Frisco3 - A three-fluid model with continuous freeze-out for intermediate energy heavy-ion collisions

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Motivation

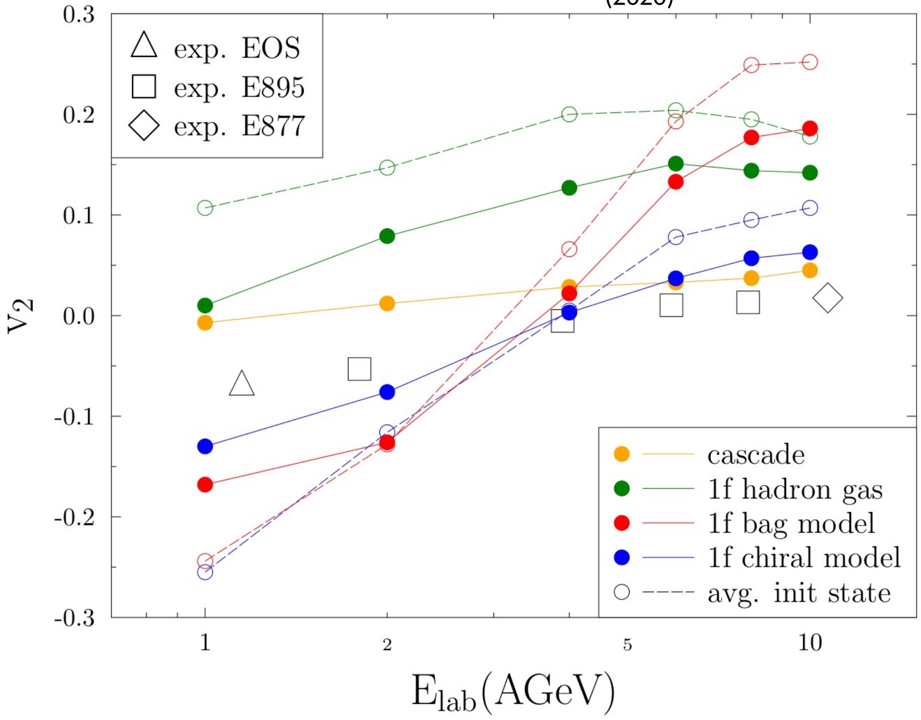
Why fluid dynamical models for simulation of h.i.c.?

- Minimize numbers of degrees of freedom while preserving essential physical features of the interacting system
- Allow systematic studies of different equations of state on observables (under the ceteris paribus assumption)
- Different fluid dynamical models have proven successful in describing different experimentally observable aspects of h.i.c. at different energies
- However, often high energy hydrodynamic models use ad hoc and fixed time initial conditions for a single fluid
  - not sufficient at intermediate energies
  - calls for dynamical production of the fireball fluid

A little detour to one fluid

- Probing the EoS with elliptic flow

C. Spieles and M. Bleicher, Eur. Phys. J. ST 229, 3537 (2020)

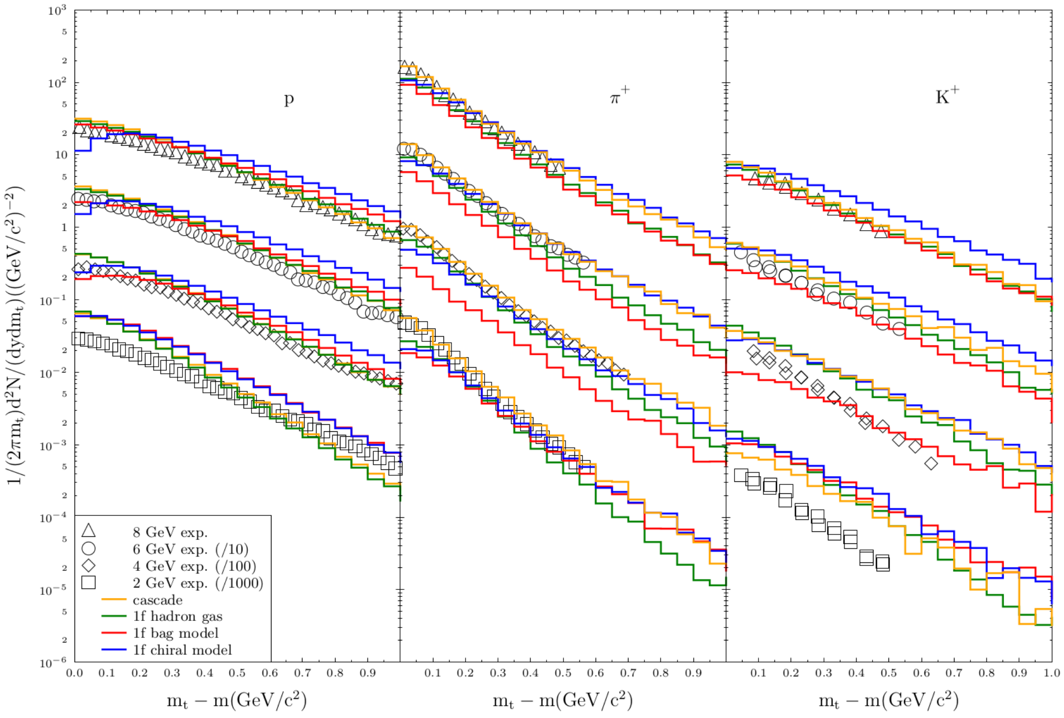


Au+Au b=7 fm

- 1fluid model results “favor” an EoS with a crossover phase transition
- Importance of initial state fluctuations

Still the detour to one fluid

- Applying a relavistically invariant particlization scheme (Cooper-Frye) yields surprisingly good agreement with data – in particular at 6-8 GeV!
- But: What to do with part of the system that never crosses CF surface?



central Au+Au

data from E895, E866, E917

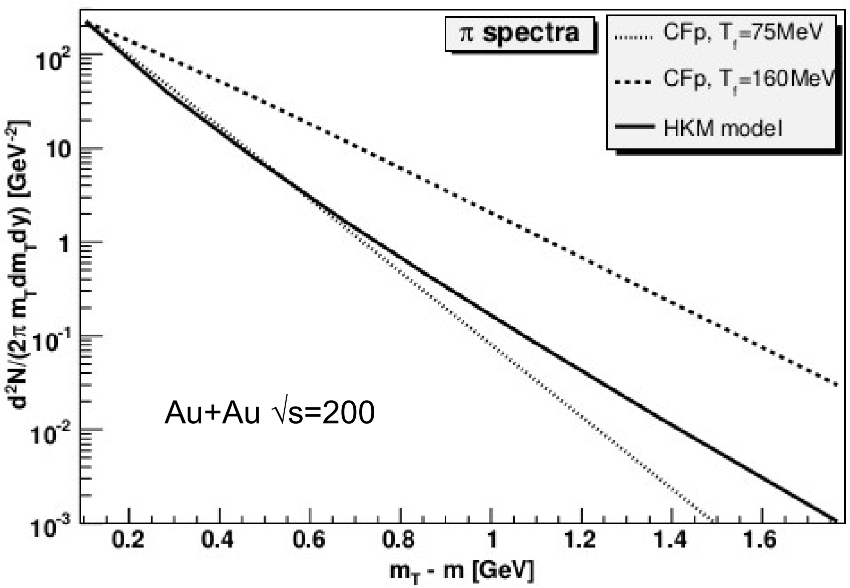
C. Spieles and M. Bleicher, Eur. Phys. J. ST 229, 3537 (2020)

Motivation

- Include consistent space and time dependent initial conditions for the fireball fluid
  - ➔ The observable finite stopping power / non-instantaneous equilibration must be accounted for in a comprehensive description → extend to a three fluid model with friction and unification
- Include proper boundary conditions for hydrodynamics for the decoupling, conserving energy and quantum numbers
(as opposed to the standard Cooper-Frye prescription which does not couple back to hydrodynamics!)
  - ➔ implement a relativistic invariant, continuous freeze-out coupling based on local scattering rates, varying in space and time

Continuous freeze-out of particles from fluids is not a new idea

- Qualitatively different spectra as compared to a Cooper-Frye ansatz must be expected in a more realistic scenario



S. V. Akkelin, Y. Hama, I. A. Karpenko, and Y. M. Sinyukov, Phys. Rev. C **78**, 034906 (2008), arXiv:0804.4104 [nucl-th].

see also: F. Grassi, Y. Hama, and T. Kodama, Z. Phys. C **73**, 153 (1996).
C. M. Hung and E. V. Shuryak, Phys. Rev. C **57**, 1891 (1998), arXiv:hep-ph/9709264.

H. Holopainen and P. Huovinen, J. Phys. Conf. Ser. 509 (2014)012114

„frisco3“ – outline of the model

- Relativistic 3+1d three-fluid model (fl=projectile, target, fireball) with a local coupling to the free-streaming regime

ideal hydro (r.h.s.=0) | friction | unification | continuous freeze-out

energy and momentum:

$$\partial_\mu T^{\mu\nu}_{\mathrm{fl}} = \tilde{F}^{\nu}_{\mathrm{fl}} + \check{F}^{\nu}_{\mathrm{fl}} + \hat{F}^{\nu}_{\mathrm{fl}}$$

baryon number:

$$\partial_\mu N^{\mu}_{\mathrm{fl}} = \tilde{S}_{\mathrm{fl}} + \check{S}_{\mathrm{fl}} + \hat{S}_{\mathrm{fl}}$$

- Feedback of continuous freeze-out on the fluids is taken into account dynamically
- Total energy, momentum and baryon number (three fluids plus free streaming particles) are conserved at all times

Initializing projectile and target fluid fields

$$\partial_\mu T^{\mu\nu}_{\mathrm{fl}} = \tilde{F}^\nu_{\mathrm{fl}} + \check{F}^\nu_{\mathrm{fl}} + \hat{F}^\nu_{\mathrm{fl}}$$

$$\partial_\mu N^\mu_{\mathrm{fl}} = \tilde{S}_{\mathrm{fl}} + \check{S}_{\mathrm{fl}} + \hat{S}_{\mathrm{fl}}$$

- Monte-Carlo sampling of nucleons within cold nuclei according to Woods-Saxon profiles
- Sum-up individual Lorentz-contracted 3d-Gaussians and map on spatial fluid grid using the UrQMD hybrid model procedure
- Ensemble averaged mode or ‘event-by-event’ mode allow for impact analysis of initial state fluctuations

H. Petersen, J. Steinheimer, G. Burau, M. Bleicher, and H. Stocker, Phys. Rev. **C78**, 044901 (2008), arXiv:0806.1695 [nucl-th].

Equations of State and propagation

$$\partial_\mu T^{\mu\nu}_{\text{fl}} = \tilde{F}^\nu_{\text{fl}} + \check{F}^\nu_{\text{fl}} + \hat{F}^\nu_{\text{fl}}$$
$$\partial_\mu N^\mu_{\text{fl}} = \tilde{S}_{\text{fl}} + \check{S}_{\text{fl}} + \hat{S}_{\text{fl}}$$

- Projectile and target fluids are modelled as a monocomponent nucleonic ideal gas, interpolating the non-realtivistic and ultrarelativistic limit as $p(e,n) = \min(\frac{2}{3}(e - n\, m_{\text{N}}), \frac{1}{3}e)$
- The 3rd fluid (fireball) can use different EoS (CMF, Bag model, HG,…). For the current test we use an ideal gas of hadronic resonances. The g.c. pressure $p(T,\mu_{\text{B}},\mu_{\text{S}}) = \sum_i \frac{g_i}{6\pi^2}\int_0^\infty \frac{dp}{E_i}\frac{p^4}{\exp[(E_i-\mu_i)/T]\pm 1}$ and the expressions for $e(T,\mu_{\text{B}},\mu_{\text{S}})$ and $n(T,\mu_{\text{B}},\mu_{\text{S}})$ are inverted numerically in order to get required EoS as $p(e,n)$
- Hydrodynamic equations are solved for individual fluids by using the SHASTA algorithm, allowing the description of shock wave formation

$$\partial_t T^{00} + \vec{\nabla}\cdot(T^{00}\vec{v}) = -\vec{\nabla}\cdot(p\vec{v})$$
$$\partial_t T^{0i} + \partial_i(\sum T^{0j}v_j) = -\partial_i p \quad \text{for} \quad i = 1,2,3$$
$$\partial_t N^0 + \vec{\nabla}\cdot(N^0\vec{v})) = 0$$

D. H. Rischke, S. Bernard, and J. A. Maruhn, Nucl. Phys. A **595**, 346 (1995).
D. H. Rischke, Y. Pürsün, and J. A. Maruhn, Nucl. Phys. A **595**, 383 (1995).

- Fluid dynamical propagation stops, when all particles have been emitted (other criteria can be used also)

Creating the fireball: The friction term

$$\partial_\mu T^{\mu\nu}_{\text{fl}} = \tilde{F}^\nu_{\text{fl}} + \check{F}^\nu_{\text{fl}} + \hat{F}^\nu_{\text{fl}}$$
$$\partial_\mu N^\mu_{\text{fl}} = \tilde{S}_{\text{fl}} + \check{S}_{\text{fl}} + \hat{S}_{\text{fl}}$$

- The model parametrization of the four momentum transfer from fluid 1 and 2 to the fireball is based on elementary differential NN cross sections

I. N. Mishustin, V. N. Russkikh, and L. M. Satarov, Nucl. Phys. A **494**, 595 (1989).

$$\tilde{F}^\nu_{\text{fl}} = N_1 N_2 \langle v_{\text{Møller}} \int d\sigma_{\text{NN}\to\text{XN}} (p' - p)^\nu \rangle$$
$$(\text{fl} = 1, 2)$$

$$\tilde{F}^\nu_3 = -(\tilde{F}^\nu_1 + \tilde{F}^\nu_2)$$

- Simplifying assumption: Friction transfers only energy and momentum to the third fluid, i.e. $\tilde{S}_{\text{fl}} = 0$

One-fluid transition: Unification

$$\partial_\mu T^{\mu\nu}_{\rm fl} = \tilde{F}^\nu_{\rm fl} + \check{F}^\nu_{\rm fl} + \hat{F}^\nu_{\rm fl}$$
$$\partial_\mu N^\mu_{\rm fl} = \tilde{S}_{\rm fl} + \check{S}_{\rm fl} + \hat{S}_{\rm fl}$$

- For small relative velocities, the friction term vanishes. However, fluid elements of different fluids at the same space-time point with small relative velocities should thermalize with each other and become one fluid *J. Brachmann et al., Nucl. Phys. A619 (1997) 391*

➔ Instantaneous and complete transferral of four-momentum and baryon number from fluid 1 and fluid 2 into fluid 3, when the local rapidity difference between the fluids is in the range of a typical thermal velocity $\Delta y_{\rm th}$

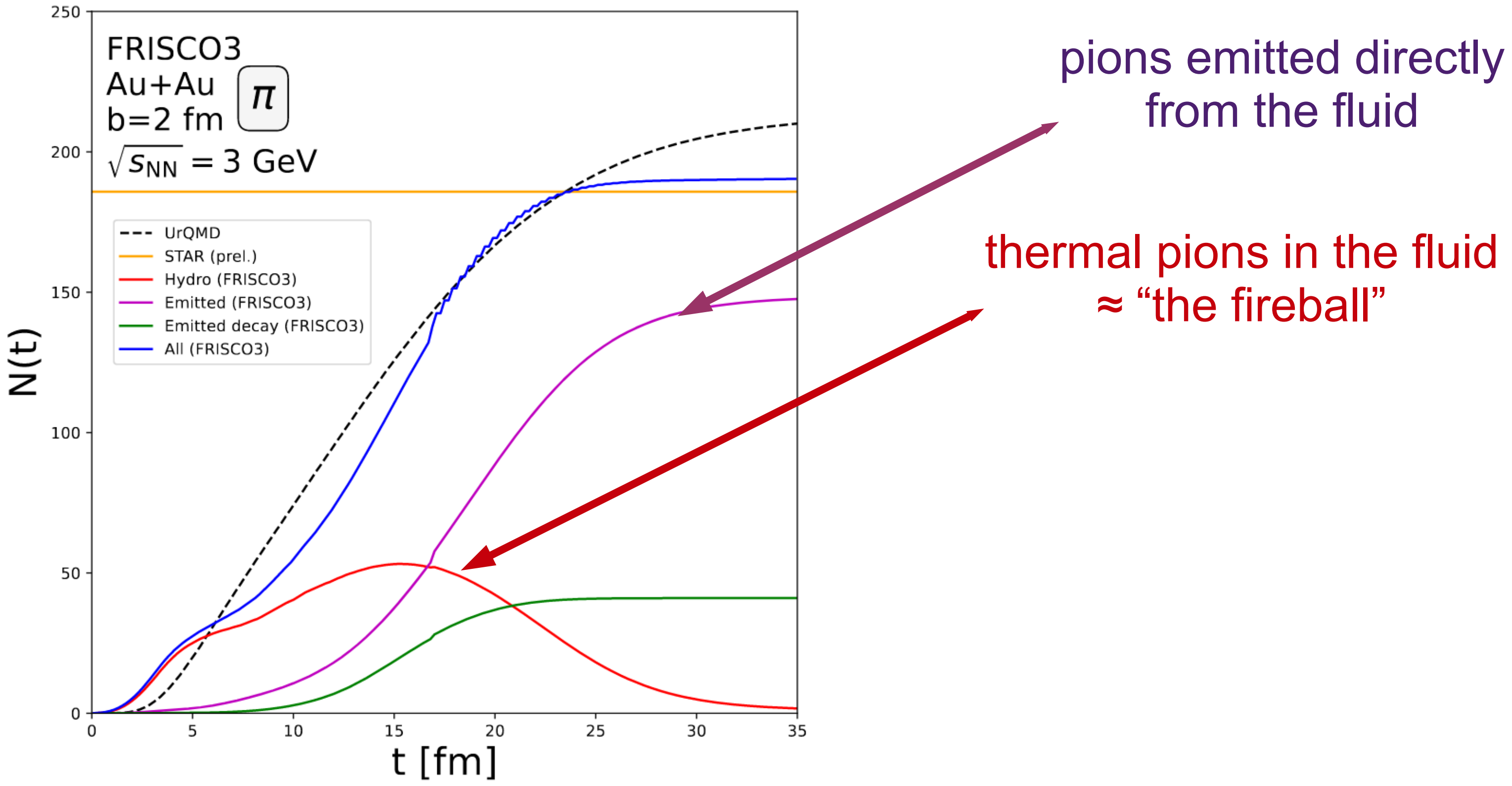
$$\check{F}^\nu_{\rm fl} = -T^{0\nu}_{\rm fl}\Theta(1 - \frac{|y_1 - y_2|}{\Delta y_{\rm th}}) \quad ({\rm fl} = 1, 2)$$

$$\check{S}_{\rm fl} = -N_{\rm fl}\Theta(1 - \frac{|y_1 - y_2|}{\Delta y_{\rm th}}) \quad ({\rm fl} = 1, 2)$$

$$\check{F}^\nu_3 = -(\check{F}^\nu_1 + \check{F}^\nu_2)$$

$$\check{S}_3 = -(\check{S}_1 + \check{S}_2)$$

Time evolution of the fireball in frisco3



- At no point in time the fireball contains more than 1/3 of the final number of pions because of continuous drain (and also due to resonance decay)

Continuous freeze-out

$$\partial_\mu T^{\mu\nu}_{\mathrm{fl}} = \tilde{F}^\nu_{\mathrm{fl}} + \check{F}^\nu_{\mathrm{fl}} + \hat{F}^\nu_{\mathrm{fl}}$$
$$\partial_\mu N^{\mu}_{\mathrm{fl}} = \tilde{S}_{\mathrm{fl}} + \check{S}_{\mathrm{fl}} + \hat{S}_{\mathrm{fl}}$$

- Drain terms as products of local (thermal) emissivity $\mathcal{E}$ of the fluid cells and the probability $\mathcal{P}$ of produced particles to escape the surrounding medium without scattering:

$$\hat{F}^\nu_{\mathrm{fl}} = -p^\nu \mathcal{E}(x,p,t)\mathcal{P}(x,p,t)$$
$$\hat{S}_{\mathrm{fl}} = -\mathcal{E}(x,p,t)\mathcal{P}(x,p,t)$$

(only for fl=3)

- Construct emissivity analogous to a collision term in the relaxation time approxiomation of kinetic Boltzmann-Type models:

$$\mathcal{E}(x,p,t) = \frac{U^\alpha p_\alpha}{p^0} \frac{f_{\mathrm{eq}}(T(x,t),\mu(x,t),U^\alpha(x,t))}{\tau_0}$$

- The relaxation time scale τ_0 depends on the scattering rate and therefore on the particle’s momentum
- Everything above depends on the particle type → sum over all hadrons in the resonance gas

Probability of escape

$$\partial_\mu T^{\mu\nu}_{\mathrm{fl}} = \tilde{F}^{\nu}_{\mathrm{fl}} + \check{F}^{\nu}_{\mathrm{fl}} + \hat{F}^{\nu}_{\mathrm{fl}}$$
$$\partial_\mu N^{\mu}_{\mathrm{fl}} = \tilde{S}_{\mathrm{fl}} + \check{S}_{\mathrm{fl}} + \hat{S}_{\mathrm{fl}}$$

- The probability of escape is determined by the future trajectory of the prospective free hadron through the evolving medium:

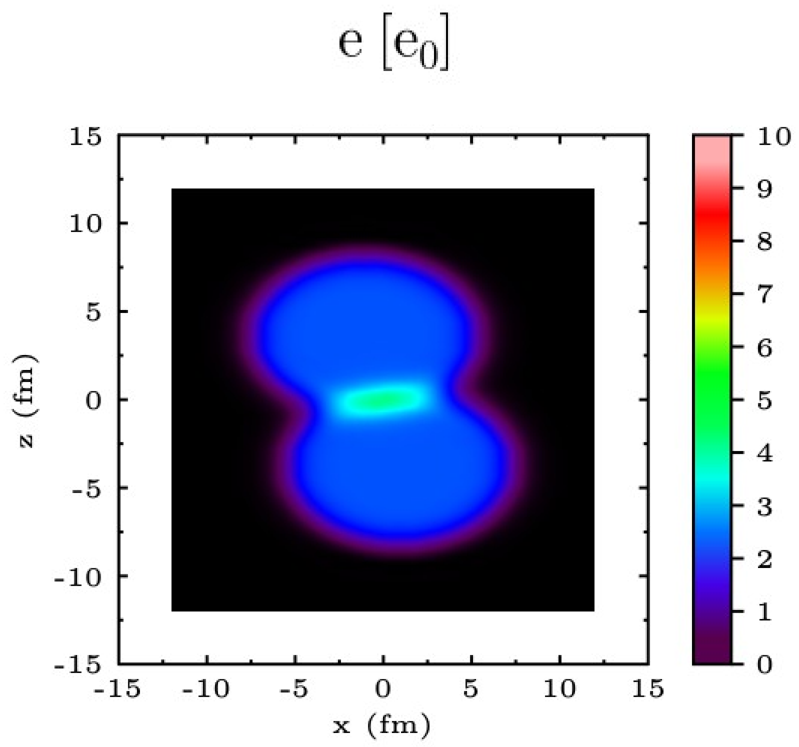
$$\mathcal{P}(x,p,t) = \exp[-\int_t^\infty v_{\mathrm{rel}}(r(t'),p,t')\sigma\rho(r(t'),t')\mathrm{d}t']$$

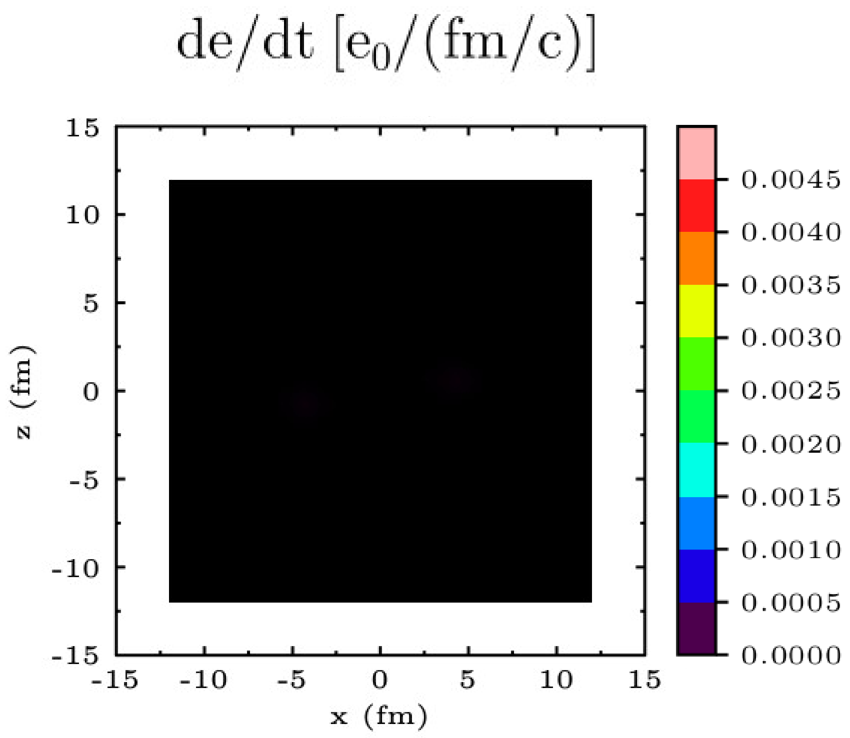
- Energy dependent elementary cross sections of all possible combinations of hadron-hadron interactions must be evaluated! (parametrizations from UrQMD)
- Thermal motion within the medium must not be neglected!
- Simplification: Non-escaping hadrons are assumed to be held back in the emitting fluid cell, when in fact they should reappear at the place and time of absorption (according to the assumed classical propagation)

Qualitative behaviour

central Au+Au at 2 AGeV

reaction plane at t =4 fm/c





energy density (sum of all fluids)

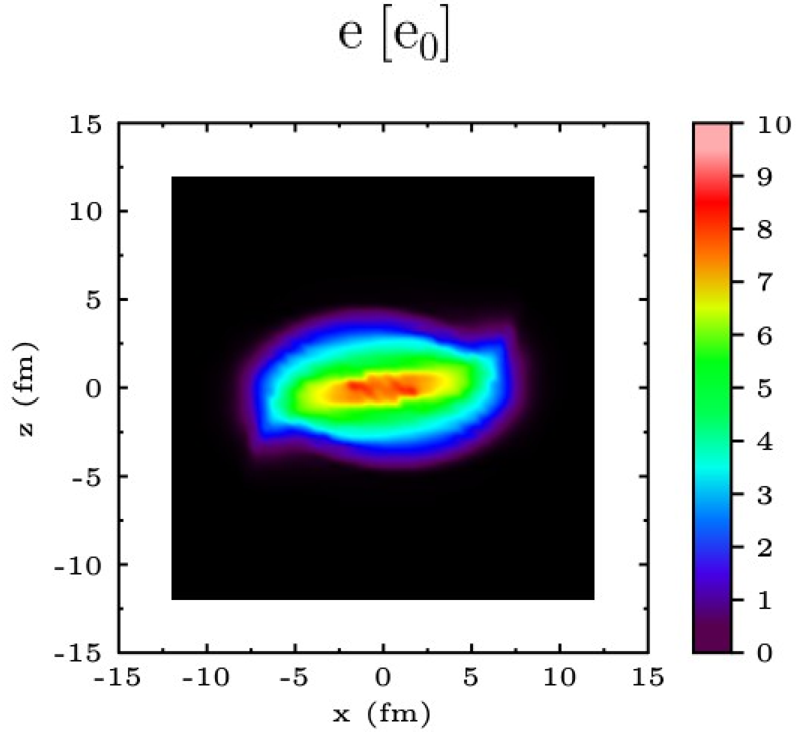
rate of energy density loss

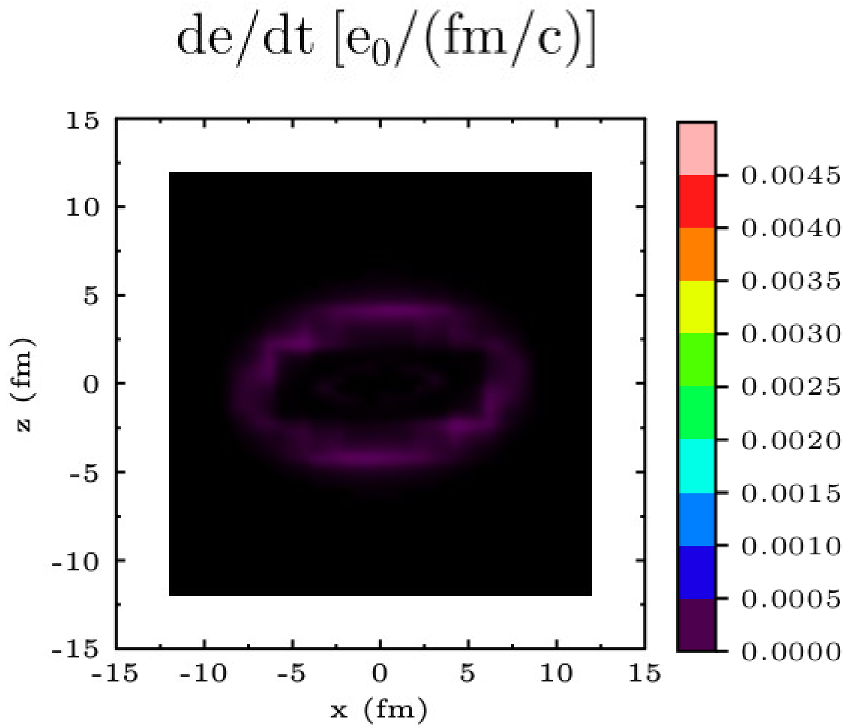
(continuous freeze-out)

Qualitative behaviour

central Au+Au at 2 AGeV

reaction plane at t =10 fm/c

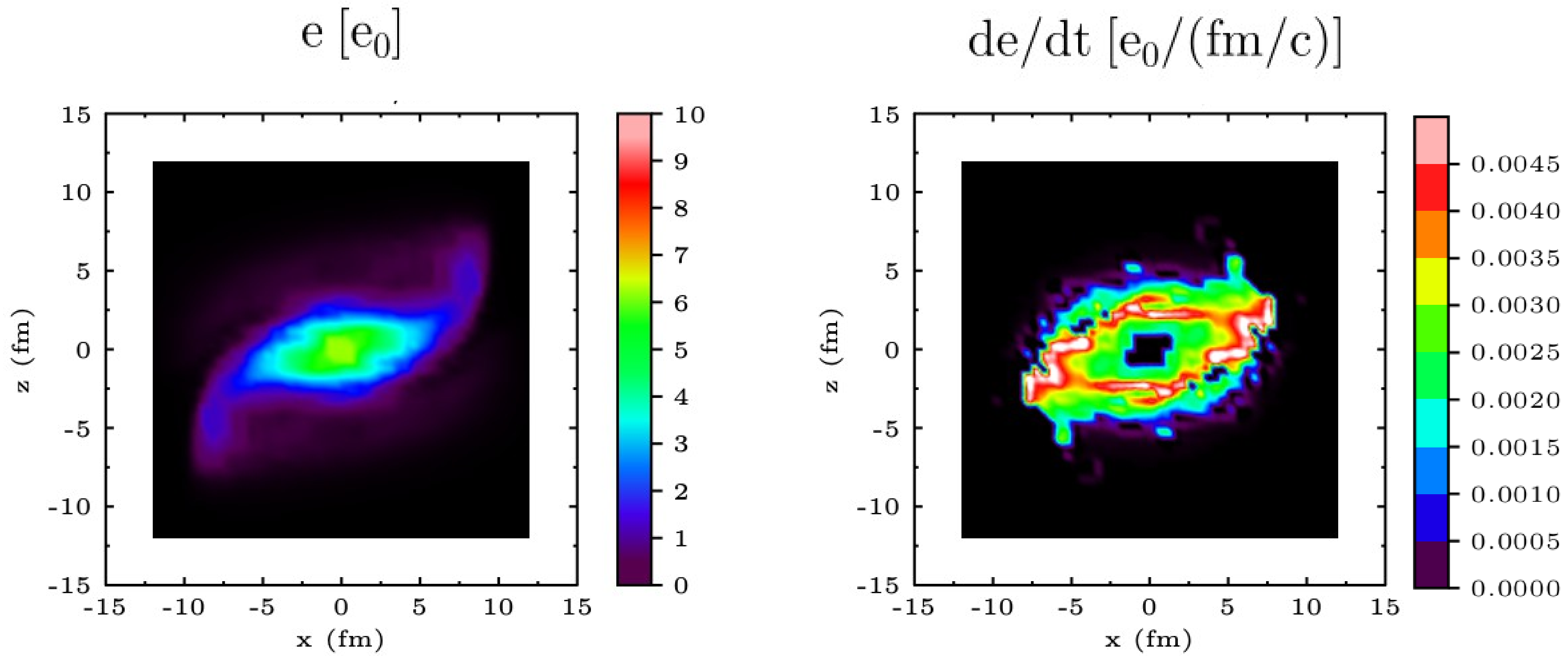




Qualitative behaviour

central Au+Au at 2 AGeV

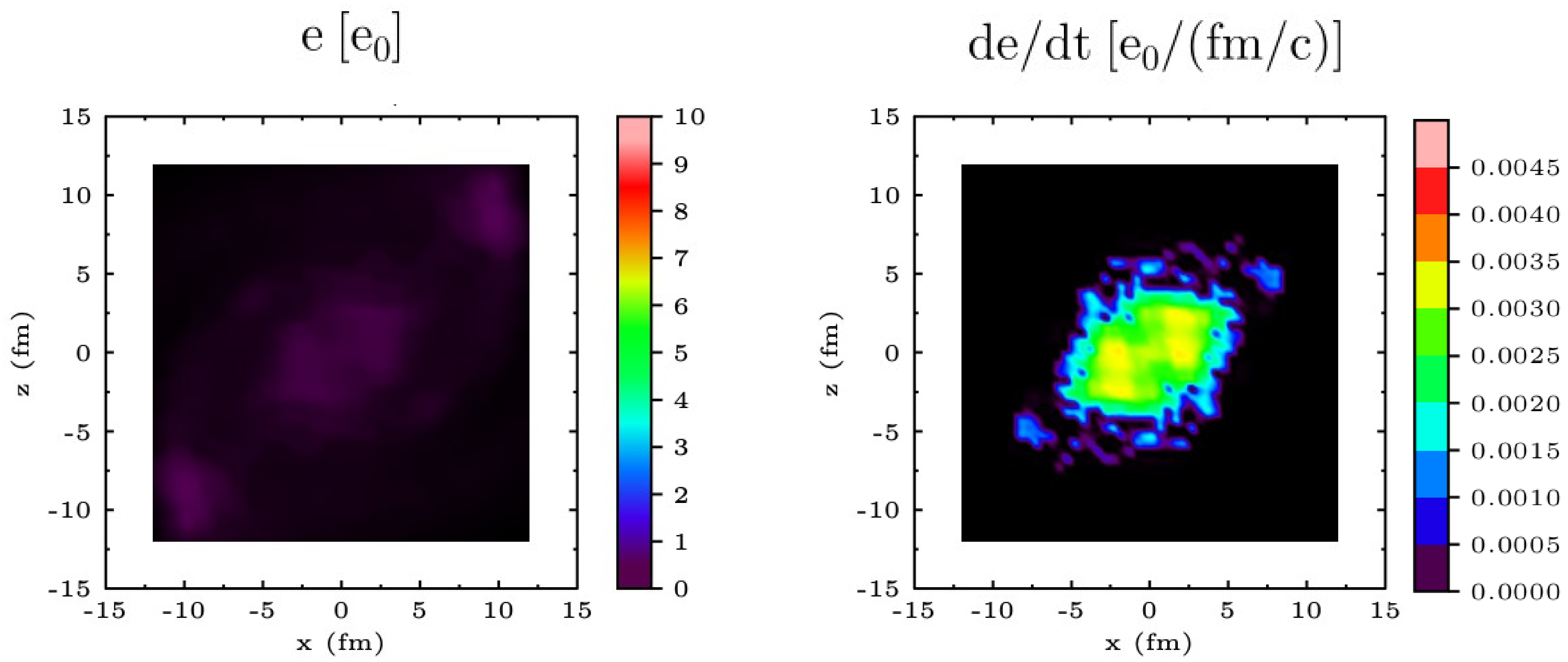
reaction plane at t =16 fm/c



Qualitative behaviour

central Au+Au at 2 AGeV

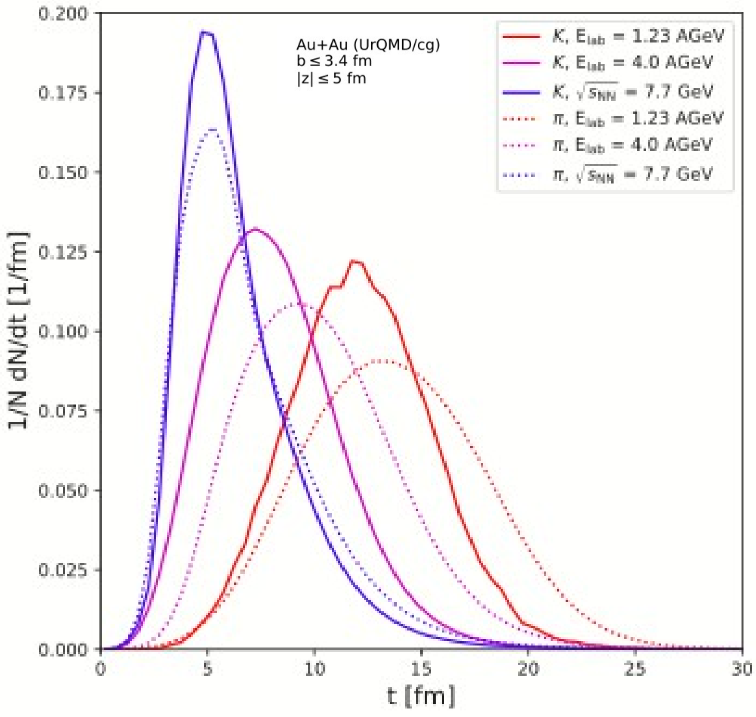
reaction plane at t =22 fm/c

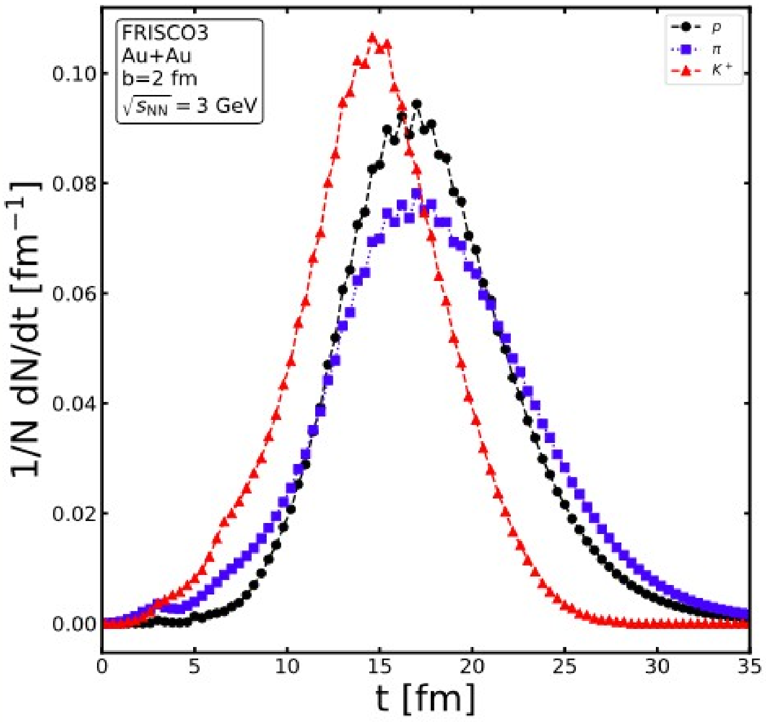


Quantitative behaviour

- Actual freeze-out profiles differ for different hadron types in frisco3 as in UrQMD
- Kaons are emitted at significantly earlier times than pions (on the average)

UrQMD
T.Reichert, G. Inghirami, M. Bleicher
EPJ Web Conf. 259 (2022) 10005

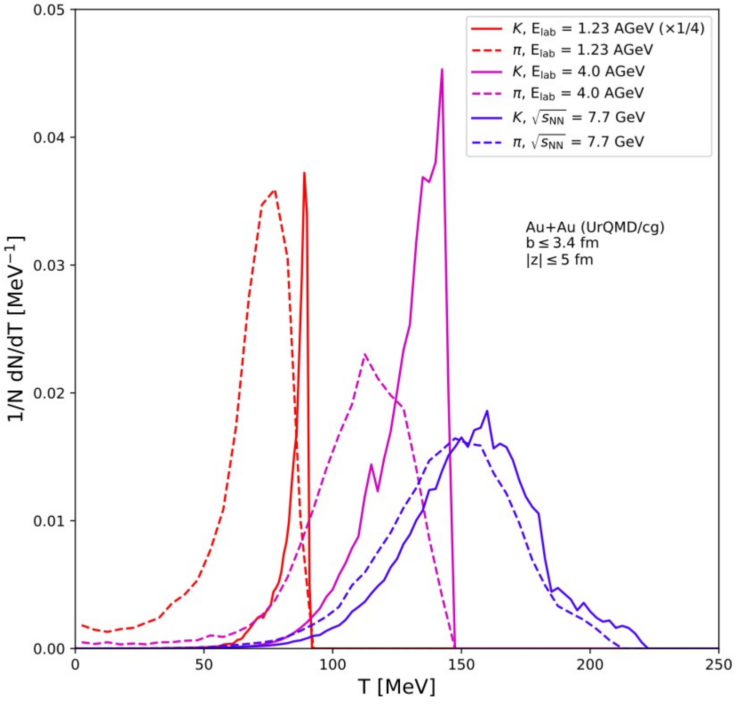


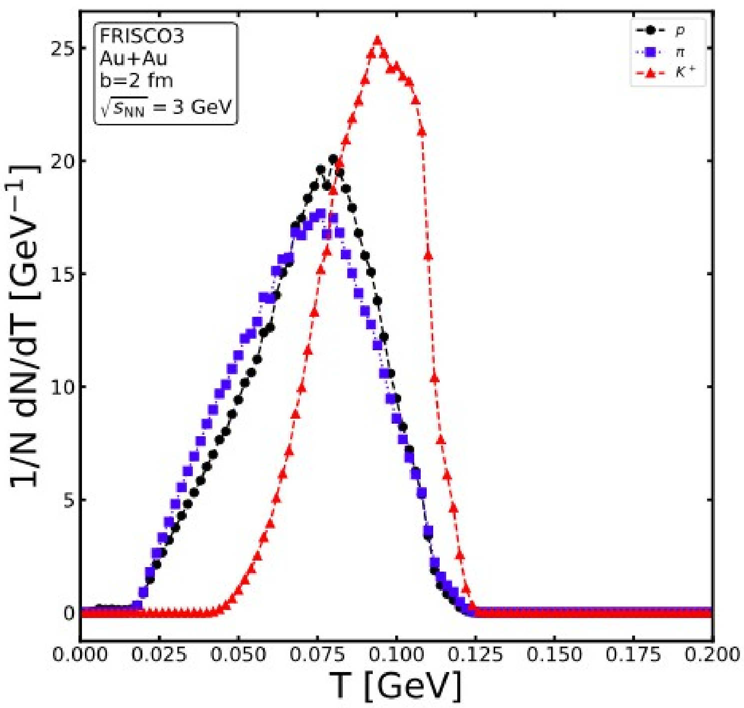


Quantitative behaviour

- Actual freeze-out profiles differ for different hadron types in frisco3 as in UrQMD
- Kaons are emitted at significantly higher temperatures than pions (on the average)

UrQMD
T.Reichert, G. Inghirami, M. Bleicher
EPJ Web Conf. 259 (2022) 10005

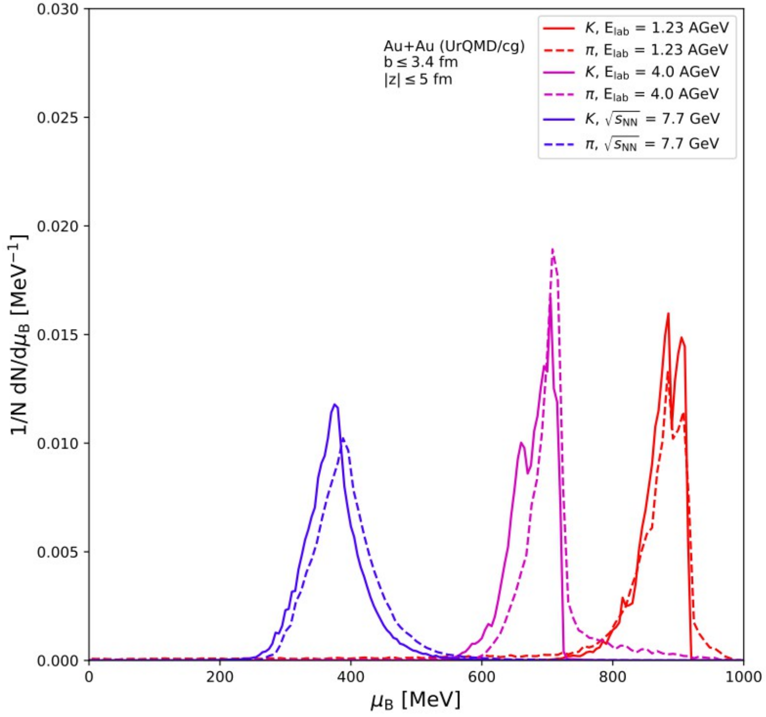


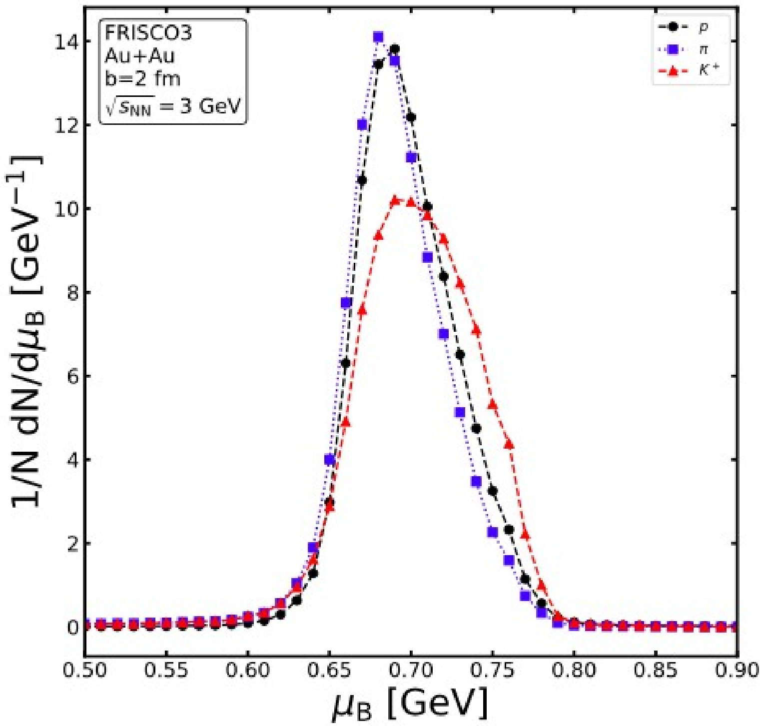


Quantitative behaviour

- Only weak variation of baryochemical potential at freeze-out for different hadron types in frisco3 as in UrQMD (on the average)

UrQMD
T.Reichert, G. Inghirami, M. Bleicher
EPJ Web Conf. 259 (2022) 10005





Challenges to overcome: Challenge 1

$$\partial_\mu T^{\mu\nu}_{\text{fl}} = \tilde{F}^{\nu}_{\text{fl}} + \check{F}^{\nu}_{\text{fl}} + \hat{F}^{\nu}_{\text{fl}}$$
$$\partial_\mu N^{\mu}_{\text{fl}} = \tilde{S}_{\text{fl}} + \check{S}_{\text{fl}} + \hat{S}_{\text{fl}}$$

- How to incorporate information about the future states of the system S(t'>t) in a hydrodynamic transport model of type S(t) → S(t+Δt) ?
- Solution: Precalculation of the complete time evolution of the system
  - i.e. the three fluids' fields of densities, velocities, temperatures, chemical potentials etc. as functions of time - within the same model, but *without the continuous freeze-out term*:

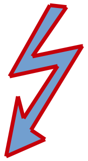
$$\partial_\mu T^{\mu\nu}_{\text{fl}} = \tilde{F}^{\nu}_{\text{fl}} + \check{F}^{\nu}_{\text{fl}} + \hat{F}^{\nu}_{\text{fl}}$$
$$\partial_\mu N^{\mu}_{\text{fl}} = \tilde{S}_{\text{fl}} + \check{S}_{\text{fl}} + \hat{S}_{\text{fl}}$$

→ S*(t') for all t'

Challenges to overcome: Challenge 2

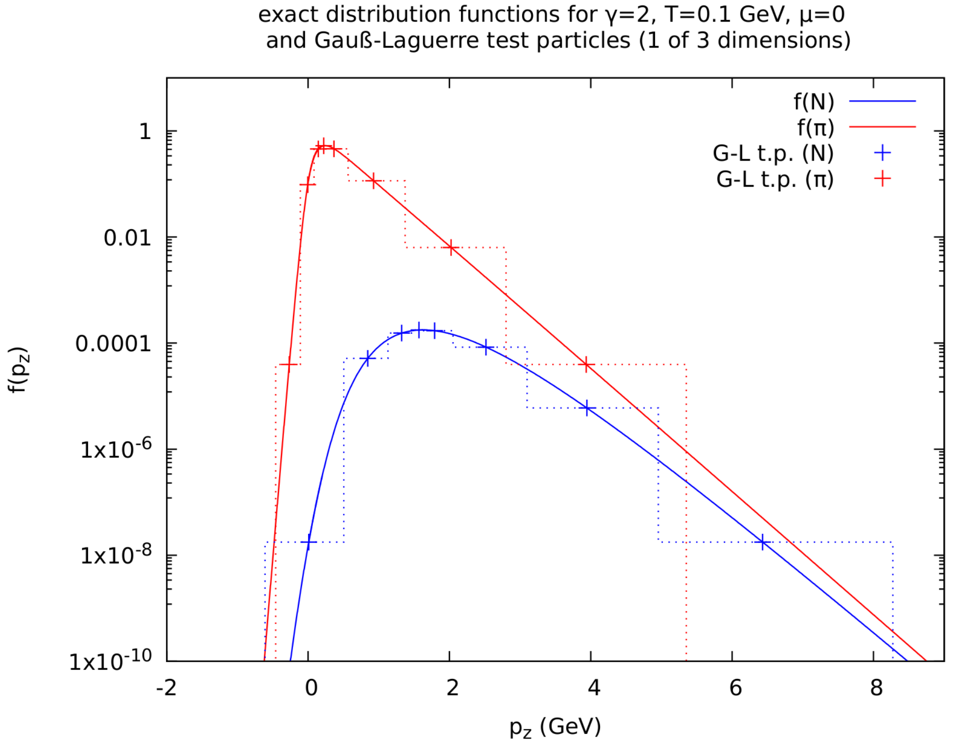
$$\partial_\mu T^{\mu\nu}_{\mathrm{fl}} = \tilde{F}^{\nu}_{\mathrm{fl}} + \check{F}^{\nu}_{\mathrm{fl}} + \hat{F}^{\nu}_{\mathrm{fl}}$$
$$\partial_\mu N^{\mu}_{\mathrm{fl}} = \tilde{S}_{\mathrm{fl}} + \check{S}_{\mathrm{fl}} + \hat{S}_{\mathrm{fl}}$$

- The drain term needs to be calculated differentially over the whole momentum space for each fluid cell and each time step and for each hadron type
- Numerical standard integration over the convoluted functions of thermal probability distributions, the particle trajectories and the spatio-temporal density profile of the medium cannot be done in reasonable time
- A Monte-Carlo sampling ansatz leads to inevitable fluctuations and discontinuities within the remaining fluids → hydro fails numerically!



- Solution: Represent the hadrons' distribution functions in the computational frame by a limited (fixed) number of test particles on an adaptive grid in momentum space

Reducing test particle numbers: Gauss-Laguerre test particle weights

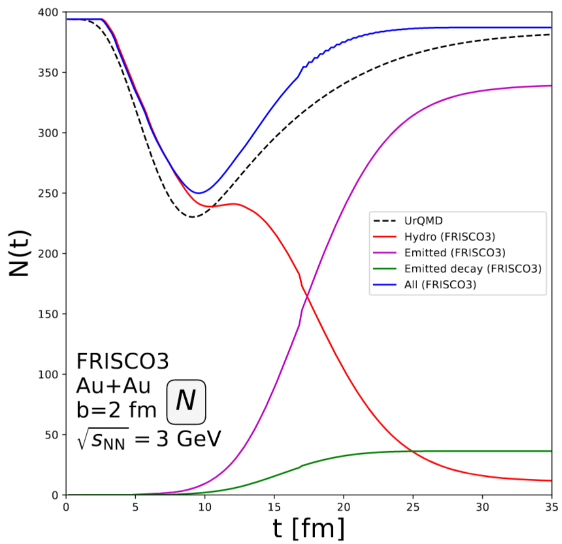


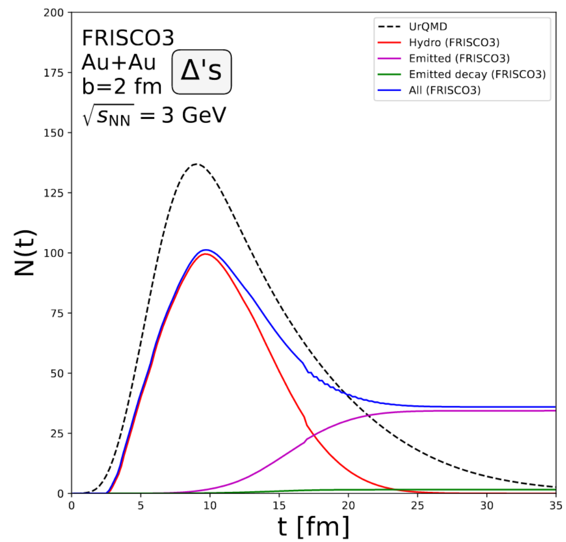
$$\int_0^{+\infty} e^{-x} f(x)\, dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

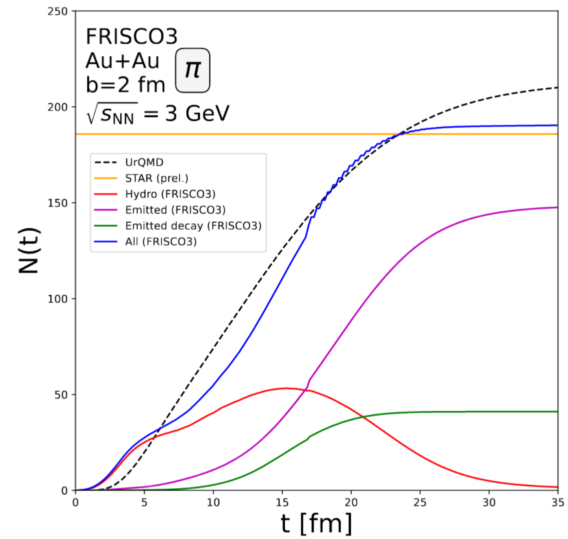
$$w_i = \frac{x_i}{(n+1)^2 [L_{n+1}(x_i)]^2}$$

- Freeze-out integral uses weighted test particle method
- Using MC technique would need 10^6 test particles for 1 % accuracy
- GL weighted test particles allow to reduce to 8*8*8=512 G-L test particles with 0.1 % accuracy
- Dramatic speed-up of calculation

Comparison of the time evolution of particle yields: UrQMD vs. Frisco3 - non-strange particles

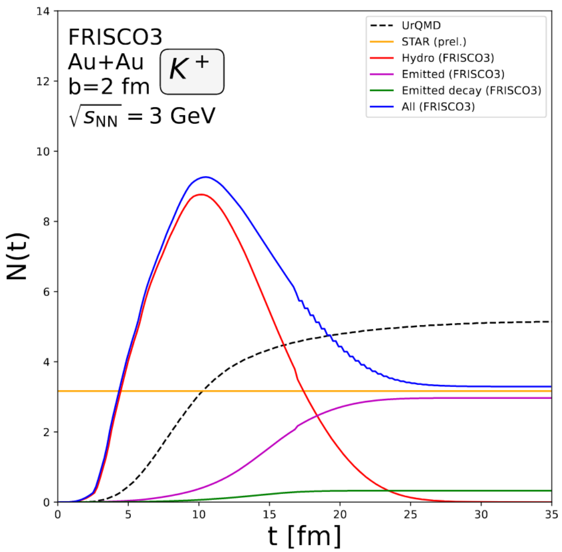


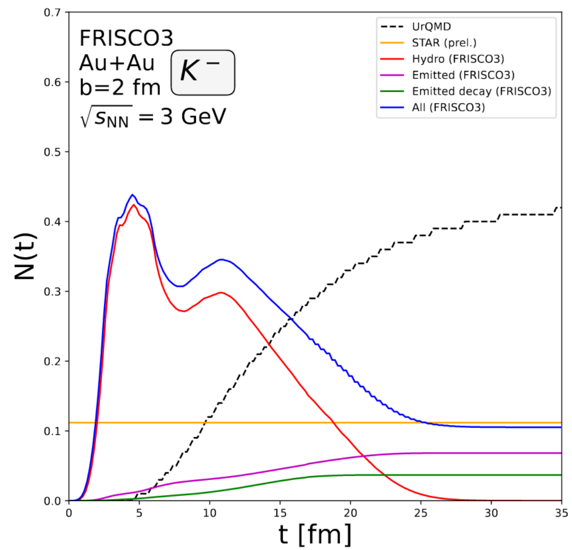


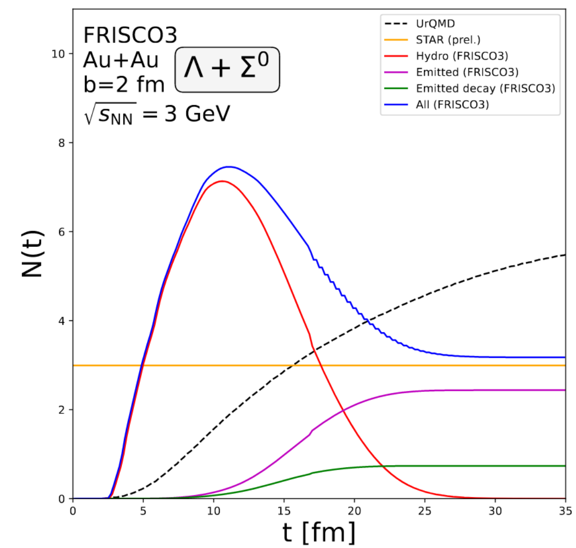


- Frisco3 yields similar time evolution to UrQMD → similar T, μ_B evolution
- Shuffling of baryon number between nucleons and resonances
- Similar pion production indicates similar entropy production

Comparison of the time evolution of particle yields: UrQMD vs. Frisco3 - strange particles

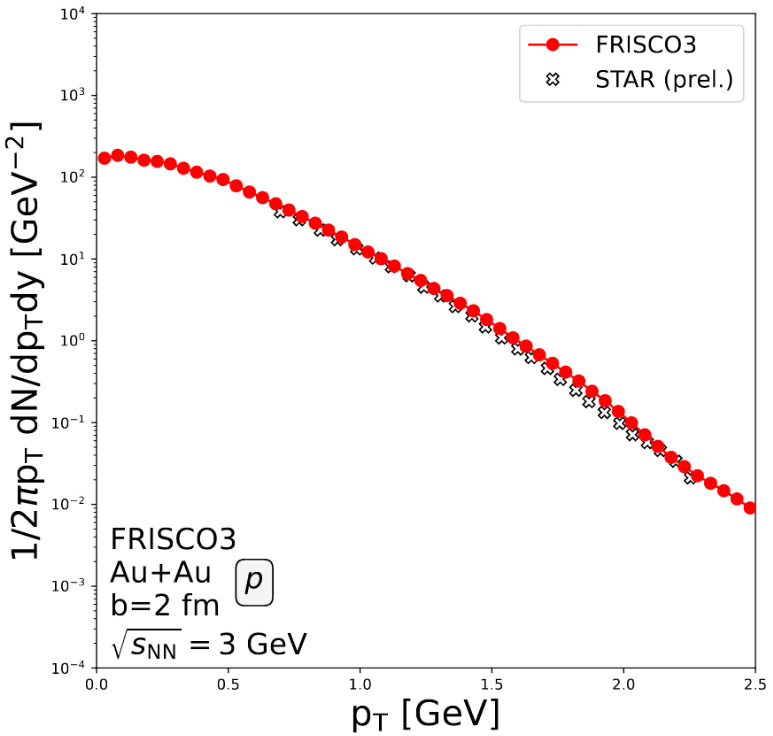


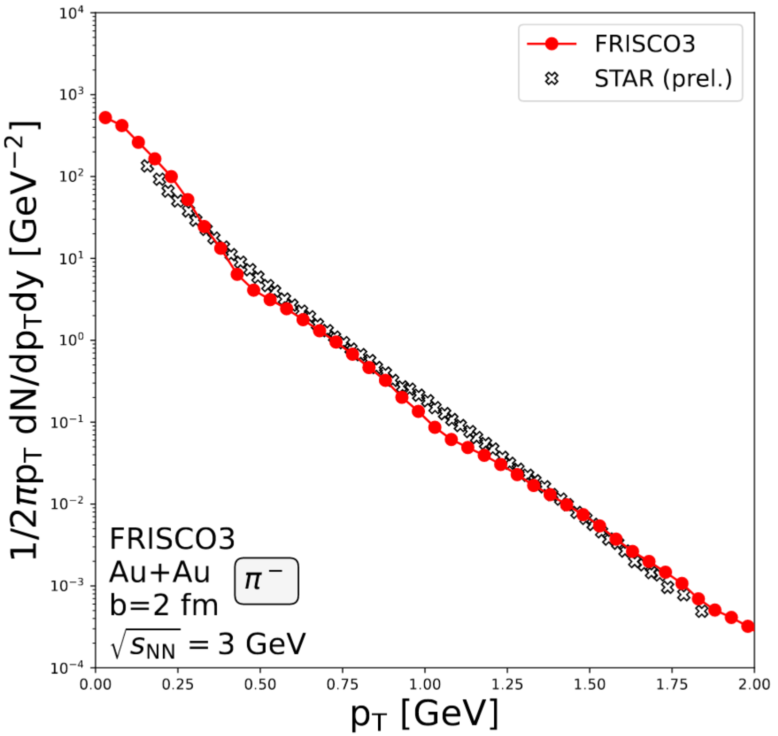




- Frisco3 yields different time evolution to UrQMD for strange particles
  ⟶ still similar T, μ_B evolution
  ⟶ BUT: local strangeness equilibration (Frisco3) or not (UrQMD)?
- At the moment no strangeness current in Frisco3, only local thermal strangeness densities with <s-sbar>=0

First results - particle spectra

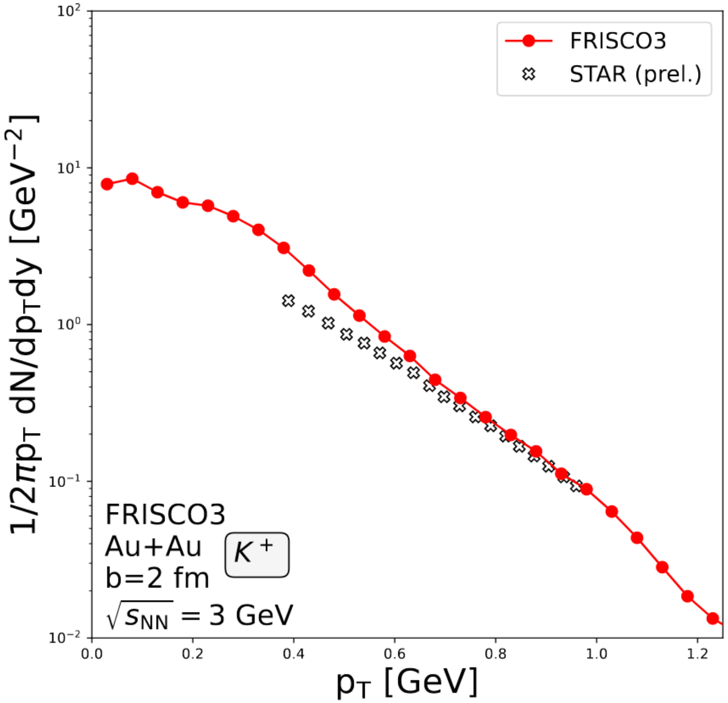


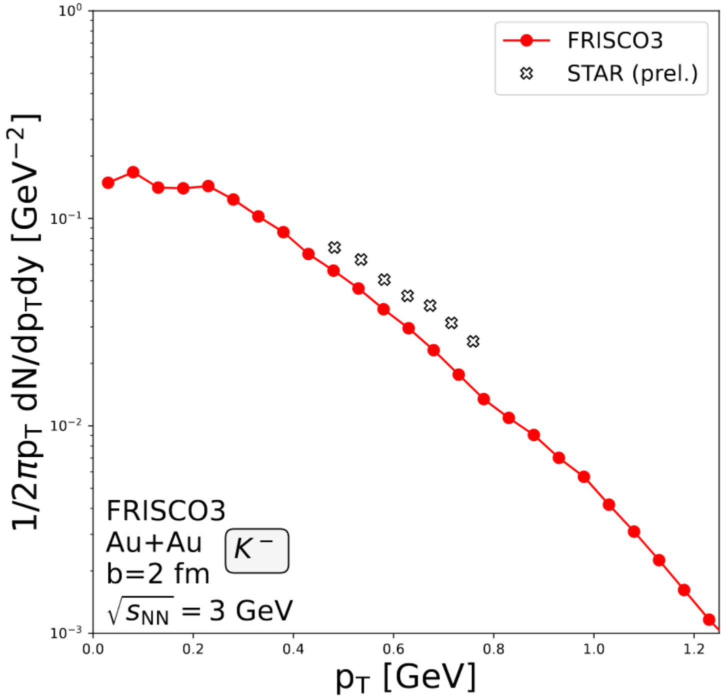


- General expansion dynamics of nucleons and pions looks ok
- General multiplicities look ok

- Remember: Different decoupling times pick up different flow velocities!

Particle spectra - kaons

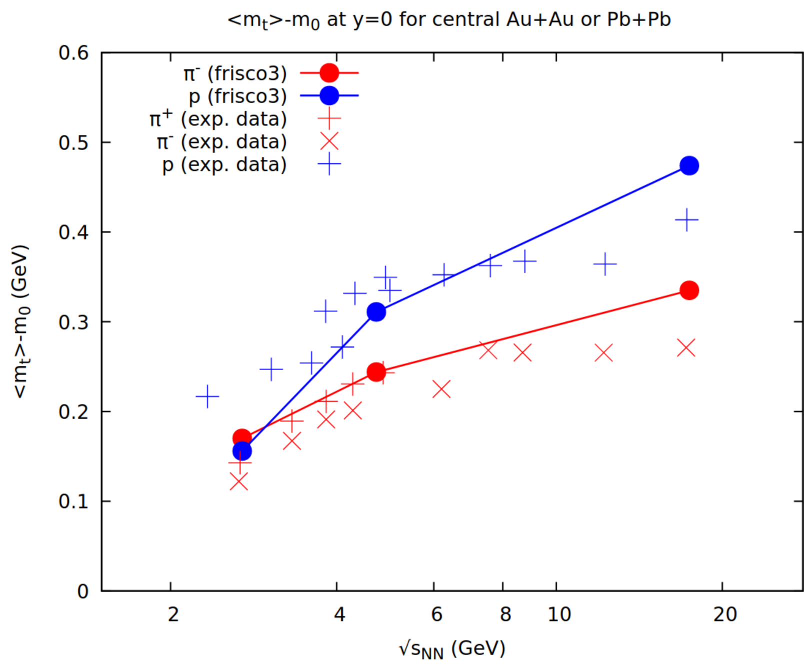


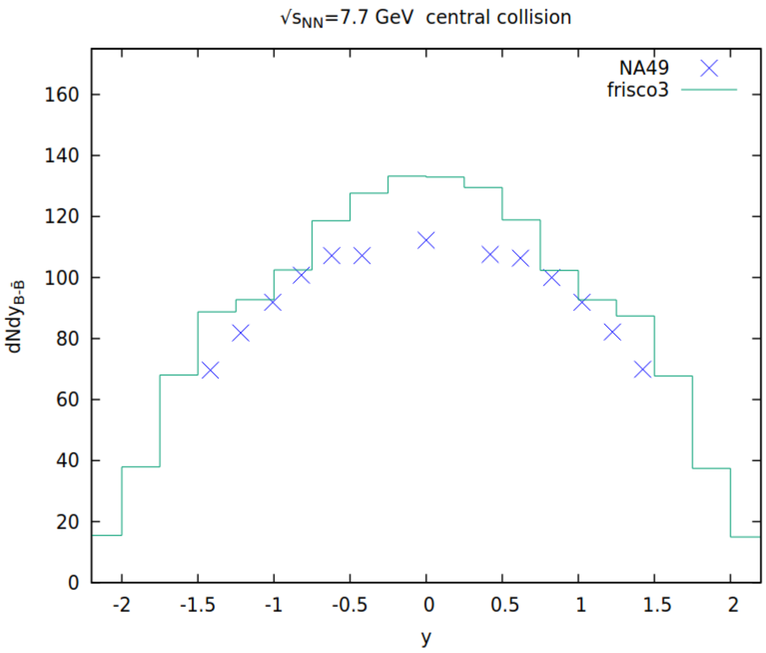


- General expansion dynamics of (anti-)kaons looks ok
- Remember: Different decoupling times pick up different flow velocities!

- More detailed tests are coming

Preliminary results at higher energies





- First results show reasonable transverse momenta and baryon stopping

Summary

- Problems of current hydrodynamics approaches:
  - Fixed time initial state
  - Unclear distribution of initial energy density and baryon density in space
  - Simplified freeze-out using fixed ε, T… criterion and Cooper-Frye
  - No backreaction from freeze-out to fluid evolution
- To overcome these problems we developed a new multi-fluid approach to describe heavy-ion collisions consistently from the initial state (cold nuclei) through the reaction to the final state of free streaming hadrons:
  - Friction and unification for the dynamical creation of the fireball fluid
  - Replacement of the simple Cooper-Frye freeze-out using a decoupling based on the scattering rate (dependent on hadron species and momentum)
  - Space-time dependent backreaction of the freeze-out on the fluid-system is taken into account
  - Total energy, momentum and baryon number is is exactly conserved at all times