

cost
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

HFHF Helmholtz
Forschungsakademie
Hessen für FAIR

CRC-TR 211
HIC for **FAIR**
Helmholtz International Center

GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

DFG Deutsche
Forschungsgemeinschaft

DAAD

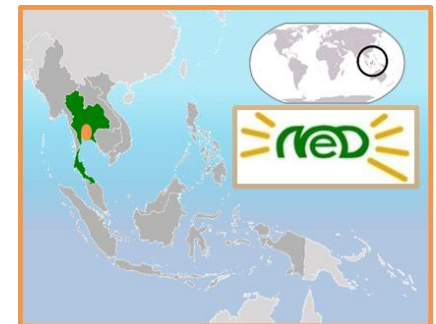
STRONG
2020

Properties of strongly interacting matter

Elena Bratkovskaya
(GSI, Darmstadt & Uni. Frankfurt)
for the **PHSD** group

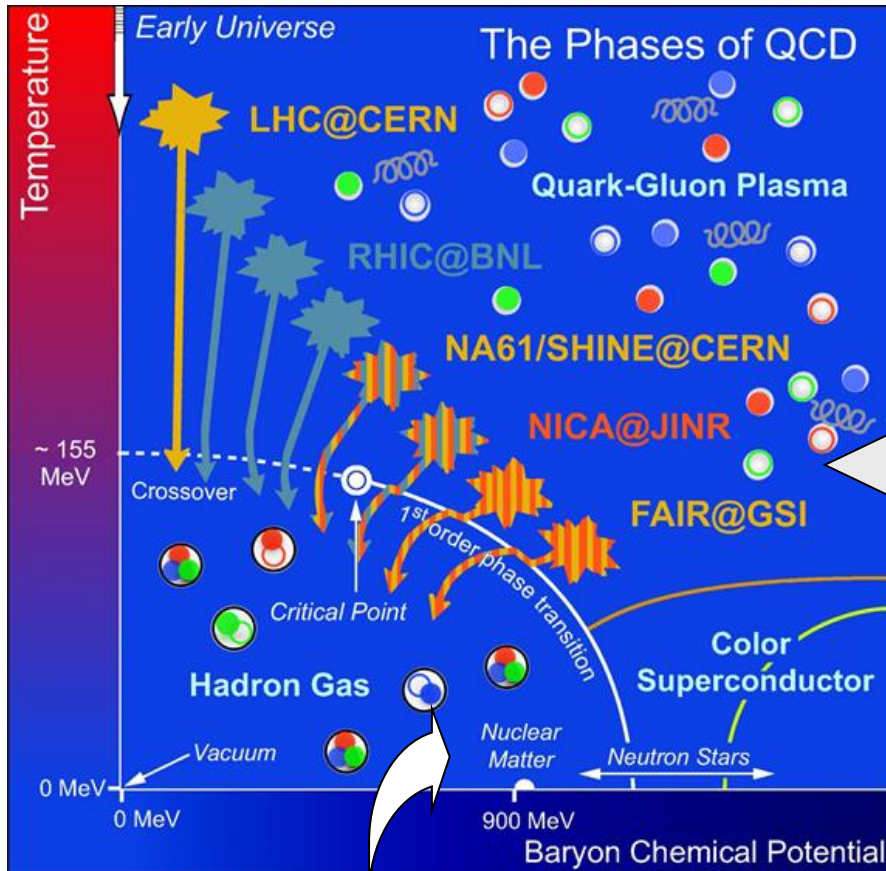


9th International Symposium on Non-equilibrium
Dynamics (NeD-2022)
28 November - 2 December, 2022
Krabi, Thailand



The ,holy grail' of heavy-ion physics:

The phase diagram of QCD → thermal properties of QCD in the (T, μ_B) plain



- **Equation-of-State** of hot and dense matter?
- Study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma**



- Search for a **critical point**
- Search for signatures of **chiral symmetry restoration**
- Study of the **in-medium properties of hadrons** at high baryon density and temperature

Dynamical Models → PHSD

The goal:

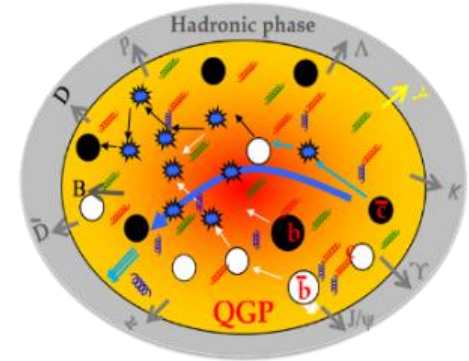
to describe the dynamics of hadrons and partons in all phases of HICs on a **microscopic basis**

Realization:

a **dynamical non-equilibrium transport approach**

- applicable for **strongly interacting systems**,
- which includes a **phase transition** from hadronic matter to QGP

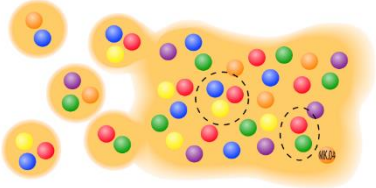
The tool: PHSD approach



- Baryons
- Antibaryons
- Mesons
- Quarks
- Gluons

$\text{Au} + \text{Au} \sqrt{s_{\text{NN}}} = 200 \text{ GeV}$





Degrees-of-freedom of QGP

For the microscopic transport description of the system one **needs to know all degrees of freedom** as well as their properties and interactions!

❖ IQCD gives QGP EoS at finite μ_B



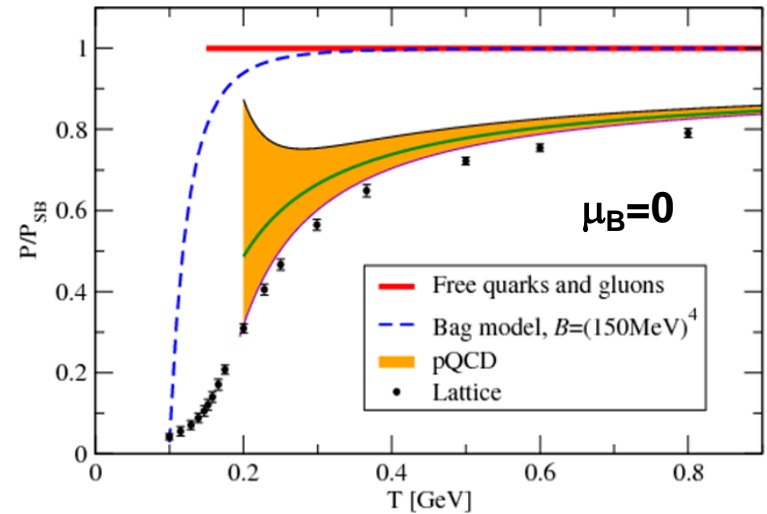
! need to be interpreted in terms of degrees-of-freedom

pQCD:

- weakly interacting system
- massless quarks and gluons

How to learn about the degrees-of-freedom of the QGP from HICs?

- ➔ microscopic transport approaches
- ➔ comparison to HIC experiments



Non-perturbative QCD ← pQCD

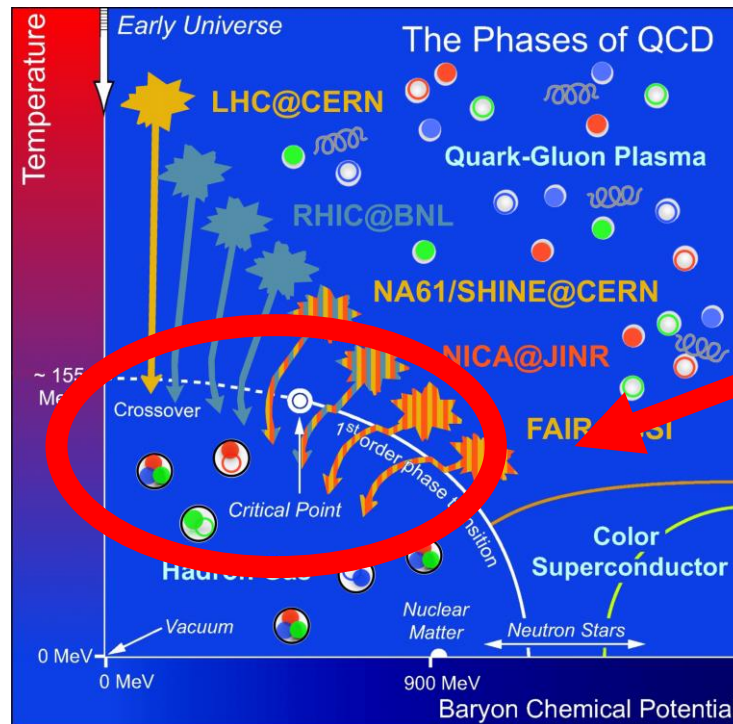
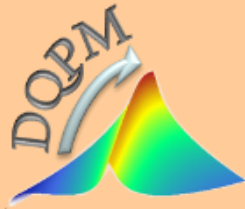
Thermal QCD

= QCD at high parton densities:

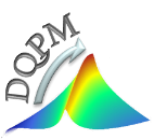
- strongly interacting system
- massive quarks and gluons
- ➔ quasiparticles
- = effective degrees-of-freedom

Thermal QCD →

DQPM (T, μ_q)



finite T, μ_q



Dynamical QuasiParticle Model (DQPM)

DQPM – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **IQCD EoS**

Degrees-of-freedom: strongly interacting **dynamical quasiparticles** - quarks and gluons

Theoretical basis :

□ ,resummed‘ single-particle Green’s functions → quark (gluon) propagator (2PI) :

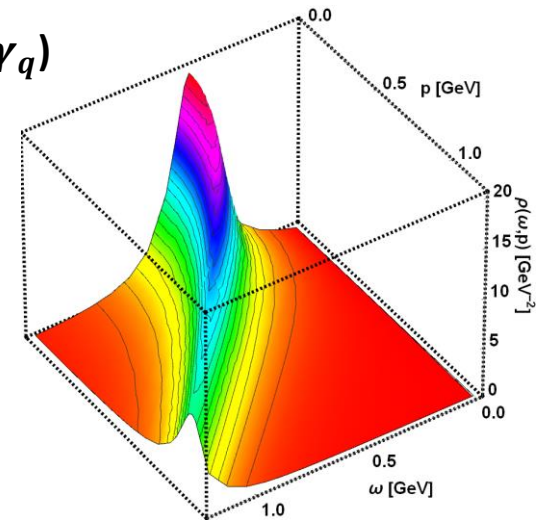
$$\begin{aligned} \text{gluon propagator: } \Delta^{-1} = P^2 - \Pi \quad & \& \quad \text{quark propagator } S_q^{-1} = P^2 - \Sigma_q \\ \text{gluon self-energy: } \Pi = M_g^2 - i2\gamma_g\omega \quad & \& \quad \text{quark self-energy: } \Sigma_q = M_q^2 - i2\gamma_q\omega \end{aligned}$$

Properties of the quasiparticles are specified by scalar **complex self-energies:**

$Re\Sigma_q$: **thermal masses** (M_g, M_q); $Im\Sigma_q$: **interaction widths** (γ_g, γ_q)

→ spectral functions $\rho_q = -2ImS_q$ → Lorentzian form:

$$\begin{aligned} \rho_j(\omega, \mathbf{p}) &= \frac{\gamma_j}{\tilde{E}_j} \left(\frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2} \right) \\ &\equiv \frac{4\omega\gamma_j}{(\omega^2 - \mathbf{p}^2 - M_j^2)^2 + 4\gamma_j^2\omega^2} \quad \tilde{E}_j^2(\mathbf{p}) = \mathbf{p}^2 + M_j^2 - \gamma_j^2 \end{aligned}$$



Parton properties

- Modeling of the quark/gluon **masses** and **widths** (ansatz inspired by HTL calculations)

Masses:

$$M_{q(\bar{q})}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} g^2(T, \mu_B) \left(T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

$$M_g^2(T, \mu_B) = \frac{g^2(T, \mu_B)}{6} \left(\left(N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

Widths:

$$\gamma_{q(\bar{q})}(T, \mu_B) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2(T, \mu_B) T}{8\pi} \ln \left(\frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

$$\gamma_g(T, \mu_B) = \frac{1}{3} N_c \frac{g^2(T, \mu_B) T}{8\pi} \ln \left(\frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

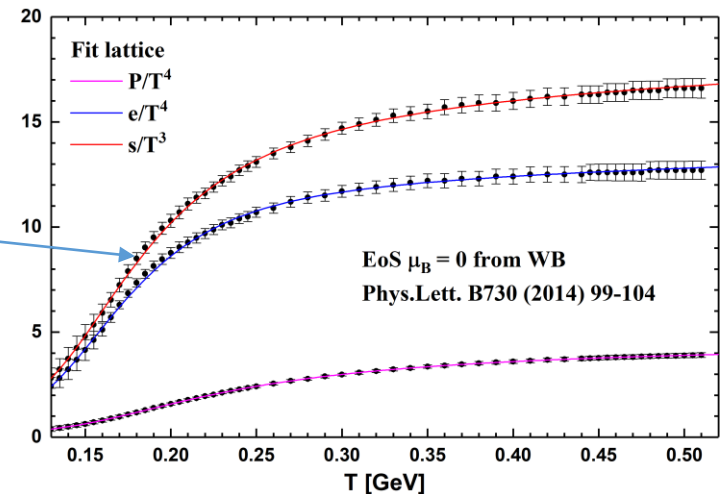
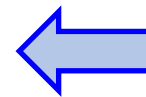
→ **DQPM :**

only **one parameter** ($c = 14.4$)
+ (T, μ_B) - dependent **coupling constant** has to be determined from lattice results

- **Coupling g:** input - IQCD **entropy density s** function of T at $\mu_B=0$

$$g^2(s/s_{SB}) = d \left((s/s_{SB})^e - 1 \right)^f$$

$$s_{SB}^{QCD} = 19/9 \pi^2 T^3$$



DQPM at finite (T, μ_q) : scaling hypothesis

- Scaling hypothesis for the effective temperature T^* for $N_f = N_c = 3$

W. Cassing, NPA 791 (2007) 365

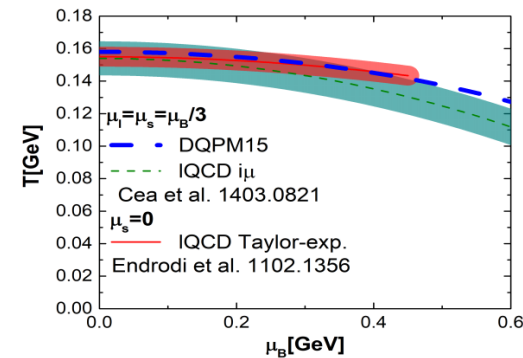
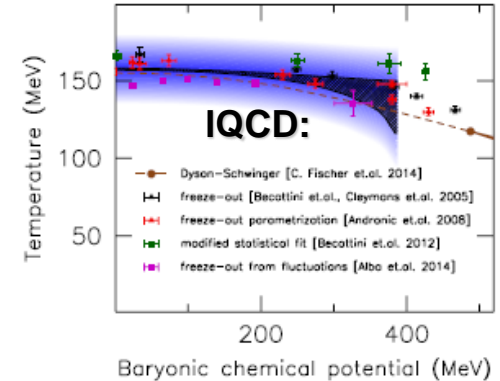
$$\mu_u = \mu_d = \mu_s = \mu_q$$

$$T^{*2} = T^2 + \frac{\mu_q^2}{\pi^2}$$

- Coupling:

$$g(T/T_c(\mu=0)) \longrightarrow g(T^*/T_c(\mu))$$

- Critical temperature $T_c(\mu_q)$ in crossover region: obtained by assuming a constant energy density ε along a critical line $T=T_c(\mu_q)$, where ε at $T_c(\mu_q=0)=156$ GeV is fixed by IQCD at $\mu_q=0$



$$\frac{T_c(\mu_q)}{T_c(\mu_q=0)} = \sqrt{1 - \alpha \mu_q^2} \approx 1 - \alpha/2 \mu_q^2 + \dots$$

$$\alpha \approx 8.79 \text{ GeV}^{-2}$$

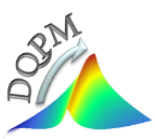
! Consistent with lattice QCD:

IQCD: C. Bonati et al., PRC90 (2014) 114025

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c} \right)^2 + \dots$$

$$\text{IQCD } \kappa = 0.013(2) \longleftrightarrow \kappa_{DQPM} \approx 0.0122$$

H. Berrehrah et al, PRC 93 (2016) 044914, Int.J.Mod.Phys. E25 (2016) 1642003,



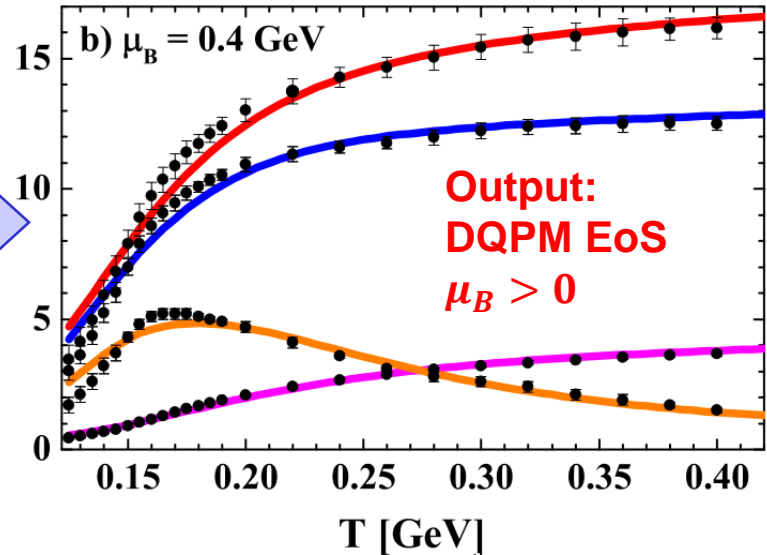
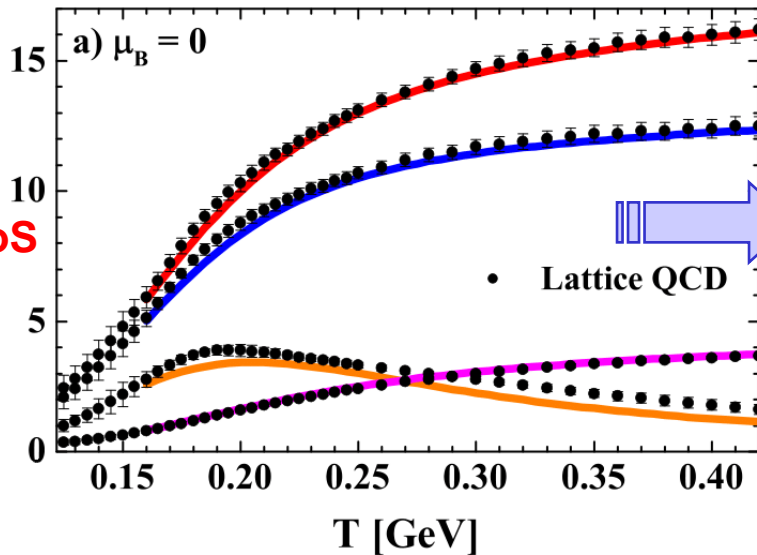
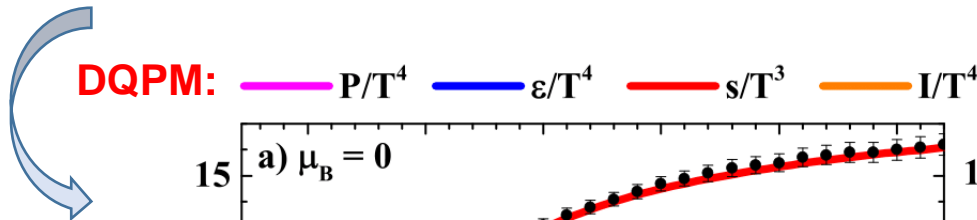
DQPM thermodynamics at finite (T, μ_q)

- Entropy and baryon density in the quasiparticle limit (G. Baym 1998):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[d_q \frac{\partial n_B}{\partial T} (\text{Im}(\ln -\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta) \right. \\ \left. + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[\sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

B. Vanderheyden, G. Baym, J. Stat. Phys. 93 (1998) 843
Blazot, Iancu, Rebhan, Phys. Rev. D 63 (2001) 065003

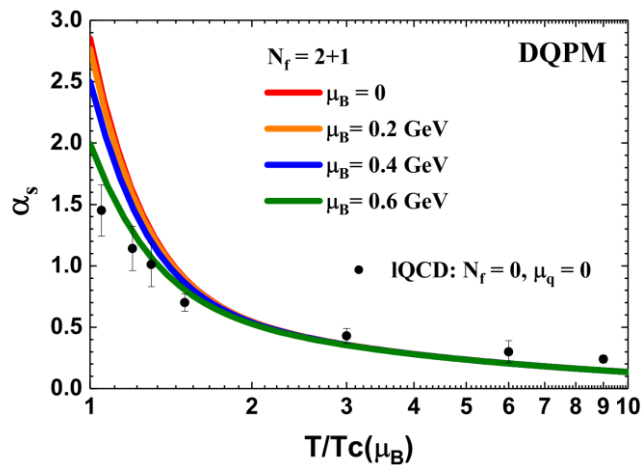


Input:
lattice EoS
 $\mu_B = 0$

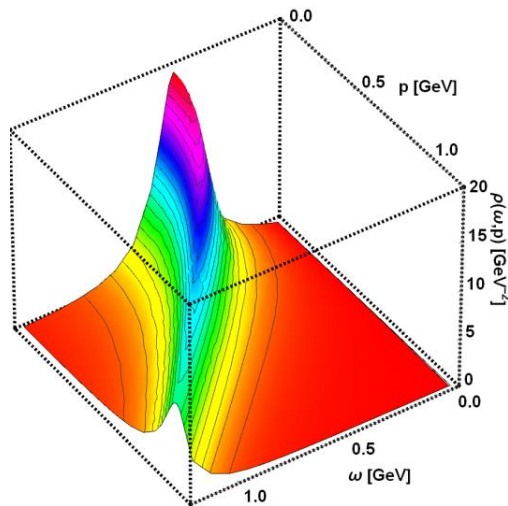
Output:
DQPM EoS
 $\mu_B > 0$

DQPM: parton properties

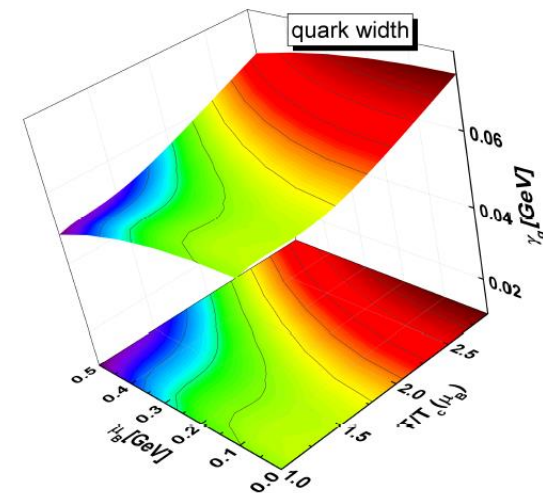
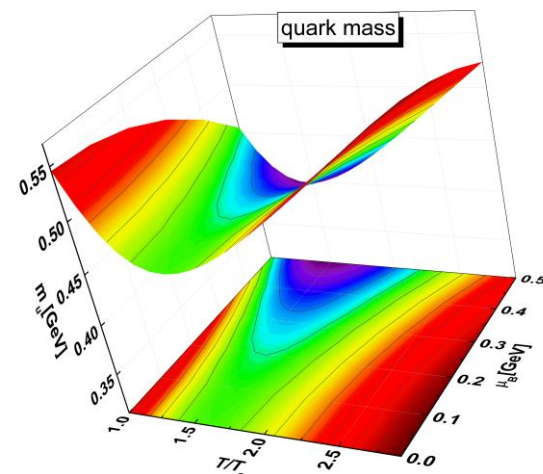
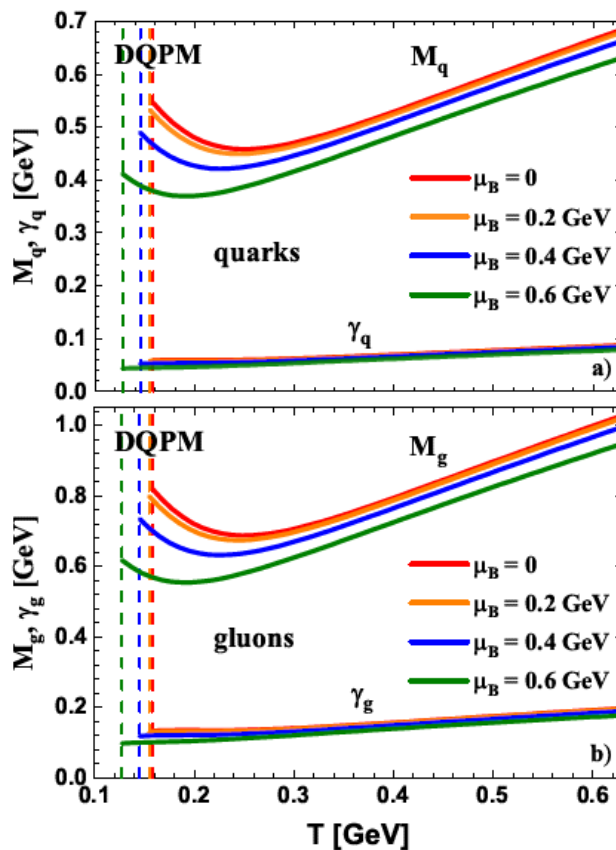
Coupling as a function of (T, μ_B)



→ Lorentzian spectral function:



Pole masses and widths vs (T, μ_B)



Partonic interactions: matrix elements

DQPM partonic cross sections \rightarrow **leading order diagrams**

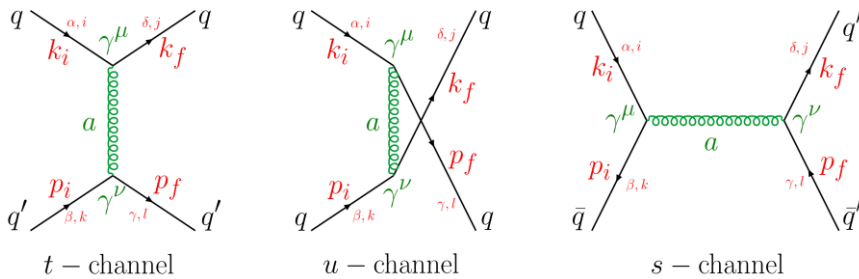
Propagators for massive bosons and fermions:

$$\frac{\mu, a \quad \nu, b}{q} = -i\delta_{ab} \frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

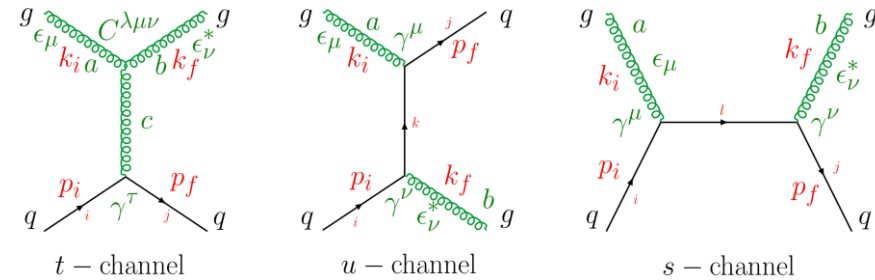
(Quasi-) elastic channels:

$$\begin{array}{c} i \\ \longrightarrow \\ q \end{array} \begin{array}{c} j \\ \longrightarrow \\ q \end{array} = i\delta_{ij} \frac{q + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

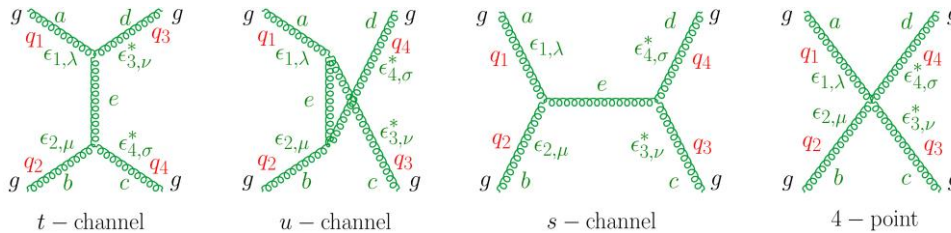
qq' \rightarrow qq' scattering



gq \rightarrow gq scattering



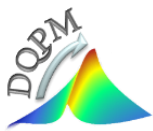
gg \rightarrow gg scattering



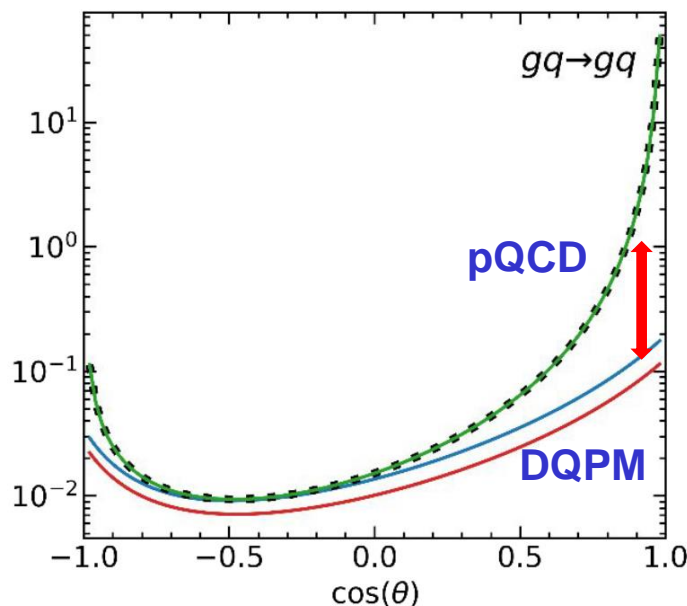
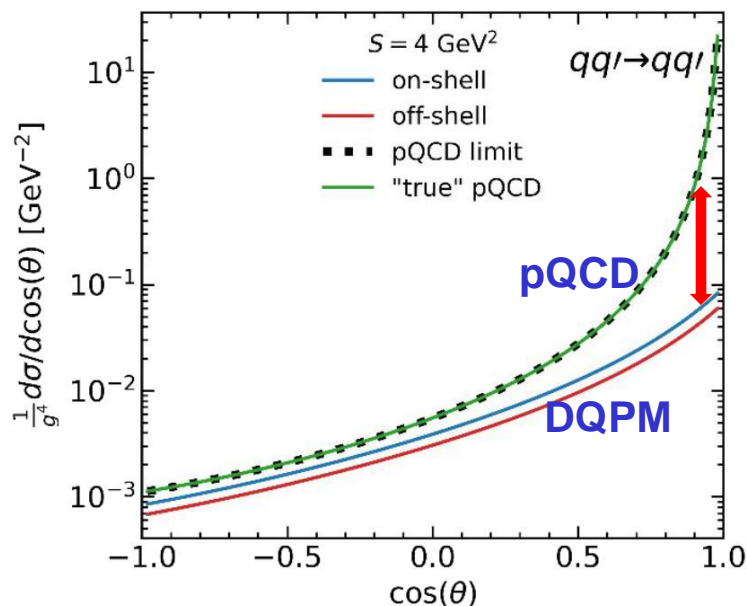
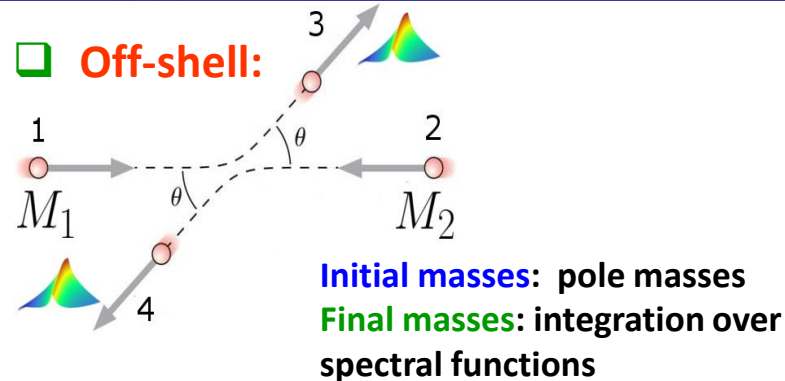
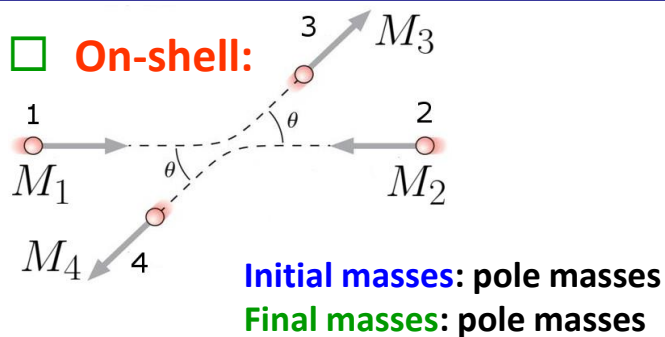
Inelastic channels:

$$\begin{aligned} q + \bar{q} &\rightarrow g \\ g &\rightarrow q + \bar{q} \end{aligned}$$





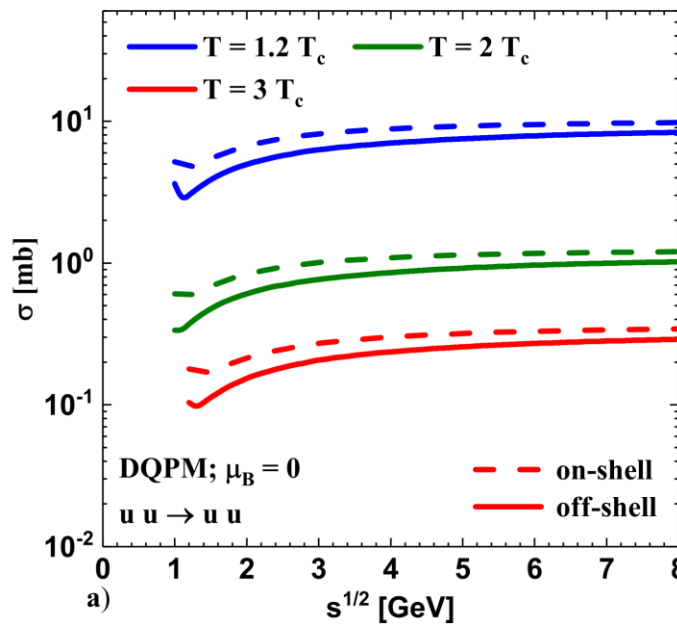
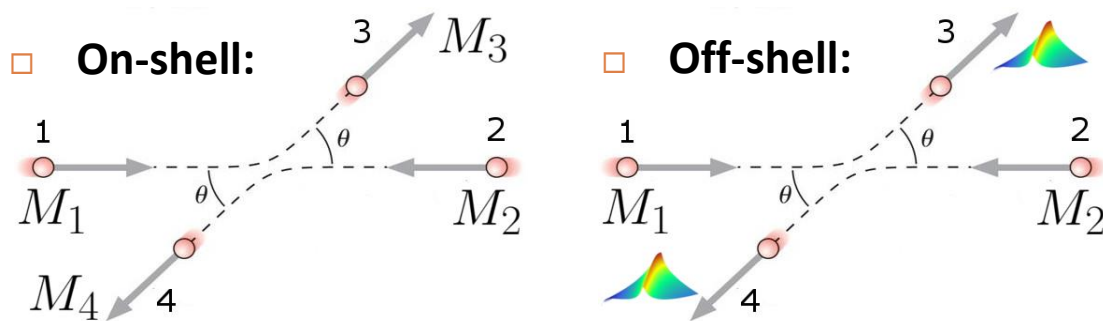
Differential cross sections



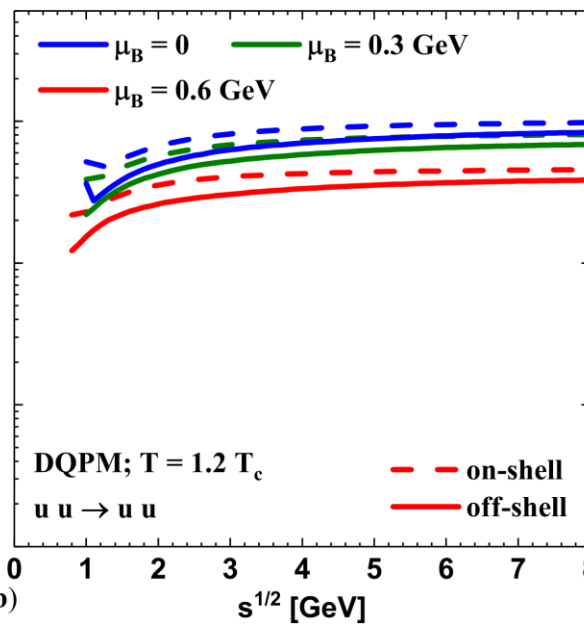
Plot by Ilia Grishmanovskii

- DQPM: $M \rightarrow 0, \gamma \rightarrow 0 \rightarrow$ reproduces pQCD limits
- Differences between DQPM and pQCD : less forward peaked angular distribution leads to more efficient momentum transfer

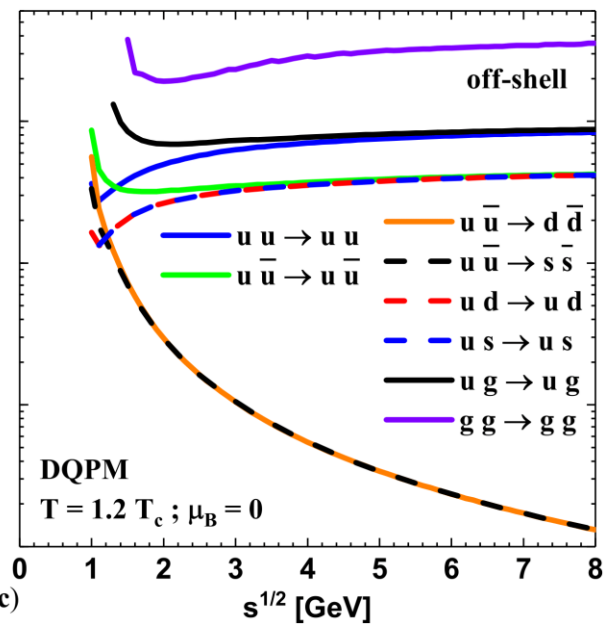
Total cross section



strong T dependence



weak μ_B dependence



strong channel dependence

DQPM: Mean-field potential for quasiparticles

Space-like part of energy-momentum tensor $T_{\mu\nu}$ defines the **potential energy density**:

$$V_p(T, \mu_q) = T_{g-}^{00}(T, \mu_q) + T_{q-}^{00}(T, \mu_q) + T_{\bar{q}-}^{00}(T, \mu_q)$$

space-like gluons + space-like quarks+antiquarks

$$\tilde{T}_{r_g^\pm} \dots = d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_g(\omega) \Theta(\omega) n_B(\omega/T) \Theta(\pm P^2) \dots$$

$$\tilde{T}_{r_q^\pm} \dots = d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_q(\omega) \Theta(\omega) n_F((\omega - \mu_q)/T) \Theta(\pm P^2) \dots$$

$$\tilde{T}_{r_{\bar{q}}^\pm} \dots = d_{\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} 2\omega \rho_{\bar{q}}(\omega) \Theta(\omega) n_F((\omega + \mu_q)/T) \Theta(\pm P^2) \dots$$

→ **Mean-field scalar potential (1PI) for quarks and gluons (U_q, U_g) vs parton scalar density ρ_s :**

$$U_s(\rho_s) = \frac{dV_p(\rho_s)}{d\rho_s} \quad \rho_s = N_g^+ + N_q^+ + N_{\bar{q}}^+$$

$$U_q = U_s, \quad U_g \sim 2U_s$$

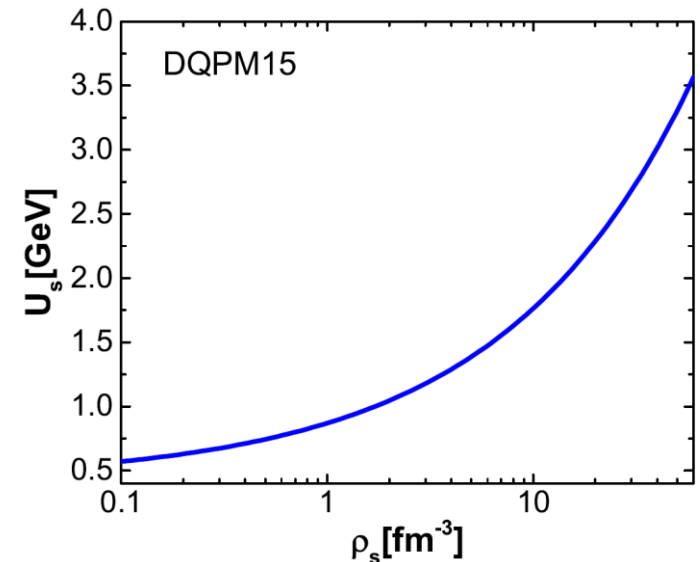
Quasiparticle **potentials** (U_q, U_g) are **repulsive** !

→ **the force** acting on a quasiparticle j :

$$F \sim M_j/E_j \nabla U_s(x) = M_j/E_j dU_s/d\rho_s \nabla \rho_s(x)$$

$$j = g, q, \bar{q}$$

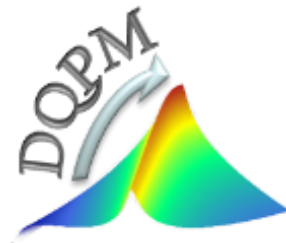
→ **accelerates** particles



QGP near equilibrium

DQPM (T, μ_q):

transport properties at finite (T, μ_q)



The properties of QGP in HICs → transport coefficients

Properties of the QGP near equilibrium are characterized by **transport coefficients**

Shear η , bulk viscosity ζ , ... are 'input' for the **viscous hydrodynamic models!**

Hydrodynamics

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \Delta T^{\mu\nu}$$

$$J_B^\mu = n_B u^\mu + \Delta J_B^\mu$$

$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases}$$

input for hydro simulations

$$\Delta T^{\mu\nu} = \eta \left(D^\mu u^\nu + D^\nu u^\mu + \frac{2}{3} \Delta^{\mu\nu} \partial_\rho u^\rho \right) - \zeta \Delta^{\mu\nu} \partial_\rho u^\rho$$

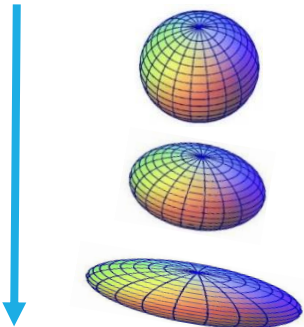
$$\Delta J_B^\mu = \kappa_B D^\mu \left(\frac{\mu_B}{T} \right)$$

$$D^\mu = \Delta^{\alpha\nu} \partial_\nu \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Shear viscosity

Resistance to 'deformation'

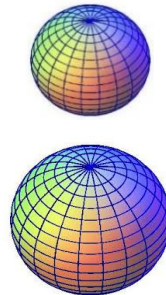
$$\eta \nabla^{\langle\mu} u^{\nu\rangle}$$



Bulk viscosity

Resistance to expansion

$$-\zeta \nabla u$$

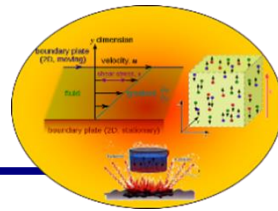


Baryon/electric charge
diffusion coefficients

$$\kappa_B \nabla^\mu \frac{\mu_B}{T}$$

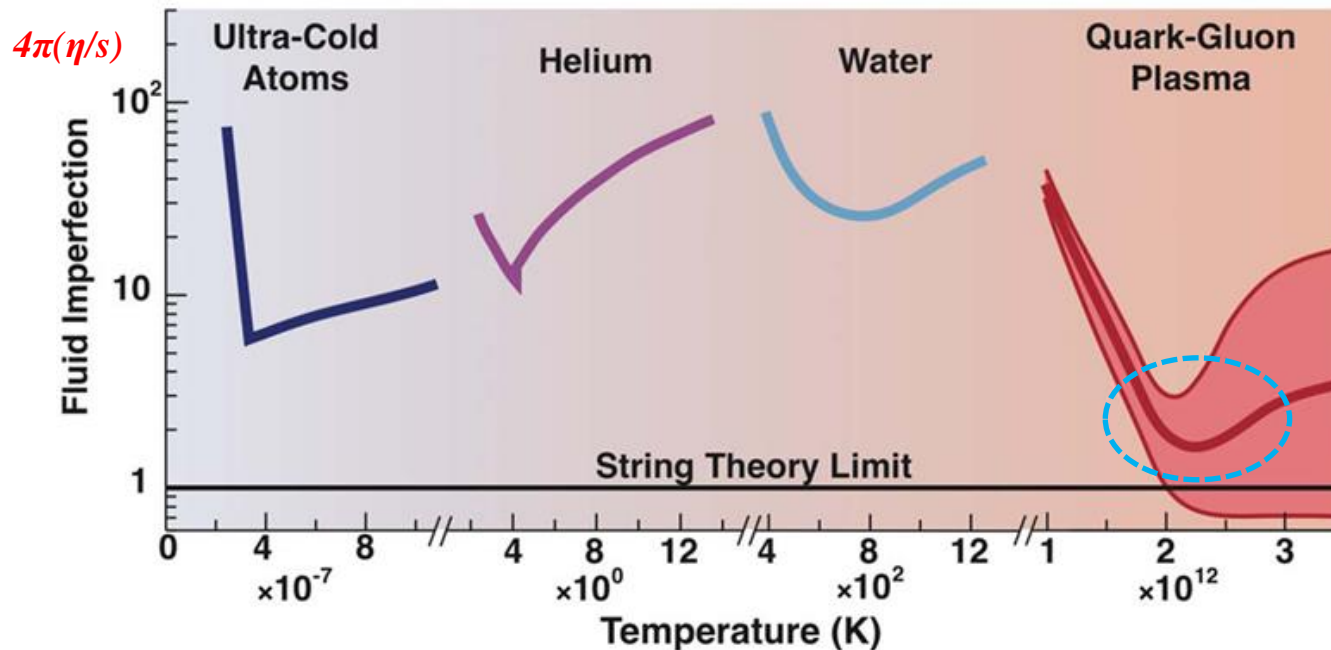


The properties of QGP from HIC - shear viscosity



The **shear viscosity** of a system measures its **resistance** to ‘deformation’, i.e. to flow

Compilation of the ratio of shear viscosity to entropy density (η/s) for various substances:



Exp. data + IQCD: η/s near T_c is very small !

→ **QGP** : close to an **ideal liquid**, not a gas of weakly interacting quarks and gluons

→ **QGP**: **strongly-interacting matter**

pQCD: shear viscosity η

QCD: Pure Yang-Mills (only gluons)

LO (Leading order) perturbative QCD calculations:

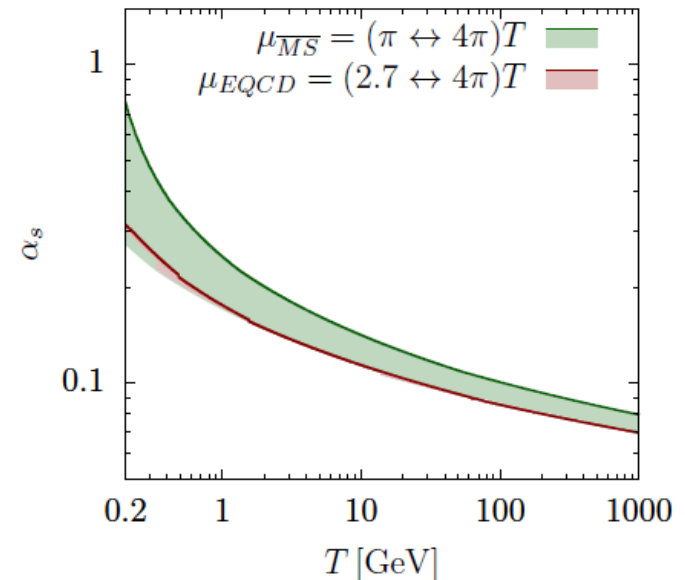
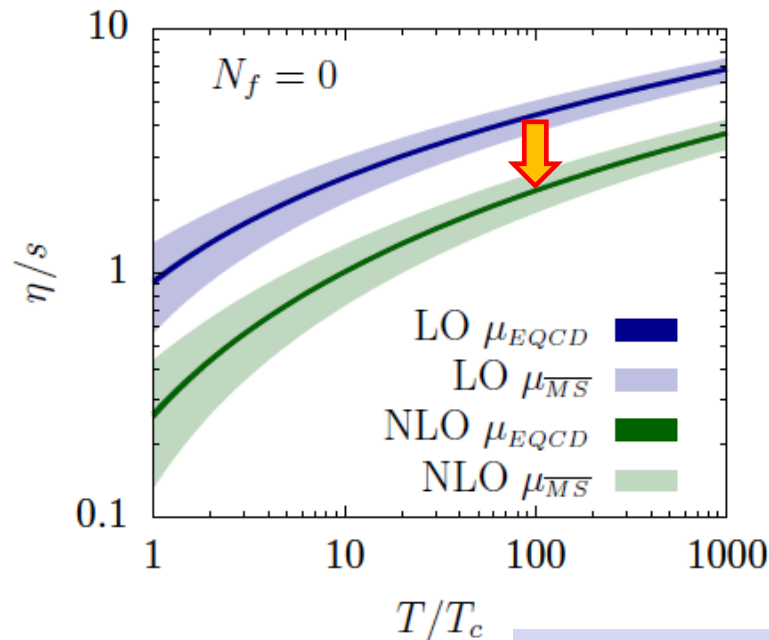
$\eta/s > 0.5$ at T near T_c 'AMY': P.B. Arnold, G.D. Moore and L.G. Yaffe, JHEP 11 (2000) 001)

NLO (Next-to-leading order):

J. Ghiglieri, G.D. Moore, D. Teaney, JHEP 1803 (2018) 179 :

“The next-to-leading order corrections are large and bring η/s down by more than a factor of 3 at physically relevant couplings.

The perturbative expansion is problematic even at $T \sim 100$ GeV”



→ from pQCD to effective models of QCD!

Transport coefficients: shear viscosity η

➤ Relaxation Time Approximation

$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_i (1 \pm f_i) f_i$$

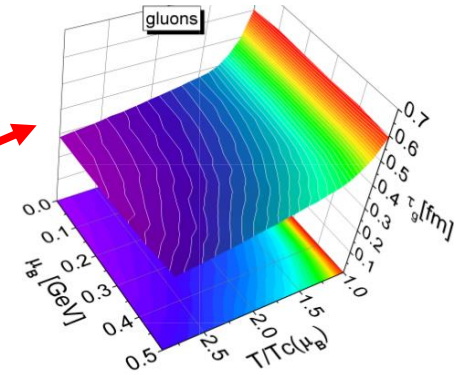
Interaction rate:

$$\Gamma_i^{\text{on}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q, \bar{q}, g} \int \frac{d^3p_j}{(2\pi)^3 2E_j} d_j f_j(E_j, T, \mu_q) \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4) |\bar{\mathcal{M}}|^2(p_i, p_j, p_3, p_4) (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)$$

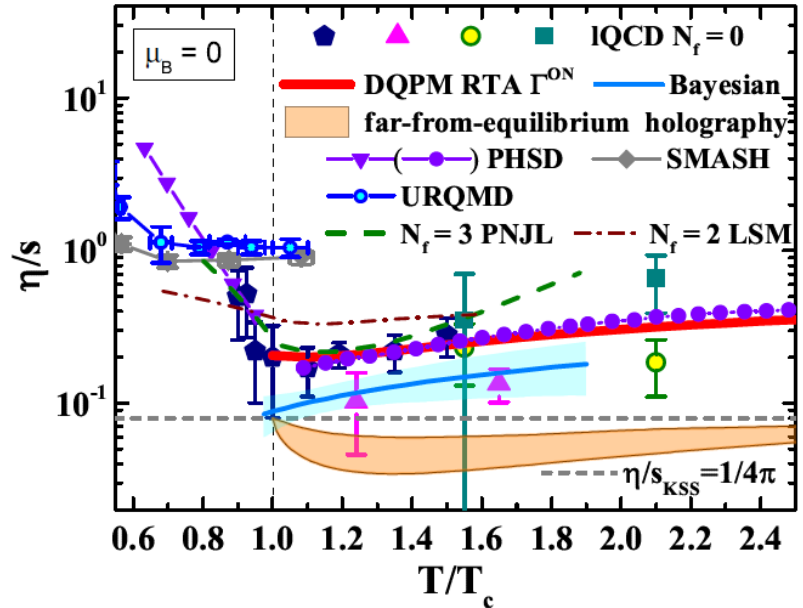
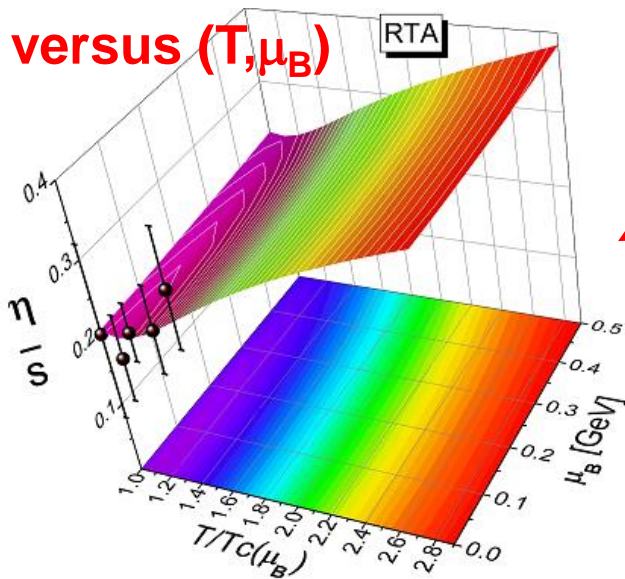
Relaxation time:

$$1) \tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

$$2) \tau_i(T, \mu_B) = \frac{1}{2\gamma_i(T, \mu_B)}$$



η/s versus (T, μ_B)



- Good agreement with IQCD
- Light increase of shear viscosity with μ_B

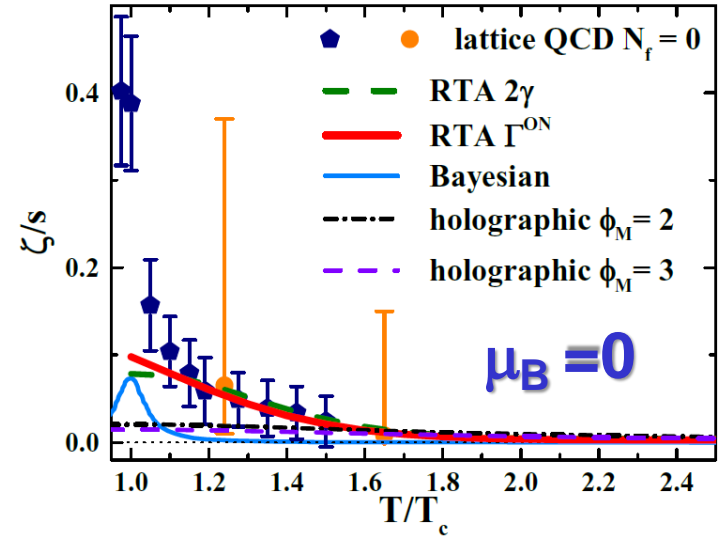
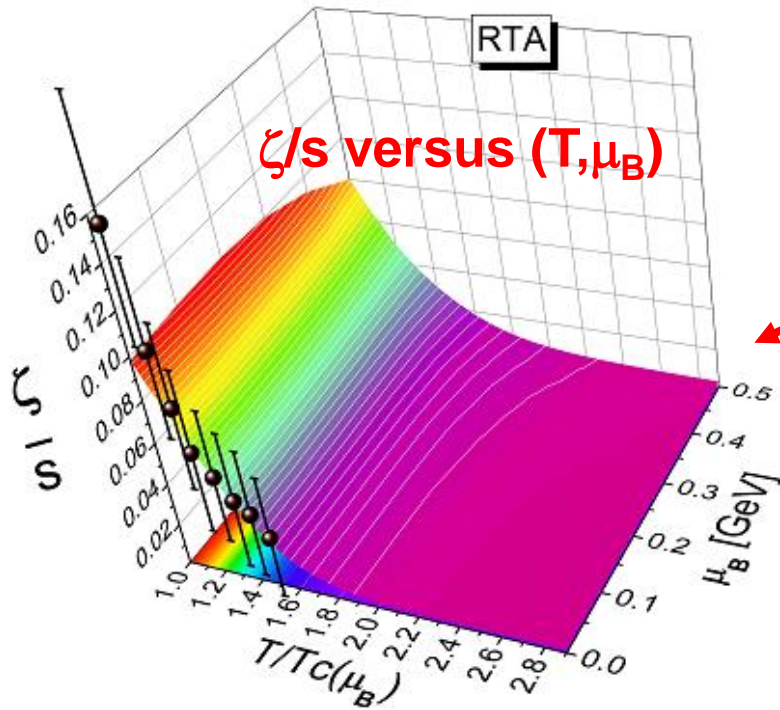
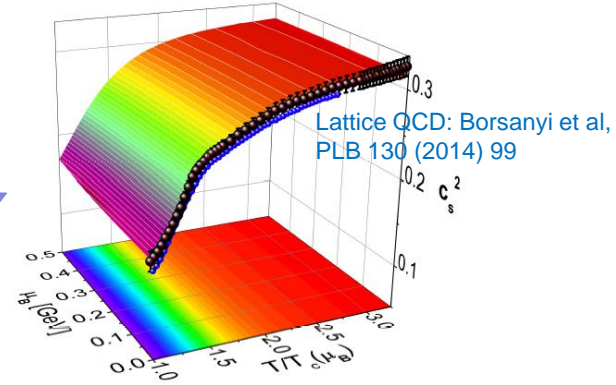
Transport coefficients: bulk viscosity ζ

➤ Relaxation Time Approximation

$$\zeta^{\text{RTA}}(T, \mu_B) = \frac{1}{9T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \tau_i(\mathbf{p}, T, \mu_B) \times \frac{d_i(1 \pm f_i)f_i}{E_i^2} \left(\mathbf{p}^2 - 3c_s^2 \left(E_i^2 - T^2 \frac{dm_i^2}{dT^2} \right) \right)^2$$

Speed of sound
Speed of sound c_s^2

Speed of sound c_s^2



- Good agreement with IQCD
- Weak dependence of bulk viscosity on μ_B

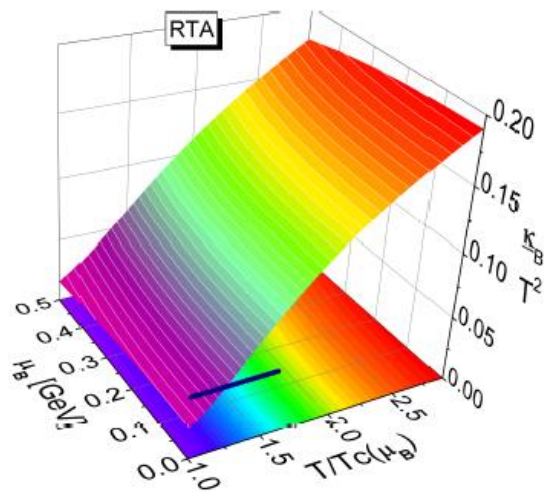
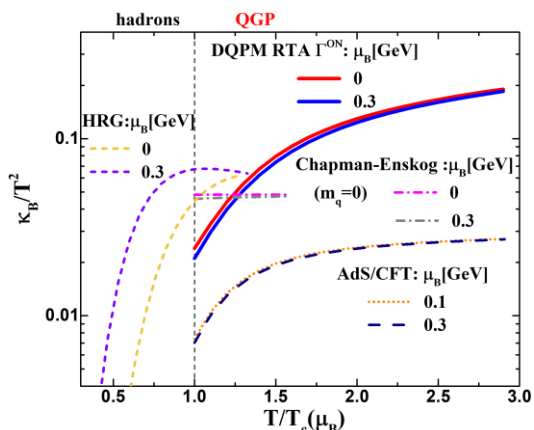
Transport diffusion coefficients

- $\kappa_{qq'}$ ($q; q' = B; S; Q$) - **diffusion coefficient matrix** for the baryon (B), strange (S) and electric (Q) charges using Chapman-Enskog method (CE) & RTA

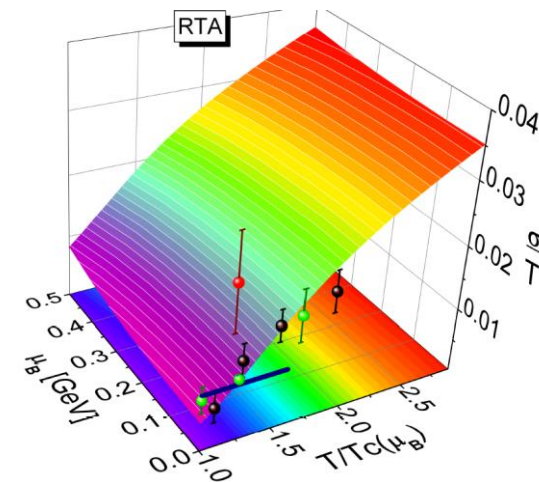
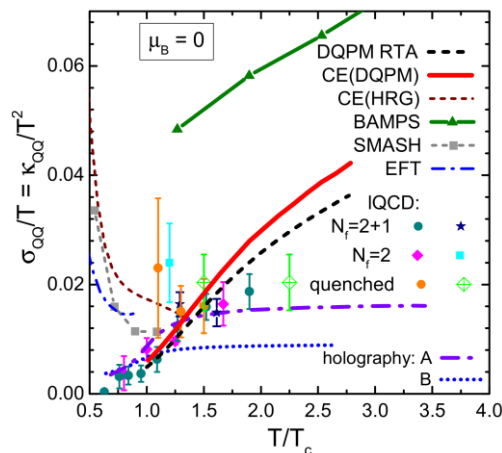
J. A. Fotakis, O. Soloveva, C. Greiner, O. Kaczmarek and E. B., PRD 104 (2021) , 034014

$$\begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix}$$

Baryon diffusion coefficient κ_B/T^2

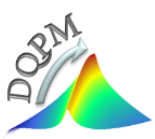


Electric conductivity σ_e/T



HRG: J. A. Fotakis et al, PRD 101 (2020) 7, 076007
AdS/CFT: T. Son and A. O. Starinets, JHEP 0603, 052 (2006)

□ Baryon diffusion coefficients decrease with μ_B

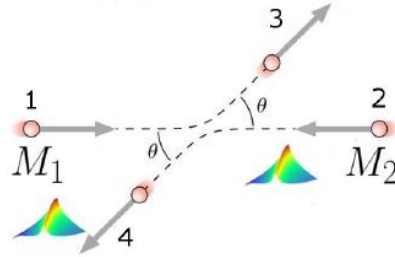


Transport coefficients: \hat{q}

The jet transport coefficient \hat{q} in non-perturbative strongly interacting QGP (DQPM):

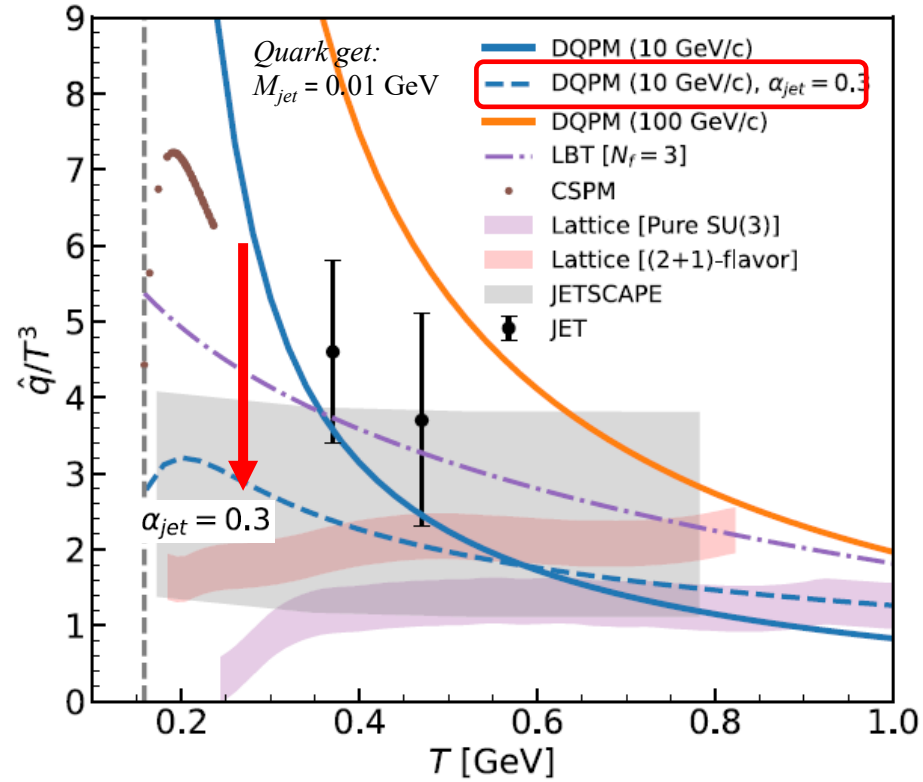
$$\hat{q}(p) = \frac{d\langle(\Delta p_T)^2\rangle}{dx}$$

Elastic scattering of jet parton with off-shell partons from sQGP:



$$\begin{aligned} \langle\langle O^* \rangle\rangle &\equiv \frac{1}{2E_p} \sum_{i=q,\bar{q},g} \int dm \rho_i(m) \int dm' \rho_i(m') \\ &\times \int \frac{d^3k}{(2\pi)^3 2E} f_i(k, m_i) \int \frac{d^3k'}{(2\pi)^3 2E'} \int \frac{d^3p'}{(2\pi)^3 2E'_p} \\ &\times O^* (2\pi)^4 \delta^{(4)}(p + q - p' - q') \frac{|M_{ic}|^2}{\gamma_c}, \end{aligned}$$

$\rho_i(m)$ – parton spectral function

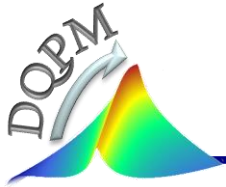


Quark jet :

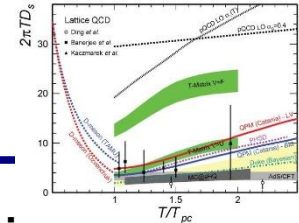
$$\begin{aligned} \langle O \rangle &= \langle O \rangle_{uu \rightarrow uu} + \langle O \rangle_{u\bar{u} \rightarrow u\bar{u}} + 2\langle O \rangle_{ud \rightarrow ud} \\ &+ 2\langle O \rangle_{us \rightarrow us} + \langle O \rangle_{ug \rightarrow ug}. \end{aligned}$$

Gluon jet :

$$\langle O \rangle = 4\langle O \rangle_{gu \rightarrow gu} + 2\langle O \rangle_{gs \rightarrow gs} + \langle O \rangle_{gg \rightarrow gg}.$$

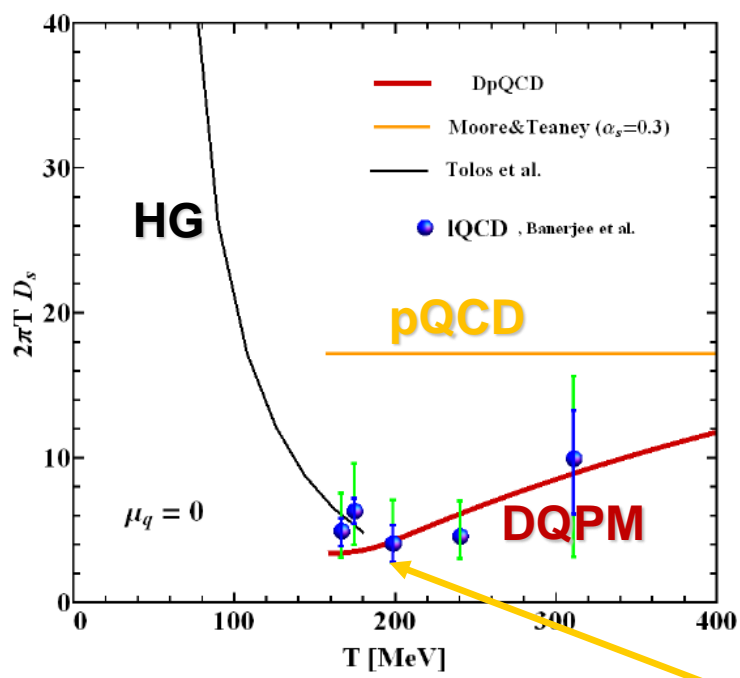


Charm spatial diffusion coefficient D_s

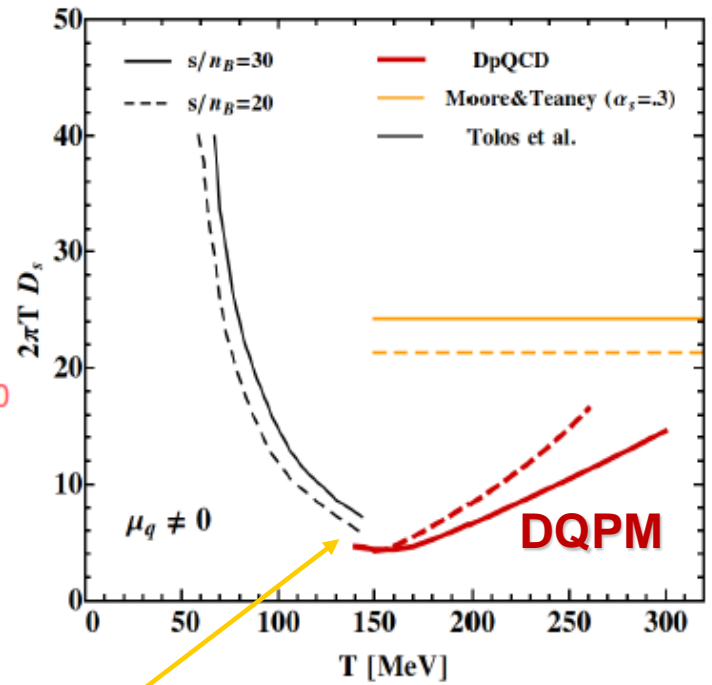


- D_s for heavy quarks as a function of T for $\mu_q=0$ and finite μ_q assuming adiabatic trajectories (constant entropy per net baryon s/n_B) for the expansion

$$D_s = \lim(\vec{p} \rightarrow 0) \frac{T}{M\eta_D} \quad \text{where } \eta_D = A/p ; A(p,T) = \text{drag coefficient}$$



$\Rightarrow \mu_q \neq 0$



□ $T < T_c$: hadronic D_s

➔ Continuous transition at T_c !

L. Tolos, J. M. Torres-Rincon, PRD 88 (2013) 074019
 V. Ozvenchuk et al., PRC90 (2014) 054909

H. Berrehrah et al, PRC 90 (2014) 051901, arXiv:1406.5322

Modeling of the 1st order phase transition:



PNJL



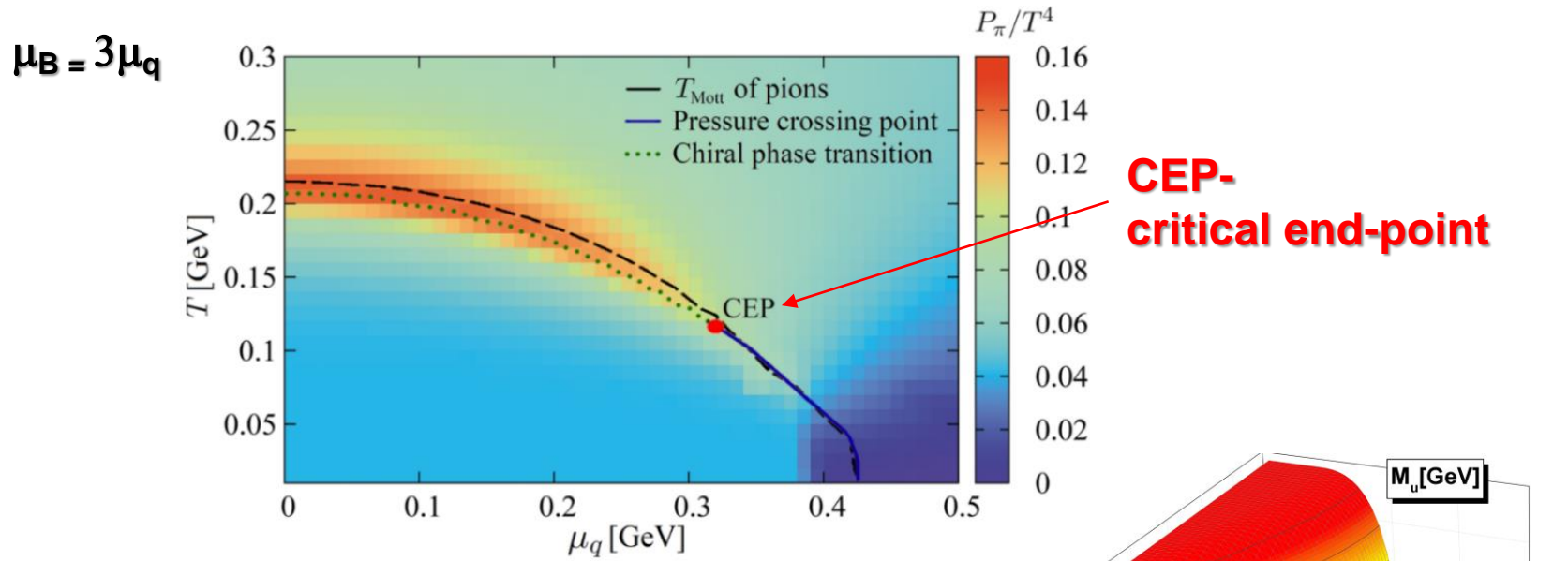
DQPM-CP(T, μ_q) –

DQPM with **critical end-point at high μ_q**

QGP in the Polyakov extended NJL model

D. Fuseau, T. Steinert, J. Aichelin PRC 101 (2020) 6 065203

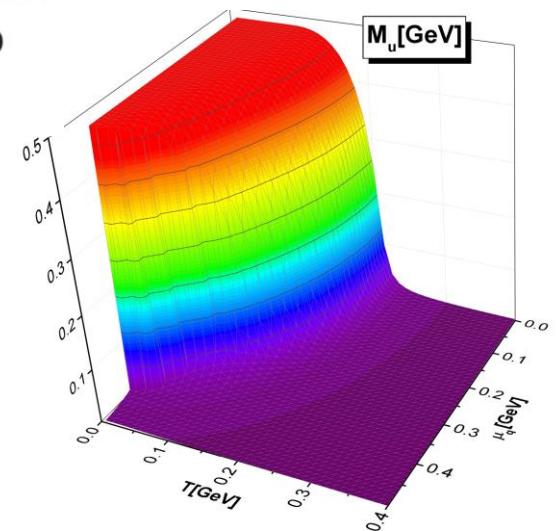
- PNJL allows for prediction of macroscopic properties of QGP at finite T and large μ_B
- & QGP transport coefficients for $0 \leq \mu_B \leq 1.2$ GeV



- **CEP:** $(T, \mu_B) = (110, 960)$ MeV, $\mu_B/T = 8.73$
- **1st order phase transition** at high μ_B
- same symmetries for the quarks as QCD

Chiral masses (M_l, M_s)

$$m_i = m_{0i} - 4G \langle \langle \bar{\psi}_i \psi_i \rangle \rangle + 2K \langle \langle \bar{\psi}_j \psi_j \rangle \rangle \langle \langle \bar{\psi}_k \psi_k \rangle \rangle$$



PNJL: Shear viscosity at high μ_B

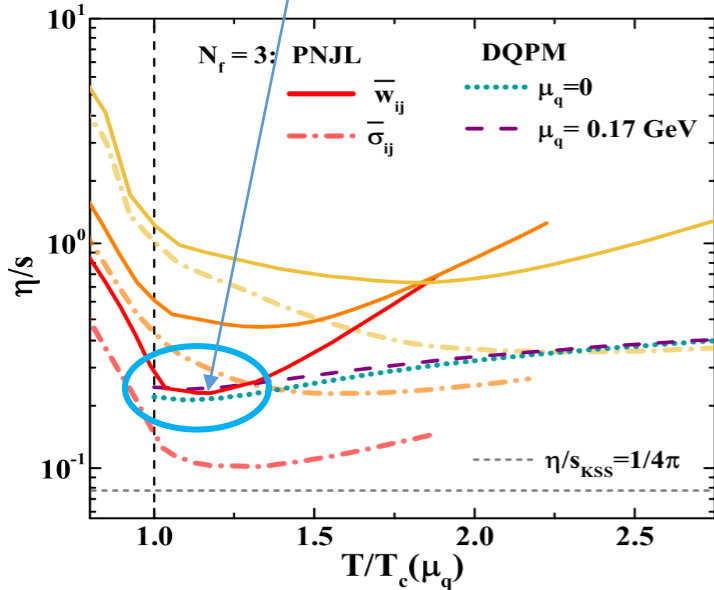
O. Soloveva, D. Fuseau, J. Aichelin and E. B., PRC 103 (2021) no.5, 054901

$$\eta^{\text{RTA}}(T, \mu_B) = \frac{1}{15T} \sum_{i=q, \bar{q}, g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) d_q f_i^\phi$$

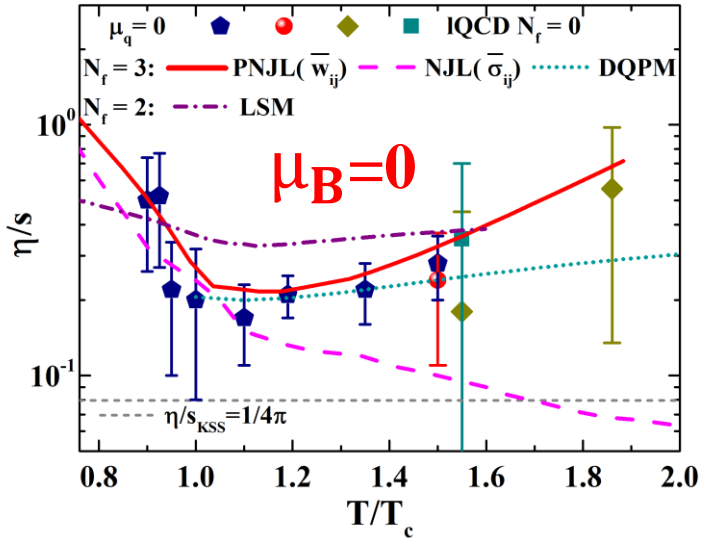
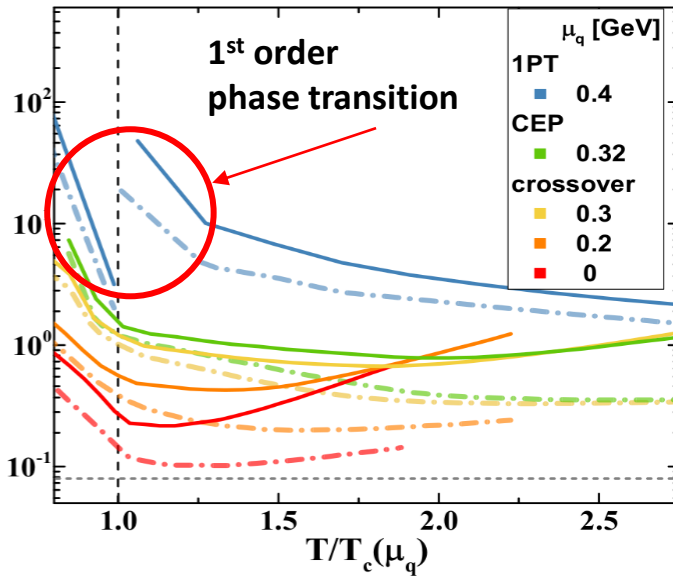
➤ **CEP:** $(T, \mu_B) = (110, 960) \text{ MeV}$, $\mu_B/T = 8.73$

$\mu_B > 0$

Comparison of η/s from PNJL and DQPM (wo CEP)



PNJL



Relaxation time:

1) DQPM ~ rate w :

$$\tau_i^{-1}(T, \mu_q) = \sum_{j=q, \bar{q}} n_j(T, \mu_q) \bar{w}_{ij}$$

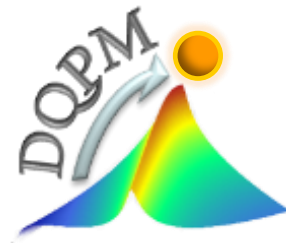
2) NJL- Sasaki ~ σ :

$$\tau_i^{-1}(T, \mu_q) = \sum_{j=q, \bar{q}} n_j(T, \mu_q) \bar{\sigma}_{ij}(T, \mu_q)$$

In agreement with $N_f=2$ NJL results C. Sasaki et al., NPA 832 (2010)

DQPM-CP(T, μ_q) :

DQPM with **critical end-point at high μ_q**

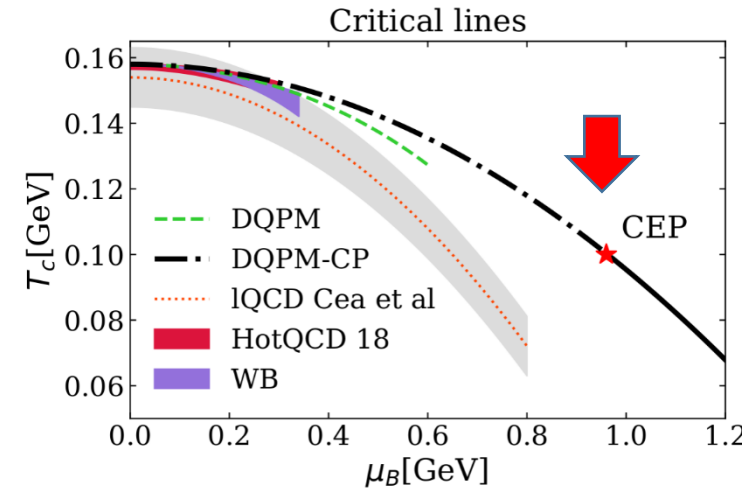




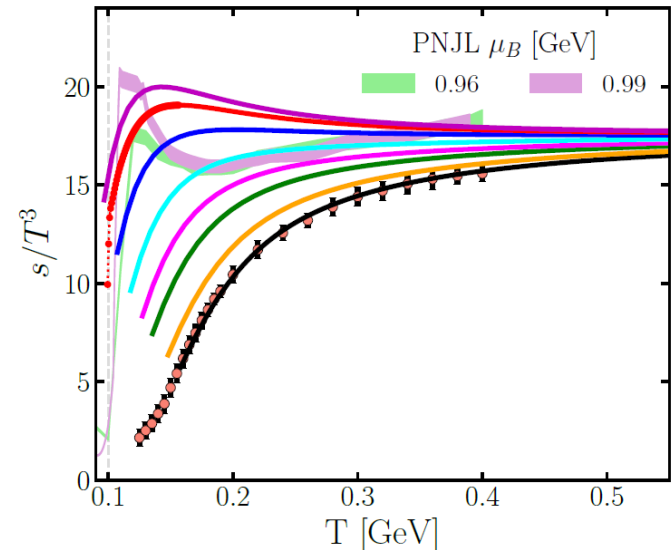
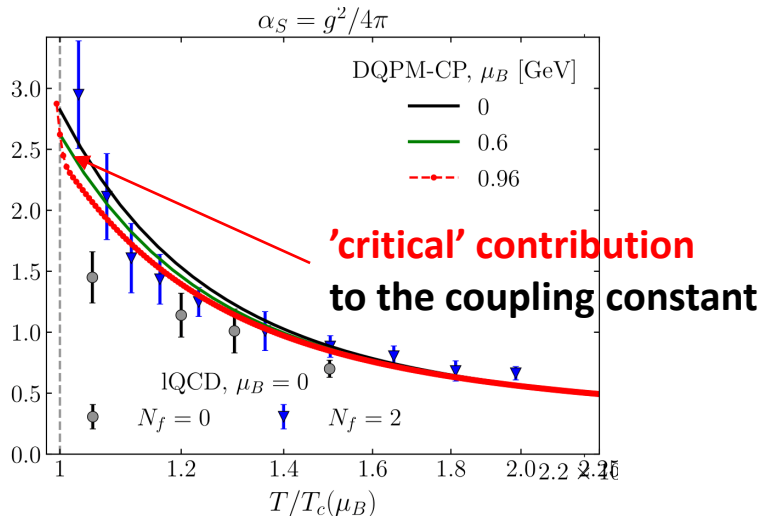
Quasiparticle model with CEP at high μ_B

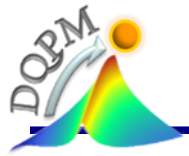
O. Soloveva, J. Aichelin and E. B., PRD 105 (2022) 054011

- ❑ **DQPM-CP** for high μ_B , including the **CEP** region based on the scaling properties of the entropy density from the PNJL model
 - ❑ **DQPM-CP** interpolates EoS and microscopic properties between two asymptotics – high $T \gg T_c, \mu_B = 0$ and $T > T_c, \mu_B \gg T$
 - ❑ **EoS** and transport coefficients of the QGP phase for the wide range of $T > T_c, m_B$
- **CEP**: $(T, \mu_B) = (100, 960)$ MeV, $\mu_B/T = 9.6$
 - **EoS**: for $\mu_B/T < 2$ agreement with IQCD for $\mu_B/T > 6$ agreement with pQCD



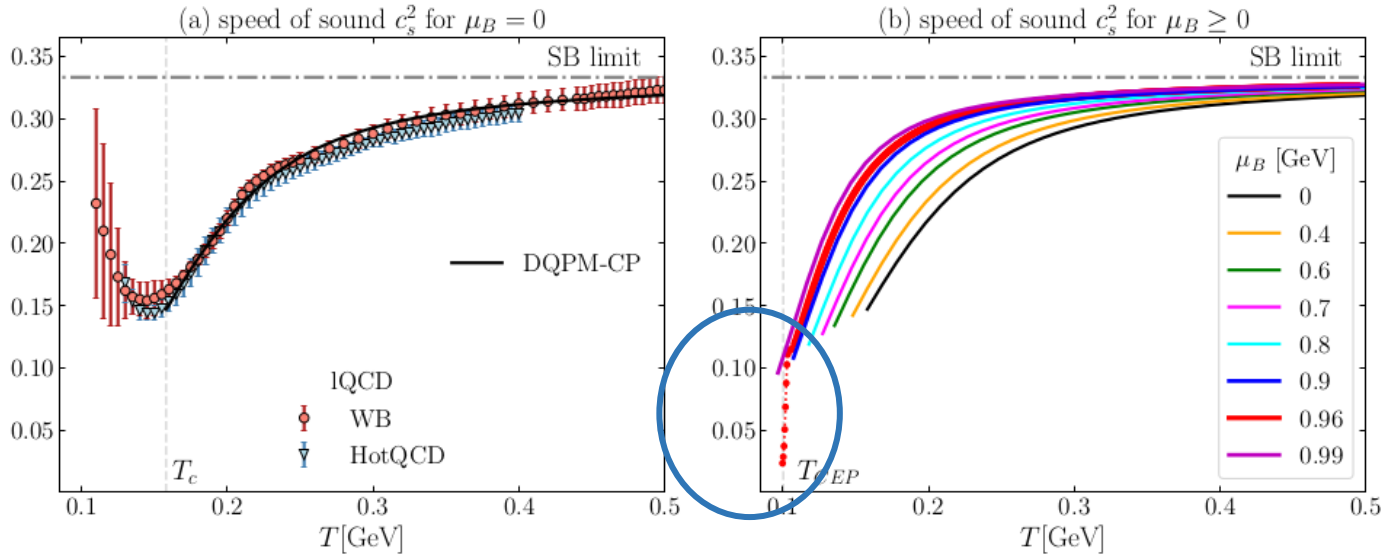
Near CEP: $g^2 = f(s^{PNJL}(T/T_c)) \rightarrow g^2(T/T_c)$



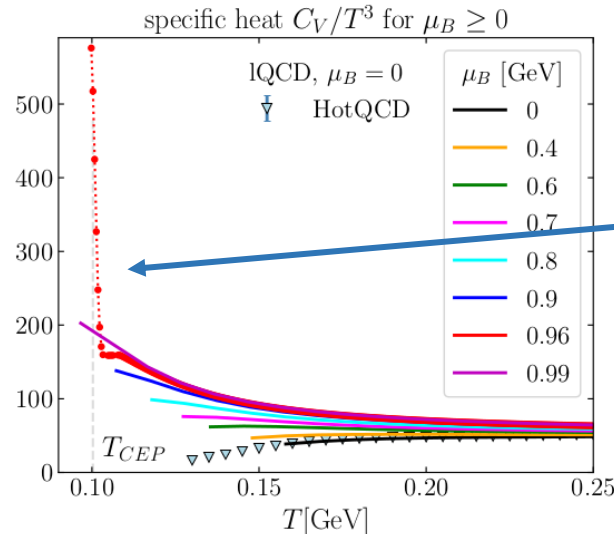


Speed of sound c_s and specific heat C_V

➤ **EoS** : for $\mu_B/T < 2$ agreement with IQCD for $\mu_B/T > 6$ agreement with pQCD



$$c_s^2 = \frac{dp}{d\epsilon} = \frac{dp/dT}{d\epsilon/dT} = \frac{s}{C_V}$$



Near CEP critical scaling can be seen:

$$\ln(C_V) = -\alpha \cdot \ln(T - T_{CEP}) + const$$

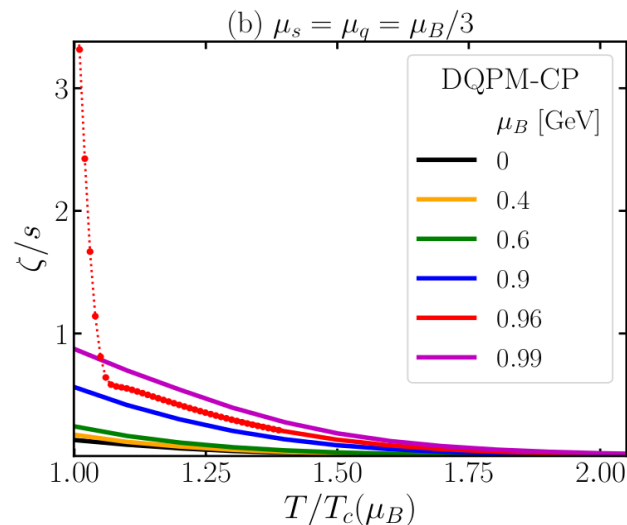
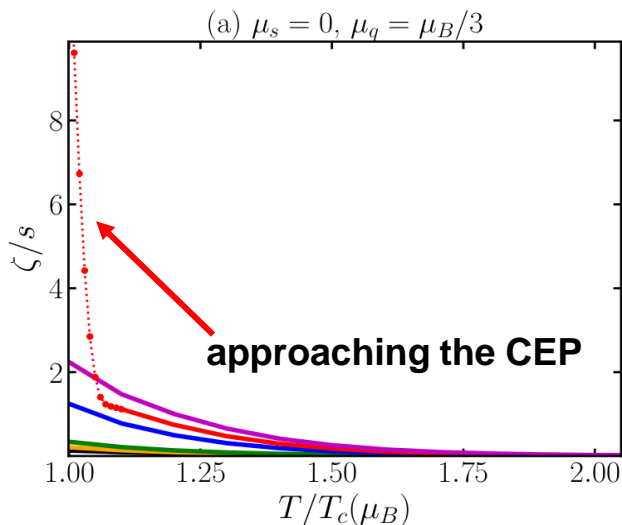
α – critical exponent

➔ **At CEP, where the transition of second order, c_s^2 vanishes, while C_V diverges**

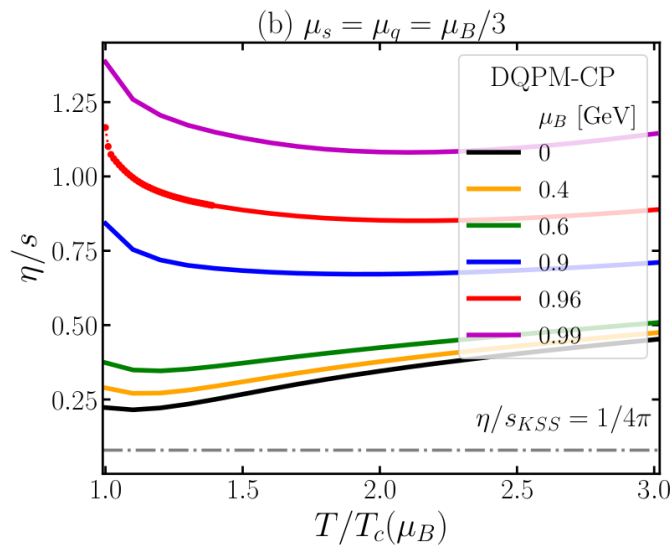
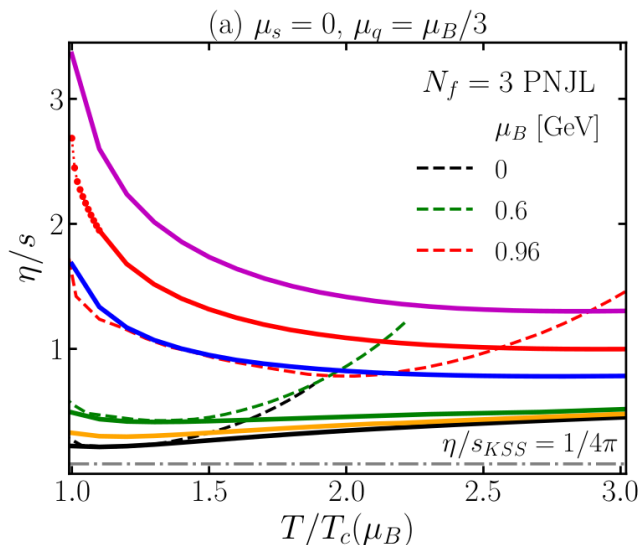


Shear and bulk viscosities near the CEP

Bulk viscosities

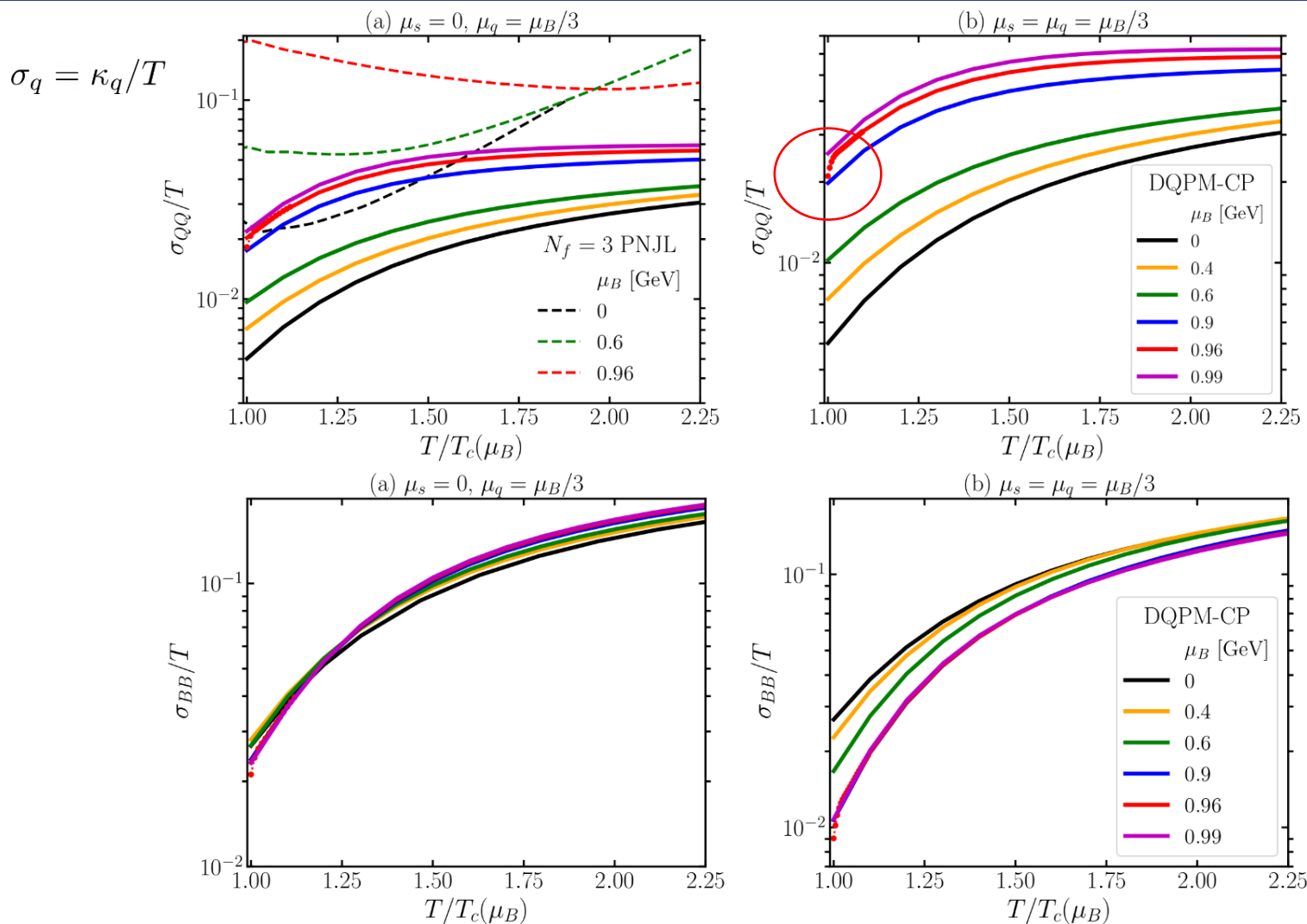


Shear viscosities



- Sudden rise of specific bulk viscosity approaching the CEP

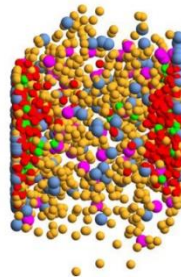
Diffusion transport coefficients near the CEP



- **B,Q,S** diffusion coefficients have pronounced μ_B, μ_S -dependence
- **Only small increase** approaching the CEP

QGP:
in-equilibrium → off-equilibrium

Microscopic transport theory!





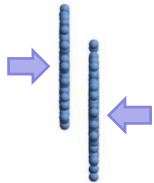
Parton-Hadron-String-Dynamics (PHSD)



PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A collision

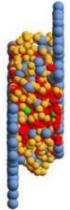


□ **Initial A+A collisions** :
 $N+N \rightarrow$ **string formation** \rightarrow decay to pre-hadrons + leading hadrons

□ **Formation of QGP stage** if local $\varepsilon > \varepsilon_{\text{critical}}$:
 dissolution of **pre-hadrons** \rightarrow partons

□ **Partonic phase - QGP:**
 QGP is described by the **Dynamical QuasiParticle Model (DQPM)** matched to reproduce **lattice QCD EoS** for finite T and μ_B (crossover)

Partonic phase



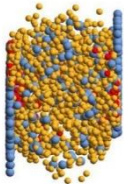
- **Degrees-of-freedom:** strongly interacting quasiparticles: **massive quarks and gluons (g, q, q_{bar})** with sizeable collisional widths in a self-generated mean-field potential

- **Interactions:** (quasi-)elastic and inelastic collisions of partons

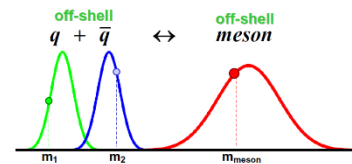
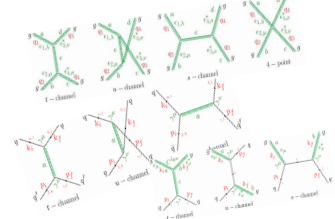
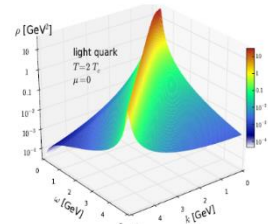
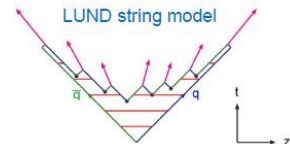
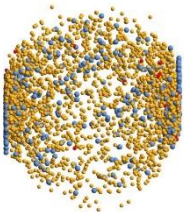
□ **Hadronization** to colorless **off-shell mesons and baryons:**
 Strict 4-momentum and quantum number conservation

□ **Hadronic phase:** hadron-hadron interactions – **off-shell HSD**

Hadronization



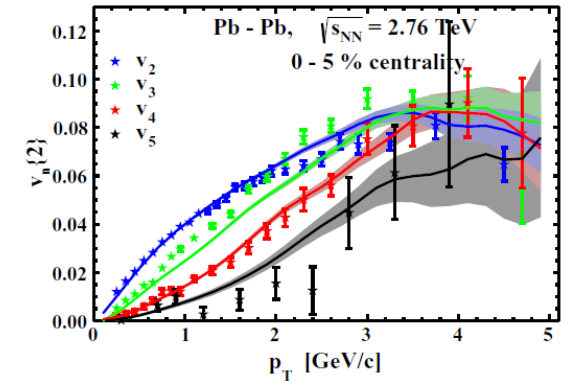
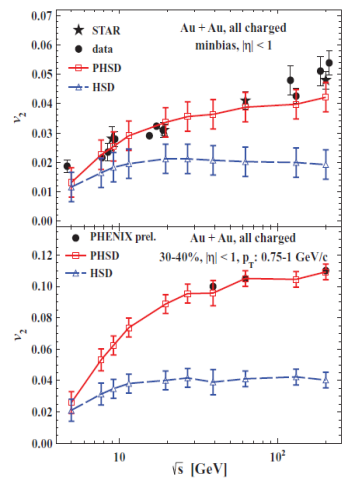
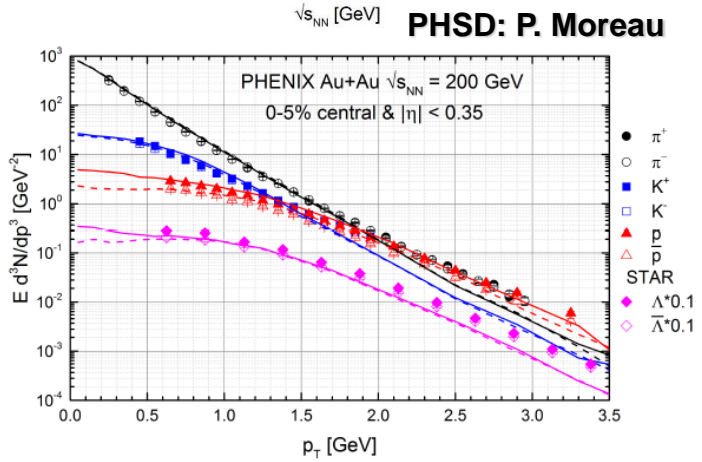
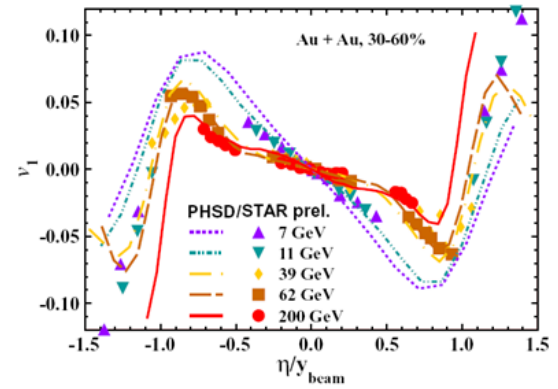
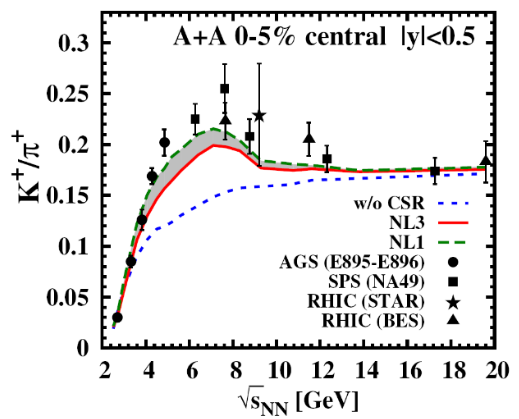
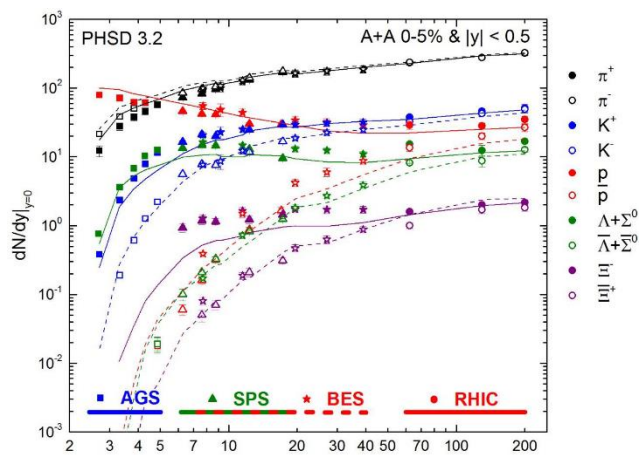
Hadronic phase





Non-equilibrium dynamics: description of A+A with PHSD

Important: to be conclusive on charm observables, the **light quark dynamics** must be well under control!

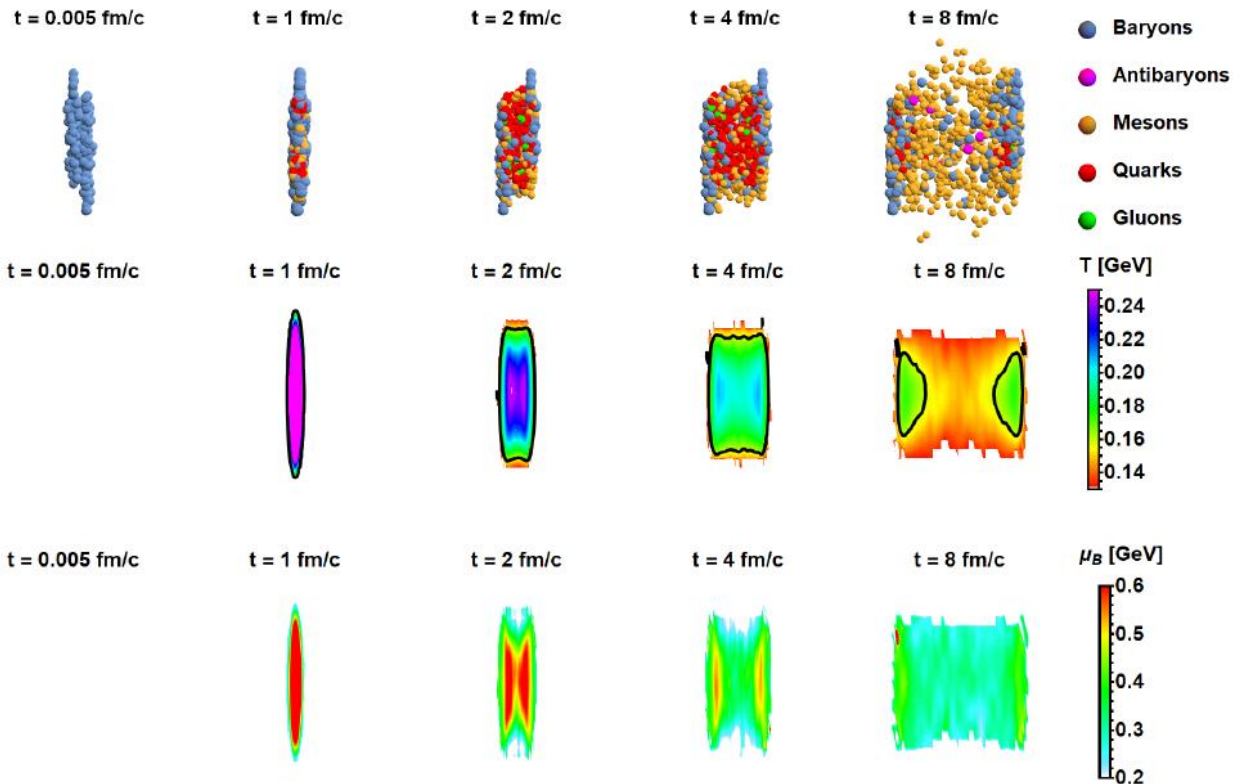


V. Konchakovski et al., PRC 85 (2012) 011902; JPG42 (2015) 055106

PHSD provides a **good description of 'bulk' observables** (y -, p_T -distributions, flow coefficients v_n , ...) from SIS to LHC energies

Traces of the QGP at finite μ_q in observables in high energy heavy-ion collisions

Au + Au $\sqrt{s_{NN}} = 19.6$ GeV – $b = 2$ fm – Section view





PHSD: QGP evolution in HICs

Input:
 ϵ^{PHSD} and n_B^{PHSD}

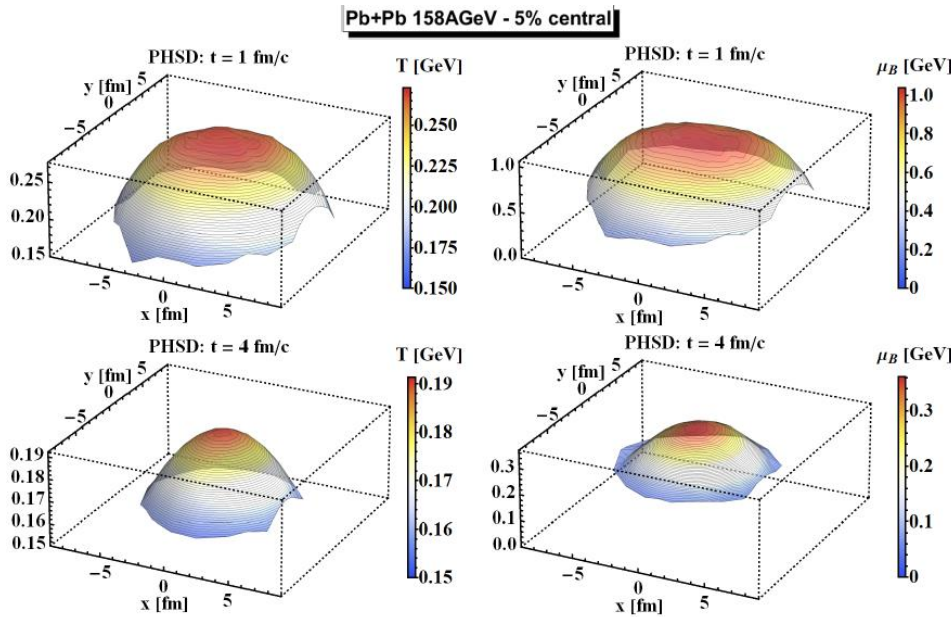
IQCD

$$\frac{n_B}{T^3} \approx \chi_2^B(T) \left(\frac{\mu_B}{T} \right) + \dots$$

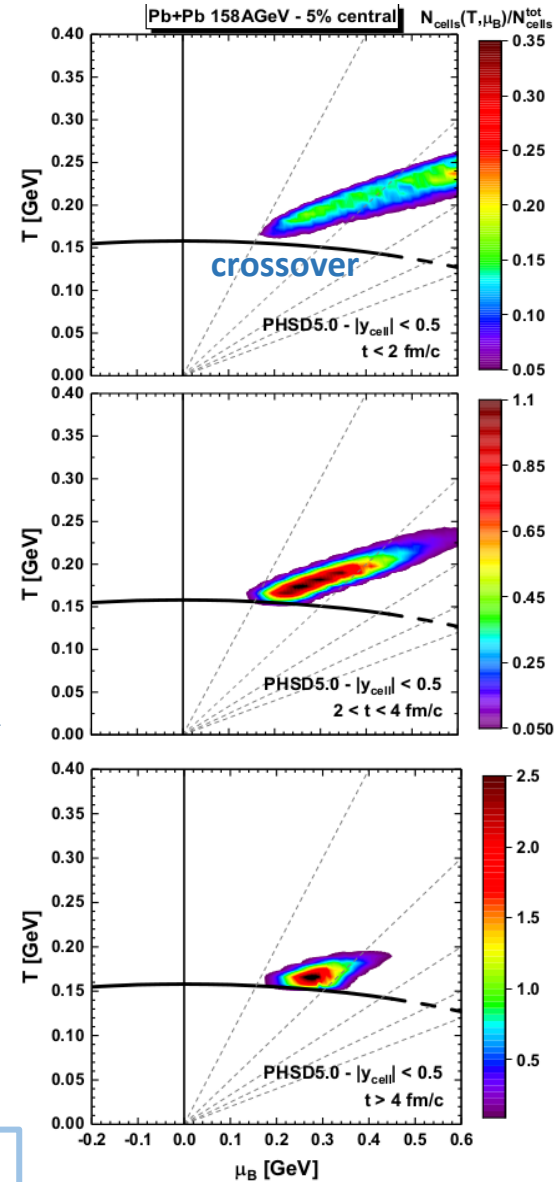
$$\Delta\epsilon/T^4 \approx \frac{1}{2} \left(T \frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T) \right) \left(\frac{\mu_B}{T} \right)^2 + \dots$$

Output:
 T, μ_B

T-profile in (x;y) & μ_B profile in (x;y)
at midrapidity ($|y_{\text{cell}}| < 1$) at fixed times (1 and 4 fm/c)



time evolution



Path through the phase diagram is not trivial and not localized

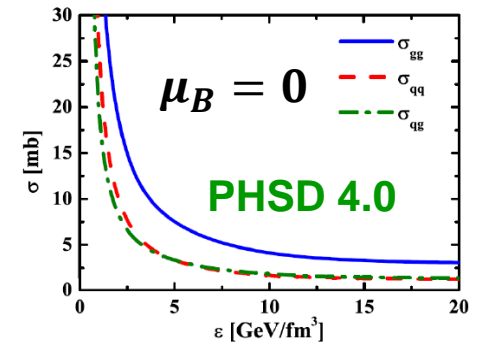
Results for HICs from PHSD 4.0 and 5.0

Comparison between three different results:

- PHSD 4.0 : only $\sigma(T)$ and $\rho(T)$

$\sigma(T)$ – parton interaction cross sections

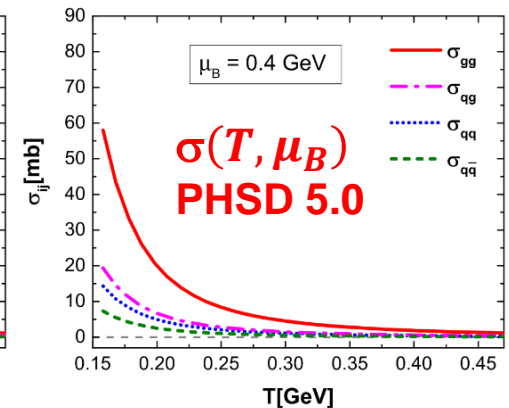
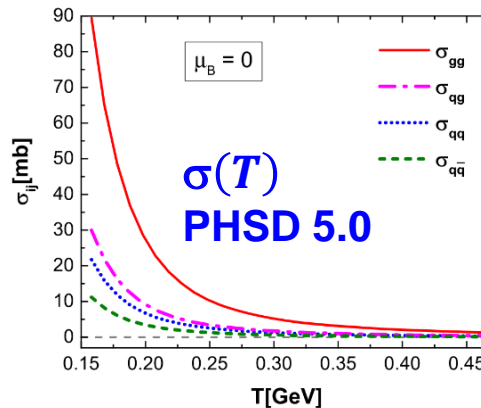
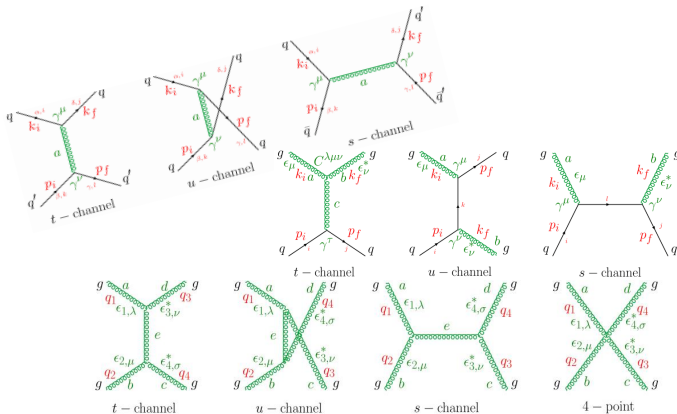
$\rho(T)$ – spectral function of partons (masses and widths)



new PHSD 5.0 : \sqrt{s} + μ_B + angular dependence of $d\sigma/d\cos\theta$

- PHSD 5.0 : with $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B = 0)$ and $\rho(T, \mu_B = 0)$ with $d\sigma/d\cos\theta$

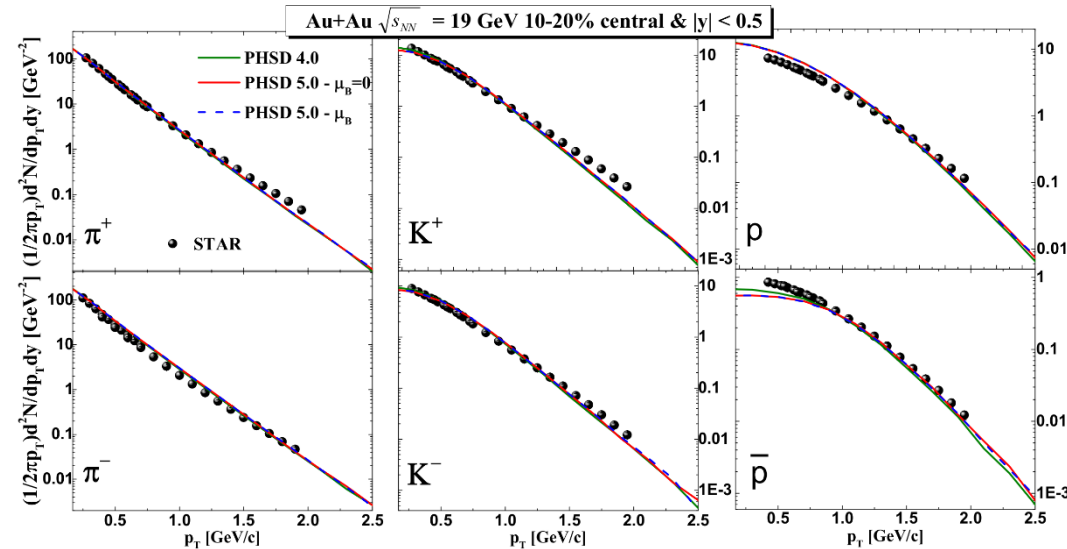
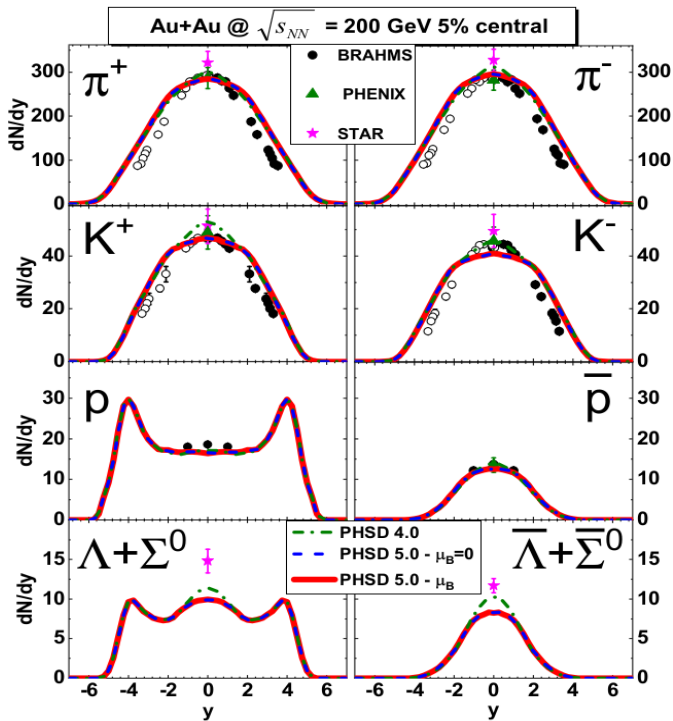
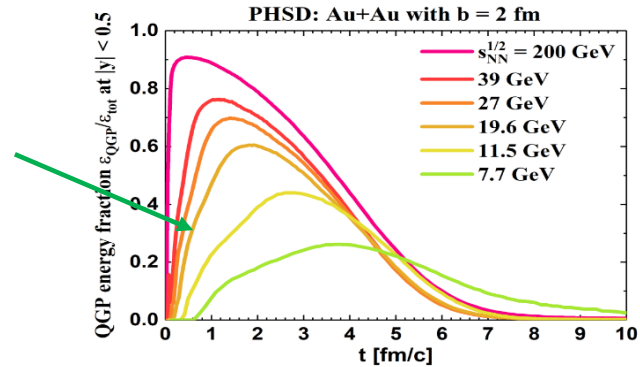
- PHSD 5.0 : with $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$ and $\rho(T, \mu_B)$ with $d\sigma/d\cos\theta$



Results for ($\sqrt{s_{NN}} = 200 \text{ GeV} - 7 \text{ GeV}$)

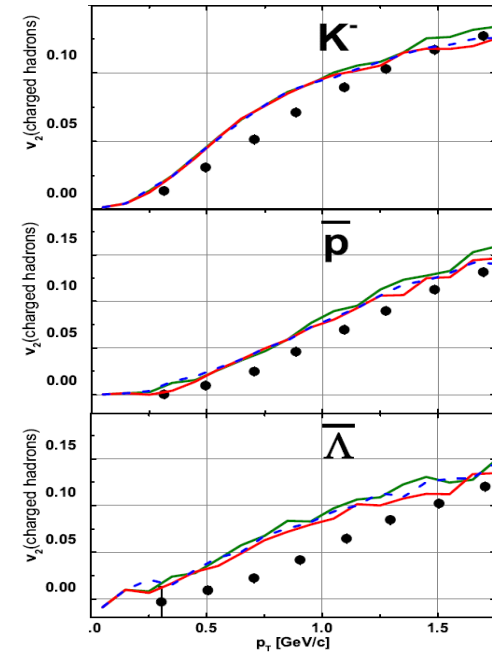
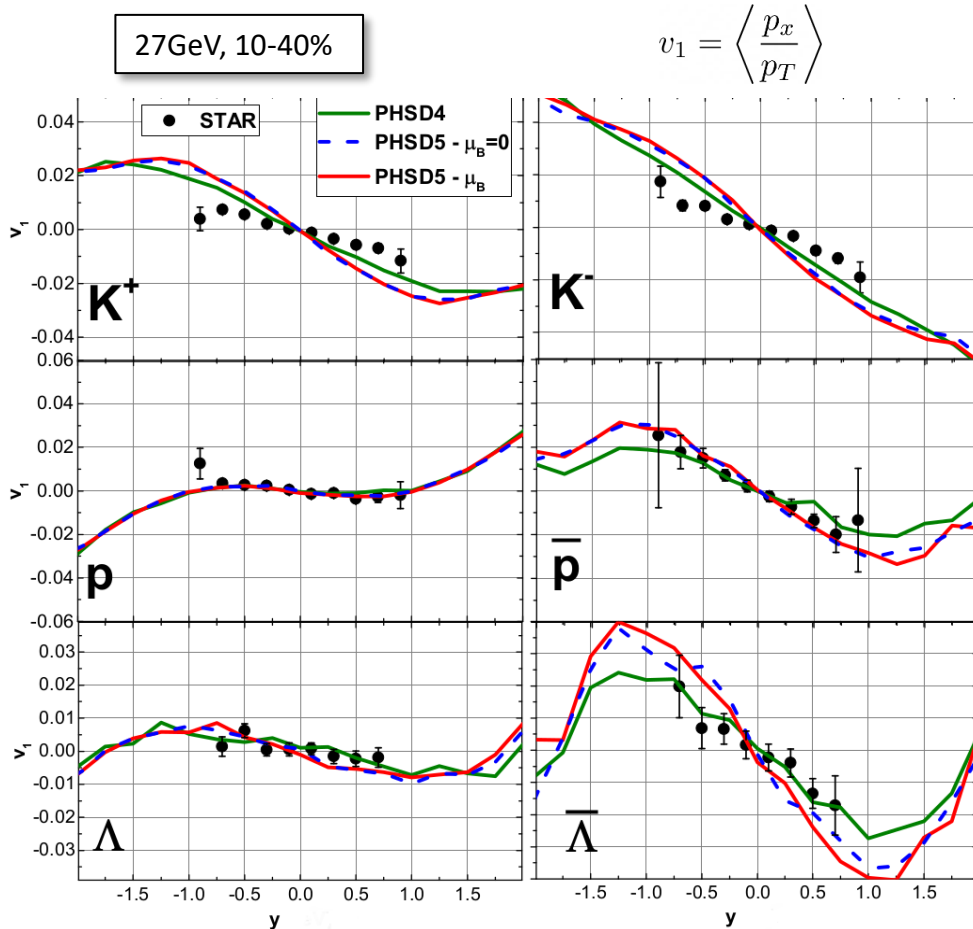
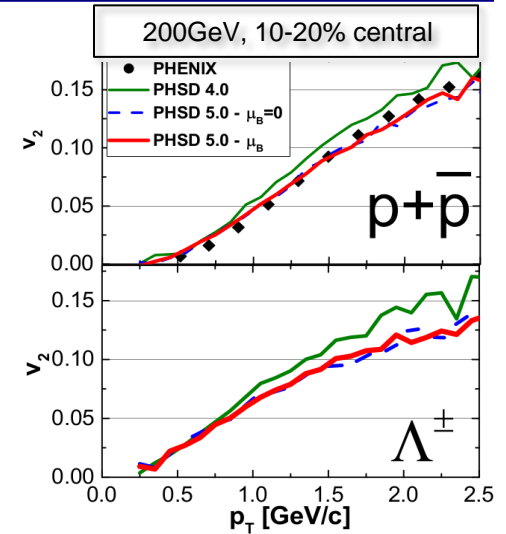
- No visible effects on p_T -spectra, dN/dy of μ_B -dependence
- Small effect of the angular dependence of $d\sigma/d\cos\theta$

at high $\sqrt{s_{NN}}$ - low μ_B
! QGP fraction is small at low $\sqrt{s_{NN}}$



Elliptic flow ($\sqrt{s_{NN}} = 200 \text{ GeV} - 27 \text{ GeV}$)

- **Weak μ_B -dependence** – small fraction of QGP or low μ_B
- Small effect of the angular dependence of $d\sigma/d\cos\theta$
- Strong flavor dependence





Messages from studies of QGP at T, μ_B

- ❑ (T, μ_B) -dependent partonic cross sections and masses/widths of quarks and gluons have been implemented in PHSD
- ❑ High- μ_B region is probed at low bombarding energies or high rapidity regions
- ❑ But, QGP fraction is small at low bombarding energies:
 - ➔ no effects of (T, μ_B) -dependent partonic cross sections and masses/widths seen in 'bulk' observables – dN/dy , p_T -spectra
- ❑ Flow harmonics v_1, v_2 show :
visible sensitivity to the explicit \sqrt{s} -dependence of total partonic cross sections σ + angular dependence of $d\sigma/d\cos\theta$, however, weak dependence on μ_B

- ❑ Outlook:
 - More precise EoS at large μ_B
 - Possible 1st order phase transition at even larger μ_B ?!

High- μ_B region of QCD phase diagram ➔ challenge for FAIR, NICA, BES RHIC