Ideal hydrodynamics limit extensions explored

Fluctuations, Polarization and gauge theory

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Based on 1810.12468,1807.02796,1701.08263,1604.05291,1502.05421,1112.4086 (PRD,PRC, last two as yet unpublished) with D.Montenegro,L.Tinti

What is this talk about

The necessity of a field theory perspective
Hydrodynamics is neither transport nor string theory!

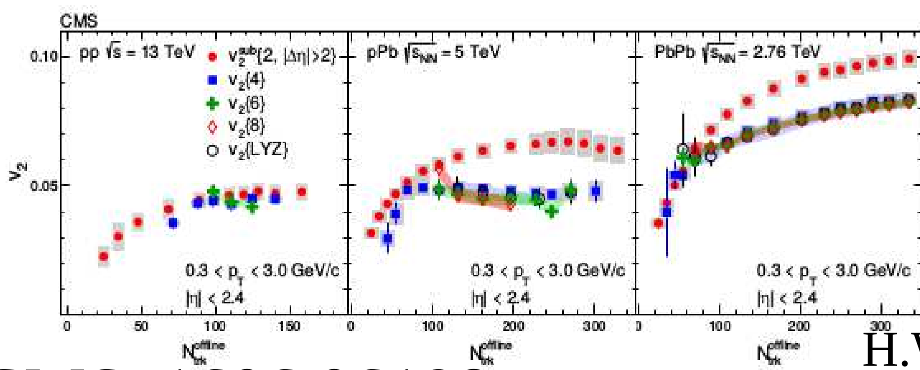
Introduction to the field theory of hydrodynamics
Our knowledge of hydrodynamics rewritten as symmetries
Perhaps not ideal for solving problems, but worth thinking about!

Extending hydrodynamics I Polarization

Extending hydrodynamics II Gauge symmetries

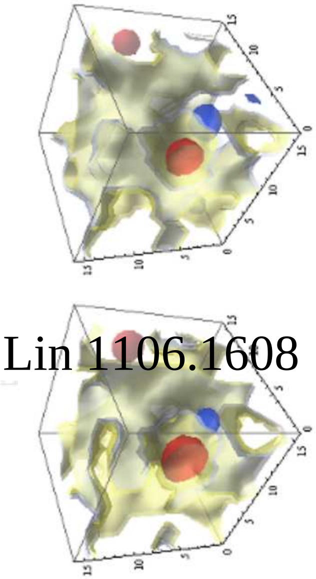
PS: So far, theory only!

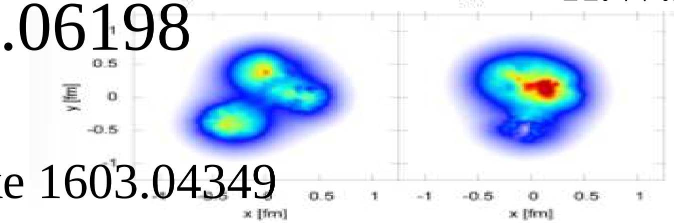
A spectacular experimental result



CMS 1606.06198

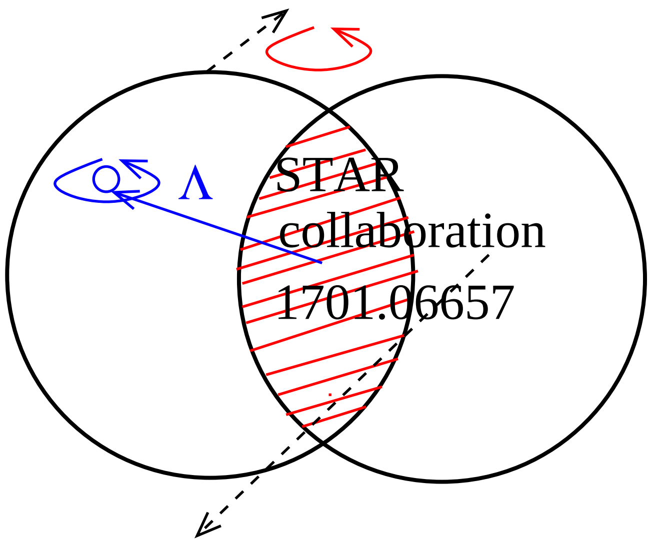
H.W.Lin 1106.1608

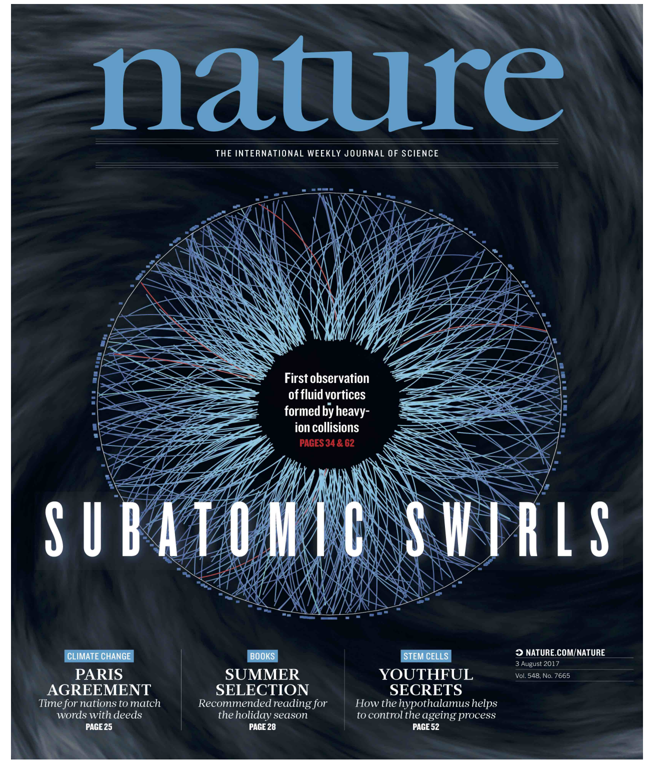




BSchenke 1603.04349

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. So mean free path is really small. What about thermal fluctuations? Nothing here is infinite, not even N_c Also hydro applicability scale below color domain scale. colored hydro?

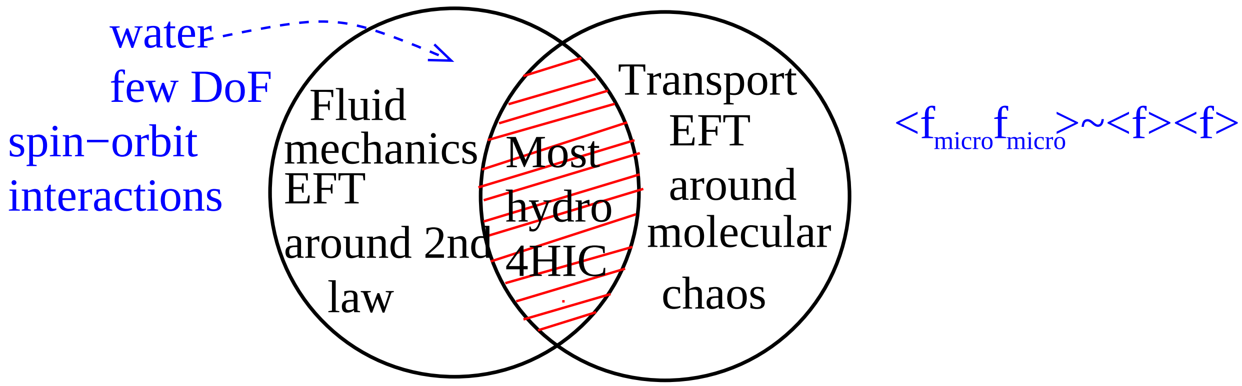




Another spectacular experimental result. But what have we learned about theory? My take is...

Hydro is not (just) transport! Nor string theory! Hydro is hydro!

Its constitutents are usually neither billiard balls not black holes!

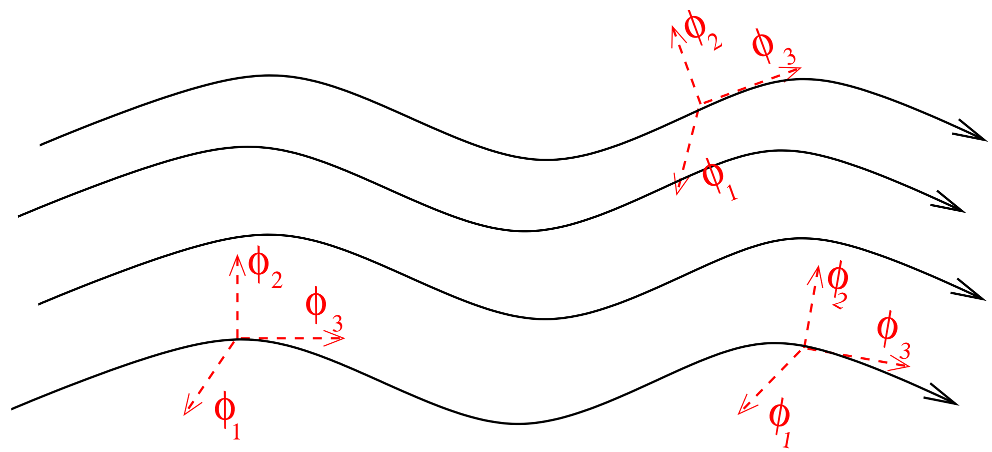


Hydro very good but "micro" and "macro" DoFs talk to each other!
Take water: Boltzmann equation breaks down as particles tightly correlated, hydro works well! AdS/CFT also suspect, this is N_c suppressed.

Note: Existing models produce polarization at freezout, violate detailed balance. No one knows how spin propagates during fluid evolution!
(Longitudinal vortical flow and spin alignment non-description?)

Lets set-up EFT around local equilibrium (Nicolis et al,1011.6396 (JHEP))

Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0,x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame) Solutions generally break these, Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons".

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \det B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b = \sqrt{B}$)
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\, \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu \phi_I}\partial_\nu \phi_I = u^\mu u^\nu - g^{\mu\nu}$)
Equation of state chosen by specifying $F(b)$. "Ideal": $\Leftrightarrow F(B) \propto b^{2/3}$
b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.
u^μ fixed by $u^\mu \partial_\mu \phi^{\forall I} = 0$. Vortices become Noether currents of diffeomorphisms!
This is all really smart, but why?

Hydrodynamics is based on <u>three</u> scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

l_{micro} stochastic, l_{mfp} dissipative. If $l_{micro} \sim l_{mfp}$ soundwaves

Of amplitude so that <u>momentum</u> $P_{sound} \sim (area)\lambda\,(\delta\rho)\,c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

Transport: Beyond <u>Molecular chaos</u> AdS/CFT: Beyond <u>large N_c</u>
It turns out Polarization, gauge symmetries mess this l_{micro} hyerarchy!

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^4}\right) \quad , \quad B = T_0^4 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength <u>and</u> $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!

Kn behaves as a gradient, T_0 as a Planck constant!!!

At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, "weighted" by the usual path integral prescription!

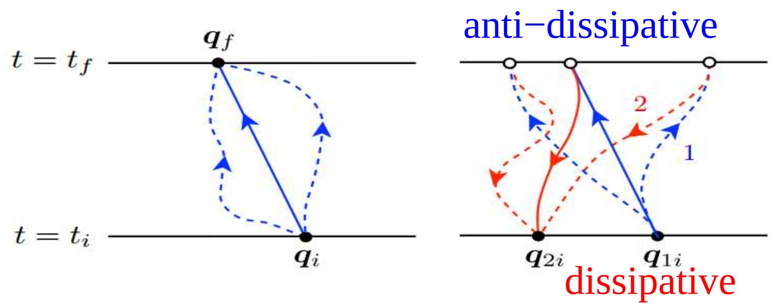
$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x\right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial \ldots}$$

$$\left(eg. \quad \left\langle T_{\mu\nu}^{x} T_{\mu\nu}^{x'} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu\nu}(x')} \right)$$

$T_0 \sim n^{-1/3}$, unlike Knudsen number, behaves as a "Planck constant".EFT expansion and lattice techniques should give all allowed terms and correlators. Coarse-graining will be handled here!

The big problem with Lagrangians... usually only non-dissipative terms
But there are a few ways to fix it. We focus on coordinate doubling (Galley, but before Morse+Feschbach)

Dissipative extension of Hamiltons principle

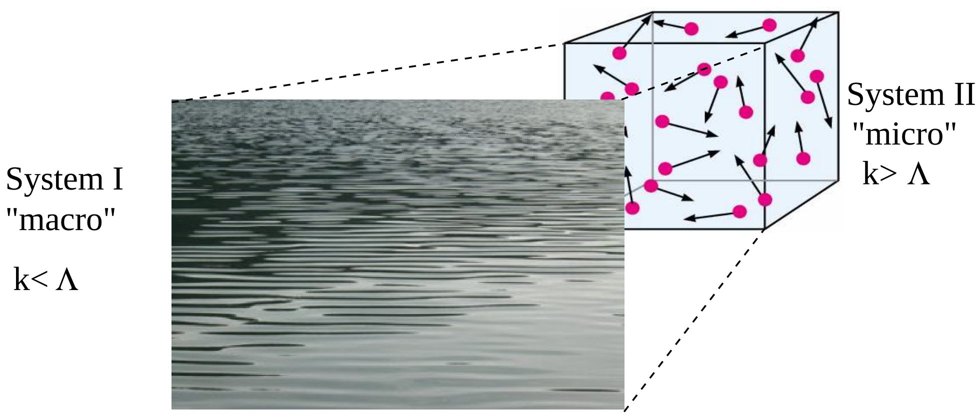


$$L = \frac{1}{2}\Big(\underbrace{m\dot{x}^2 - w x^2}_{SHO}\Big) \to \underbrace{\left(m\dot{x}_+^{\,2} - w x_+^2\right)}_{\mathcal{L}_1} - \underbrace{\left(m\dot{x}_-^{\,2} - w x_-^2\right)}_{\mathcal{L}_2} + \underbrace{\alpha\left(\dot{x}_+ x_- - \dot{x}_- x_+\right)}_{\mathcal{K}}$$

two sets of equations, one with a damped harmonic oscillator, the other "anti-damped". Navier-Stokes and Israel-Stewart (GT, D.Montenegro, PRD, (2016)) Functional integrals/Lattice also possible!

For analytical calculations fluid can be perturbed around a hydrostatic ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$



Kolmogorov cascade, Viscosity from turbulence when frequency≃ energy?

And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2(\nabla.\vec{\pi}_L)^2}_{sound\quad wave} + \underbrace{\dot{\pi_T}^2}_{vortex} + Interactions(\mathcal{O}\left(\pi^3, \partial\pi^3, ...\right))$$

Unlike sound waves , Vortices can not give you "free particles", since they do not propagate: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions. This does not mean the theory is ill-defined, just that its strongly non-perturbative!
Lattice: Tommy Burch,GT, 1502.05421 In ideal limit, Indications of a 1st order transition between turbulent and hydrostatic phases! Need viscous corrections, fluctuation/dissipation on lattice (BIG project!) But also Polarization might help here!

And chemical potential?

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

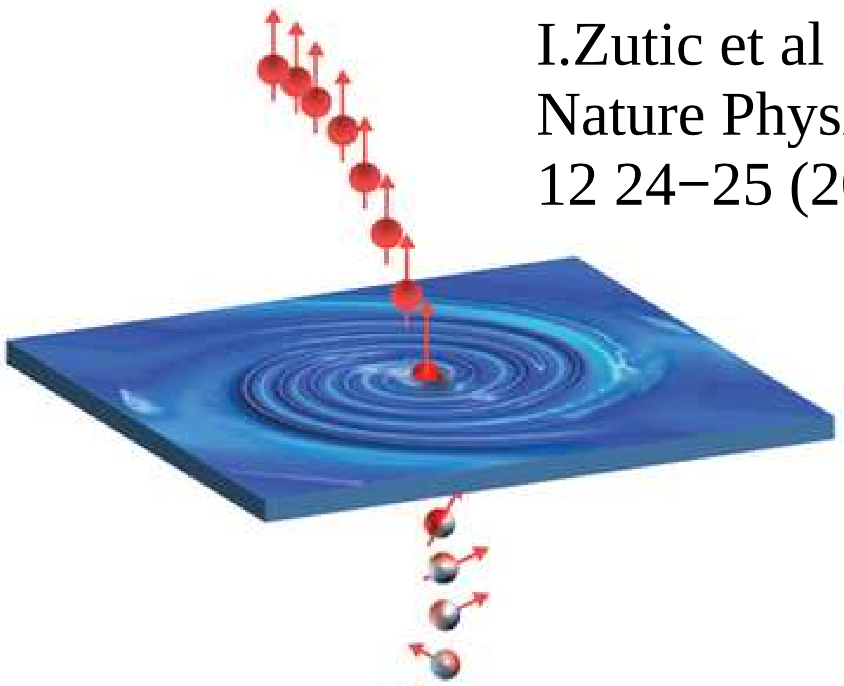
generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

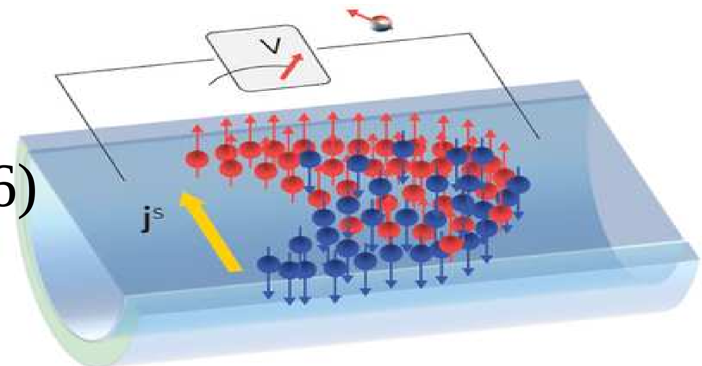
A comparison with the usual thermodynamics gives us

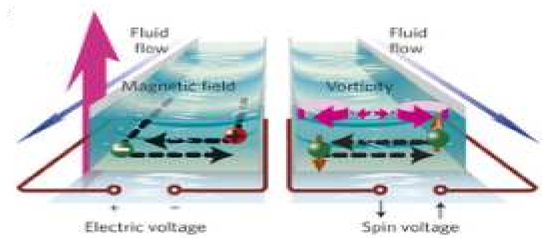
$$\mu = y \quad , \quad n = dF/dy$$

Application 1: Hydro with polarization



I.Zutic et al
Nature Physics
12 24−25 (2016)





Ultracold atoms: Zutic, Matos-Abiague, "Spin Hydrodynamics", Nature Physics **12** 24-25 Takahashi et al", Nature Physics **12** 52-56 (2016)

Combining polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^μ (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

(i-iii) ,with symmetries and EFT define the theory

So how do we implement polarization?

In comoving frame, polarization described by a representation of a "little group" of the volume element.

Need local $\sim SO(3)$ charges <u>and</u> unambiguus definition of u^μ $(s^\mu \propto J^\mu)$

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[- \sum_{i=1,2,3} \alpha_i(\phi_I) \hat{T}_i^{\mu\nu} \right]$$

For particle spinor, vector, tensor... repreentations possible.

For "many incoherent particles" RPA means only vector representation remains

Chemical shift symmetry, $SO(3)_{\alpha_{1,2,3}} \to SO(3)_{\alpha_{1,2,3}(\phi^I)}$

$$\alpha_i \to \alpha_i + \Delta\alpha_i\left(\phi_I\right) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

$y_{\mu\nu} \equiv \mu_i$ for polarization vector components in comoving frame

This way we ensured spin current flows with u^μ.

Note that it is not a proper chemical potential (it it would be there would be 3 phases attached to each ϕ_I) as $y_{\mu\nu}$ not invariant under symmetries of ϕ_I. $y_{\mu\nu}$ "auxiliary" polarization field

How to combine polarization with local equilibrium?

Since polarization decreases the entropy by an amount proportinal to the DoFs and independent of polarization direction

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \qquad , \qquad F(b) \to F(b, y) = F\left(b\left((1 - c y^2\right)\right)$$

.

Other terms break requirement (i)

First law of thermodynamics,

$$dE = TdS - pdV - Jd\Omega \to dF(b) = db\frac{dF}{db} + dy\frac{dF}{d(yb)}$$

Energy-momentum tensor Not uniquely defined

Canonical Noether charge for translations, could be asymmetric $\sim \frac{\partial L}{\partial(\partial\psi_i)}\partial\psi_j$

Belinfante-Rosenfeld $\sim \frac{\delta S}{\delta g_{\mu\nu}}$ independent of spin, no non-relativistic limit

Which is the source for $\partial_\mu T^{\mu\nu} = 0$? Not clear but irrelevant since hydro+polarization can't be written in terms of conservation laws

8 degrees of freedom,5 equations $(e, p, u_{x,y,z}, y^{\mu\nu})$. One can include the antisymmetric part of $T_{\mu\nu}$ and match equations but...

No entropy maximization If spin waves and sound waves separated, in comoving volume their ratio is arbitrary... but it should be decided by entropy maximization!

Consistent lagrangian exists if polarization always proportional to vorticity,

$$y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^{\mu}u^{\nu} - \partial^{\nu}u^{\mu}\right)$$

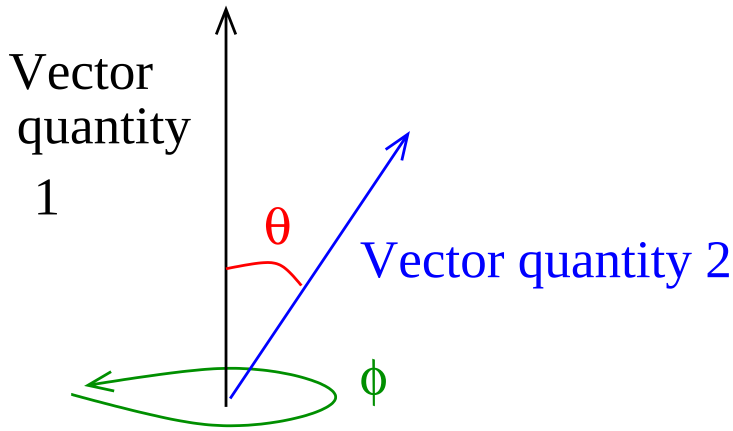
extension of Gibbs-Duhem to angular momentum uniquely fixes χ via entropy maximization. For a free energy $\mathcal{F}$ to be minimized

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V}dV + \frac{\partial \mathcal{F}}{\partial e}de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]}d\left[\Omega_{\mu\nu}\right] = 0$$

where $[\Omega_{\mu\nu}]$ is the vorticity in the comoving frame.
THis fixes χ . It also constrains the Lagrangian to be a Legendre transform of the free energy just as in the chemical potential case, in a straightforward generalization of Nicolis,Dubovsky et al. Free energy always at (local) minimum! (requirement (ii))

A qualitative explanation

Instant thermalization means vorticity instantly adjusts to angular momentum, and is parallel to angular momentum. Corrections to this will be of the relaxation type a-la Israel-Stewart



Note that microscopic physics could allow an arbitrary angle between vorticity and polarization. but such systems would have no hydrodynamic limit due to requirement (iii) and the necessity for stability of relaxation dynamics

These techniques lead to a well-defined Euler-Lagrange equation of motion

$$\left\{g_b(1-cy)\partial_\nu b + g_y 4y^\beta_\alpha \chi(T)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)}\partial^\nu(\partial_\lambda\phi^I)\right\}\times$$

$$\times[(1-cy)\frac{\partial b}{\partial(\partial_\nu\phi^I)} - (8cb)y^\beta_\alpha\chi(T)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu\phi^I)}] + g(b,y)\times$$

$$\times(1-cy)\partial_\nu(\frac{\partial b}{\partial(\partial_\nu\phi)}) - 8c\chi(T)g(b,y)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda\phi^I)}\left[\frac{y^\beta_\alpha}{2}\partial_\nu\partial_\lambda\phi^I\times\right.$$

$$\times\frac{\partial b}{\partial(\partial_\nu\phi^I)} + (\partial^\nu b)4y^\beta_\alpha\delta^\lambda_\nu + b\chi(T)(\frac{\partial(\partial^\beta u_\alpha)}{\partial(\partial_\nu\phi^I)} + \frac{\partial(\partial_\alpha u^\beta)}{\partial(\partial_\nu\phi^I)})\times$$

$$\left.\times\partial_\nu(\partial_\lambda\phi^I) + by^\beta_\alpha\partial_\nu ln\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu\phi^I)}\right] = 0$$

NB depends on accelleration, so $\Delta S = 0 \Rightarrow \partial_\mu\partial_\nu\frac{\partial F}{\partial(\partial_\mu\partial_\nu\phi^I)} = \partial_\mu\frac{\partial F}{\partial(\partial_\mu\phi^I)}$

Which can be linearized, $\phi_I = X_I + \pi_I$
The "free" (sound wave and vortex kinetic terms) part of the equation will be

$$\mathcal{L} = (-F'(1))\left\{\frac{1}{2}(\dot{\pi})^2 - c_s^2[\partial\pi]^2\right\}+$$

$$+f\zeta\left\{\ddot{\pi}^i\partial_i\dot{\pi}_j + \ddot{\pi}_i\ddot{\pi}_j + \partial_j\dot{\pi}^i\partial_i\dot{\pi}_j + \partial_j\dot{\pi}_i\ddot{\pi}_j+\right.$$

$$\left.+(2\ddot{\pi}^i\partial_j\dot{\pi}_i - 2\ddot{\pi}_j\partial^i\dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j\dot{\pi}_i^2 - \partial_i\dot{\pi}_j^2)\right\}$$

- Accelleration terms survive linearization
- Vortices and sound wave modes mix at "leading" order. Change in temperature due to sound wave changes polarizability, and that changes vorticity

We decompose perturbation into sound and vortex $\phi_I = \nabla\phi + \nabla \times \vec{\Omega}$

$$\left(\begin{array}{c} \varphi \\ \vec{\Omega} \end{array} \right) = \int dw d^3k \left(\begin{array}{c} \varphi_0 \\ \vec{\Omega}_0 \end{array} \right) \exp\left[i \left(\vec{k_{\phi,\Omega}}.\vec{x} - w_{\phi,\Omega} t \right) \right]$$

Dispersion relation for parallel to k ("sound-wave") and vector part ("vortex")

$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0 \quad , \quad (3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4 \Omega = 0$$

Dispersion relations show violation of causality! $(dw/dk \geq 1)$

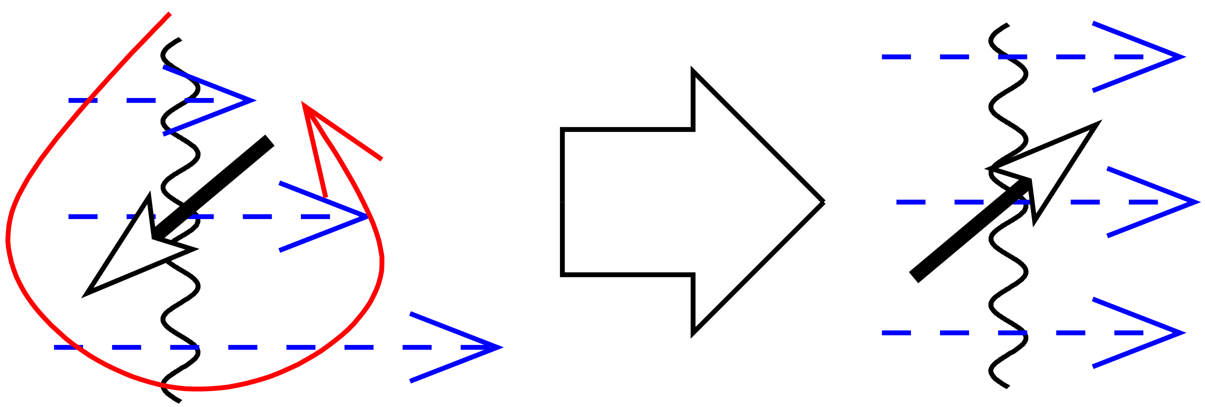
Linearization leads to violation of causality because of Ostrogradski's theorem (Lagrangian depends on accelleration)

To fix this, we need polarization to relax to vorticity, a la Israel-Stewart

$$y_{\mu\nu} = \chi(T,y)\Omega_{\mu\nu} \Rightarrow \tau_\Omega u_\alpha \partial^\alpha y_{\mu\nu} + y_{\mu\nu} = \chi(T,y)\Omega_{\mu\nu}$$

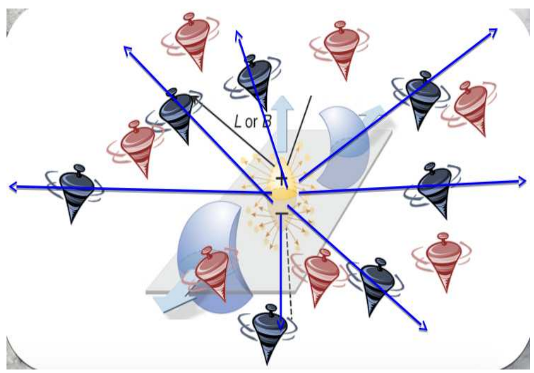
GT,D Montenegro, 1807.02796 lower limit to η/s in polarizeable fluids

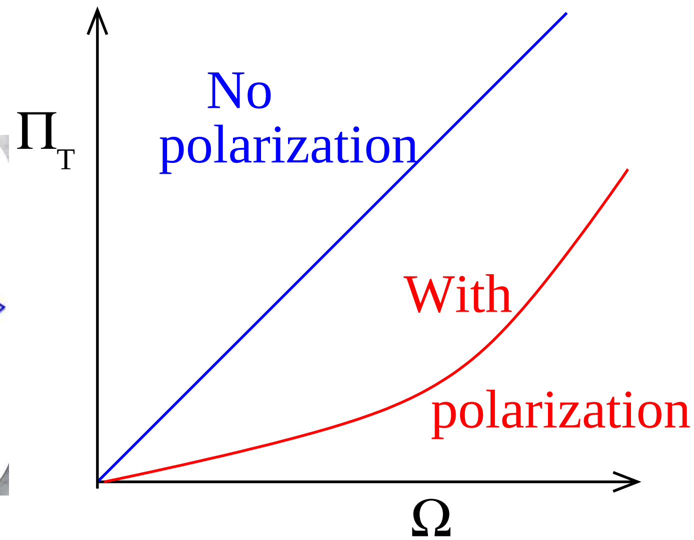
$$\tau_Y^2 \geq \frac{8c\chi^2(b_o,0)}{(1-c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$



Polarization and vortex stability

Fluctuation-dissipation: $\tau_{\Omega} \sim \lim_{\omega \to 0} \omega^{-1} \int dt \left\langle y_{\mu\nu} \Omega_{\mu\nu} \right\rangle \exp(i\omega t)$





Polarization makes vorticity aquire a "soft gap" wrt angular momentum. At small amplitudes, creating polarization is more advantageus than creating vorticity. This means small amplitude vortices get quenched. Stabilizes theory against vortice instabilities

Gauge theory and local thermalization

The formalism we introduced earlier is ok for quark polarization but problematic for gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for transversely polarized spin states. So vorticity vs polarization is ambiguus

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \to T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda}\right)$$

where Φ is fully antisymmetric. $\delta S/\delta g_{\mu\nu}$ and canonical tensors are lmits of choice of Φ . But vorticity global (and gauge invariant), $y_{\mu\nu}$ local (and gauge dependent). Affects EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators)

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \qquad , \qquad \exp(i\alpha) \to \exp\left(i\sum_i \alpha_i \hat{T}_i\right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J^\mu_a \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U^{-1}_{ac}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i \right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U^{-1}_{ac}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} U^{-1}_{fg} \partial_\mu \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} \partial_\mu \left(U^{-1}_{fg} \alpha_d U_{bd}(x) \right) - J^{\mu}_{a} \left(U \partial_\mu U \right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J^{\mu}_{j} \partial_\mu \alpha_i\right) \to F\left(b, J^{\mu}_{j} \left(\partial_\mu - U(x) \partial_\mu U(x)\right) \alpha_i\right)$$

Which is totally unexpected, profound and crazy

The swimming ghost!

$$F\left(b, J_j^{\mu} \partial_{\mu} \alpha_i\right) \rightarrow F\left(b, J_j^{\mu}\left(\partial_{\mu} - U(x) \partial_{\mu} U(x)\right) \alpha_i\right)$$

Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl
the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \rightarrow 1 \quad , \quad y_{ab} \rightarrow \mu_Q \quad , \quad u_{\mu} \partial^{\mu} \alpha_i \rightarrow A_{\tau}$$

So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...

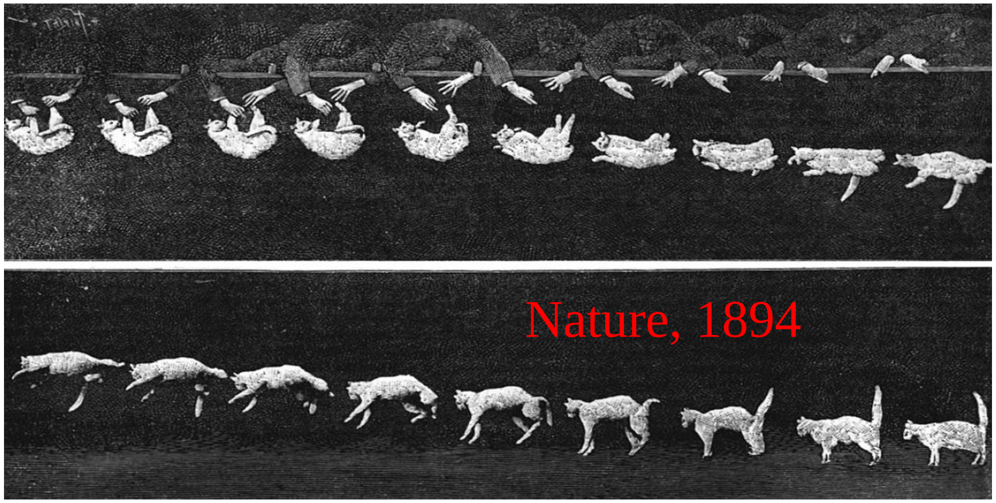
The swirling ghost

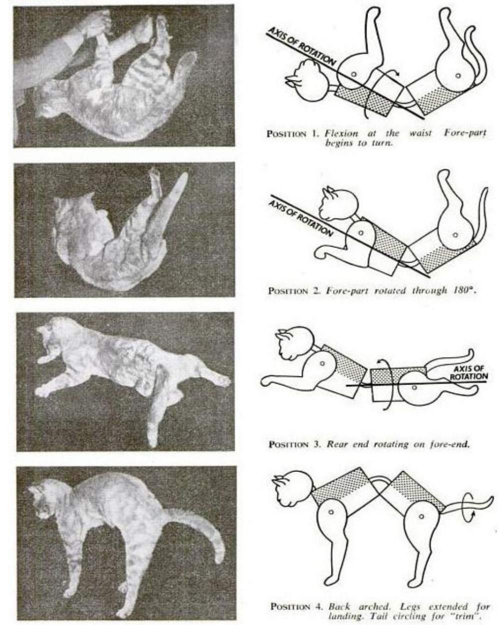
Since $u^\mu \partial_\mu$ is in the Lagrangian,let us compare vorticity and Wilson loops!

$$\mathrm{Vorticity}: \oint J_\mu dx^\mu \neq 0 \quad , \quad \mathrm{Wilsonloop}: \oint dx_\mu \partial^\mu U_{ab} \equiv \int_\Sigma d\Sigma_{\mu\nu} F^{\mu\nu}_{ab}$$

Lagrangian will in general have gauge-invariant terms proportional to $Tr_a \omega_{\mu\nu a} F^{\mu\nu}_a$ Unlike in Jackiw et al , $F_{\mu\nu}$ is not field strength but just a polarization tensor, whose value is set by entropy maximization.

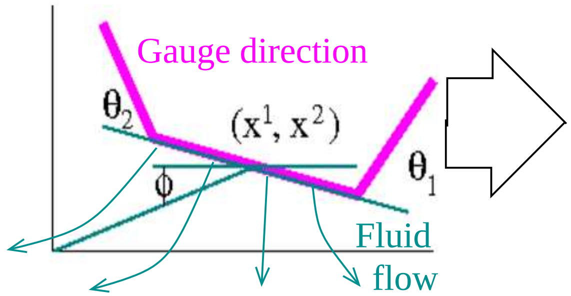
But circular modes correlating angle around vortex of u_μ and direction a of $F^a_{\mu\nu}$ non-dissipative (unlike in polarization hydro described earlier)





S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a "set of gauges" a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a “swimmer”, with arms and legs outstretched in “gauge” directions...

Hydrostatic vacuum unstable against purcell swimmers in Gauge space!

A statistical mechanics/Gauge explanation

Hydrodynamic limit: $\partial^\mu s_\mu \equiv \partial^\mu \left(u_\mu \ln N_{microstates} \right) = 0$

In thermal Gauge theory microstates contain gauge redundancies,

$N_{microstates} \to N_{microstates} - N_{gauge}$ But s_μ^{real} not parallel to s_μ^{gauge}

so no local equilibrium!. recall hydrostatic limit perturbation

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential". Let us vary the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

"color susceptibility" typically negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes "work" from "heat" is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Development of EoMs, linearization, etc. of this theory in progress!

A crazy guess, speculation Remember that all flow dependence through μ_{ab} color chemical potentials. What if local equilibrium happens when they go to zero, i.e. color density is neutral.

Could colored-swimming ghosts quickly be produced, and then locally thermalize and color-neutralize the QGP?

Similar to Positivity violation picture of confinement (Alkofer)

What about gauge-gravity duality?

Large N non-hydrodynamic modes go away in the planar limit
There are N ghost modes and N^2 degrees of freedom

Conformal fixed point most likely means ghosts non-dynamical
Not yet sure of this, but conformal invariance reduces pseudo-Gauge transformations to

$$\Phi_{\lambda,\mu\nu} \underbrace{\rightarrow}_{conformal} g_{\sigma\mu}\partial_{\nu}\phi - g_{\sigma\mu}\partial_{\mu}\phi$$

where ϕ is a scalar function. Irrelevant for dynamics.
As shown in Capri et al (1404.7163) Gribov copies for a Yang-Mills theory non-dynamical there. It would be a huge job to do this for hydrodynamics.

Conclusions

Hydrodynamics is not a limit of transport, AdS/CFT or any other microscopic theory

Hydrodynamics is an EFT built around symmetries and entropy maximization and should be treated as such

Once you realize this , generalizing it to theories with extra DoFs, symmetries etc. becomes straight-forward.

Lots of things to do Gauge symmetry looks particularly interesting!

Can we do better? Put theory on the lattice, work with Thiago Nunes