Light nuclei production in heavy-ion collisions

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&
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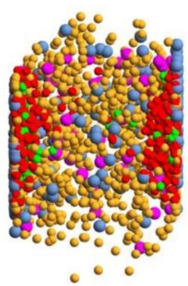
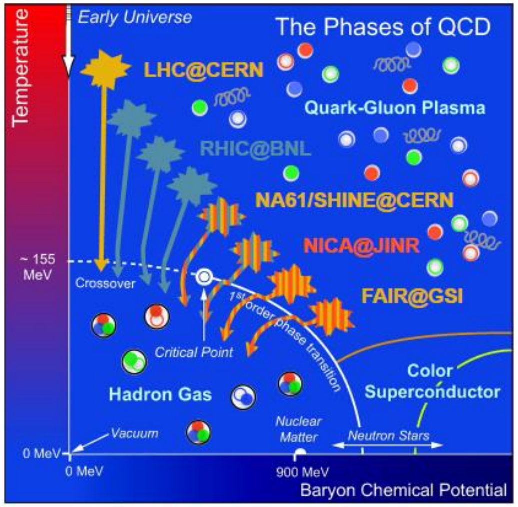
Workshop on Nonequilibrium Dynamics
Nov. 28 – Dec.2, 2022
Krabi/Thailand



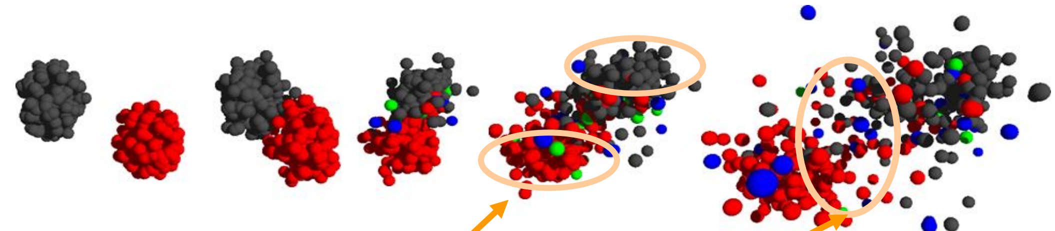
The enigma of heavy-ion physics:

The phase diagram of QCD



Experimental observables:

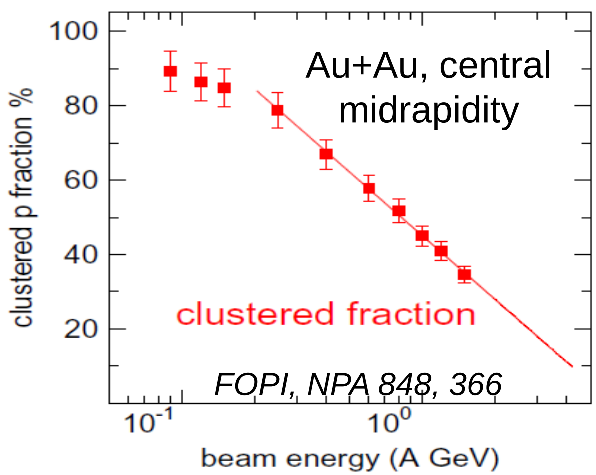
... Clusters and (anti-) hypernuclei



- projectile/target spectators➔ heavy cluster formation
- midrapidity➔ light clusters

! Hyperons are created in participant zone

- Clusters are very **abundant at low energy**

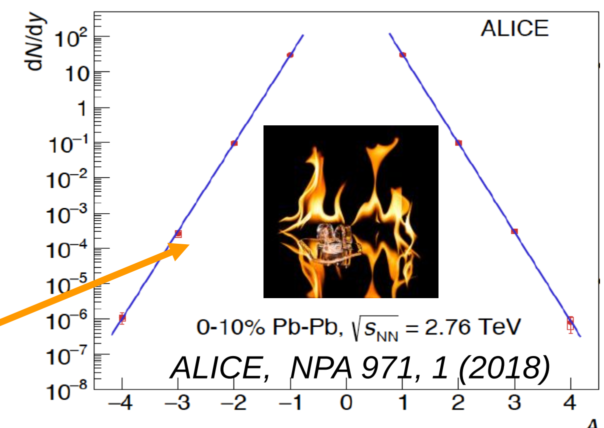


(Anti-) hypernuclei production:

- at mid-rapidity by coalescence of Λ with nucleons during expansion
- at projectile/target rapidity by rescattering/absorption of Λ by spectators

High energy HIC:

‚Ice in a fire' puzzle: how the weakly bound objects can be formed and survive in a hot enviroment ?!



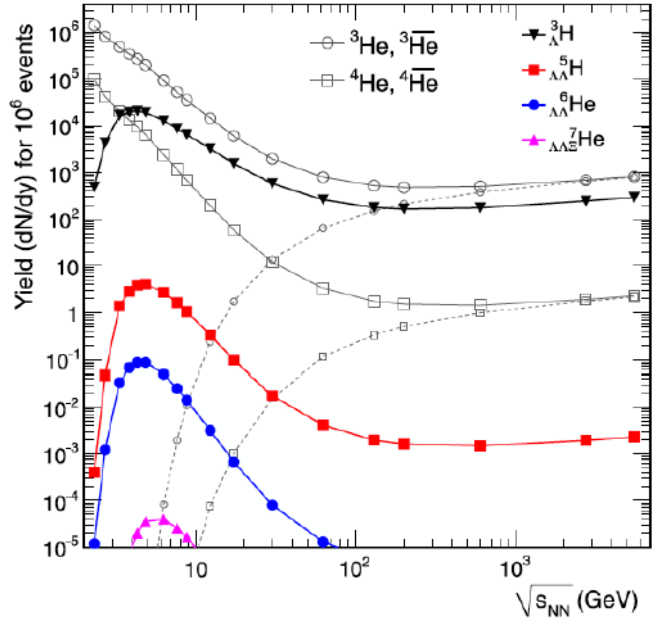
Modeling of cluster and hypernuclei formation

Existing models for cluster formation:

- **statistical model:**
  - assumption of thermal equilibrium
- **coalescence model:**
  - determination of clusters at freeze-out time by coalescence radii in coordinate and momentum space (limited to A=2,3)

➔ **don‘t provide information on the dynamical origin of cluster formation**

A. Andronic et al., PLB 697, 203 (2011)



In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

➔ **transport models:**
dynamical modeling of cluster formation based on interactions:

– via potential interaction - potential mechanism

-- by scattering - kinetic mechanism

PHQMD

Cluster formation: QMD vs MF

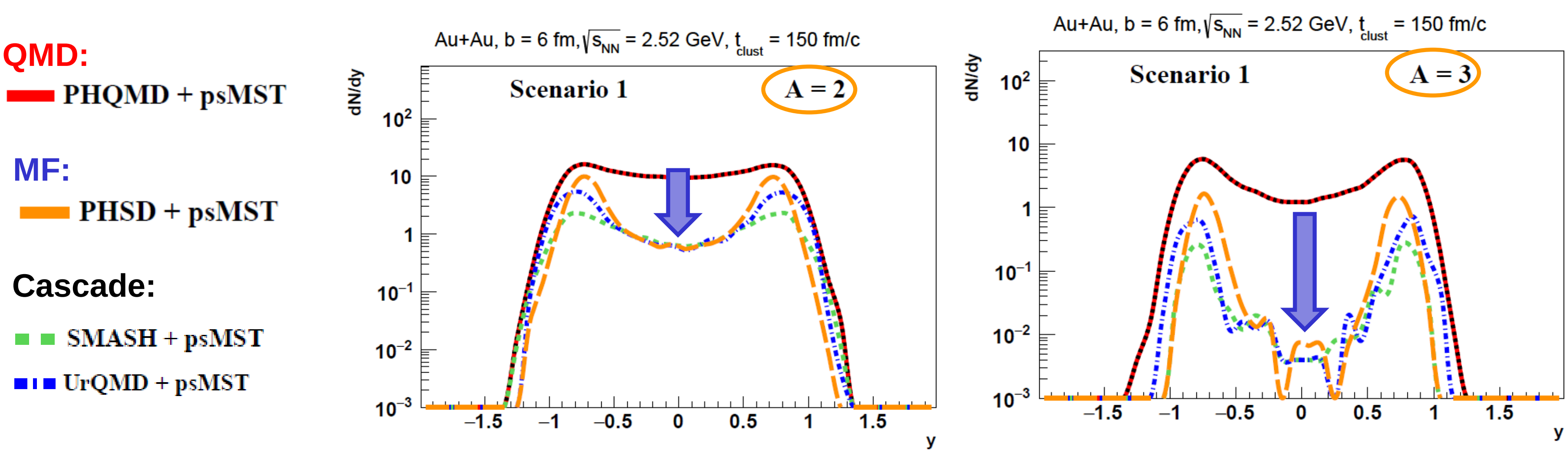
- **Cluster formation is sensitive to nucleon dynamics**

➔ **One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:**

- **QMD (quantum-molecular dynamics) – as n-body theory allows to keep correlations**
- **MF (mean-field based models) – correlations are smeared out**
- **Cascade – no potential interaction -> no binding-> diverging trajectories**

Example: Cluster stability over time:

V. Kireyeu, Phys.Rev.C 103 (2021) 5



➔ **n-body QMD dynamics necessary for the description of cluster production**

PHQMD

PHQMD: a **unified n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

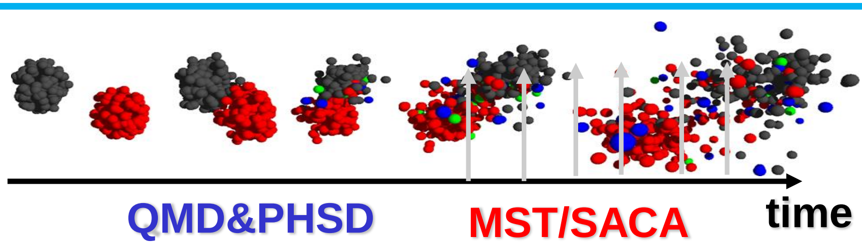
Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

Cluster recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)



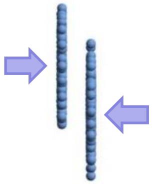
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1

PHQMD Collision Integral → from Parton-Hadron-String-Dynamics

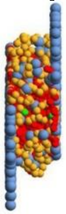
PHSD is a non-equilibrium microscopic transport approach for the description of strongly-interacting hadronic and partonic matter created in heavy-ion collisions

Dynamics: based on the solution of generalized off-shell transport equations derived from Kadanoff-Baym many-body theory

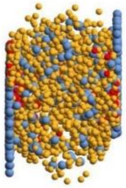
Initial A+A collision



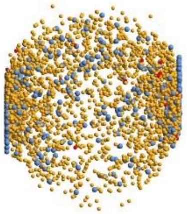
Partonic phase



Hadronization

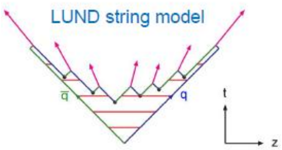


Hadronic phase

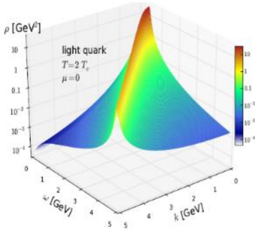


→ PHQMD **Initialization of A-nuclei + QMD propagation of baryons**

PHSD collision integral → PHQMD

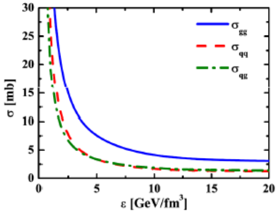


- **Initial A+A collisions :**
  N+N → string formation → decay to pre-hadrons + leading hadrons
- **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:
  dissolution of pre-hadrons → partons



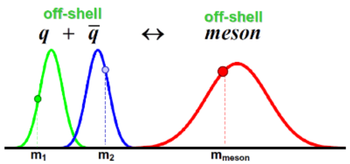
- **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

  - Degrees-of-freedom: strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential



  - Interactions: (quasi-)elastic and inelastic collisions of partons

- **Hadronization** to colorless off-shell mesons and baryons:
  Strict 4-momentum and quantum number conservation



- **Hadronic phase:** hadron-hadron interactions – off-shell HSD

PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

QMD propagation

- **Generalized Ritz variational principle:** $\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ **for N particles** **(neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

[Aichelin Phys. Rept. 202 (1991)]

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\dot{r}_{i0} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}$$

Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j\neq i} V_{i,j})$

$$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t) = V_{\mathrm{Skyrme}} + V_{\mathrm{Coul}}$$

QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H \rangle = \langle T \rangle + \langle V \rangle = \sum_i (\sqrt{p_{i0}^2 + m^2} - m) + \sum_i \langle V_{Skyrme}(\mathbf{r_{i0}}, t) \rangle$$

- **Skyrme potential** **('static') * :**

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0} \right)^{\gamma}$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

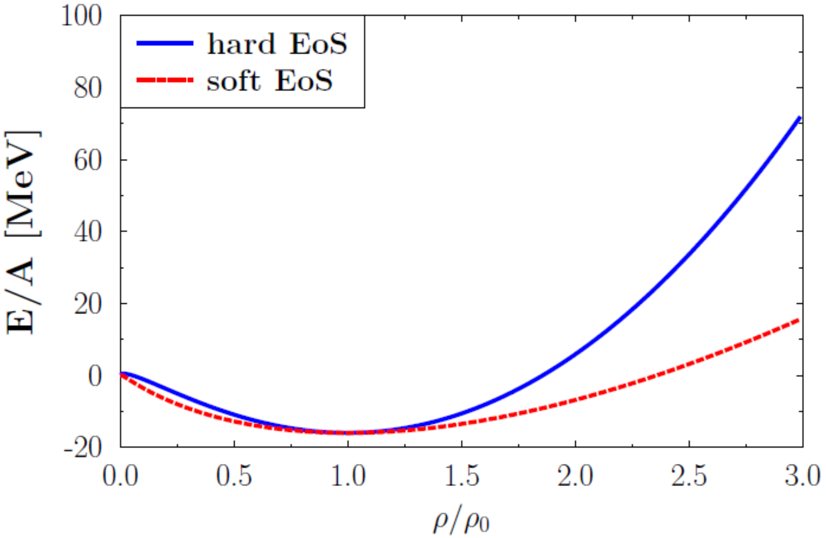
- **modifed interaction density (with relativistic extension):**

$$\rho_{int}(\mathbf{r_{i0}}, t) \rightarrow C \sum_j (\frac{4}{\pi L})^{3/2} e^{-\frac{4}{L}(\mathbf{r}_{i0}^{T}(t) - \mathbf{r}_{j0}^{T}(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^{L}(t) - \mathbf{r}_{j0}^{L}(t))^2},$$

- **HIC ←→ EoS for infinite matter at rest**
- **compression modulus K of nuclear matter:**

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

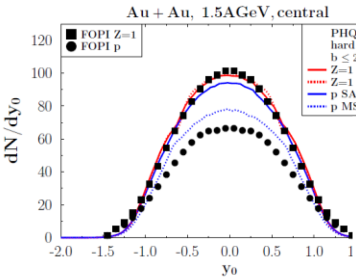
EoS for infinite matter at rest

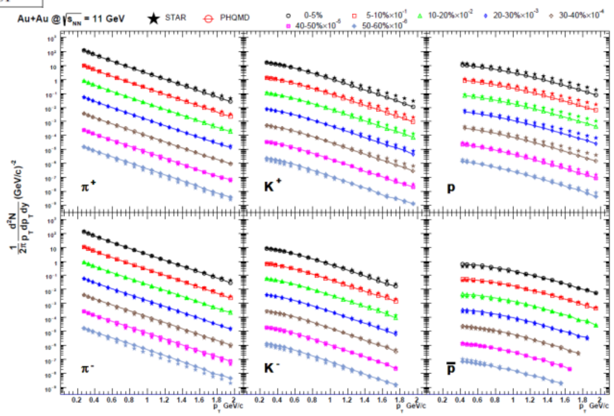


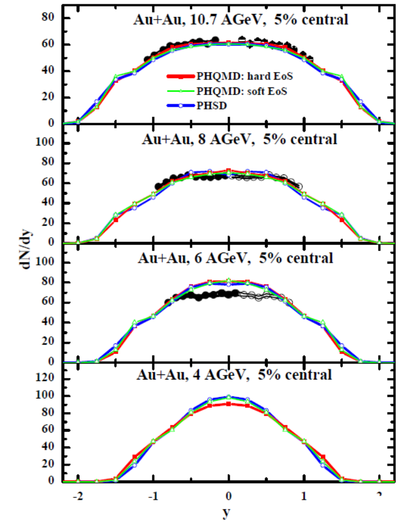
*Work in progress: implementation of momentum-dependent potential (M. Winn)

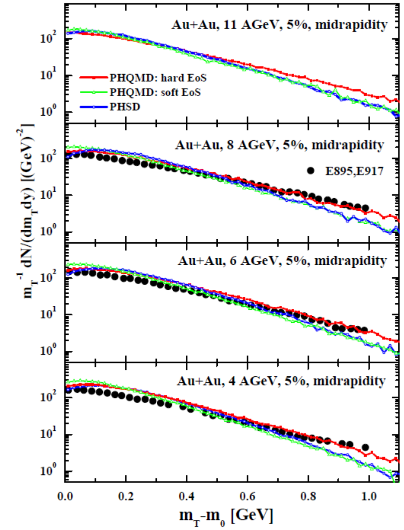
Highlights: PHQMD ‚bulk' dynamics from SIS to RHIC

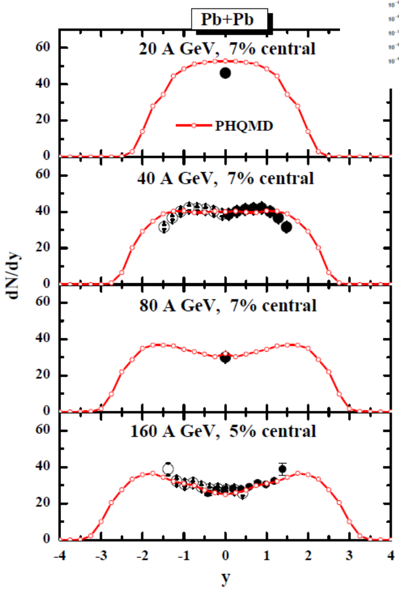
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905

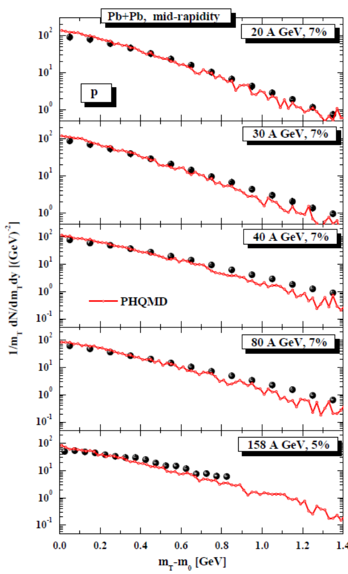


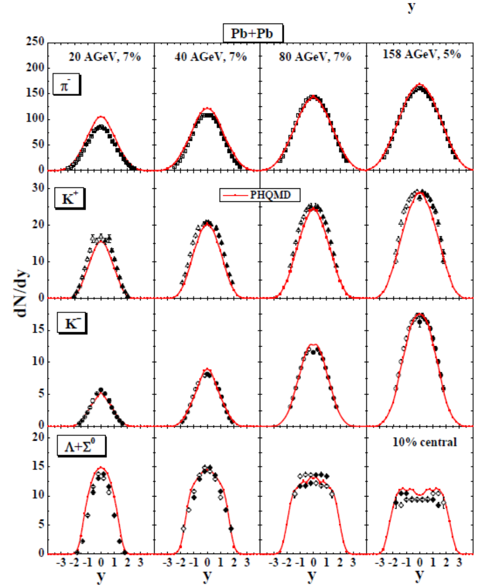


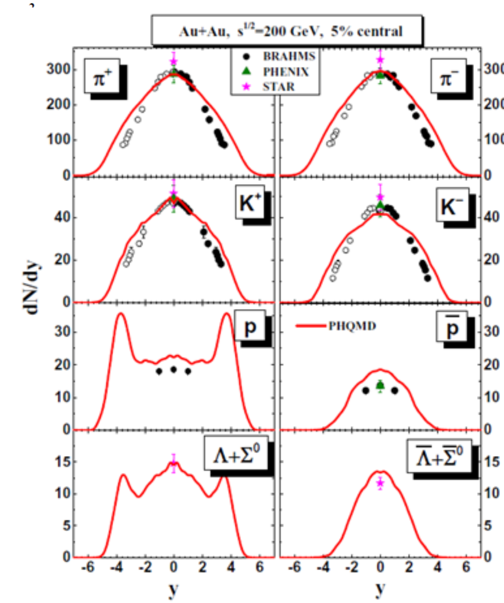










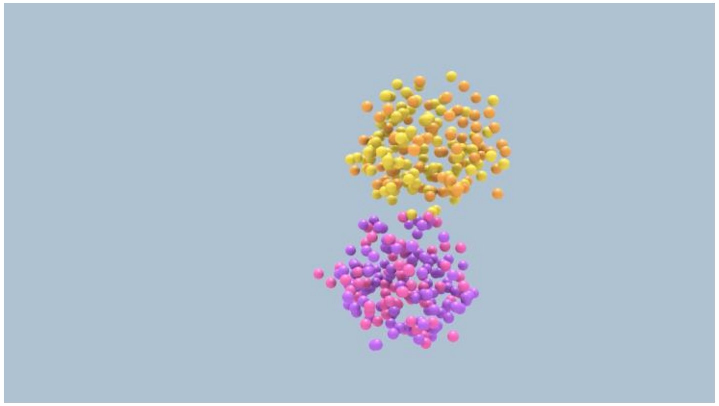


PHQMD provides a good description of hadronic 'bulk' observables from SIS to RHIC energies

I. Potential mechanism for cluster production in PHQMD: MST & SACA

PHQMD

Time evolution: Au+Au, b=2 fm, 600 AGeV



By M. Winn

Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

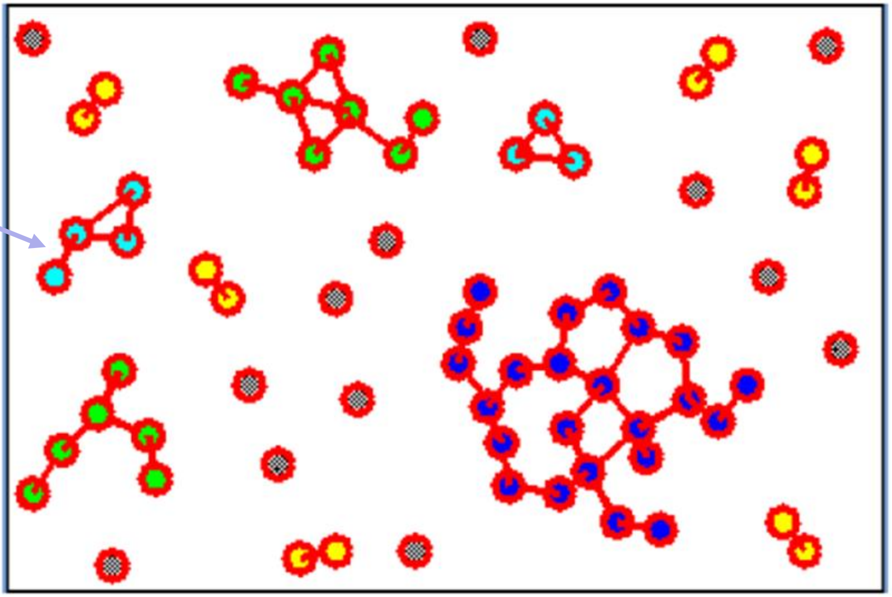
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are 'bound' if their distance in the cluster rest frame fulfills

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm}$$

2. Particle is bound to a cluster if it binds with at least one particle of the cluster.

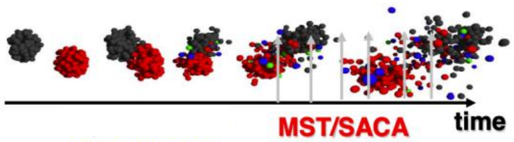


* Remark:
inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are mostly not at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)

- MST + extra condition: $E_B<0$ negative binding energy for identified clusters

Cluster stability in semi-classical models

Cluster stability problem in semi-classical models (as QMD):



QMD can not describe clusters as 'quantum objects'
➔ the cluster quantum ground state has to respect a minimal average kinetic energy of the nucleons while the semi-classical (QMD) ground state - not!
➔ nucleons may still be emitted from the QMD clusters while in the corresponding quantum system this is not possible
➔ thus, a cluster which is "bound" at time t can spontaneously dissolve at $t + \Delta t$

= QMD clusters are not fully stable over time:
➔ the multiplicity of clusters is time dependent
➔ the form of the final rapidity, p_T distribution and ratio of particles do not change with time

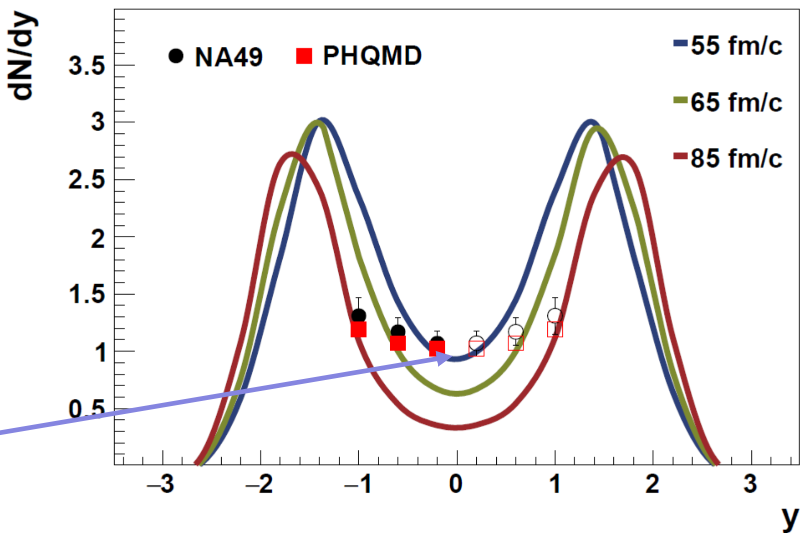
How to compare with experiments?

S. Gläßel et al., PRC 105 (2022) 1

PHQMD results are taken at 'physical time' :

$$t = t_0 \cosh(y)$$

where t_0 is the time selected as a best description of the cluster multiplicity at y=0



MST with stabilized fragments

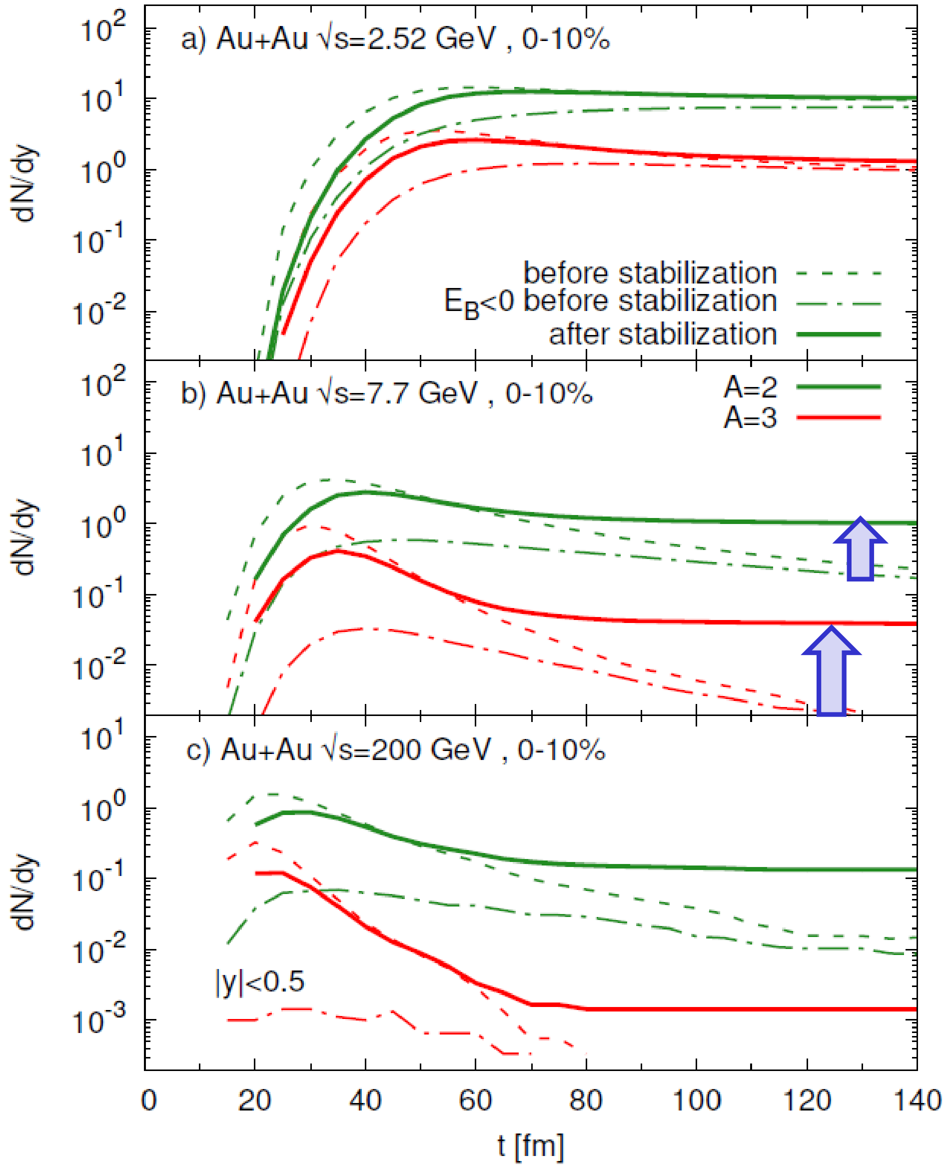
Lorentz transformation between computational frame and cm frame of clusters destabilizes cluster
→ Binding energy gets positive

Correction Procedure:

- For each nucleon in MST track the freezout-time = time at which the last collision occurred

- If after freeze out (for all N) E_B becomes positive: keep the cluster as before the disintegration

Allows to recover most of "lost" clusters and all clusters are bound

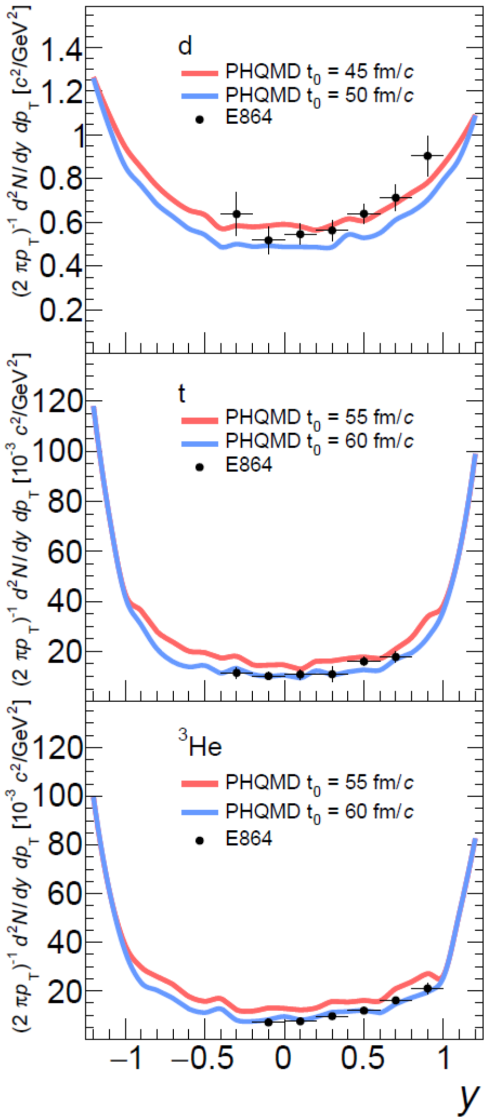


G. Coci et al., in preparation

Cluster production in HICs at AGS energies

y- distributions of d, t, ^{3}He **Scenario 1:** **p_T - distribution of deuterons**

Au+Pb@10.6 AGeV



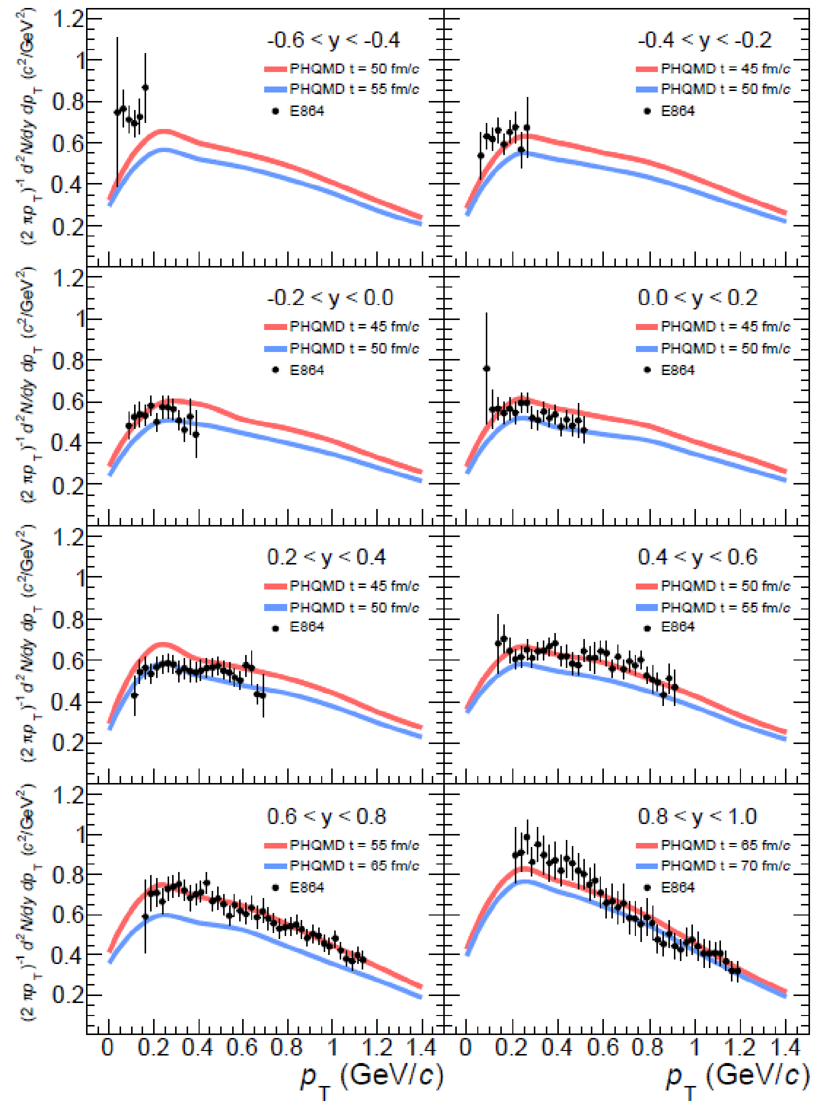
The PHQMD results for the y-distribution are taken at **'equal physical time'**

$t = t_0\cosh(y)$,

where t_0 is the time at y=0

Consider t_0 =45 and 50 fm/c

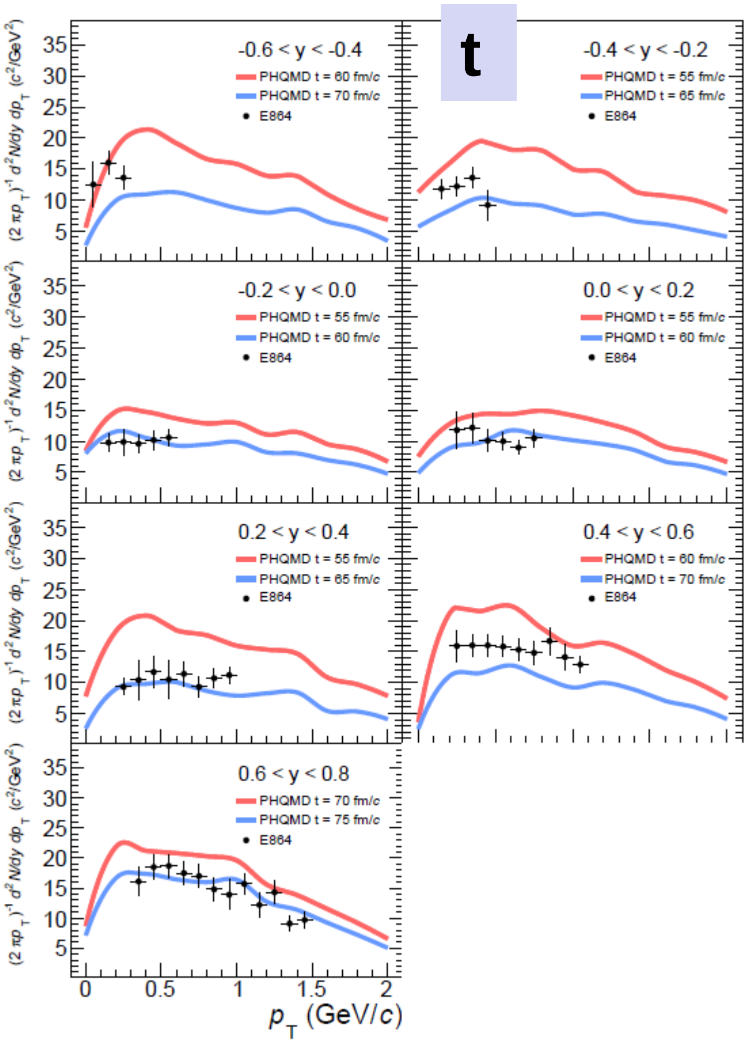
Au+Pb@10.6 AGeV

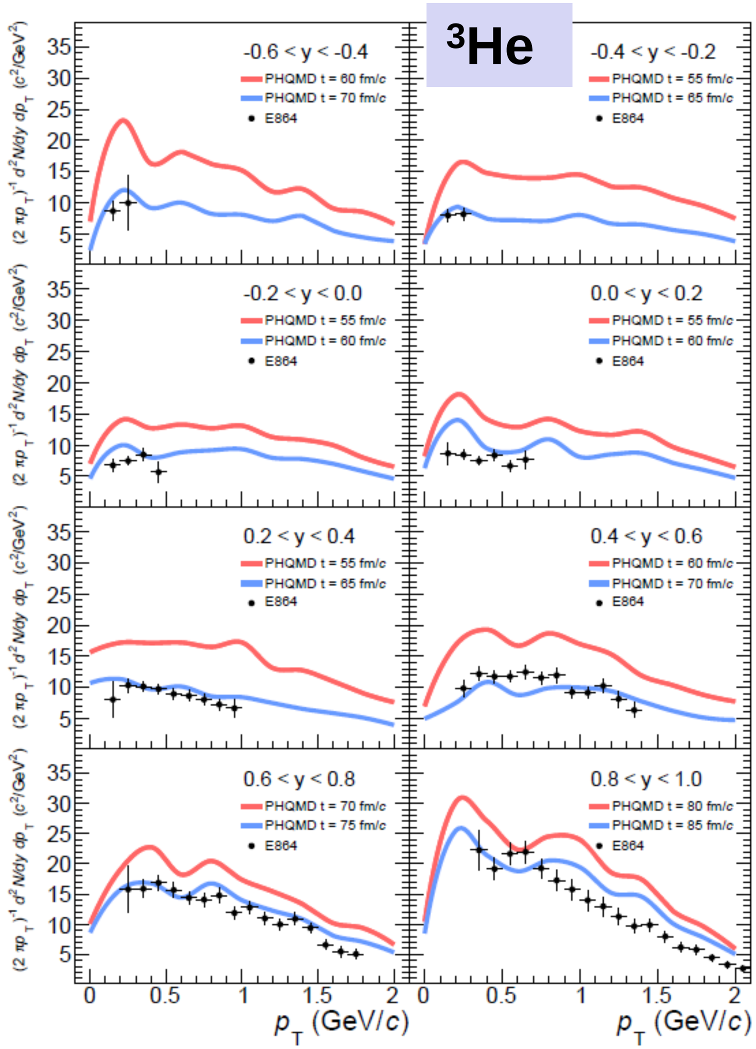


S. Gläßel et al., PRC 105 (2022) 1

Cluster production in HICs at AGS energies

The p_T - distributions of t and ^{3}He from Au+Pb at 10.6 A GeV



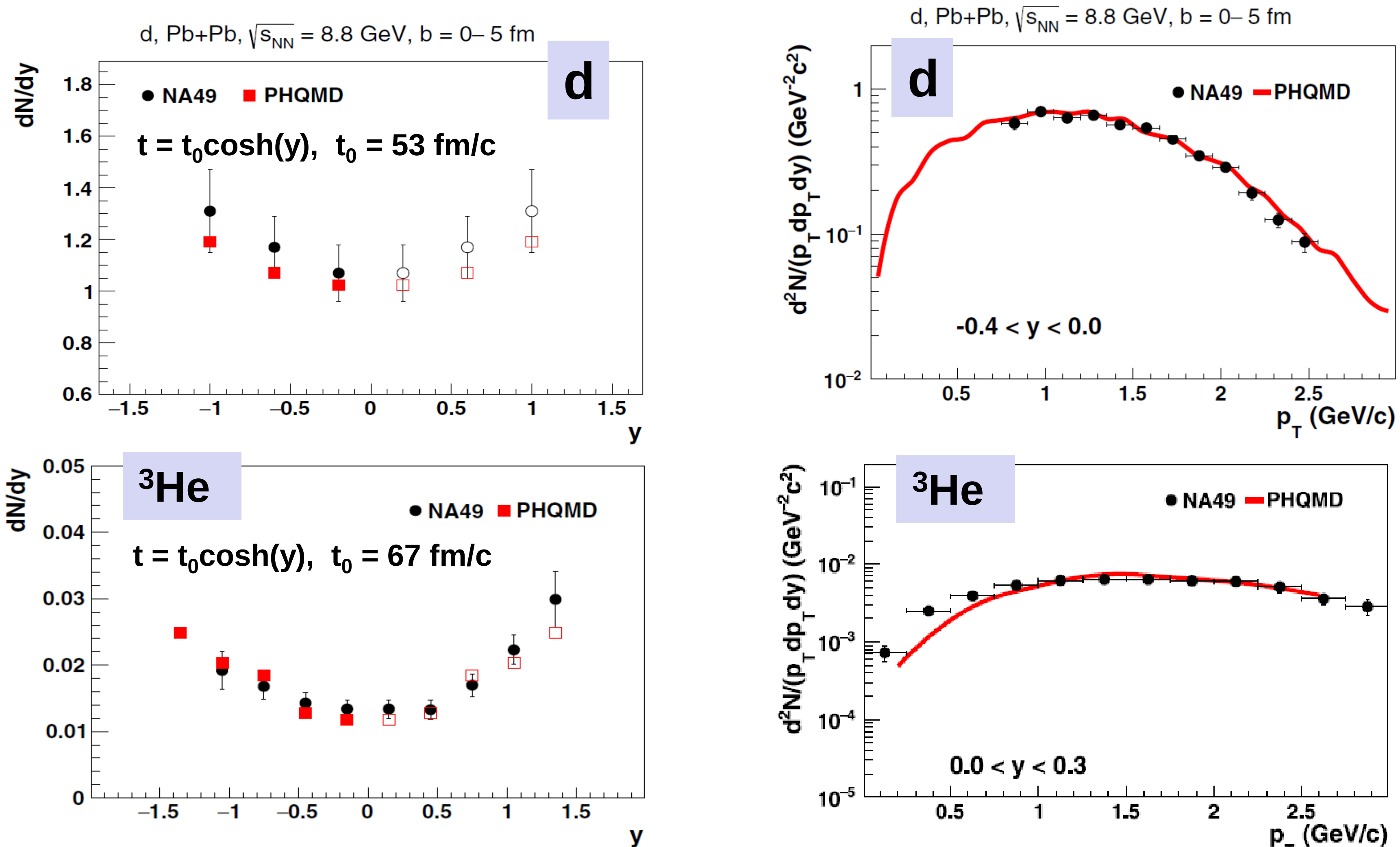


S. Gläßel et al., Phys. Rev. C 105 (2022) 1

Cluster production in HICs at SPS energies

PHQMD

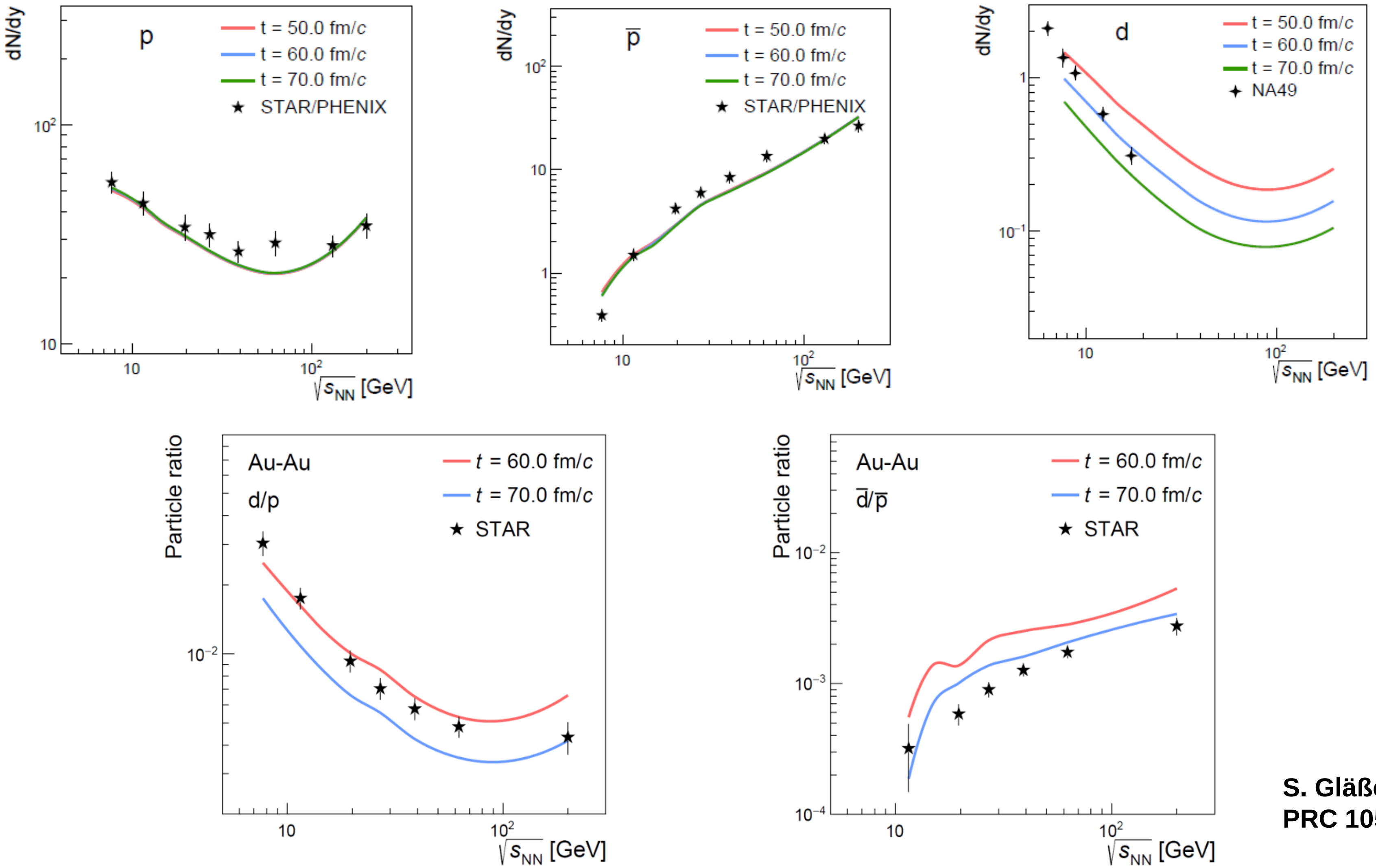
The rapidity and p_T-distributions of d and ^{3}He from Pb+Pb at 30 A GeV



The PHQMD results for d and ^{3}He agree with NA49 data

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

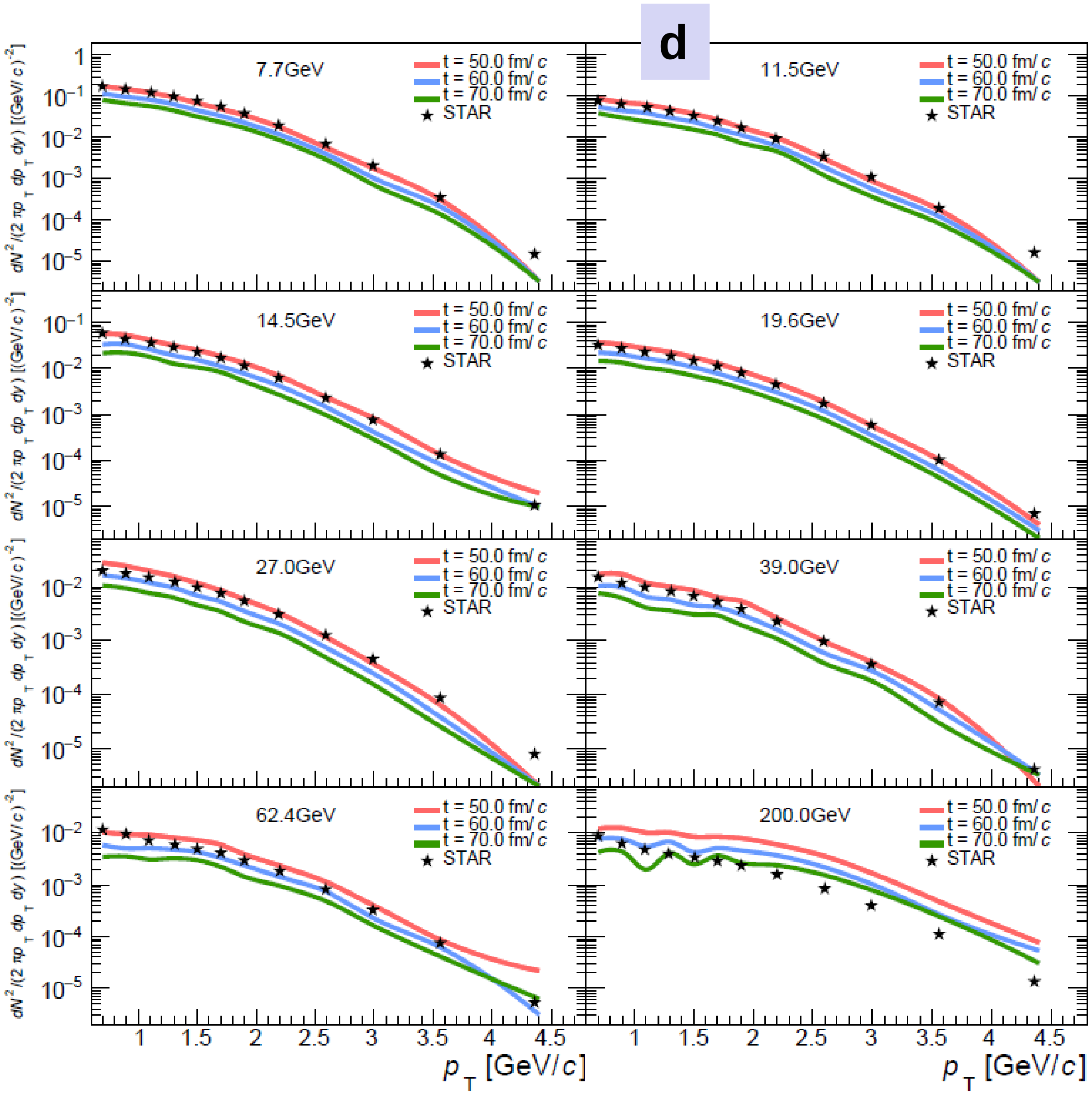
Excitation function of multiplicity of $p, \bar{p}, d, \bar{d}$



S. Gläßel et al., PRC 105 (2022) 1

The $p, \bar{p}$ yields at y~0 are stable, the $d, \bar{d}$ yields are better described at *t= 60-70* fm/c

Deuteron p_T spectra from 7.7GeV to 200 GeV



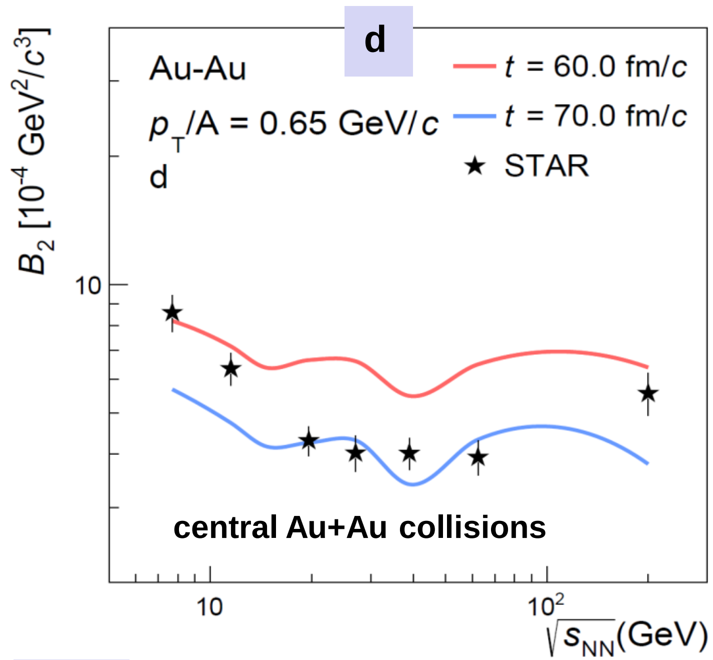
Comparison of the PHQMD results for the deuteron p_T -spectra at midrapidity with STAR data

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

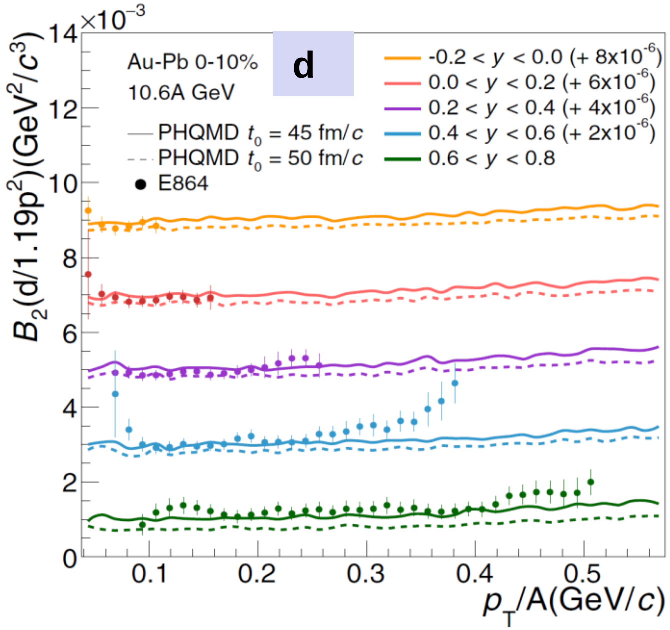
Coalescence parameter B_2 for d and ^{3}He

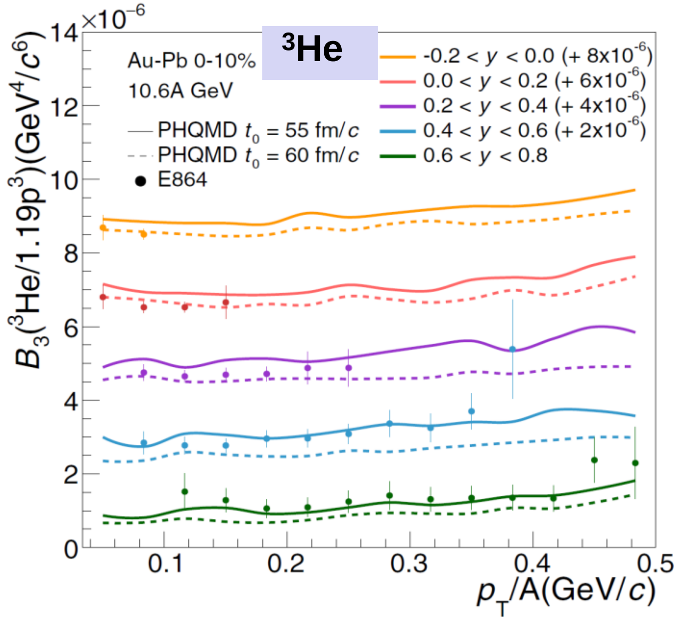
Coalescence parameter B_2:

$$B_2 = \frac{E_d \frac{d^3 N_d}{d^3 P_d}}{\left(E_p \frac{d^3 N_p}{d^3 p_p}\Big|_{p_p = P_d/2}\right)^2}$$



S. Gläßel et al., Phys. Rev. C 105 (2022) 1



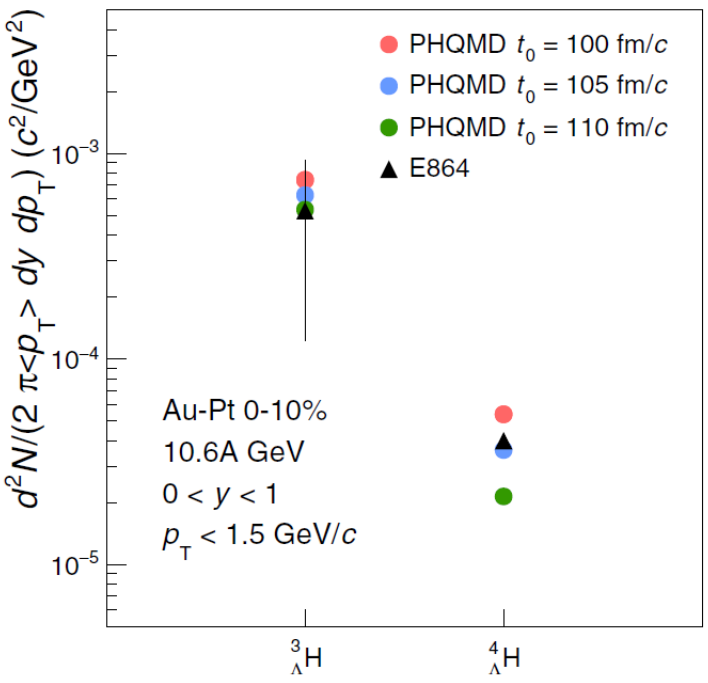


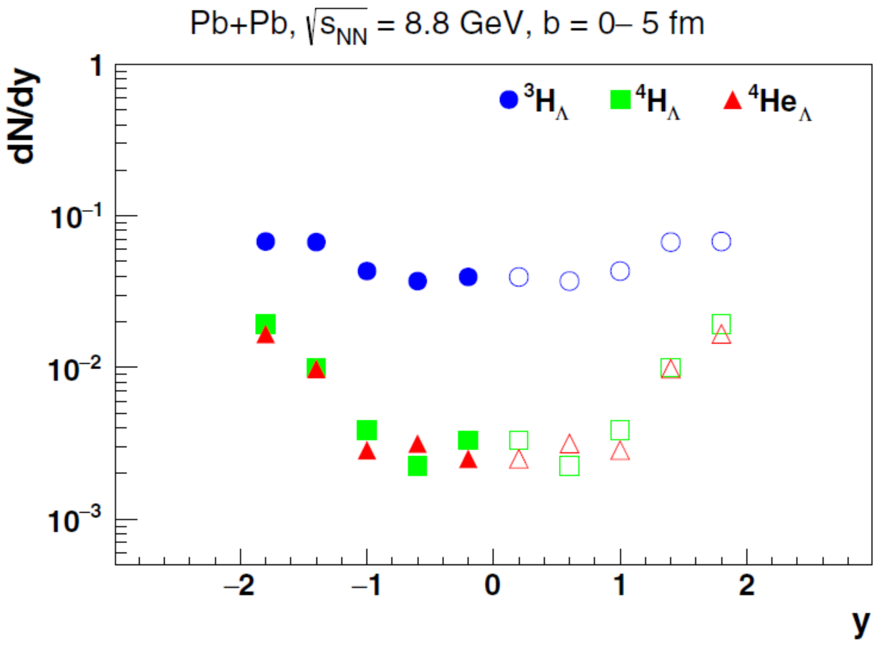
$$t = t_0 \cosh(y)$$

Hypernuclei production at 10.6 and 30 AGeV

The PHQMD results for hypernuclei production in Au+Pt central collisions at 10.6 A GeV

The PHQMD predictions for dN/dy of $^3H_\Lambda$, $^4H_\Lambda$ and $^4He_\Lambda$ from central Pb+Pb collisions at 30 A GeV ($s^{1/2}$ = 8.8 GeV)





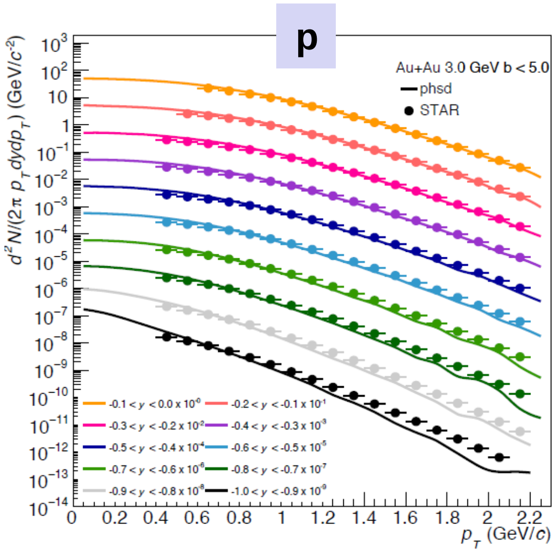
- Assumption on nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$

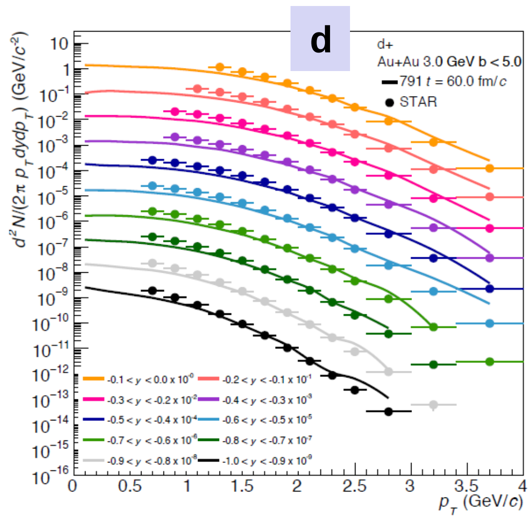
S. Gläßel et al., Phys. Rev. C 105 (2022) 1

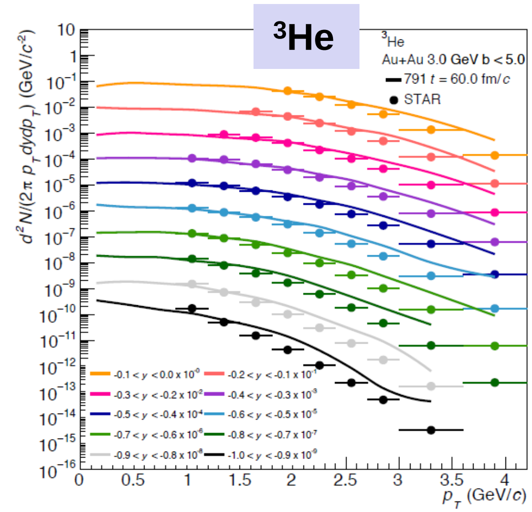
Light cluster production at $s^{1/2}$ = 3 GeV

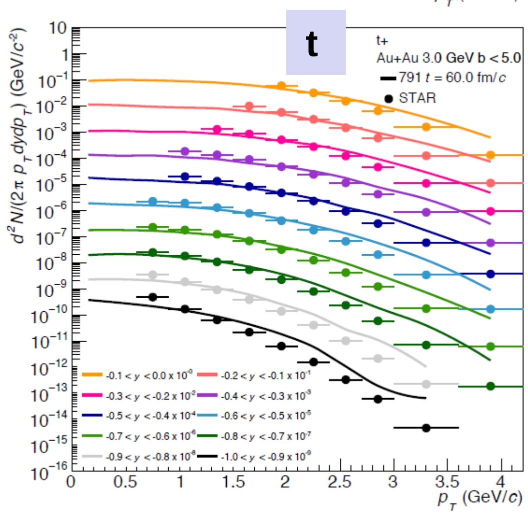
The PHQMD comparison with recent STAR fixed target p_T distribution of p, d, t, ^{3}H, ^{4}H from Au+Au central collisions at $\sqrt{s}$ =3 GeV

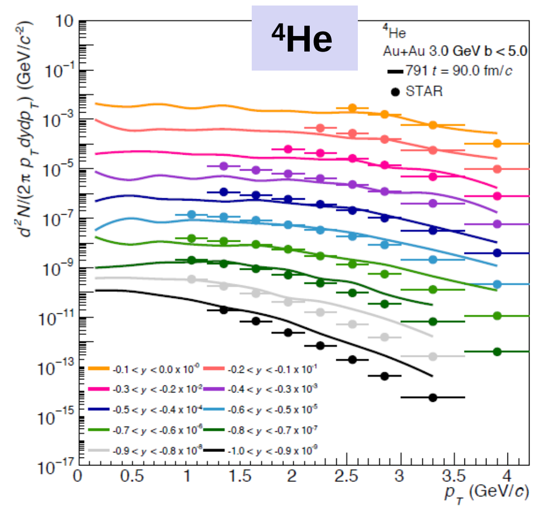
PHQMD: t = 60 fm/c











(preliminary) STAR data – talk by Hui Liu at QM'2022

➔ **Good description of cluster production**

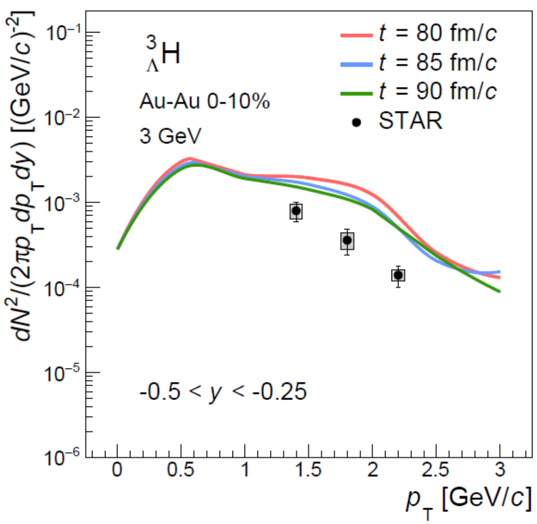
S. Gläßel, PHQMD calculations: arXiv:2208.11802

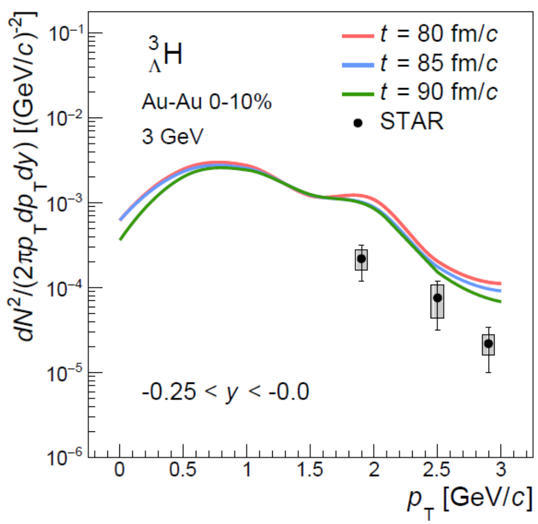
Hypernuclei production at $s^{1/2}$ = 3 GeV

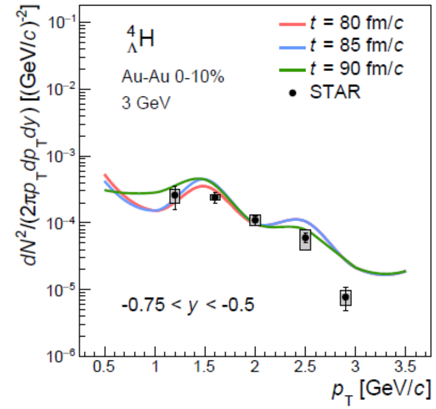
The PHQMD comparison with recent STAR fixed target p_T distribution of $^3H_\Lambda$, $^4H_\Lambda$ from Au+Au central collisions at $\sqrt{s}$ =3 GeV

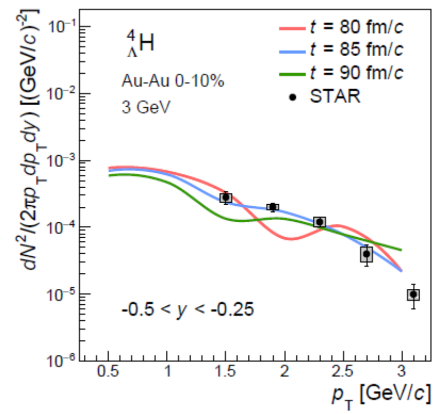
STAR: Phys. Rev. Lett. 128, 202301 (2022)

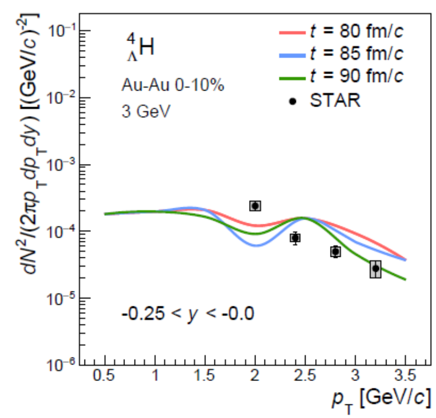
- **Assumption for nucleon-hyperon potential: $V_{N\Lambda} = 2/3\ V_{NN}$**









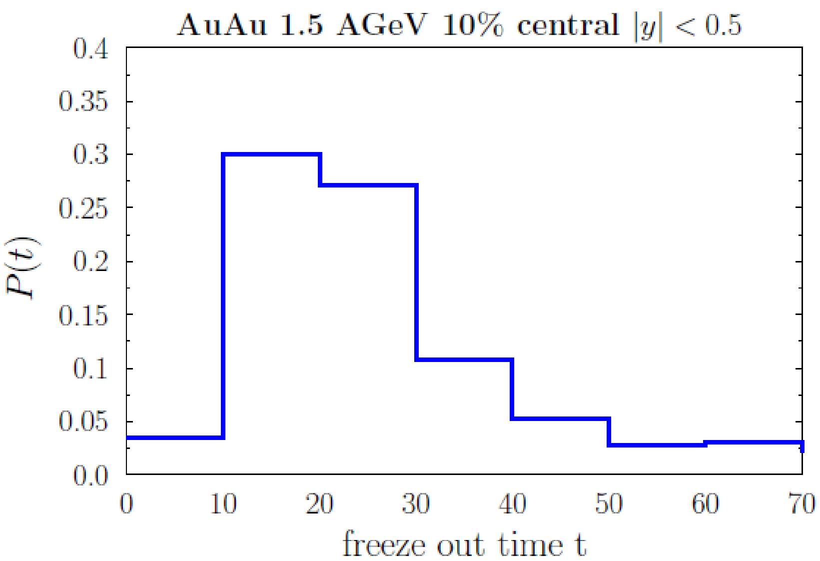


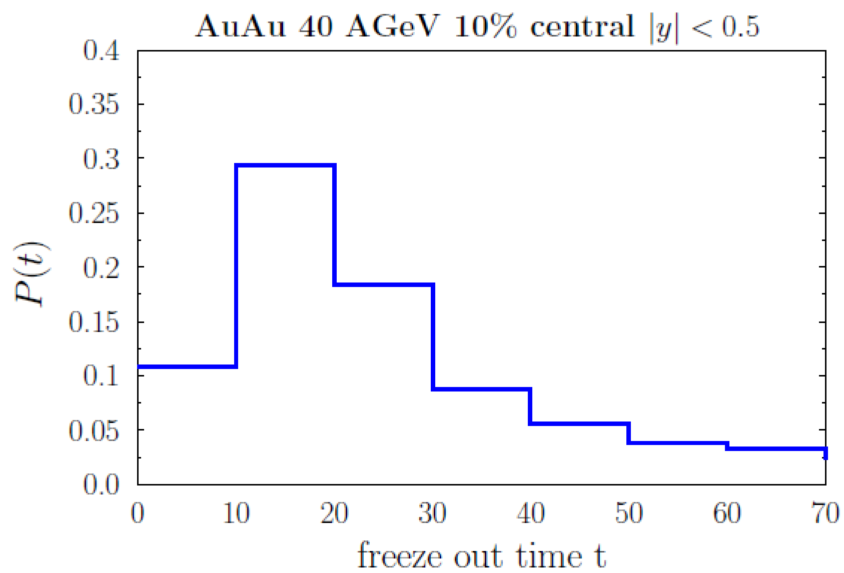
➔ **Reasonable description of hypernuclei production at $\sqrt{s}$ =3 GeV**

PHQMD: S. Gläßel et al., Phys. Rev. C 105 (2022) 1

When does the system freeze out?

- The normalized distribution of the freeze-out time of baryons (nucleons and hyperons) which are finally observed at mid-rapidity |y|<0.5
  * Here freeze-out time as defined by the last elastic or inelastic collision, after that only potential interaction between baryons occurs



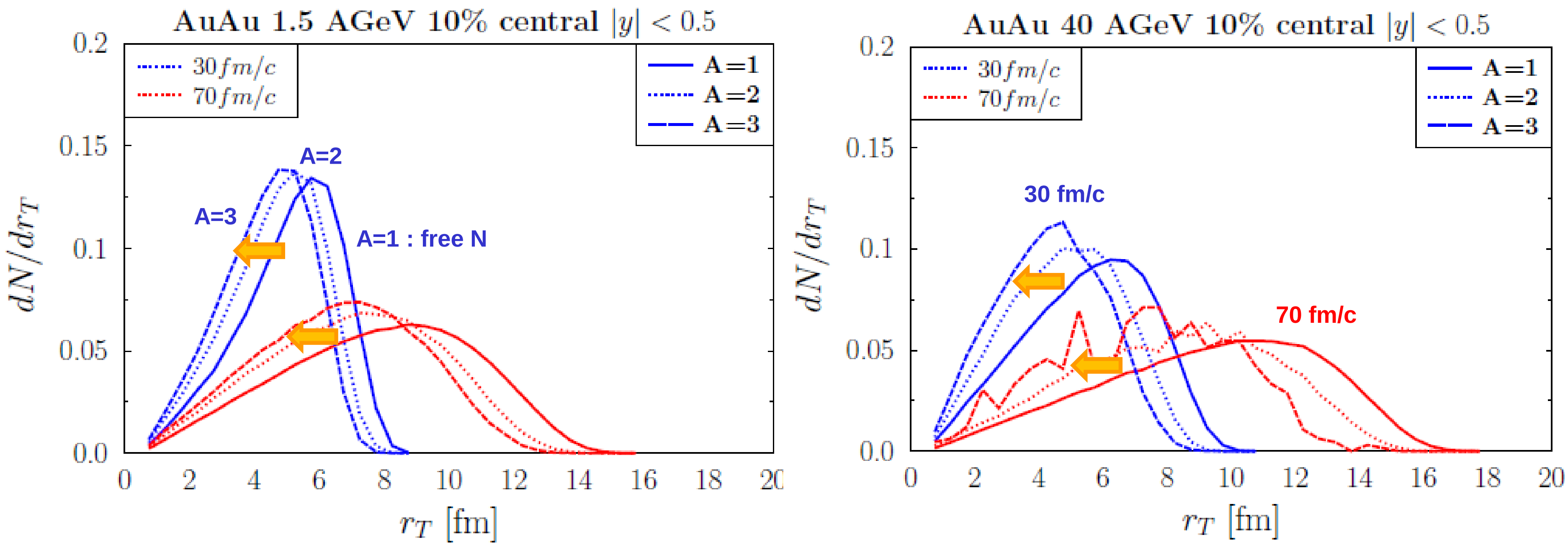


➔ **Freeze-out time of baryons in Au+Au at 1.5 AGeV and 40 AGeV:**
- **similar profile since expansion velocity of mid-rapidity fireball is roughly independent of the beam energy**

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

Where are the clusters formed?

- The snapshot (taken at time 30 and 70 fm/c) of the **normalized distribution of the transverse distance r_T of the nucleons** to the center of the fireball.
- It is shown for A=1 (free nucleons) and for the nucleons in A=2 and A=3 clusters

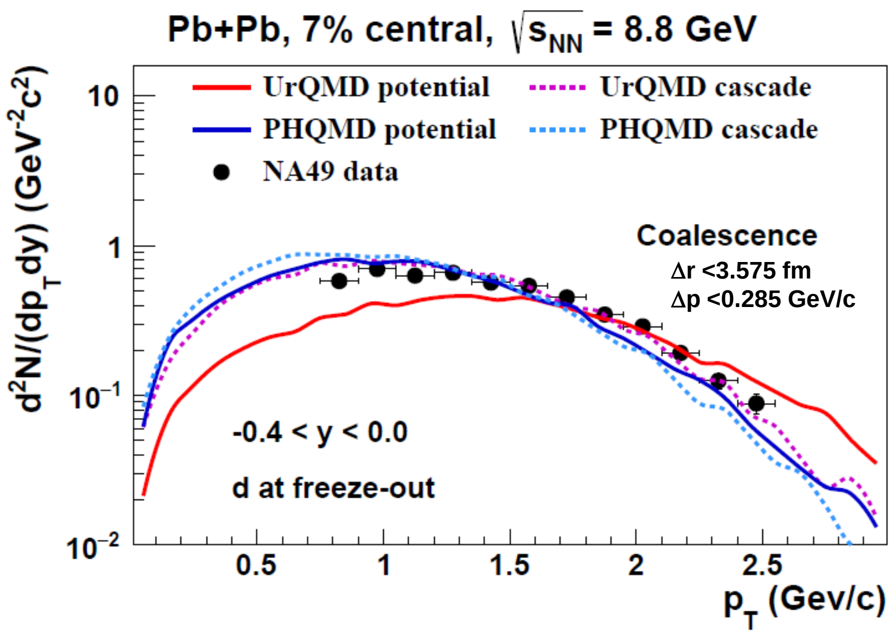


➔ **Transverse distance profile** of **free nucleons and clusters are different**!
➔ Clusters are mainly formed **behind the 'front' of free nucleons** of expanding fireball
➔ 'ice' is behind the 'fire' ➔ cluster can survive

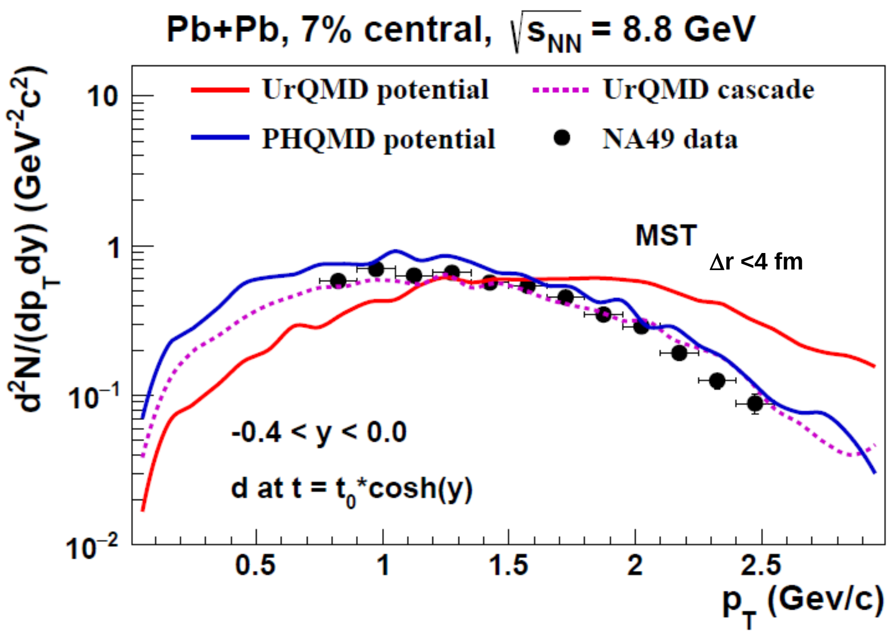
S. Gläßel et al., Phys. Rev. C 105 (2022) 1

Comparison of the coalescence and MST for d

Coalescence



MST



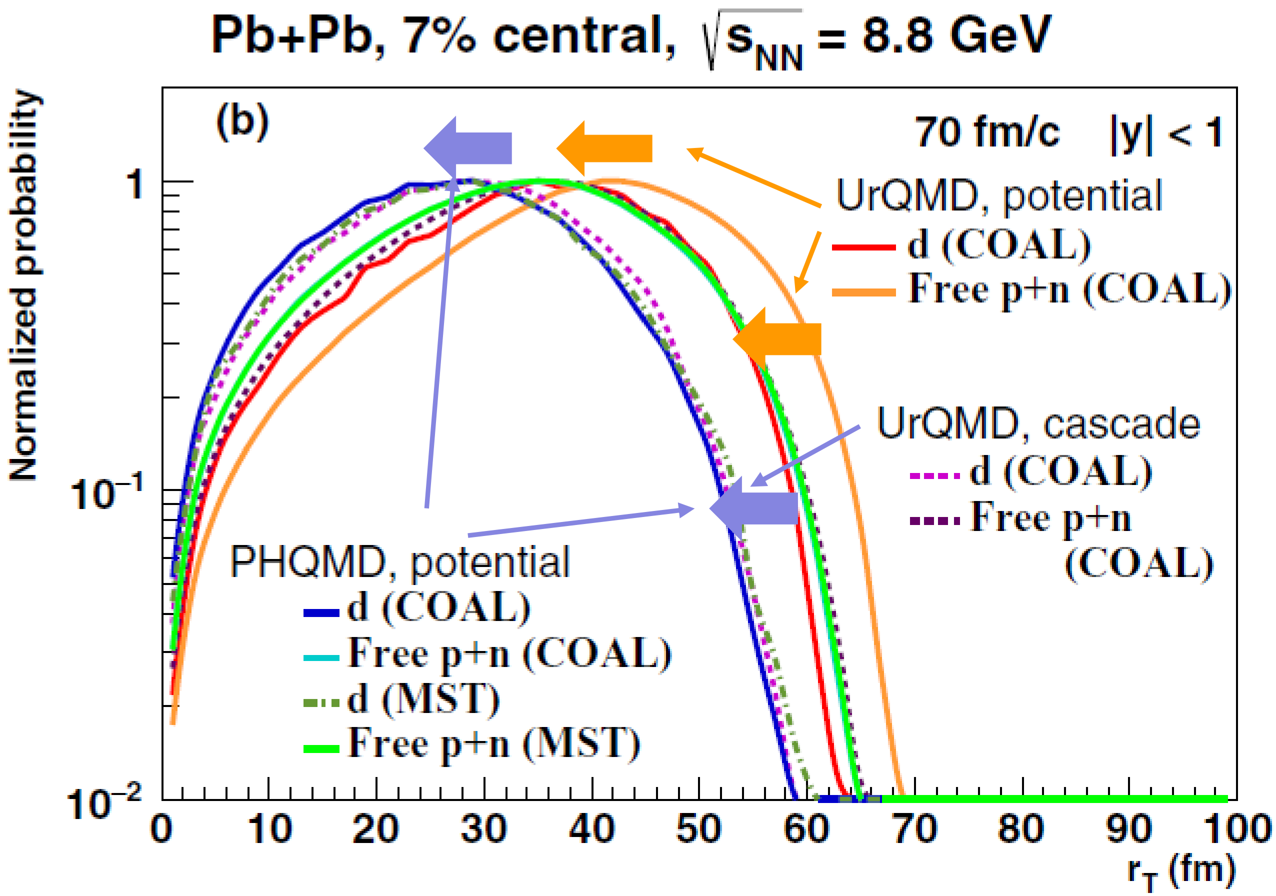
➔ Coalescence and MST give very similar multiplicities and y- and p_T –distributions

➔ PHQMD and UrQMD results in the cascade mode are very similar

➔ Deuteron production is sensitive to the realization of potential in transport approaches

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

Comparison of the coalescence and MST for d



➔ **Coalescence as well as the MST procedure show that the deuterons remain in transverse direction closer to the center of the heavy-ion collision than free nucleons**

➔ **deuterons are behind the fast nucleons (and pion wind)**

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

II. Kinetic mechanism for deuteron production in PHQMD

PHQMD

Gabriele Coci et al., in preparation

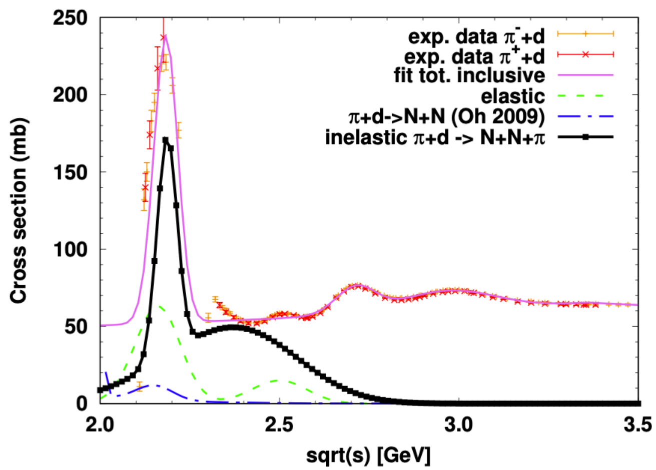
Deuteron production by hadronic reactions

"Kinetic mechanism"

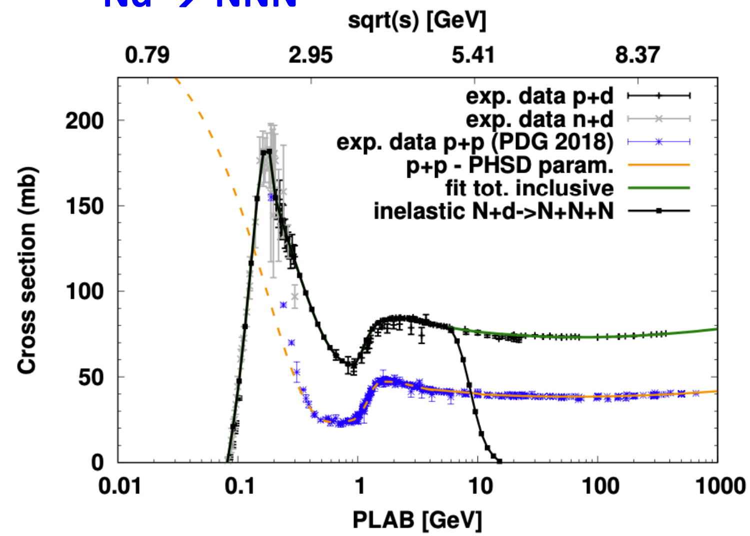
1) **hadronic inelastic reactions NN ⟷ dπ , πNN ⟷ dπ , NNN ⟷ dN**
2) **hadronic elastic π+d, N+d reactions**

- **Hadronic reactions for d+π and d+N scattering have very large cross sections σ_{peak} ≈ 200 mb**

πd → πNN



Nd → NNN

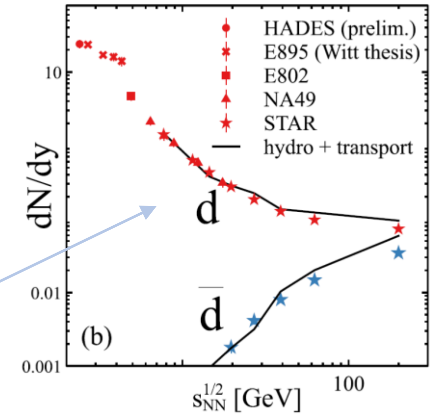


- **the rates for the inverse processes pNN →pd, NNN→dN in hadronic matter are large due to the time-reversal symmetry**

* **Kinetic production by inverse reaction N + p + n → N + d first studied in HICs at E_{Lab} ∽ 1 AGeV by P.J. Siemens, J. Kapusta PRL 43 (1979) 1486**

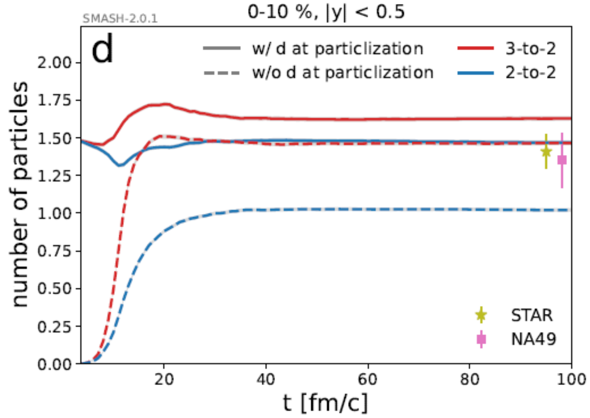
Models for deuteron production by hadronic reactions

- **SMASH, AMPT: Inverse reactions X+N+N → X+d (X=π,N with X catalyzer) important for d formation in HICs**
- **at RHIC and LHC energies: large π abundance**
  → deuterons form by π-catalysis: π+p+n → π+d
  at SIS energies: large N abundance
  → deuterons form by N-catalysis: N+p+n → N+d



- **SMASH (hydro + kinetic): $\pi NN \leftrightarrow d\pi$, $NNN \leftrightarrow dN$ are realized**

1) **via fictitious dibaryon resonance d' as two-step processes of N+N → d' and π + d' → π + d**
   D. Oliinychenko PRC 99 (2019) 4, 044907
2) **via 3 ⟷ 2 transition rates**
   J. Staudenmaier et al., PRC 104 (2021) 3, 034908



- **AMPT: $\pi NN \leftrightarrow d\pi$ via impulse approximation:**

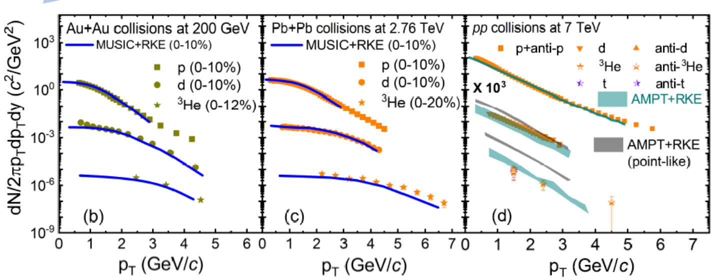
$$\mathcal{M}_{\pi d\to\pi NN} \rightarrow \langle\tilde{p}_N|\phi_d\rangle\mathcal{M}_{\pi N\to\pi N}$$

+ accounting of the finite size of deuterons via Wigner function

$$|\phi_d(\tilde{\mathbf{p}})|^2 = \int d^3\mathbf{r}\gamma_d W_d = (4\pi\sigma_d^2)^{3/2}e^{-\tilde{\mathbf{p}}^2\sigma_d^2}$$

leads to the suppression of d production in pp

K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742



Kinetic deuterons in PHQMD

Deuteron production are three body collisions (three body phase space)

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

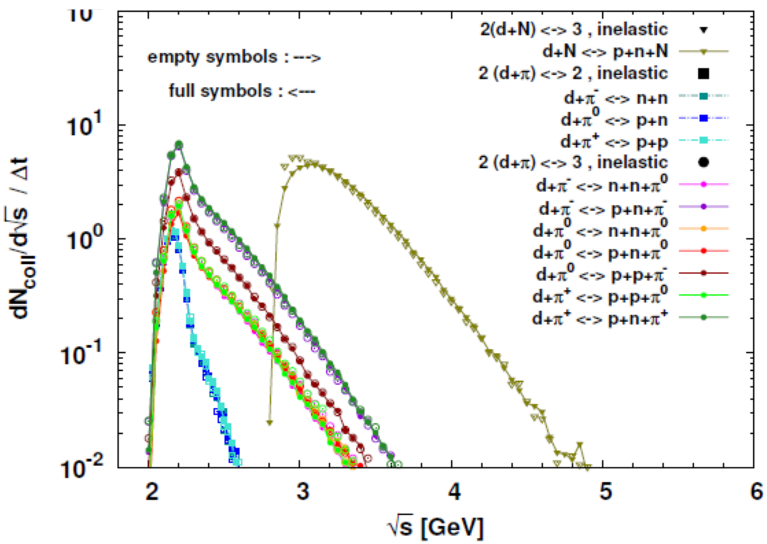
$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$

$$\pi^- + p + p \leftrightarrow \pi^0 + d$$

$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$

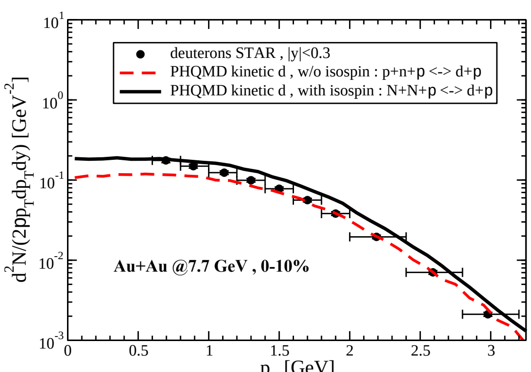
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$

$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

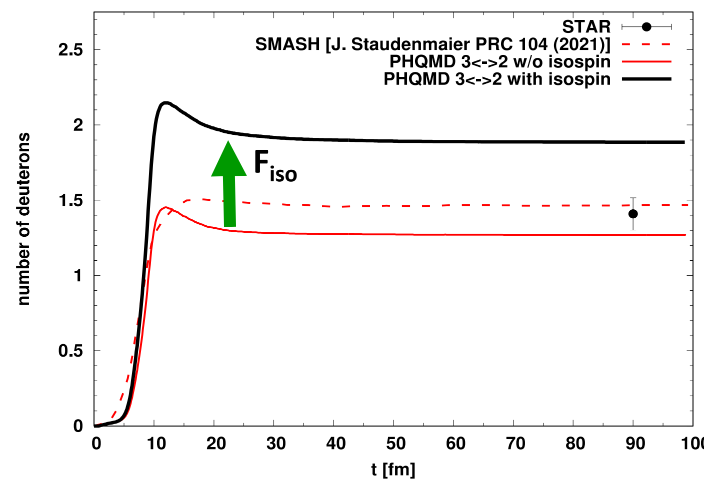


→ **Detailed balance** condition fulfilled

Including all isospin channels enhances deuteron yield by 80%



p_T slope not affected



Kinetic + potential deuterons in PHQMD

In the most recent study: Gabriele Coci et al., in preparation

- **Kinetic deuterons** (all isospin channels)

We assume that initially the nucleons have to be correlated assuming that they form a deuteron with the probability

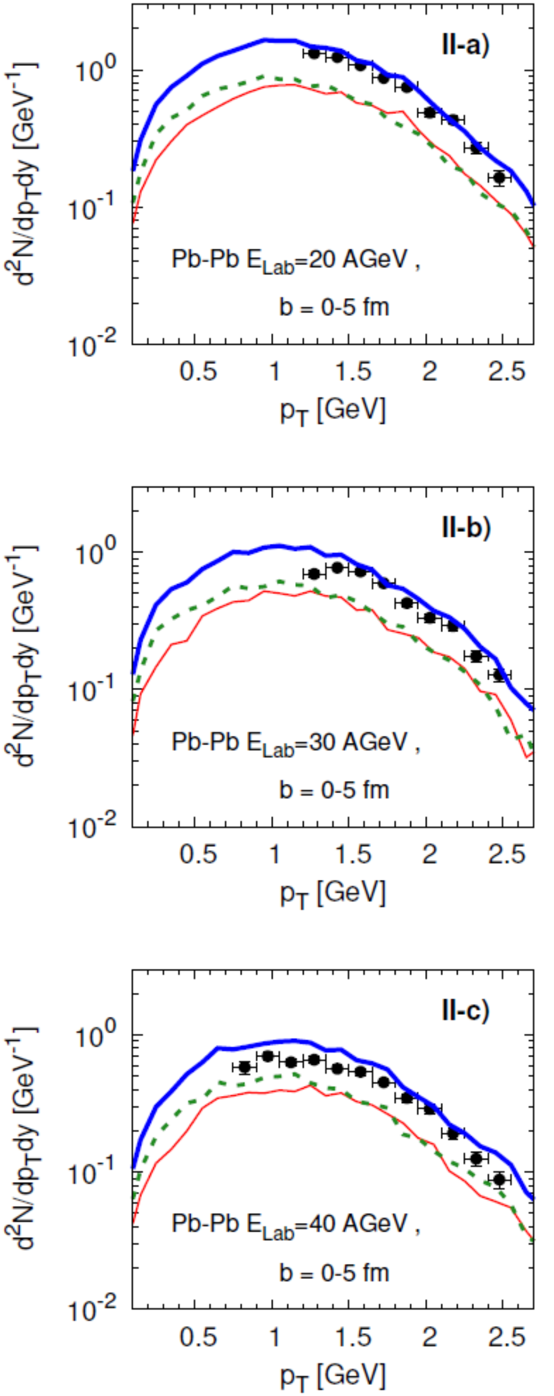
$$|\phi_d(\tilde{\mathbf{p}})|^2 = \int d^3\mathbf{r}\gamma_d W_d = (4\pi\sigma_d^2)^{3/2} e^{-\tilde{\mathbf{p}}^2\sigma_d^2}$$

- **Potential deuterons**

Identified by MST

Only clusters with negative binding energy which were stabilized (if necessary)

Both together describe quite correctly the NA49 data



Summary

- The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

- Clusters are formed dynamically by potential interactions among nucleons and hyperons and identified by Minimum Spanning Tree model

- Kinetic mechanism for deuteron production is implemented in the PHQMD with inclusion of full isospin decomposition for hadronic reactions which enhances d production
- However, accounting for the quantum properties of the deuteron, modelled by the projection of the relative momentum of the interacting pair of nucleons on the deuteron wave-function in momentum space, leads to a strong reduction of d production, especially at target/projectile rapidities

- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dp$_T$ as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from AGS to top RHIC energies.

A detailed analysis reveals that stable clusters are formed

- shortly after elastic and inelastic collisions have ceased
- behind the front of the expanding energetic hadrons
- since the 'fire' is not at the same place as the 'ice', cluster can survive

- Coalescence and MST give very similar deuteron distributions within the PHQMD and UrQMD transport approaches

Outlook

PHQMD:

- LHC energies → numerous computational efforts
- Momentum-dependent potential - important for low energies of SIS, FAIR
- Realistic description of hyperon-nucleon potential – important for hypernuclei dynamics
- Kinetic formation of light clusters like t, He ?
- Extended study of collective observables for clusters

New experimental data are needed!

y-distributions → mechanisms for cluster formation at large y

Collective observables v_1, v_2, …