pescaia, Italy

June 21st, 2019

Matthias Kaminski

University of Alabama

[Cartwright, Kaminski; arXiv (2019)]

Correlations far from equilibrium

in charged strongly coupled fluids

NED2019, 16-22 June 2019, Castiglione della Pescaia, Italy

June 21st, 2019

Matthias Kaminski

University of Alabama

[Cartwright, Kaminski; arXiv (2019)]

Correlations far from equilibrium

in charged strongly coupled fluids subjected to a strong magnetic field

NED2019, 16-22 June 2019, Castiglione della Pescaia, Italy

June 21st, 2019

Matthias Kaminski

University of Alabama

[Cartwright, Kaminski; arXiv (2019)]

Summary of results

- generated charged magnetic fluid with initial anisotropy

$$\rho \neq 0 \qquad \mathcal{B} \neq 0 \qquad\qquad \Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$$

- calculated 1- and 2-point functions
- early times: strong medium effect
- 2-point functions thermalize significantly slower than 1-point functions

	$\rho = 0$	$\rho = 0.78\rho_e$	approach to extremality → ρ
$\mathcal{B} = 0$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.4$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.4$	$t_{1pt} \to \infty$ $t_{2pt} \to \infty$
$\mathcal{B} = 1$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.9$	$t_{1pt} \approx 0.8$ $t_{2pt} \approx 2.3$	
$\mathcal{B} = 3$	saturation regime $t_{2pt} \not\approx t_{2pt}(l)$ $t_{1pt} \approx 0.6$ $t_{2pt} \approx 1.4$	$t_{1pt} \approx 0.6$ $t_{2pt} \approx 1.5$	
$\mathcal{B}$ ↓			

➡ competition of scales

*charge

*magnetic field

*initial anisotropy

➡ universal thermalization time at large $\mathcal{B}$

[Glorioso, Son; (2018)]
[Grozdanov, Poovuttikul; JHEP (2019)]

➡ large charge: thermalization times diverge

(in $\mathcal{N}$=4 Super-Yang-Mills theory in 3+1 dimensions, minimally coupled to external U(1) gauge field)

Summary of results

- generated charged magnetic fluid with initial anisotropy

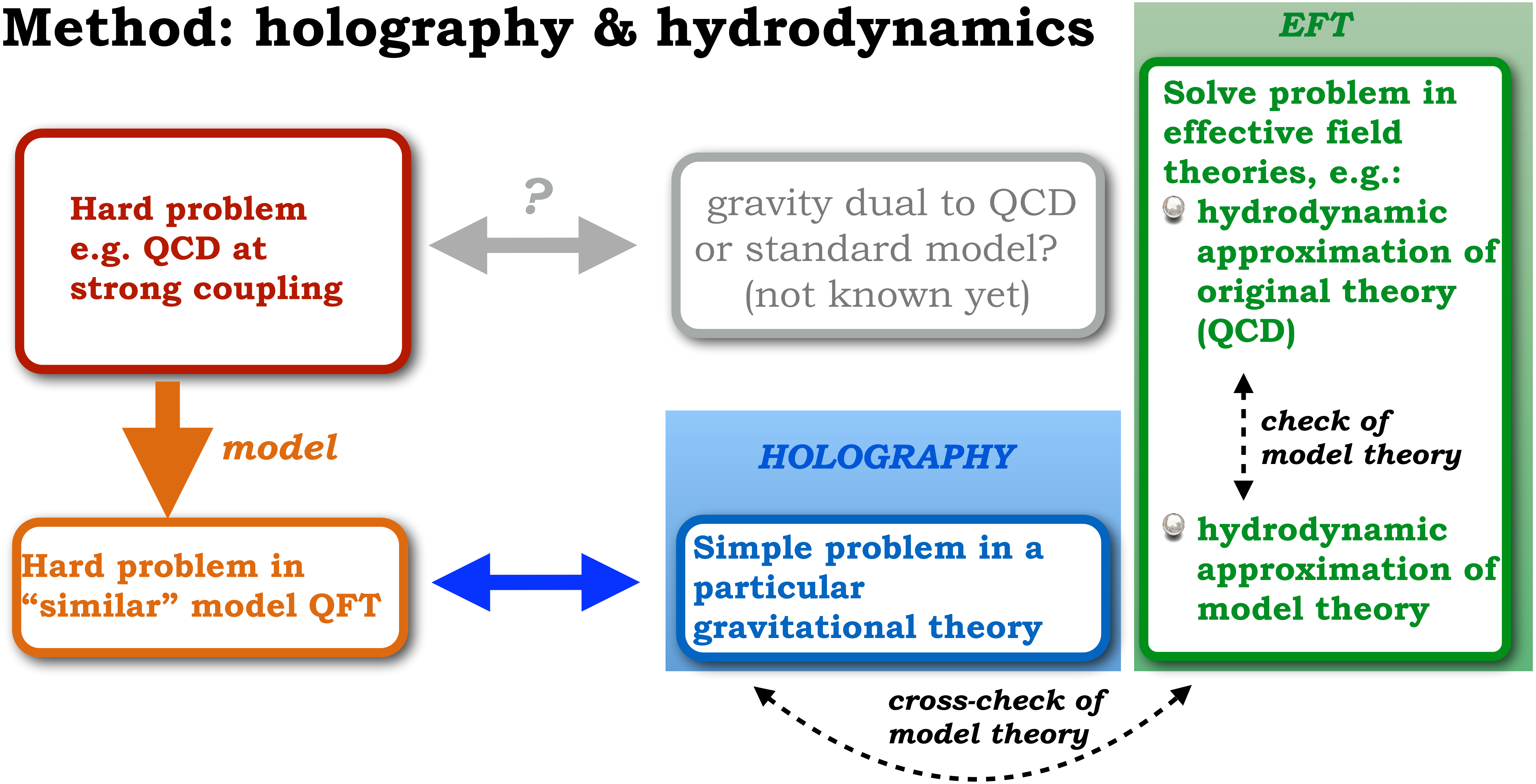
$\rho \neq 0$ $\qquad \mathcal{B} \neq 0$ $\qquad \Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$

- [redacted] nd 2-point functions
- [redacted] edium effect
- 2-[redacted] si[redacted] tions

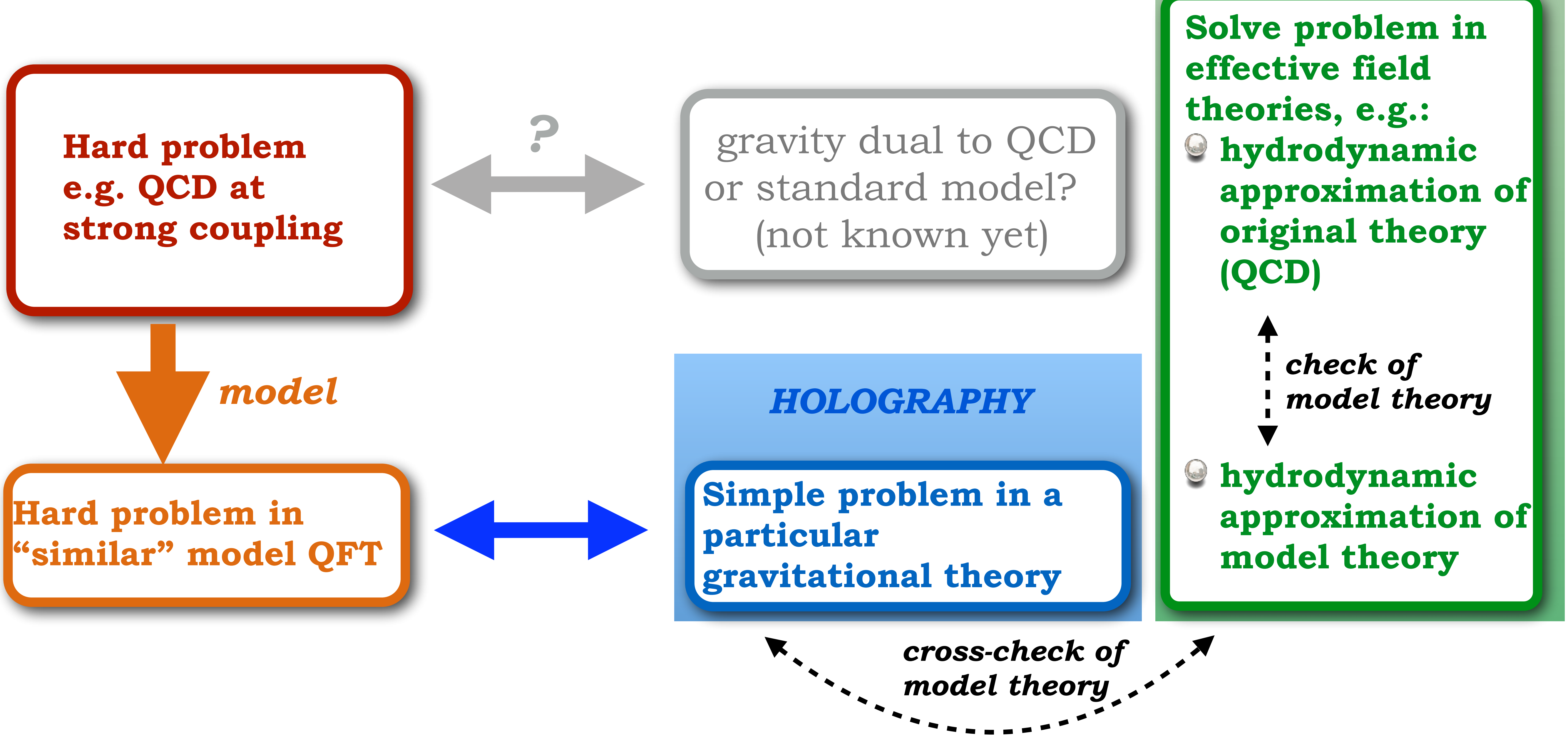
	[redacted]	[redacted]	[redacted]
$\mathcal{B} = 0$	$t_{1pt} \approx$ [redacted] $t_{2pt} \approx 1.4$	[redacted]	[redacted]
$\mathcal{B} = 1$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.9$	t_{1pt} [redacted] t_{2pt} [redacted]	[redacted]
	saturation regin[redacted] $t_{2pt} \not\approx t_{2pt}(l)$ [redacted]		
$\mathcal{B} = 3$	$t_{1pt} \approx 0.6$ $t_{2pt} \approx 1.4$	$t_{1pt} \approx$ [redacted] $t_{2pt} \approx$ [redacted]	[redacted]
$\mathcal{B}$			

(in *N*=4 Super-Yang-Mills theory in 3+1 dimensions, minimally coupled to external U(1) gauge field)

Method: holography & hydrodynamics



Method: holography & hydrodynamics



- ➡ Holography good at **qualitative** or **universal** predictions.
- ➡ Compare holographic result to hydrodynamics of model theory.
- ➡ Compare hydrodynamics of original theory to hydrodynamics of model.
- ➡ Understand holography as an **effective description**.

Motivation

1 $\mathcal{N}$=4 Super-Yang-Mills theory with a magnetic field **in equilibrium has a universal magnetoresponse** variable, which agrees *well* with its QCD equivalent

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]

2 consider this theory **near equilibrium**, compute dispersion relations & correlation functions and compare to *strong magnetic field (chiral) hydro*

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; work in progress]

[Ammon, Kaminski, Koirala, Leiber, Wu; JHEP (2017)]

3 consider this theory **far from equilibrium**

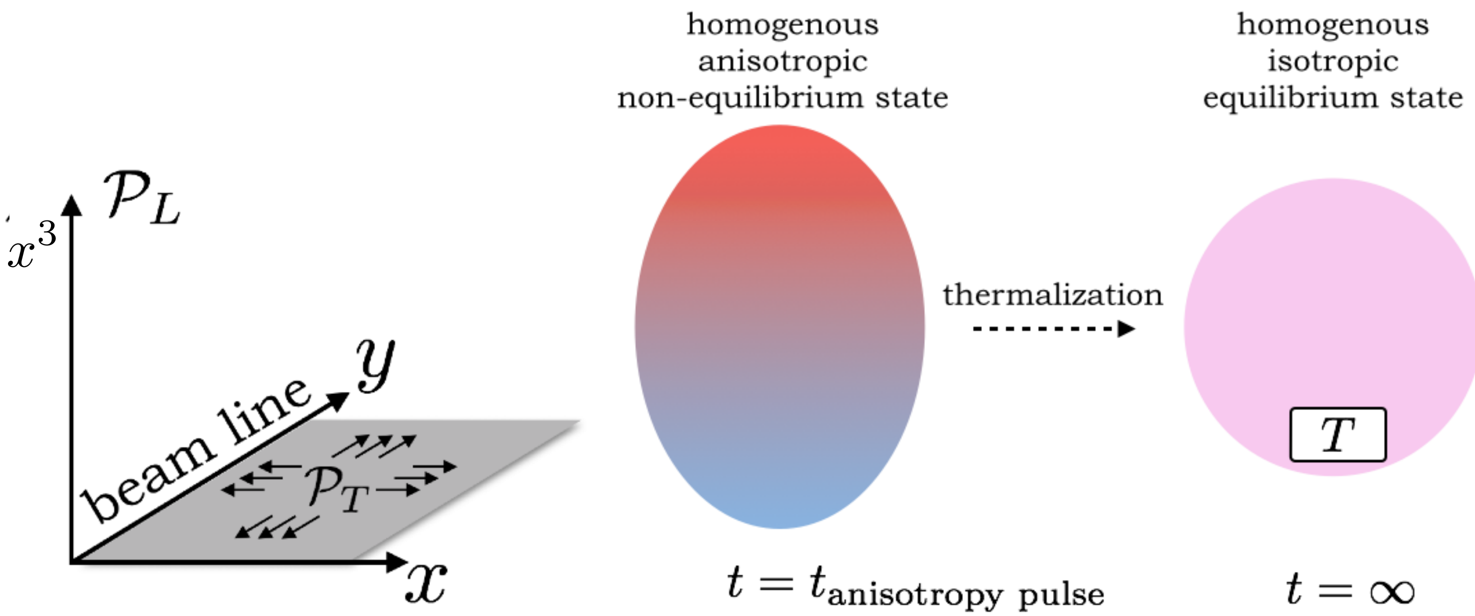
[Cartwright, Kaminski; arXiv (2019)]

Casey Cartwright
(University of Alabama)

Outline

1. Invitation
2. **Setup & Calculations**
3. Results
4. Discussion/Outlook

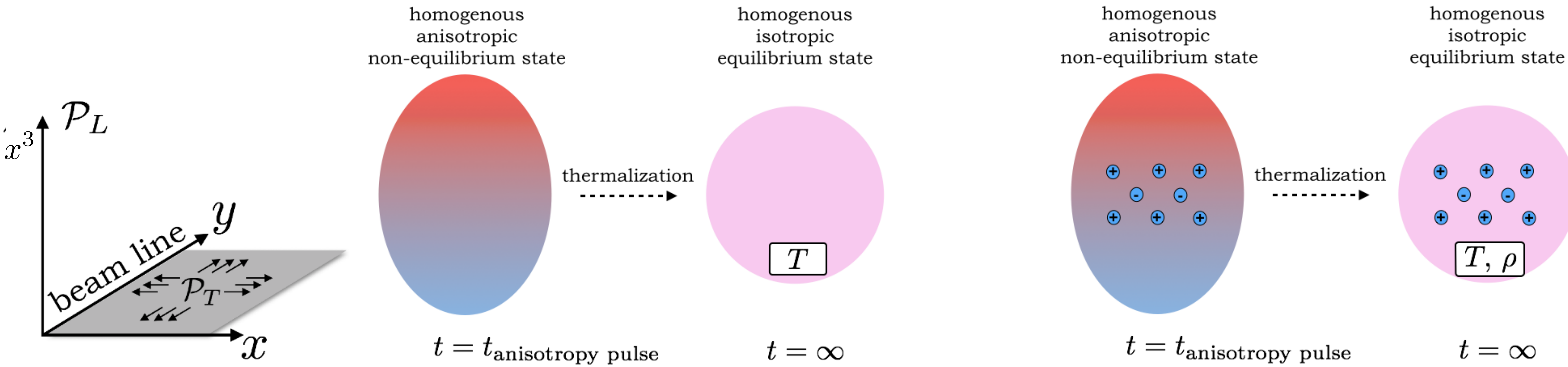
Fluid thermalizing after initial anisotropy



$\Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$

(a) neutral fluid

Fluid thermalizing after initial anisotropy

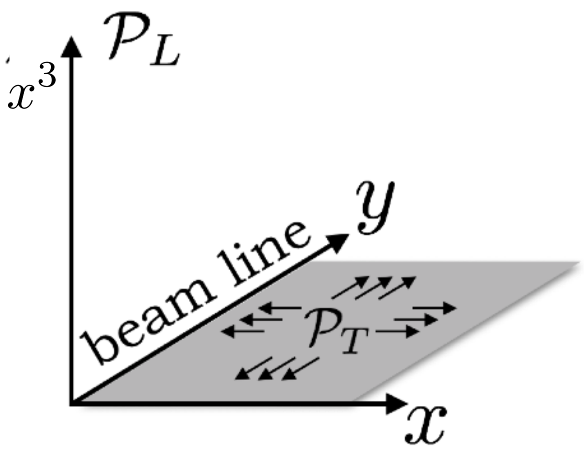


$\Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$

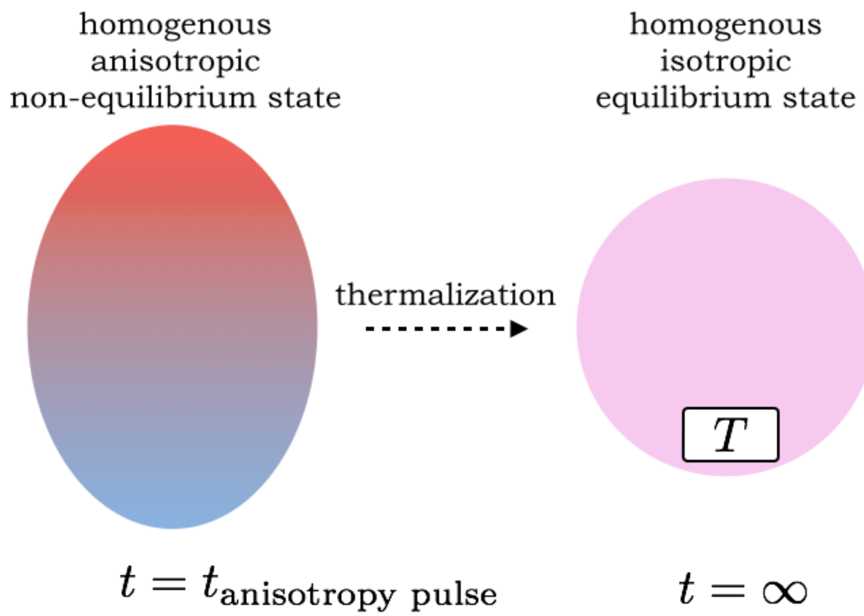
(a) neutral fluid

(b) charged fluid

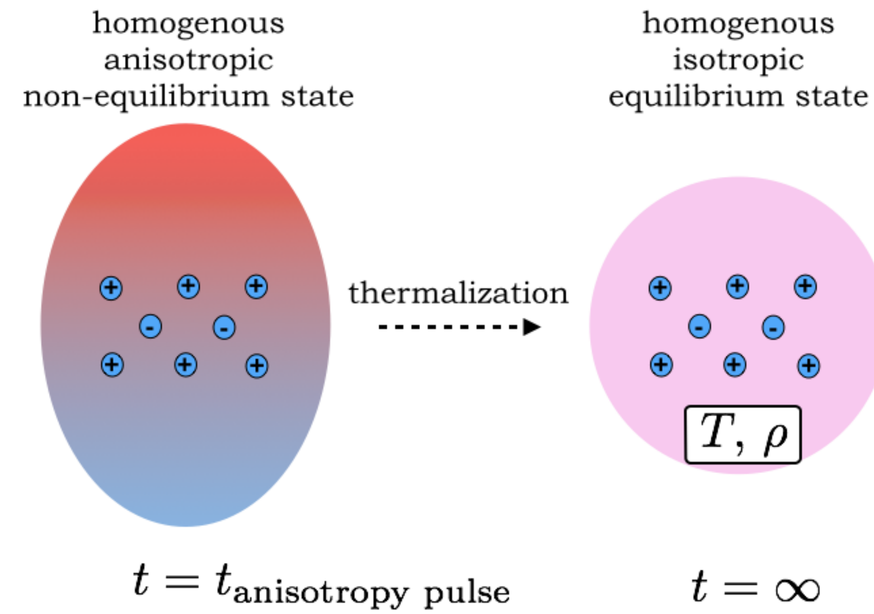
Fluid thermalizing after initial anisotropy



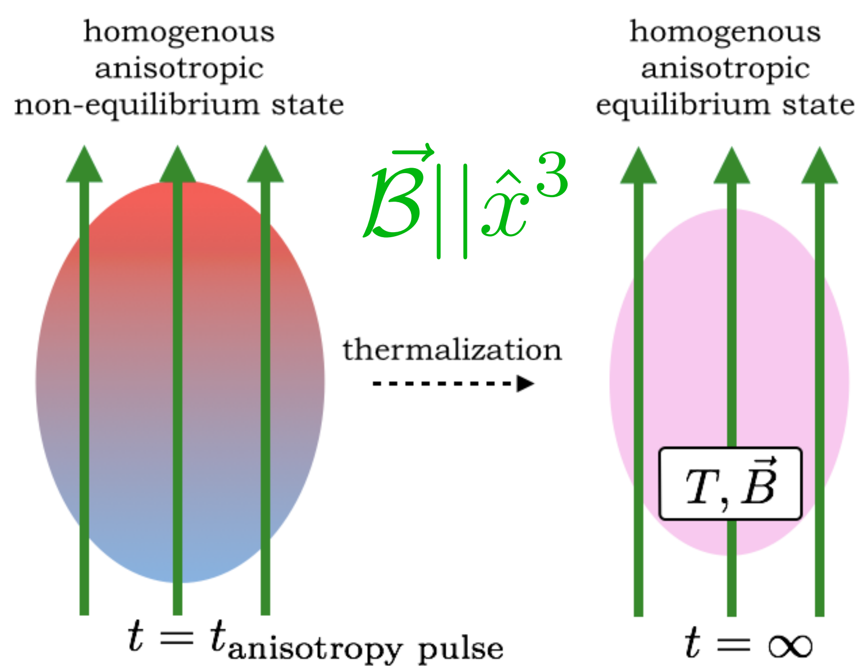
$$\Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$$



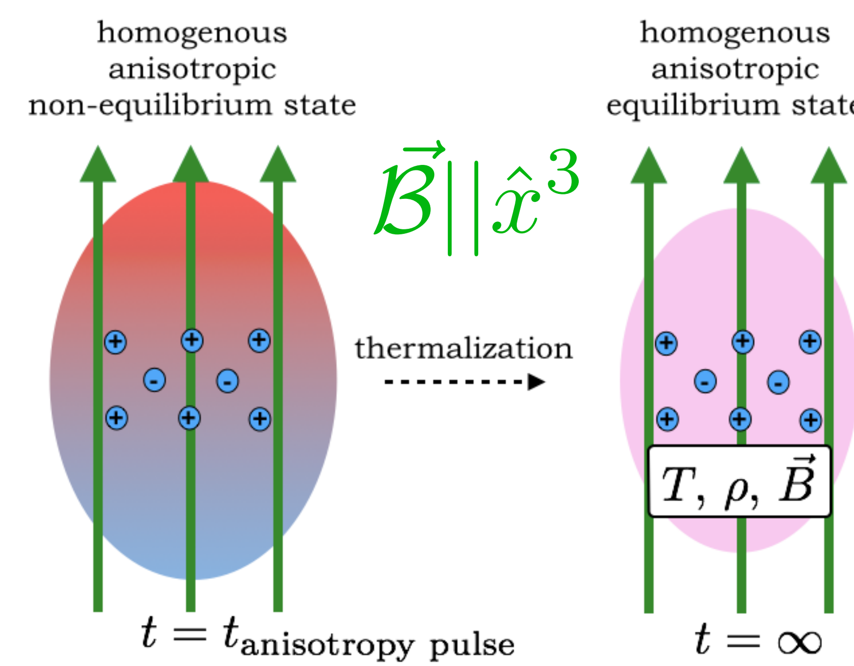
(a) neutral fluid



(b) charged fluid



(c) neutral fluid in magnetic field



(d) charged fluid in magnetic field

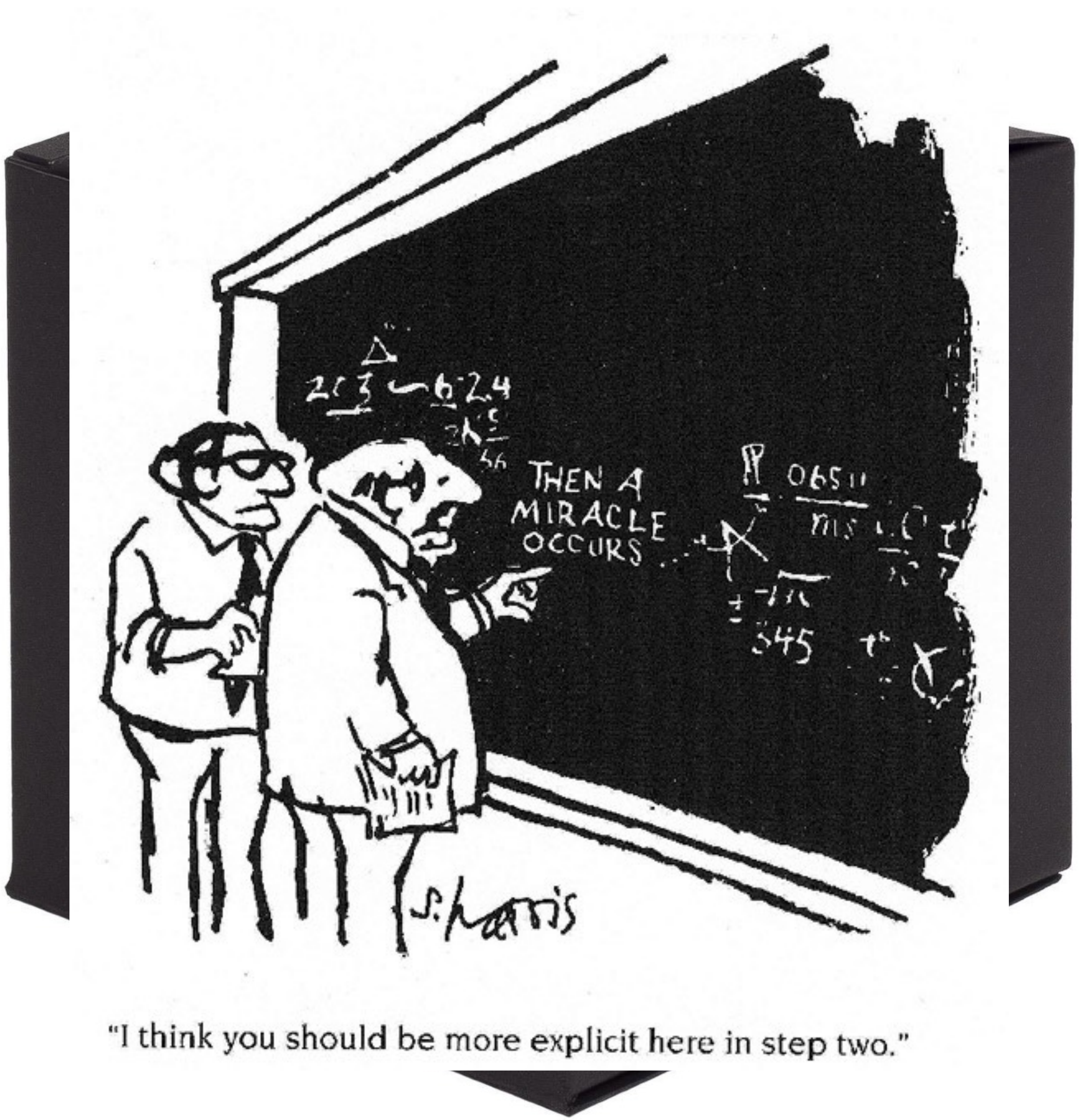
The AdS/CFT correspondence

gravity theory →

→ **gauge theory**

The AdS/CFT correspondence

gravity theory



"I think you should be more explicit here in step two."

gauge theory

Gravity setup (dual to *N*=4 SYM)

Einstein-Maxwell-Chern-Simons action:

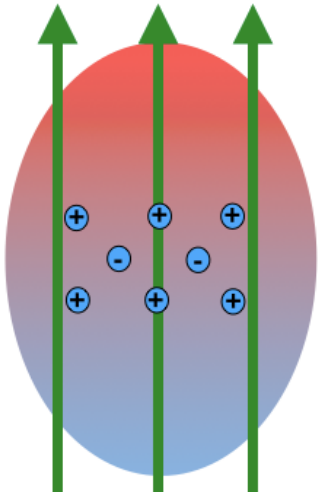
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda + F_{\mu\nu}F^{\mu\nu}) + \gamma\epsilon^{\alpha\beta\gamma\delta\eta}A_\alpha F_{\beta\gamma}F_{\delta\eta}$$

neglected in this work

Metric ansatz:

$$ds^2 = -A(r,t)dt^2 + 2drdt + S(t,r)^2(e^{B(r,t)}(dx^2 + dy^2) + e^{-2B(r,t)}dz^2)$$

homogenous
anisotropic
non-equilibrium state



$\vec{\mathcal{B}}||\hat{x}^3$

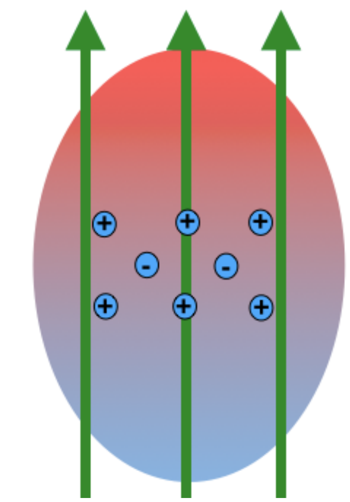
Gravity setup (dual to *N*=4 SYM)

Einstein-Maxwell-Chern-Simons action:

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,(R - 2\Lambda + F_{\mu\nu}F^{\mu\nu}) + \gamma\epsilon^{\alpha\beta\gamma\delta\eta}A_\alpha F_{\beta\gamma}F_{\delta\eta}$$

neglected in this work

homogenous
anisotropic
non-equilibrium state



$\vec{\mathcal{B}}||\hat{x}^3$

Metric ansatz:

$$\mathrm{d}s^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(t,r)^2(e^{B(r,t)}(\mathrm{d}x^2+\mathrm{d}y^2) + e^{-2B(r,t)}\mathrm{d}z^2)$$

Maxwell equations are solved by:

$$\mathscr{A}(r,t) = (0, \phi(r,t), -\frac{1}{2}y\mathscr{B}, \frac{1}{2}x\mathscr{B}, 0)$$

$$-\partial_r\phi(r,t) = \mathscr{E}(r,t) = \frac{\rho(r,t)}{S(t,r)^3}$$

Einstein equations are nested:

[Chesler, Yaffe; PRL (2009)]

$$S''(t,r) = -\frac{1}{2}B'(t,r)^2 S(t,r)$$

$$\dot{S}'(t,r) = \frac{\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^3} - \frac{2S'(t,r)\dot{S}(t,r)}{S(t,r)} + \frac{\rho^2}{3S(t,r)^5} + 2S(t,r)$$

Derivative: $\dot{f} = \partial_t f + \frac{1}{2}A\partial_r f.$

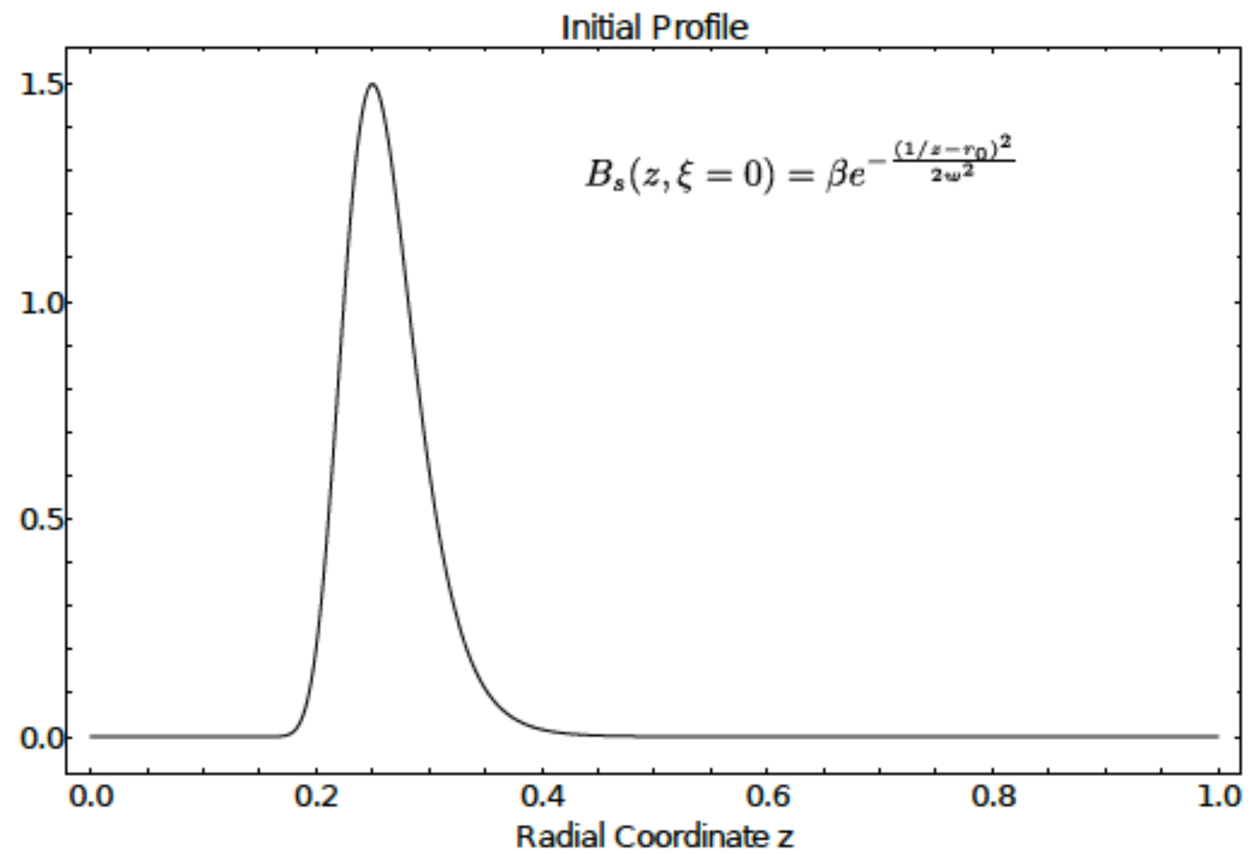
$$\dot{B}'(t,r) = -\frac{3\dot{B}(t,r)S'(t,r)}{2S(t,r)} - \frac{3B'(t,r)\dot{S}(t,r)}{2S(t,r)} + \frac{2\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4}$$

$$A''(t,r) = -3B'(t,r)\dot{B}(t,r) - \frac{10\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4} + \frac{12S'(t,r)\dot{S}(t,r)}{S(t,r)^2} - \frac{14\rho^2}{3S(t,r)^6} - 4$$

$$\ddot{S}(t,r) = \frac{1}{2}A'(t,r)\dot{S}(t,r) - \frac{1}{2}\dot{B}(t,r)^2 S(t,r).$$

Background (the state in the field theory)

$$ds^2 = -A(r,t)dt^2 + 2drdt + S(t,r)^2(e^{B_s(r,t)}(dx^2 + dy^2) + e^{-2B_s(r,t)}dz^2)$$



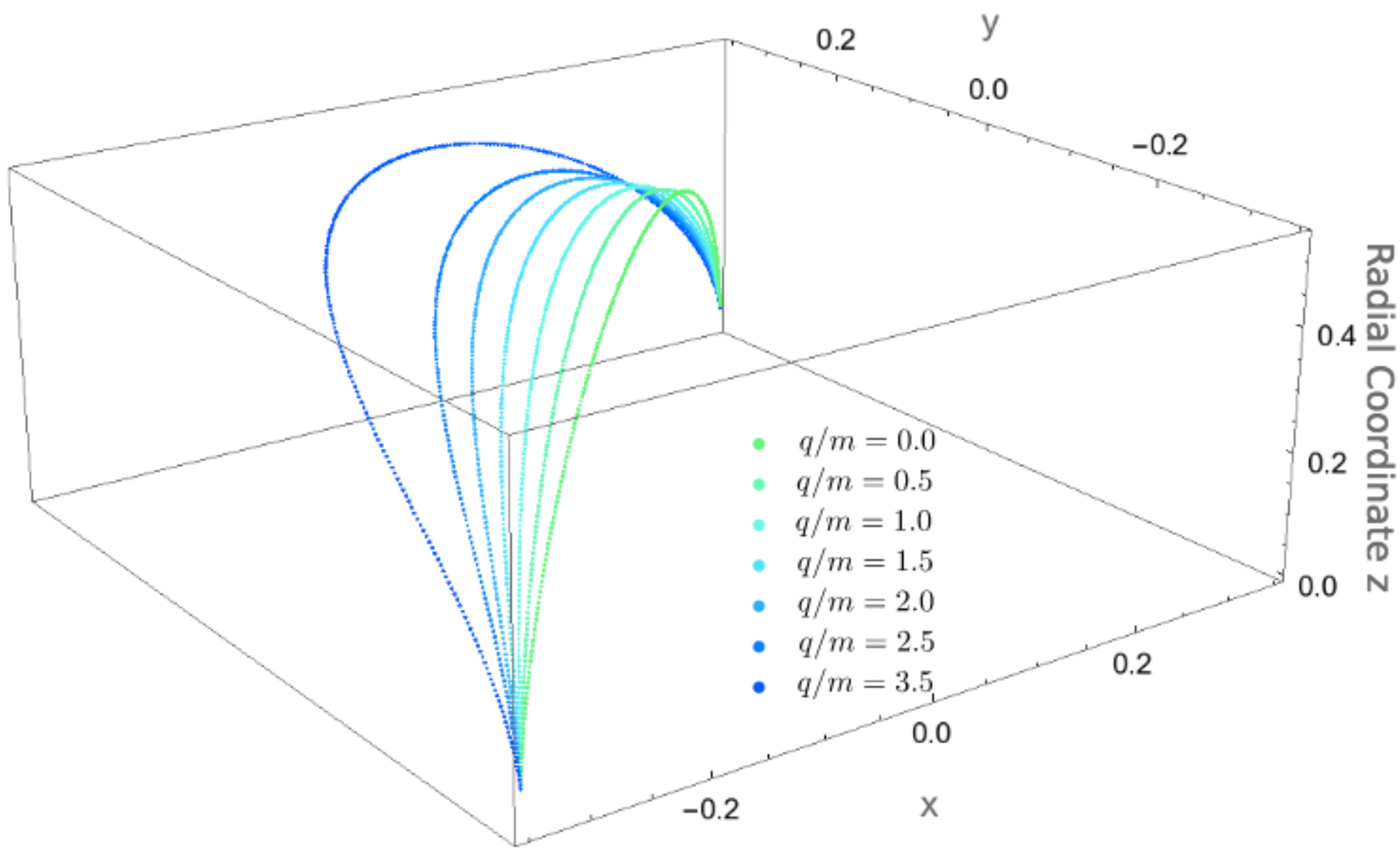
Numerical implementation- characteristic formulation

[Chesler, Yaffe; PRL (2009)]

- use (pseudo)spectral methods with Cardinal Function basis to solve ODEs in *r* at initial time for $S,\ \dot{S},\ \dot{B},\ A$ on Chebyshev grid
- time step forward using 4th order Runge-Kutta on first 4 time steps, and subsequently Adams-Bashforth
- boundary expand and solve for subtracted and scaled functions
- radial diffeomorphism used to keep horizon fixed

Correlations - geodesic approximation

[Balasubramanian, Ross; PRD(2000)]



Correlations - geodesic approximation

[Balasubramanian, Ross; PRD(2000)]

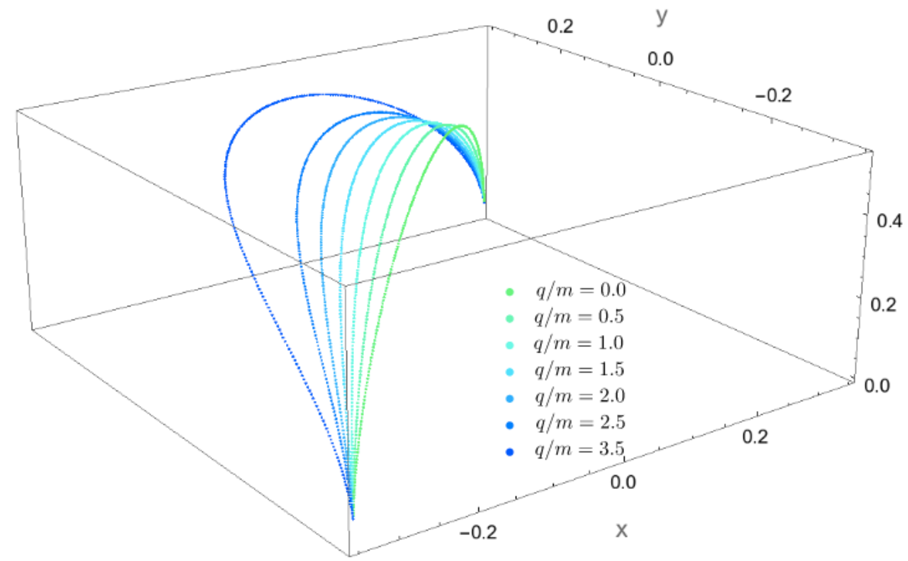
Correlator as a sum over geodesics:

$$\langle \mathcal{O}(t,\vec{x}_1)\mathcal{O}(t,\vec{x}_2)\rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L}$$

Geodesic length (Lagrangian): $\Delta L = L - L_{\text{thermalized}}$

$$L = \int \mathrm{d}\lambda \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \quad \Rightarrow \frac{d^2 x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0$$

geodesic equation



Numerical implementation - relaxation method:

[Ecker, Grumiller, Stricker; JHEP (2015)]

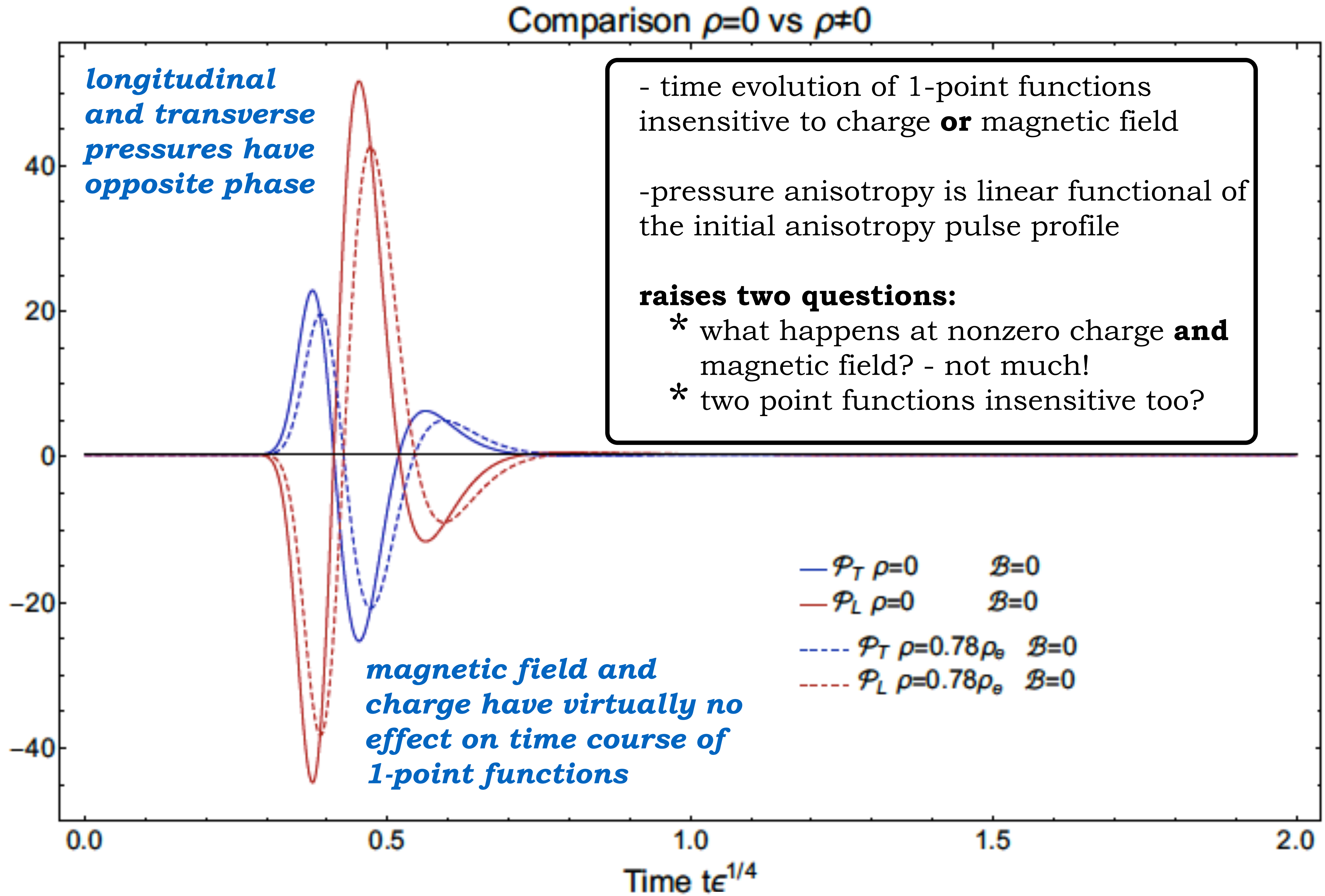
1. Generate the dynamic background
2. Generate interpolations of the metric functions
3. Discretize the geodesic equations using a relaxation scheme
4. Approximate the proper length using a Riemann sum

Outline

1. Invitation
2. Setup & Calculations
3. **Results**
4. Discussion/Outlook

Background - 1-Point Functions

reproduces and extends [Fuini, Yaffe; JHEP (2015)]



longitudinal and transverse pressures have opposite phase

- time evolution of 1-point functions insensitive to charge **or** magnetic field

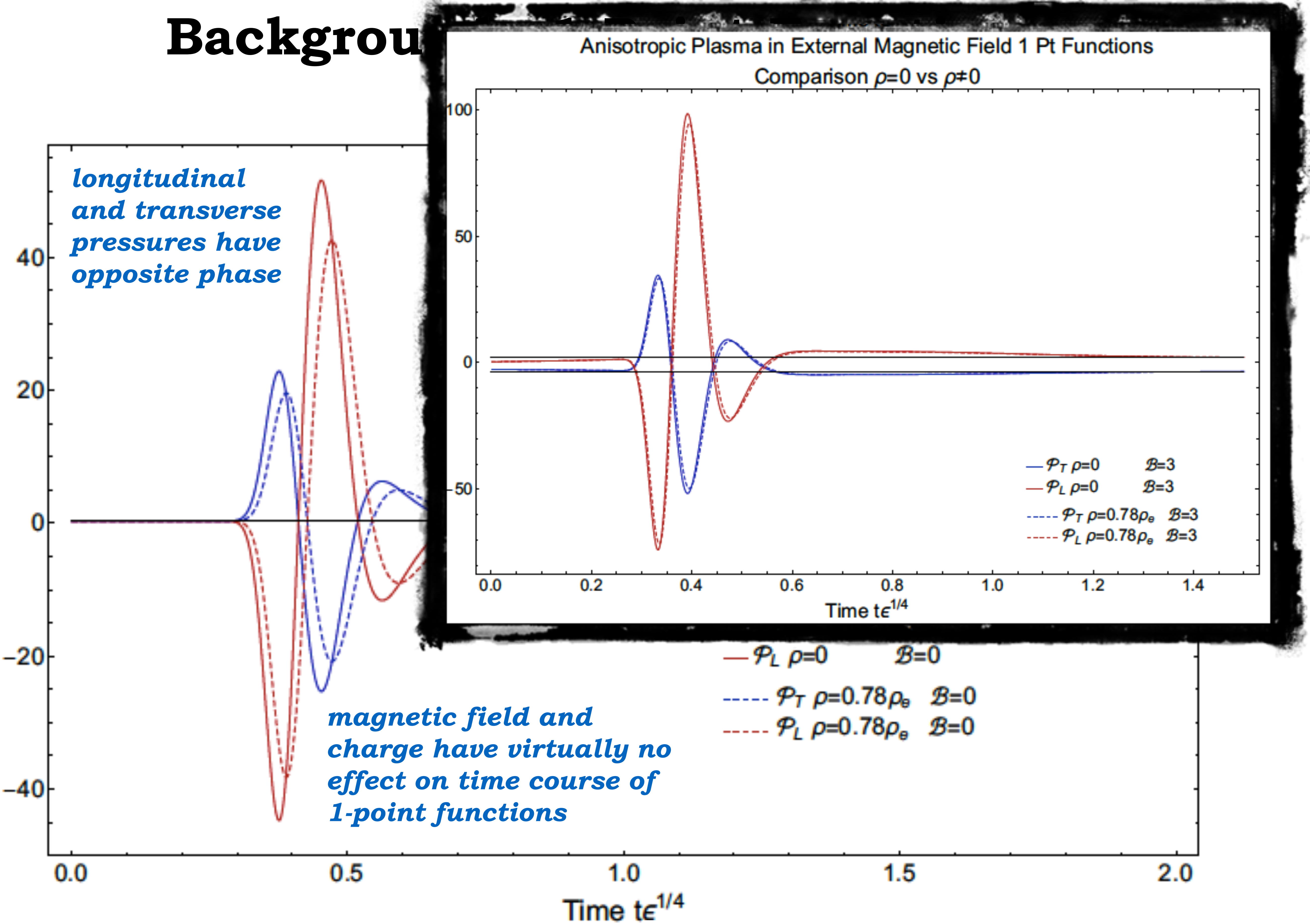
-pressure anisotropy is linear functional of the initial anisotropy pulse profile

raises two questions:

- what happens at nonzero charge **and** magnetic field? - not much!
- two point functions insensitive too?

magnetic field and charge have virtually no effect on time course of 1-point functions

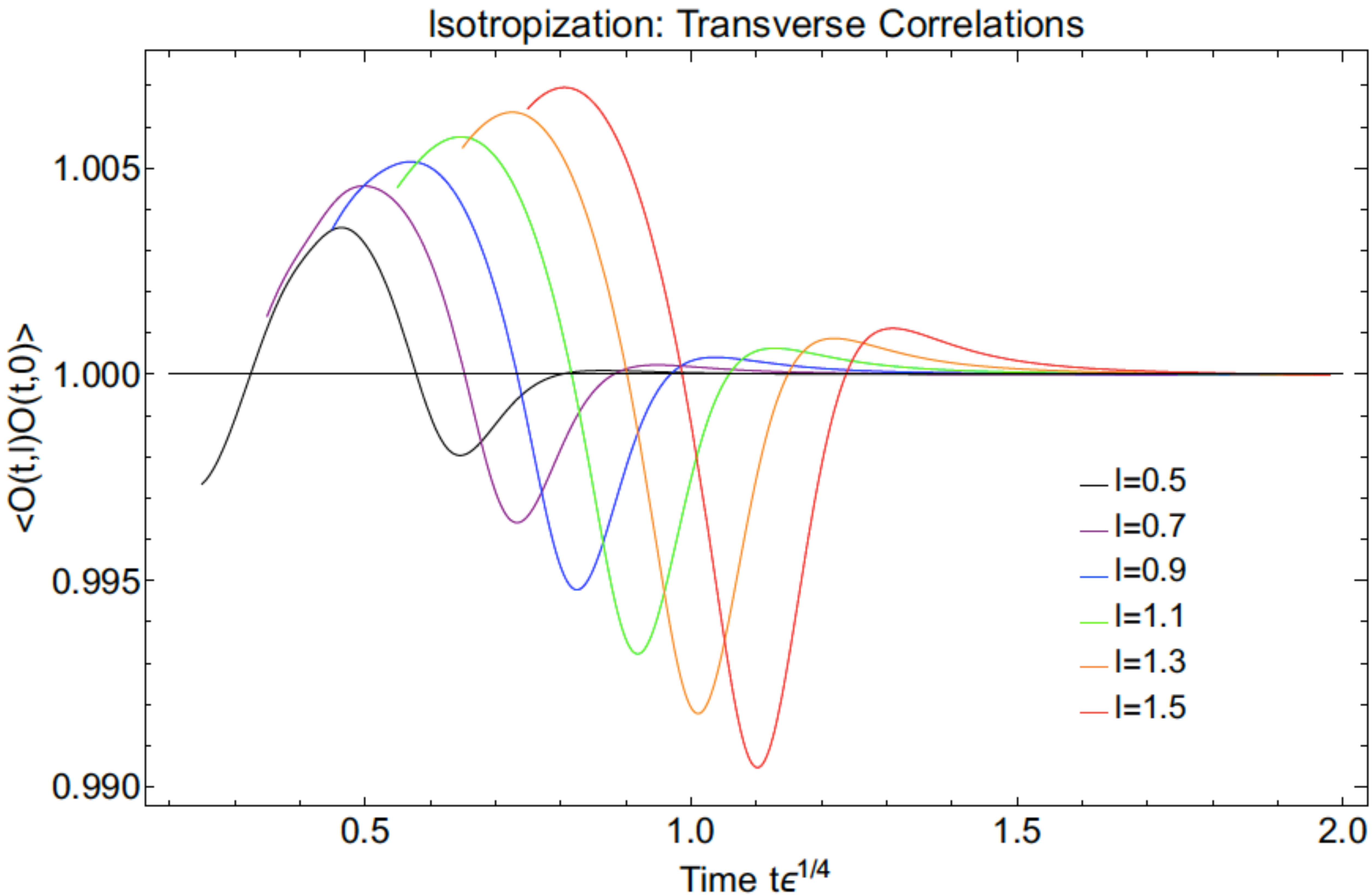
Backgrou



longitudinal and transverse pressures have opposite phase
magnetic field and charge have virtually no effect on time course of 1-point functions
40
20
0
−20
−40
0.0
0.5
1.0
1.5
2.0
Time tε^1/4
$\mathcal{P}_L$ ρ=0 $\mathcal{B}$=0
$\mathcal{P}_T$ ρ=0.78ρₑ $\mathcal{B}$=0
$\mathcal{P}_L$ ρ=0.78ρₑ $\mathcal{B}$=0
Anisotropic Plasma in External Magnetic Field 1 Pt Functions
Comparison ρ=0 vs ρ≠0
100
50
0
-50
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Time tε^1/4
$\mathcal{P}_T$ ρ=0 $\mathcal{B}$=3
$\mathcal{P}_L$ ρ=0 $\mathcal{B}$=3
$\mathcal{P}_T$ ρ=0.78ρₑ $\mathcal{B}$=3
$\mathcal{P}_L$ ρ=0.78ρₑ $\mathcal{B}$=3

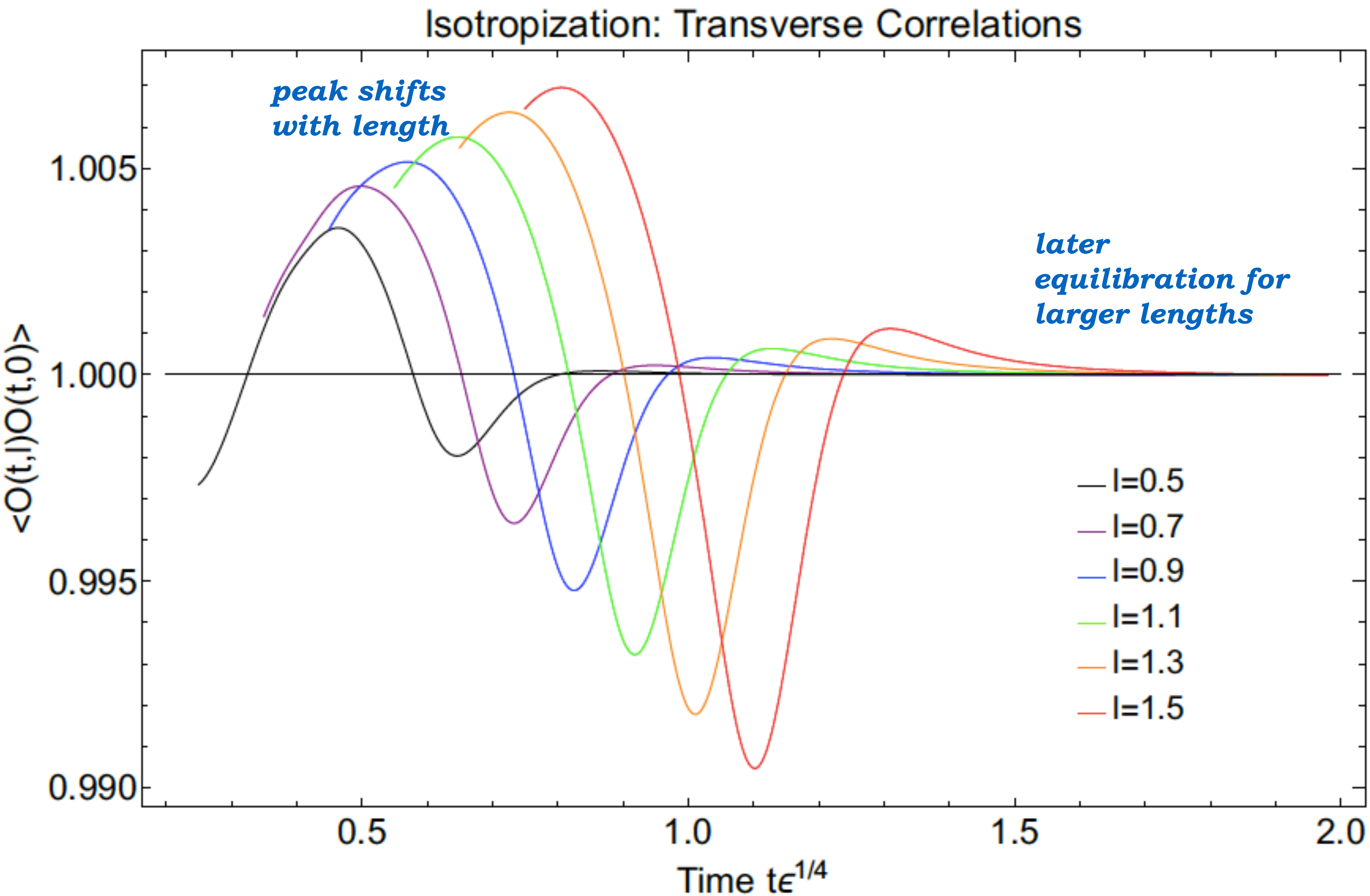
Correlations - zero charge, zero *B*

results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]



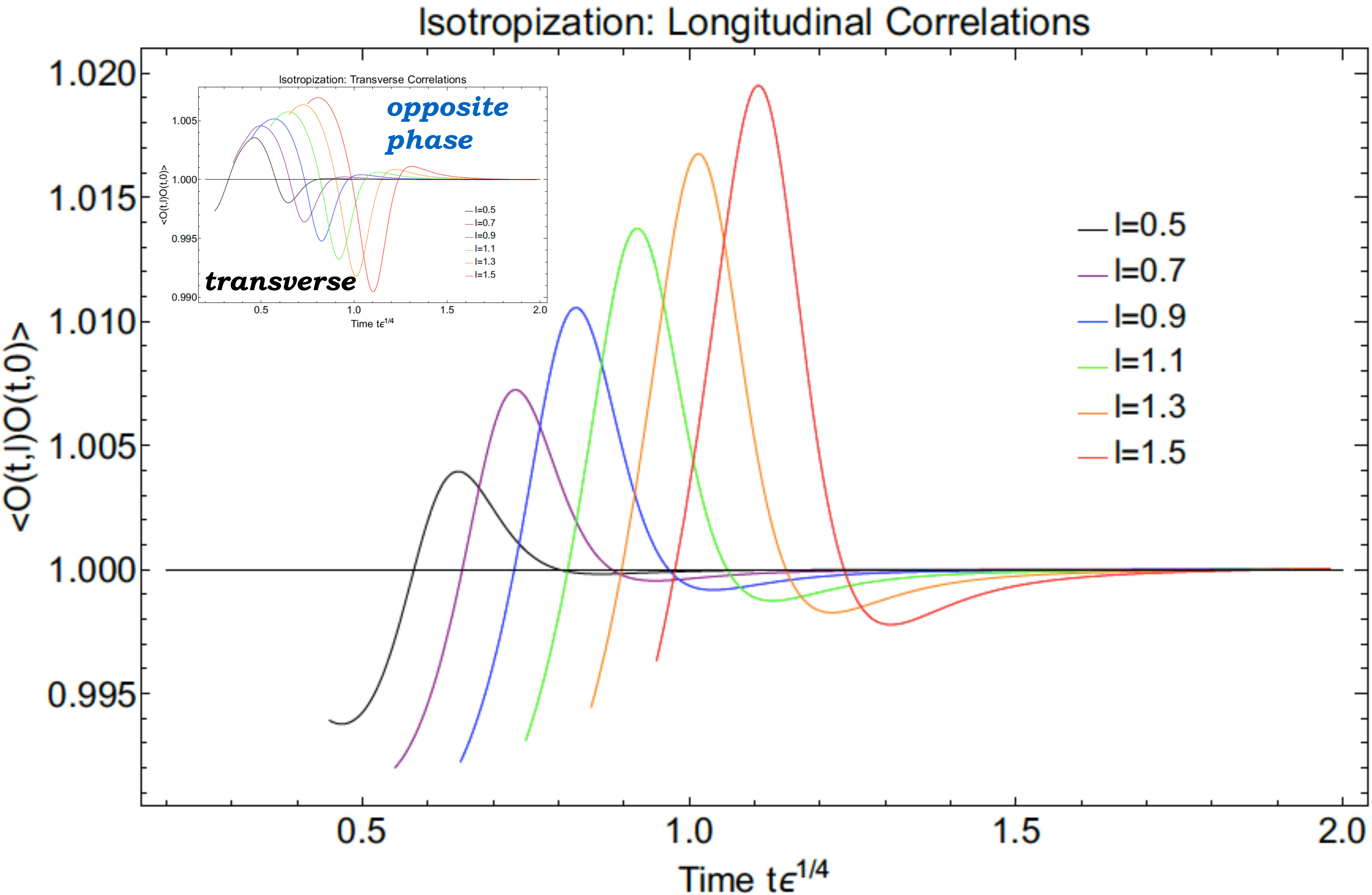
Correlations - zero charge, zero B

results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]



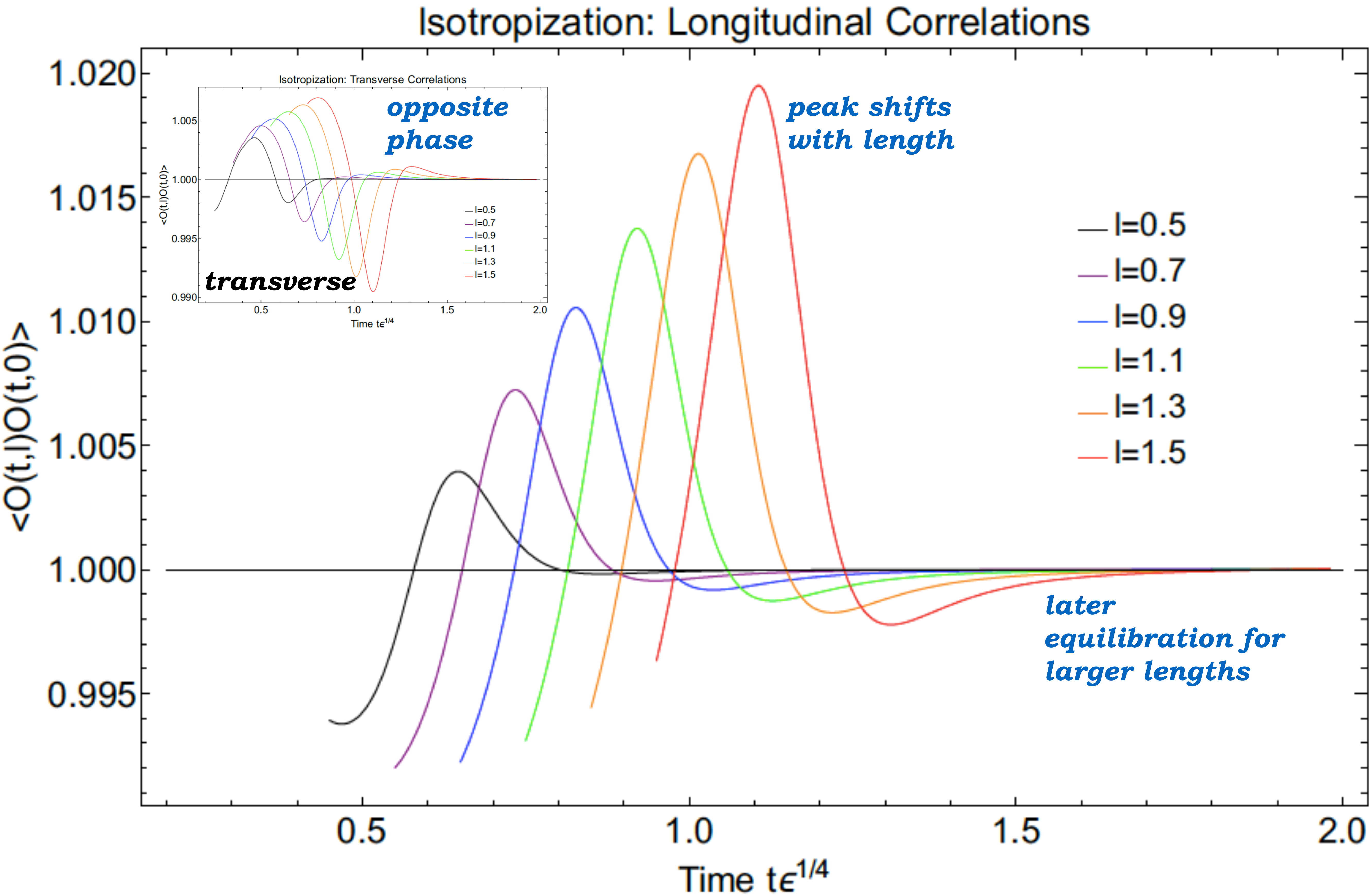
Correlations - zero charge, zero B

reproducing results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]



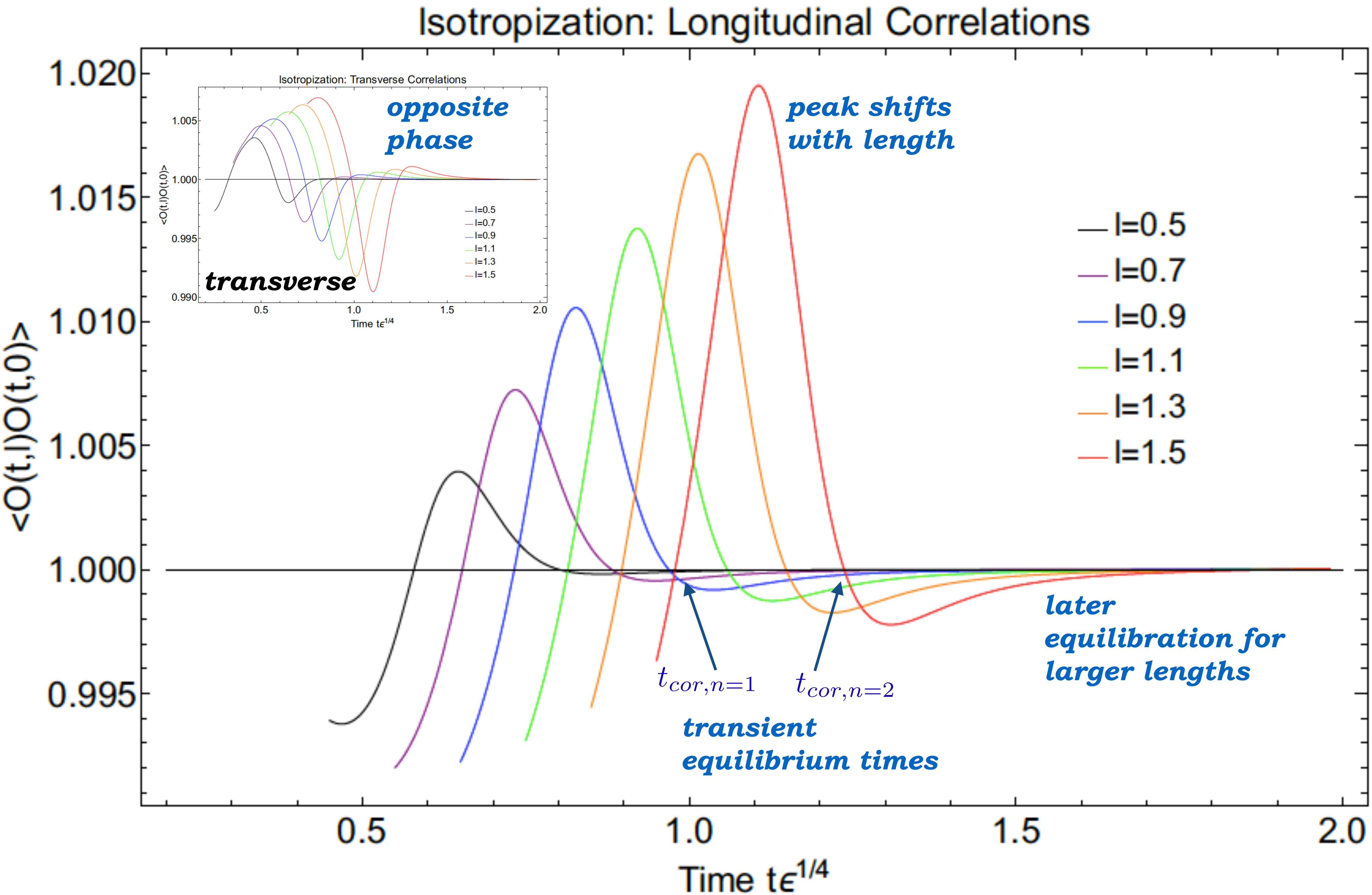
Correlations - zero charge, zero *B*

reproducing results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]

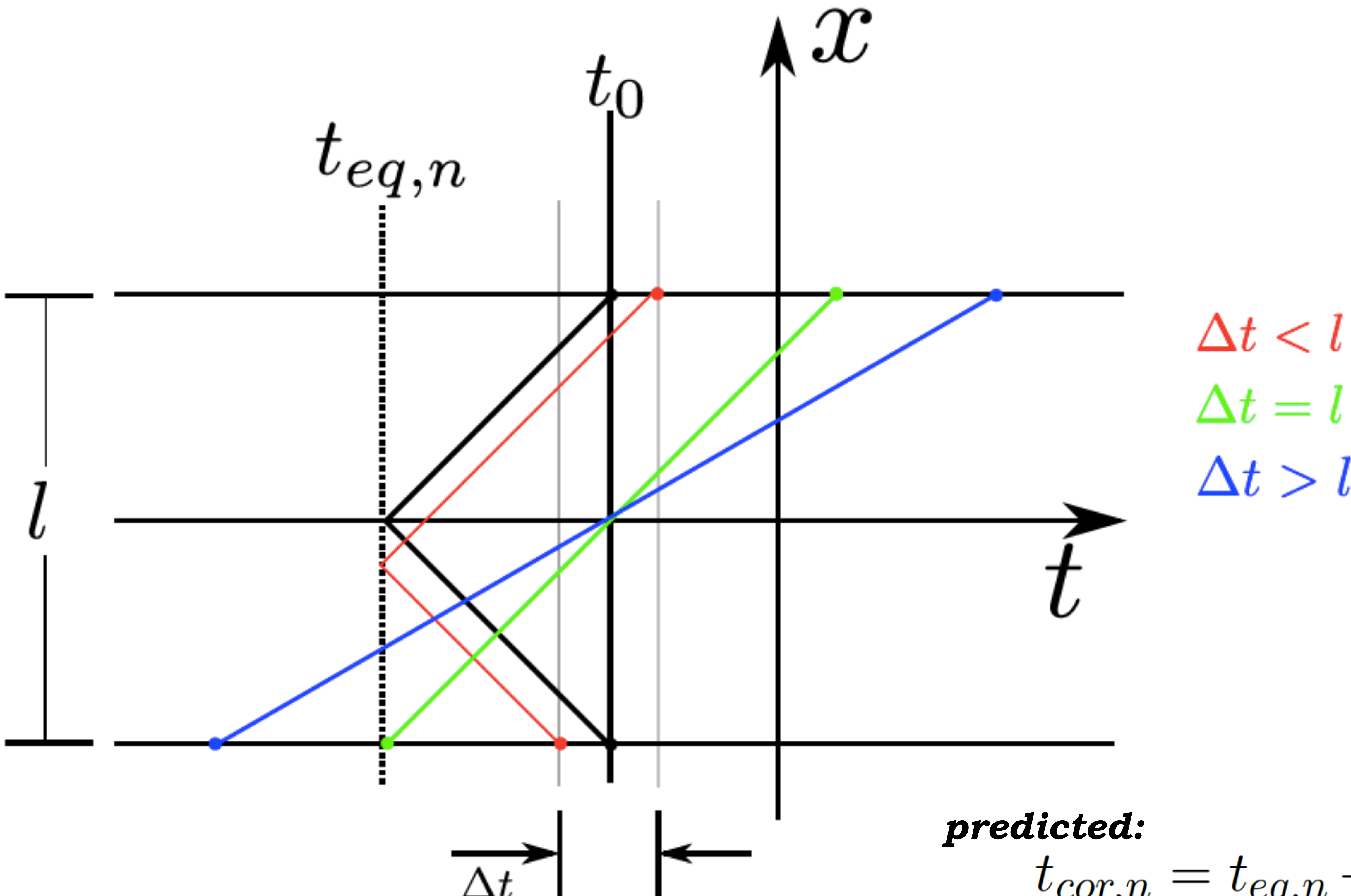


Correlations - zero charge, zero B

reproducing results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]



Transient equilibrium times



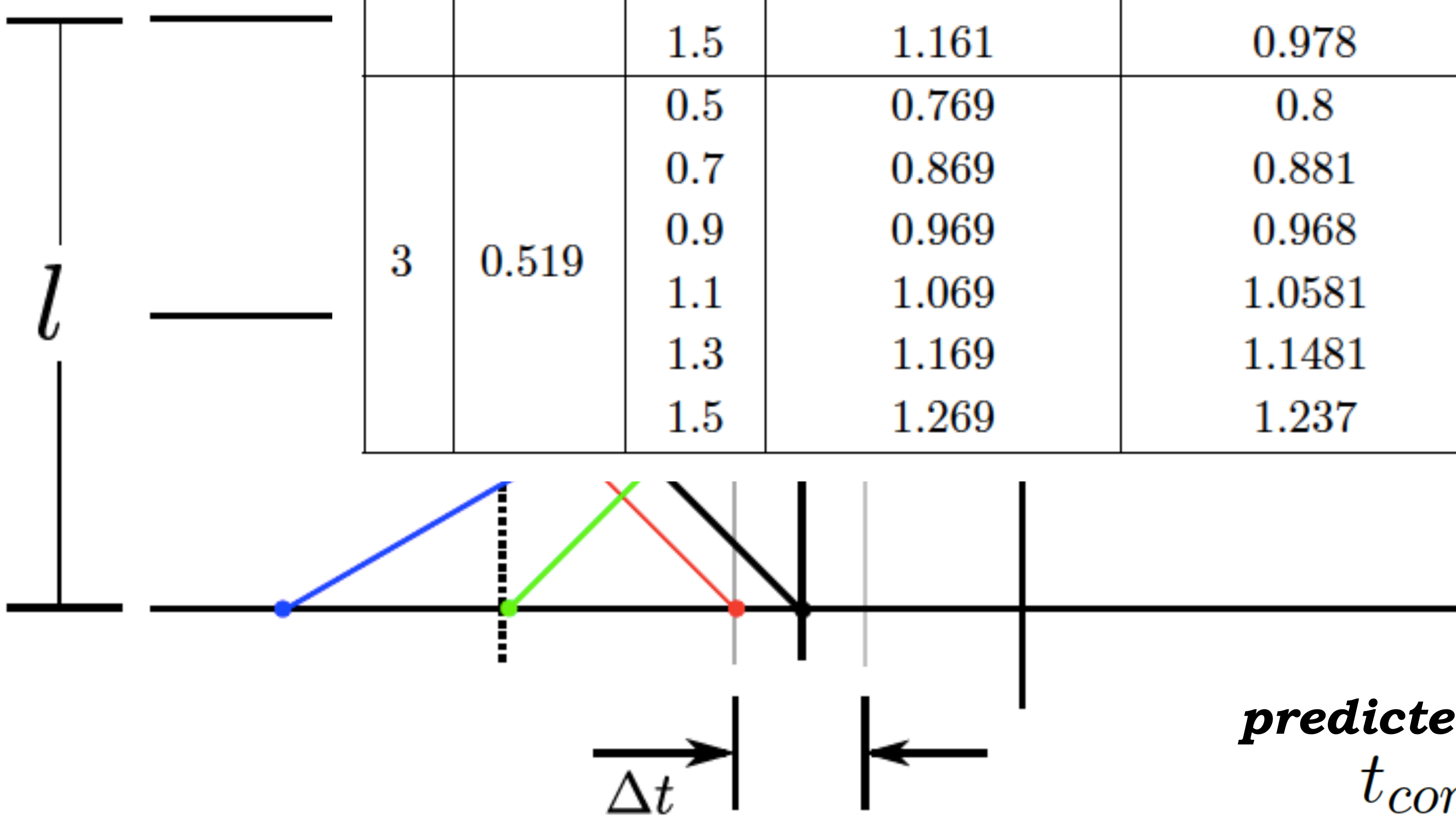
predicted:

$$t_{cor,n} = t_{eq,n} + l/2$$

Transient equilibrium times

Longitudinal separation

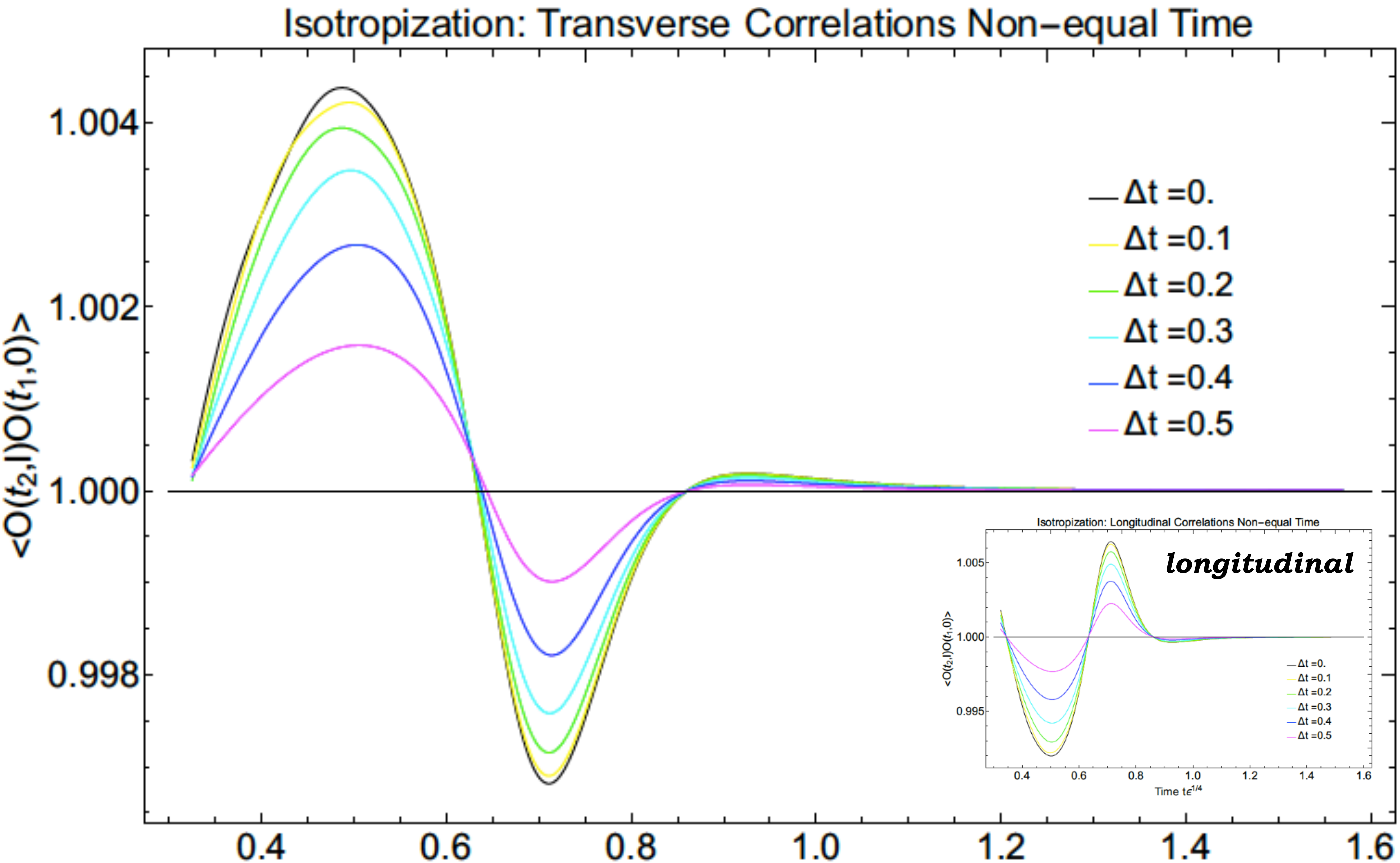
n	$t_{eq,n}$	$l\epsilon^{1/4}$	Predicted $t_{cor,n}$	Numerical $t_{cor,n}$	Relative percent error
2	0.411	0.5	0.661	0.578	13.301
		0.7	0.761	0.653	15.342
		0.9	0.861	0.732	16.188
		1.1	0.961	0.814	16.553
		1.3	1.061	0.897	16.785
		1.5	1.161	0.978	17.063
3	0.519	0.5	0.769	0.8	4.026
		0.7	0.869	0.881	1.43
		0.9	0.969	0.968	0.042
		1.1	1.069	1.0581	1.013
		1.3	1.169	1.1481	1.797
		1.5	1.269	1.237	2.553



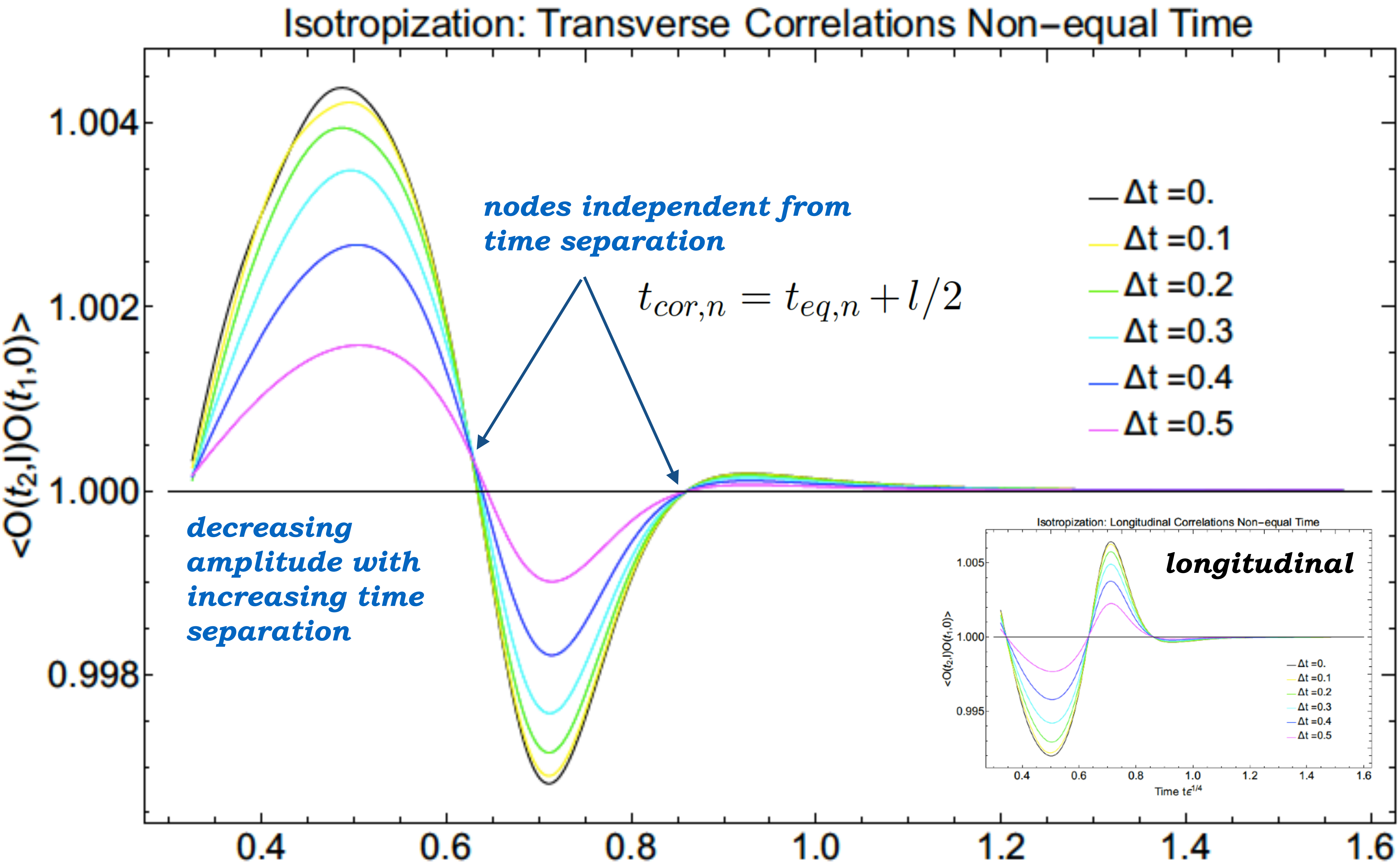
predicted:

$$t_{cor,n} = t_{eq,n} + l/2$$

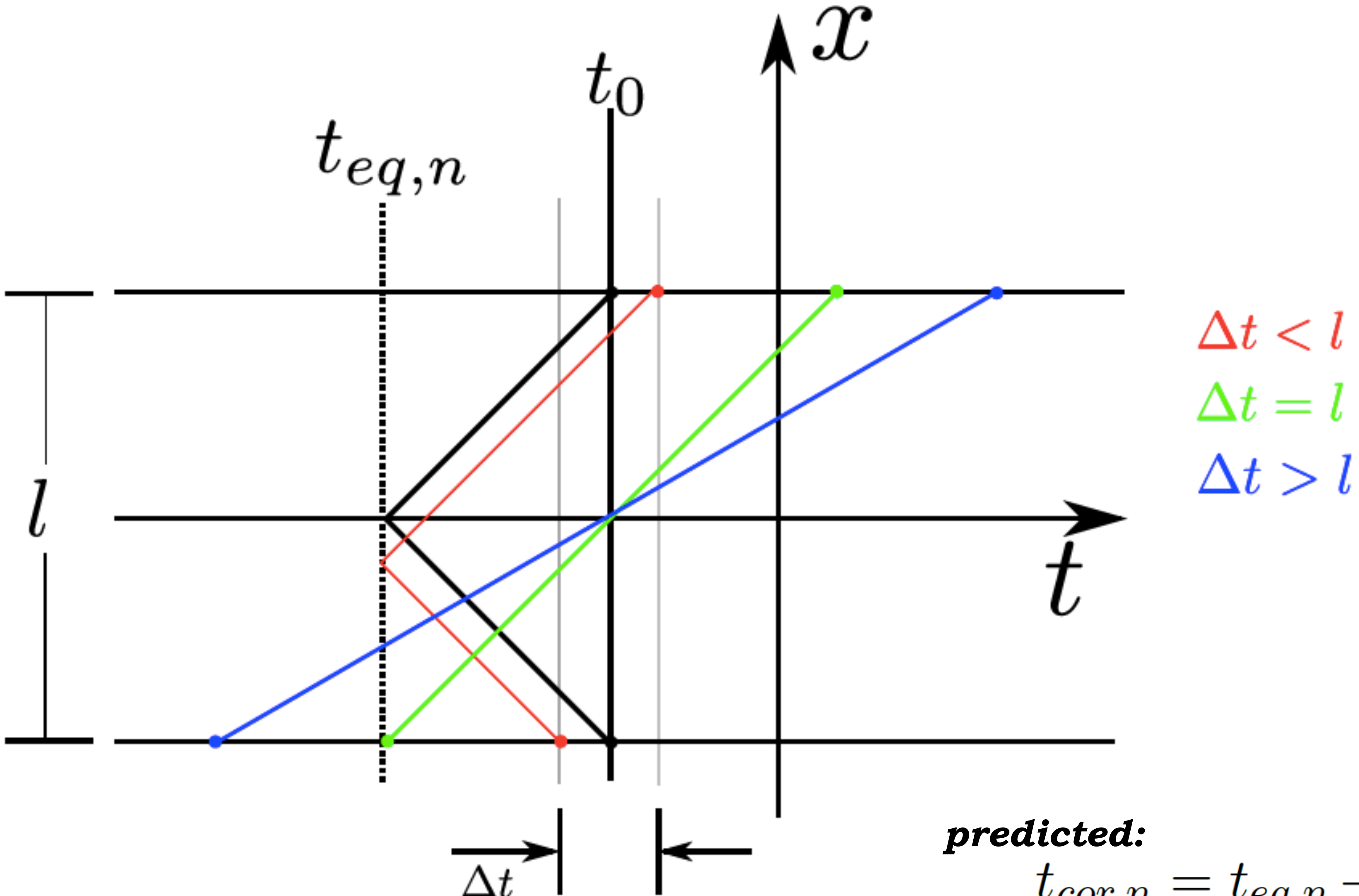
Correlations - zero charge, zero B



Correlations - zero charge, zero *B*



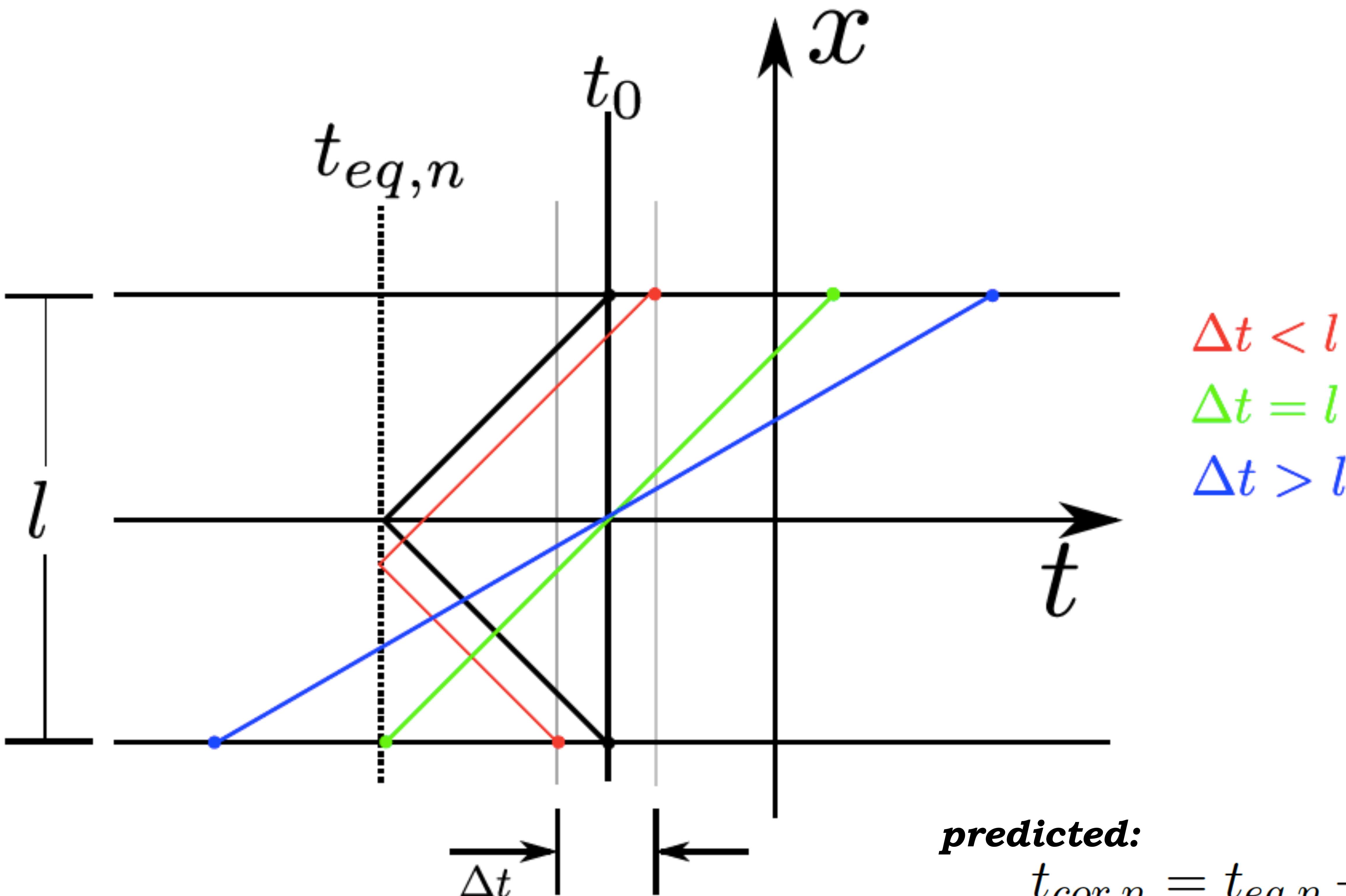
Transient equilibrium times



predicted:

$$t_{cor,n} = t_{eq,n} + l/2$$

Transient equilibrium times



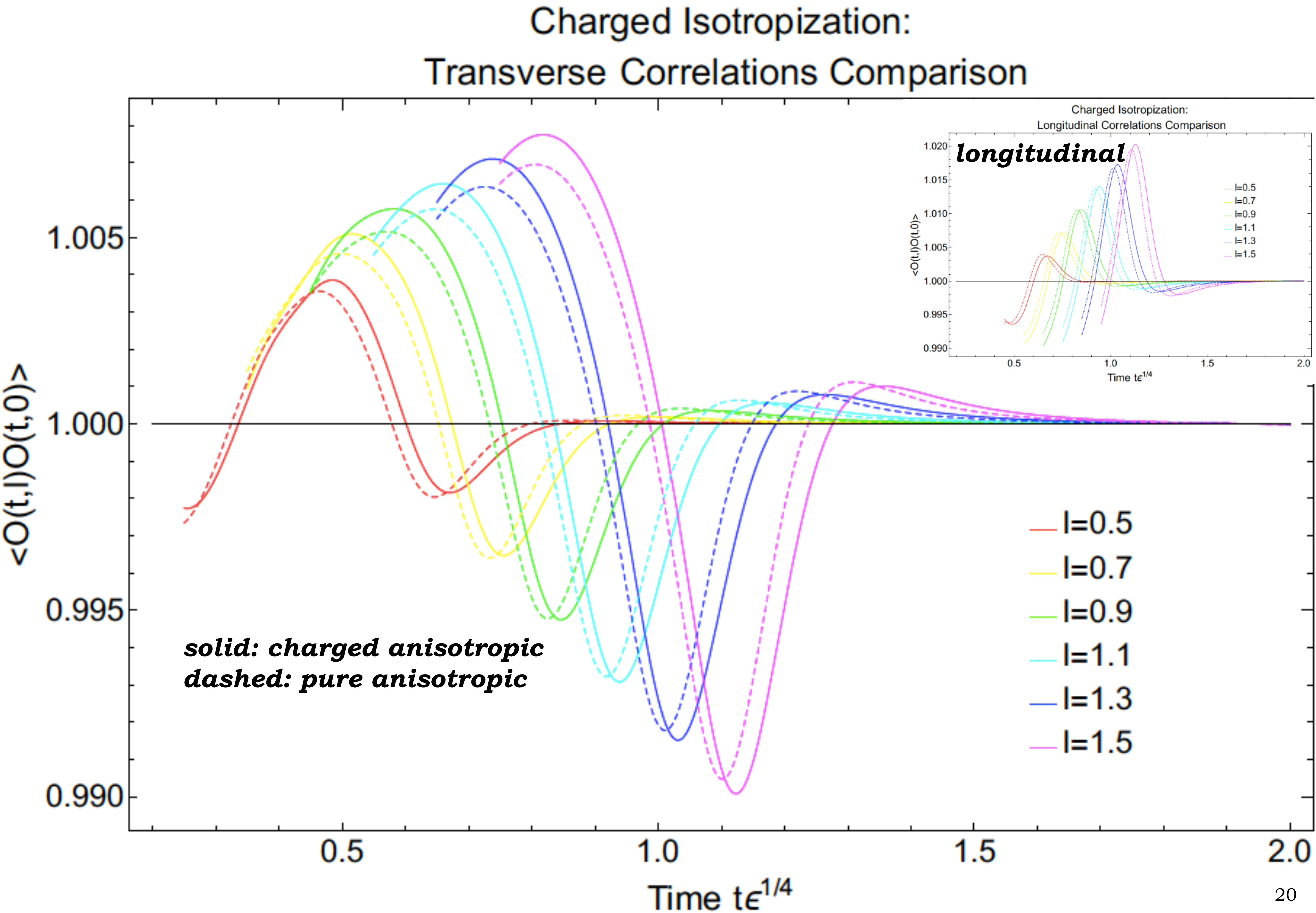
predicted:

$$t_{cor,n} = t_{eq,n} + l/2$$

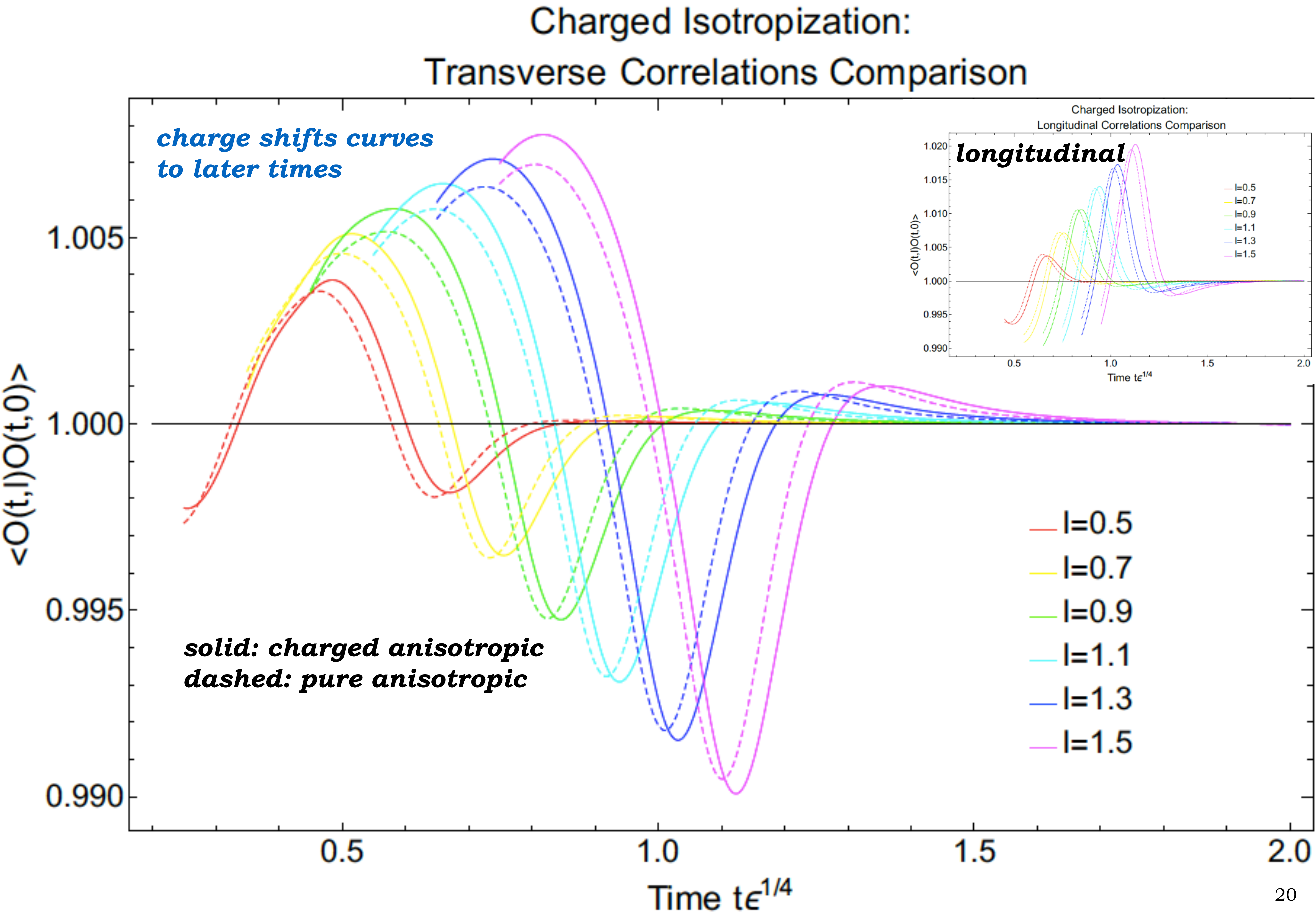
earlier node: 14.4%

later node: 2%

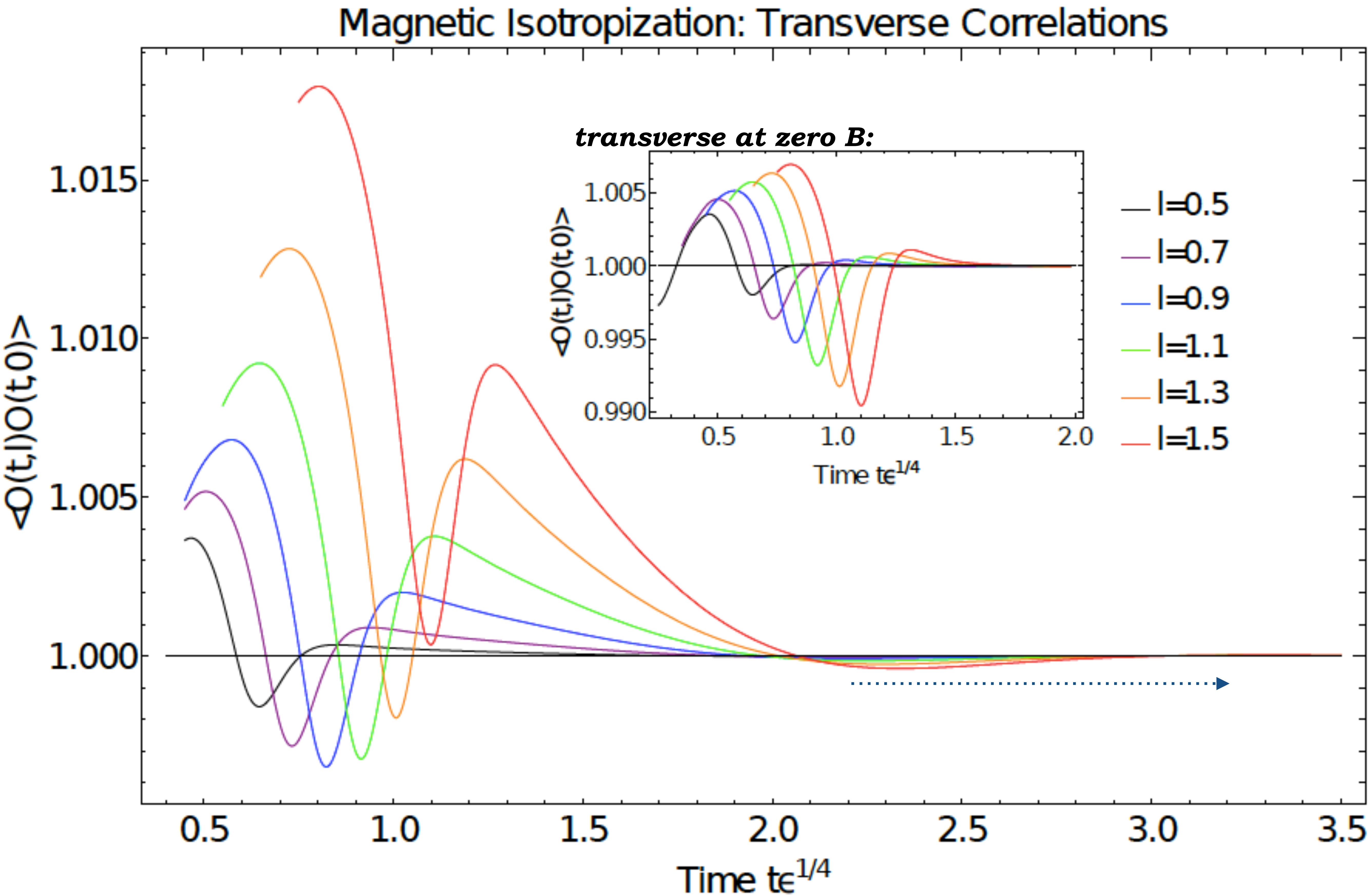
Correlations - nonzero charge, zero *B*



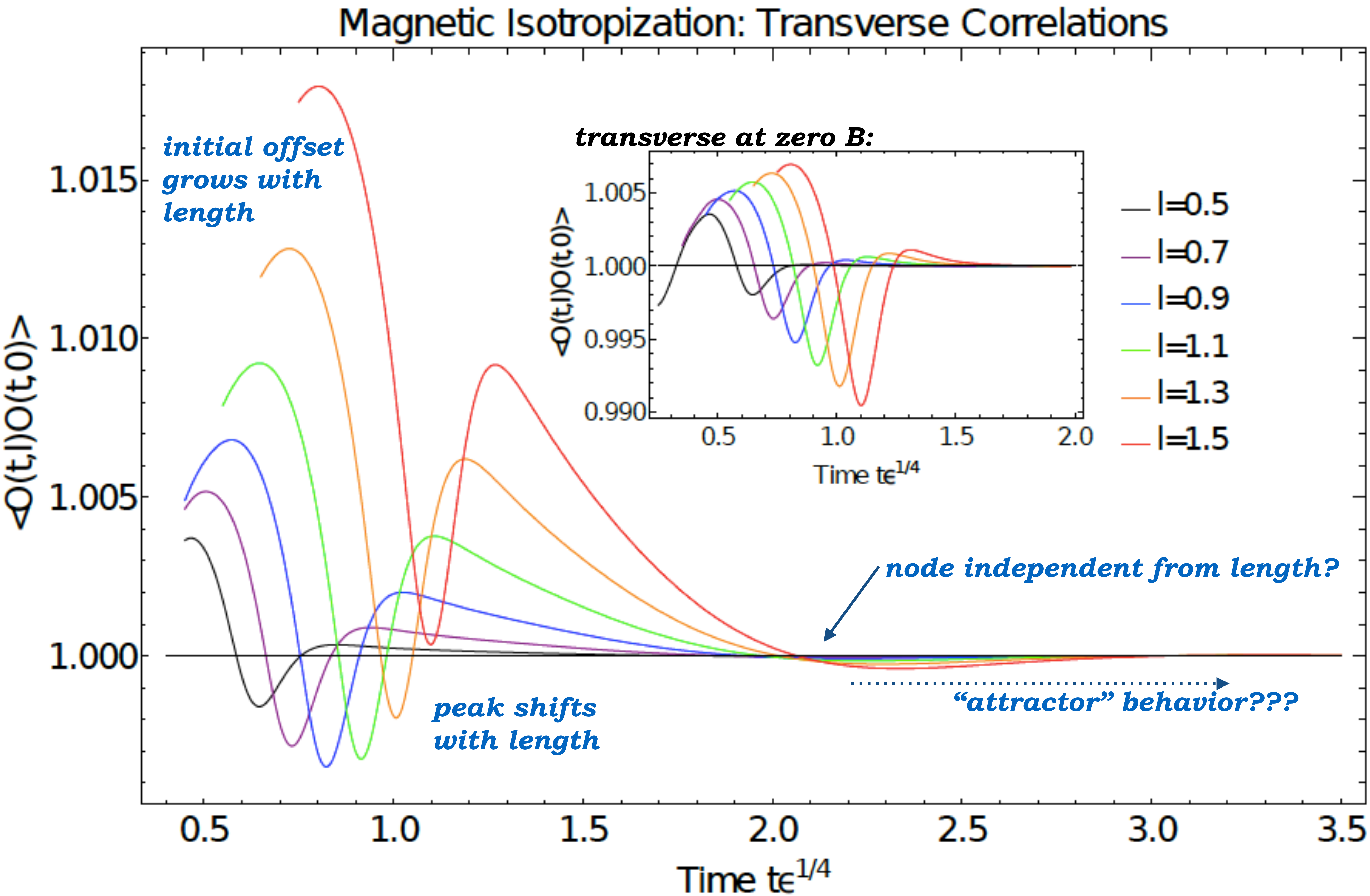
Correlations - nonzero charge, zero *B*



Correlations - zero charge, nonzero *B*



Correlations - zero charge, nonzero *B*



Transient equilibrium times
- zero charge, nonzero *B*

Longitudinal separation

n	$t_{eq,n}$	$l\epsilon^{1/4}$	Predicted $t_{cor,n}$	Numerical $t_{cor,n}$	Relative percent error
3	0.518	0.5	0.768	0.579	28.125
		0.7	0.868	0.660	27.224
		0.9	0.968	0.755	24.737
		1.1	1.068	0.863	21.322
		1.3	1.168	0.981	17.406
		1.5	1.268	D.N.E	D.N.E
4	0.716	0.5	0.966	0.782	21.091
		0.7	1.066	0.844	23.268
		0.9	1.166	0.916	24.011
		1.1	1.266	0.99	24.44
		1.3	1.366	1.061	25.137
		1.5	1.4659	D.N.E	D.N.E
5	1.896	0.5	2.146	1.909	11.69
		0.7	2.246	1.915	15.879
		0.9	2.346	1.944	18.712
		1.1	2.446	1.983	20.867
		1.3	2.546	2.03	22.517
		1.5	2.646	2.083	23.776

Thermalization times - definition

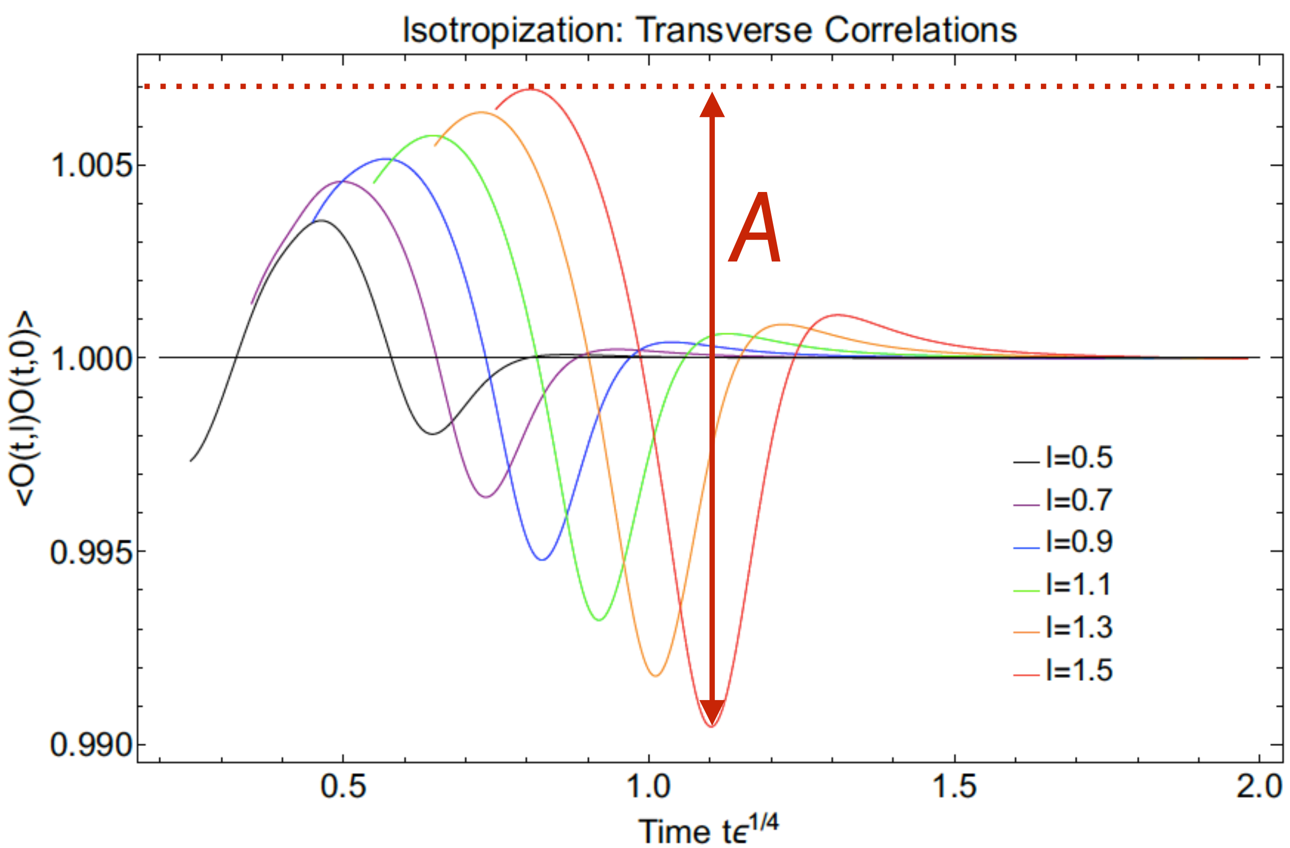
The thermalization time is that time t, for which the following equation is satisfied for all times greater than t:

$$|\Delta\langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle - \Delta\langle\mathcal{O}(t=\infty,l)\mathcal{O}(t=\infty,0)\rangle| \leq 0.01A$$

Peak to peak amplitude: $A = \max(\Delta\langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle) - \min(\Delta\langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle)$

Anisotropic part:

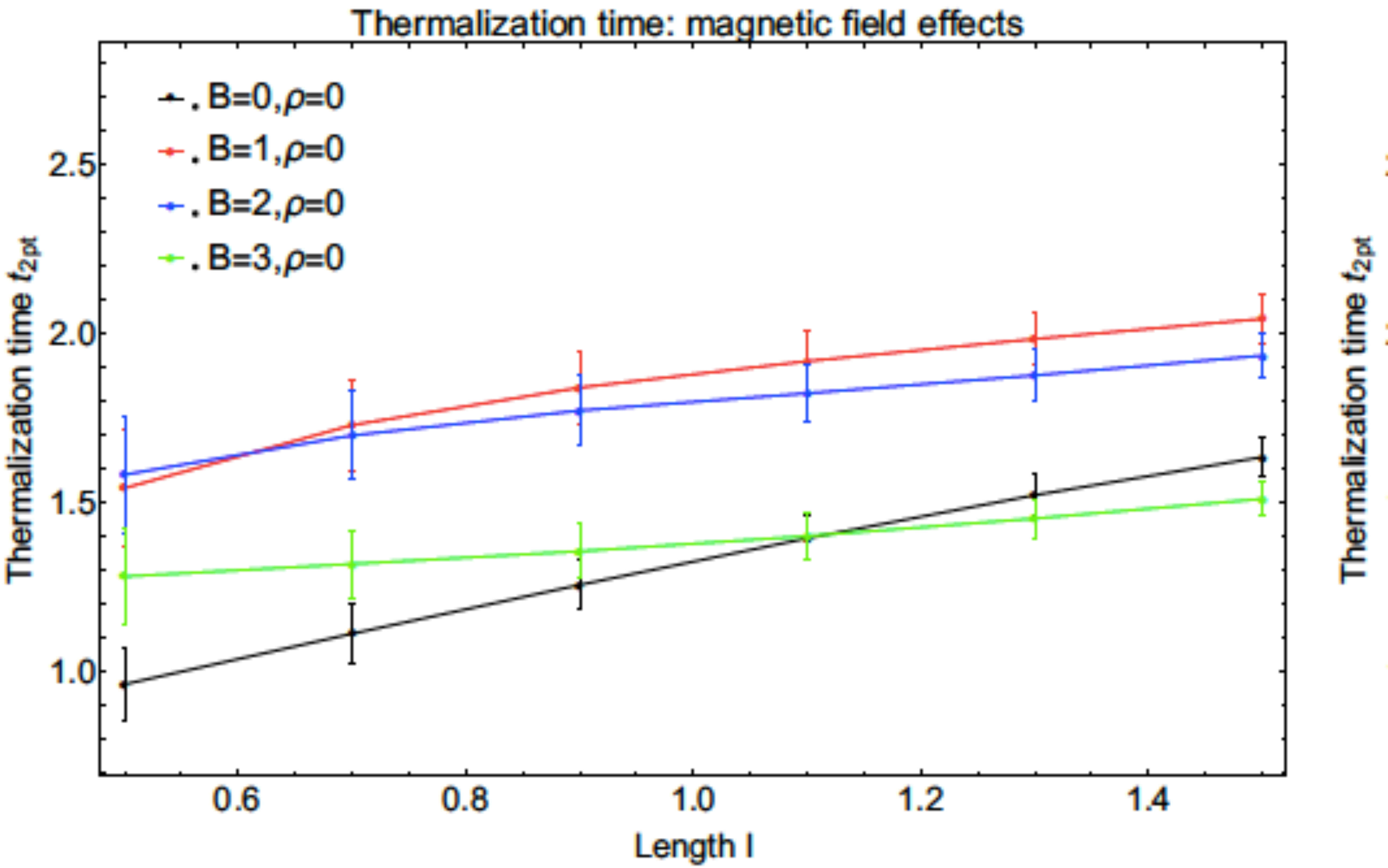
$$\Delta\langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle = (\langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle_T - \langle\mathcal{O}(t,l)\mathcal{O}(t,0)\rangle_L)$$



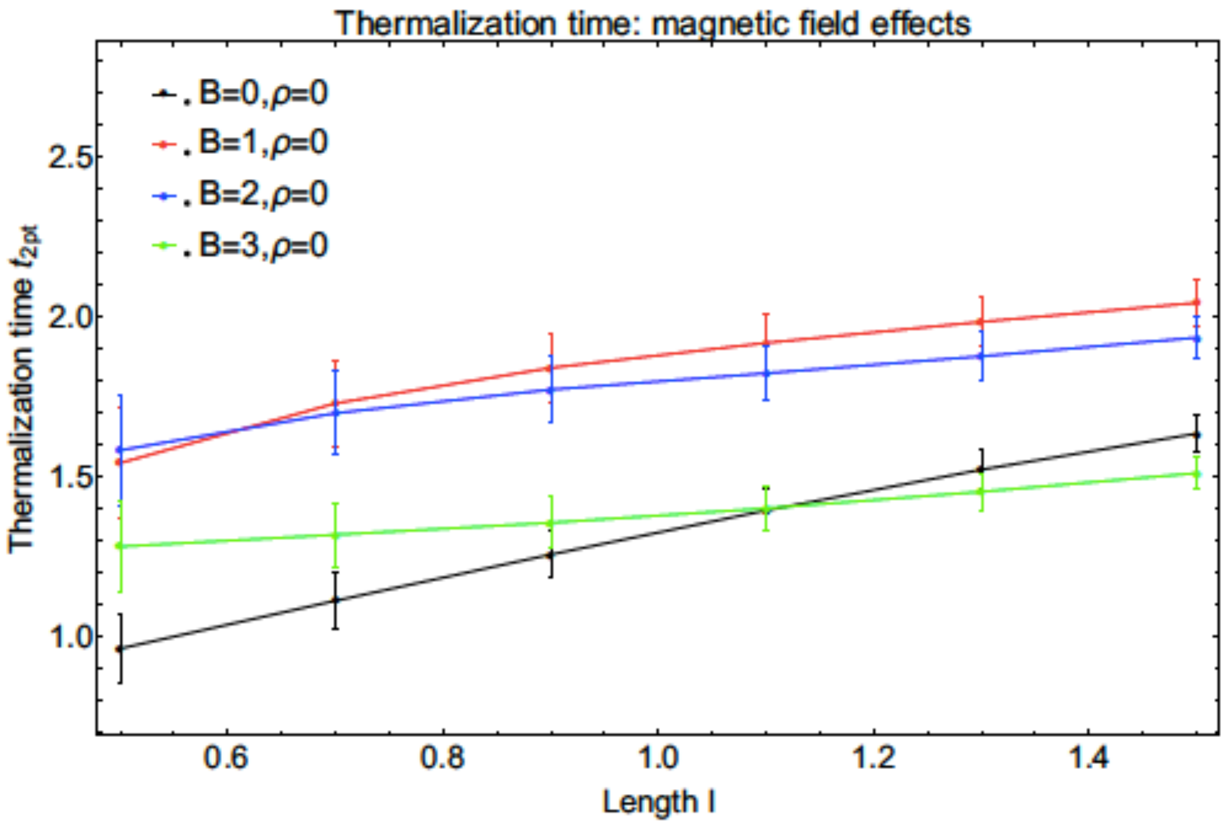
Recall 1-point functions:

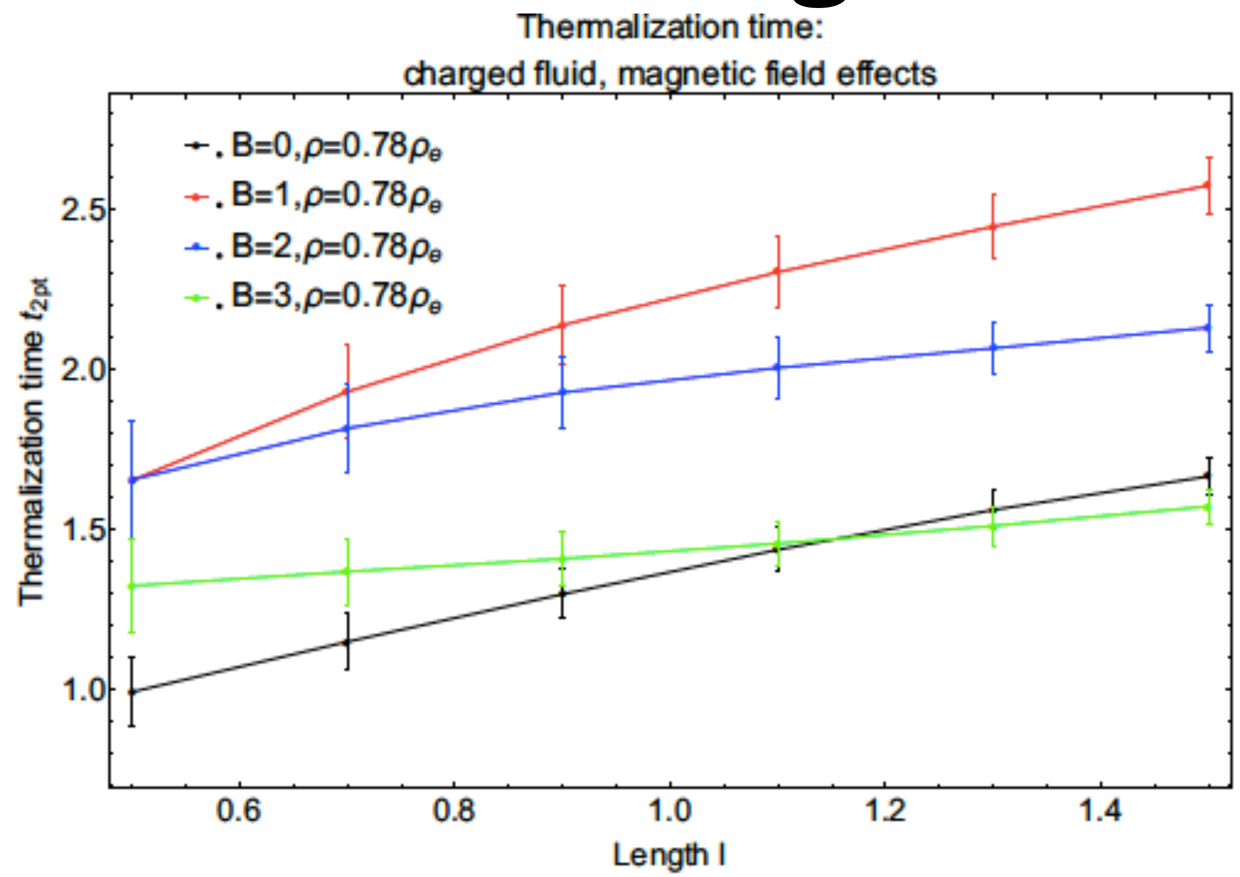
$$\Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$$

Thermalization times as function of length and $\mathcal{B}$

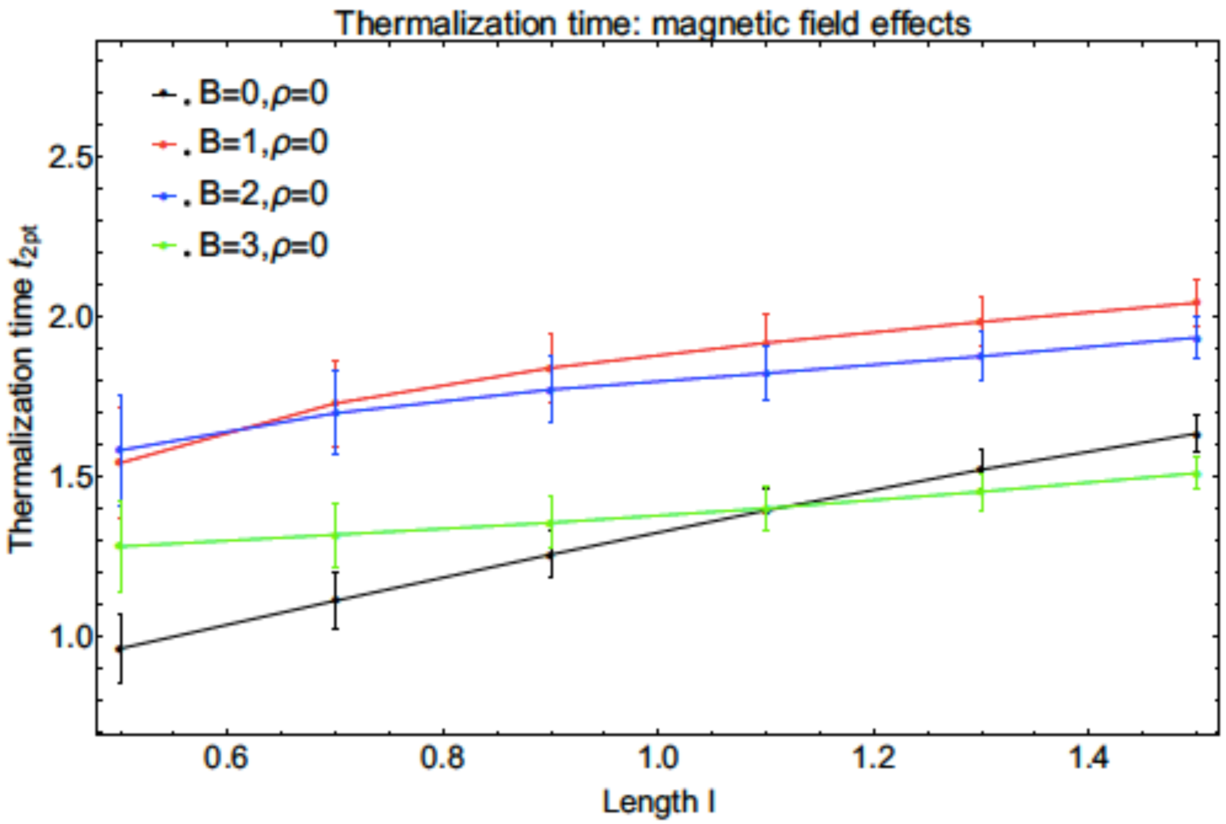


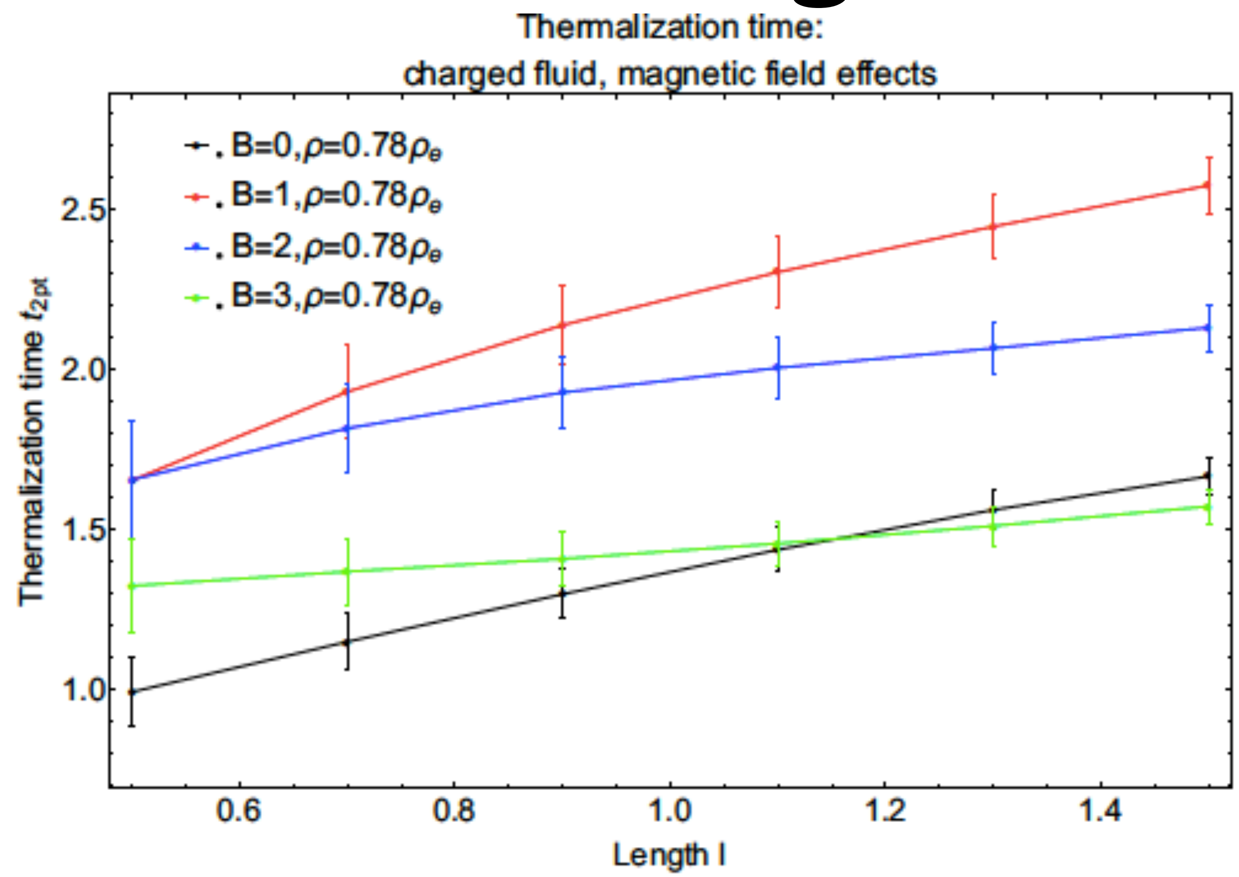
Thermalization times as function of length and $\mathcal{B}$

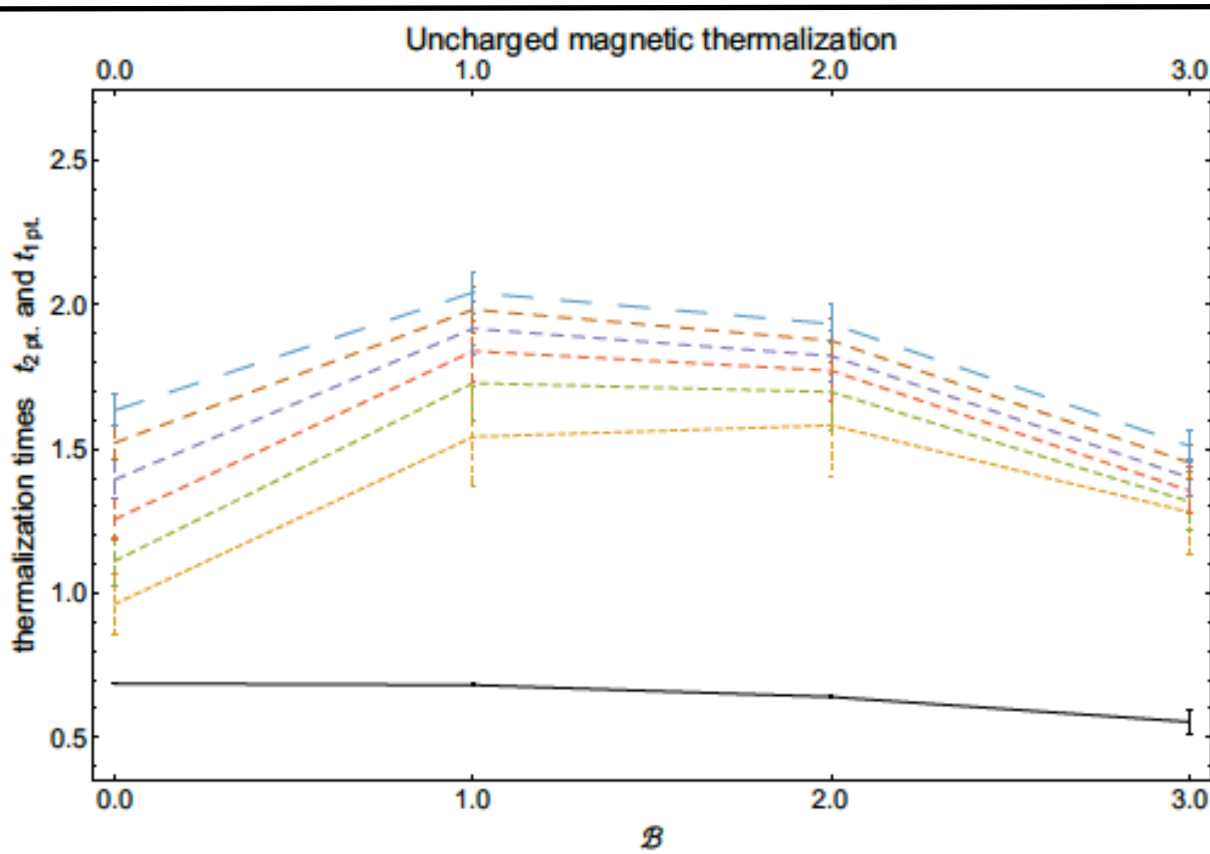


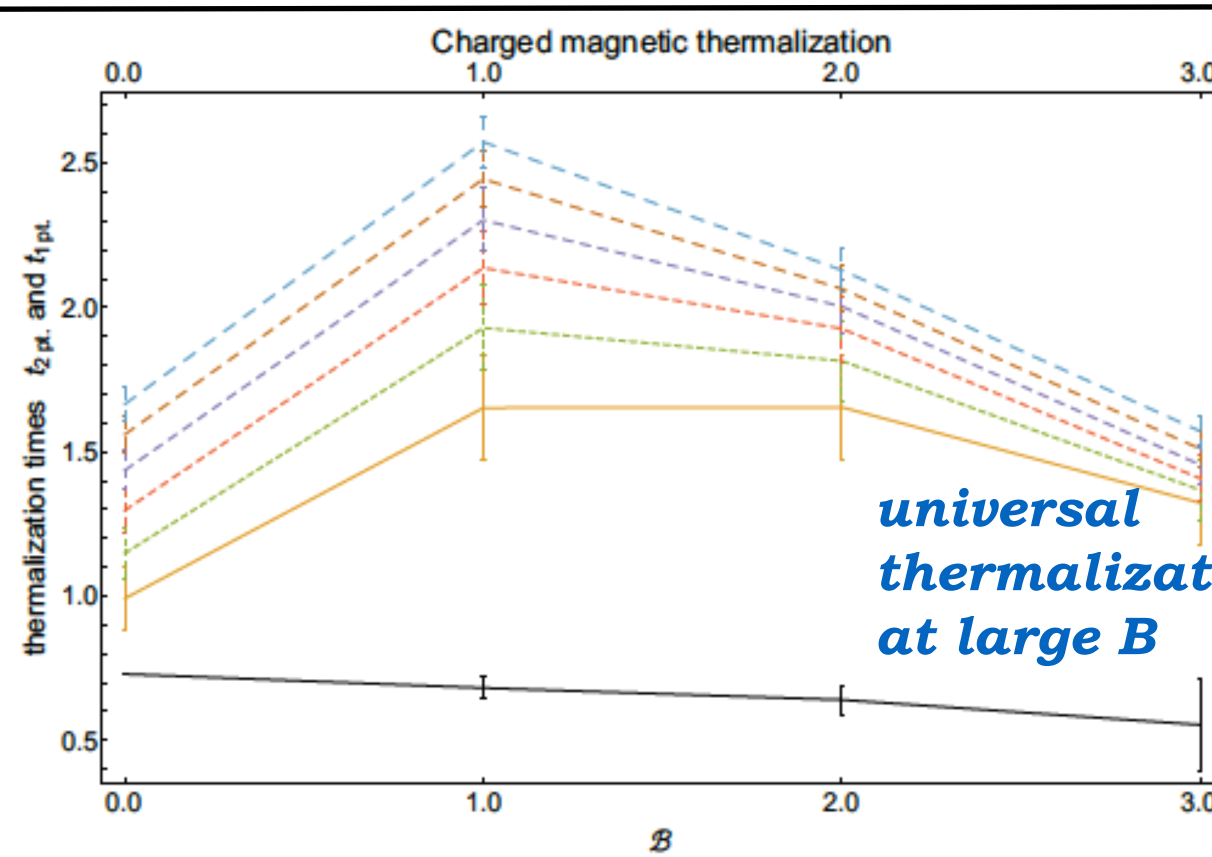


Thermalization times as function of length and $\mathcal{B}$









universal thermalization at large B

Outline

1. Invitation
2. Setup & Calculations
3. Results
4. **Discussion/Outlook**

Summary of results

- generated charged magnetic fluid with initial anisotropy

  $\rho \neq 0 \qquad \mathcal{B} \neq 0 \qquad\qquad \Delta\mathcal{P} = \mathcal{P}_T - \mathcal{P}_L$

- calculated 1- and 2-point functions
- early times: strong medium effect
- 2-point functions thermalize significantly slower than 1-point functions

	$\rho = 0$	$\rho = 0.78\rho_e$	approach to extremality → ρ
$\mathcal{B} = 0$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.4$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.4$	$t_{1pt} \to \infty$ $t_{2pt} \to \infty$
$\mathcal{B} = 1$	$t_{1pt} \approx 0.7$ $t_{2pt} \approx 1.9$	$t_{1pt} \approx 0.8$ $t_{2pt} \approx 2.3$	
$\mathcal{B} = 3$	saturation regime $t_{2pt} \not\approx t_{2pt}(l)$ $t_{1pt} \approx 0.6$ $t_{2pt} \approx 1.4$	$t_{1pt} \approx 0.6$ $t_{2pt} \approx 1.5$	
↓ $\mathcal{B}$			

➡ competition of scales

*charge

*magnetic field

*initial anisotropy

➡ universal thermalization time at large $\mathcal{B}$

[Glorioso, Son; (2018)]
[Grozdanov, Poovuttikul; JHEP (2019)]

➡ large charge: thermalization times diverge

(in $\mathcal{N}$=4 Super-Yang-Mills theory in 3+1 dimensions, minimally coupled to external U(1) gauge field)

Discussion I

- comparison to **chiral hydrodynamics at strong *B***

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; work in progress]

$$\langle J^z J^z \rangle(\omega, \mathbf{k}=0) \sim \sigma_{||}$$

$$\langle J^x J^x \rangle(\omega, \mathbf{k}=0) \sim \rho_{\perp}$$

$$w = \epsilon + P_{||}$$

preliminary results

... so, correlators receive **altered physical interpretation**

- **effective field theory** of fluid far from equilibrium

[Romatschke; PRL (2017)]

[Heller, Spalinski; PRL (2015)]

[Cartwright, Kaminski et al.; work in progress]

Discussion I

- comparison to **chiral hydrodynamics at strong *B***

[Ammon, Kaminski et al.; JHEP (2017)]

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; work in progress]

$$\langle J^z J^z\rangle(\omega, \mathbf{k}=0) \sim \sigma_{||}$$ ***parallel conductivity***

$$\langle J^x J^x\rangle(\omega, \mathbf{k}=0) \sim \rho_{\perp}$$ ***perpendicular resistivity***

$$\langle J^x J^y\rangle(\omega=0, \mathbf{k}) \sim -ik \underbrace{\xi_B}_{C\mu}$$ *anomaly type*

$$\langle J^x J^y\rangle(\omega, \mathbf{k}=0) \sim \frac{n}{B} - \omega^2 \frac{w^2}{B^4}\tilde{\rho}_{\perp} + \dots$$ *Hall type*

preliminary results

$$w = \epsilon + P_{||}$$

… so, correlators receive **altered physical interpretation**

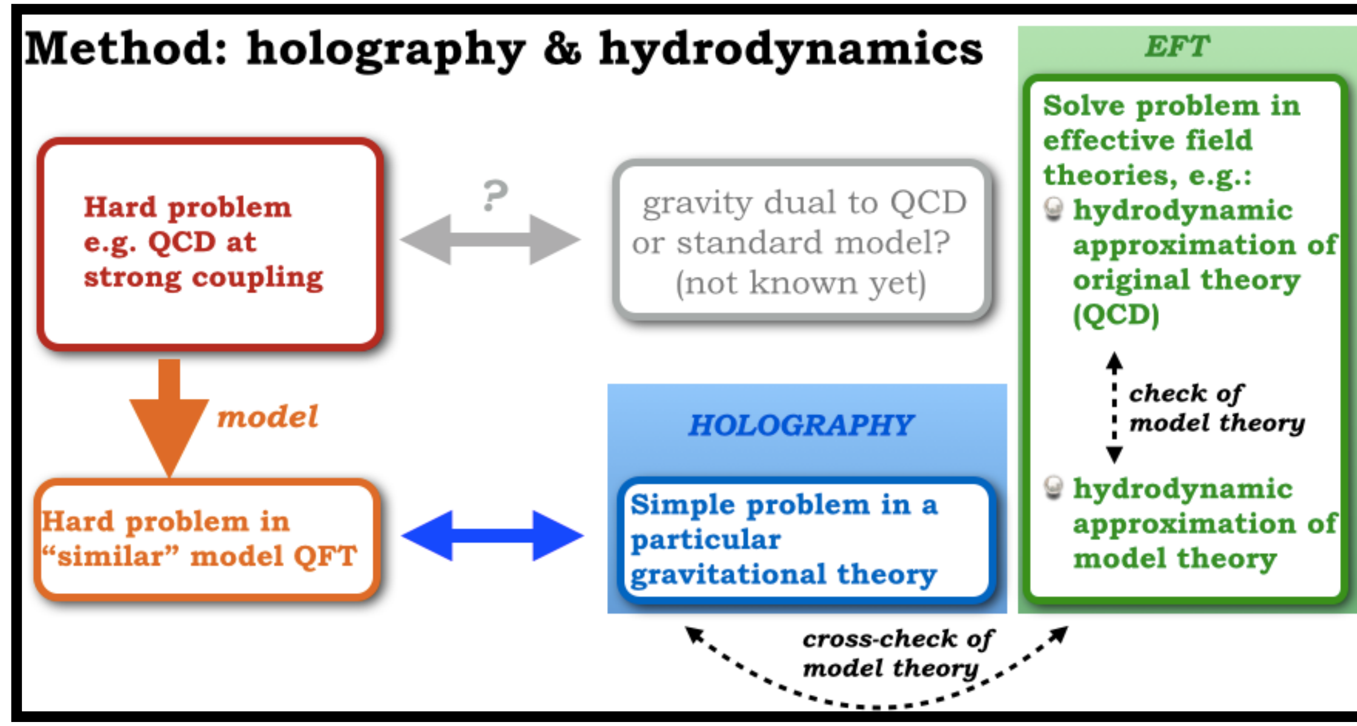
- **effective field theory** of fluid far from equilibrium

[Romatschke; PRL (2017)]

[Heller, Spalinski; PRL (2015)]

[Cartwright, Kaminski et al.; work in progress]

Discussion II



- **entanglement** entropy
  [Cartwright, Kaminski et al.; work in progress]
- **initial state** —>v_n
- **shear viscosity far from eq.**
  [Wondrak, Kaminski,Bleicher; in progress]
- correlations in **plasma with dynamical electromagnetic** fields
- test/construct **"magnetohydrodynamics"**

[Hernandez, Kovtun; JHEP (2017)]
[Grozdanov, Hofman, Iqbal; PRD (2017)]
[Hattori, Hirono, Yee, Yin; (2017)]

- **chiral transport** far from equilibrium;
  e.g. chiral magnetic effect and chiral vortical effect

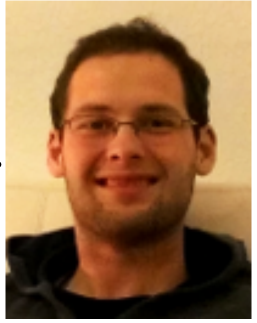
[Kharzeev; (2004)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]
[Banerjee et al; JHEP (2011)]
[Son,Surowka; PRL (2009)]

Collaborators

Perimeter, Canada
Juan Hernandez

Friedrich-Schiller University of Jena, Germany


Prof. Dr. Martin Ammon


Dr. Julian Leiber


Sebastian Grieninger

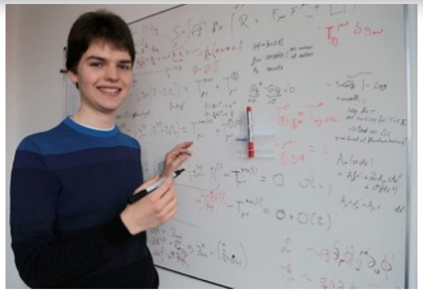
Regensburg University, Germany


Prof. Dr. Andreas Schäfer

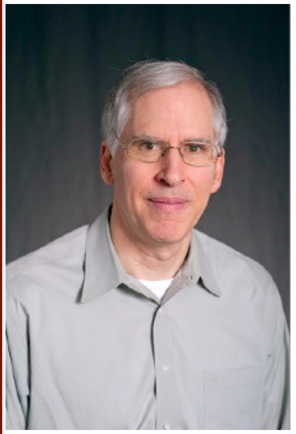
Goethe University, Frankfurt, Germany


Dr. Gergely Endrödi


Prof. Dr. Marcus Bleicher


Michael Wondrak

University of Washington, Seattle, USA


Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA


Roshan Koirala

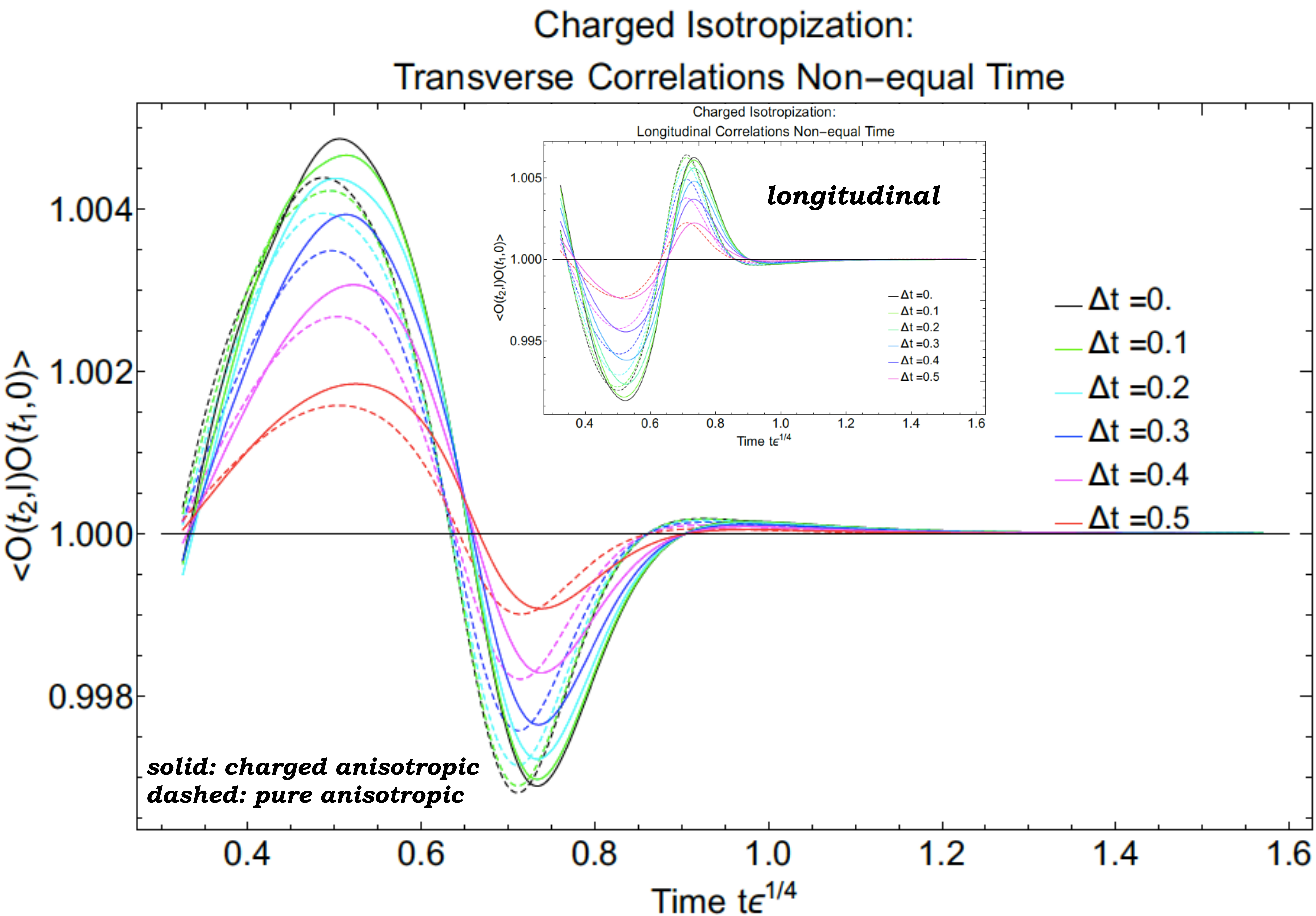

Markus Garbiso


Dr. Jackson Wu

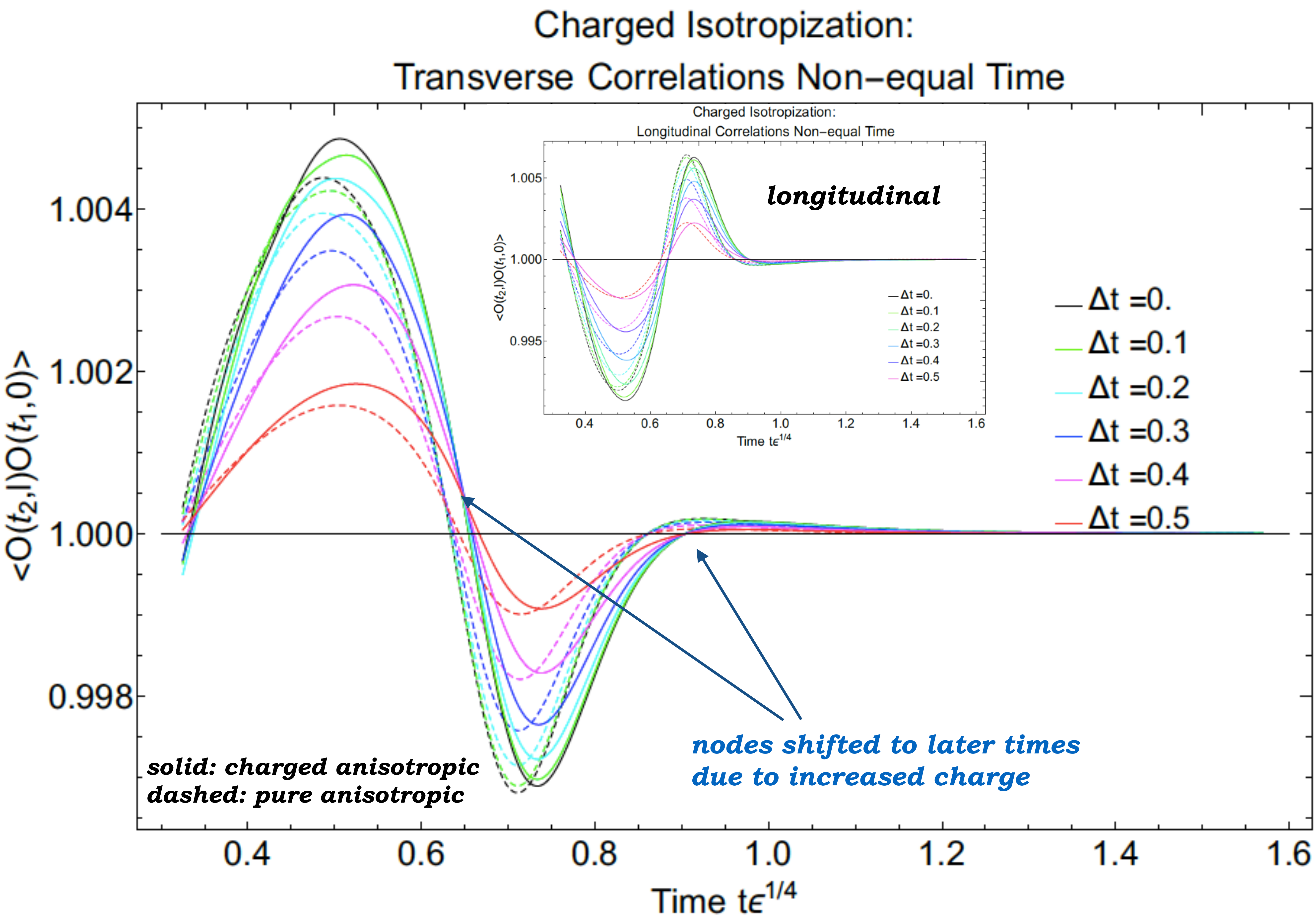

Casey Cartwright

APPENDIX

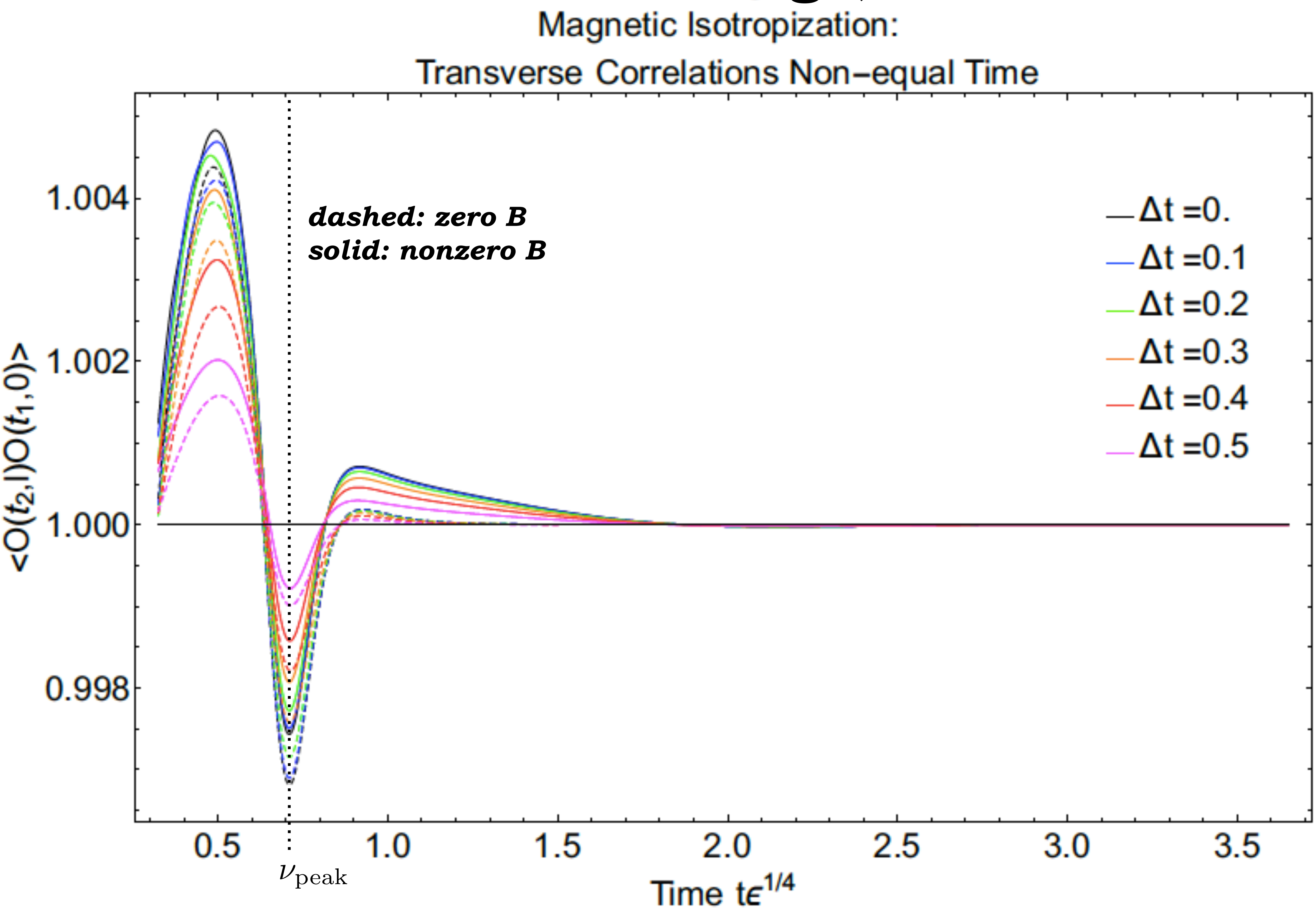
Correlations - nonzero charge, zero B



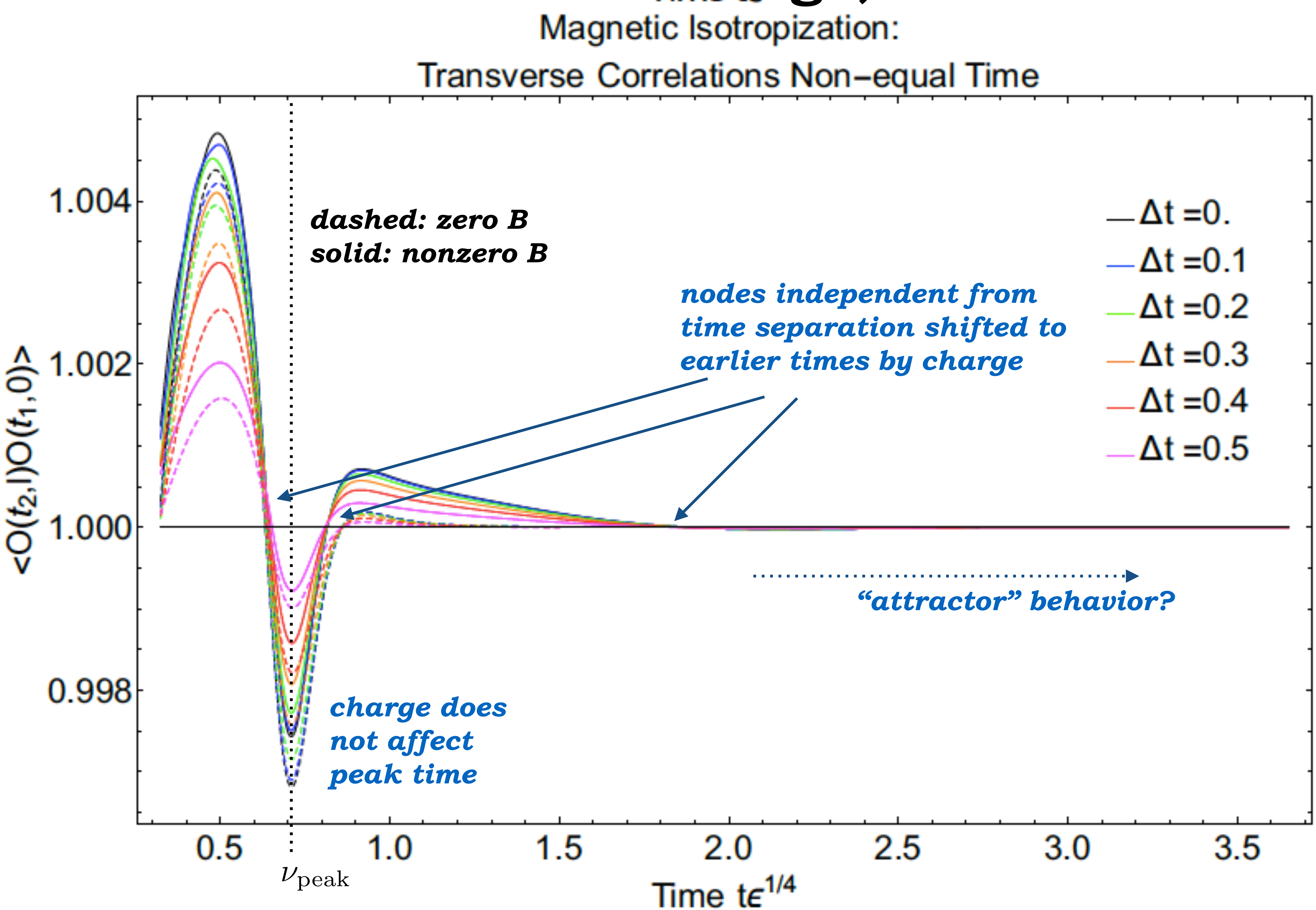
Correlations - nonzero charge, zero *B*



Correlations - zero charge, nonzero B



Correlations - zero charge, nonzero B



Technicalities I

Chebyshev representation of functions:

$$f(r) \approx \sum_{i=0}^{N} T_i(r) a_i, \qquad C_j(r) = \frac{2}{N p_j} \sum_{m=0}^{N} \frac{1}{p_m} T_m(r_j) T_m(r)$$

Derivatives:

$$D_{ij} = \frac{\mathrm{d}C_j(r)}{\mathrm{d}r}\Big|_{r=r_i}, \qquad D^2 = D \circ D$$

Radial shift invariance: $r \to r + \xi$ to fix $z_h = 1/r_h = 1$

Chebyshev grid: $r_i = \frac{1}{2}(a+b) + \frac{1}{2}(a-b)\cos(i\pi/(N-1))$

Boundary expansions:

$$S(v,r) = r + \xi + \mathcal{O}(r^{-7}), \tag{2.12a}$$

$$B(v,r) = \log(r)\left(-\frac{20\mathcal{B}^2\xi(v)^3}{3r^7} + \frac{10\mathcal{B}^2\xi(v)^2}{3r^6} - \frac{4\mathcal{B}^2\xi(v)}{3r^5} + \frac{\mathcal{B}^2}{3r^4}\right) + \frac{b_4(v)}{r^4} + \mathcal{O}(r^{-8}), \tag{2.12b}$$

$$A(v,r) = (r+\xi(v))^2 - 2\xi'(v) + \frac{a_4(v)}{r^2} + \log(r)\left(\frac{8\mathcal{B}^2\xi(v)^3}{3r^5} - \frac{2\mathcal{B}^2\xi(v)^2}{r^4} + \frac{4\mathcal{B}^2\xi(v)}{3r^3} - \frac{2\mathcal{B}^2}{3r^2}\right) + \mathcal{O}(r^{-6}). \tag{2.12c}$$

Technicalities II

Work with subtracted functions:

$$S(v,r) = \frac{1}{r^4} S_s(v,r) + r + \xi,$$

$$B(v,r) = \frac{1}{r^4} B_s(v,r) + \log(r)\left(-\frac{20\mathcal{B}^2\xi(v)^7}{3r^7} + \frac{10\mathcal{B}^2\xi(v)^2}{3r^6} - \frac{4\mathcal{B}^2\xi(v)}{3r^5} + \frac{\mathcal{B}^2}{3r^4}\right),$$

$$A(v,r) = \frac{1}{r^2} A_s(v,r) + (r+\xi(v))^2 - 2\xi'(v)$$

$$+ \log(r)\left(-\frac{10\mathcal{B}^2\xi(v)^4}{3r^6} + \frac{8\mathcal{B}^2\xi(v)^3}{3r^5} - \frac{2\mathcal{B}^2\xi(v)^2}{r^4} + \frac{4\mathcal{B}^2\xi(v)}{3r^3} - \frac{2\mathcal{B}^2}{3r^2}\right).$$

How to fix the horizon position:

$\dot{S} = 0$ defines location of horizon

$\partial_v \dot{S}(v,r)\Big|_{r=r_h} = 0.$ shall not change over time

writing this out gives:

$$A(v,r) + \dot{B}(v,r)^2 \frac{3S(v,r)^6}{(6S(v,r)^6 - \rho^2 - e^{-2B(v,r)}S(v,r)^2\mathcal{B}^2)}\Bigg|_{r=r_h} = 0.$$ first order ODE for $\xi(v)$

Procedure: 1. First time step: guess initial shift to put horizon at 1 and iteratively improve at shifted *r*

2. All time steps after that: solve ODE

YET: horizon drift!

Technicalities III

Unique solutions and scalings:

$$\epsilon_{\mathcal{B}} = \epsilon + \frac{1}{4}\mathcal{B}^2 \ln|\mathcal{B}| \qquad T_\alpha{}^\alpha = -\frac{1}{2}\kappa\mathcal{B}^2$$

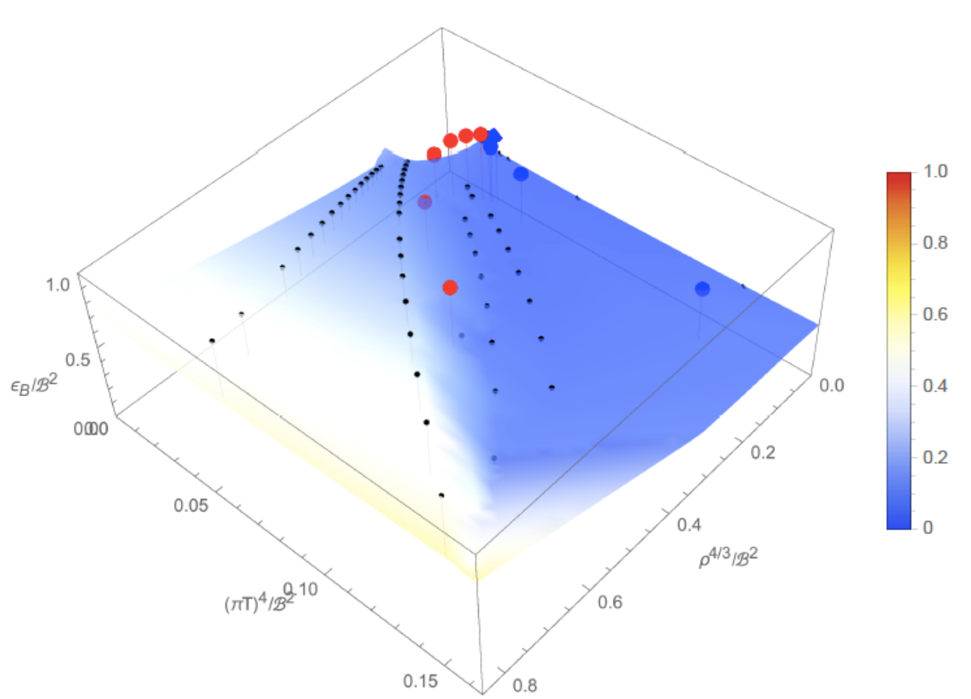
$$\rho^{4/3}/\mathcal{B}^2 \qquad (\pi T)^4/\mathcal{B}^2$$

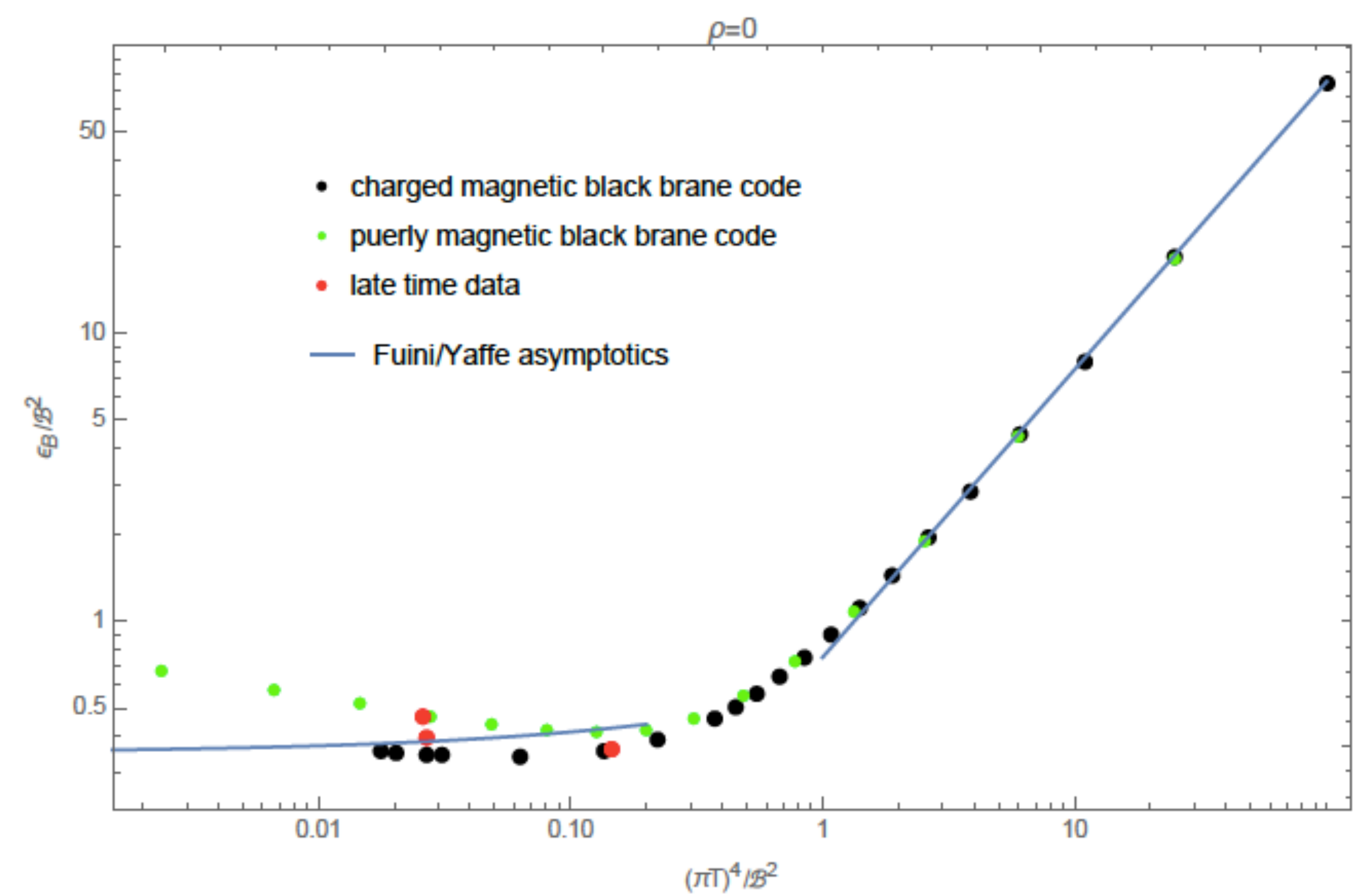
Renormalization scale dependent energy-momentum tensor:

$$\langle T_{00}\rangle = -\frac{3}{4}a_4 + \frac{1}{2}\mathcal{B}^2 \ln \mu L$$

$$\langle T_{11}\rangle = \langle T_{22}\rangle = -\frac{1}{4}a_4 + b_4 - \frac{1}{4}\mathcal{B}^2 + \frac{1}{2}\mathcal{B}^2 \ln \mu L$$

$$\langle T_{33}\rangle = -\frac{1}{4}a_4 - 2b_4 - \frac{1}{2}\mathcal{B}^2 \ln \mu L$$

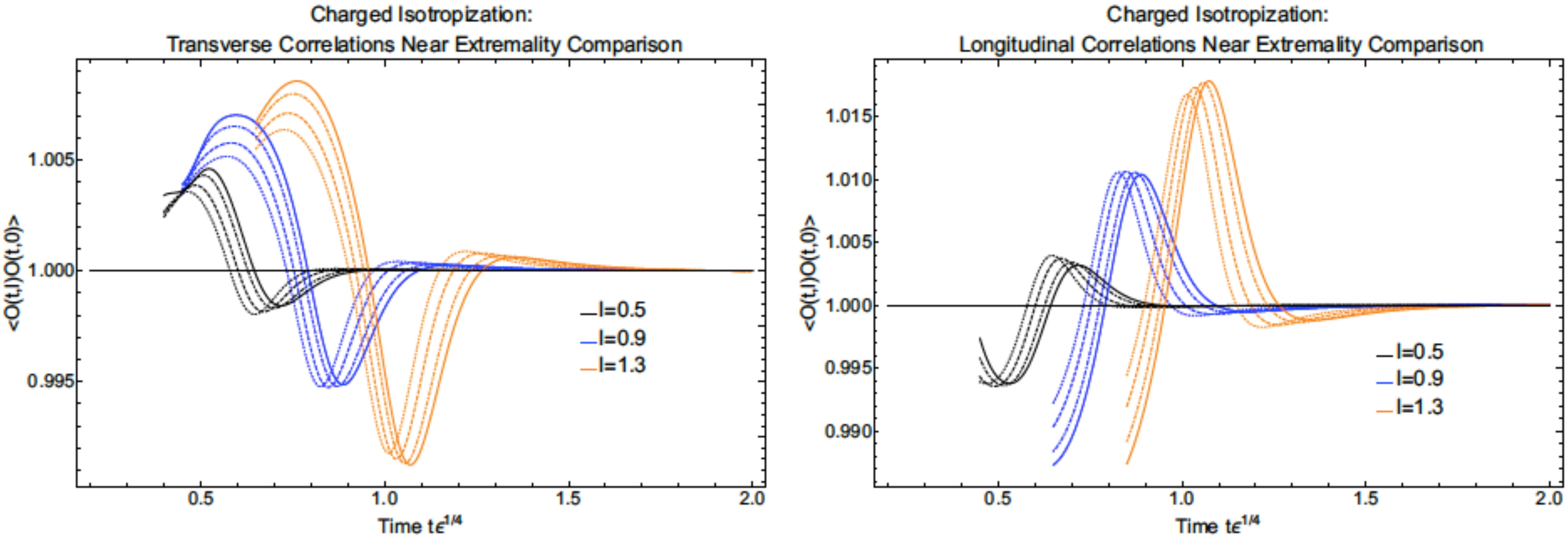




Thermalization times as function of length and $\mathcal{B}$

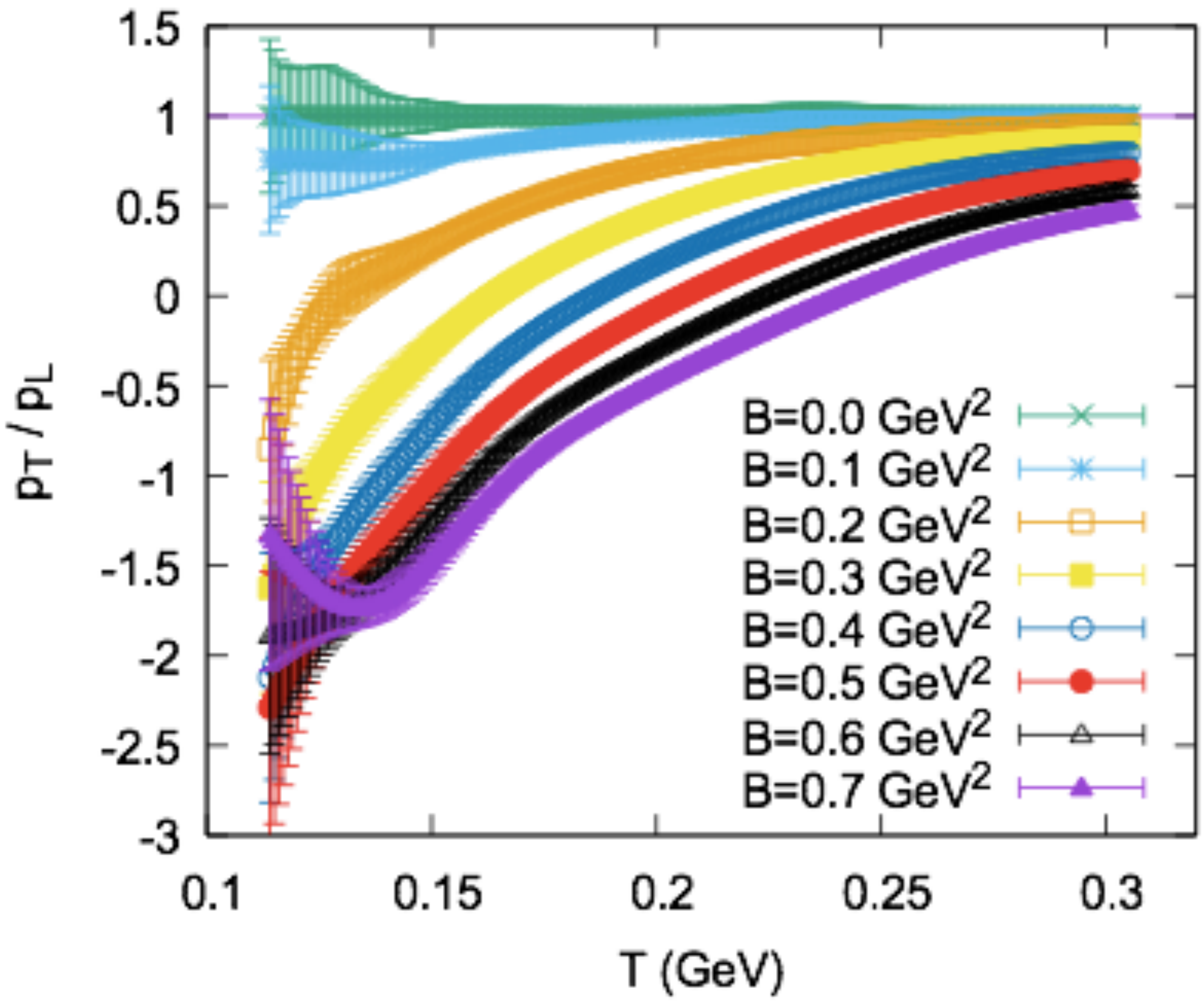
	$t_{1pt.}$	$t_{2pt.}$		
case		$l = 0.5$	$l = 1.1$	$l = 1.5$
(a) $\rho = 0$, $\mathcal{B} = 0$	0.6869	0.9624	1.3942	1.6346
(b) $\rho \neq 0$, $\mathcal{B} = 0$	0.7297	0.9917	1.4360	1.6656
(c) $\rho = 0$, $\mathcal{B} = 1$	0.6815	1.5425	1.9180	2.0433
(c) $\rho = 0$, $\mathcal{B} = 2$	0.6403	1.5823	1.8229	1.9345
(c) $\rho = 0$, $\mathcal{B} = 3$	0.5537	1.2811	1.4007	1.5108
(d) $\rho \neq 0$, $\mathcal{B} = 1$	0.7746	1.6526	2.3034	2.5742
(d) $\rho \neq 0$, $\mathcal{B} = 2$	0.6803	1.6547	2.0043	2.130
(d) $\rho \neq 0$, $\mathcal{B} = 3$	0.5609	1.3232	1.4555	1.5716

Charged 2-pt functions towards extremality



Scale invariance in LQCD with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$$

longitudinal pressure: $$p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$$

F_{QCD} ... *free energy*

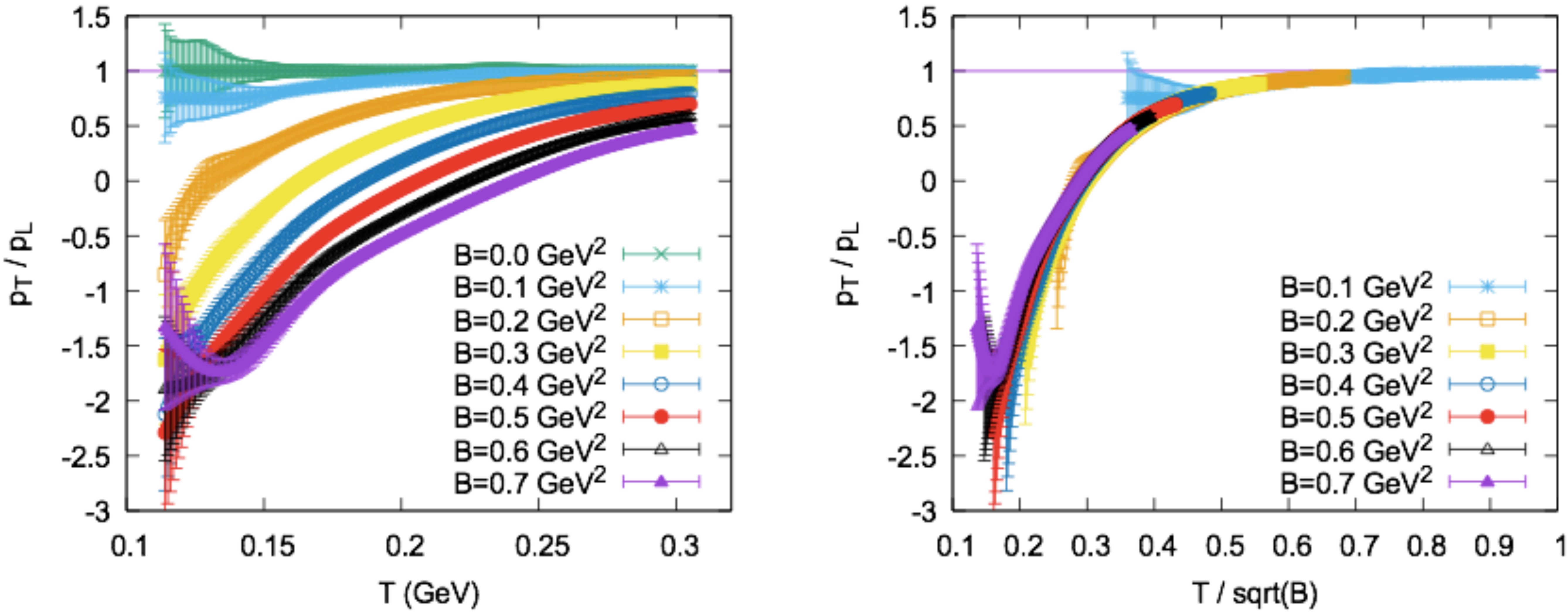
L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

Scale invariance in LQCD with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\rm T} = -\frac{L_{\rm T}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm T}}$

longitudinal pressure: $p_{\rm L} = -\frac{L_{\rm L}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm L}}$

$F_{\rm QCD}$ ... *free energy*

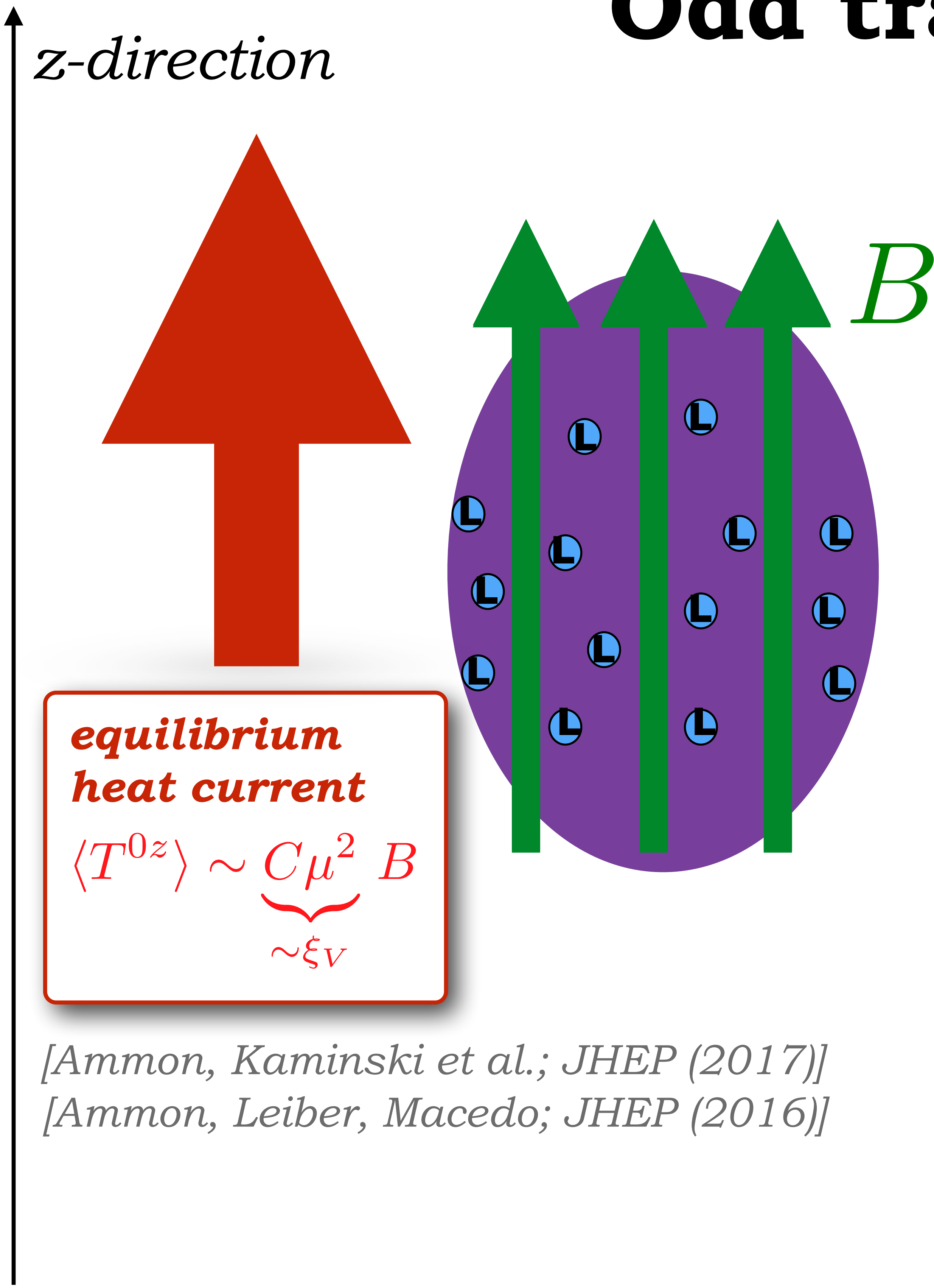
$L_{\rm T}$ ... *transverse system size*

$L_{\rm L}$ ... *longitudinal system size*

V ... *system volume*

Odd transport

Odd transport



$\parallel$ *parallel*

$\perp$ *perpendicular*

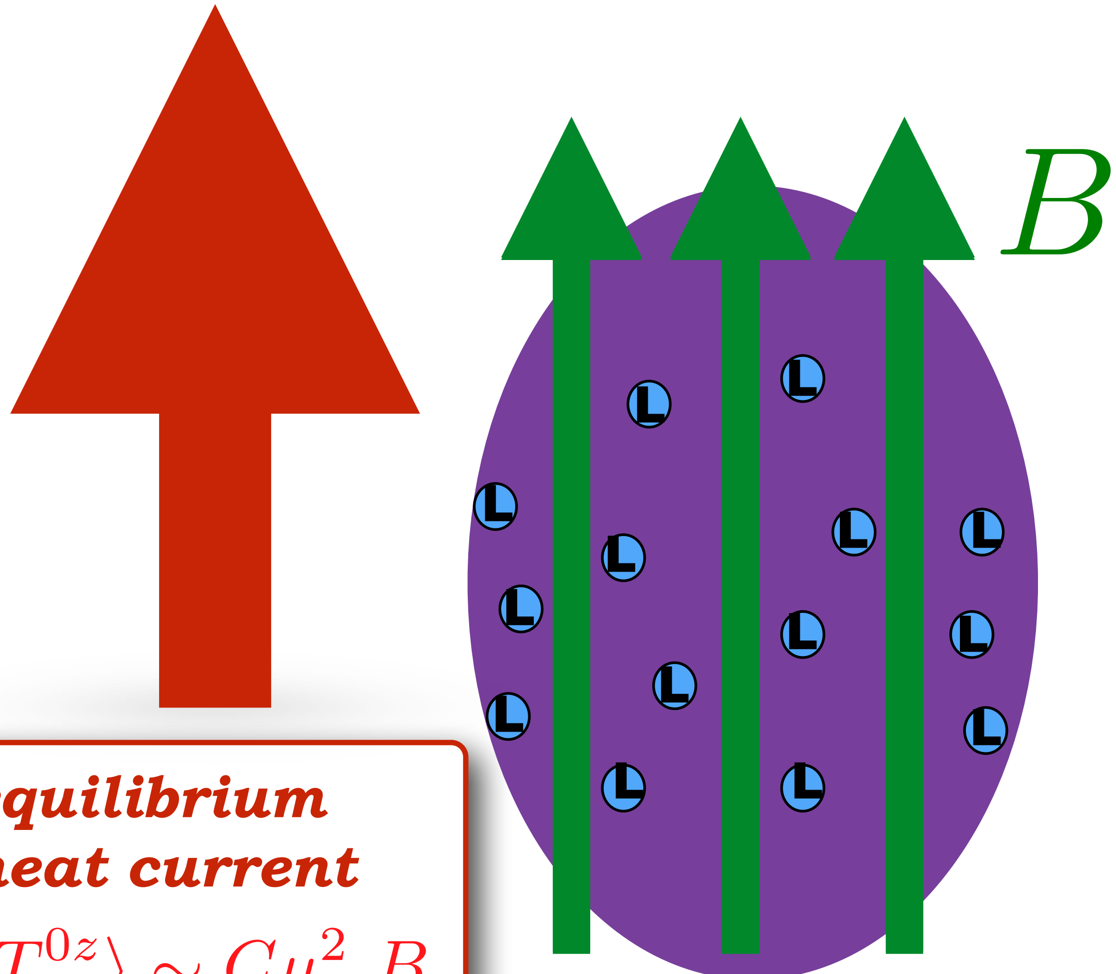
equilibrium heat current

$$\langle T^{0z}\rangle \sim \underbrace{C\mu^2}_{\sim \xi_V} B$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Odd transport

z-direction



$||$ *parallel*

$\perp$ *perpendicular*

equilibrium heat current

$$\langle T^{0z} \rangle \sim \underbrace{C\mu^2}_{\sim \xi_V} B$$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

non-equilibrium parallel conductivity / perpendicular resistivity

$$\langle J^z J^z \rangle (\omega, \mathbf{k} = 0) \sim \sigma_{||}$$

$$\langle J^x J^x \rangle (\omega, \mathbf{k} = 0) \sim \rho_{\perp}$$

non-equilibrium parity-odd transport

$$\langle J^x J^y \rangle (\omega, \mathbf{k} = 0) \sim \frac{n}{B} - \omega^2 \frac{w^2}{B^4} \tilde{\rho}_{\perp} + \dots$$ *Hall type*

$$\langle J^x J^y \rangle (\omega = 0, \mathbf{k}) \sim -ik \underbrace{\xi_B}_{C\mu}$$ *anomaly type*

EFT result I: strong B thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$$B \sim \mathcal{O}(1)$$

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

Axial current:

$$\langle J^{\mu}_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

based on previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

EFT result I: strong B thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$B \sim \mathcal{O}(1)$

equilibrium heat current

$$\langle T^{\mu\nu}_{\mathrm{EFT}} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

Axial current:

"magnetic pressure shift"

$$\langle J^{\mu}_{\mathrm{EFT}} \rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B} \right) + \mathcal{O}(\partial)$$

equilibrium charge current

based on previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

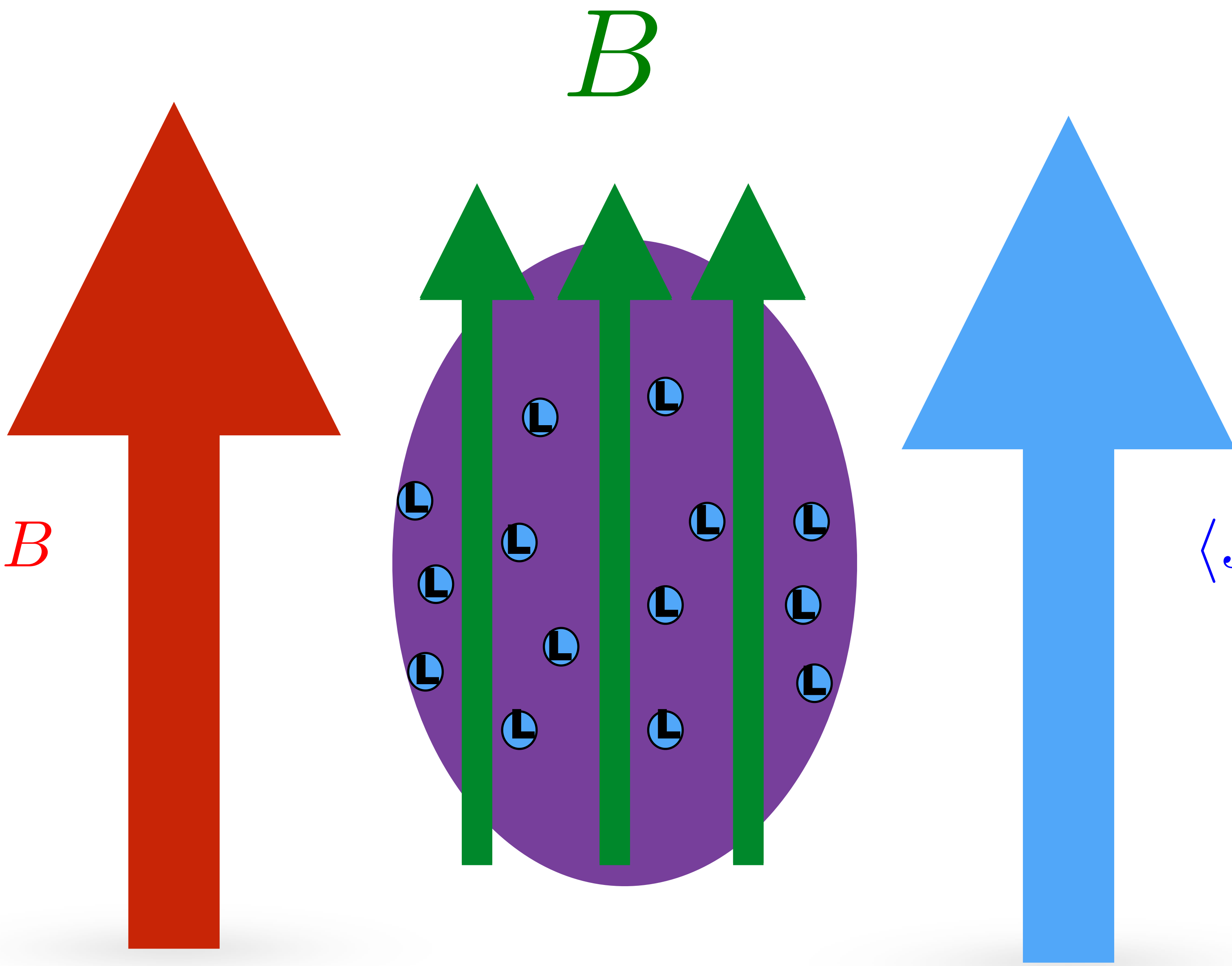
[Israel; Gen.Rel.Grav. (1978)]

➡ new contributions to thermodynamic equilibrium observables

Currents in equilibrium

$$\langle T^{0z} \rangle = \xi_V^{(0)} B$$

heat current



$$\langle J^z \rangle = \xi_B^{(0)} B$$

axial current

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]

[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]
[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^{\mu}\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB}B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB}B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^{\mu}_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

➡ **agrees in form with strong *B* thermodynamics from EFT**

EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \qquad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$\mathfrak{s}_0 = s_0/n_0$
$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$

EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$\mathfrak{s}_0 = s_0 / n_0$

$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$

EFT result II: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0 / n_0$$

$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

EFT result II: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0 / n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s} / \partial T)_P$$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$
$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**
[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$
$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

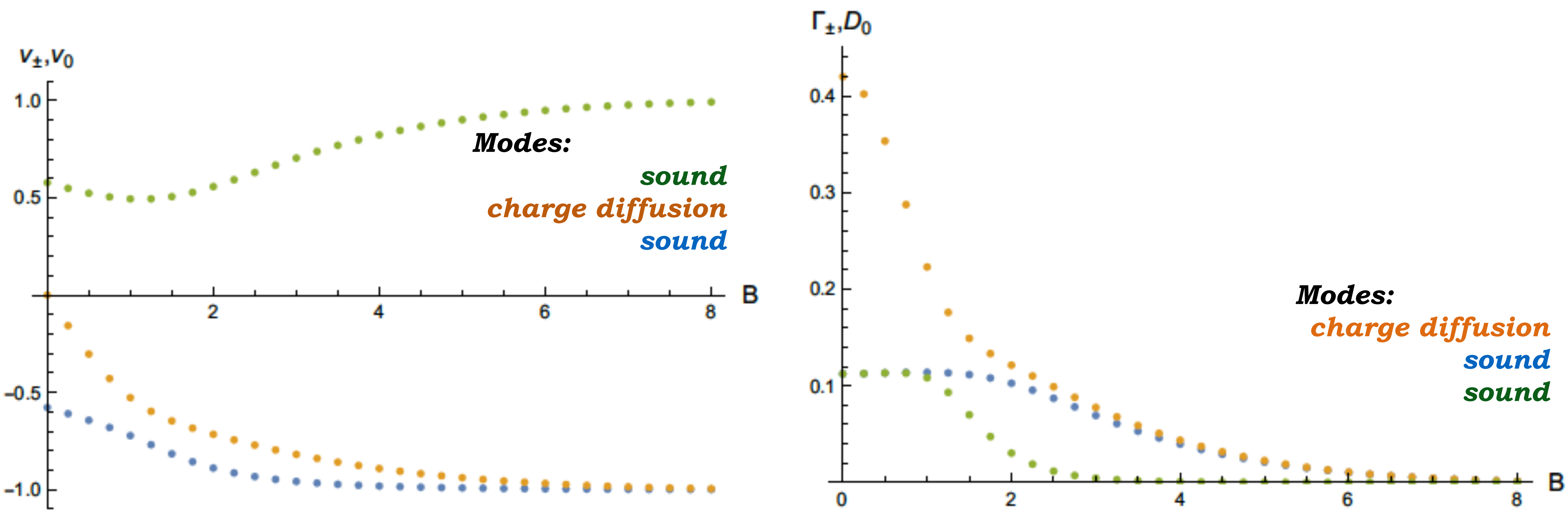
➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in "agreement" with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at ...**

the speed of light **and without attenuation**



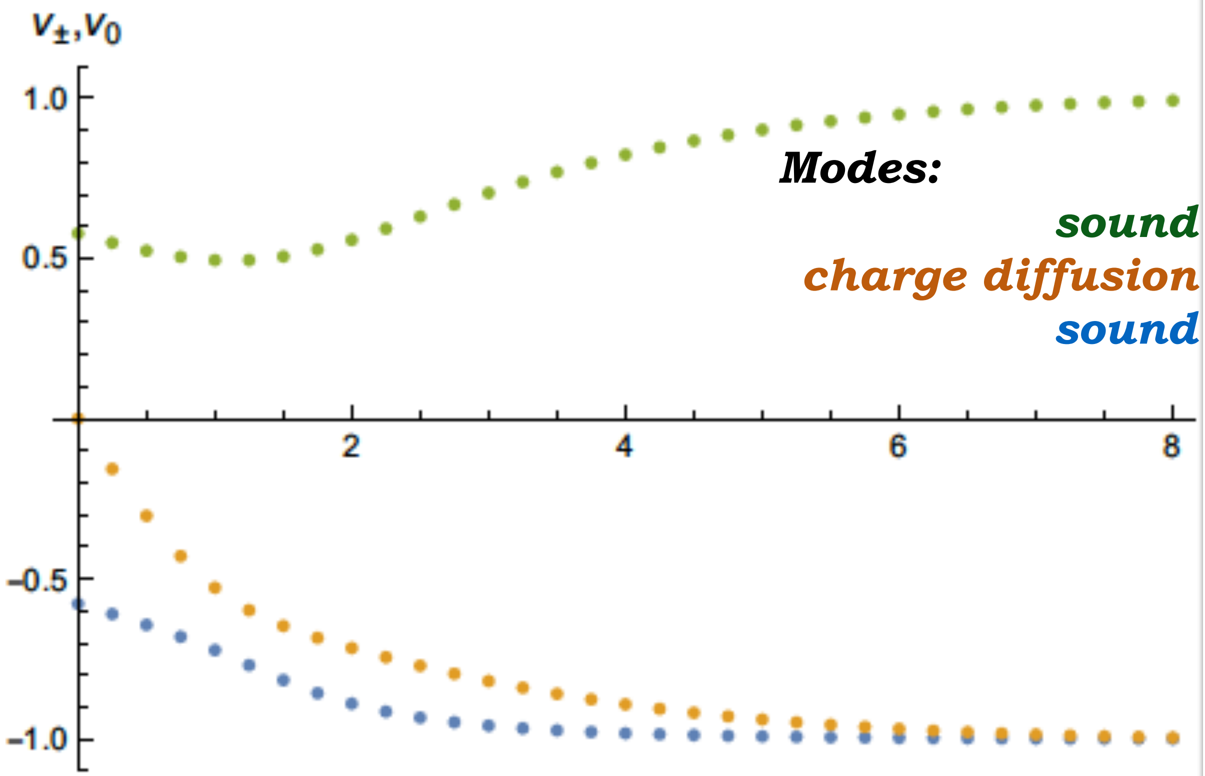
confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in “agreement” with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

the speed of light **and without attenuation**



RECALL: weak B hydrodynamic pole

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$ *former sound modes*

confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in "agreement" with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

the speed of light **and without attenuation**

RECALL: weak B hydrodynamic poles

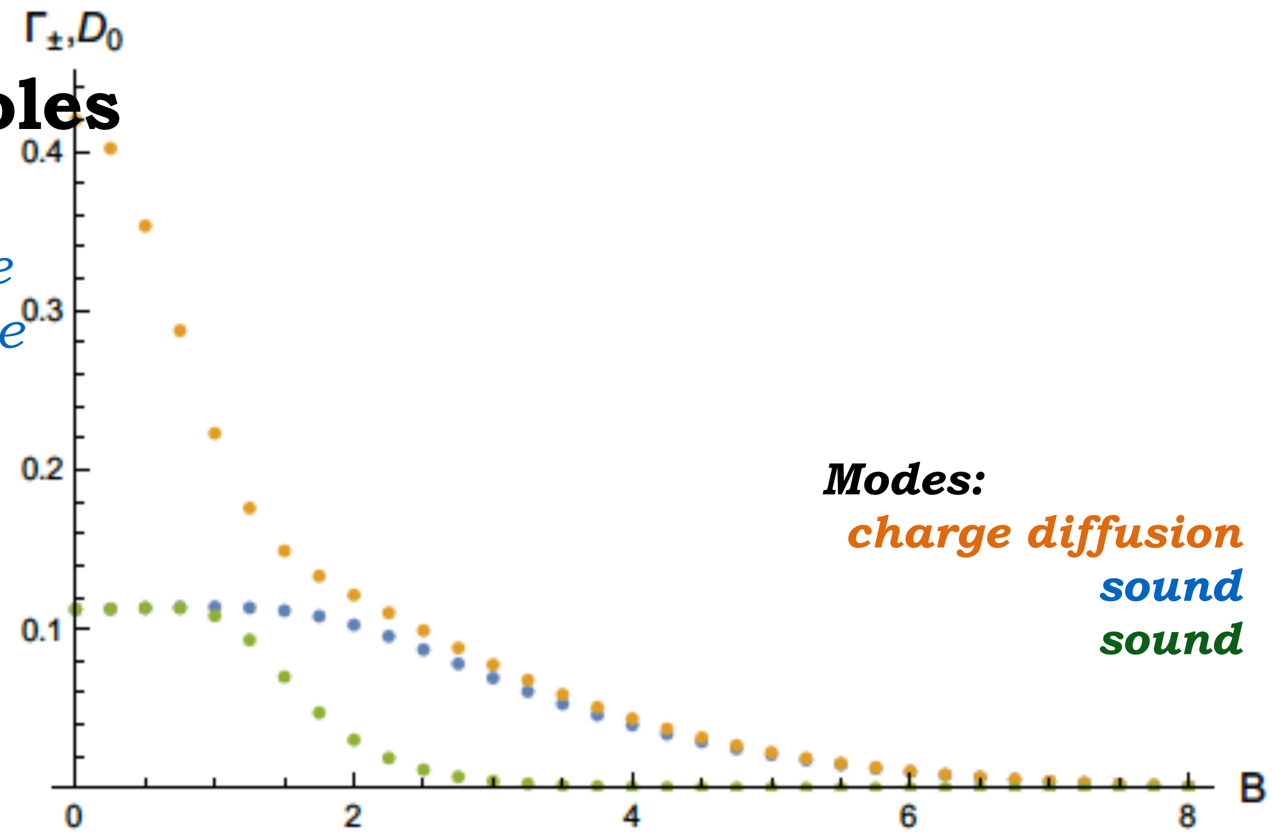
$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes



confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*