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Some critical remarks about QCD critical point

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FIAS Frankfurt Institute for Advanced Studies

HIC for FAIR Helmholtz International Center

NATIONAL RESEARCH CENTER KURCHATOV INSTITUTE

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- Few remarks about Liquid-Gas phase transition
- Effective thermodynamic potential for Chiral phase Transition
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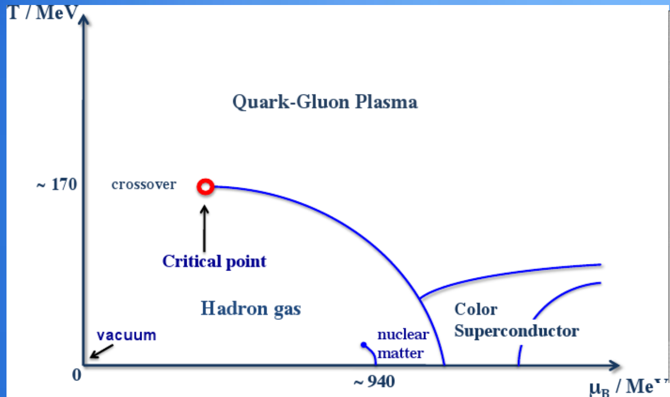
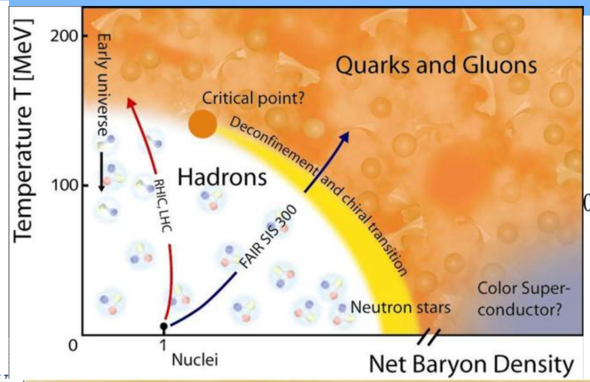
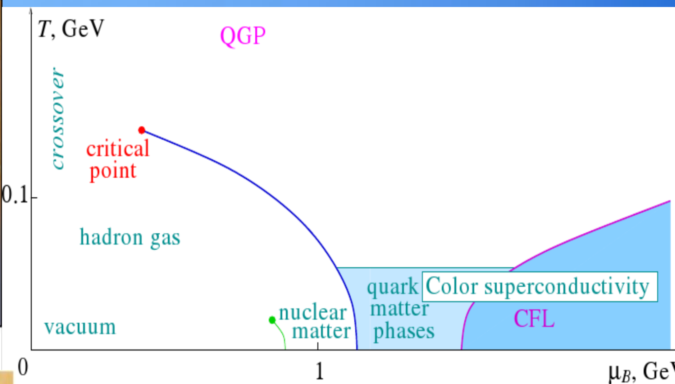
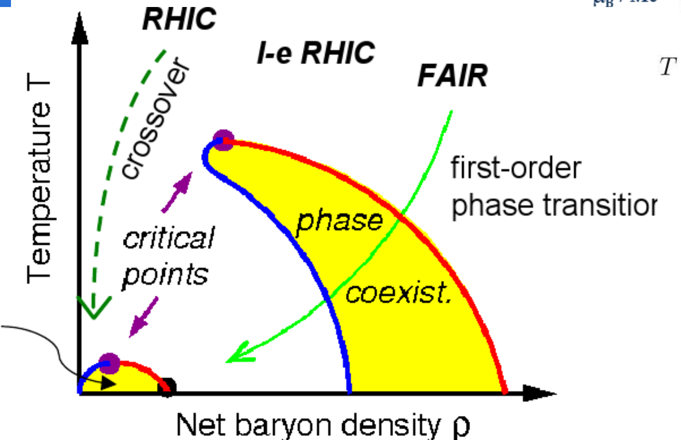
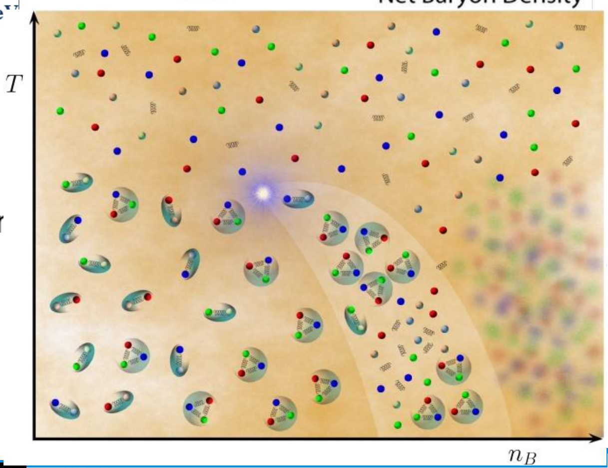
Introduction: "QCD" phase diagram ?

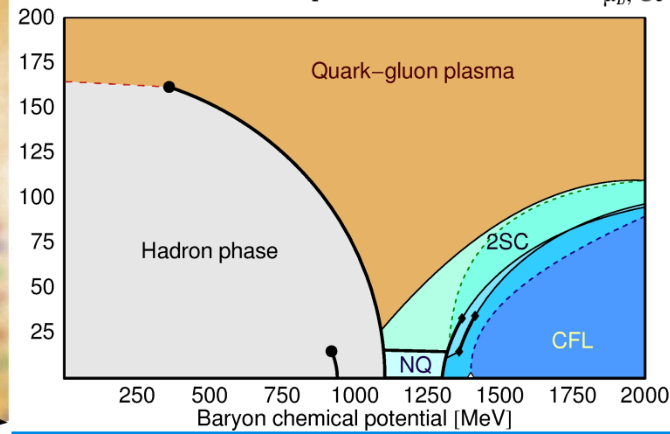


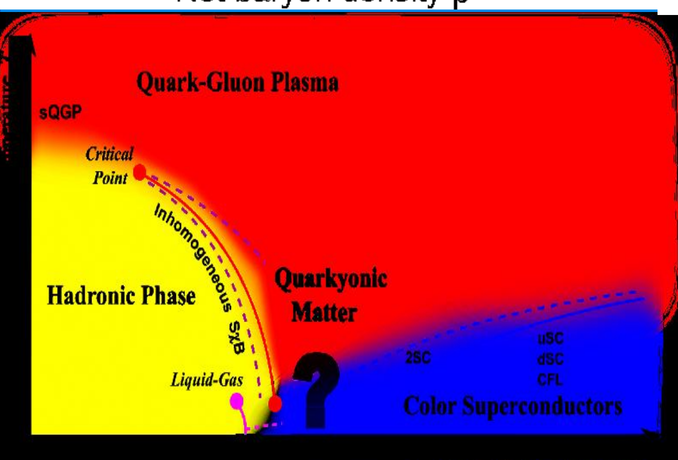


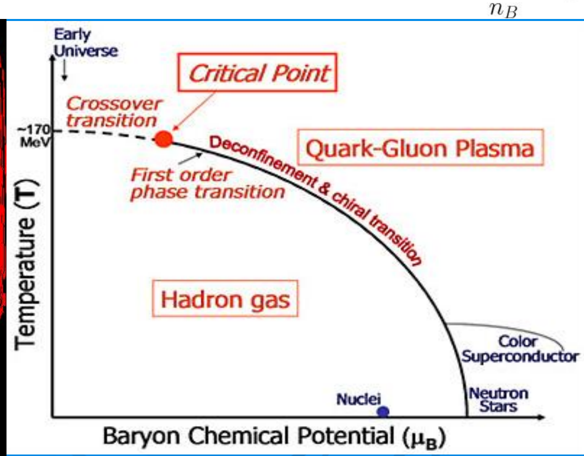


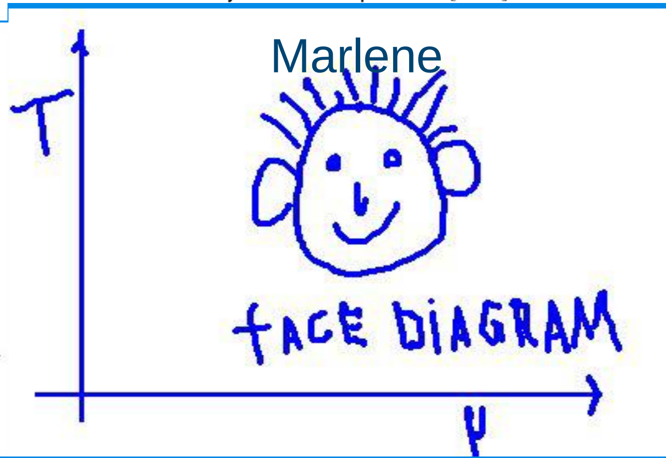




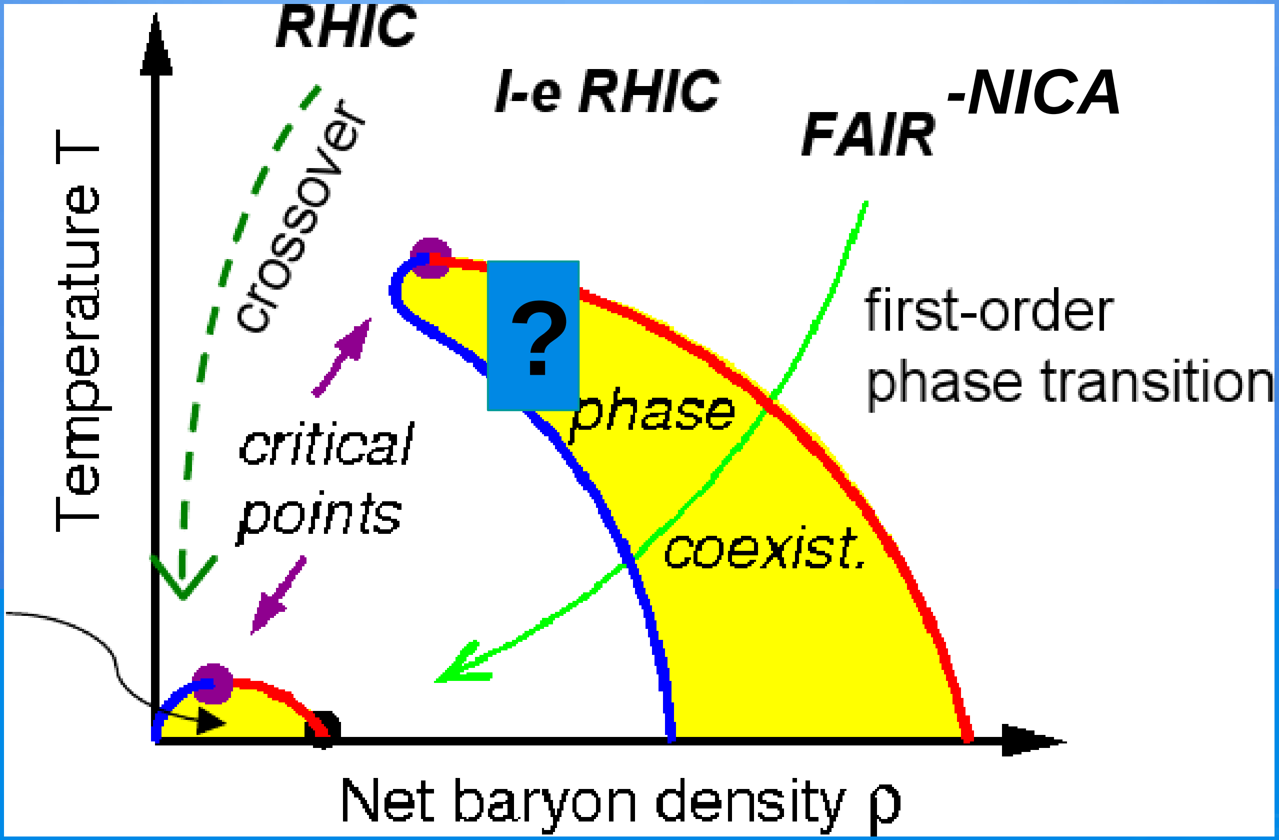






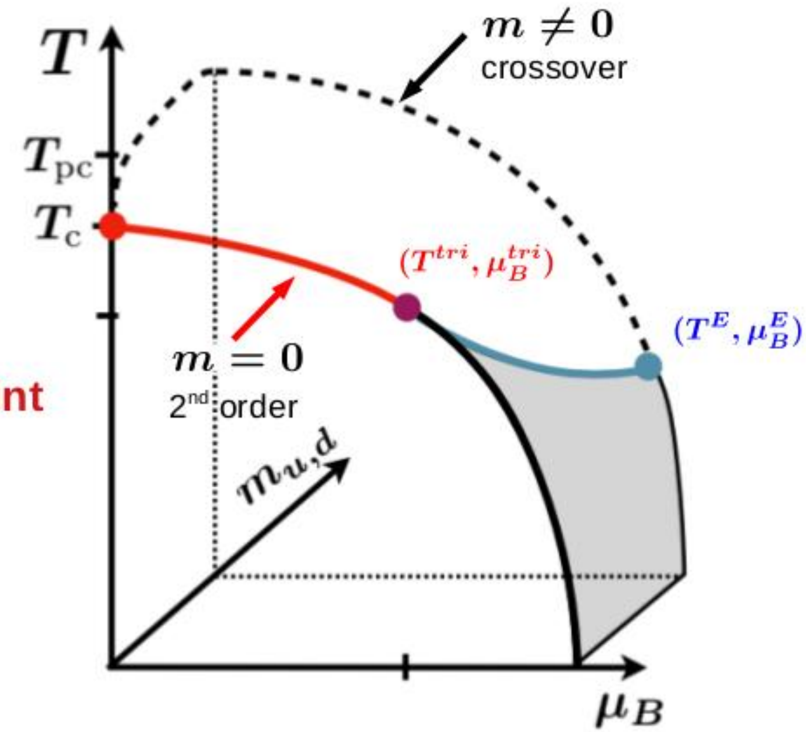


Phase diagram of strongly-interacting matter –a dream or reality?



RHIC/LHC experiments didn't find any clear evidence for QCD critical point. On the other hand, the L-G phase transition is well established experimentally.

Crossover, chiral phase transition at $\mu_B = 0$ and the (tri)-critical point at $\mu_B > 0$



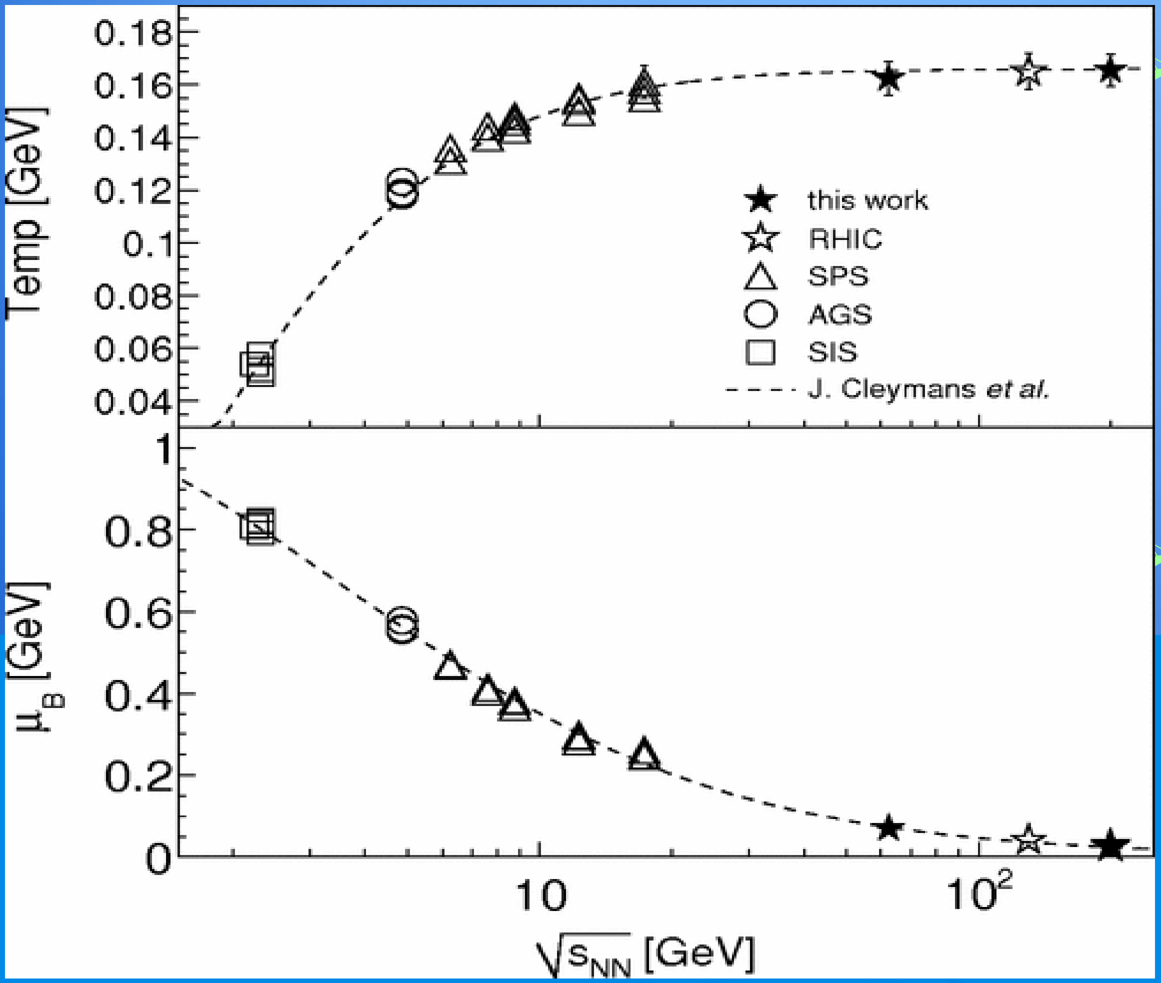
a possible tri-critical point will show up only at

$T < 135$ MeV

the critical point is likely to be located at $T < 130$ MeV

Random Matrix Model	A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.,M. Verbaarschot, Phys. Rev. D58 (1998) 096007
QCD	M. Stephanov, Phys. Rev. D73 (2006) 094508
NJL	M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Chemical freeze-out parameters



- At lower beam energies s<(7 AGeV)² the freeze-out points lie in the domain of nuclear physics:
  - T<100 MeV
  - mu>0.6 GeV
- The critical behavior for liquid-gas phase transition may already show up!

Few remarks about Liquid-Gas transition

Liquid-gas phase transition in nuclear matter

Follows from VdW character of nuclear forces: repulsion ($r<r_c$)+attraction ($r>r_c$)

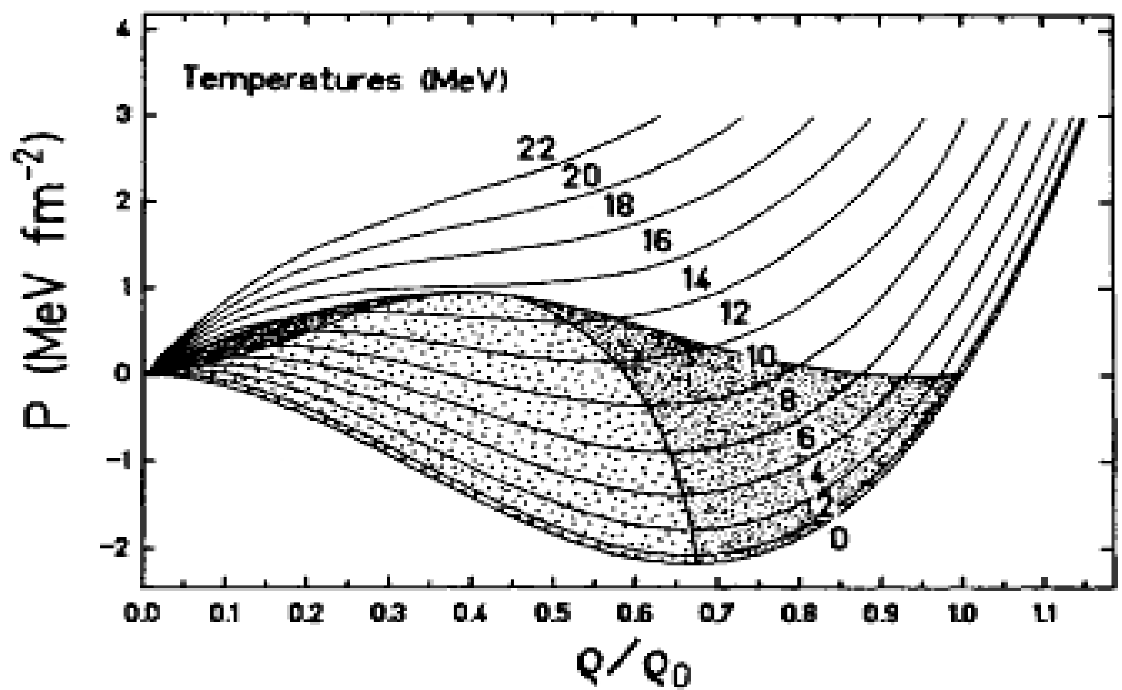
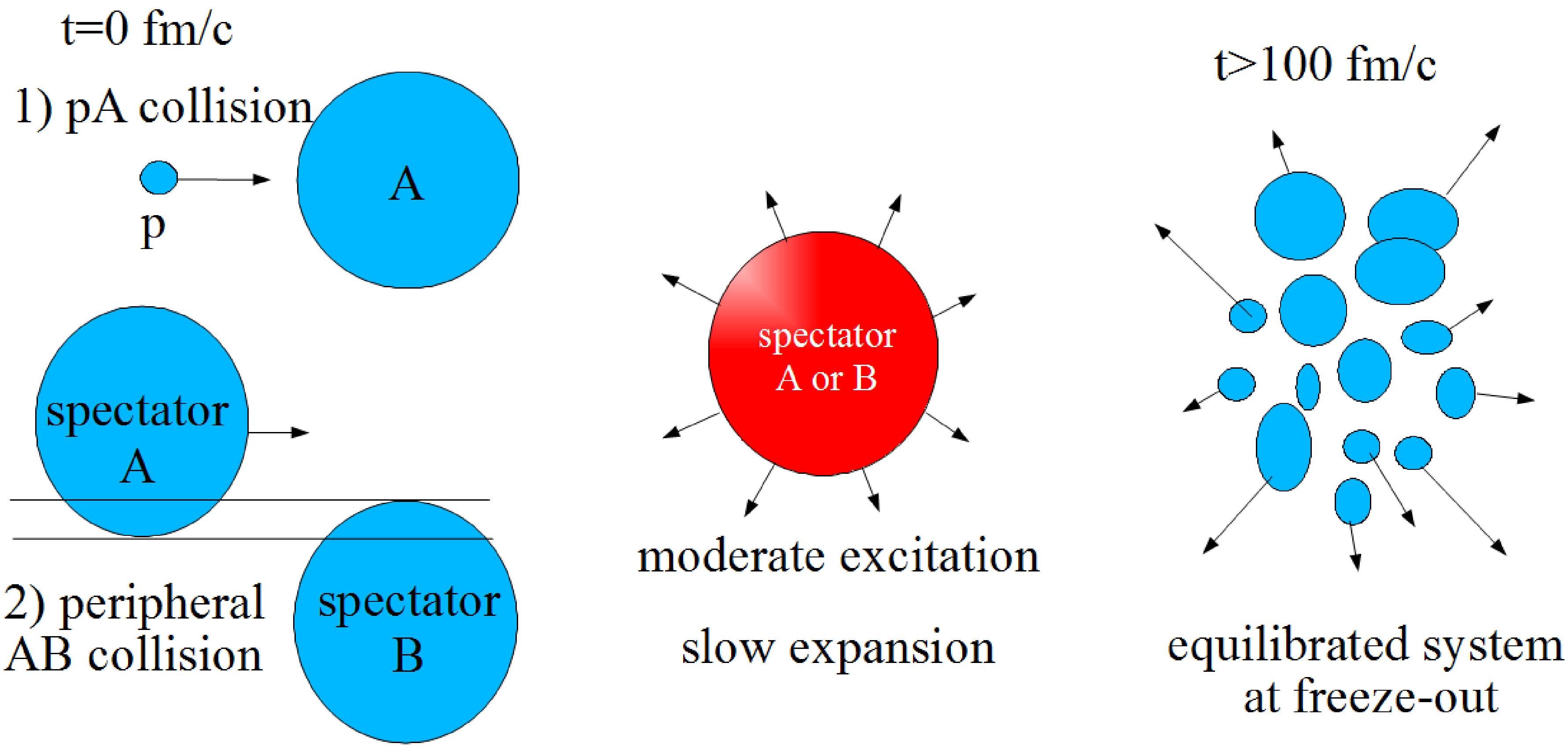


FIG. 1. Pressure curves shown as a function of density for the infinite matter case. The instability area (thin dotted) and the upper line of phase separation are included besides the 12 isotherms: The coexistence region is defined for positive pressure and bounded by the line of phase separation.

B.J.Strack, Phys. Rev. C35, 691 (1987

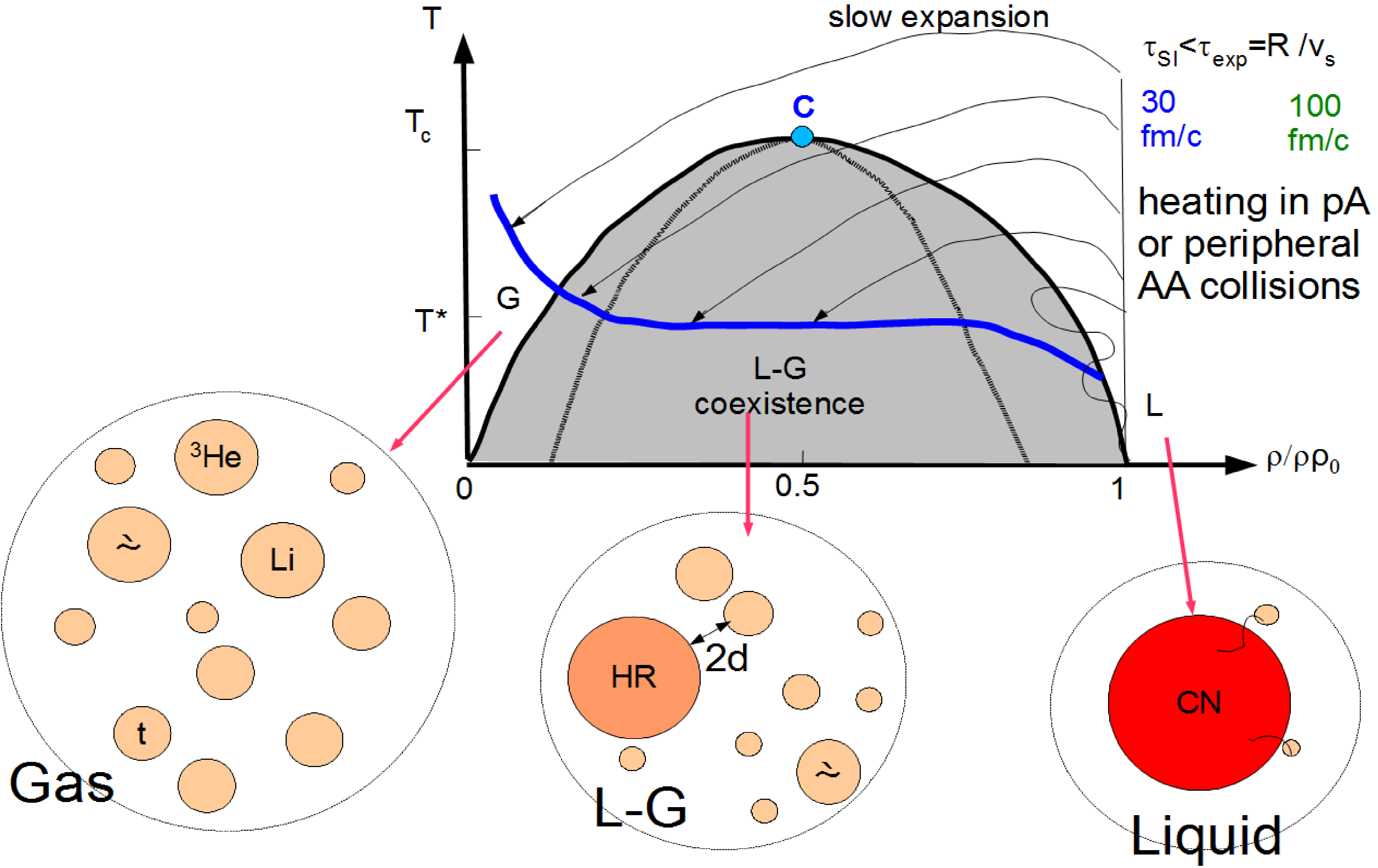
Nuclear break-up: multifragmentation



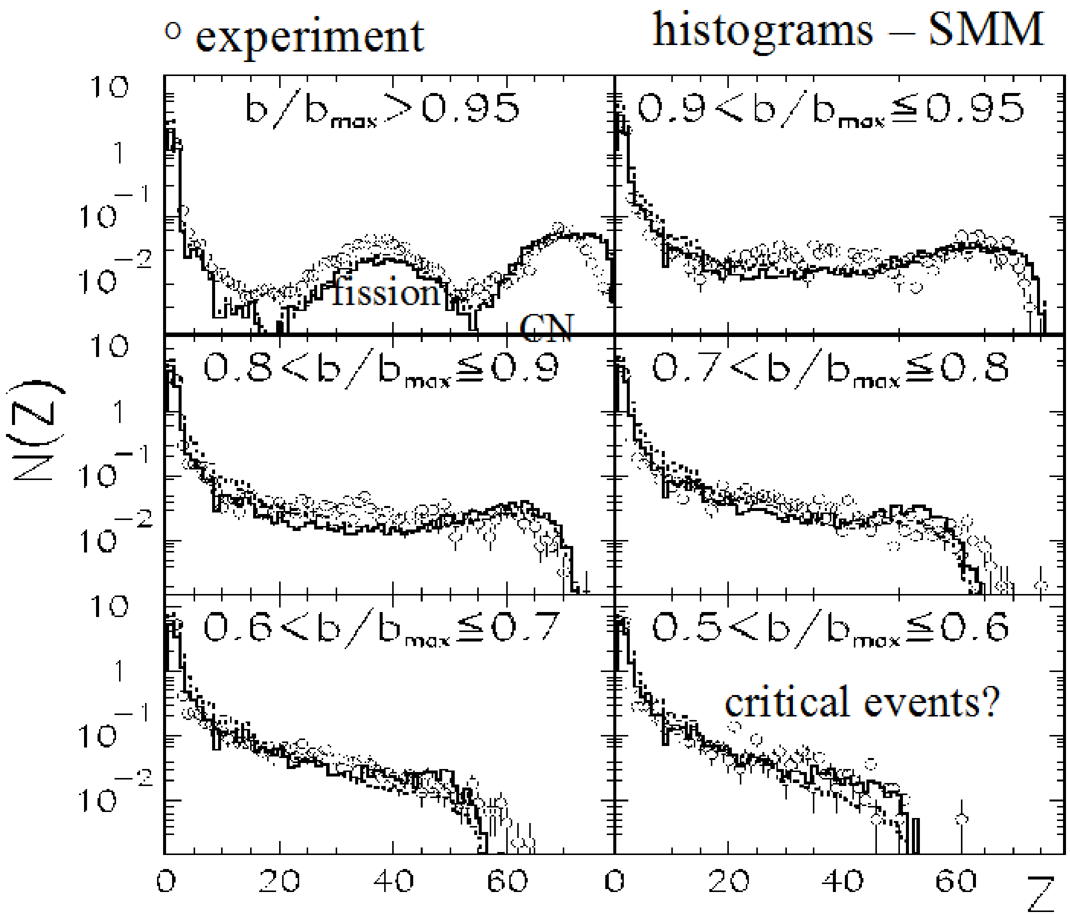
Power-low fragment mass distribution around critical point, $Y(A) \sim A^{-\tau}$
Can be well understood within an equilibrium statistical approach

Early 80s: Randrup&Koonin; D. Gross et al; Bondorf-Mishustin-Botvina; Hahn-Stoecker;
Later: S. Das Gupta et al.; Gulminelli&Raduta et al,...

Multifragmentation is a manifestation of the Liquid-Gas p. t. in finite nuclei



Evolution of partitions with E*



(Statistical Multifragmentation Model)

power-law distribution $Y(Z) \sim Z^{-\tau}$ with $\tau \approx 2$ (Fisher, 1962)

E*/A in spectators is growing towards central collisions (smaller b)

Peripheral Au+Au collisions at 35 AMeV

M. D'Agostino et al., Nucl. Phys. A650, 329 (1999)

Nuclear caloric curve

Predicted in 1985 within the SMM
Bondorf, Donangelo, Mishustin, Schulz
NPA 444 (1985) 460

Experimental discovery
Pochodzalla and ALADIN collaboration,
PRL 75 (1995) 1040

J. Bondorf et al. / Statistical multifragmentation 469

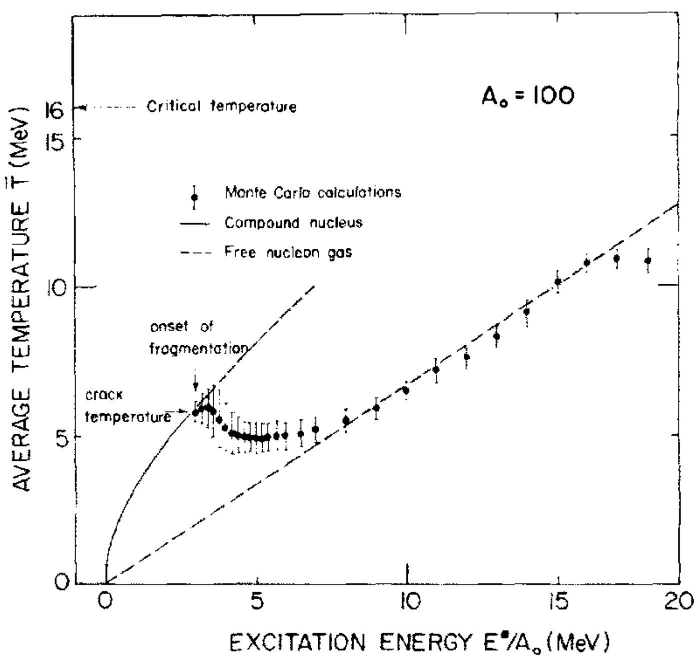


Fig. 4. The average temperature T as a function of the excitation energy E^*/A_0. The dashed line illustrates the temperature of a free nucleon gas.

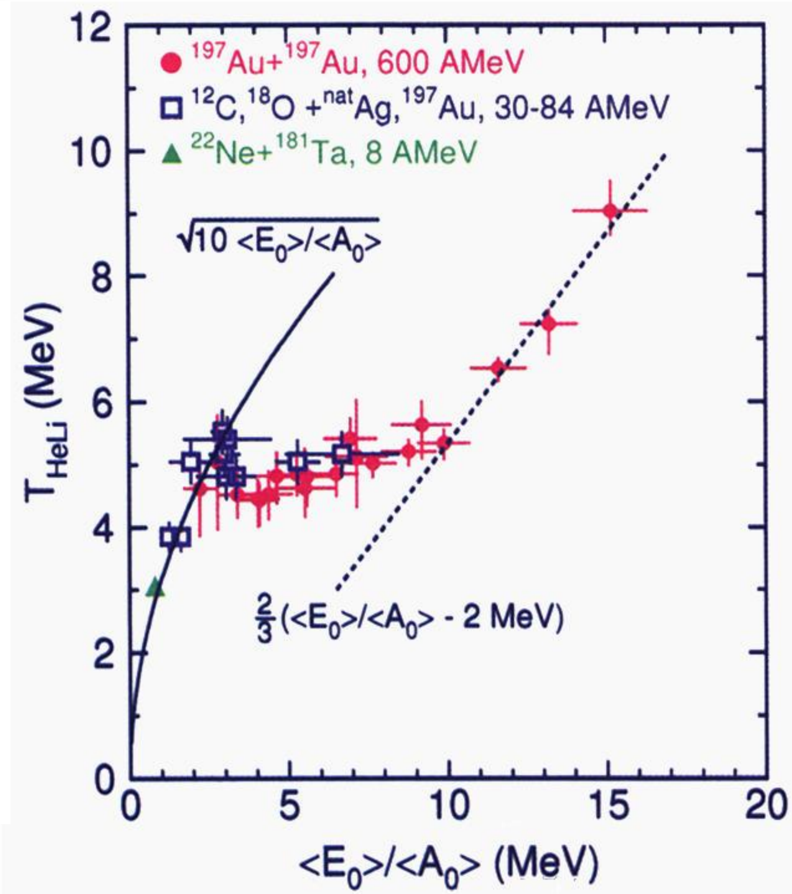


FIG. 2. Caloric curve of nuclei determined by the dependence of the isotope temperature T_{HeLi} on the excitation energy per nucleon. The lines are explained in the text.

Theoretical prediction has been confirmed only 10 years later!

Effective thermodynamic potential for 1st and 2nd order phase transitions

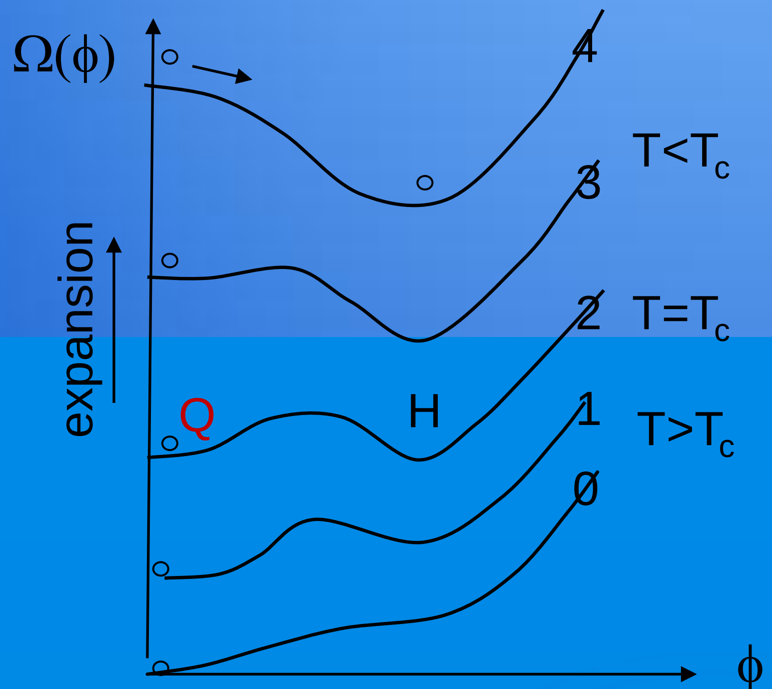
Effects of fast dynamics

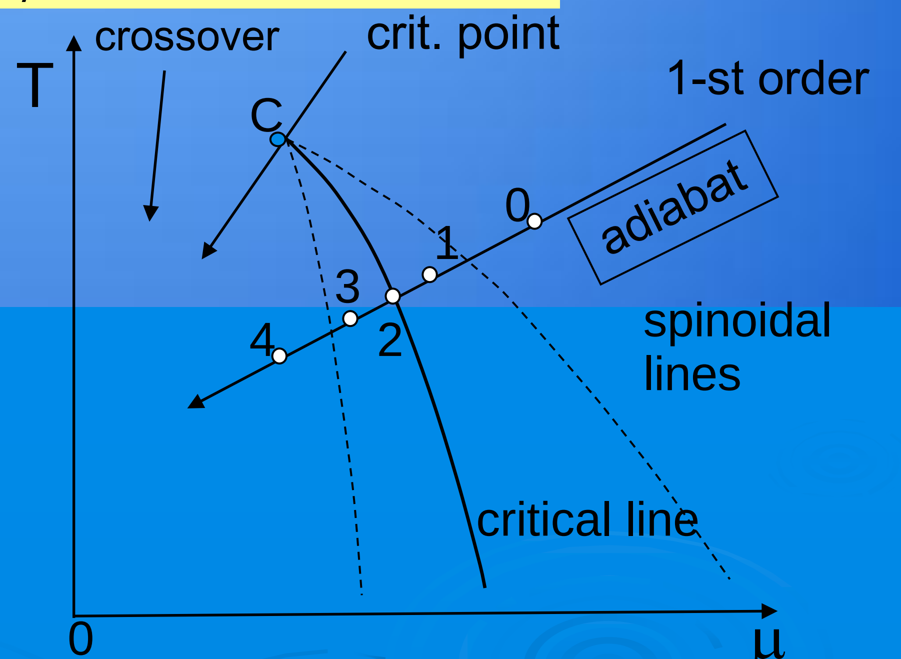
Effective thermodyn. potential for a 1st order phase transition

$$\Omega(\phi;T,\mu) = \Omega_0(T,\mu) + \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 + \frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by $\frac{\partial\Omega}{\partial\phi} = 0 \Rightarrow P = -\Omega(\phi_{eq})$





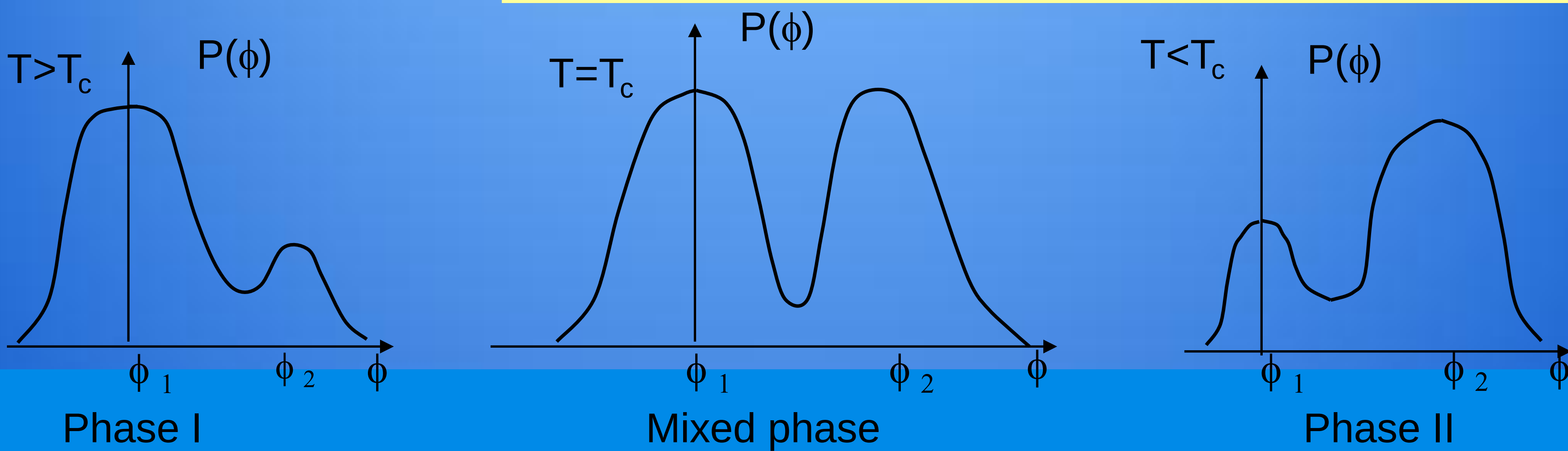
In rapidly expanding system, 1-st order phase transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Equilibrium fluctuations of order parameter in 1st order phase transition

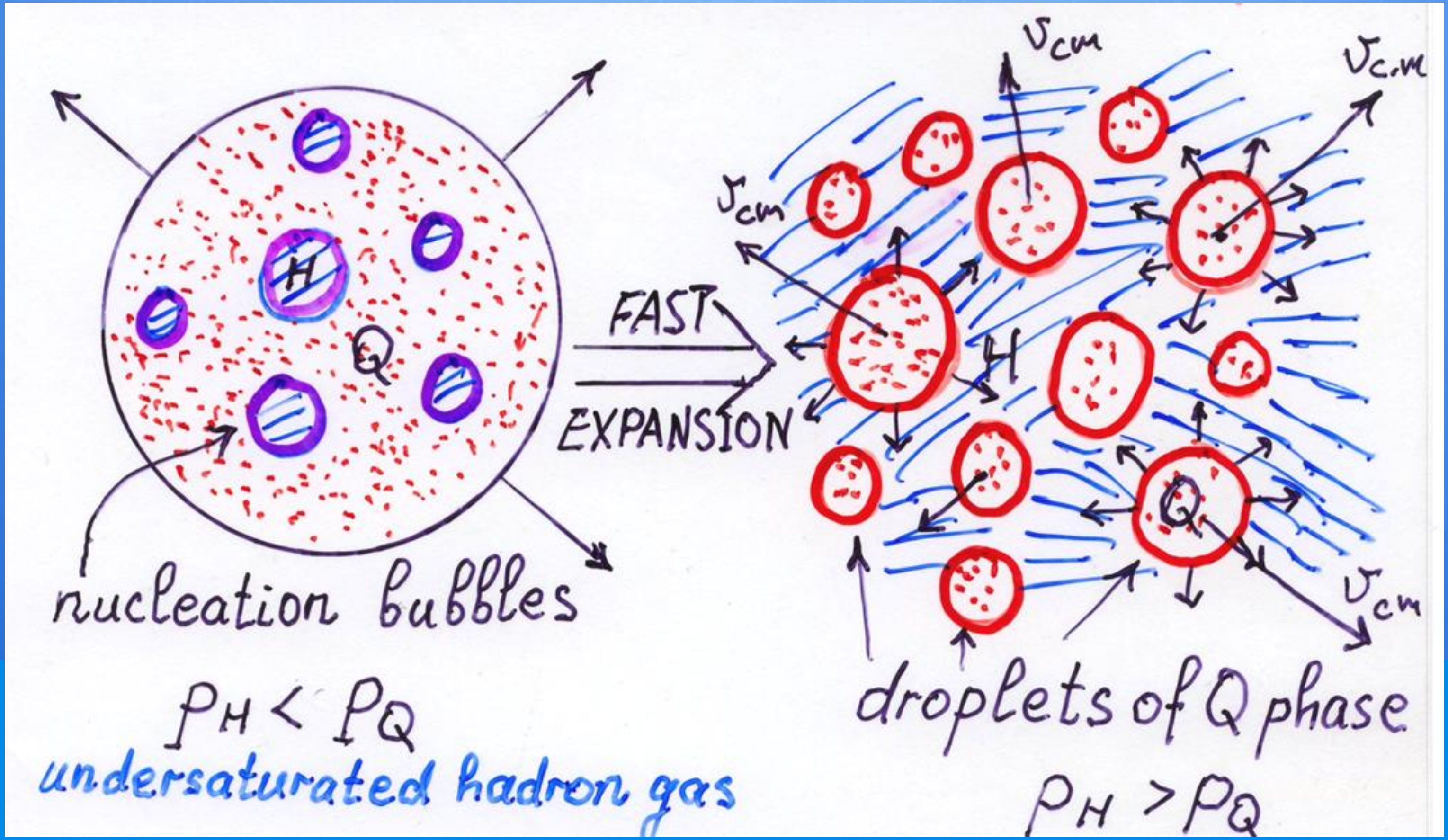
Probability distribution of fluctuations

$$P(\delta\phi) \sim \exp\left(-\frac{\Delta\Omega(\delta\phi)V}{T}\right), \quad \delta\phi = \phi - \langle\phi\rangle$$



- **In an equilibrated system fluctuations of the order parameter, i.e. Polyakov loop, should demonstrate bi-modal distributions (lattice calculations?);**
- **In a rapidly evolving system these fluctuations will be out of equilibrium;**
- **During supercooling process strong fluctuations may develop in the form of droplets of a metastable phase.**

Rapid expansion through a 1st order phase transition



The system is trapped in a metastable state until it enters the spinodal instability region, when Q phase becomes unstable and splits into droplets

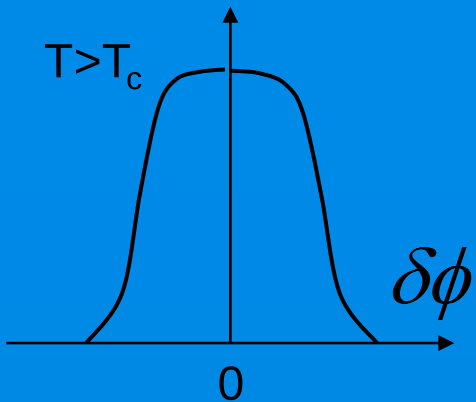
Csernai&Mishustin, 1995; Mishustin, 1999; Rafelski et al. 2000; Randrup, 2003; Steinheimer&Randrup 2013; …

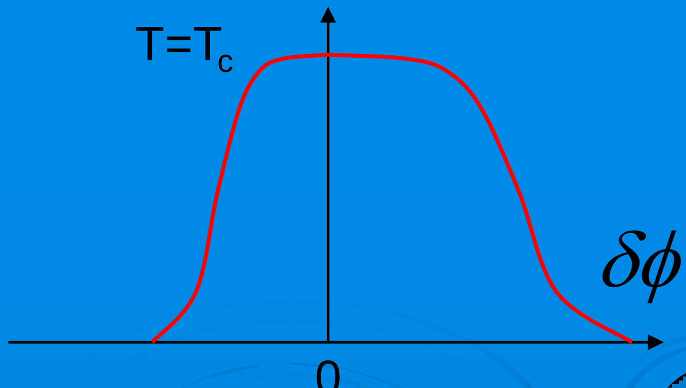
Evolution of equilibrium fluctuations in 2nd order phase transition

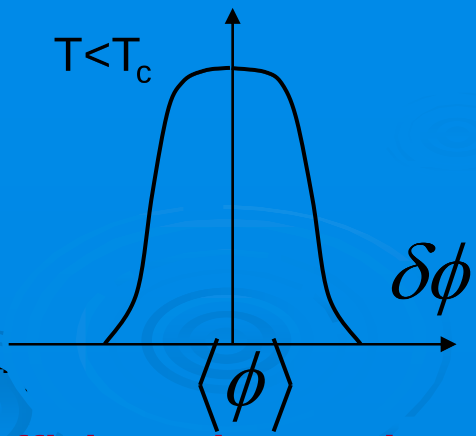
$$\Omega(\phi) = \frac{1}{2}a(T)\phi^2 + \frac{1}{2}b(\nabla\phi)^2 + \frac{\lambda}{4}\phi^4, \quad a(T) = a_0(T - T_c)$$

$$\langle\phi\rangle = \frac{a(T)}{\lambda}, \; T < T_c \text{ and } \langle\phi\rangle = 0, \; T > T_c, \quad \delta\phi = \phi - \langle\phi\rangle$$

$$\text{Distribution of fluctuations} \quad P(\delta\phi) \sim \exp\left[-\frac{\Delta\Omega(\delta\phi)V}{T}\right]$$

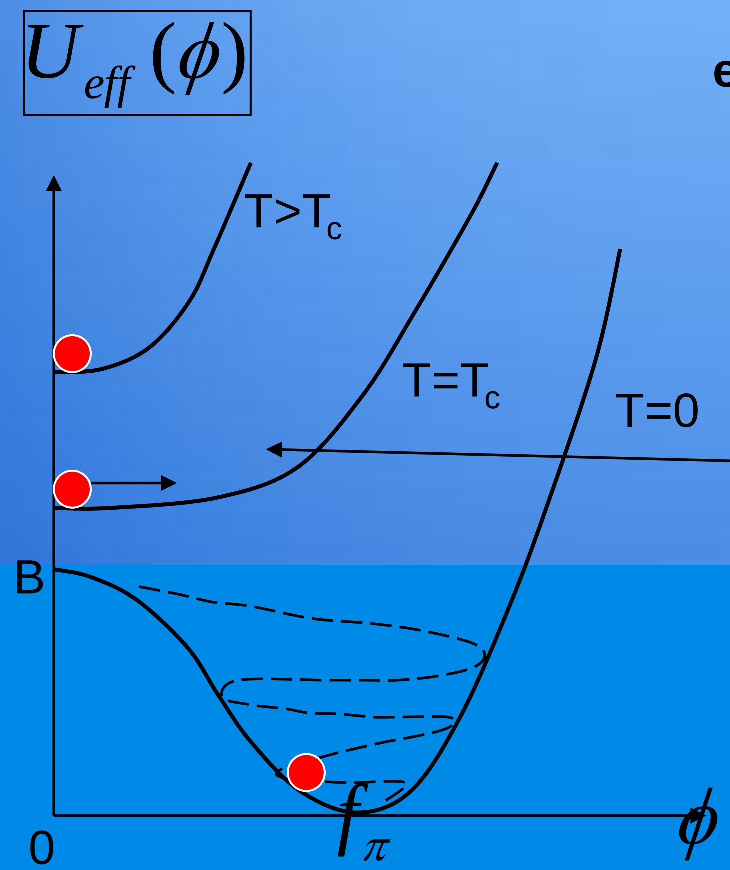






In rapidly expanding system critical fluctuations have not sufficient time to develop

Critical slowing down in the 2nd order phase transition



Fluctuations of the order parameter evolve according to the relaxation equation

$$\frac{d\delta\phi}{dt} = -\gamma \frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\mathrm{rel}}}$$

In the vicinity of the critical point the relaxation time for the order parameter diverges - no restoring force

$$\tau_{\mathrm{rel}}(T) \sim \frac{1}{\left|T - T_c\right|^{\nu}} \to \infty, \quad \nu \sim 2$$

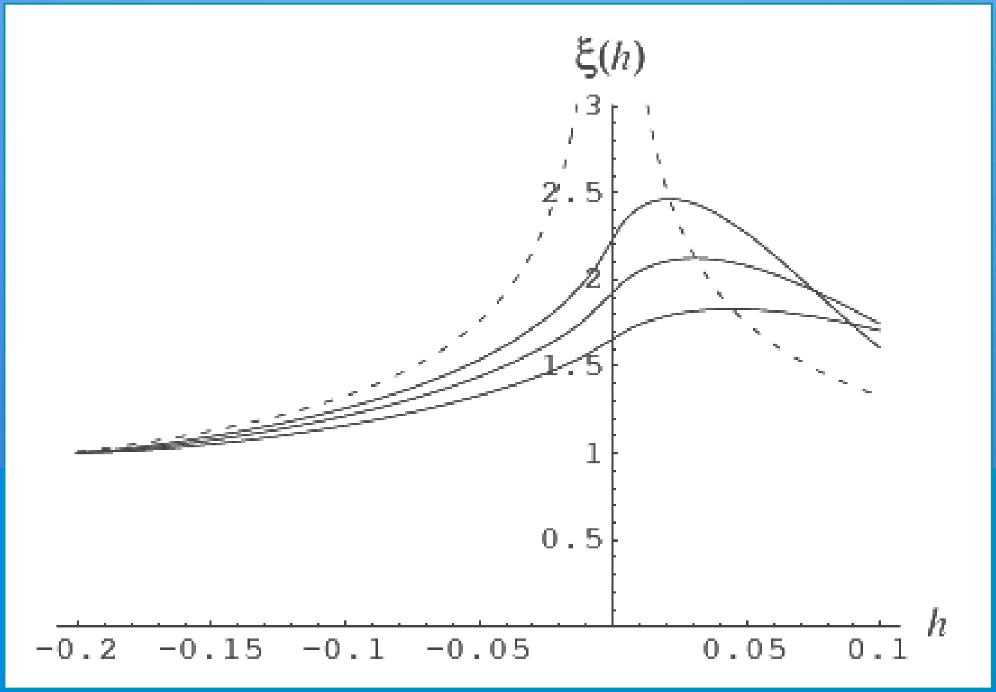
(Landau&Lifshitz, vol. X, Physical kinetics)

"Rolling down" from the top of the potential is similar to spinodal decomposition (Csernai&Mishustin 1995)

Critical slowing down 2

B. Berdnikov, K. Rajagopal, Phys. Rec. D61 (2000)

Correlation length as function of parameter h (characterizing the closeness to the critical point) for different expansion rates



One can expect only a factor 2 enhancement in the correlation length even for slow cooling rate, dT/dt=10 MeV/fm. Critical fluctuations have not enough time to build up!

Modeling fluctuations in dynamical environments

Simple model for chiral phase transition

Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}),$$

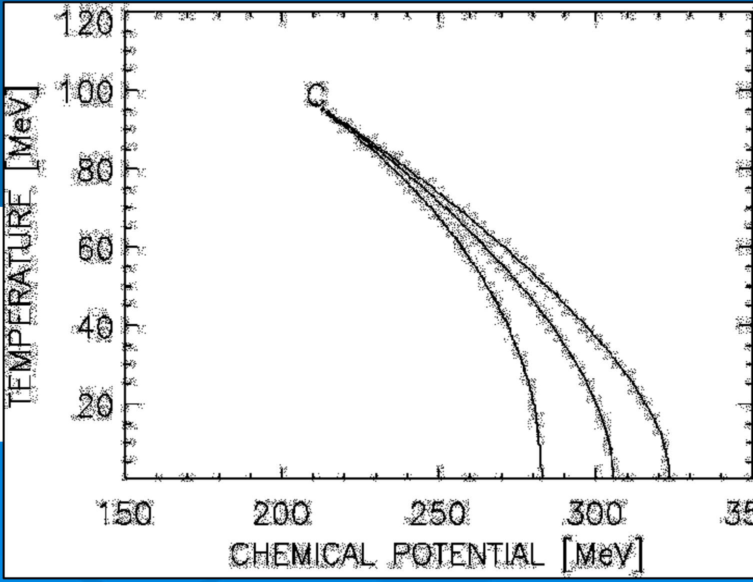
$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \ \ <\sigma>_{\text{vac}} = f_\pi \rightarrow H = f_\pi m_\pi^2$$

Effective thermodynamic potential contains contributions of mean field σ and quark-antiquark fluid:

$$U_{\text{eff}}(\sigma; T, \mu) = U(\sigma, \boldsymbol{\pi}) + \Omega_q(m; T, \mu)$$

$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2), \ \ \boldsymbol{\pi} \approx 0$$

Phase diagram for g=3.3



CO, 2nd and 1st order chiral transitions can be obtained by choosing coupling g.

Non-equilibrium Chiral Fluid Dynamics

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134;
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907;
M. Nahrgang, C. Herold, S. Leupold, , C. Herold, M. Bleicher, Phys. Rev. C 84 (2011) 024912;
M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, J. Phys. G40 (2014) 055108.

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system “fluid+field”

$$\partial_\nu (T_{\text{fluid}}{}^{\mu\nu} + T_{\text{field}}{}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}{}^{\mu\nu} = -\partial_\mu T_{\text{field}}{}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma})\partial^\nu\sigma = (g\rho_s + \eta\partial_t\sigma)\partial^\nu\sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu\partial^\mu\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma} + g < \bar{q}q > + \eta\partial_t\sigma = \xi$$

Langevin equation for the order parameter

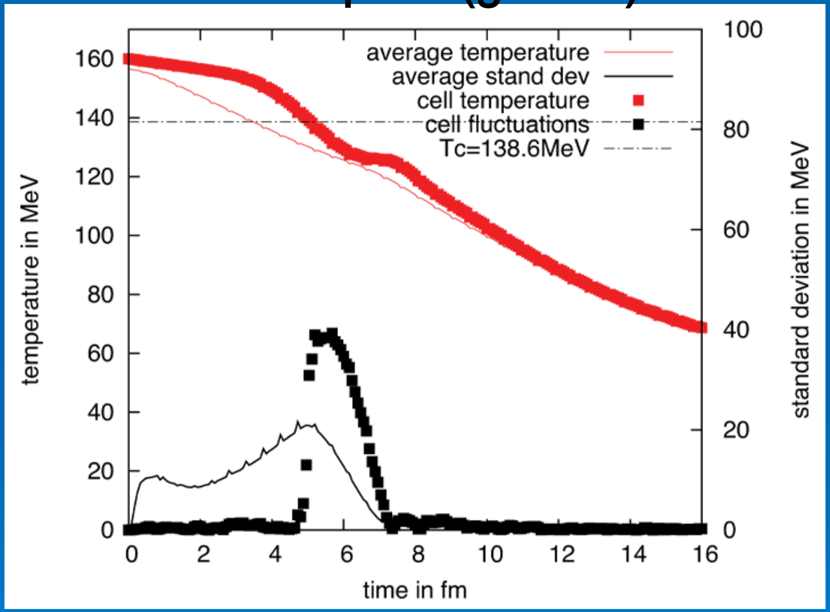
$$< \xi(t,\vec{r}) >= 0, \quad < \xi(t,r)\xi(t',r') >= \frac{1}{V} m_\sigma \eta \delta(t-t')\delta(r-r')\coth\left(\frac{m_\sigma}{2T}\right)$$

Bjorken expansion through a phase transition

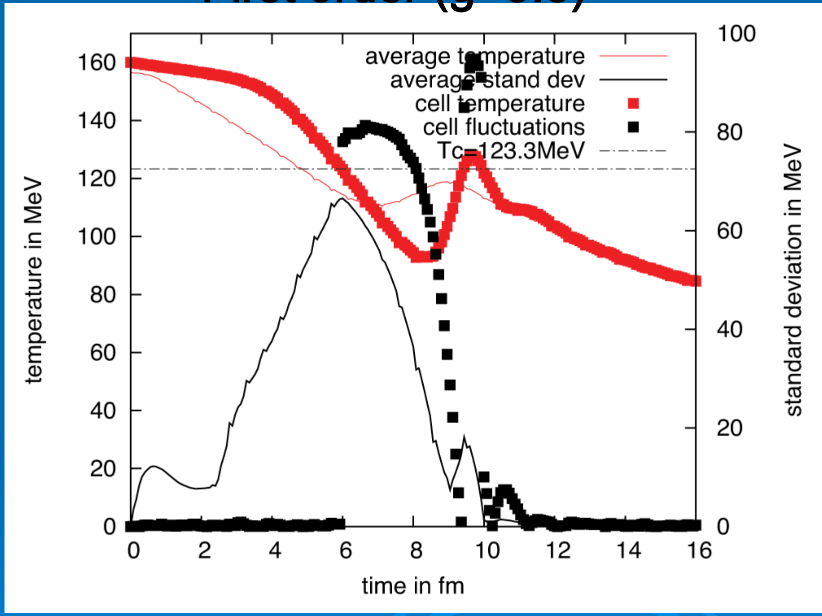
Initial state: cylinder of length L with linear velocity profile in z direction, ellipsoidal cross section in x-y plane

$$\text{At } t=0: \ v(z)=\frac{2z}{L}0.2c, \ \ -\frac{L}{2}<z<\frac{L}{2}; \ \ v_x=v_y=0; \ T=160\ MeV$$

Critical point (g=3.63)



First order (g=5.5)

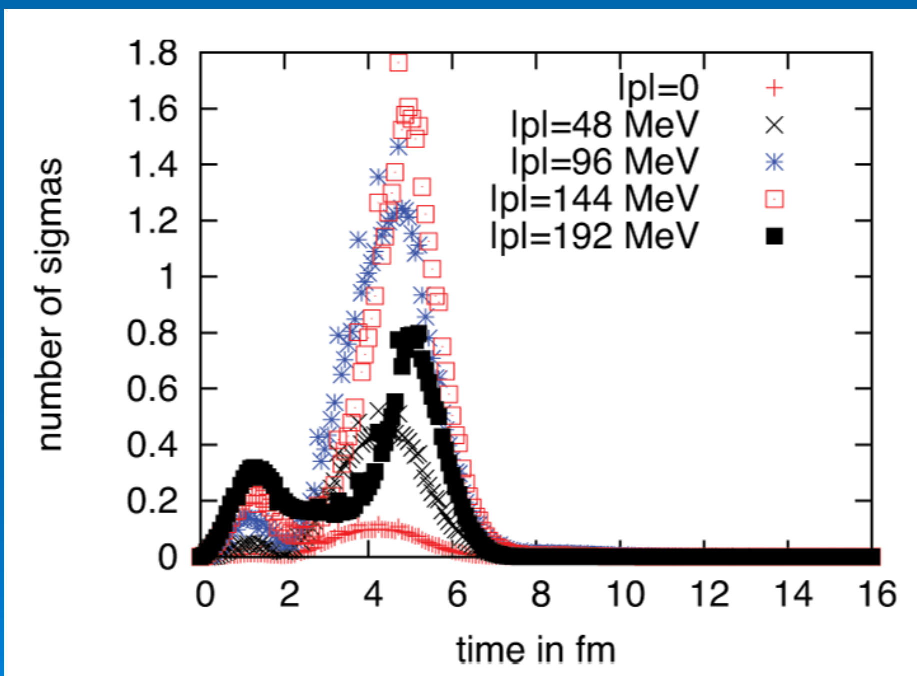


Mean values and standard deviation of T for the whole system and for a central cell (1 fm^3) are shown as a function of time. Supercooling and reheating effects are clearly seen in the 1-st order transition. Fluctuations are much stronger in the case of 1^{st} order phase transition (right) as compared with the critical point (left).

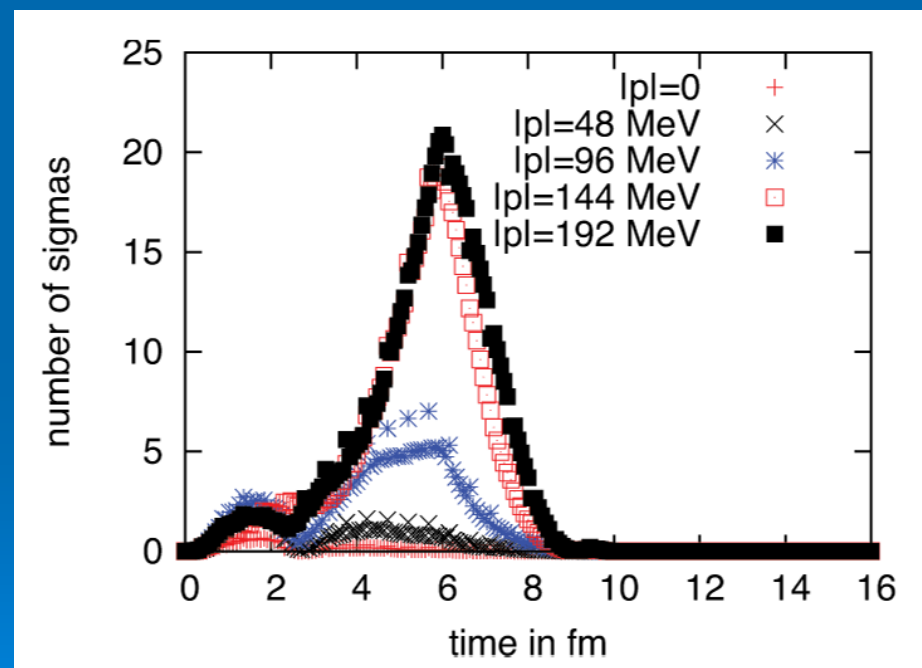
Sigma fluctuations in expanding fireball

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 \,|\sigma_k|^2 + |\dot{\sigma}_k|^2], \;\; \omega_k = \sqrt{m_\sigma^2 + k^2}, \;\; m_\sigma^2 = \frac{\partial^2 U_{\mathrm{eff}}}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}$$

Critical point (g=3.63)



First order (g=5.5)



Fluctuations are rather weak at critical point (left), but increase strongly at the 1st order transition (right) after 4 fm/c

Extension to finite baryon densities

Model 1: Polyakov-Quark-Meson Model (PQM)

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Nucl. Phys. A 925 (2014) 14;

- Include µ-dependence in Polyakov loop potential, (cf. Schäfer, Pawlowski, Wambach Fukushima)

$$\mathcal{U}(\ell, T, T_0)\ , \quad T_0 \to T_0(\mu)$$

- Calculate grand canonical potential for finite chemical potential

$$\Omega_{q\bar{q}} = -2N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \left(\ln\left[1 + 3\ell \mathrm{e}^{-\beta(E-\mu)} + 3\ell \mathrm{e}^{-2\beta(E-\mu)} + \mathrm{e}^{-3\beta(E-\mu)}\right] + (\mu \to -\mu) \right\} \right.$$

- Propagate (net) baryon density in the hydro sector

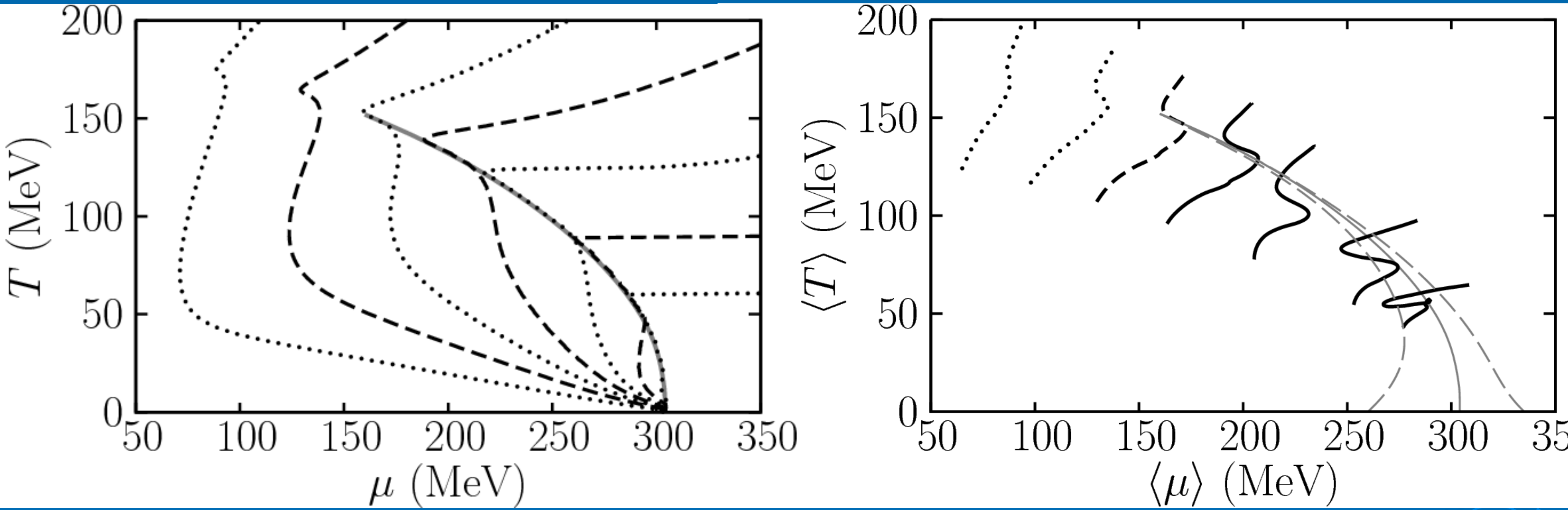
$$\partial_\mu n^\mu = 0\ , \quad n^\mu = \rho u^\mu$$

- Generate baryon number fluctuations via Langeven

Trajectories in the T-μ plane

CFD calculations are done for spherical fireball of R=4 fm

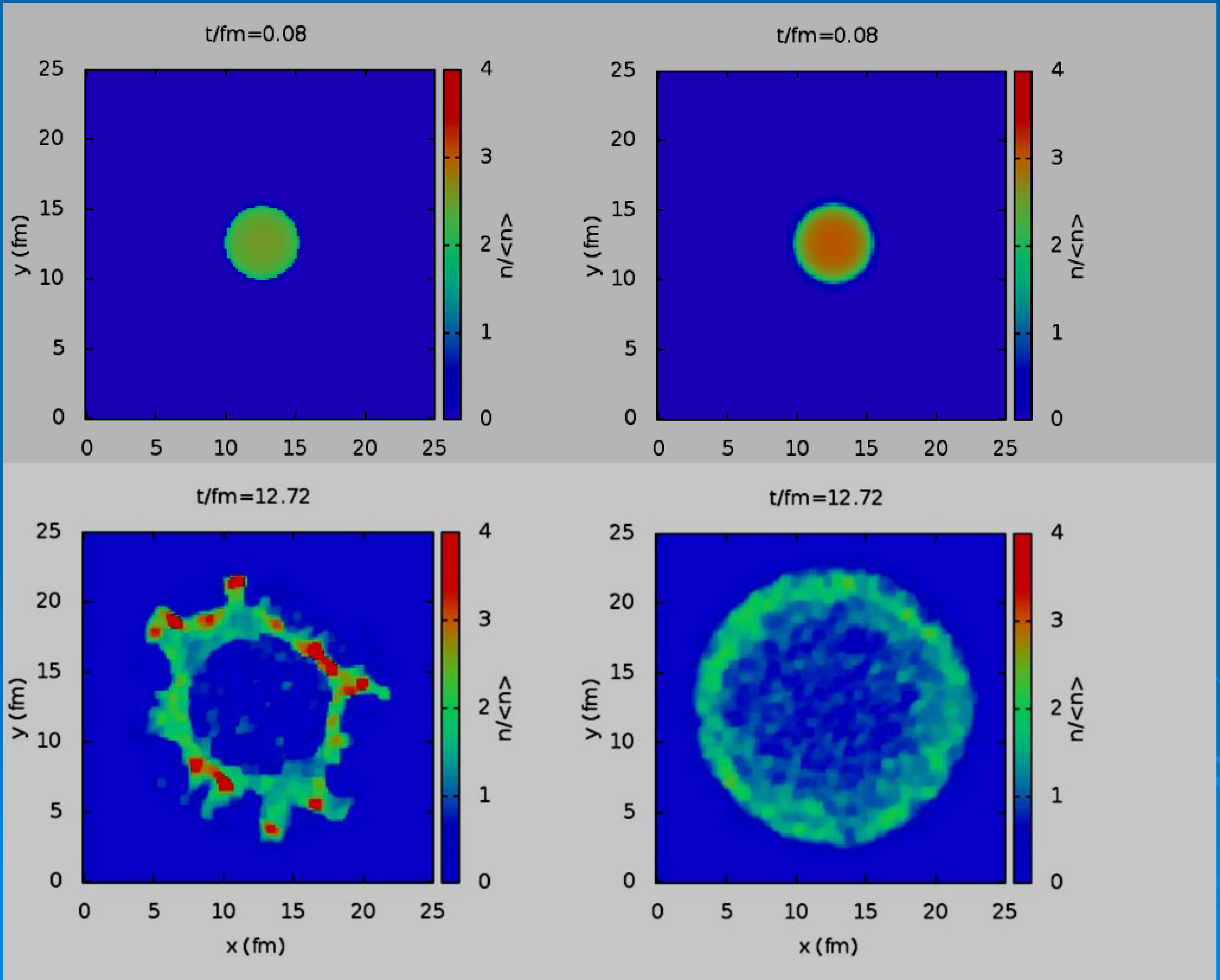
Isentropic expansion Hydrodynamic evolution



- Trajectories are close to isentropes for crossover and CP;
- Non-equilibrium "back-bending" is clearly seen in FO case;
- In the case of strong FO transition (solid lines, right) the system is trapped in spinodal region for a significant time

Dynamical simulation of fast expansion

First order | **Critical point**



Dynamical droplet formation

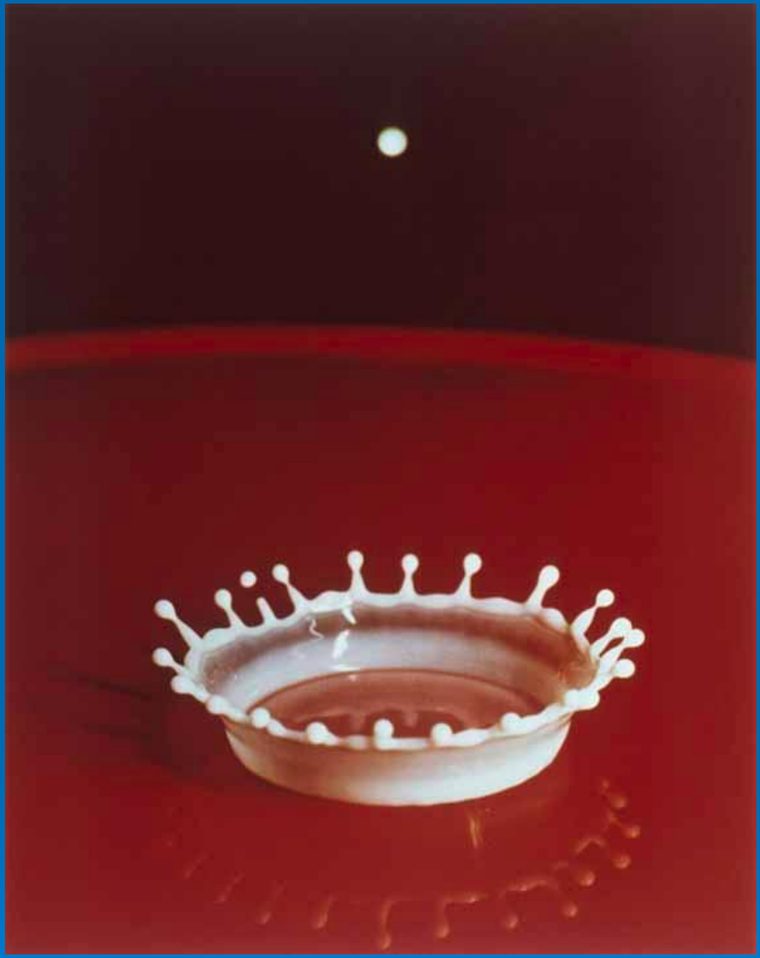
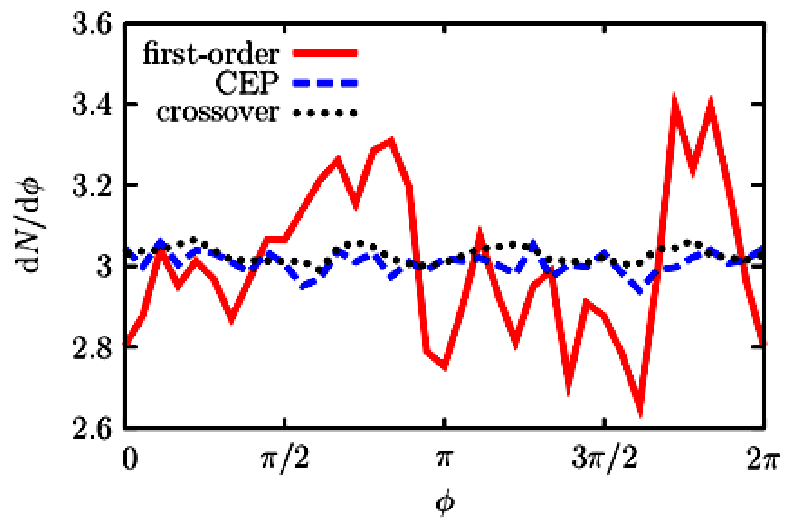
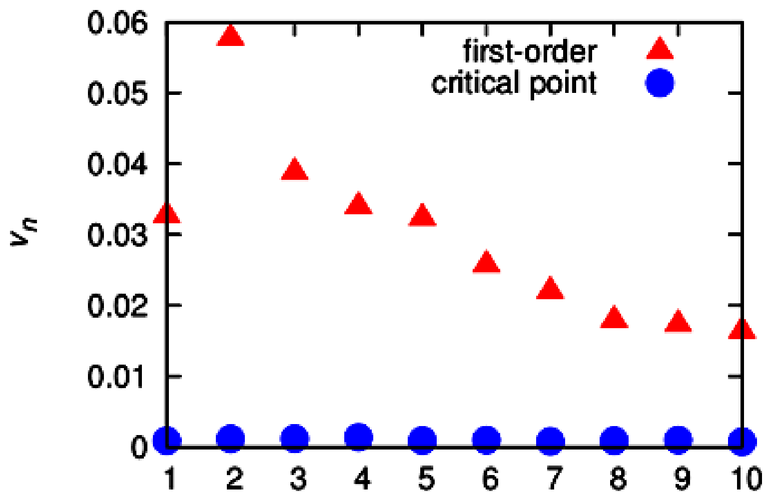


Photo: HEE-NC-57001

Splash of a milk drop

Observable signatures of baryon density fluctuations





Azimuthal fluctuations of net-B In single events: enhanced production of light nuclei (d, t,He)

High harmonics of baryonic flow (averaged over many events):

$v_n=<\cos[n(\varphi-\varphi_n)]>$

More realistic calculations

- **In the previous calculations the EOS had a P=0 point at a finite baryon density (like the MIT bag model), that makes possible stable quark droplets**

- **It is interesting to see what happens in a more realistic case when quark droplets are unstable at zero pressure** **(J. Steinheimer et al, PRC 89 (2014) 034901)**

- **There exist several models which have such a property, in particular so called Quark-Hadron Model** **(S. Schramm et al.) or** **Quark-Dilaton Model**
- **(Ch. Sasaki and I. Mishustin. Phys. Rev. C85, (2012) 025202).**

Model 2: SU(3) chiral quark-hadron (QH) model

V. Dexheimer, S. Schramm, Phys. Rev. C 81 (2010) 045201

Includes: a) 3 quarks (u,d,s) plus baryon octet,
b) scalar mesons (σ, ς), vector meson (ω)
c) Polyakov loop (l)

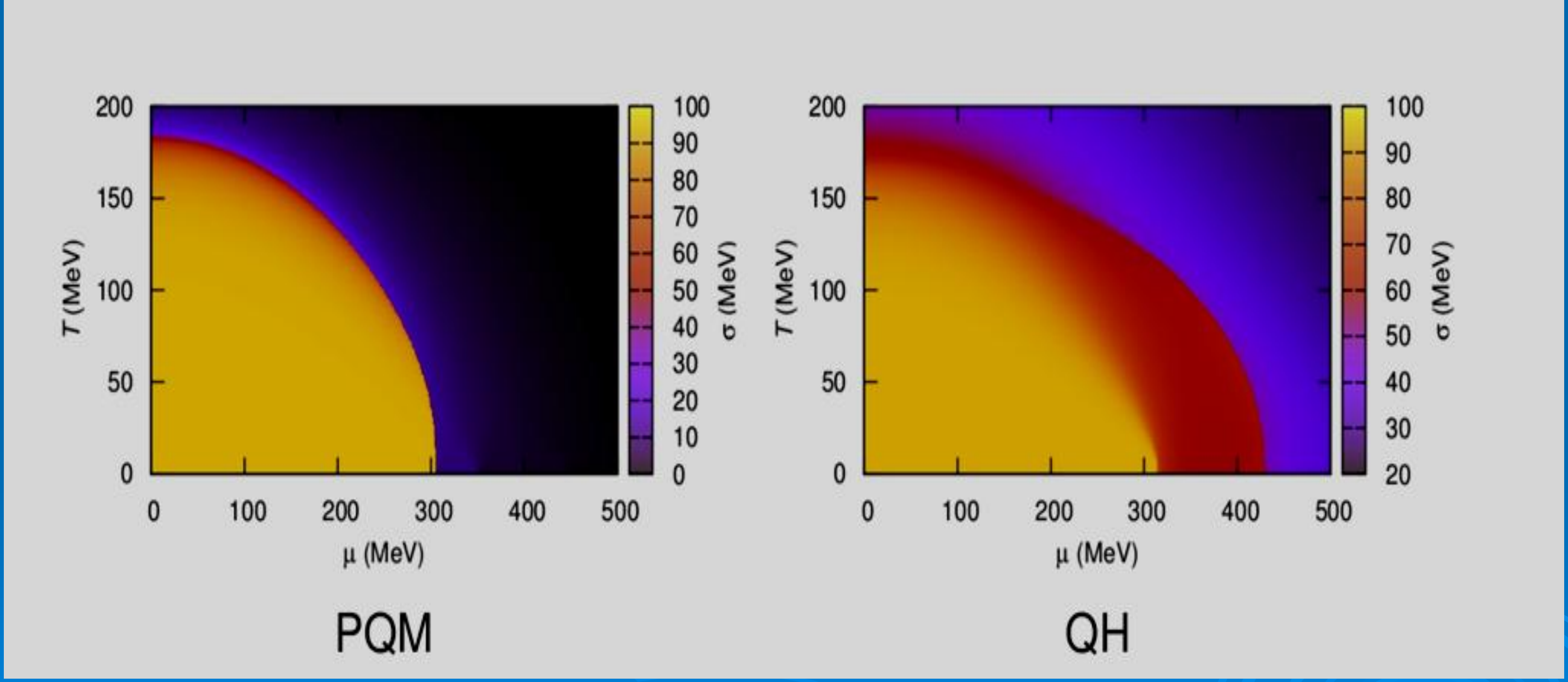
$$\mathcal{L} = \sum_i \overline{\psi}_i \left(i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i\right)\psi_i + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - U\left(\sigma, \zeta, \omega\right) - \mathcal{U}(\ell)$$

Effective masses:

$$\begin{aligned} M_q &= g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell) \\ M_B &= g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{B\ell}\ell^2 \end{aligned}$$

QHM predicts two phase transitions

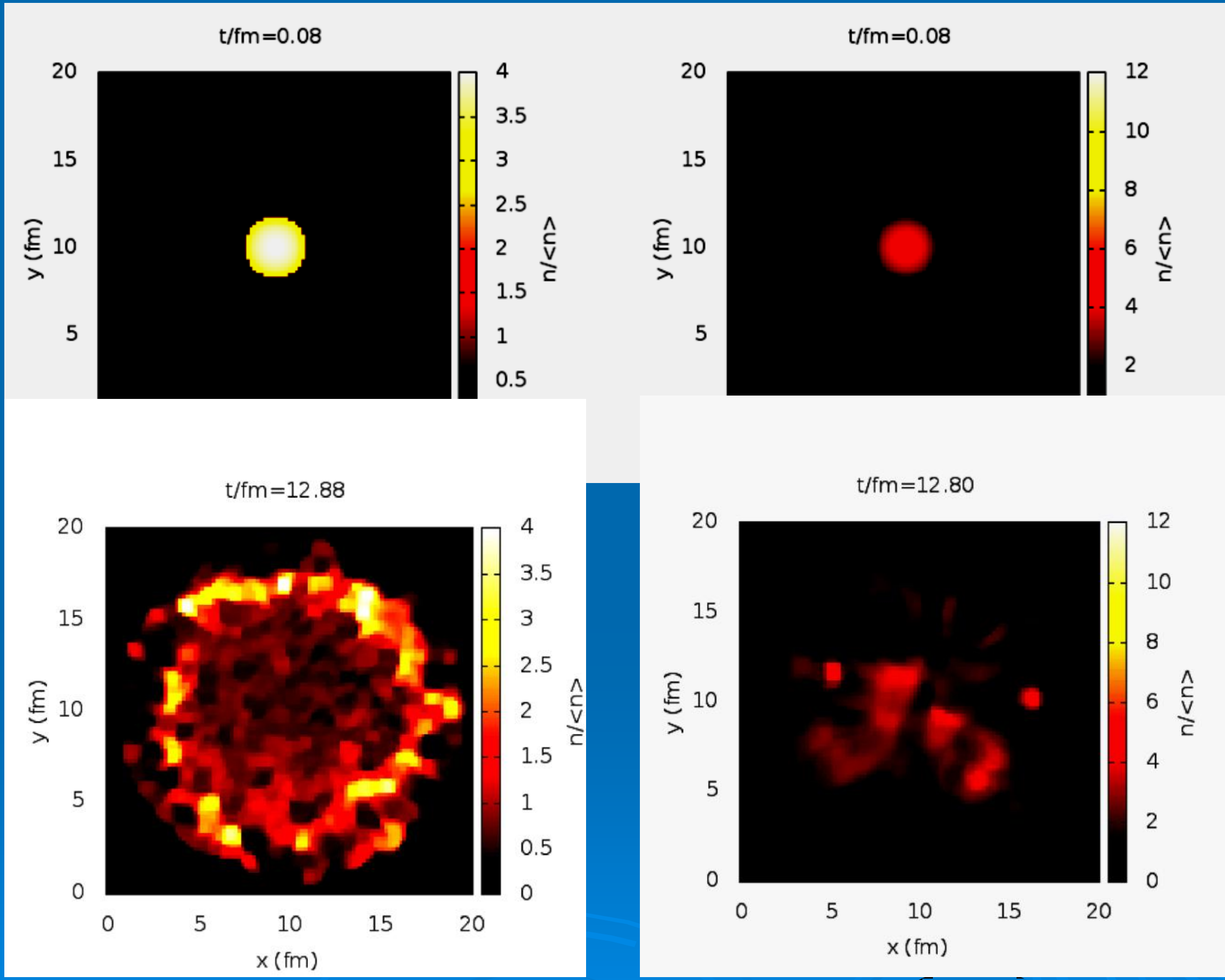
1) Nuclear ground state at μ_N=3μ≈m_N reproduced correctly
2) liquid-gas PT at μ≈300 MeV, and
3) deconfinement/chiral PT at higher μ≈450 MeV



Ch. Herold et al., Seam Pacific Conference 2014

PQM vs. QHM: domain formation

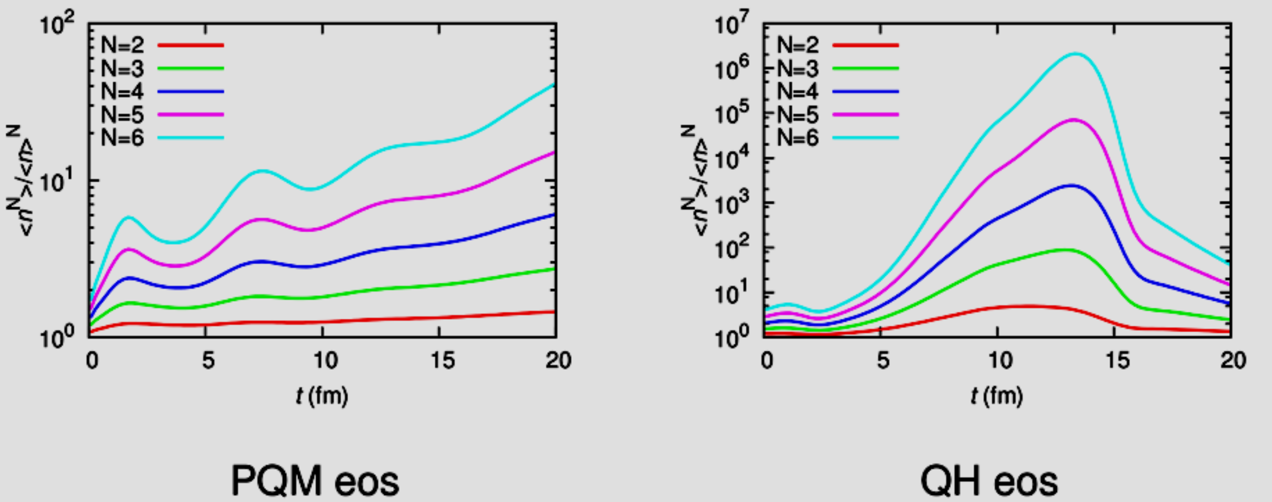
Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014



QH predicts domains with much higher densities!

PQM vs. QHM: density moments

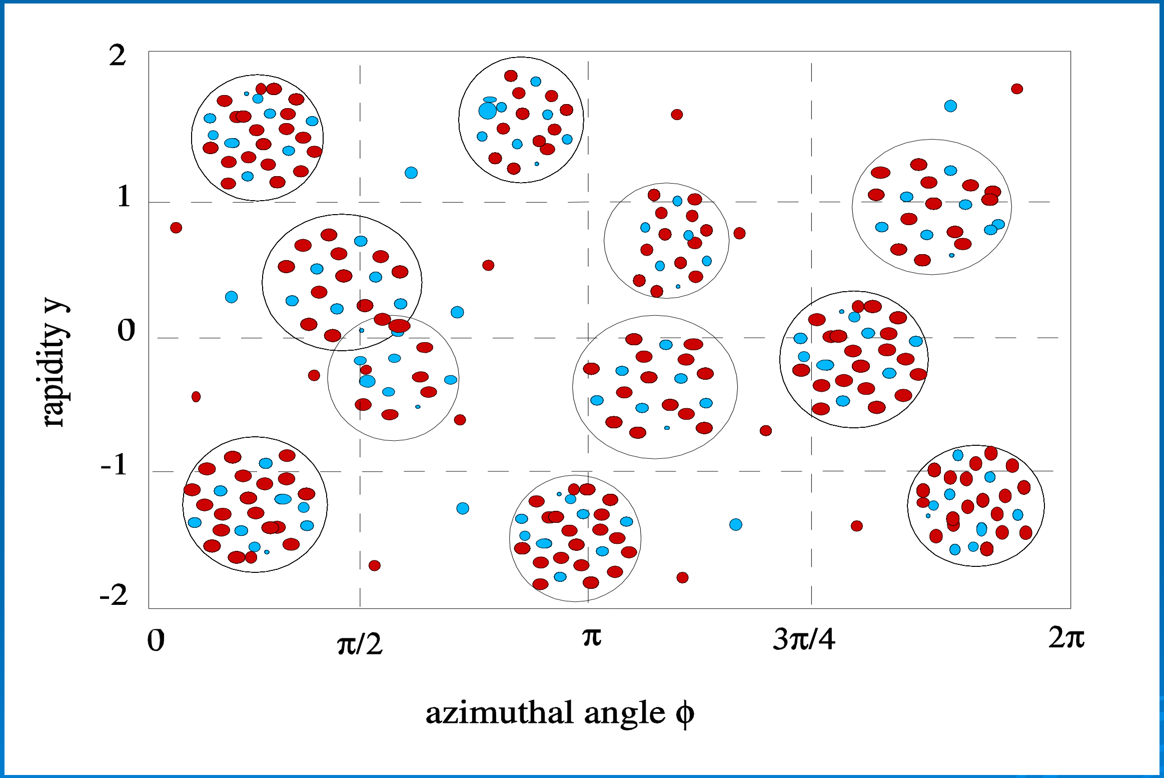
$$\langle n^N \rangle = \int d^3x \, n(x)^N P_n(x) \qquad \text{with} \qquad P_n(x) = \frac{n(x)}{\int d^3x \, n(x)}$$



In PQM density contrast grows towards freeze-out stage, but in QHM it has a maximum at the intermediate dense stage. But strong clustering effect is washed out at t>15 fm/c!

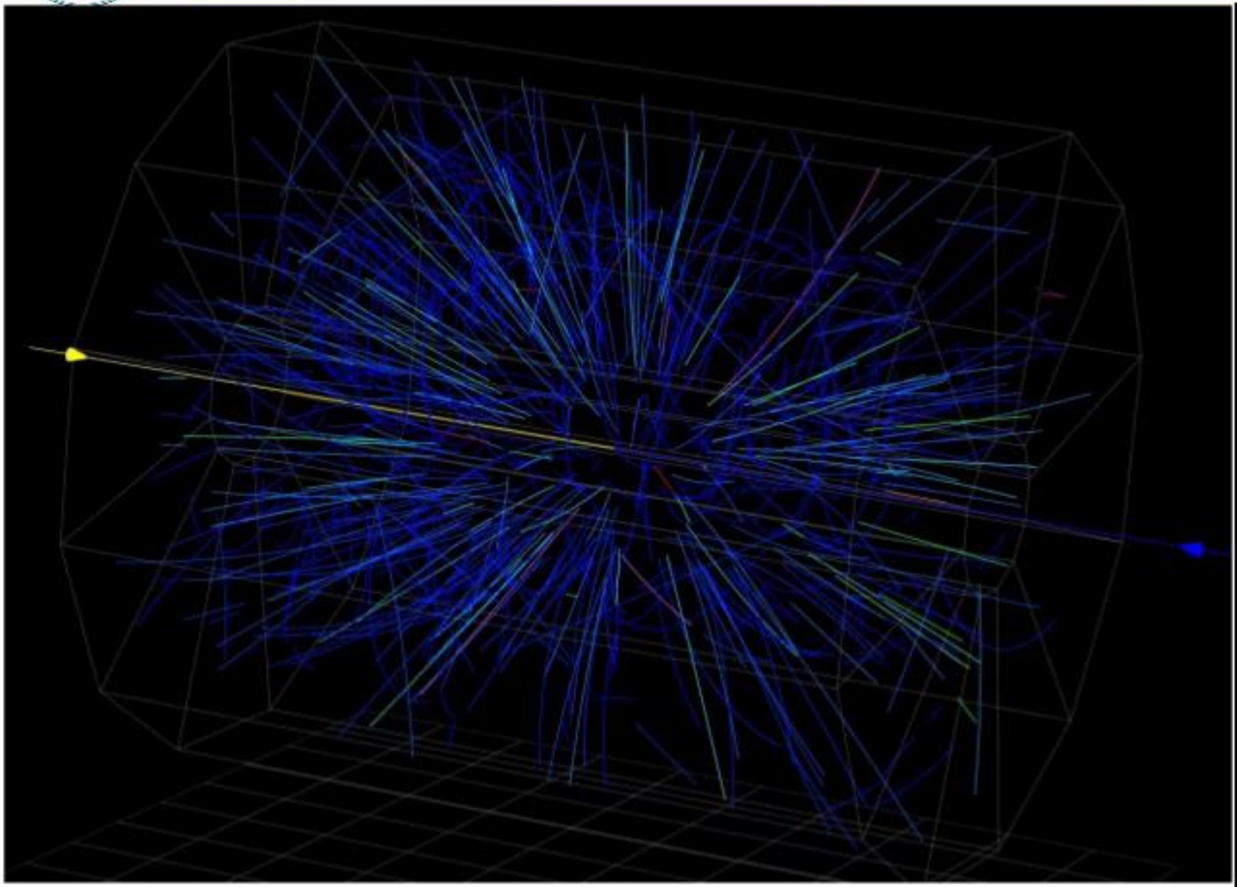
Experimental signatures of metastable domains

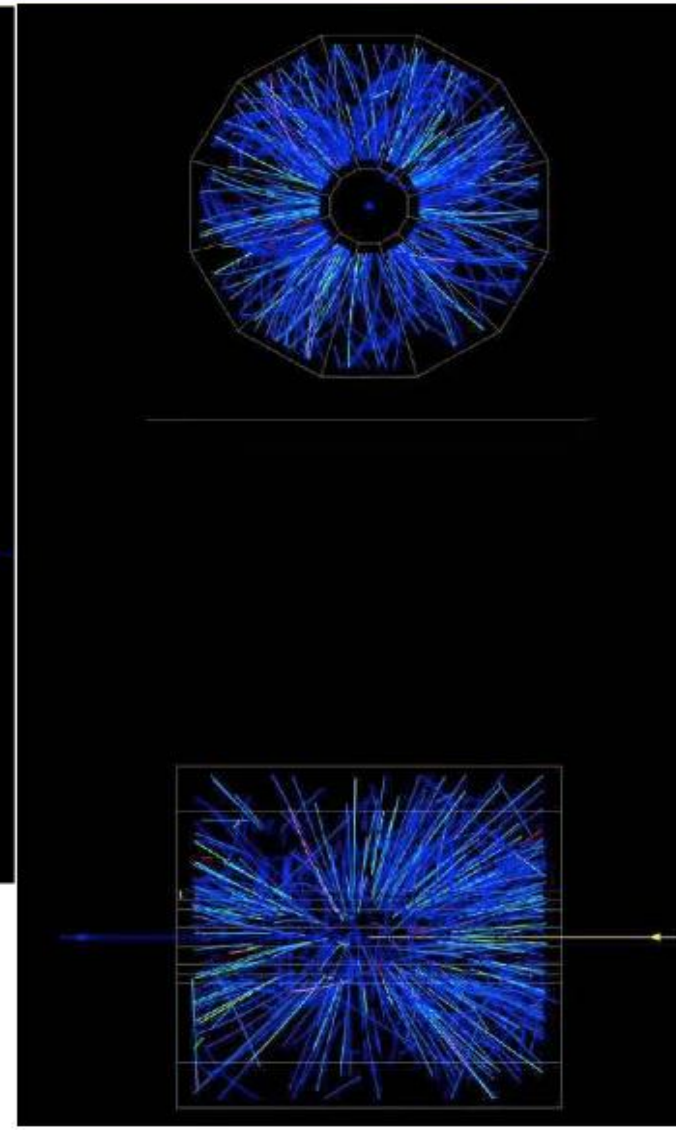
Look for bumpiness in distributions of net baryons in individual events, i. e. in azimuthal angle or rapidity



The bumps correspond to the emission from individual domains.

3D Event Display at STAR





BES-II, Au+Au collisions at 19.6 GeV.

Higher-order cummulants

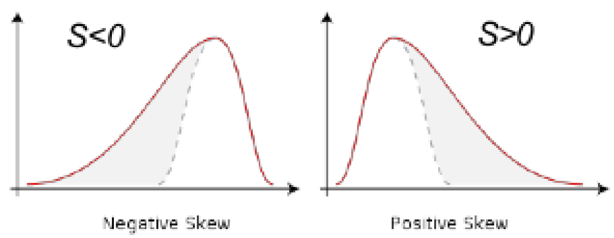
Higher Moments of Conserved Quantities (B, Q, S)

1. Higher order cumulants/moments: describe the shape of distributions and quantify fluctuations. (sensitive to the correlation length (ξ))

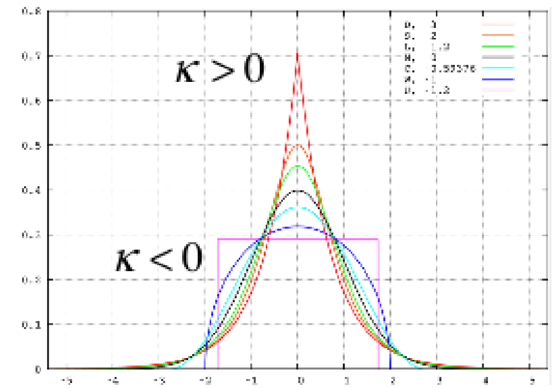
$$< \delta N >= N - < N >$$
$$C_1 = M =< N >$$
$$C_2 = \sigma^2 =< (\delta N)^2 >$$
$$C_3 = S\sigma^3 =< (\delta N)^3 >$$
$$C_4 = \kappa\sigma^4 =< (\delta N)^4 > -3 < (\delta N)^2 >^2$$

$$\left\langle (\delta N)^3 \right\rangle_c \approx \xi^{4.5}, \quad \left\langle (\delta N)^4 \right\rangle_c \approx \xi^7$$

Skewness (S) → asymmetry



Kurtosis (κ) → Sharpness

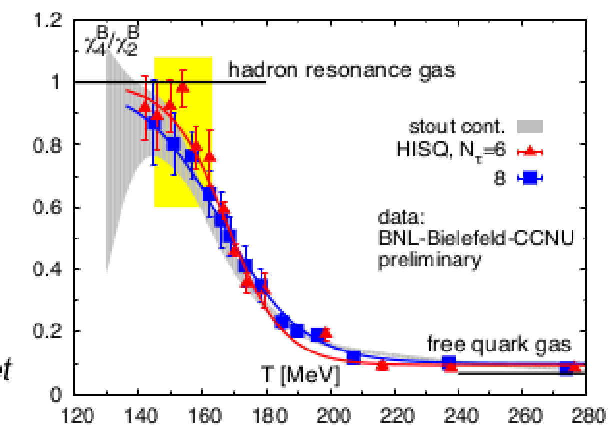


M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); 107, 052301 (2011).
M.Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009).

2. Direct connect to the susceptibility of the system.

$$\frac{\chi_q^4}{\chi_q^2} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \qquad \frac{\chi_q^3}{\chi_q^2} = S\sigma = \frac{C_{3,q}}{C_{2,q}},$$

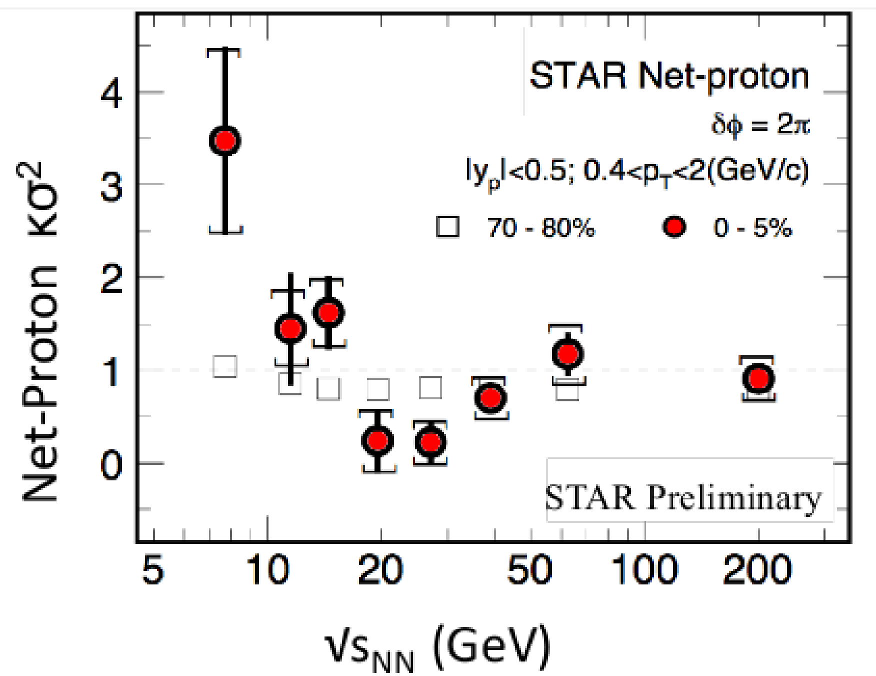
$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial (\mu_q)^n}, q = B,Q,S$$



S. Ejiri et al, Phys.Lett. B 633 (2006) 275. Cheng et al, PRD (2009) 074505. B. Friman et al., EPJC 71 (2011) 1694. F. Karsch and K. Redlich, PLB 695, 136 (2011). S. Gupta, et al., Science, 332, 1525(2012). A. Bazavov et al., PRL109, 192302(12) // S. Borsanyi et al., PRL111, 062005(13)

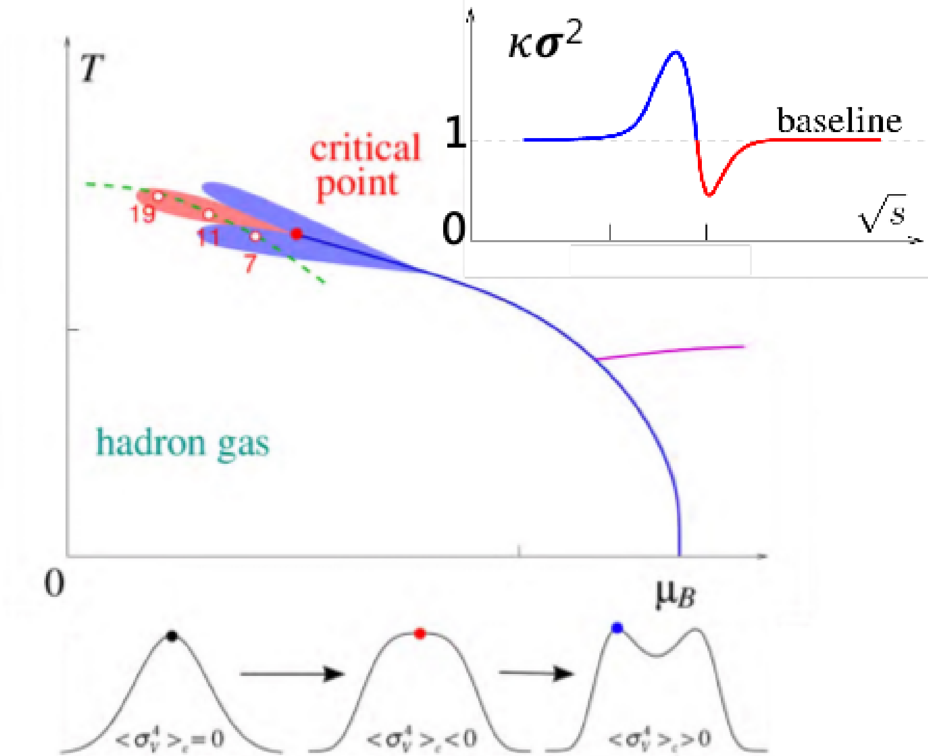
Net-Proton Fluctuations

Experimental Measure



STAR: Phys. Rev. Lett. 105, 022302 (2010).
Phys. Rev. Lett. 112, 032302 (2014).
PoS CPOD2014 (2015) 019.

Theoretical calculations

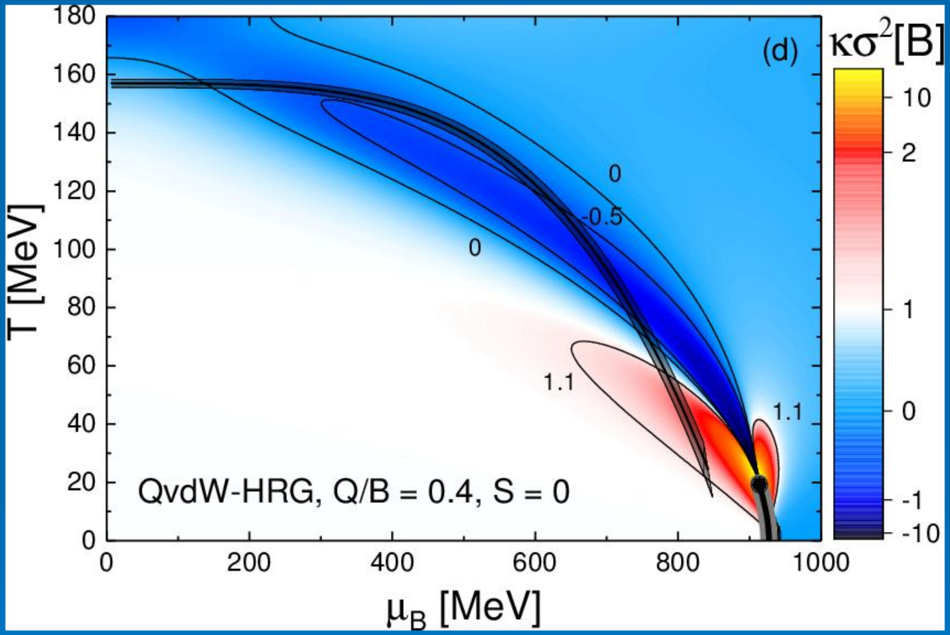


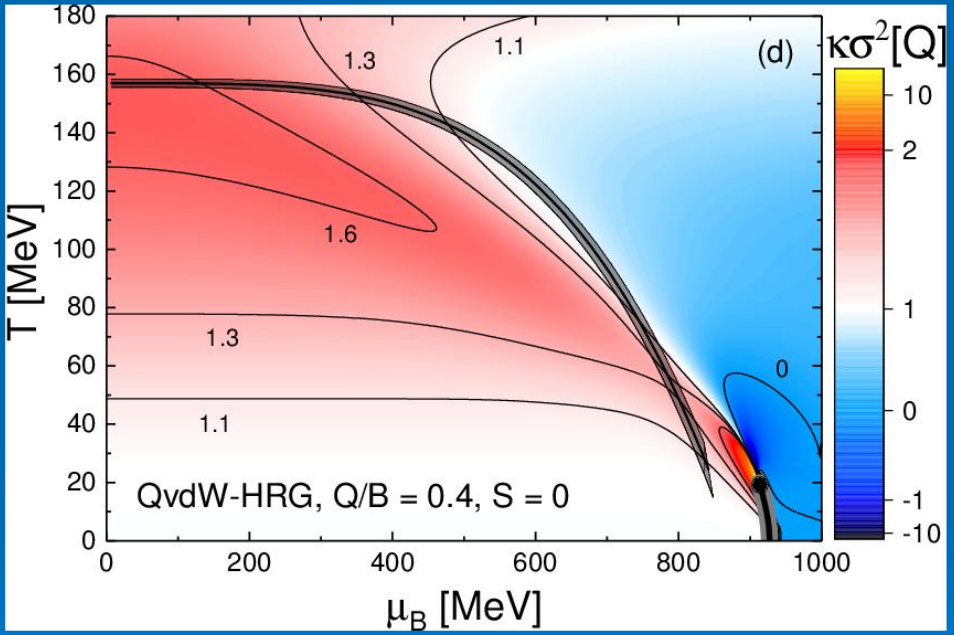
M. Stephanov, PRL107, 052301(2011)
J. Phys. G: 38, 124147 (2011).

- First observation of the non-monotonic energy dependence of fourth order net-proton fluctuations. Hint of entering Critical Region ?

Fluctuations in the HRG with hard-core repulsion (QvdW)

R. Poberezhnyak, V. Vovchenko, A. Motornenko, M. Gorenstein, H.Stoecker,
Chemical freeze-out conditions and fluctuations of conserved charges, arXiv:1906.01954



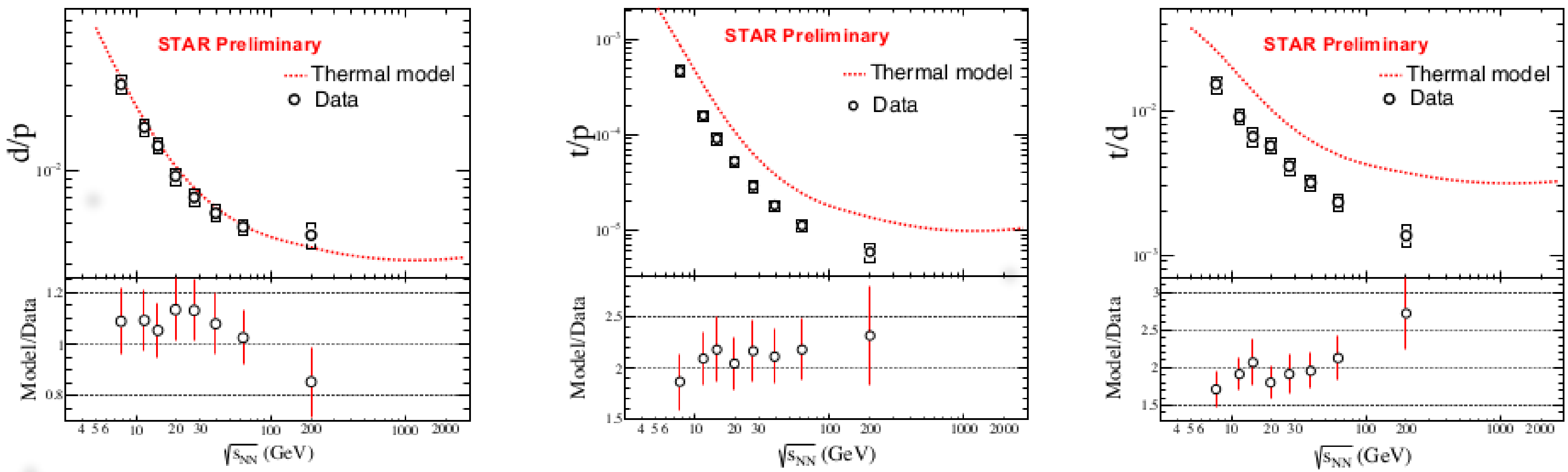


Noticeable fluctuations persist even at temperatures of about 100 MeV, i.e. much higher than the critical point for L-G phase transition! This may explain some anomalies observed by BES STAR at RHIC.

Conclusions

- Nuclear L-G phase transition is well established in intermediate-energy HI collisions (slow expansion)
- In relativistic heavy-ion collisions, because of rapid expansion, phase transitions will proceed out of equilibrium
- 2nd order phase transition (with CEP) is too weak to produce significant observable effects in fast dynamics
- Non-equilibrium signatures of a1st order transition (dynamical domain formation) may show up in data only
- if they occur close to freeze-out stage
- At present there exist no convincing evidences for a critical point or 1st order phase transition above nuclear saturation density

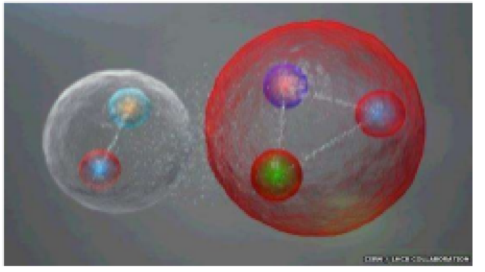
Light Nuclei Yield Ratio Vs. Thermal model



- At RHIC energies, thermal model can describe the d/p ratios, but can not describe the t/p, t/d ratios.
- If deuteron is formed at very late stage via nucleon cola., why it can be described by thermal model ?

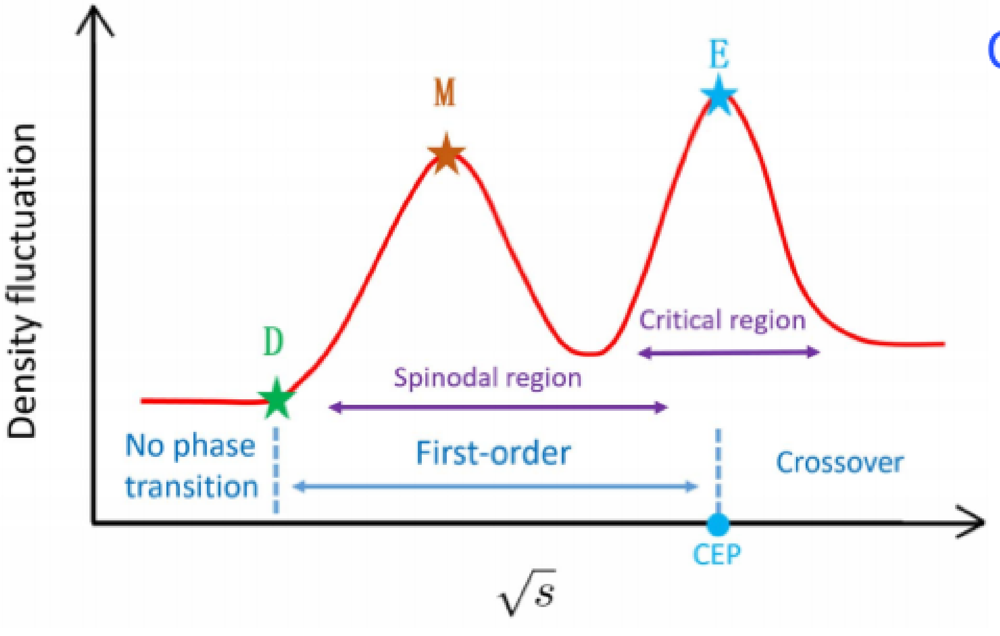
New Observable for CP: Light Nuclei Production

Near CP or 1st order phase transition, baryon density fluctuation become large.



Light nuclei production

(Baryon Clustering)



Coalescence + nucleon density flu.

$$N_d = \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3/2} N_p\langle n\rangle(1+\alpha\Delta n),$$

$$N_{^3{\rm H}} = \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3} N_p\langle n\rangle^2[1+(1+2\alpha)\Delta n],$$

$$\mathrm{N_t \cdot N_p / N_d^2 \approx g(1+\Delta n)}$$

Neutron density fluctuations:

$$\Delta \mathrm{n} = \langle(\delta \mathrm{n})^2\rangle / \langle \mathrm{n}\rangle^2$$

K. J. Sun, L. W. Chen, C. M. Ko, Z. Xu, Phys. Lett. B774, 103 (2017).
K. J. Sun, L. W. Chen, C. M. Ko, J. Pu, Z. Xu, Phys. Lett. B781, 499 (2018).
Edward Shuryak and Juan M. Torres-Rincon, arXiv:1805.04444

Calculation of damping term

T.Biro and C. Greiner, PRL, 79. 3138 (1997)

M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC 84, 024912 (2011)

The damping is associated with the processes:

$$\sigma \to \bar{q}q,\ \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2}\left[1 - 2n_F\left(\frac{m_\sigma}{2}\right)\right]\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2}$$

Around Tc the damping is due to the pion modes, η=2.2/fm