



Non-equilibrium Dynamics (NeD-2019)

Harmonic flow coefficients and Hydrodynamic Events

Takeshi Kodama

Collaboration with

Dan Wen, Wei-Liang Qian and Yojiro Hama





Harmonic flow coefficients and Hydrodynamic Events

- 
1. Coarse Grainings and Hydrodynamics
 2. Event by Event Hydrodynamics and Granularity
 3. Degenerate Nature of Harmonic Coefficients
 4. Quantum effects?

Hydrodynamic Variables

$$(T^{\mu\nu}) = \begin{pmatrix} (\varepsilon + P) \gamma^2 - P & (\varepsilon + P) \gamma^2 \boldsymbol{\beta} \\ (\varepsilon + P) \gamma^2 \boldsymbol{\beta} & (\varepsilon + P) \gamma^2 \boldsymbol{\beta} \boldsymbol{\beta}^T + P \hat{\mathbf{I}} \end{pmatrix}$$

$$T_{\mu\nu} = T_{\mu\nu}(x)$$

(ideal case)

$$\partial^\mu T_{\mu\nu} = 0 \quad + \quad \text{Equation of States}$$

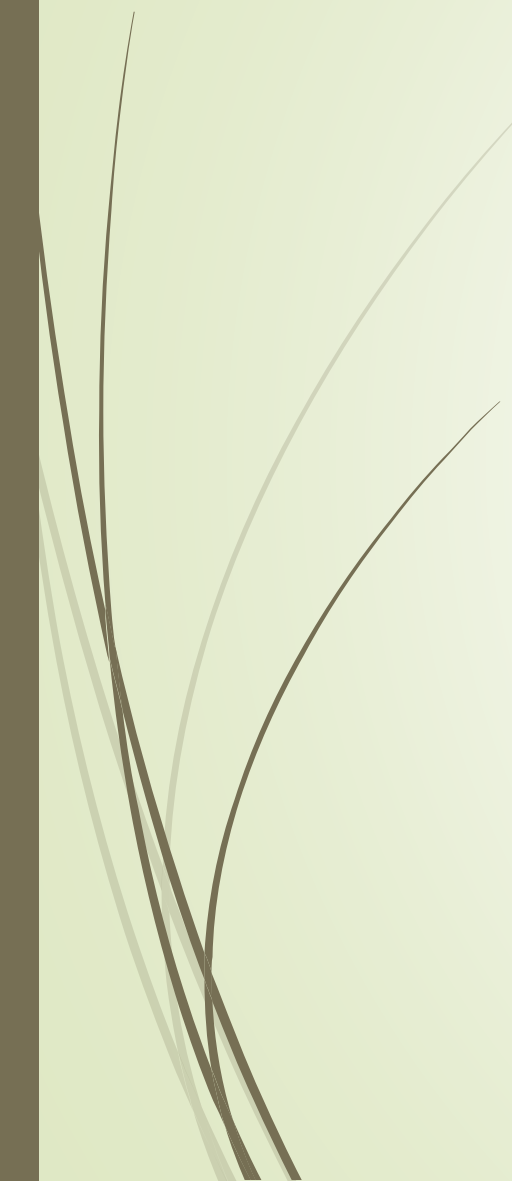
Hydrodynamics = Dynamics of Continuum media

For macroscopic objects, we do not much about the validity....





Several Types of Coarse Graining

- Space-like coarse graining
 - Time-like coarse graining
 - Nature of observables (event average)
- 



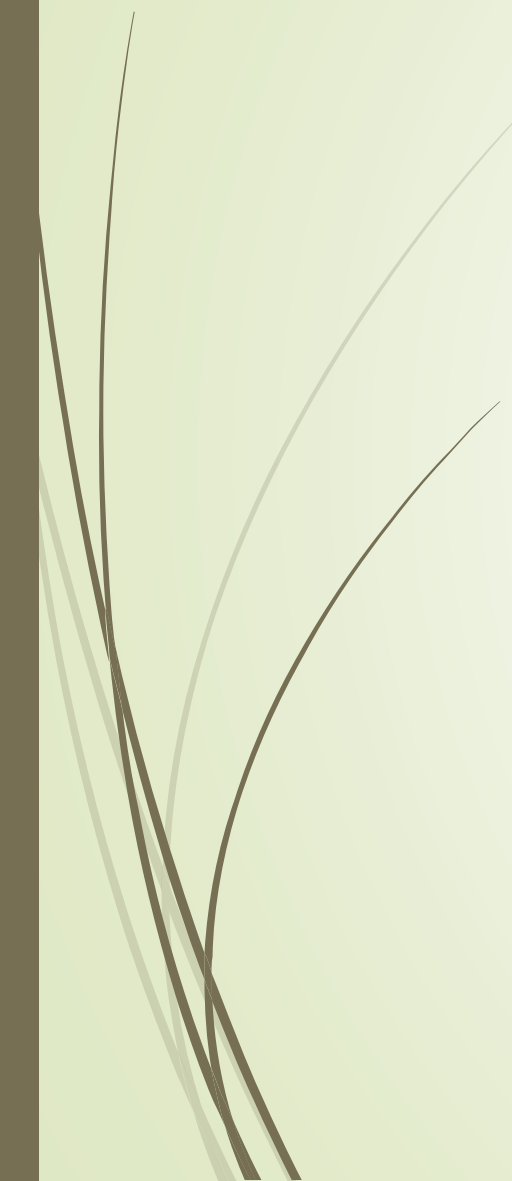
Pre-harmonic flow analysis



Starting from Landau, a huge list of hydrodynamic approach on particle Productions in high energy pp or nuclear collisions



Several Types of Coarse Graining

- Space-like coarse graining
 - Time-like coarse graining
 - Nature of observables
- 



Several Types of Coarse Graining

- Space-like coarse graining
- Time-like coarse graining
- Nature of observables (**event average**)

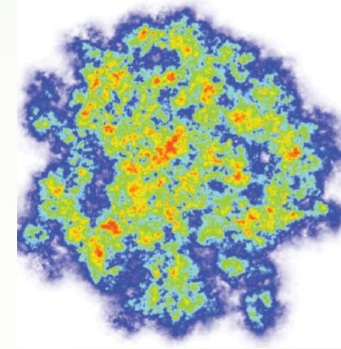
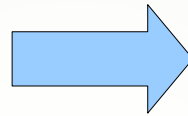
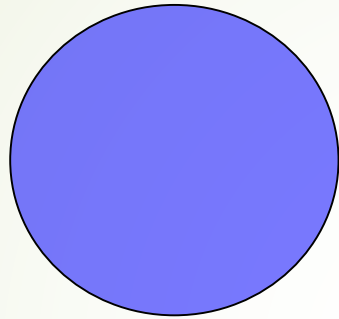
Phys. Rev. C 55, 1455 (1997), S. Paiva, Y. Hama, and T. Kodama,
JPhys.G: Nuclear and Particle Physics 27, 75, 2001, T Osada, CE Aguiar, Y Hama, T
Kodama

- **Thanks to Klaus Werner for the suggestion of use of event generator, NEXUS !!**

Coarse Graining for event by event fluctuations

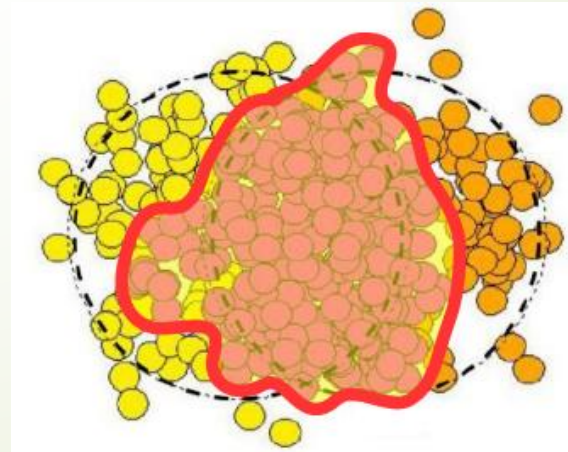
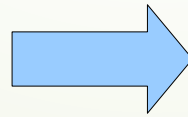
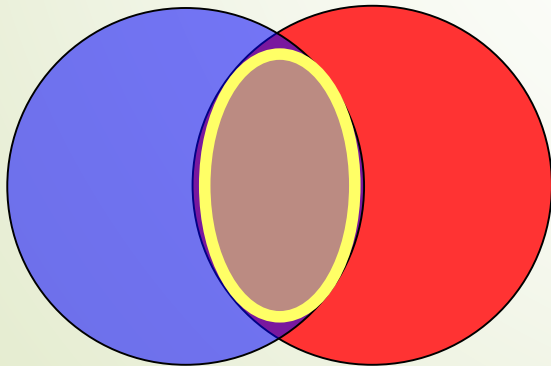
Nucleus shape fluctuates in every collision

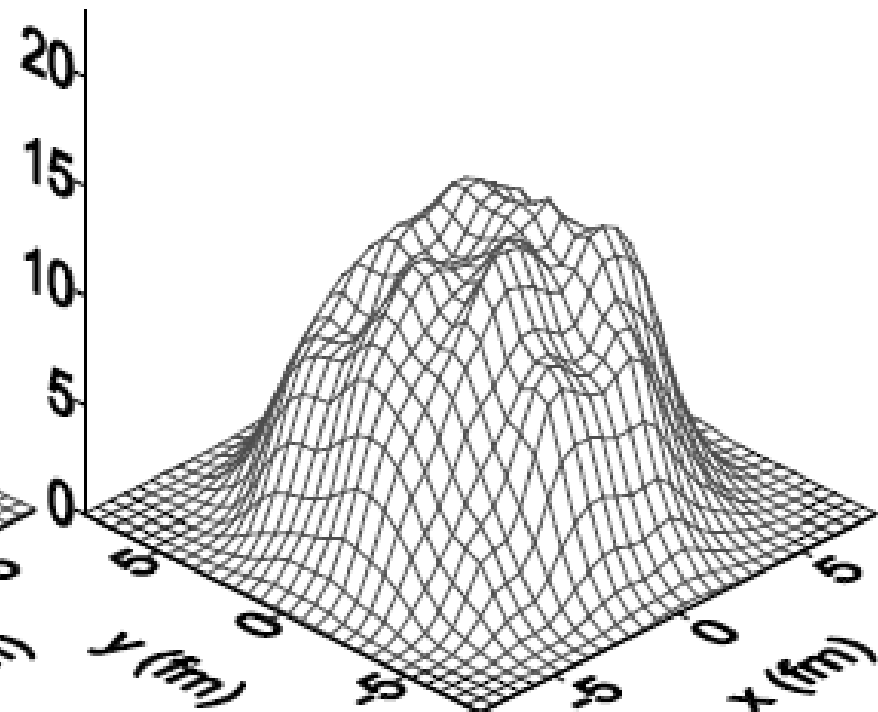
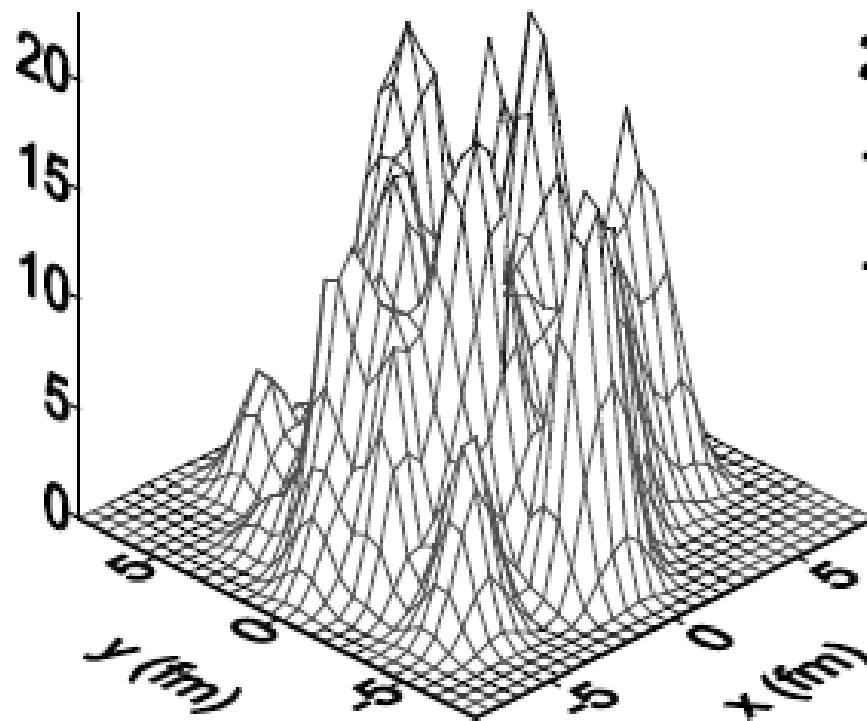
Pb

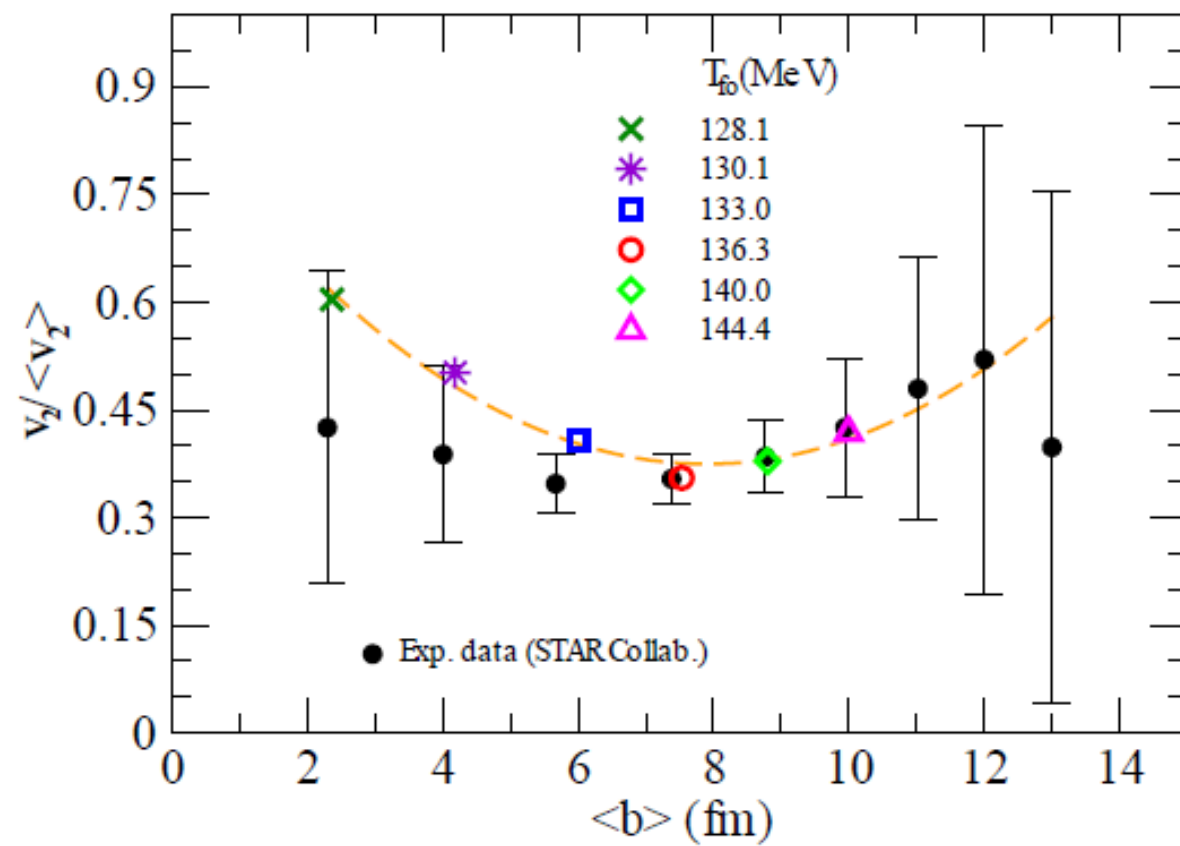


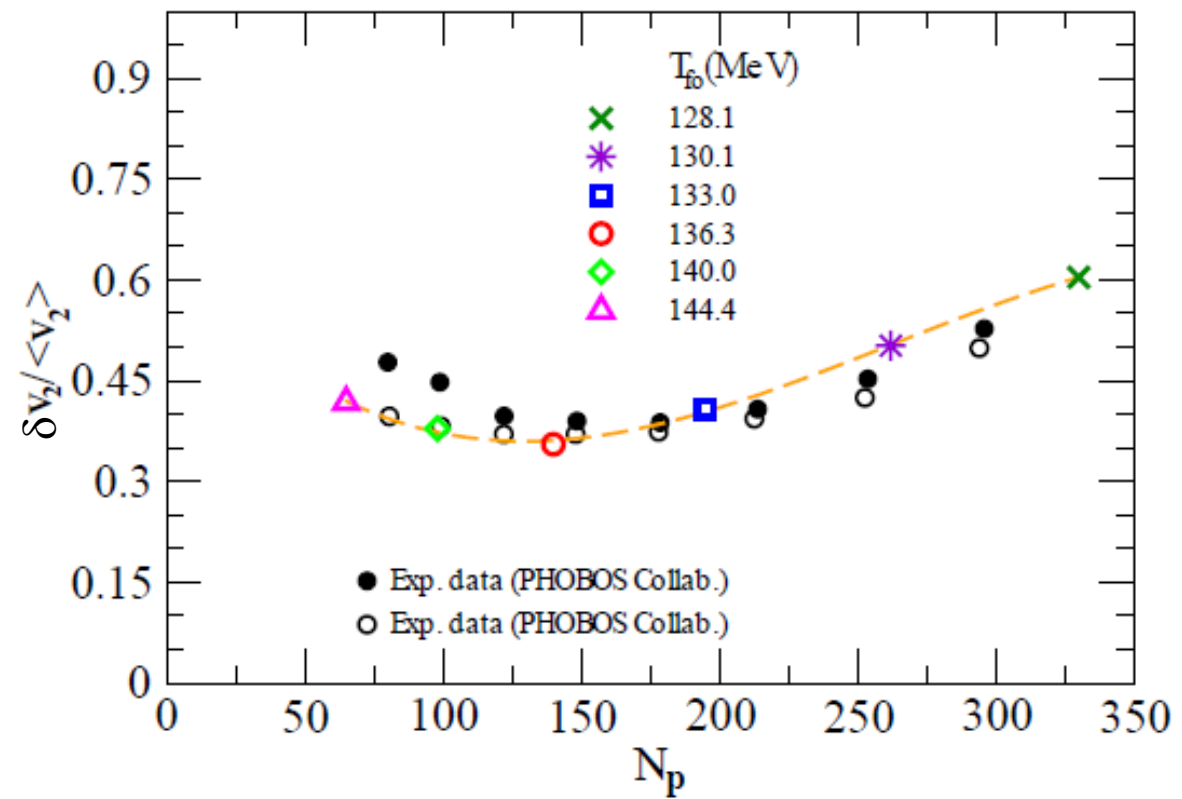
Collision Geometry is dramatically affected

For v_2 scale, we look at the scale of

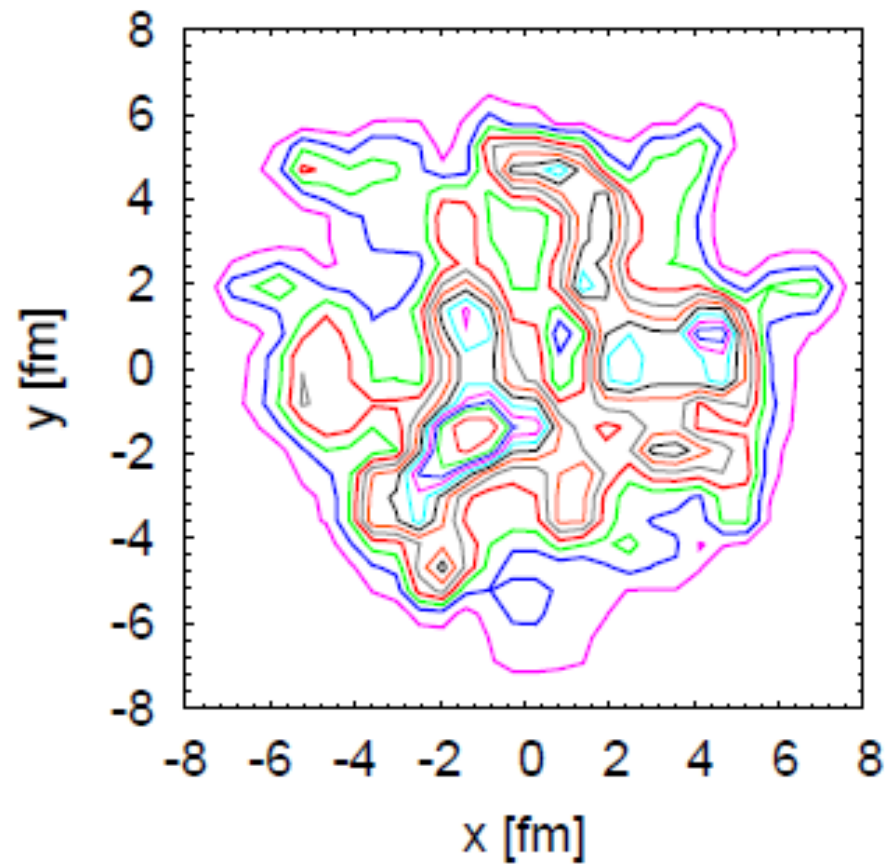




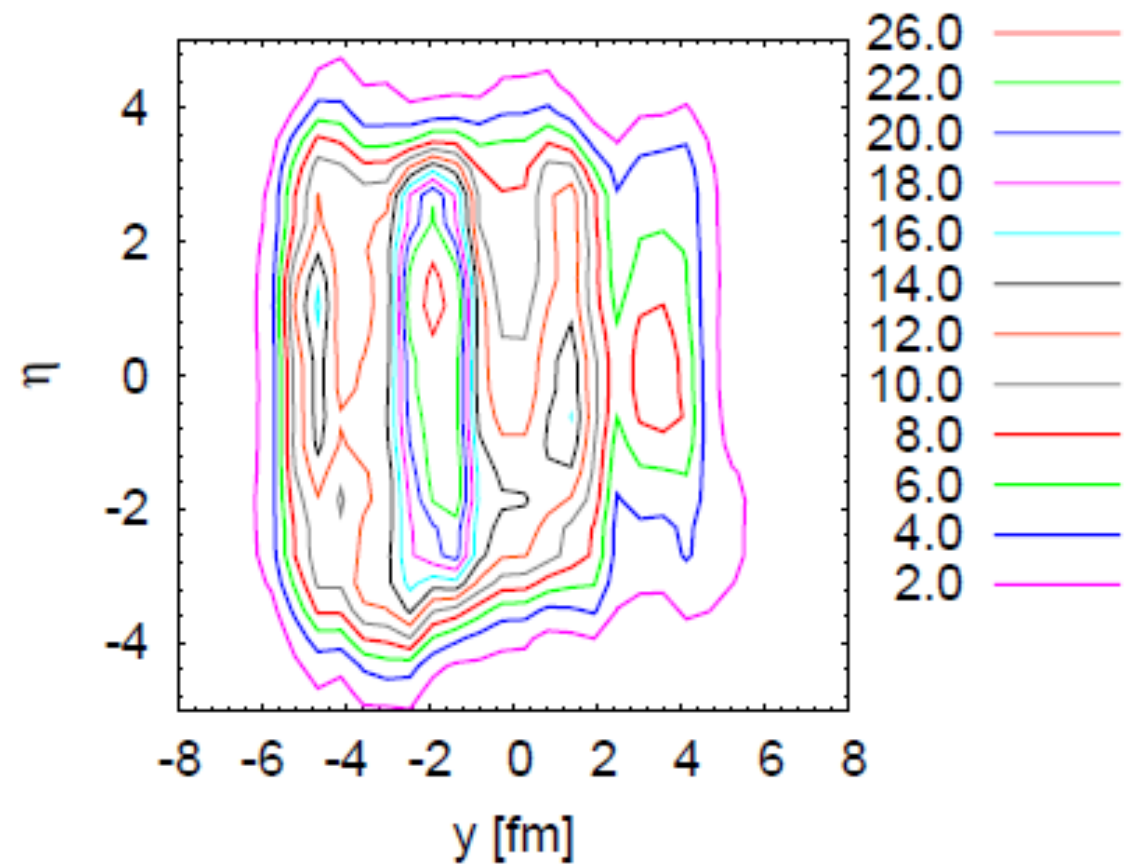


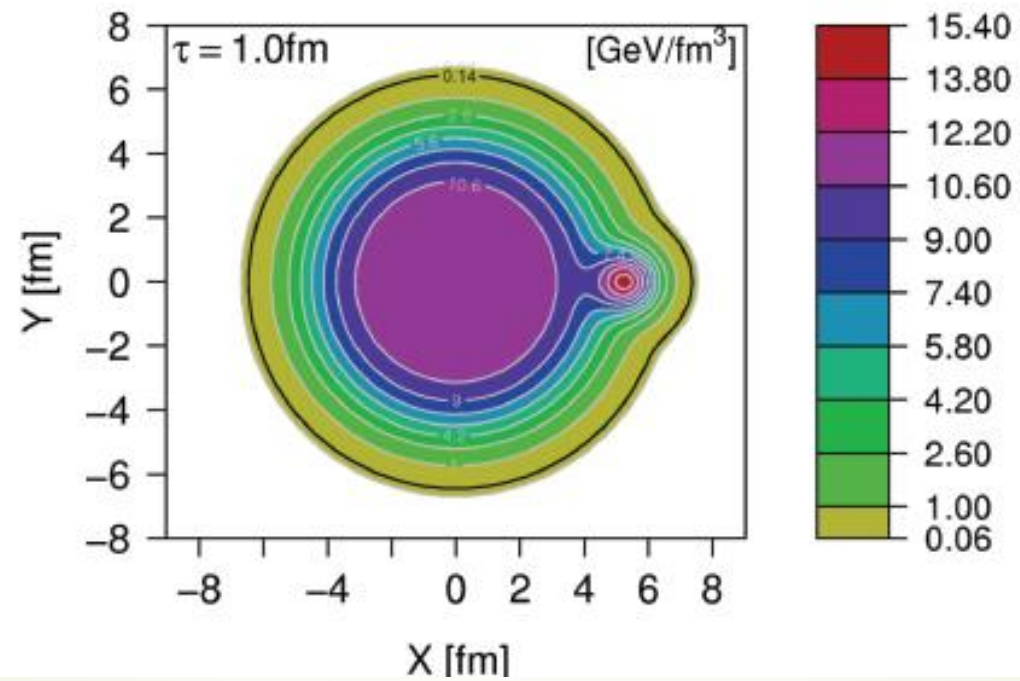


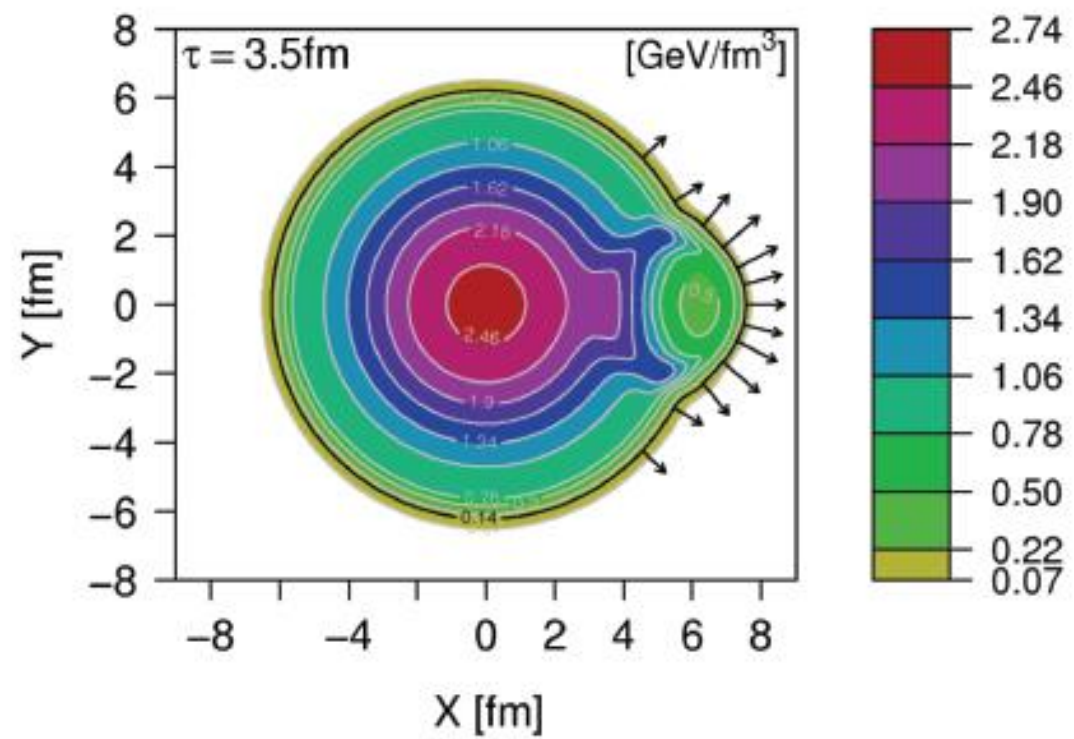
$\tau=1\text{fm}$, $b=0\text{fm}$, $\eta=0$

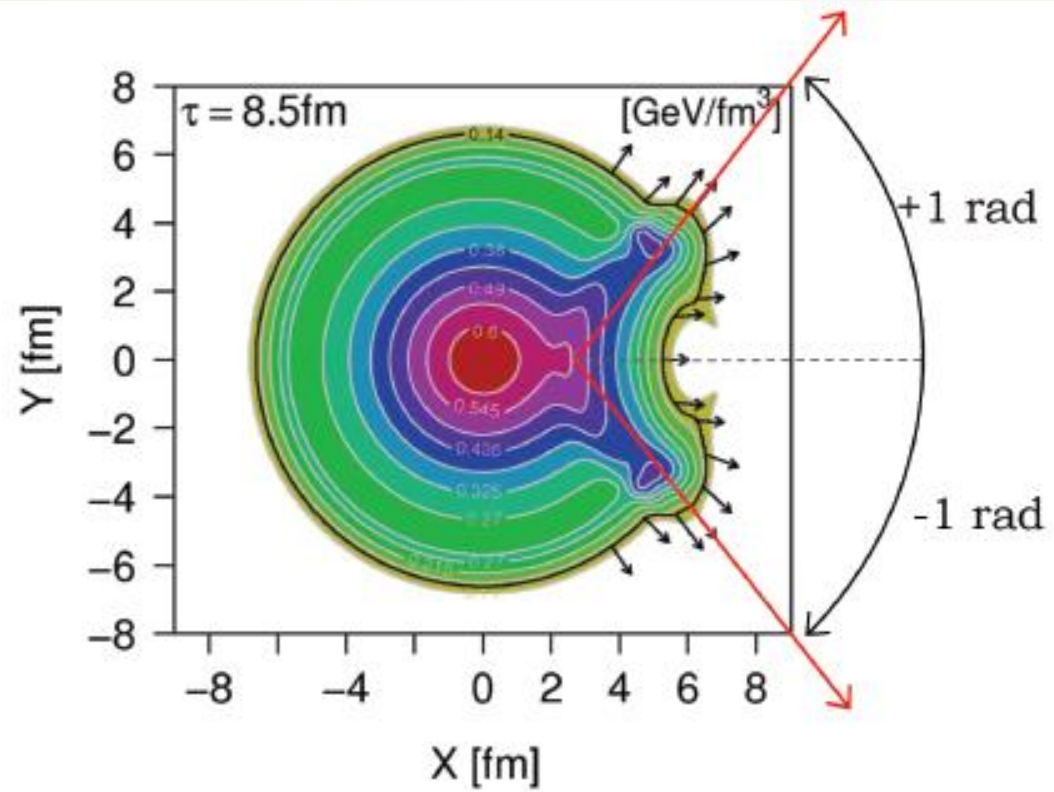


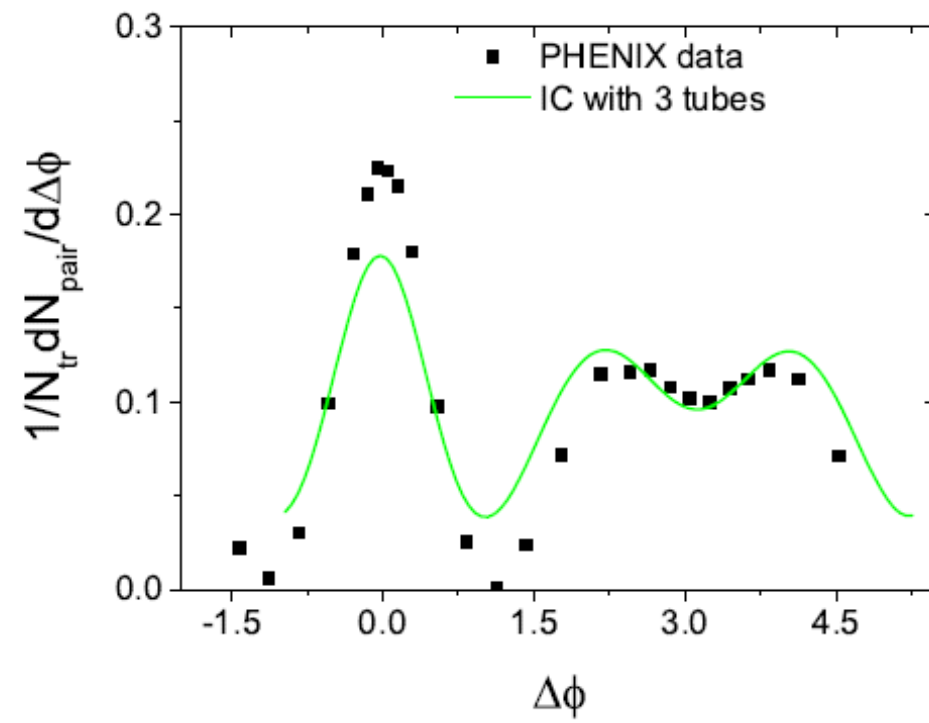
$\tau=1\text{fm}$, $b=0\text{fm}$, $x=-2$





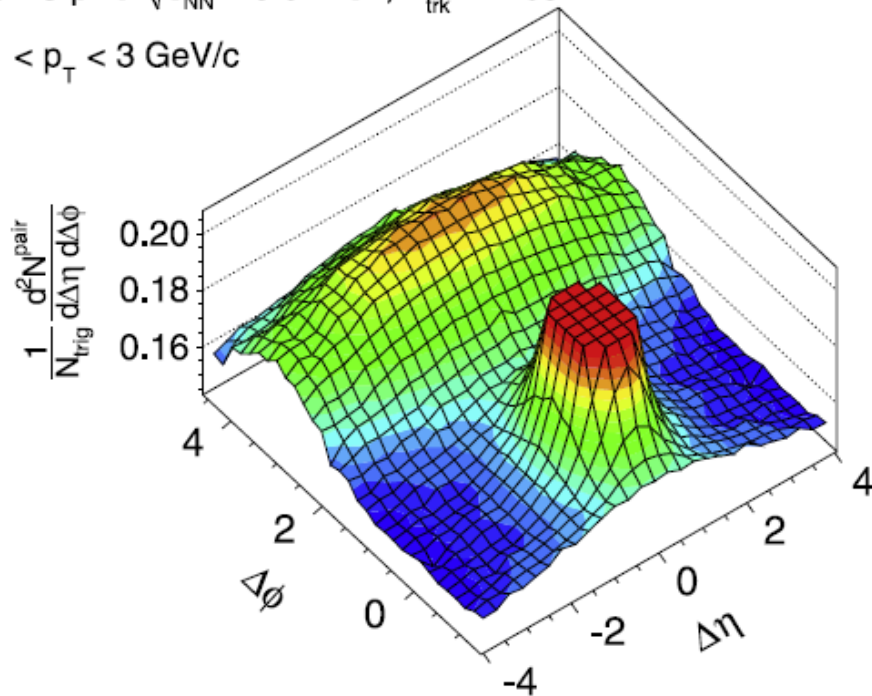




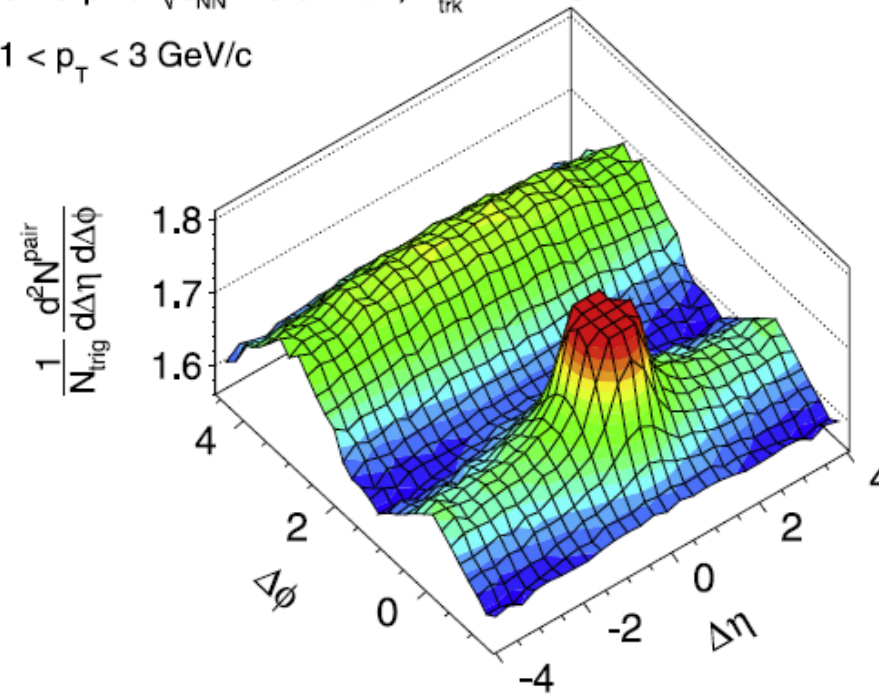


Also for Ridge

a CMS pPb $\sqrt{s_{NN}} = 5.02$ TeV, $N_{\text{trk}}^{\text{offline}} < 35$
 $1 < p_T < 3$ GeV/c

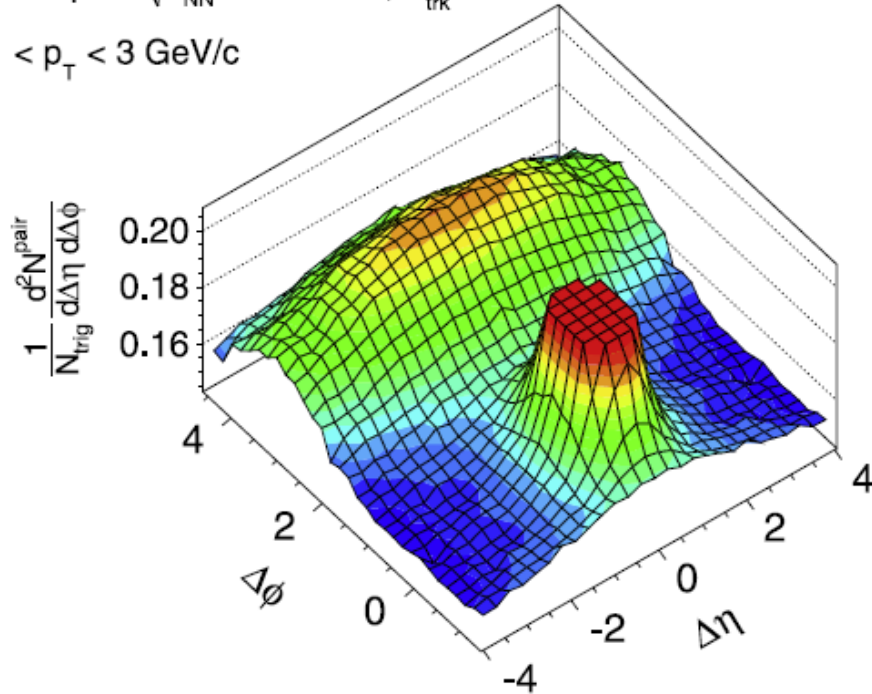


b CMS pPb $\sqrt{s_{NN}} = 5.02$ TeV, $N_{\text{trk}}^{\text{offline}} \geq 110$
 $1 < p_T < 3$ GeV/c

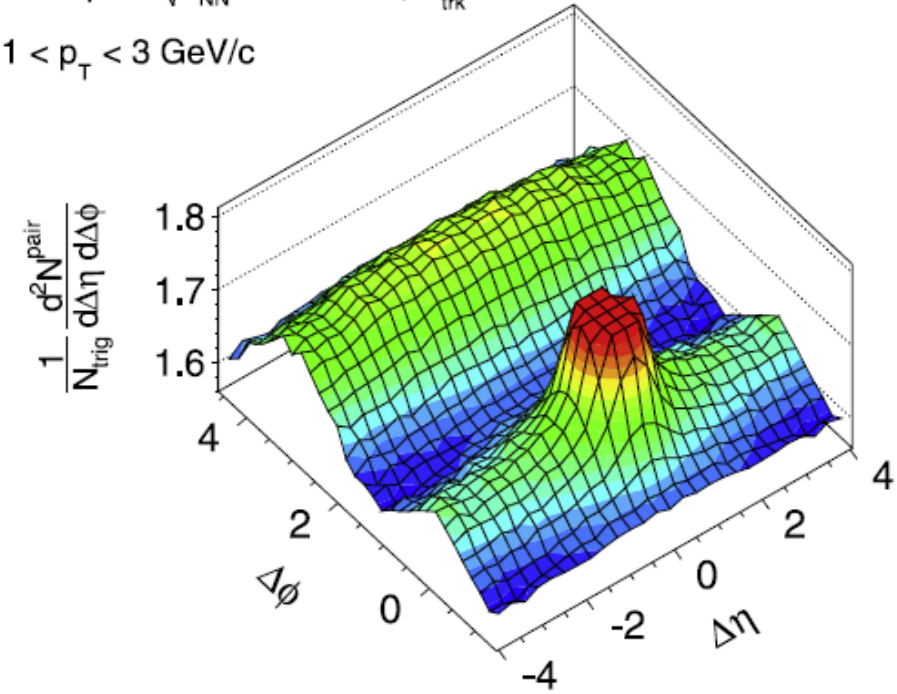


Also for Ridge

a CMS pPb $\sqrt{s_{NN}} = 5.02$ TeV, $N_{\text{trk}}^{\text{offline}} < 35$
 $1 < p_T < 3$ GeV/c



b CMS pPb $\sqrt{s_{NN}} = 5.02$ TeV, $N_{\text{trk}}^{\text{offline}} \geq 110$
 $1 < p_T < 3$ GeV/c



Thus we thought the presence of granularities in EbE initial conditions are fundamental... but

Harmonic Flow Analysis (now the basis of Hydrodynamic Model)

$$\frac{dN}{dyd\phi} = \frac{dN}{dy} [1 + 2v_1 \cos(\phi - \psi_1) + 2v_2 \cos[2(\phi - \psi_2)] + \dots],$$
$$v_n = \frac{\int d\phi \cos[n(\phi - \psi_n)] \frac{dN}{dyd\phi}}{\int d\phi \frac{dN}{dyd\phi}} = \langle \cos[n(\phi - \psi_n)] \rangle,$$

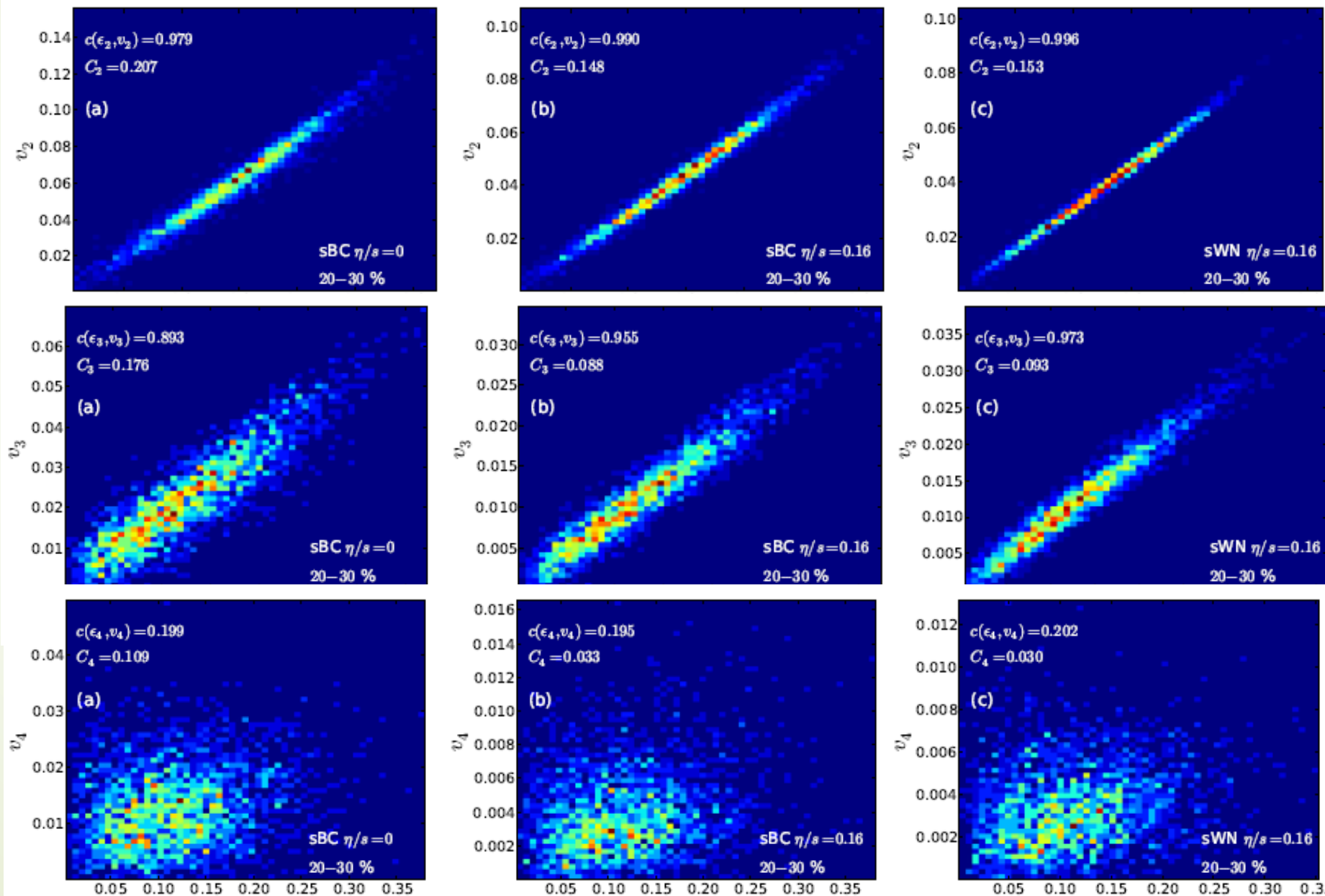
$$v_n = v_n(p_T; b)$$

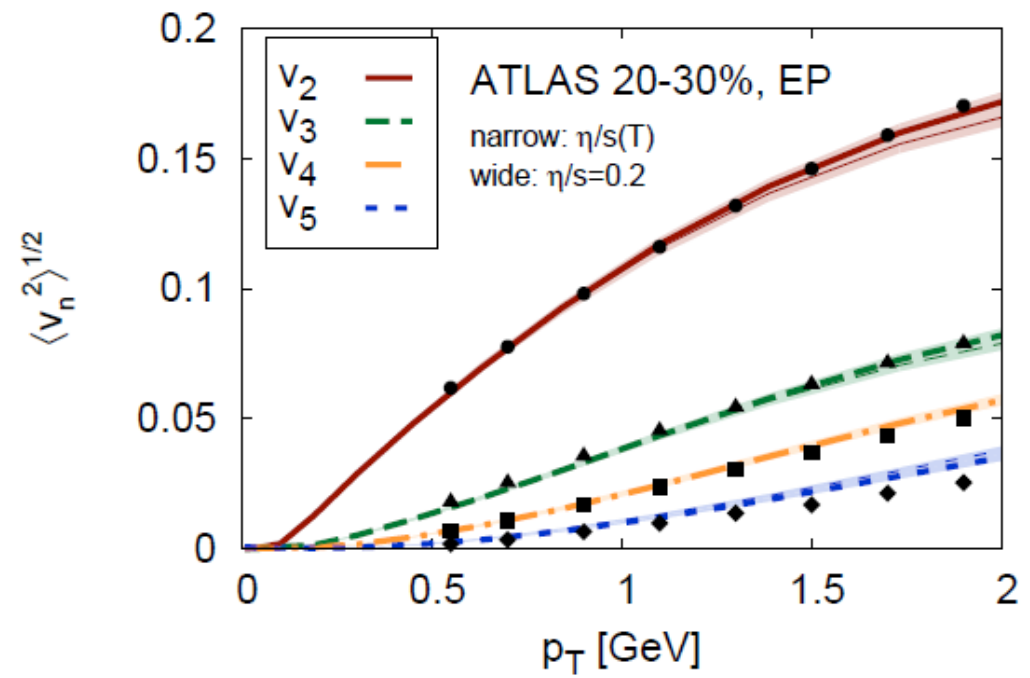
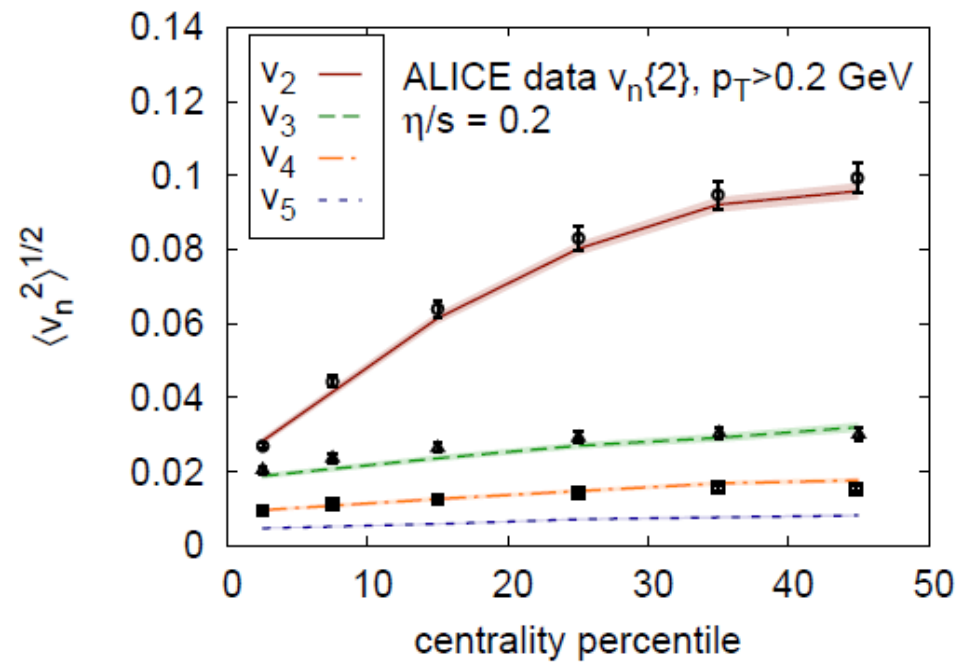
B.Alver, G.Roland (2010)

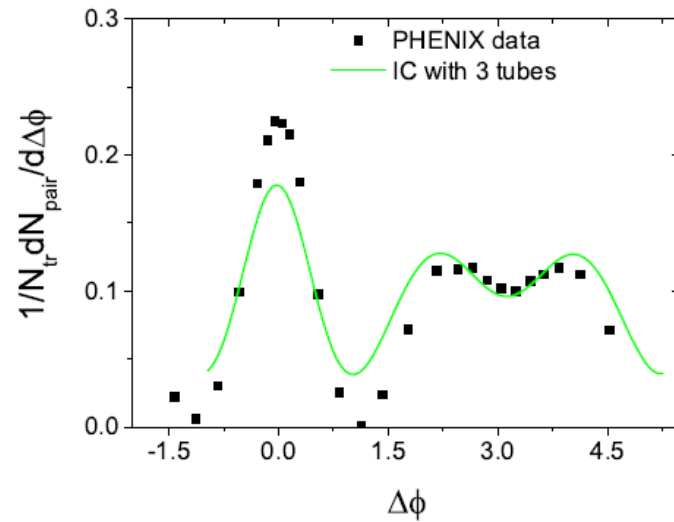
Great Success and can be used to determine
the EoS and transport coefficients !

EbE Correlation ($\epsilon_n - v_n$)

H. Niemi, G. S. Denicol,
H. Holopain, P. Huovinen
(2012)







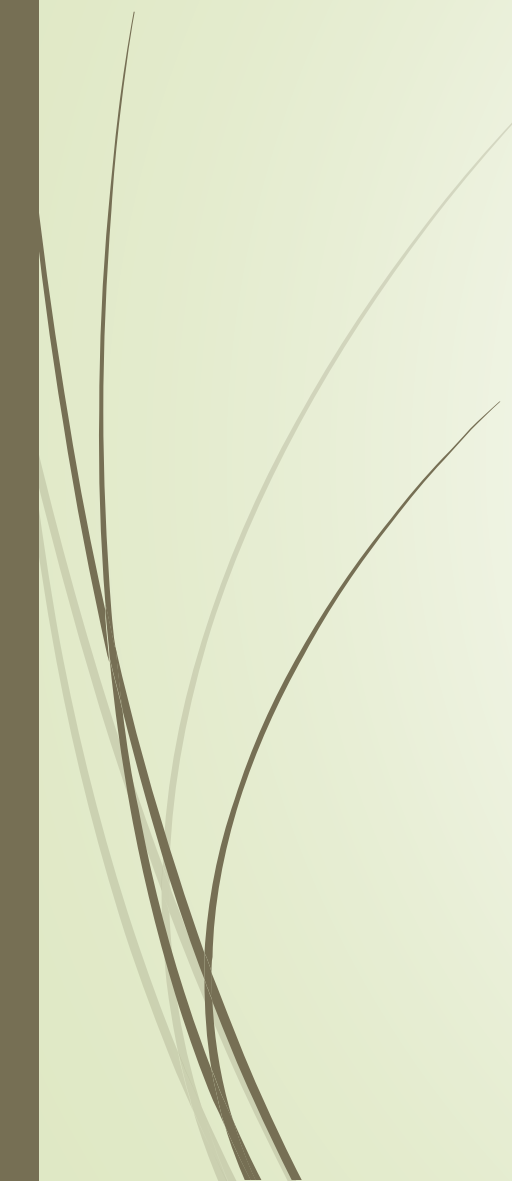
Two hadron correlation $\rightarrow$ Just a triangular flow V_3



Success of Hydrodynamic Approach



Determine EoS and Transport coefficients

- Search for the critical point
 - EoS for Baryon Rich Matter
- 

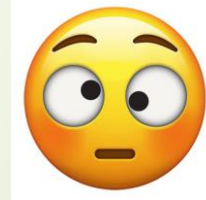
Success of Hydrodynamic Approach



Determine EoS and Transport coefficients

- Search for the critical point
- EoS for Baryon Rich Matter

EbE Fluctuation vs Use of EoS



Success of Hydrodynamic Approach



Determine EoS and Transport coefficients

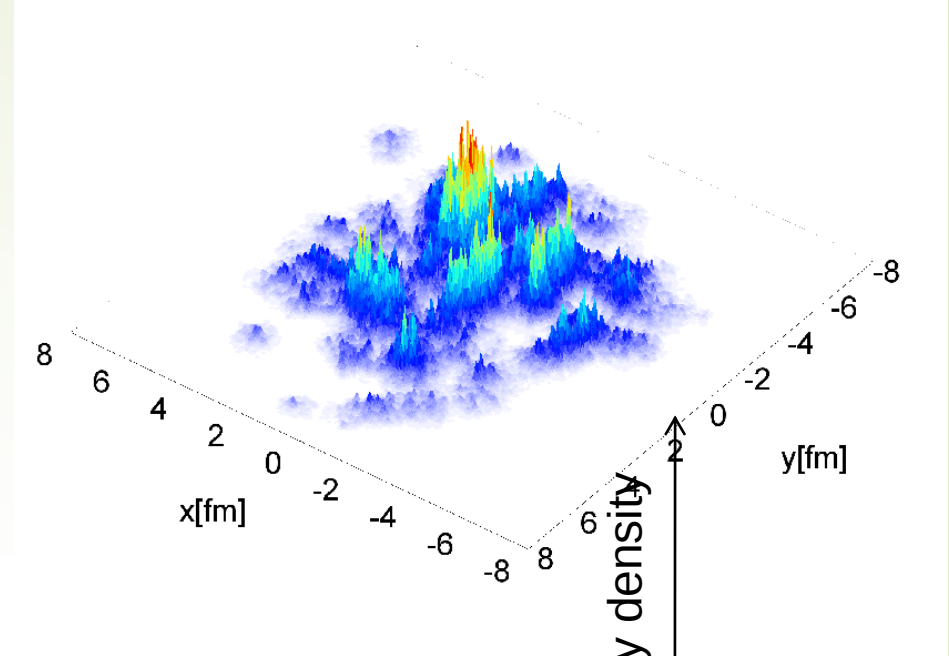
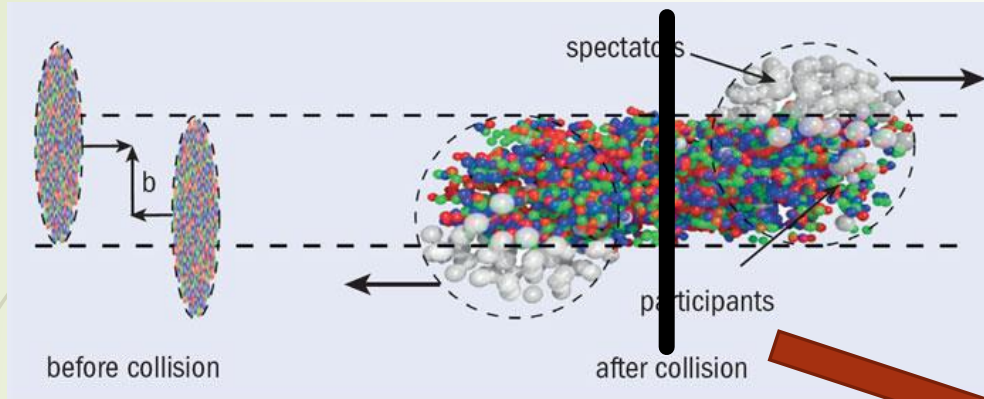
- Search for the critical point
- EoS for Baryon Rich Matter

EbE Fluctuation vs Use of EoS



How quantitatively precise?

Initial condition in HIC

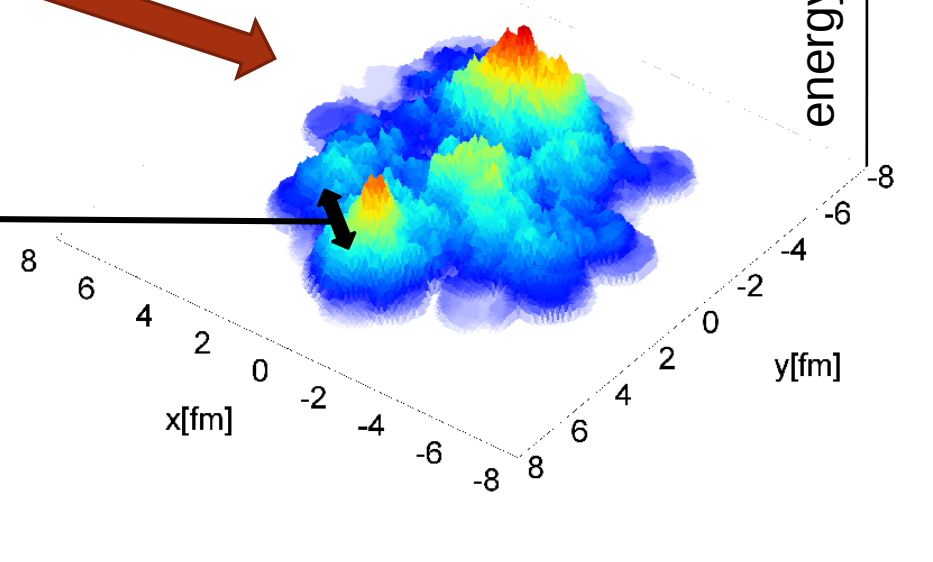


The width of one peak

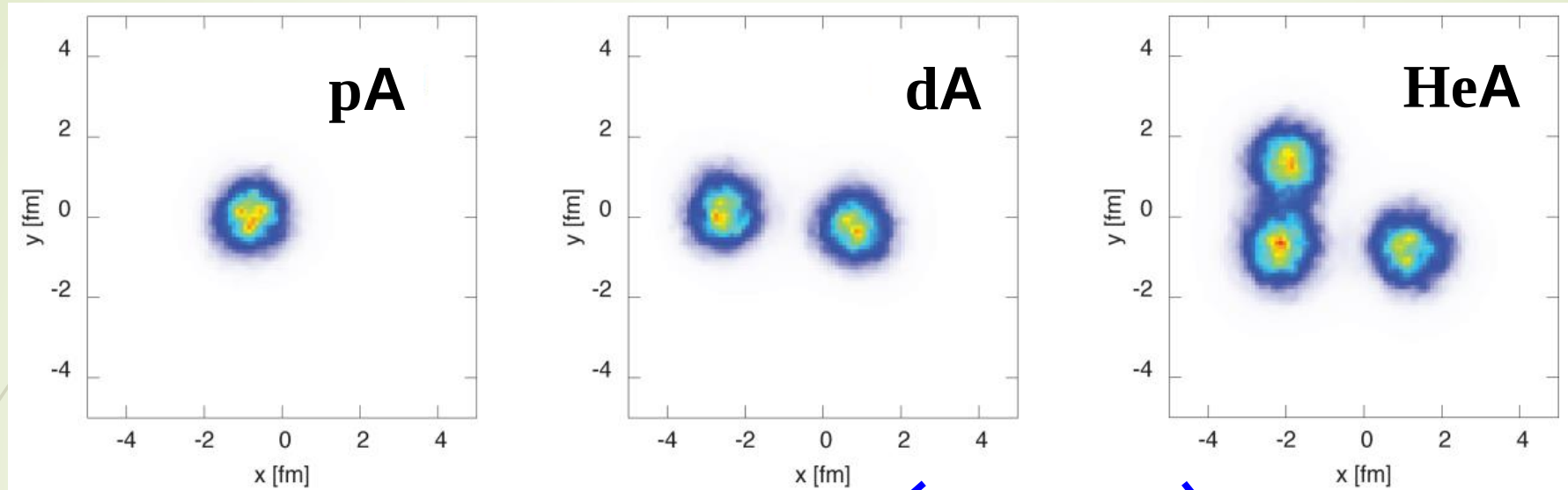
$\sim 1 \text{ fm} (= 10^{-15} \text{ m})$

The grid size for hydro simulation

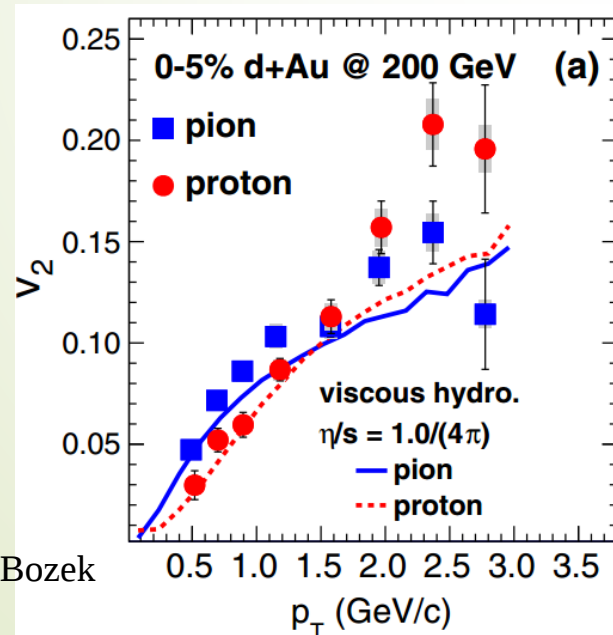
$\leq 0.1 \text{ fm}$



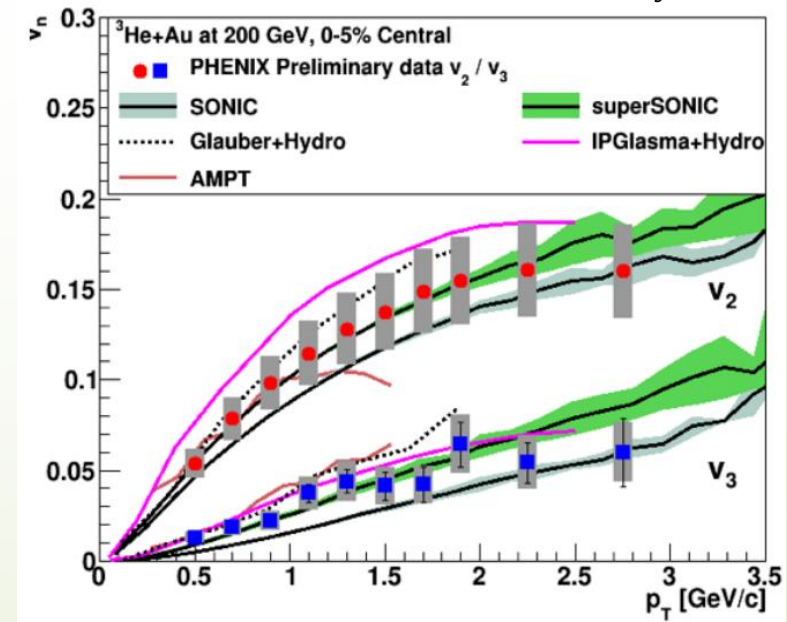
Hydrodynamic description of small systems?



Good agreement with calculations



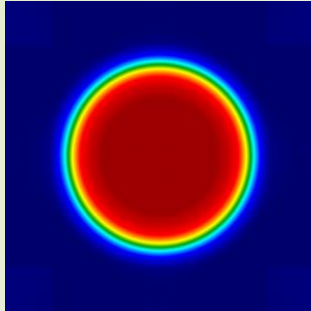
Calculation by Bozek



Calculation by Romatschke

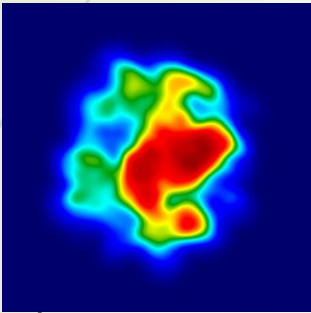
Fluid-dynamics is being applied to describe small distance/time scales

30



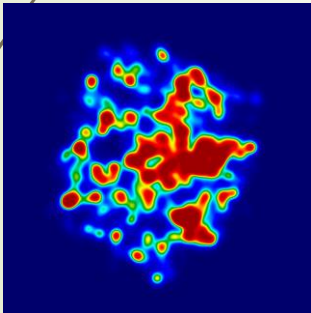
Smooth Initial conditions

$\lambda \sim 5$ fm



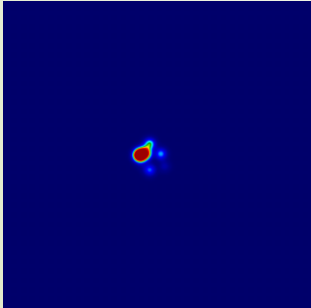
Fluctuations in nucleon positions

$\lambda \sim 1$ fm



**Local entropy fluctuations (NBD)
sub-fermionic fluctuations**

$\lambda \sim 0.3$ fm



proton-nucleus collisions

$\lambda \sim 0.3$ fm
or smaller



Hydrodynamic Evolution

$$T_{\mu\nu} = T_{\mu\nu}(x)$$

$$\partial^\mu T_{\mu\nu} = 0 \quad + \quad \text{Equation of States}$$

Hydrodynamic Evolution

$$T_{\mu\nu} = T_{\mu\nu}(x)$$

$$\partial^\mu T_{\mu\nu} = 0$$

- + Equation of States (Local equilibrium)
- + Dissipative Correction
- + Freezeout Process

Hydrodynamic Evolution

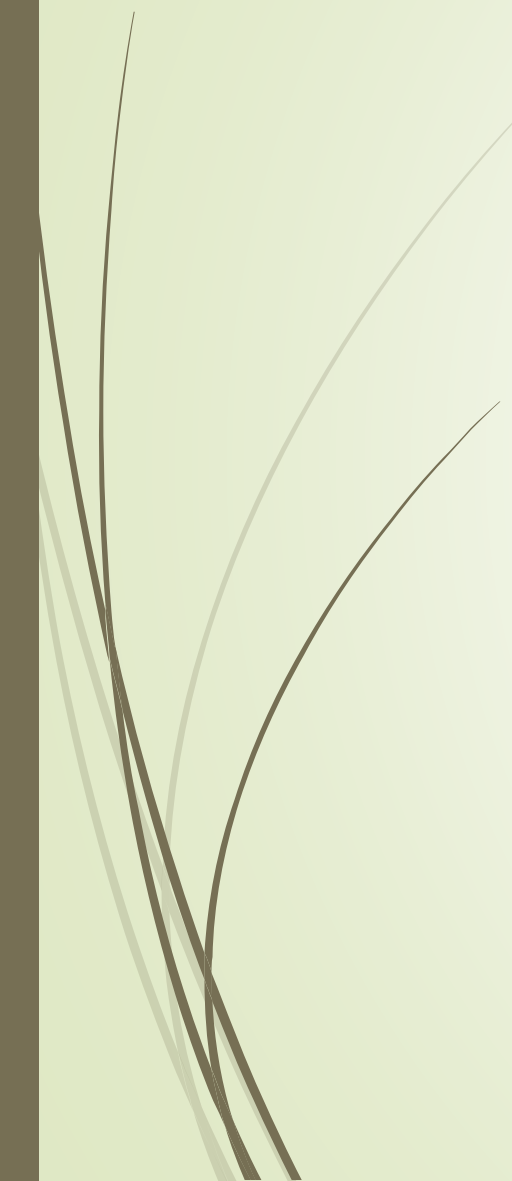
$$T_{\mu\nu} = T_{\mu\nu}(x)$$

$$\partial^\mu T_{\mu\nu} = 0 \quad + \quad \begin{array}{l} \text{Equation of States (Local equilibrium)} \\ \text{Dissipative Correction} \\ \text{Freezeout Process} \end{array}$$

We enforce to close the system in terms of energy-momentum tensor...



Several Types of Coarse Graining

- Space-like coarse graining
 - Time-like coarse graining
 - Event average (observables)
- 

Hydrodynamic Evolution

$$T_{\mu\nu} = T_{\mu\nu}(x)$$

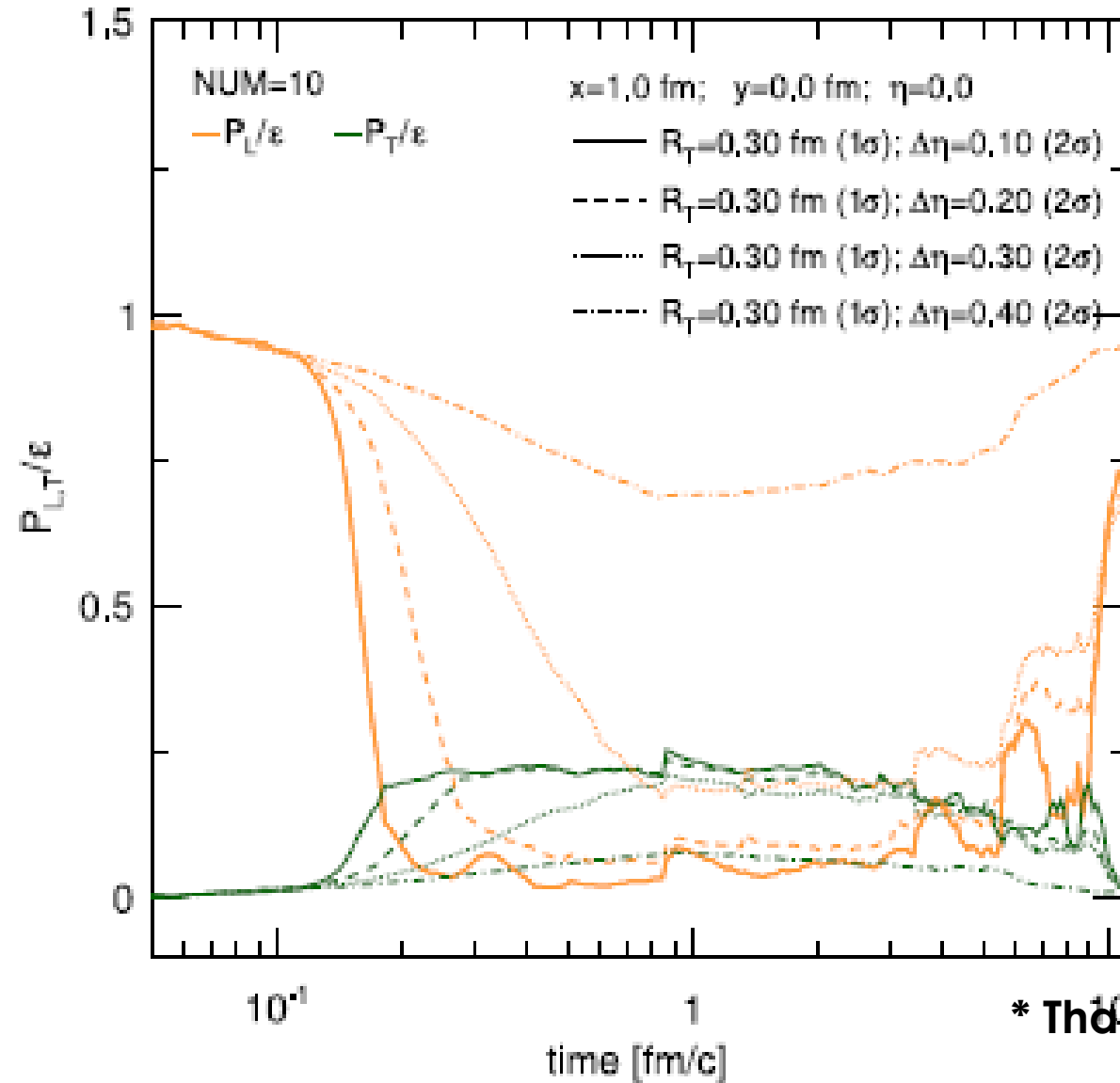
$$\partial^\mu T_{\mu\nu} = 0$$

- + Equation of States (Local equilibrium)
- + Dissipative Correction
- + Freezeout Process

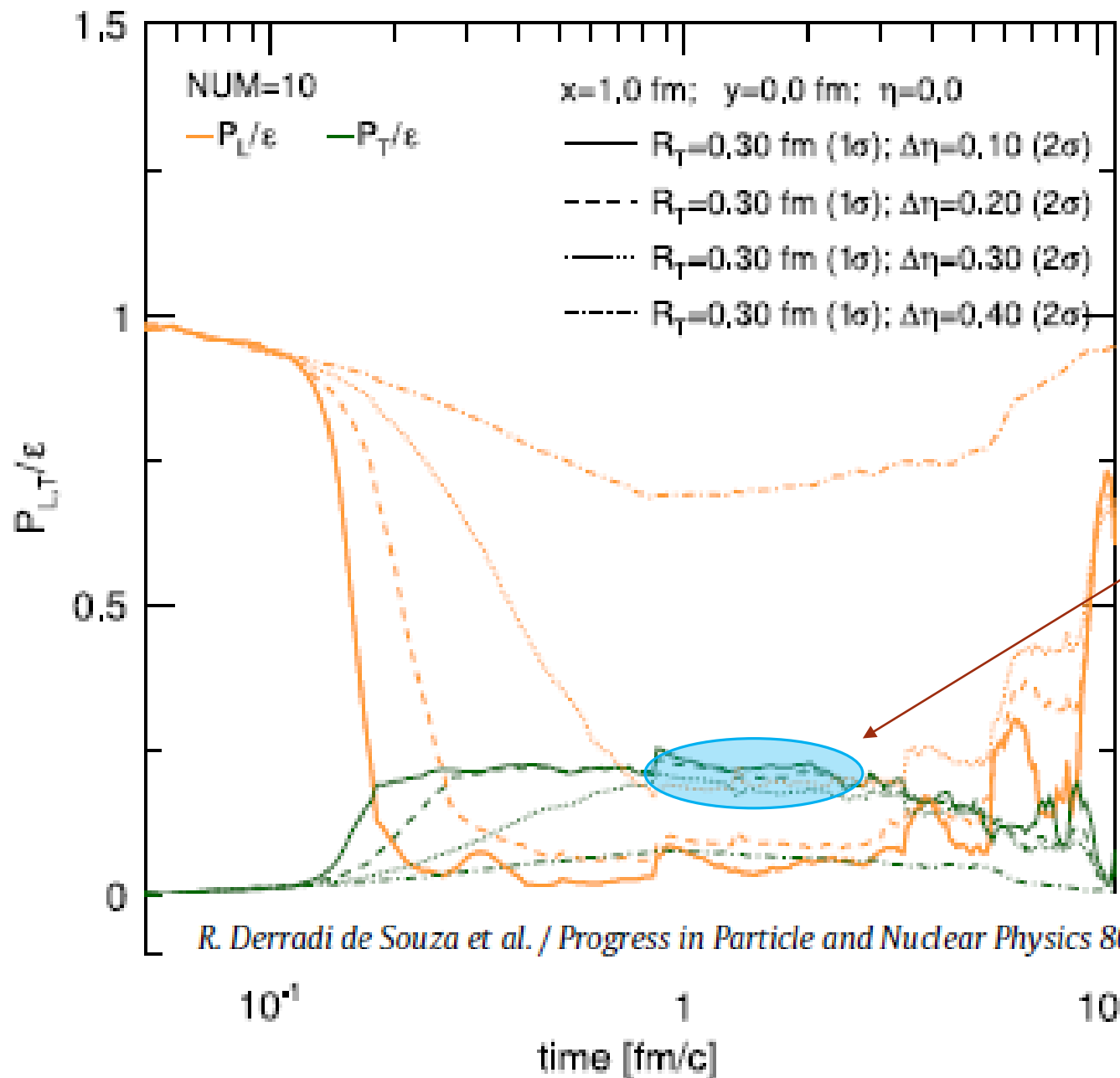
What happens with the “Event Average?”



PHSD Representation of Energy&Momentum Tensor



* Thanks to Elena and collaborators



Thermal equilibrium:

Only very small
fraction of space-
time domain

R. Derradi de Souza et al. / Progress in Particle and Nuclear Physics 86 (2016) 35–85



Basic Question:
In Hydrodynamic description,

The harmonic coefficients really
carry sufficient Information for the
quantitative analysis??





Basic Question:
In Hydrodynamic description,

The harmonic coefficients really
carry sufficient Information for a
quantitative analysis??



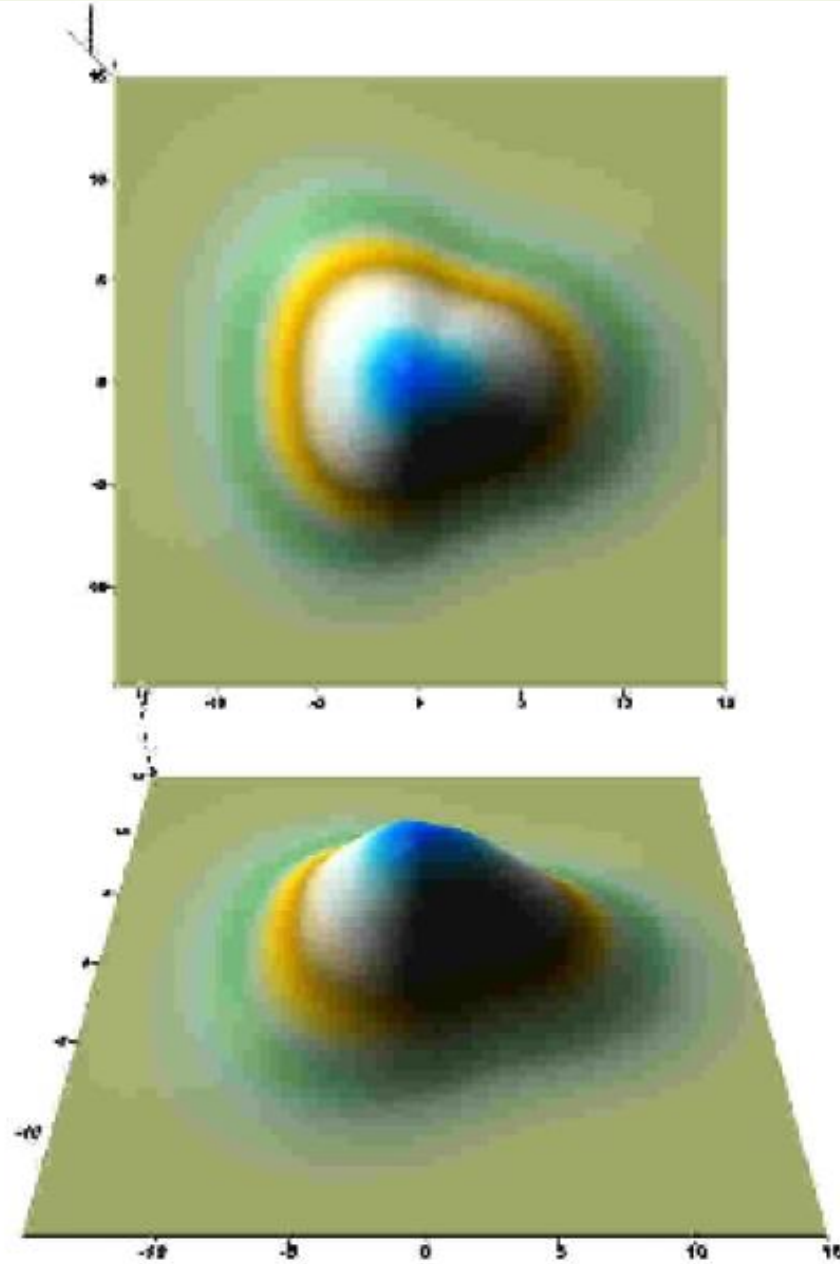
Are there other initial states which
give rise to the same behavior of
the harmonic coefficients?

An example: *Smooth*

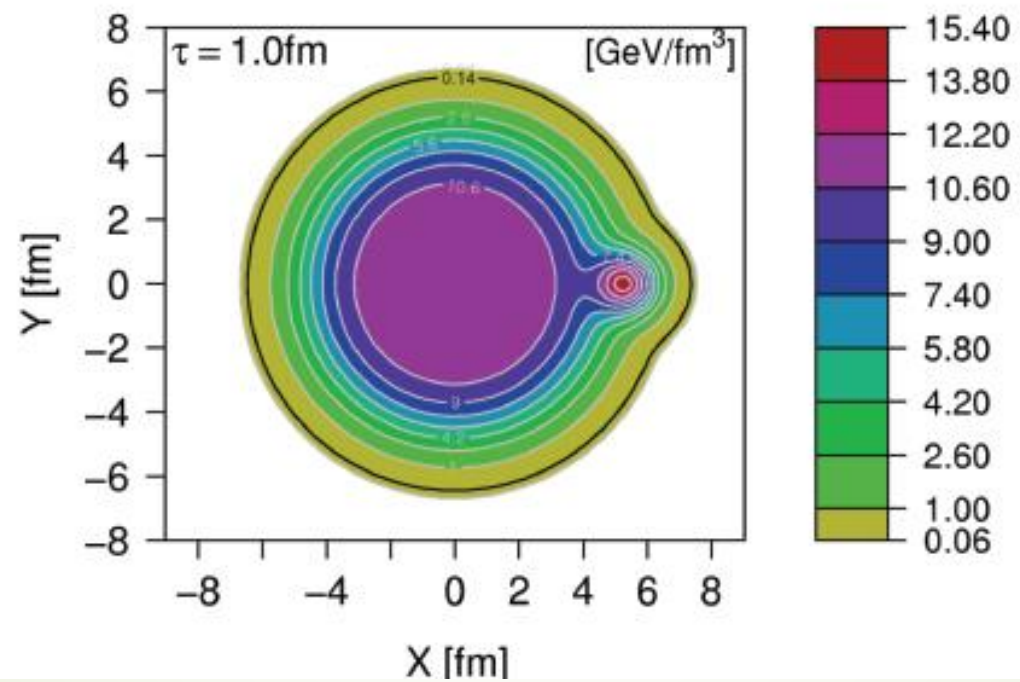
$$\varepsilon(r, \theta) = Z \exp \left\{ -\frac{r^2}{R^2(\theta)} \right\}$$

$$R(\theta) = R_0 \left(1 + \sum_{n=2} C_n \cos n(\theta - \theta_n) \right)^{1/2}$$

Smooth

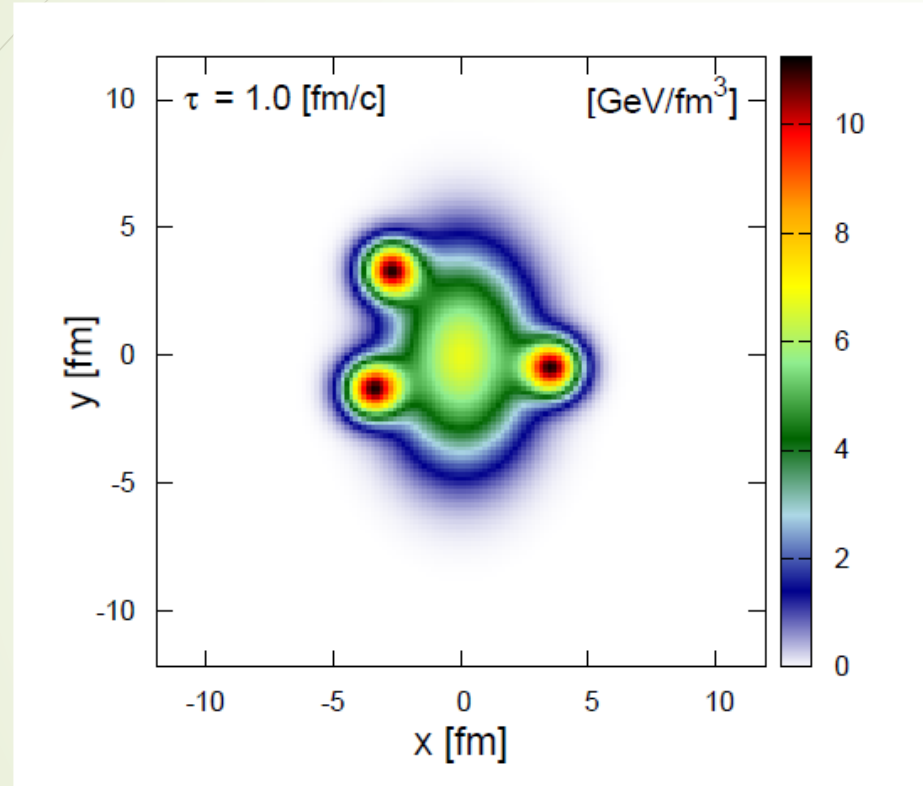


Comare with *Granular*



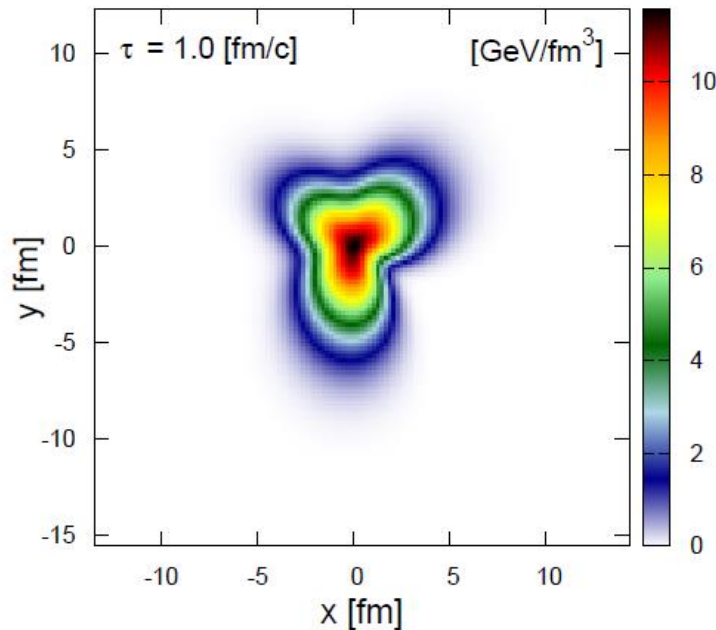
Granular

or

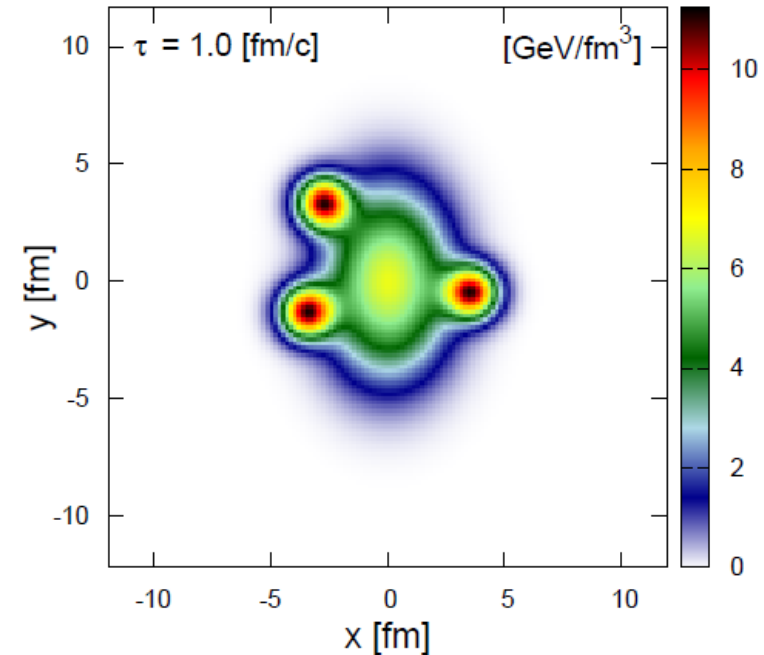


etc....

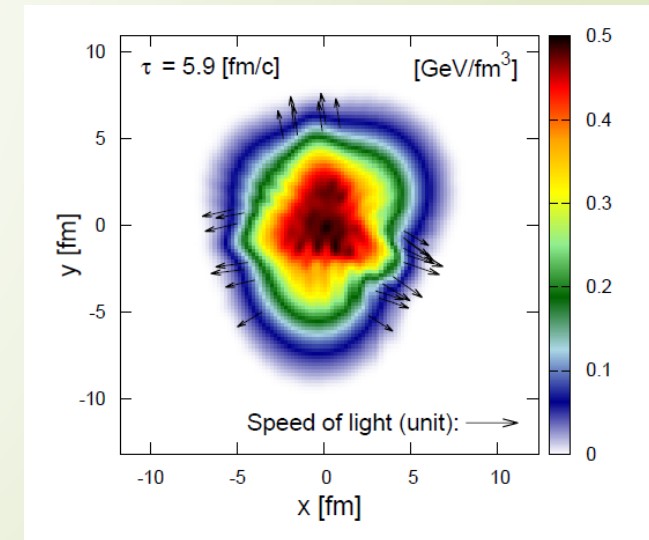
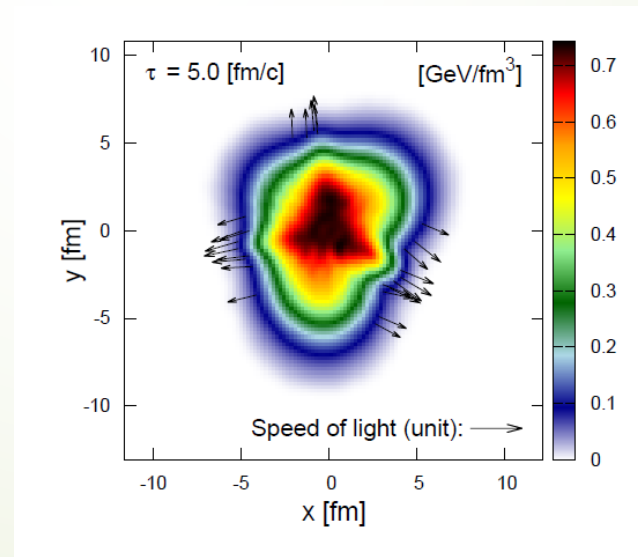
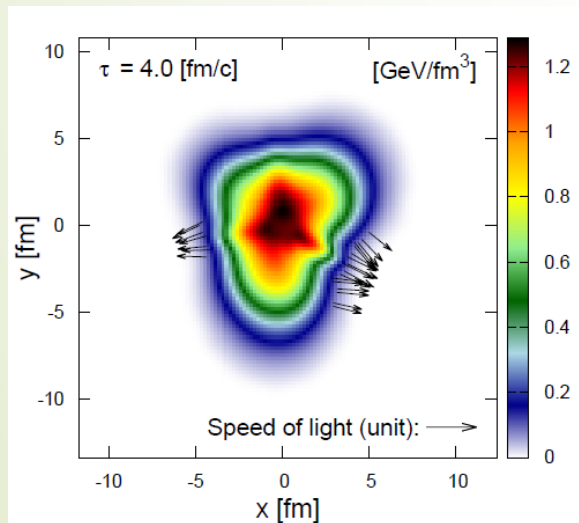
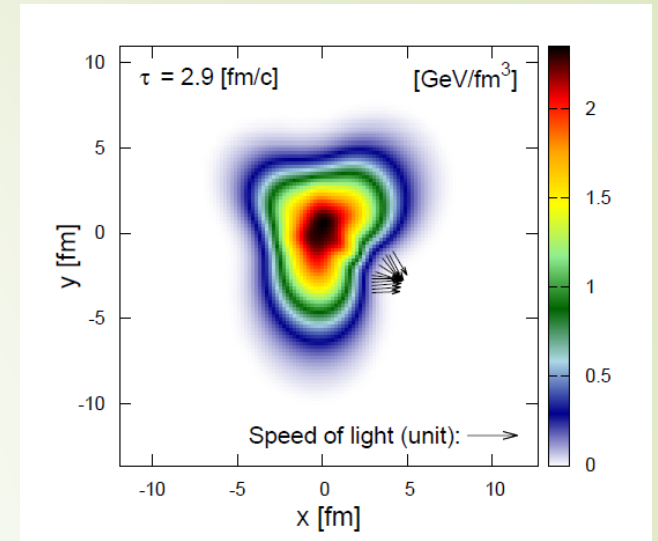
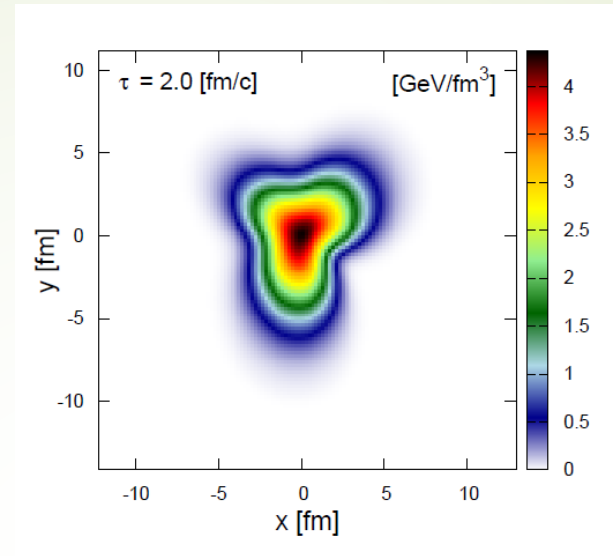
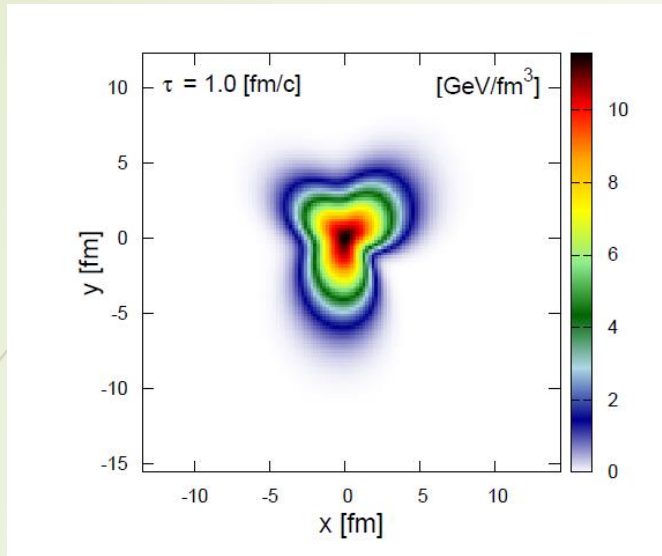
Compare two different initial conditions, having in average the same anisotropic distribution coefficients, $\{\varepsilon_n\}$, one **smooth**, other with **granularities**.



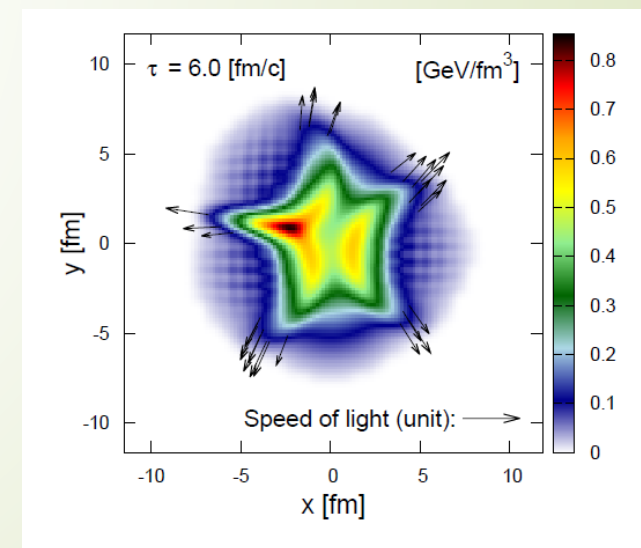
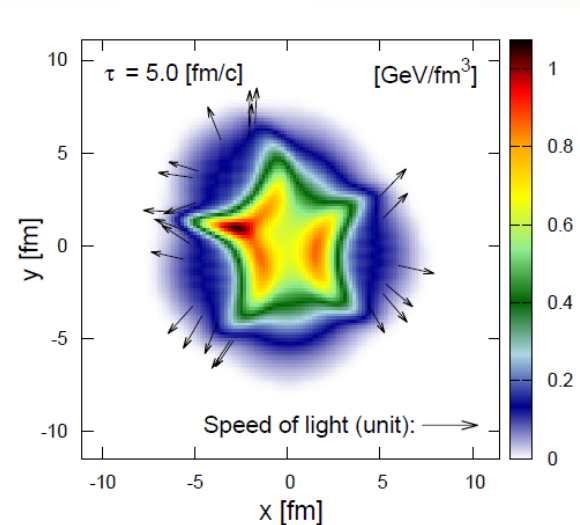
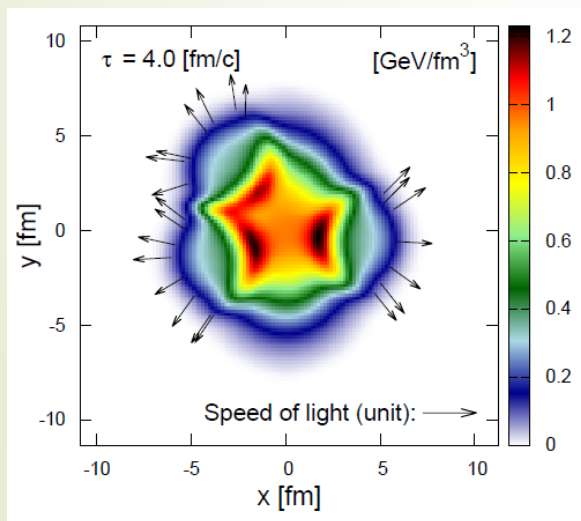
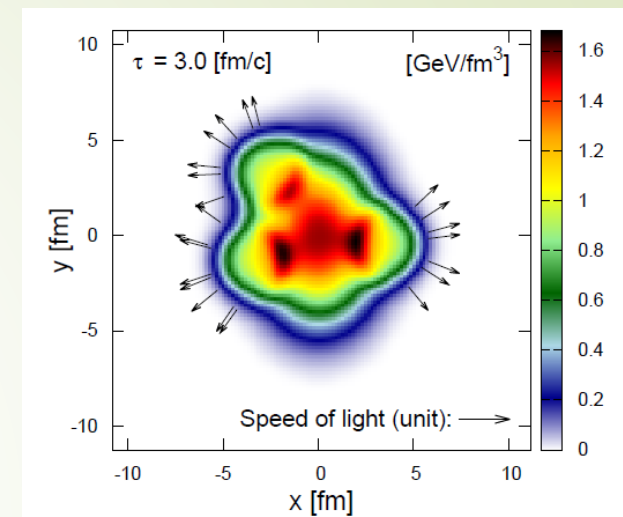
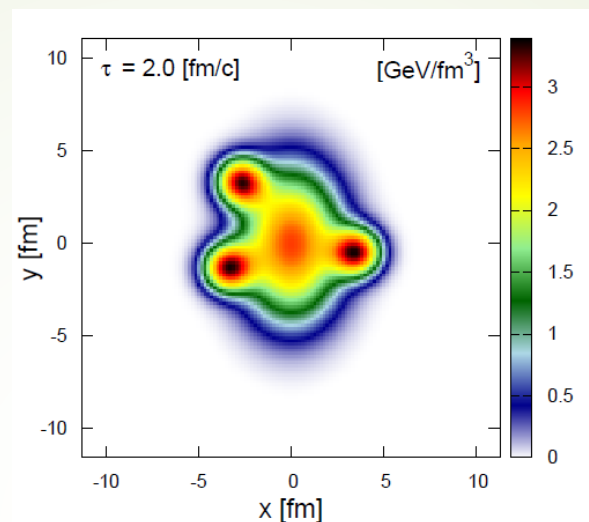
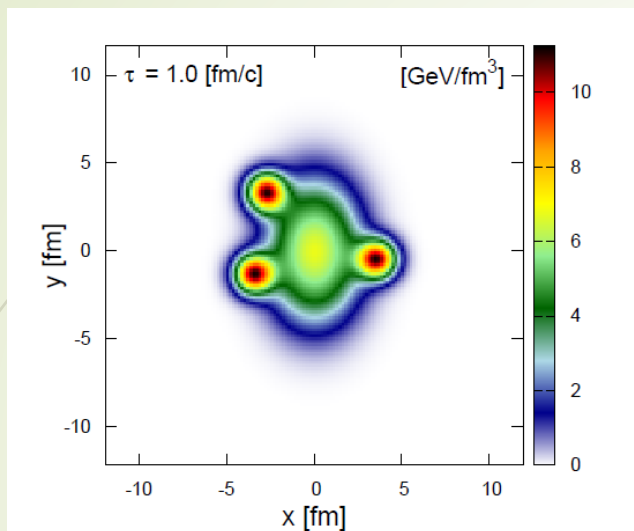
VS



Smooth



Granular



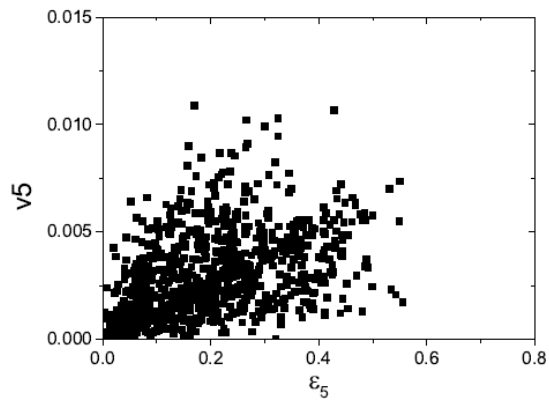
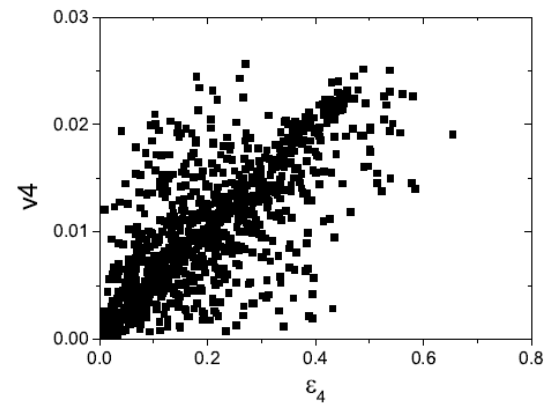
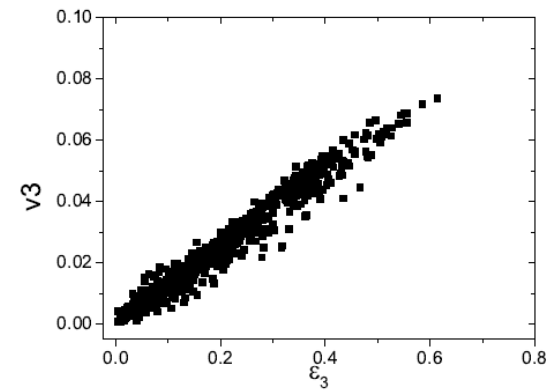
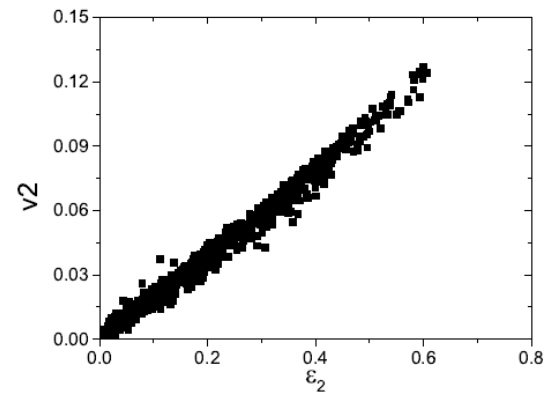


Tuning to have the same energy density anisotropy with **smooth** and **granular** initial conditions, their time evolutions are totally different.

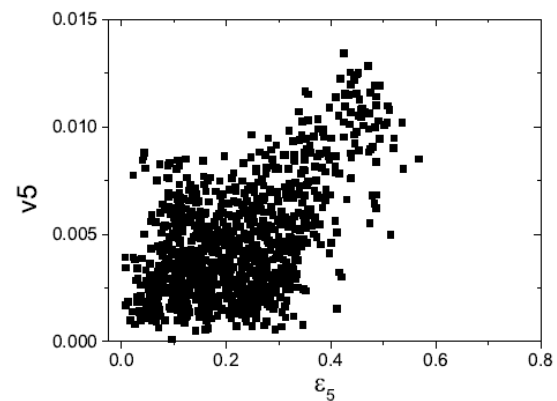
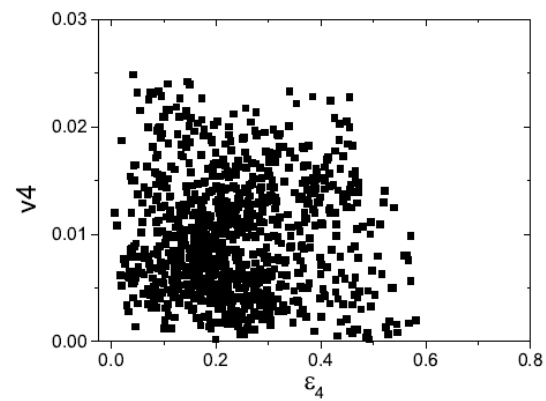
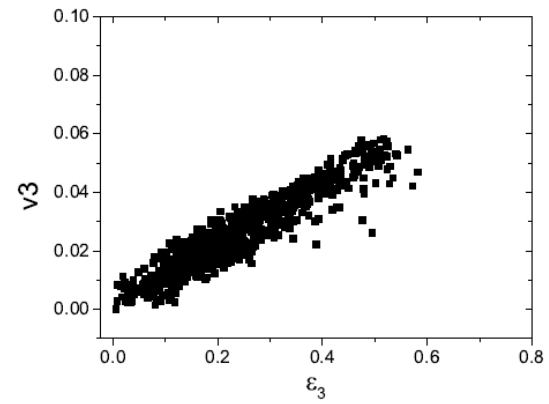
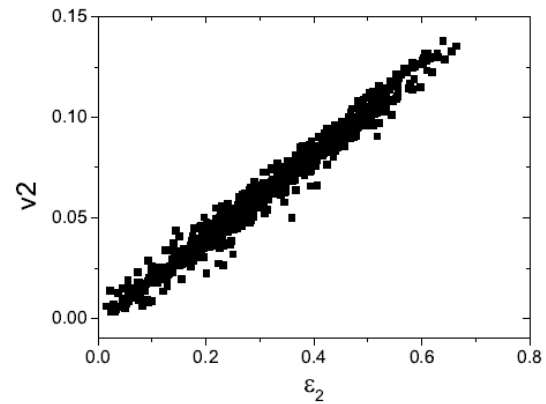
But statistically,...



Correlation between $\varepsilon_n \times v_n$

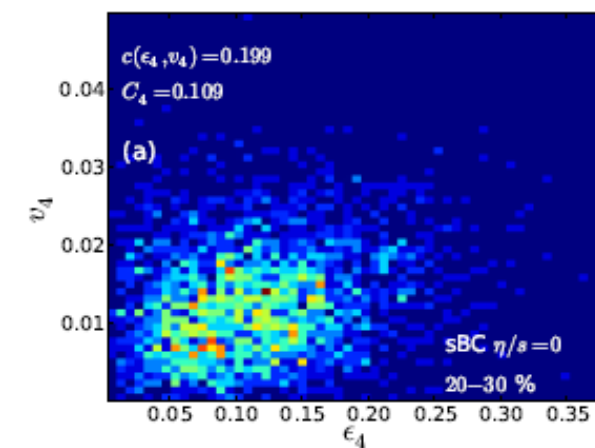
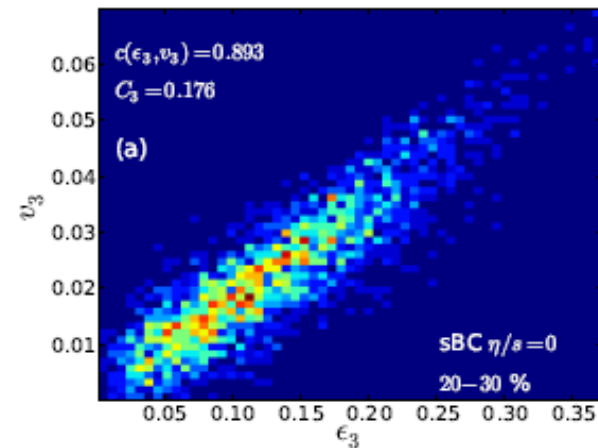
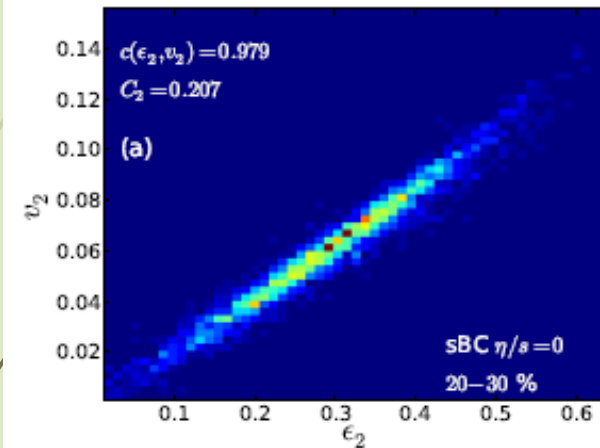


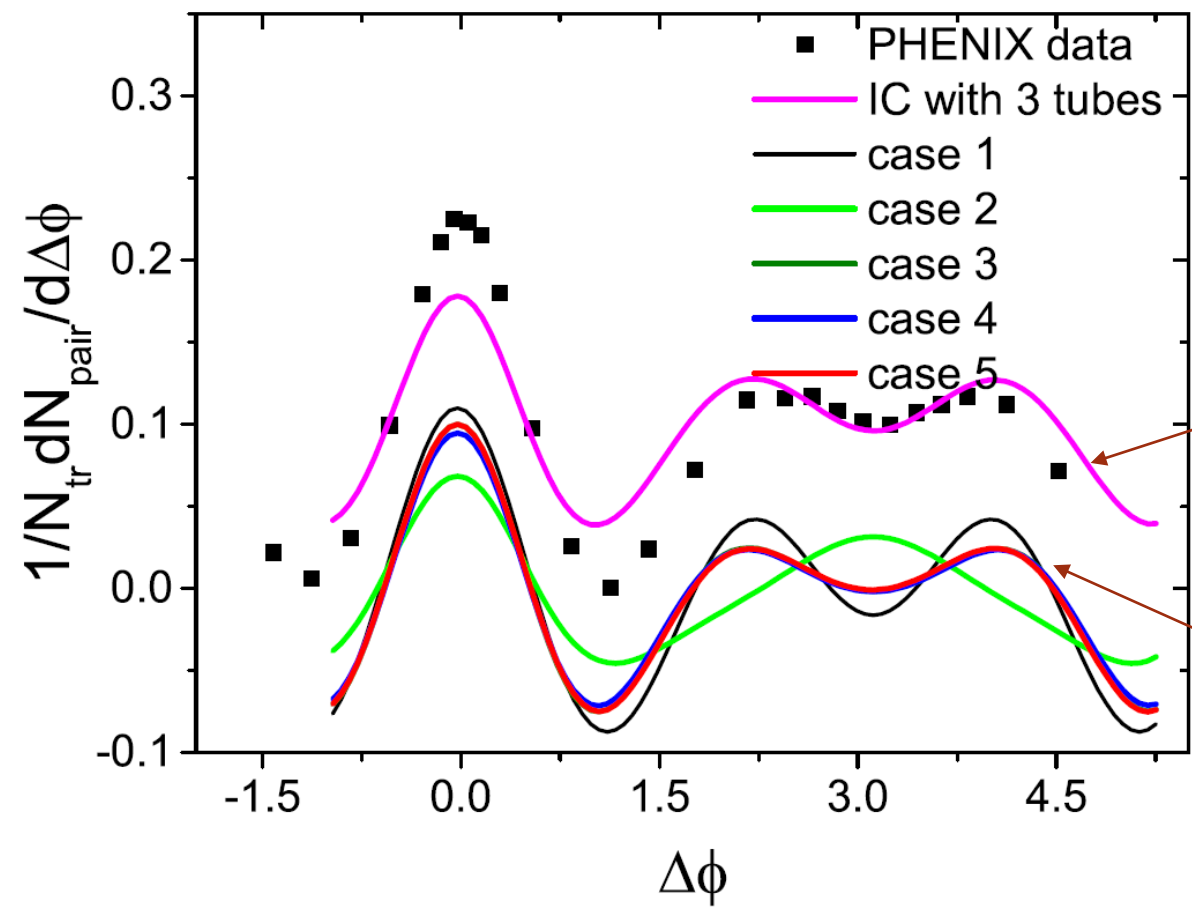
Correlation between $\varepsilon_n \times v_n$



EbE Correlation ($\epsilon_n - v_n$)

H. Niemi, G. S. Denicol,
H. Holopain, P. Huovinen
(2012)





Granular

Smooth



Further detailed studies are in progress on the effects of EoS and Transport coefficients, system size, pT and centrality dependence, etc. But, apparently only fitting Harmonic Flow coefficients is not enough to determine the dynamics on EbE basis.

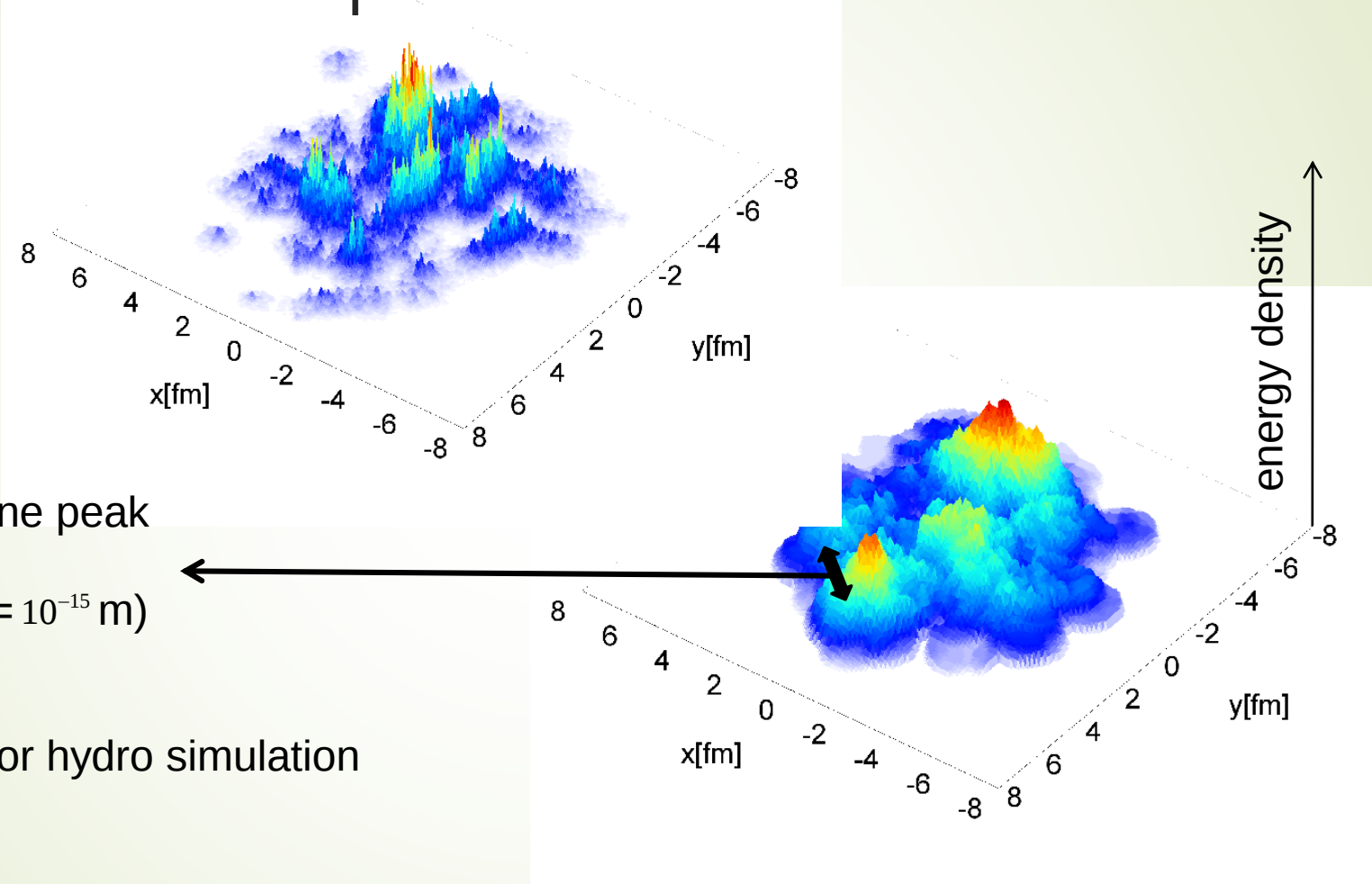
Necessity for the higher order observables to see the non-linear responses.

L. Yan, S. Pal, J.-Y. Ollitrault, Nuclear Physics A 956, 340 (2016).
L. Yan, J.-Y. Ollitrault, Physics Letters B 744, 82 (2015)
and recent proposals....

Another importante aspect:

Where are the quantum effects?

Another importante aspect: Where are the quantum effects?



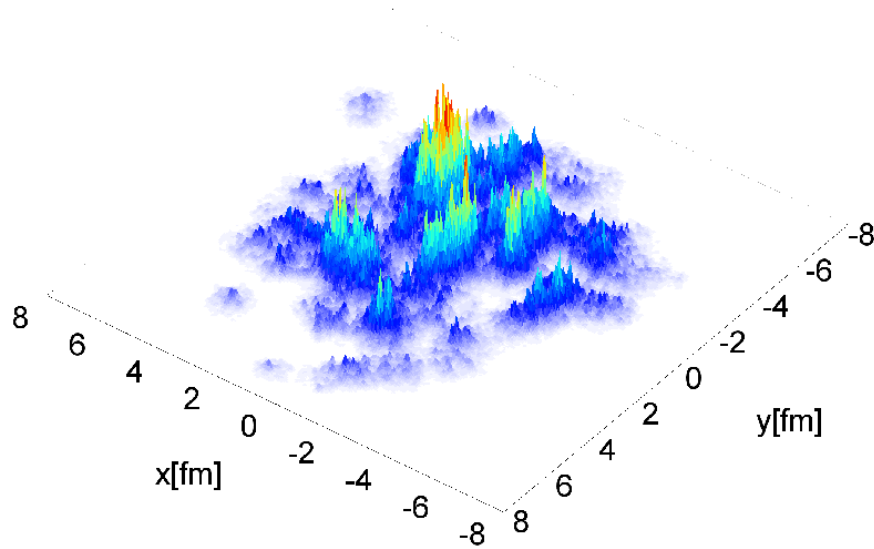
The width of one peak

$\sim 1 \text{ fm} (= 10^{-15} \text{ m})$

The grid size for hydro simulation

$\leq 0.1 \text{ fm}$

Uncertainty relation affects a lot for small systems and granualities.....

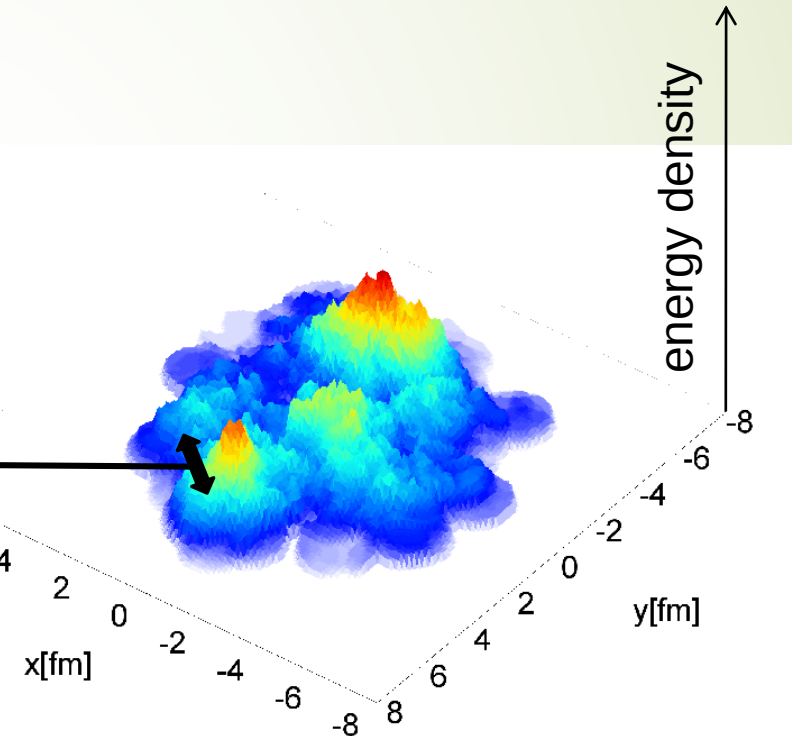


The width of one peak

$\sim 1 \text{ fm}$

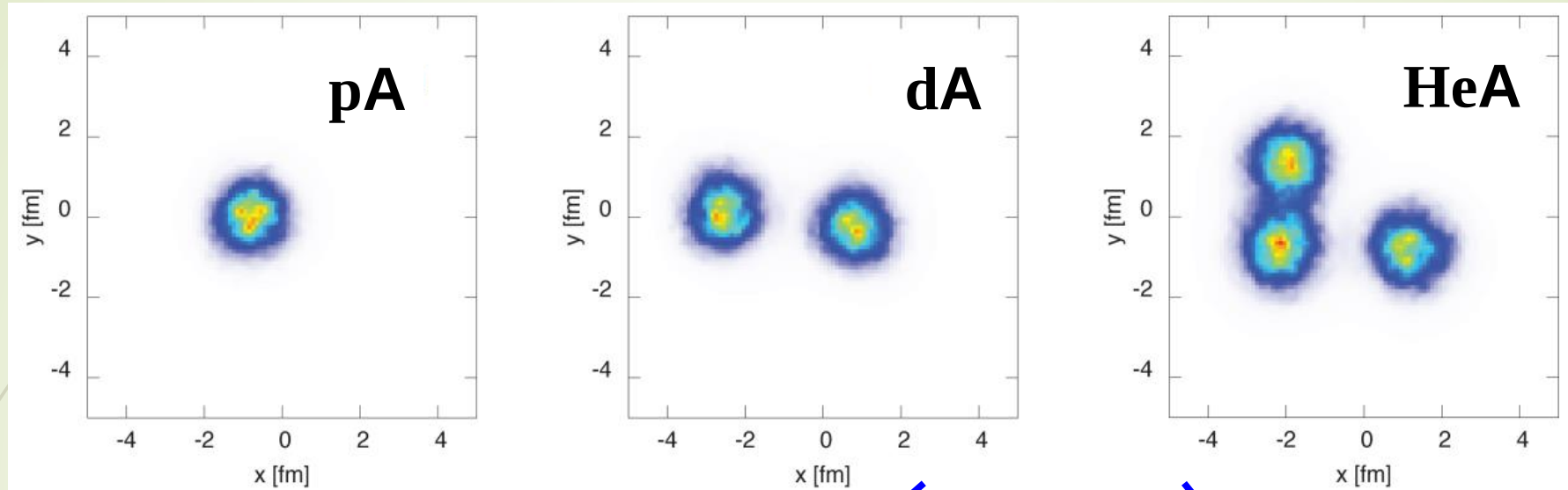
The grid size for hydro simulation

$\leq 0.1 \text{ fm}$

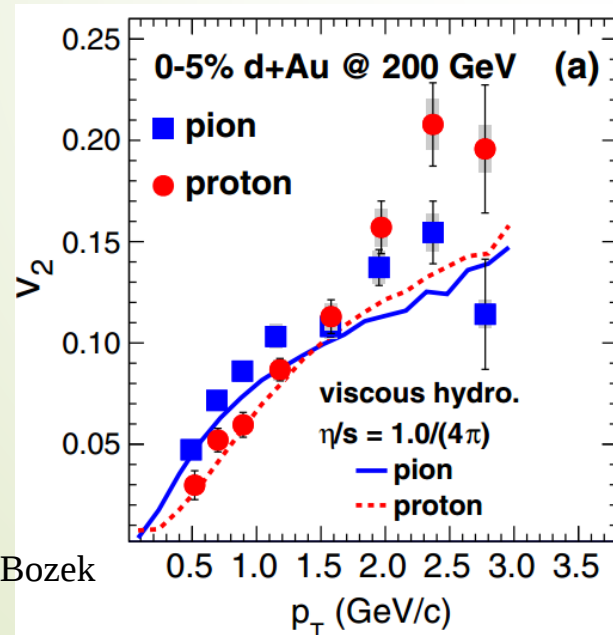


$$\Delta p > 1 \text{ GeV} !$$

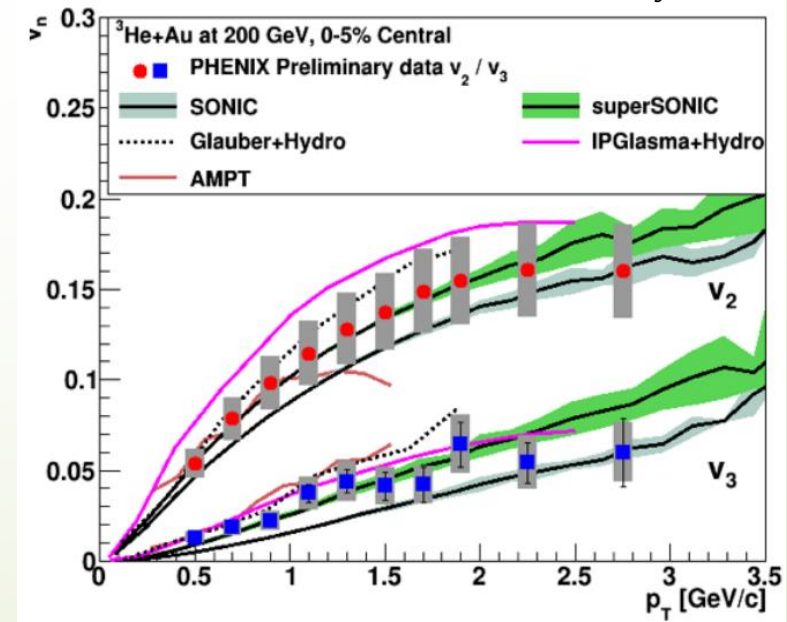
Hydrodynamic description of small systems?



Good agreement with calculations



Calculation by Bozek



Calculation by Romatschke



Quantum Effects in Hydro



Additional Quantum Pressure (the Uncertainty Relation)

T.Koide, T.K. Physics Letters A Volume 382, Issue 22, 5 June 2018, Pages 1472-1480

Well-known the common structure between
Hydrodynamics & Schroedinger Equation
(even for manybody systems – Gross-Pitaevski)

Quantum Effects in Hydro



Additional Quantum Pressure (the Uncertainty Relation)

T.Koide, T.K. Physics Letters A Volume 382, Issue 22, 5 June 2018, Pages 1472-1480

$$p_Q = -\frac{1}{2m} \frac{1}{\sqrt{\rho}} \nabla^2 \sqrt{\rho}$$

How to formulate the quantum effects
covariantly in a hydrodynamic form... ?

Maybe Giorgio ...



Thank you for your attention !