a new dynamical microscopic model for quarkonia production in Relativistic HIC

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NED Workshop

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Based on arxiv 2206.01308

With Joerg Aichelin, Denys Yen Arrebato Villar, Jiaxing Zhao

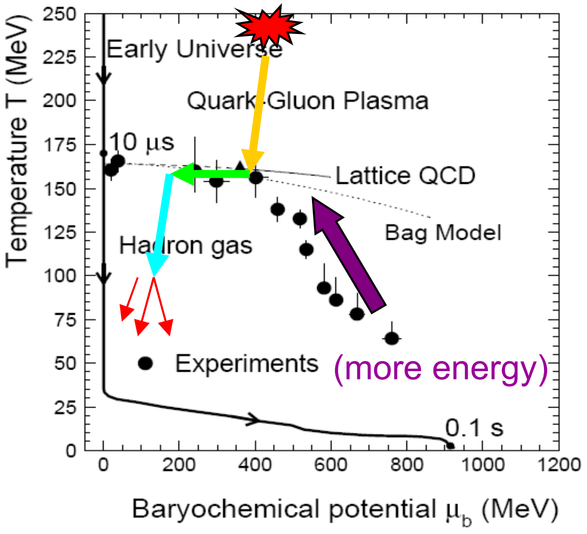
Nice discussions with Taesoo Song, Elena Bratkovskaya and Klaus Werner

Subatech and Pays de la Loire IMT Atlantique Bretagne-Pays de la Loire École Mines-Télécom cnrs IN2P3 Nantes Université STRONG 2020

Investigating the QGP, How ?

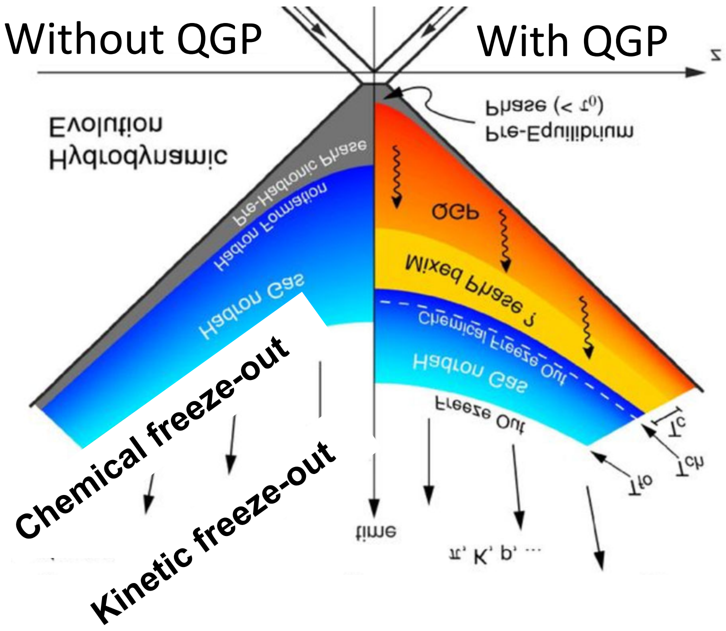
Q: Does the system created in central region reach and maintain equilibrium long enough (10 fm/c) to be understood in terms of a quasi-stationnary state ?

=> Hadro-chemistry of the final state as a *thermometer* (# and spectra):



P. Braun-Munzinger & J. Wambach (arXiv:0801.4256)

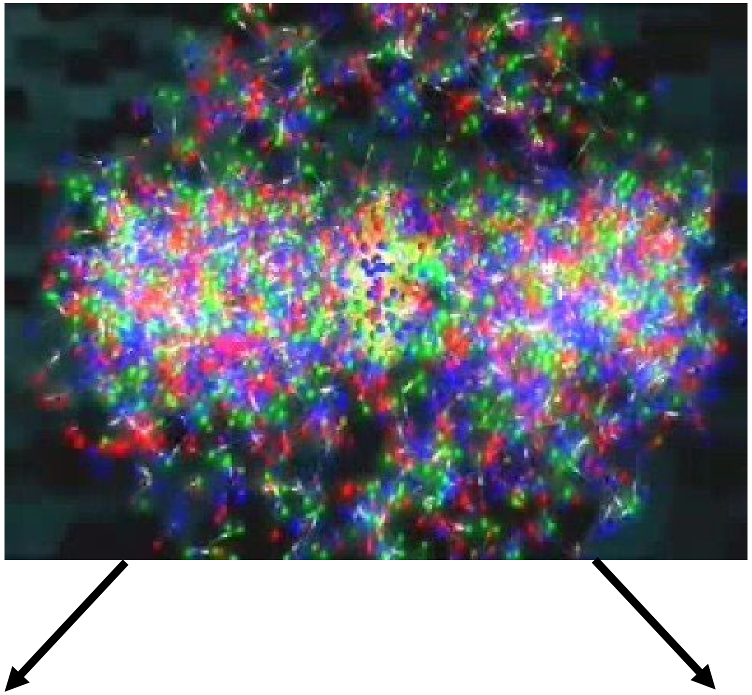
T(t)



Memory Loss !

But How can we proceed to probe the QGP “beyond the horizon”, that is “before the freeze out” ?

Investigating the QGP, How ?



Method #1 : "soft probes"

Although memory lost locally, the final stage results from the convolution of 1) initial stage & 2) **(fluid) dynamical evolution of QGP**, sensitive to various key aspects : EOS & transport coeffcients (viscosity η,...) => allows to constrain QGP properties

Method #2 : "penetrating probes"

Identify some particle / object / mode that does not (completely) loses its memory through QGP evolution.

Quarkonia suppression as HP

Expected medium effects : the « Quarkonia suppression »

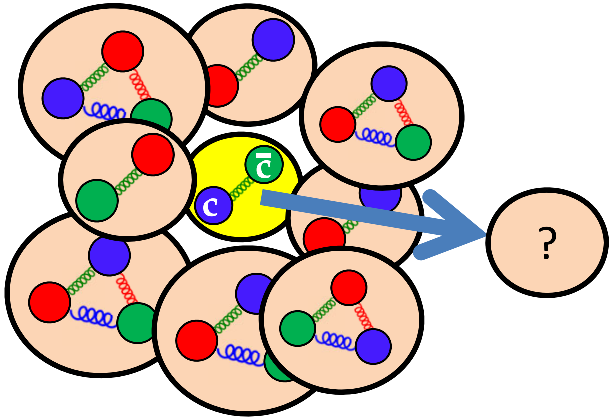
Smaller amount of quarkonia produced in heavy ion collisions per binary nucleon collision as compared to pp collisions.

Quantified with the *nuclear modfication factor:*

$$R_{\mathrm{AA}}(p_{\mathrm{T}},\eta) = \frac{dN^{\mathrm{AA}}/d^2p_{\mathrm{T}}d\eta}{\langle N_{\mathrm{coll}}\rangle\, dN^{\mathrm{pp}}/d^2p_{\mathrm{T}}d\eta}$$

Different contributions

In hadronic phases:

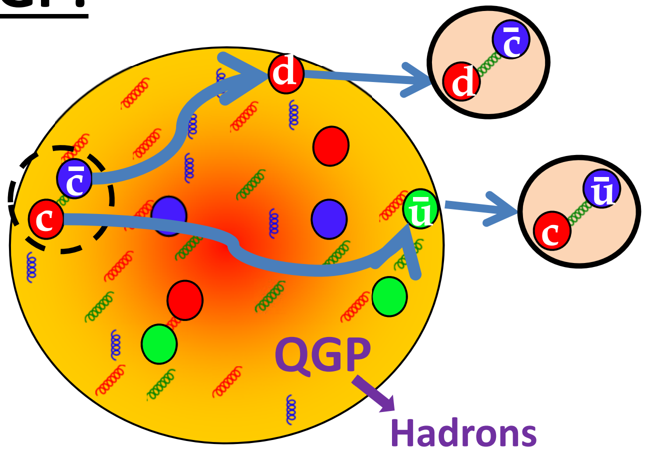


« Normal » suppression (~ small)

From Cold Nuclear Matter effects

NA 38 , 1988 on

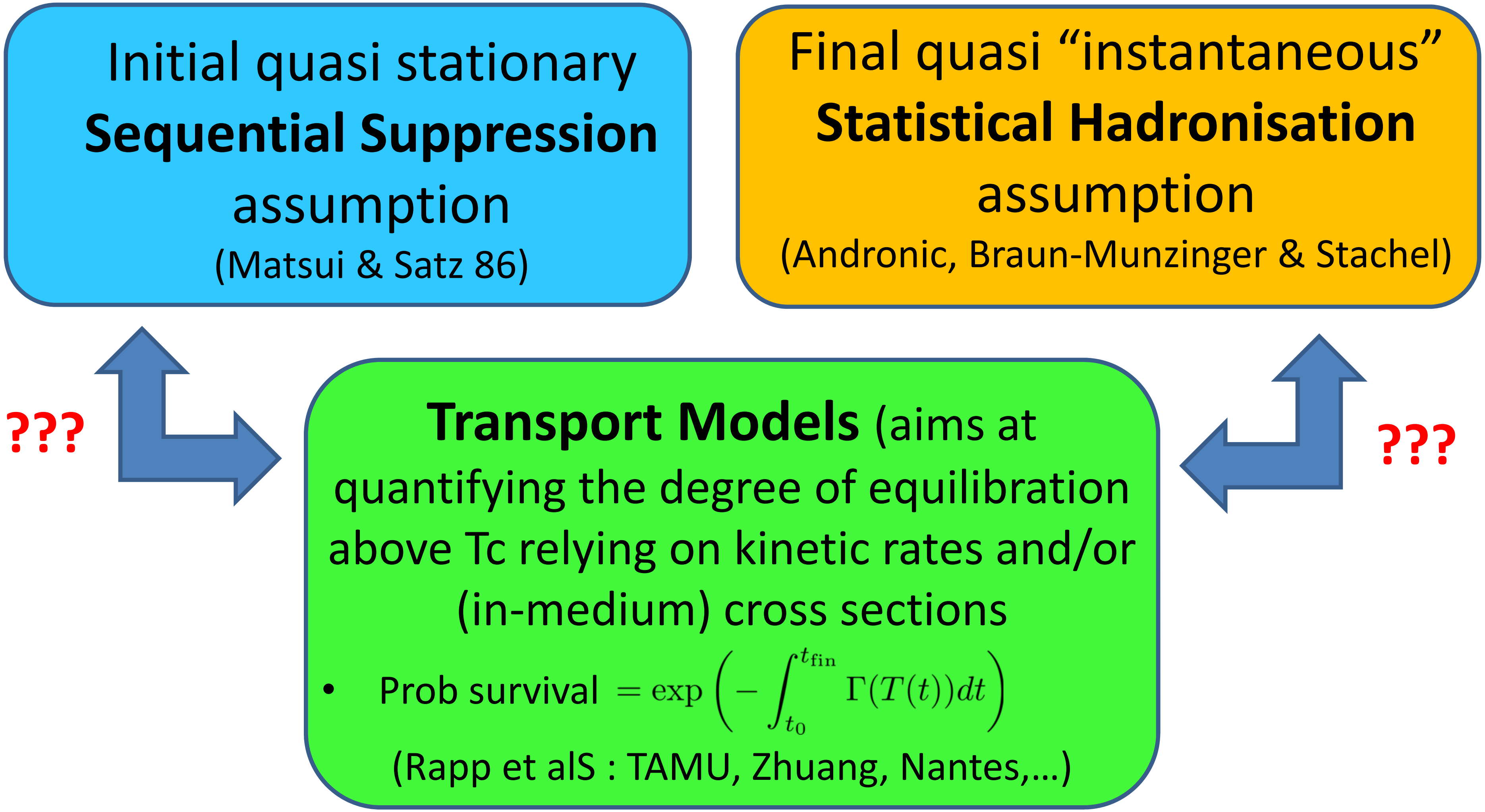
In QGP:



« Abnormal » suppression

from color screening …

Schematic summary of approaches



Initial quasi stationary **Sequential Suppression** assumption (Matsui & Satz 86)

Final quasi "instantaneous" **Statistical Hadronisation** assumption (Andronic, Braun-Munzinger & Stachel)

???

???

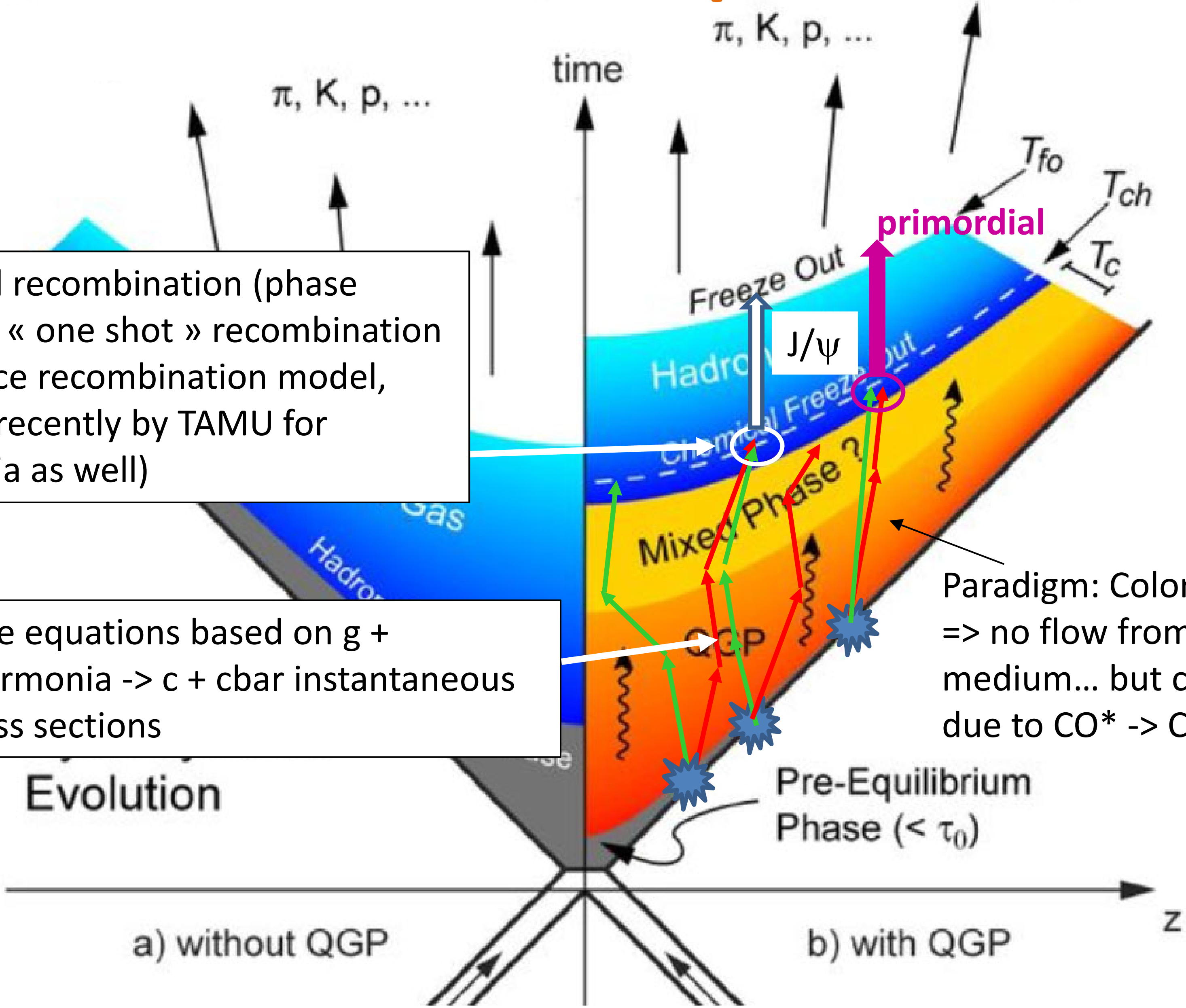
Transport Models (aims at quantifying the degree of equilibration above Tc relying on kinetic rates and/or (in-medium) cross sections

- Prob survival $= \exp\left(-\int_{t_0}^{t_{\mathrm{fin}}} \Gamma(T(t))dt\right)$

(Rapp et alS : TAMU, Zhuang, Nantes,...)

Nowadays, all state-of-the art dynamical models include both suppression and recombinaison, although not always consistently

Charmonia in transport models



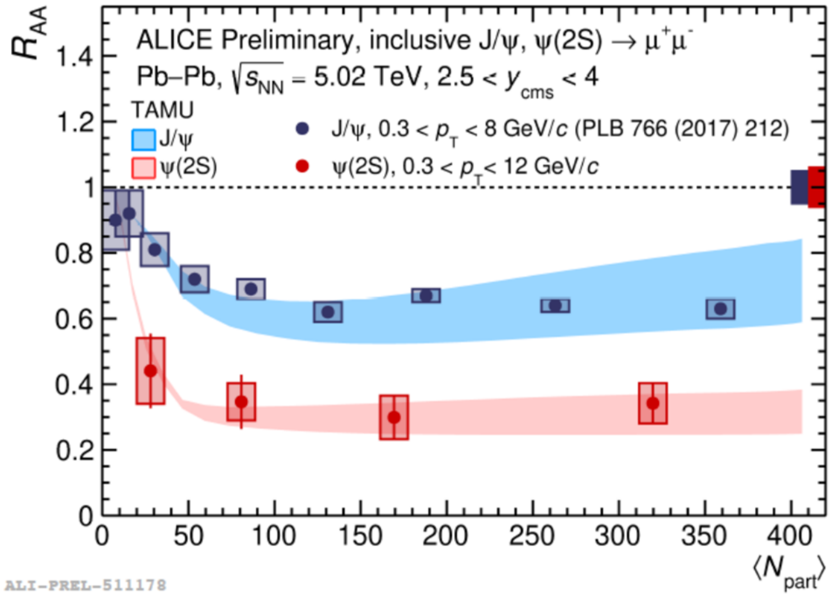
Statistical recombination (phase space) or « one shot » recombination (resonance recombination model, adopted recently by TAMU for charmonia as well)

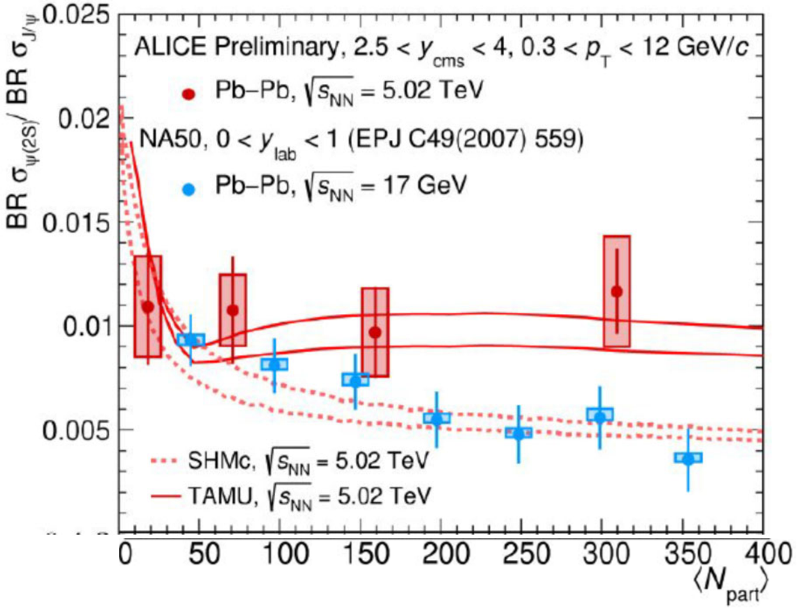
Rate equations based on g + charmonia -> c + cbar instantaneous cross sections

Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

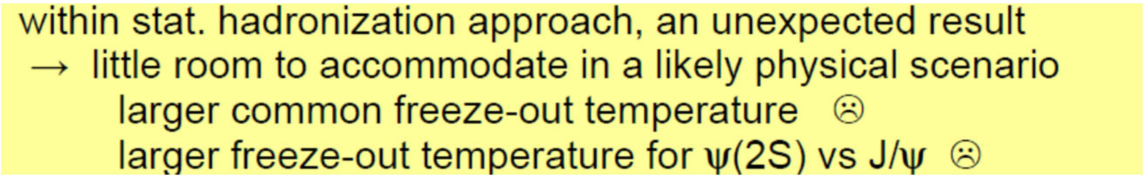
TAMU et al: some illustrative results

Recent precise measurement of ψ(2S) by ALICE

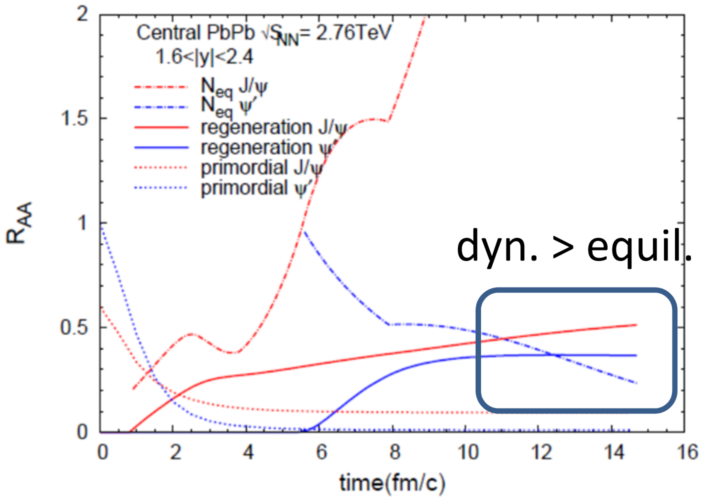




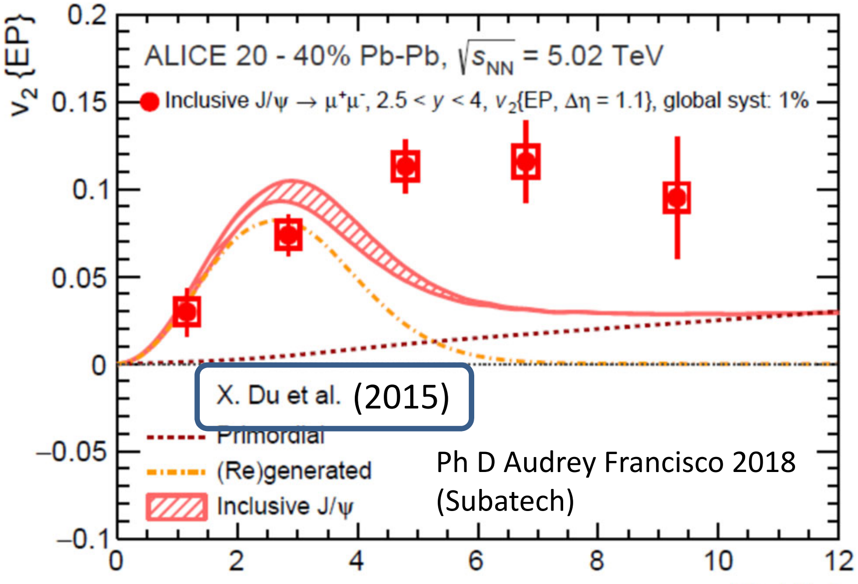
- Found in good agreement with TAMU (Du and Rapp 2015) : late formation due to hadronic channels
- In tension with SHM ; Stachel QM 2022:



within stat. hadronization approach, an unexpected result
→ little room to accommodate in a likely physical scenario
larger common freeze-out temperature ☹
larger freeze-out temperature for ψ(2S) vs J/ψ ☹

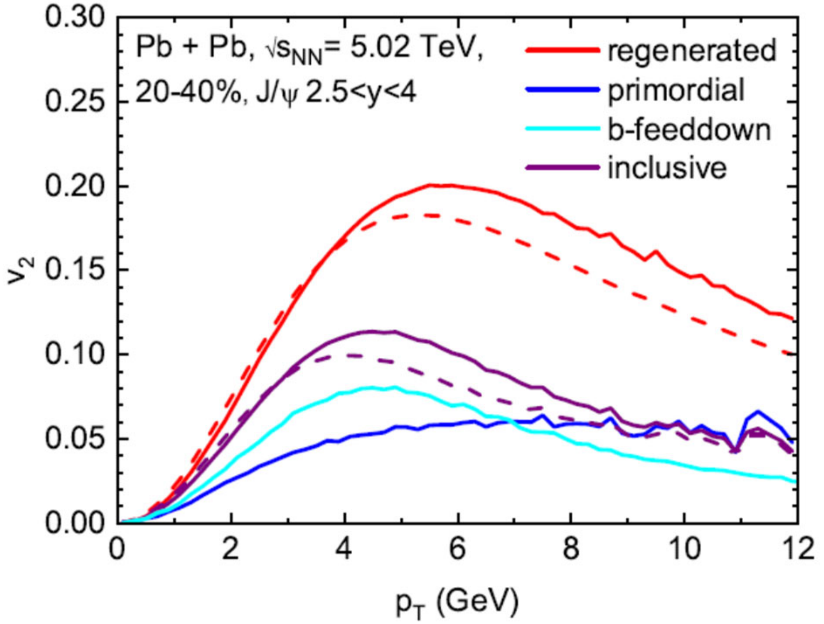


V_2 comparison from the models

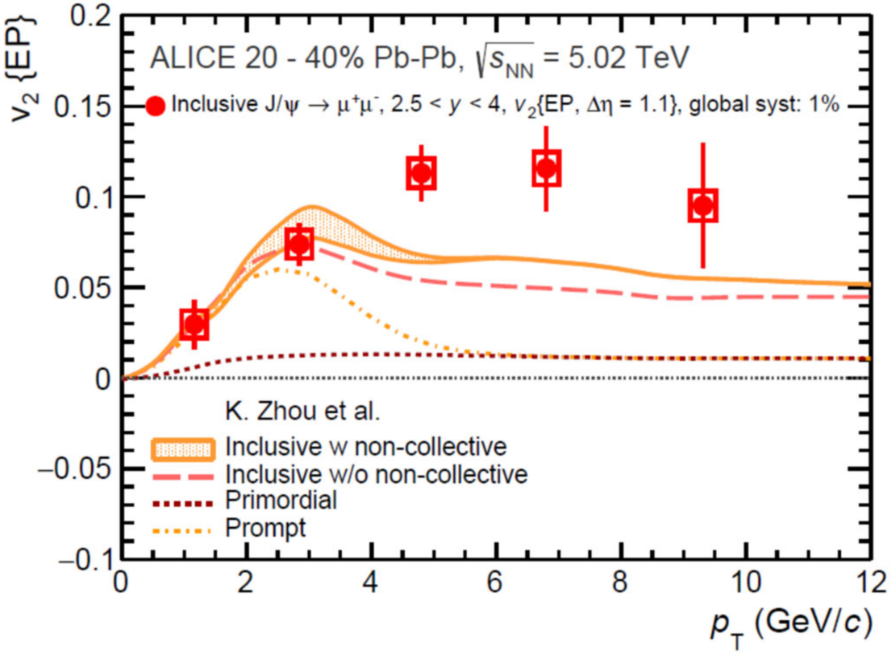


X. Du et al. (2015)

Ph D Audrey Francisco 2018 (Subatech)



He et al, Phys.Rev.Lett.128, 162301 (2022))



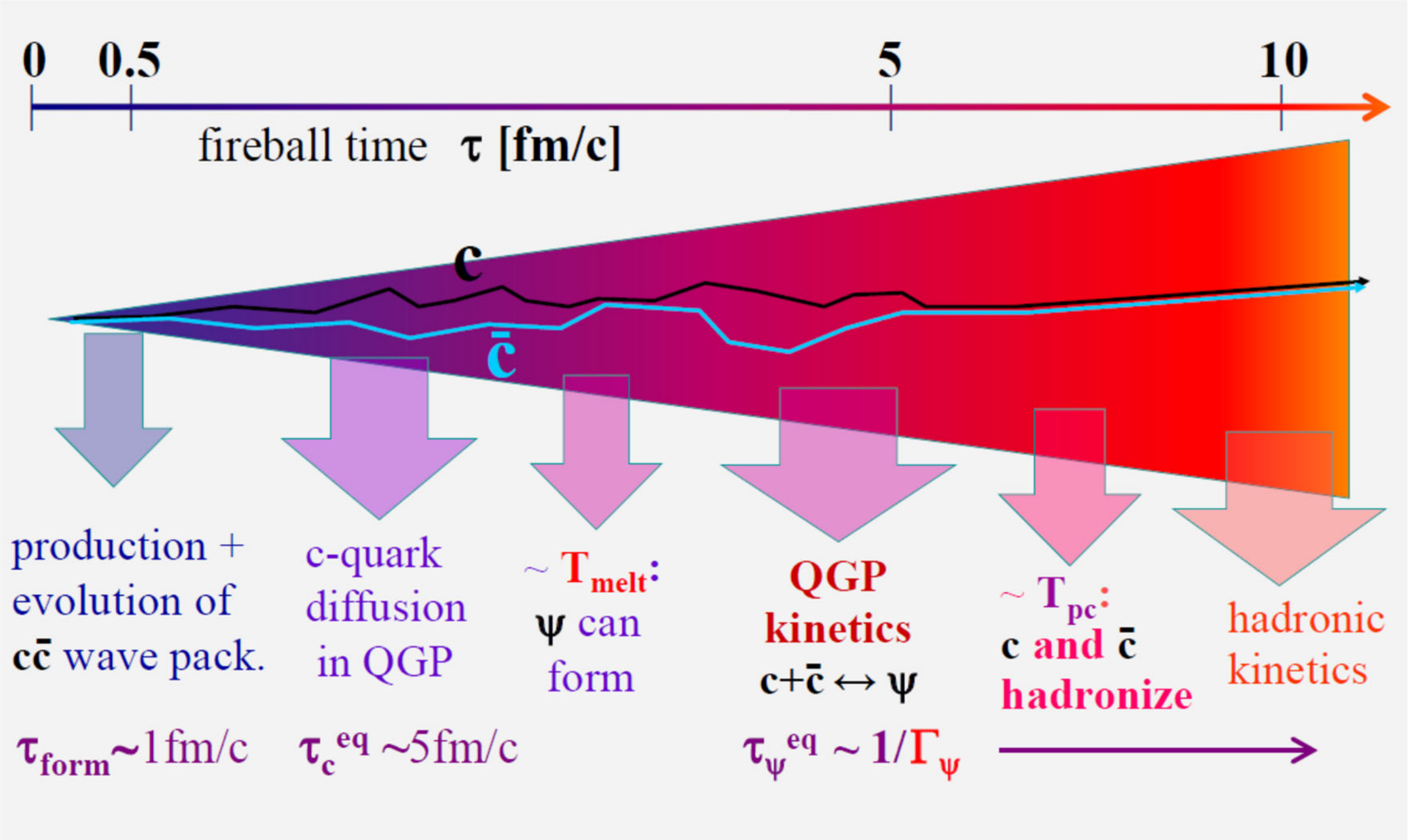
What has changed ? Need to enter the « fine details of each approach »

What should we conclude ?

Diversity of results !

Charmonia in transport models

Rapp and Du *Nucl.Phys.A* 967 (2017) 216-224



production + evolution of $c\bar{c}$ wave pack.

c-quark diffusion in QGP

~ T_{melt}: ψ can form

QGP kinetics $c+\bar{c} \leftrightarrow \psi$

~ T_{pc}: c and $\bar{c}$ hadronize

hadronic kinetics

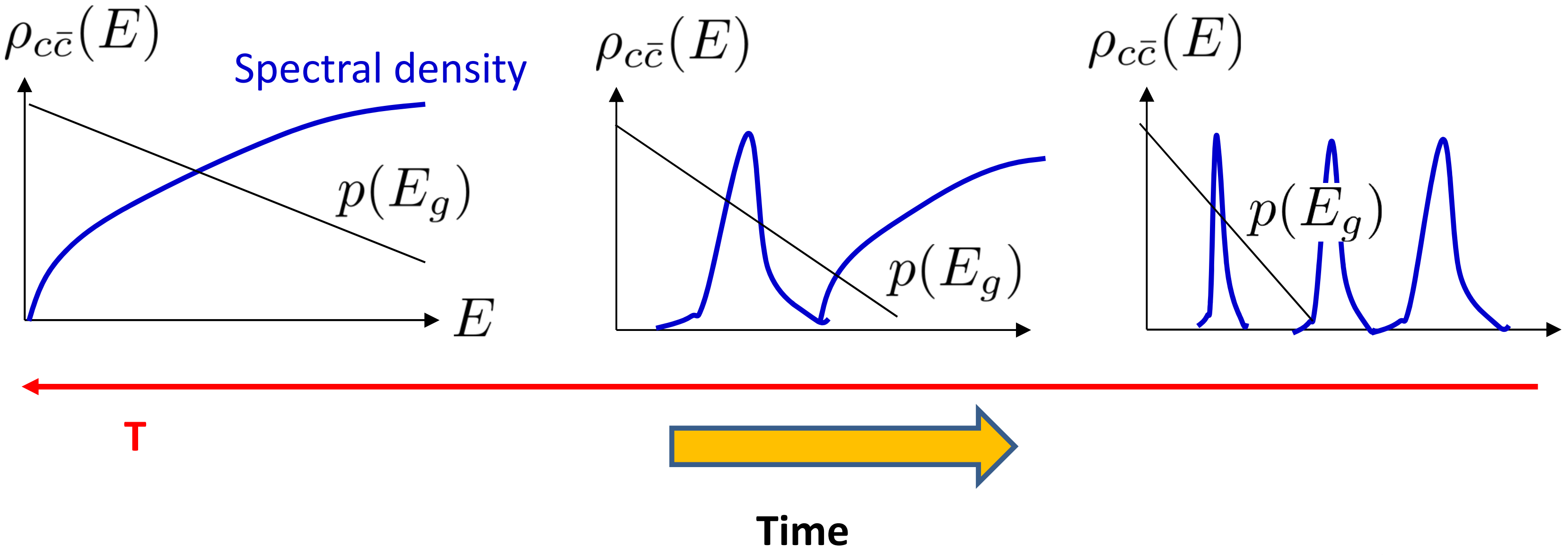
τ_{form}~1fm/c τ_c^{eq} ~5fm/c $\tau_\psi^{eq} \sim 1/\Gamma_\psi$

Charmonia does not exist

Charmonia do exist and dissociate/recombine following semi-classical laws

Charmonia in a microscopic theory

Several regimes / effects



Multiple scattering on quasi free states

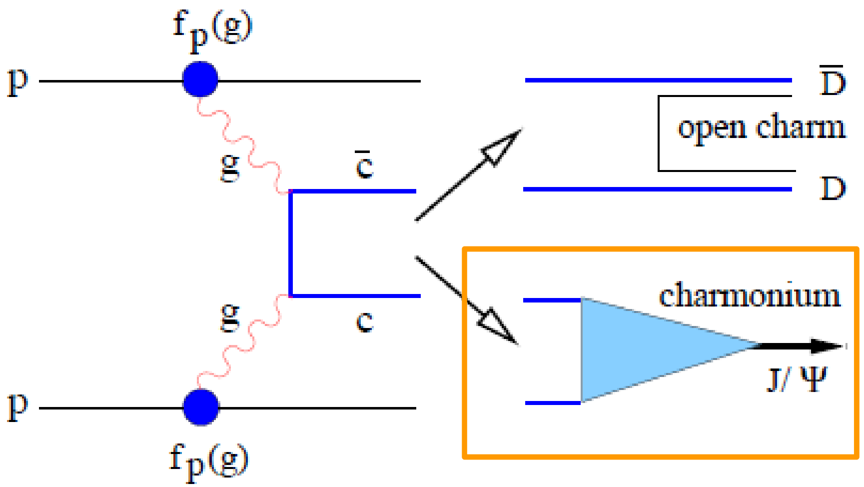
Gluo-dissociation of well identified levels by scarce “high-energy” gluons (dilute medium => cross section ok)

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Yet, still a need to define the equivalent of a formation – dissociation rate

Quantum coherence

Picture behind transport theory :



Open heavy flavor and quarkonia assumed to be uncorrelated

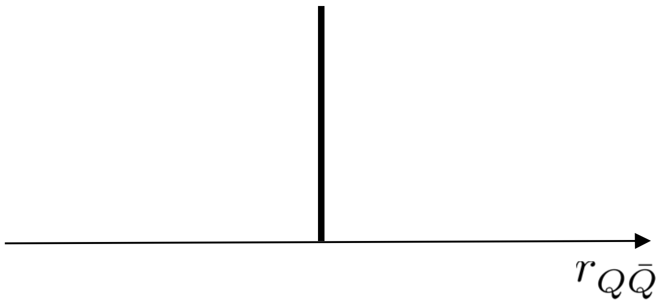
Formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Common belief in the transport community:

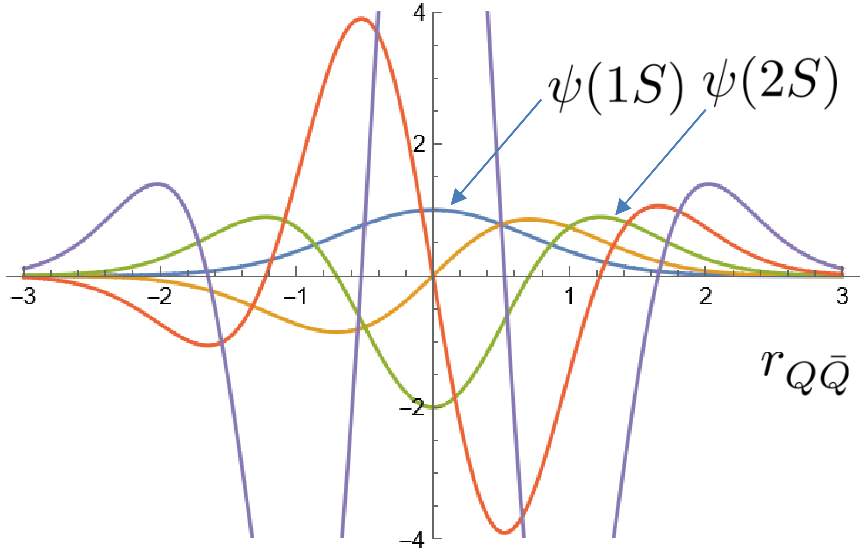
Quarkonia initially « formed » in QGP and survive with an *individual* survival probability

$$S(t) = e^{-\int_{\tau_f}^{t}\Gamma(T(t'))dt'}$$

Quantum coherence at early time



$= \Sigma$



Initial compact state : δ

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r^2_{Q\bar{Q}}$

Coherence

Neglect of coherence

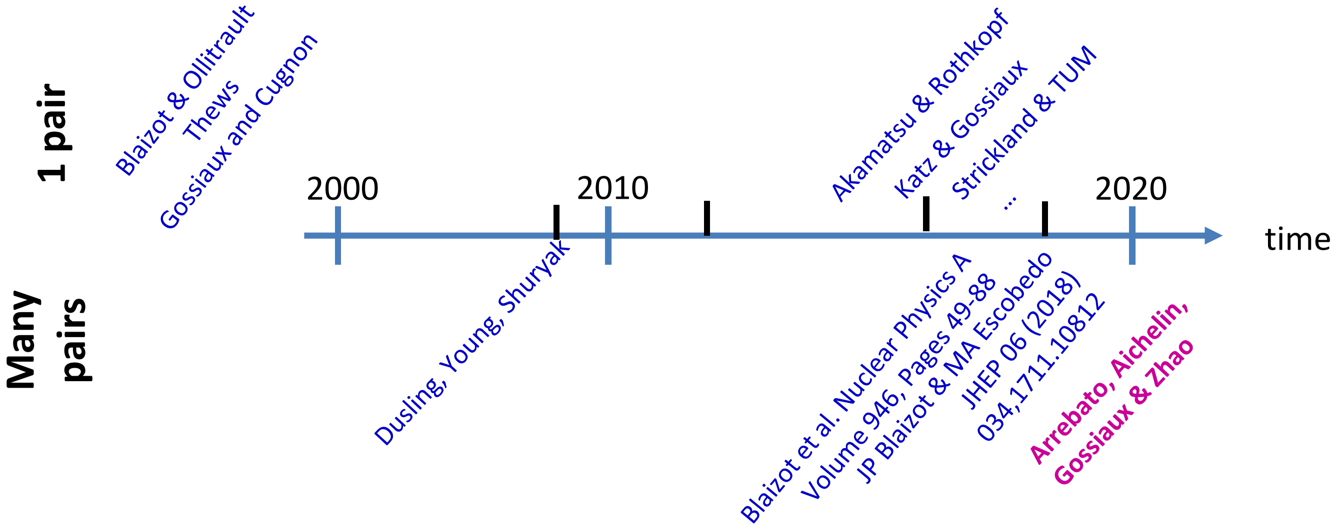
$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle \psi_j | r^2 | \psi_i \rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle \psi_i | r^2 | \psi_i \rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Crucial to include coherence !

N.B. : one can model this effect by phenomenological formation time, but lack of control

Several motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for **« from small to large »**
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

Remler’s formalism for dynamical coalescence

Generic idea : describe charmonia (Ψ) production using density matrix

$$P^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

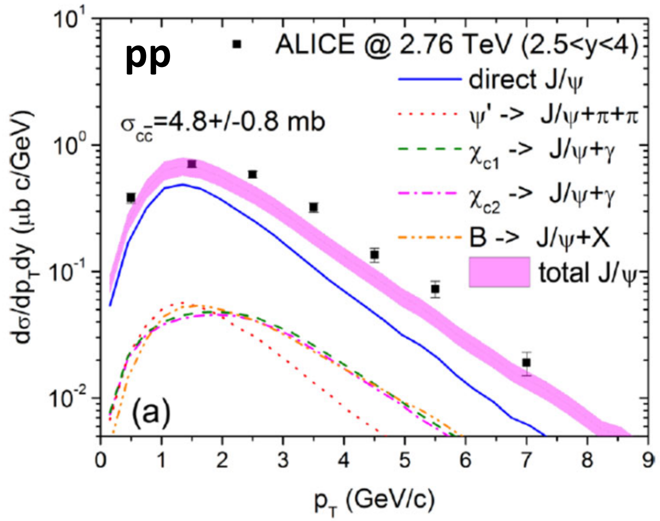
$$\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$$

Single quarkonia density operator

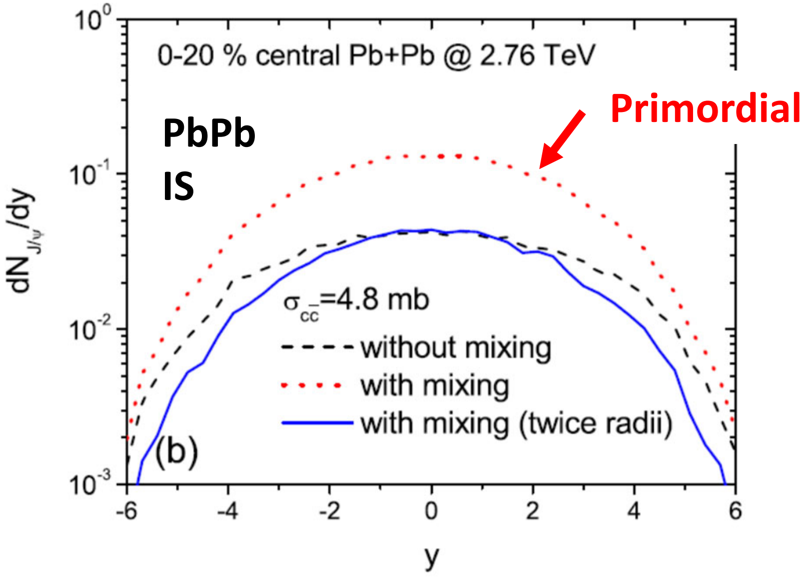
N-body density matrix (bulk partons + many c and many cbar)

“Just” looking at the initial stage brings interesting features:

T. Song, J.Aichelin and E.Bratkovskaya, PRC 96. 014907 (2017)



Good reproduction of pp -> J/ψ + x !!!



considerable enhancement of primordial J/ψ (in the **initial** state): **large off-diagonal contributions**

A bit of background

$$P^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

$$\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$$

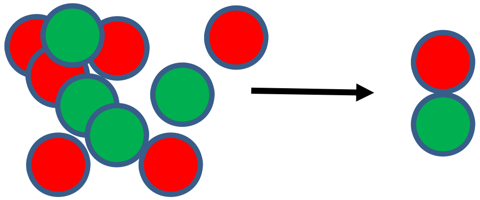
$$\frac{d\hat{\rho}_N(t)}{dt} = -i\left[\hat{H}_N, \hat{\rho}_N(t)\right]$$

Dealing with the dynamics ?

- The idea of the formalism goes back to Remler's work
- General scheme connecting composite-particle cross section and rates with time-dependent density operators
- Applied by Remler et al to the deuteron production in (low energy) AA collisions. The formalism is able to deal with many particles (nucleons -> deuterium)

E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

1980



[2020,…

Apply Remler formalism to quarkonia production in heavy ion collisions (Arrebato, Aichelin & Gossiaux, 2206.01308)

Remler formalism at work

Lessons from the past : the direct calculation is not effective for codes based on “cascade approach” (for which members of a genuine fragment are found far apart in the final stage)

⇨ Use the identity $P^{\Psi}(t) = P^{\text{prim}}(t_0) + \int_{t_0}^{t} \Gamma^{\Psi}(t^{'})dt^{'}$

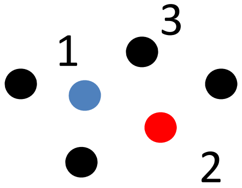
- Γ is The effective rate for quarkonia state creation (dissociation) in the medium :

$$\Gamma^{\psi}(t) = \frac{dP^{\Psi}(t)}{dt} = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}} \frac{d\hat{\rho}_N(t)}{dt}\right]$$

- $P^{\text{prim}}(t_0)$ is the production at initial time (*primordial*)

Then : Von Neumann equation + Hamiltonian composed of 2-body interactions

$$H_N = \Sigma_i K_i + \Sigma_{i>j} V_{ij} \Rightarrow H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$



1 & 2 : Q & $\bar{Q}$

3, 4, …… : light quarks

$Q\bar{Q}$ Internal Hamiltonian

Light quarks

Total interaction of Q & $\bar{Q}$ with all light partons

⇨ … ⇨ $$\Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\underset{\Sigma_{i>2}V_{i1}}{\underset{\uparrow}{\hat{U}_1}} + \underset{\Sigma_{i>2}V_{i2}}{\underset{\uparrow}{\hat{U}_2}}, \hat{\rho}_N(t)]]$$

Still quite complicated

Remler formalism at work

Passing to the Wigner representation:

$$W_N(\{r\},\{p\}) = \int \Pi d^3y e^{ipy} \langle r - \frac{y}{2}|\hat{\rho}_N|r + \frac{y}{2}\rangle$$

Direct space

$$\partial \rho_N(t)/\partial t = -i\Sigma_j[K_j, \rho_N(t)] \\ -i\Sigma_{j>k}[V_{jk}, \rho_N(t)]$$

Wigner space....

$$\partial W_N(t)/\partial t = \langle \Sigma_i v_i \cdot \partial_r W_N(\mathbf{r},\mathbf{p},t)\rangle + \\ \langle \Sigma_{i\geq j}\Sigma_n \delta(t - t_{ij}(n))\times \\ (W_N(\mathbf{r},\mathbf{p},t+\epsilon) - W_N(\mathbf{r},\mathbf{p},t-\epsilon))\rangle$$

One to one correspondance

$[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]$ and $\Gamma^\Psi(t) = -iTr[\hat{\rho}^\Psi[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$ can be modelized from *the semi-classical trajectories* evolution in Wigner space

Quarkonia: Double Gaussian approximation

$$W^\Psi_{Q\bar{Q}}(r_{\rm rel}, p_{\rm rel}) = Ce^{-r_{\rm rel}^2\sigma^2} \times e^{-\frac{p_{\rm rel}^2}{\sigma^2}}$$

Parameter: The Gaussian width σ ≈ 0.35 fm

$$[\frac{\hbar^2}{2\mu}\nabla^2 + V(r)]\Psi_{Q\bar{Q}}(r) = E_{Q\bar{Q}}\Psi_{Q\bar{Q}} \longrightarrow \langle r^2 \rangle \longrightarrow W^\Psi$$

W_N : Semi-classical approach

$$W_N = \Pi_i \delta(x_i - x_{i0}(t))\delta(p_i - p_{i0}(t))$$

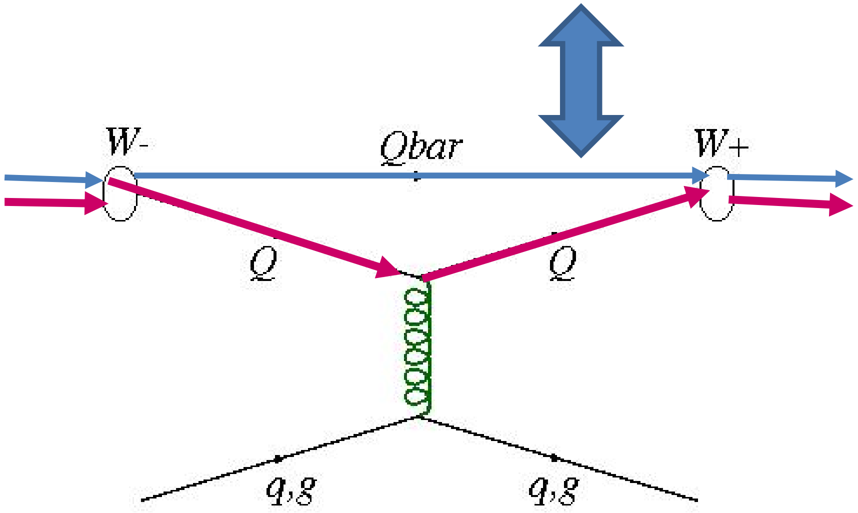
… but no explicit description of W_N required (as it appears in the trace)

Remler formalism at work

Combining the expression of the Wigner's functions and substituting in the **effective rate equation** :

$$\Gamma^{\Psi}(t) = \sum_{i=1,2} \sum_{j\geq 3} \delta(t-t_{ij}) \int \frac{d^3p_i d^3x_i}{h^3} W^{\Psi}_{Q\bar{Q}}(p_1,x_1;p_2,x_2)\,[W_N(t+\epsilon) - W_N(t-\epsilon)]$$

- The quarkonia production in this model is a three body process; the HQs interact only by collisions with the QGP !!!
- The "details" of H_{int} between HQ and bulk partons are incorporated into the evolution of W_N after each collision / time step (nice feature for the MC simulations)
- $W_N(t+\varepsilon)$ and $W_N(t-\varepsilon)$ are NOT the equivalent of gain and loss terms in usual rate equations
- Dissociation and recombination treated in the same scheme



Interaction of HQ with the QGP are carried out by EPOS2+MC@HQ (good results for D and B mesons production)

Then: $P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$

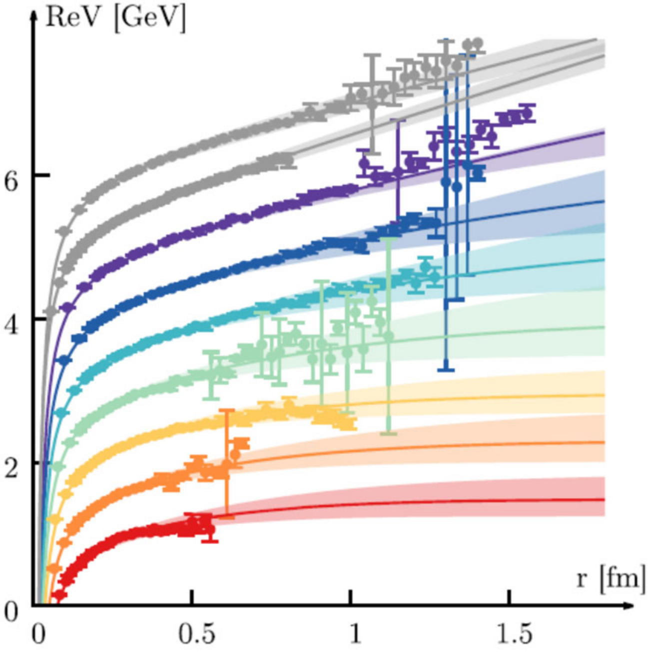
NB: Also possible to generate similar relations for differential rates

Extension of the Remler formalism

- Generalization at finite 4-velocity to relativistic Wigner density (boosted quarkonium states) … important to warrant orthogonality of states

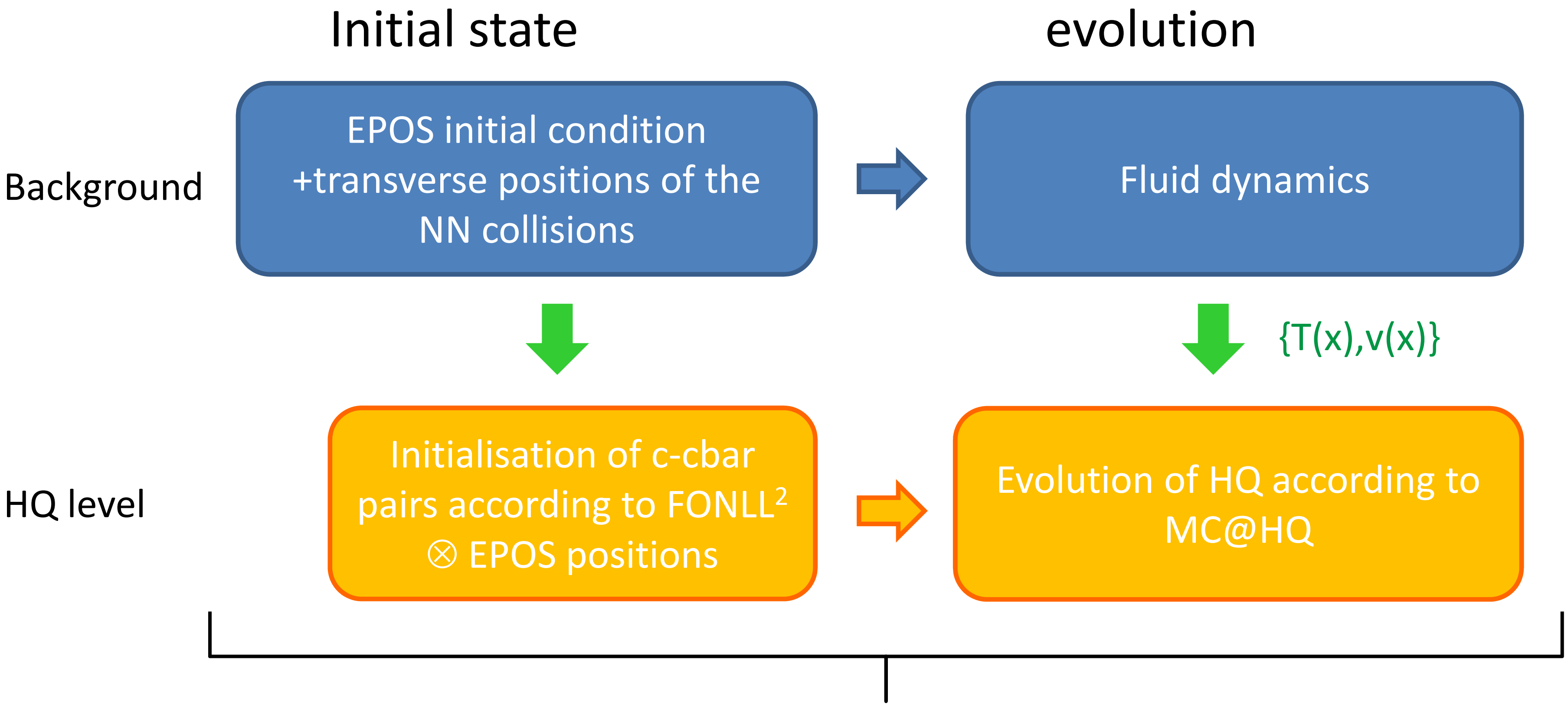
$$\mathrm{Tr}[W_u^{\Phi_i} W_{u'}^{\Phi_j}] = \delta^{(3)}(u-u')\delta_{i,j}$$

- Extra source of Γ due to “local-T” basis evolution with time : Γ^{loc}
- **Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs… and correlated trajectories.**



D. Lafferty and A. Rothkopf,
PHYS. REV. D 101, 056010 (2020)

The 3 layers of the numerical modelling



Initial state

evolution

Background

EPOS initial condition +transverse positions of the NN collisions

Fluid dynamics

{T(x),v(x)}

HQ level

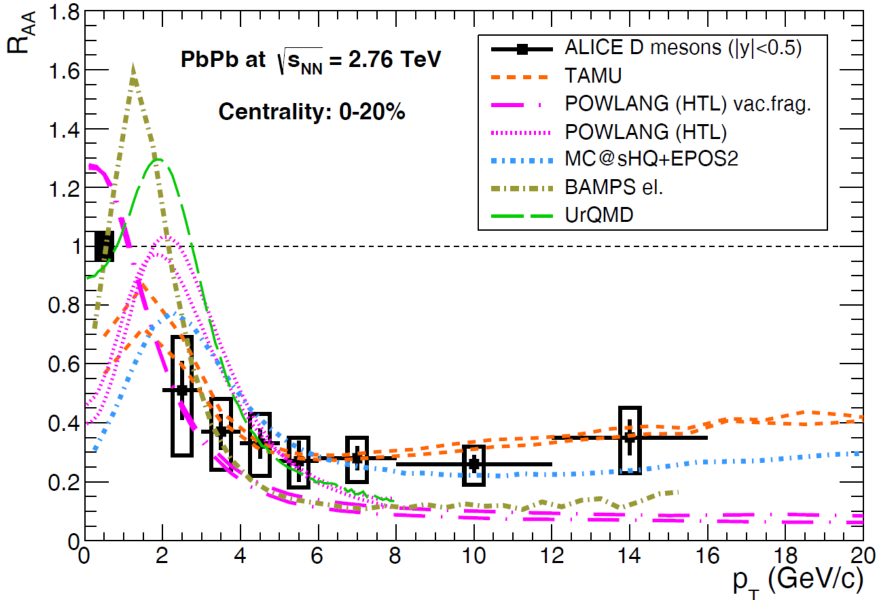
Initialisation of c-cbar pairs according to FONLL^2 ⊗ EPOS positions

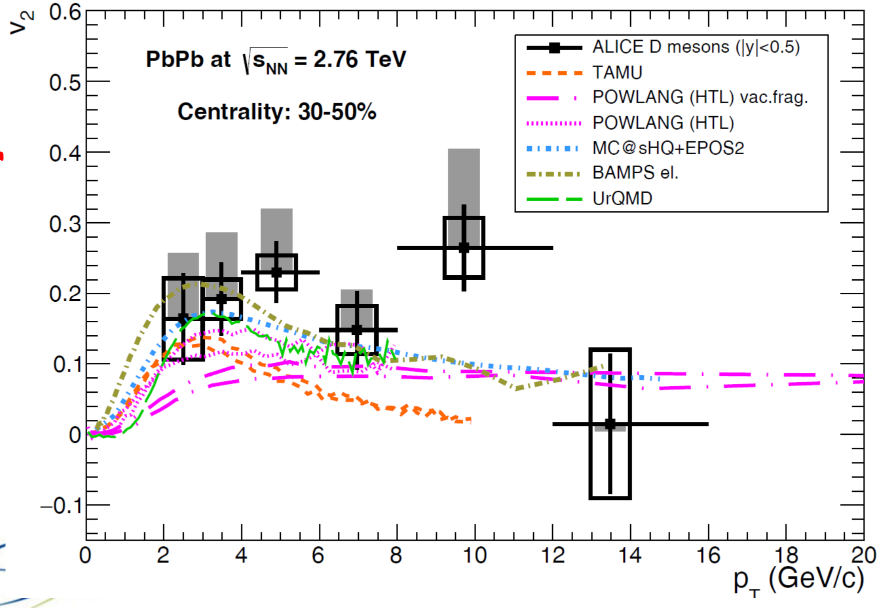
Evolution of HQ according to MC@HQ

Usual EPOS2 + MC@HQ used with some success to describe open heavy flavor production at LHC (see Eur. Phys. J. C (2016) 76:107)

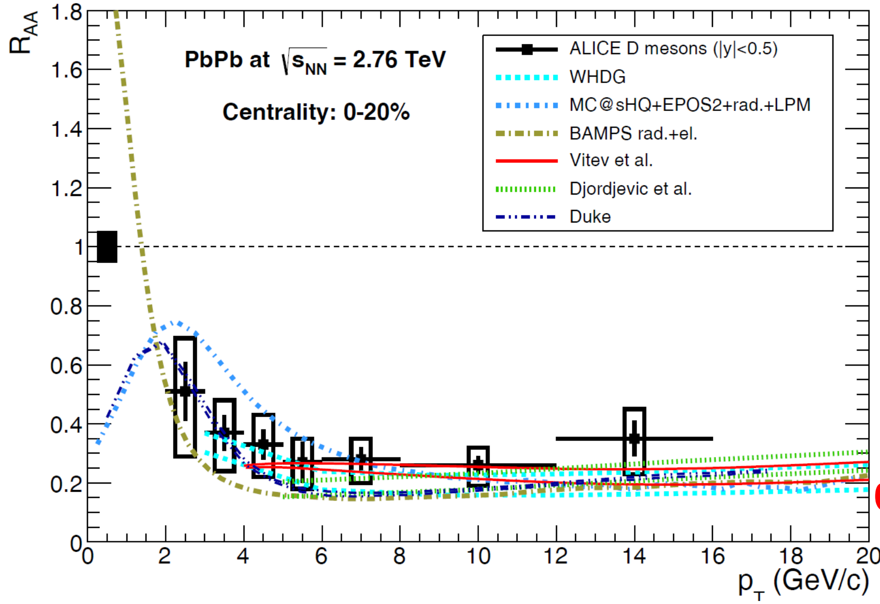
The 3 layers of the numerical modelling

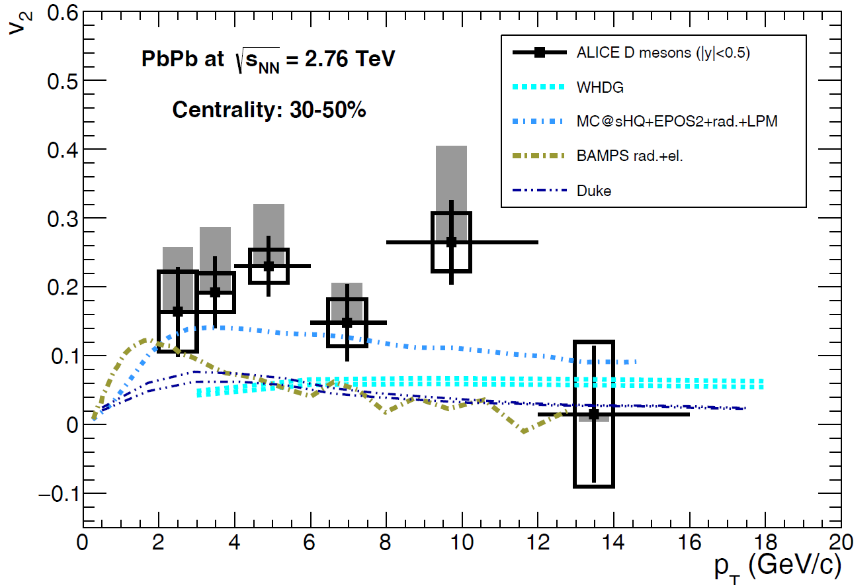
Purely elastic scatterings





Elastic scatterings + radiative energy loss





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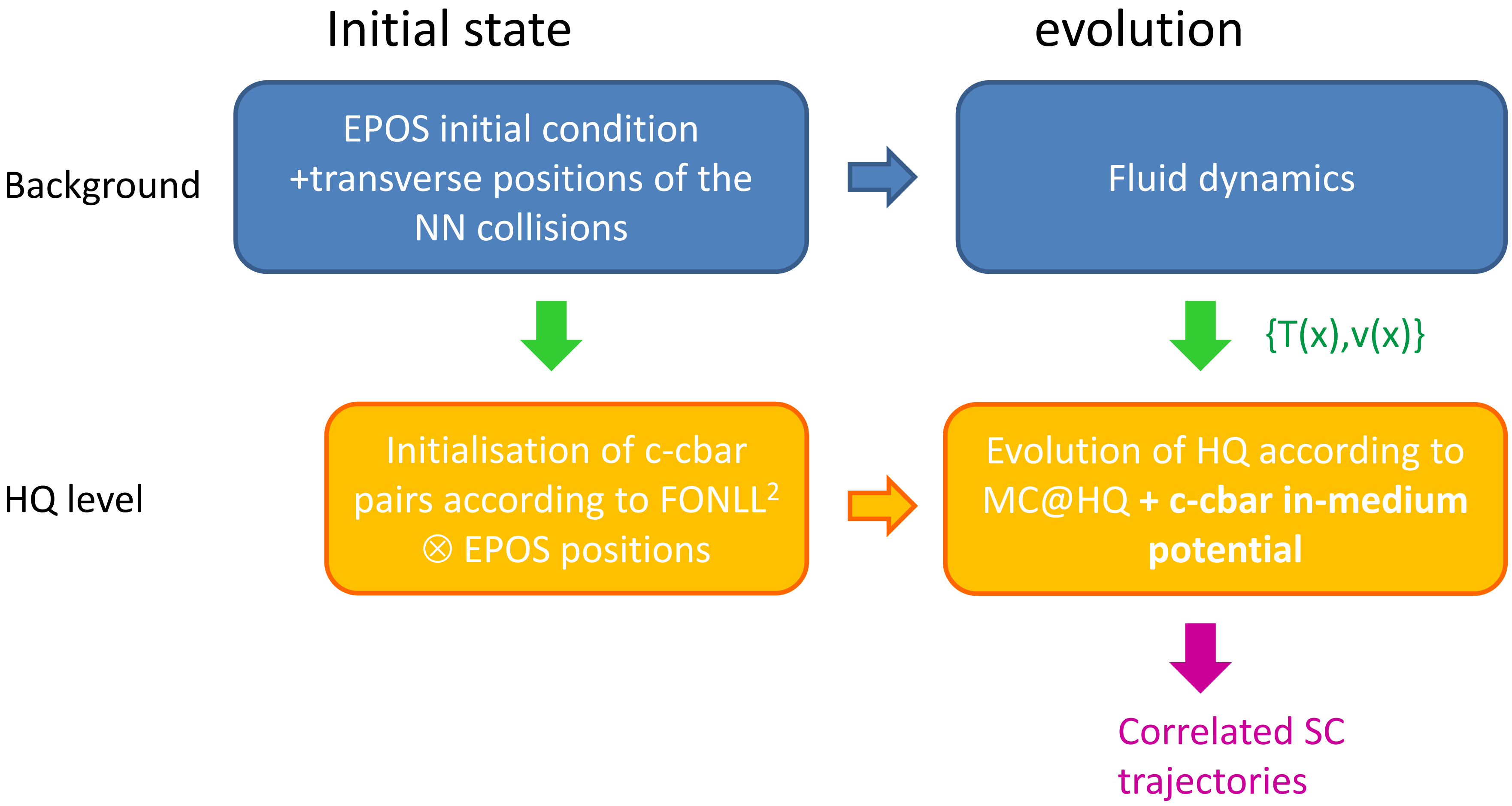

Initial state

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$\{T(x),v(x)\}$

HQ level

Initialisation of c-cbar pairs according to FONLL[2] $\otimes$ EPOS positions

Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**

Correlated SC trajectories

Extension of the Remler formalism

- Generalization to relativistic Wigner density (boosted quarkonium states)
- Extra source of Γ due to “local-T” basis evolution with time : Γ^{loc}
- **Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs… and correlated trajectories.**

D. Lafferty and A. Rothkopf,
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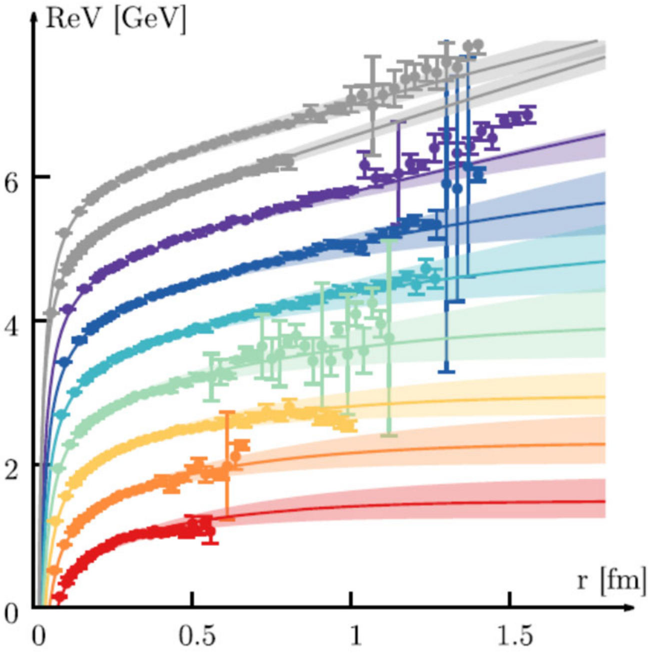
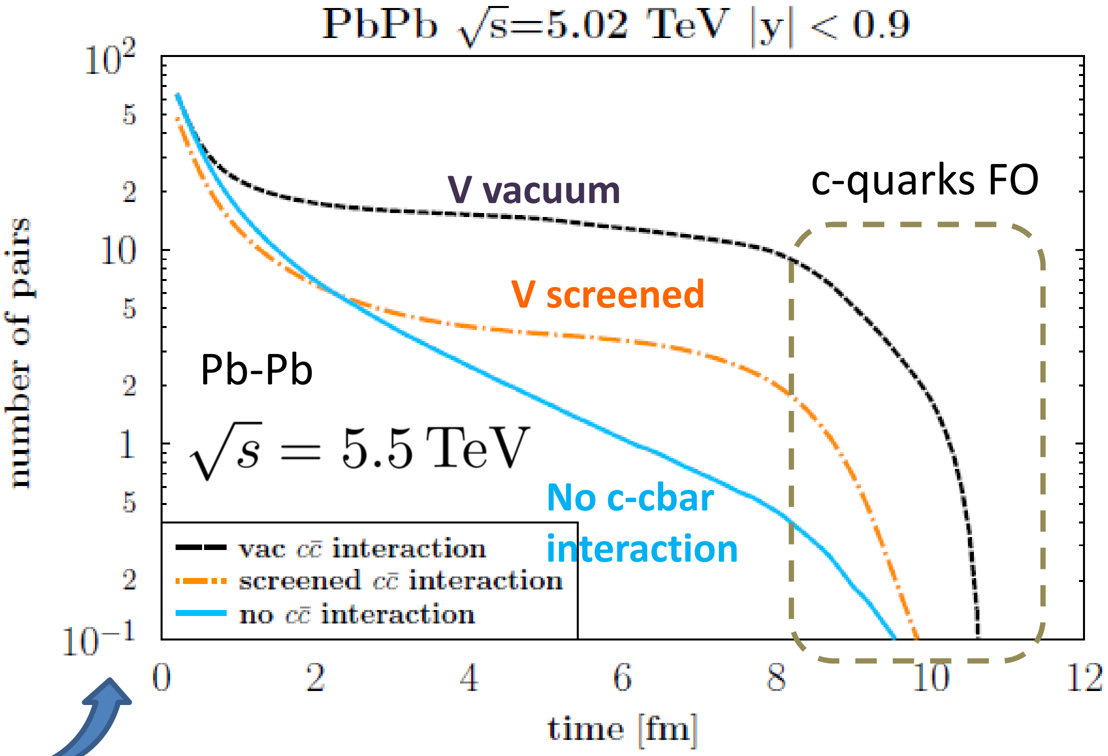
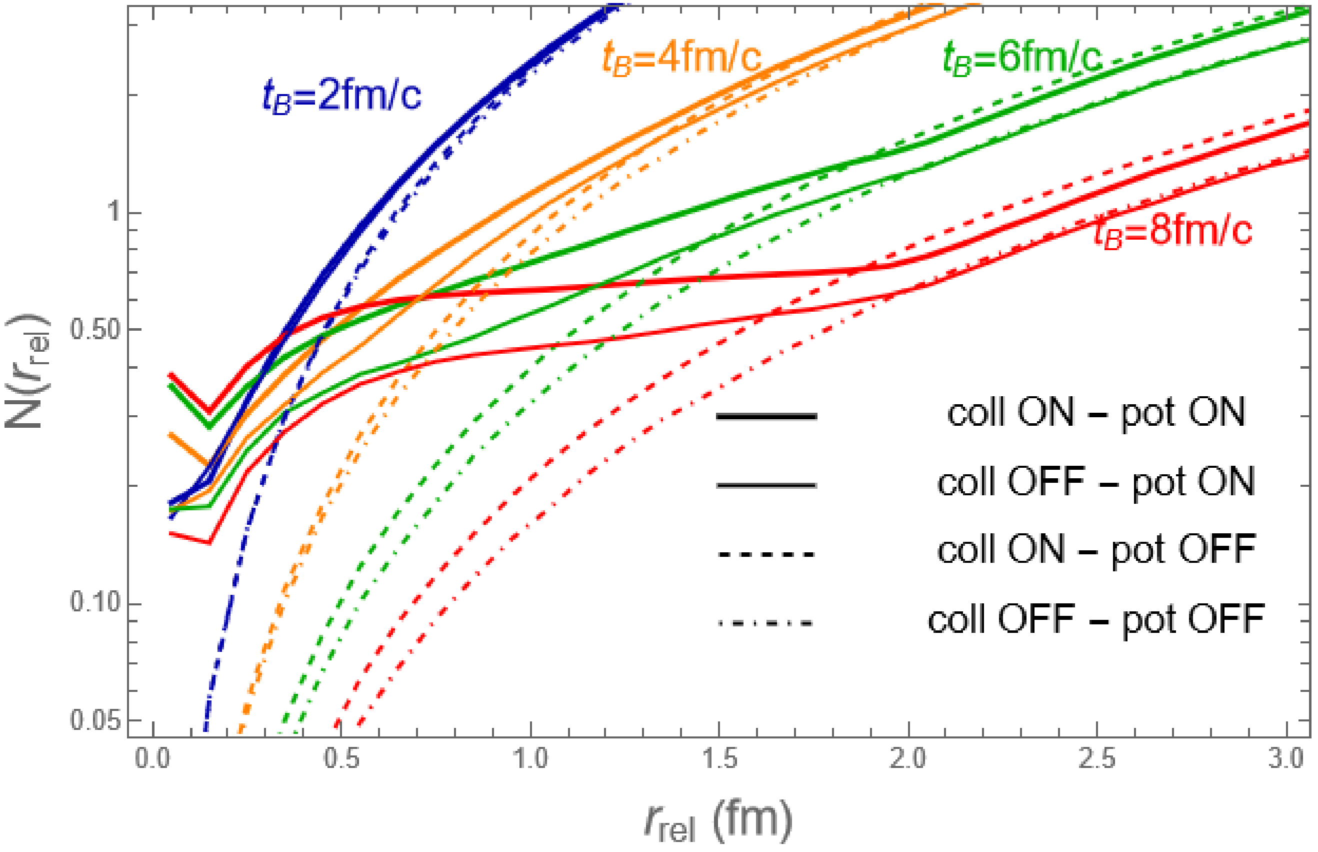


Instantaneous # of Q-Qbar at (invariant) distance < 1fm

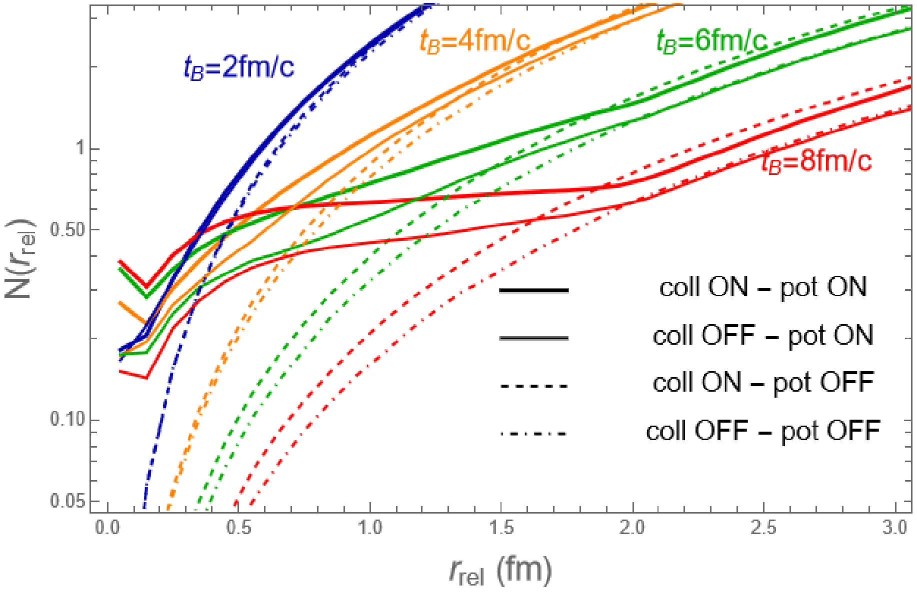


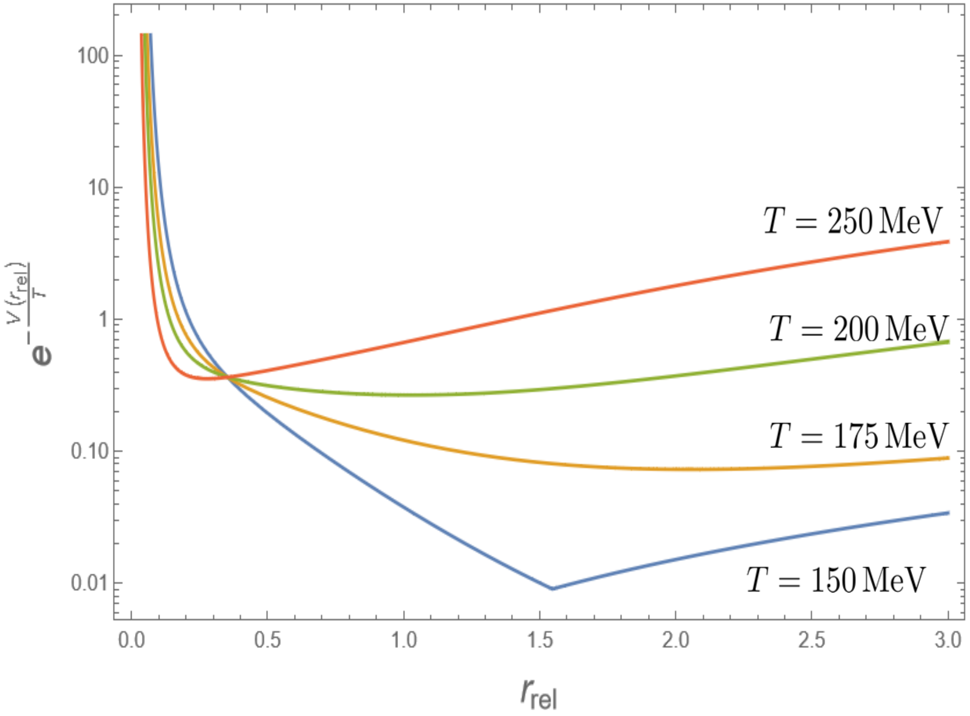
The dynamics of c-cbar correlation



- The c-cbar potential (« pot ON ») leads to a **huge increase of the c-cbar probability at close distance** at large times (not a random Poisson distribution !)…
- … Especially when the collisions with the QGP (« coll ») are switched ON as well

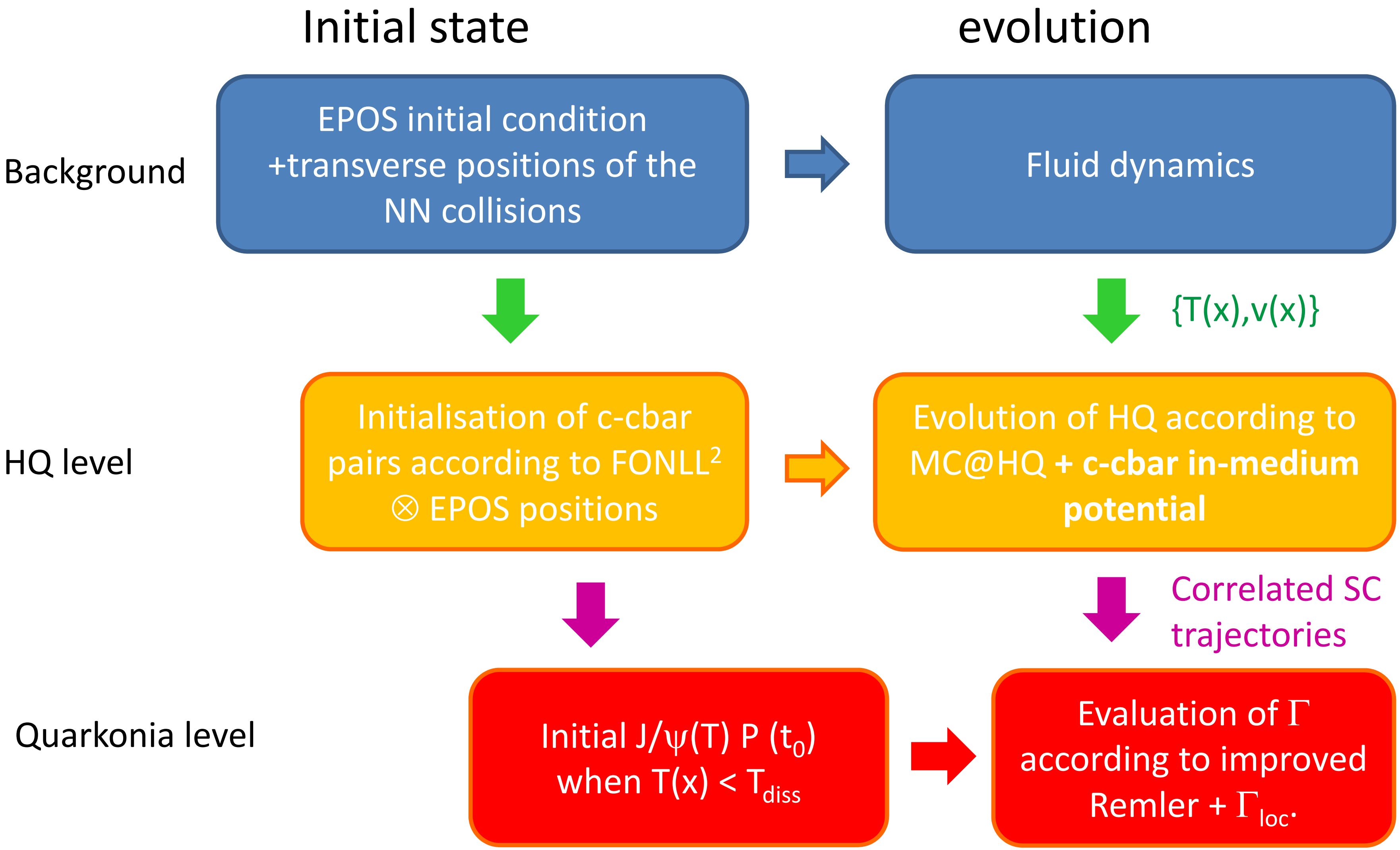
The dynamics of c-cbar correlation





- The c-cbar potential (« pot ON ») leads to a **huge increase of the c-cbar probability at close distance** at large times (not a random Poisson distribution !)…
- … Especially when the collisions with the QGP (« coll ») are switched ON as well
- … which does not correspond to the statistical distribution associated to the classical weight.

The 3 layers of the numerical modelling



Initial state

evolution

Background

EPOS initial condition +transverse positions of the NN collisions

Fluid dynamics

$\{T(x),v(x)\}$

HQ level

Initialisation of c-cbar pairs according to FONLL^2 $\otimes$ EPOS positions

Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**

Correlated SC trajectories

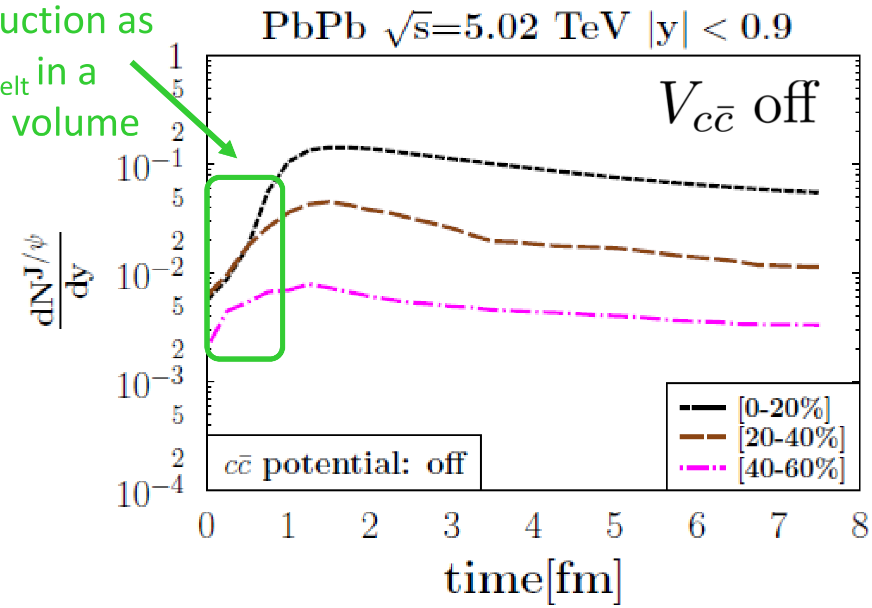
Quarkonia level

Initial $J/\psi(T)$ $P(t_0)$ when $T(x) < T_{diss}$

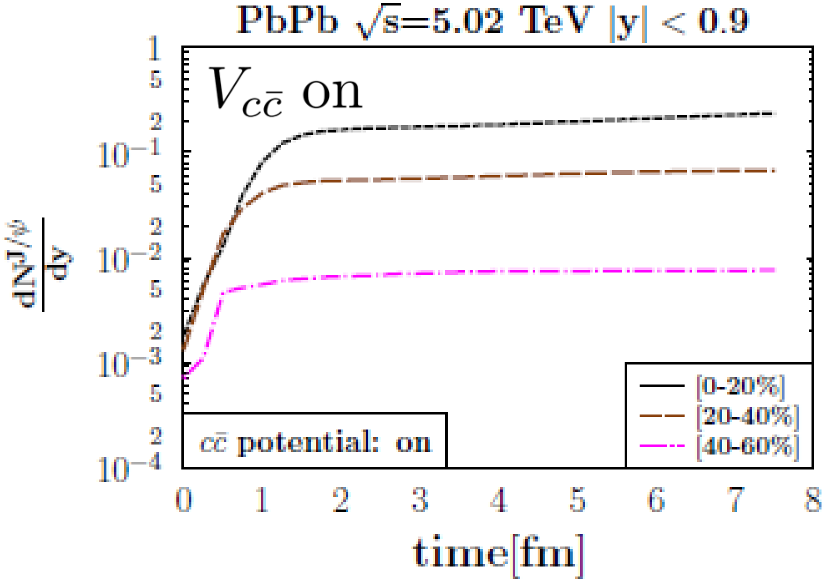
Evaluation of Γ according to improved Remler + Γ_{loc}.

We do not have J/ψ quasi particles in our approach, just correlated c-cbar trajectories

Results : J/ψ production vs time

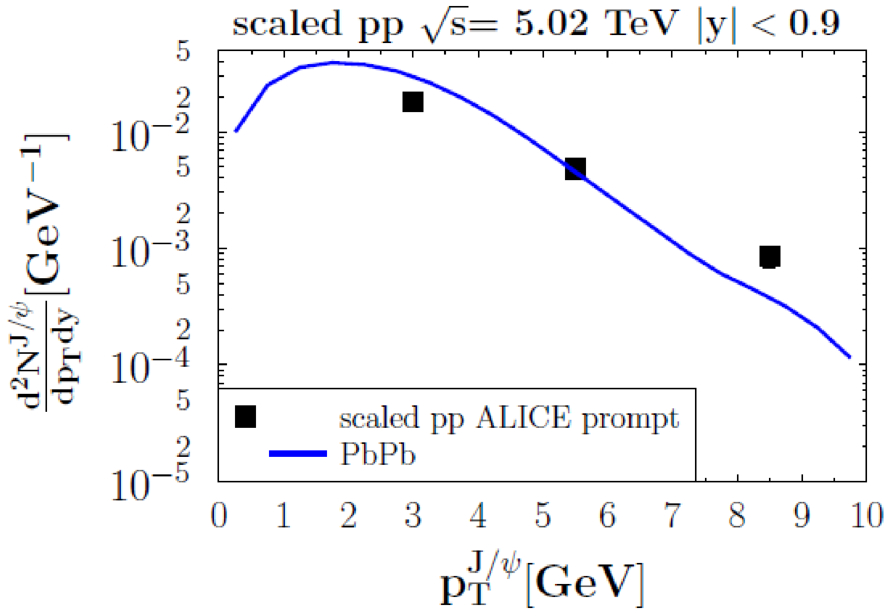


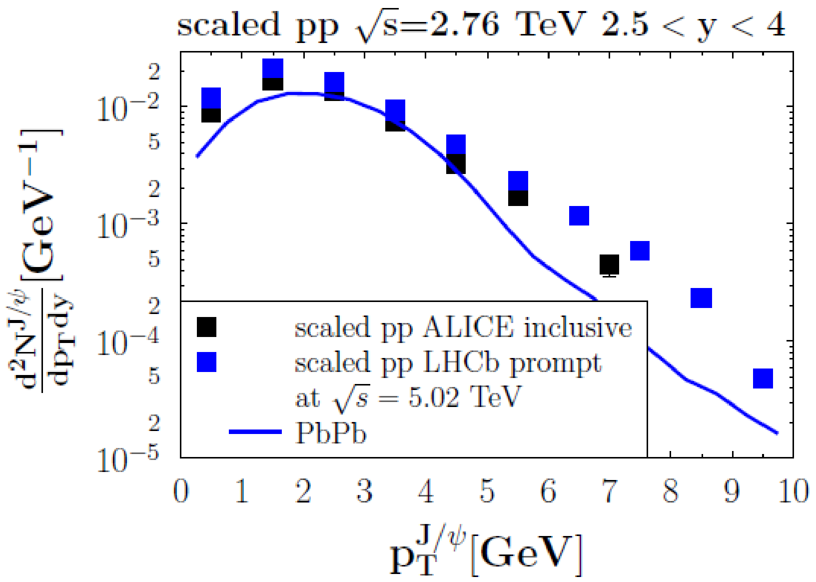
Delayed initial production as $T>T_{melt}$ in a large volume



- Without interaction potential between c and cbar, the collisions with the medium manage to destroy the native J/ψ (left)
- With the interaction potential between c and cbar « on », one observes a steady rate of J/ψ creation (increase of Γ^{col}, increase of Γ^{local})… No adiabaticity, but **no instantaneous formation** either.

Results : J/ψ production vs p_T



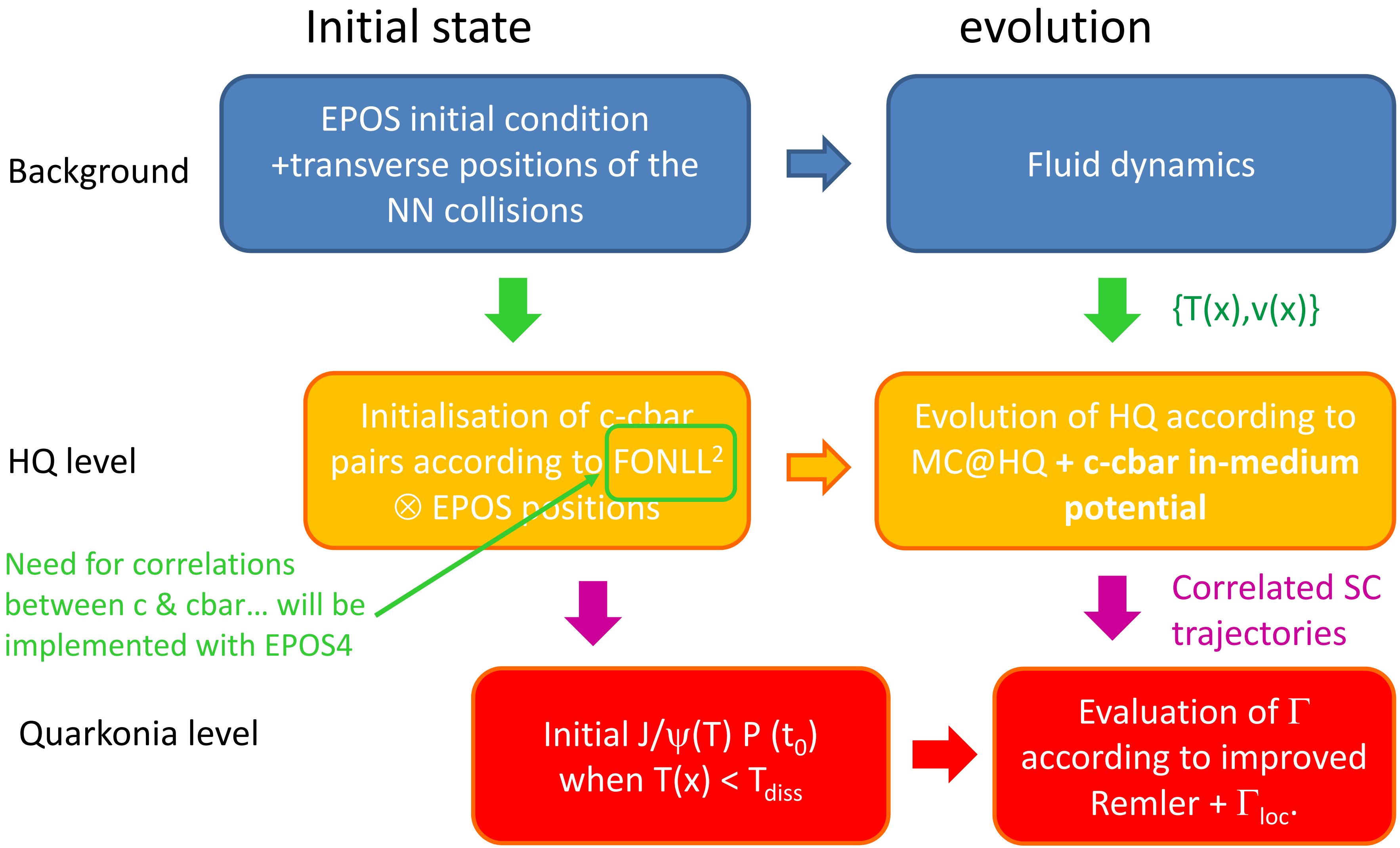


- **Equivalent pp production** (the denominator of the R_{AA}) : c-cbar according to FONLL[2] without any correlation, then coalescence with the Wigner distribution.
- No feed-down from higher states (to be implemented)

- Acceptable for p_T < 5 GeV/c, but deviations for higher p_T.

- To investigate : more appropriate scheme for c-cbar production, including c-cbar correlation.

The 3 layers of the numerical modelling



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Initialisation of c-cbar pairs according to FONLL^2 $\otimes$ EPOS positions

Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**

Need for correlations between c & cbar… will be implemented with EPOS4

Correlated SC trajectories

Quarkonia level

Initial $J/\psi(T)$ $P(t_0)$ when $T(x) < T_{diss}$

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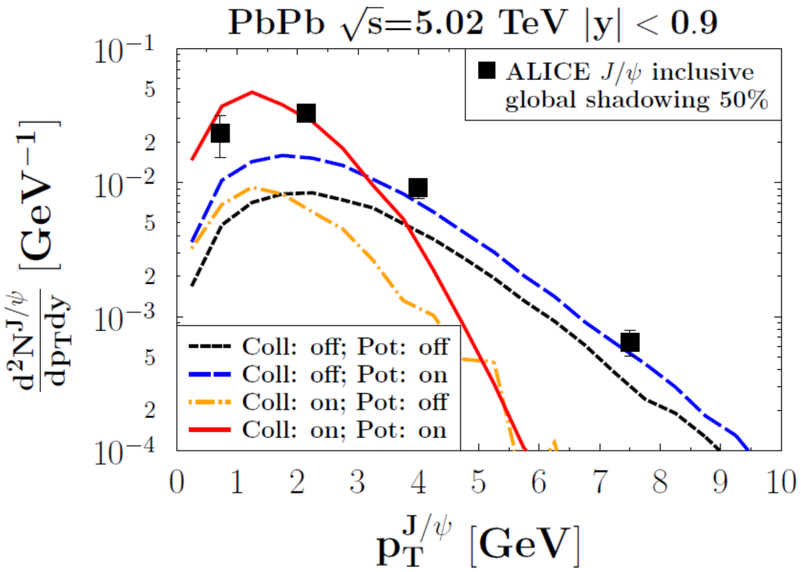
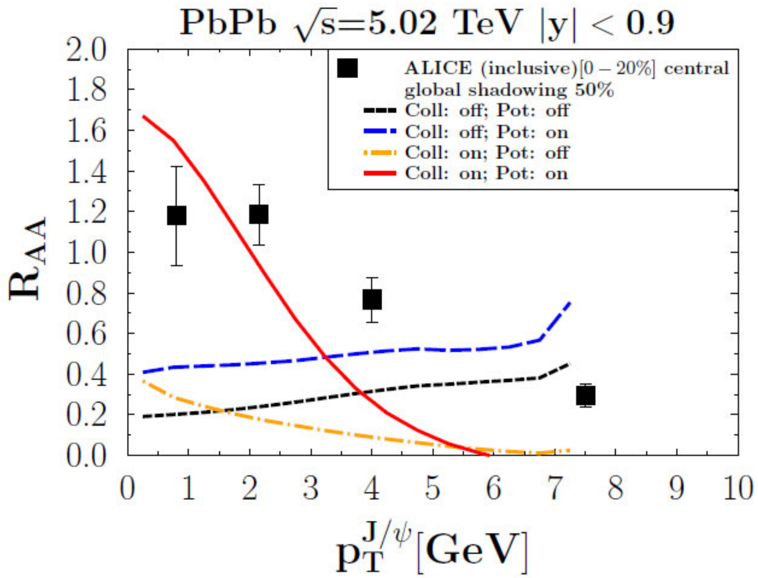
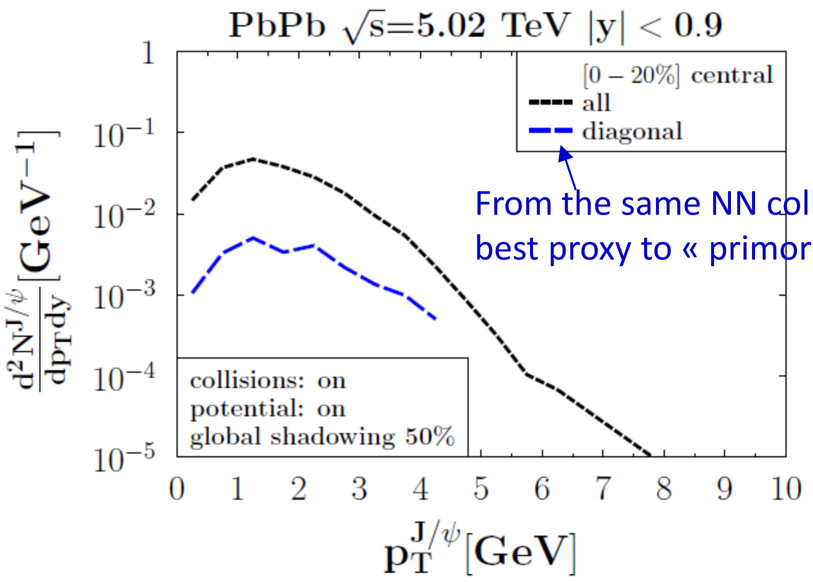




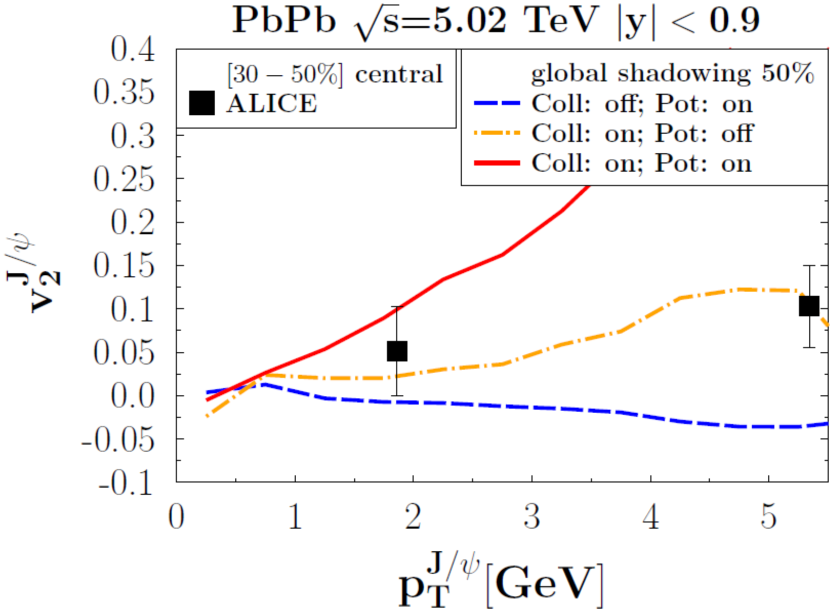


From the same NN collision, best proxy to « primordial »

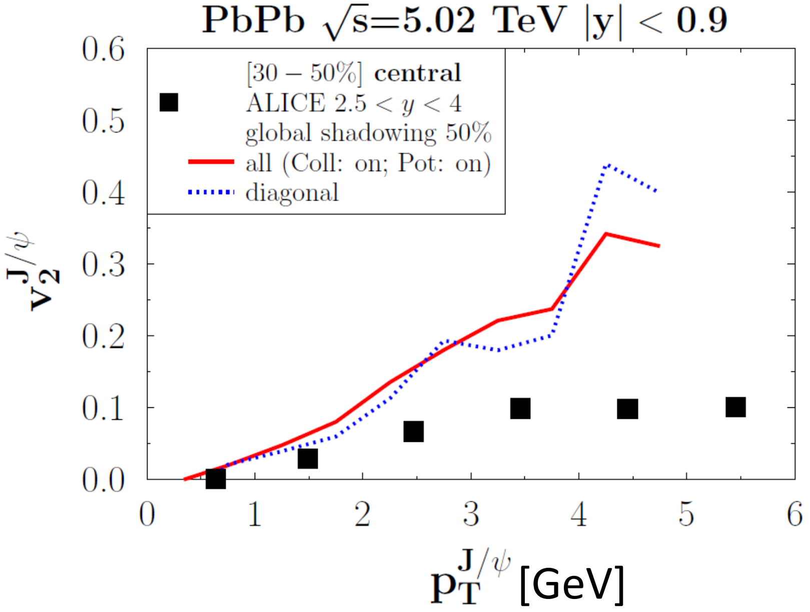
- Dynamical recombination is quite effective at low p_T
- At higher p_T, we are missing J/ψ as compared to the experimental value.
- Several possible reasons, under investigation:
  - ○ in terms of transport model : « primordial too much suppressed »
  - ○ lack of c-cbar correlation in the IS
  - ○ Higher states…

Results : J/ψ v_2

[30-50%]



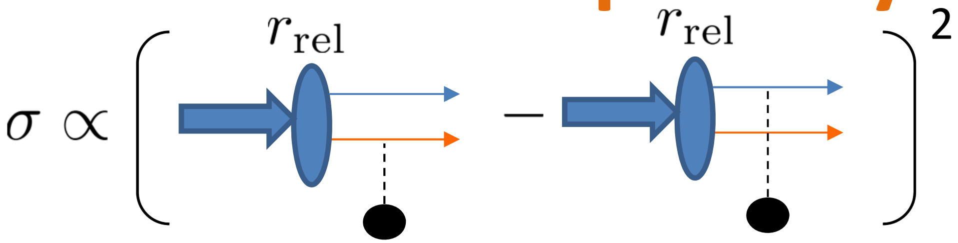
[30-50%]



- v_2 excess as compared to experimental data (**late formation** of the J/ψ due to binding potential under restoration)
- The « diagonal » contribution shows no difference wrt the full production, what is a bit conter-intuitive

The two sides of color transparency

Q-Qbar propagation in QGP.



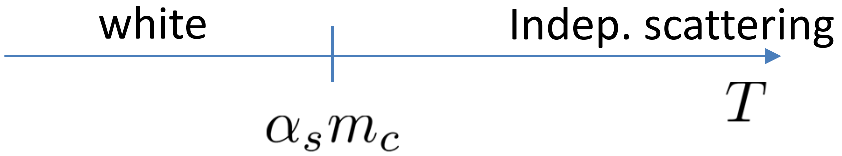
If $r_{\mathrm{rel}} << l_{\mathrm{correl}}$: **white object** => no Energy loss

If $r_{\mathrm{rel}} >= l_{\mathrm{correl}}$: 2 HQ interact individually with QGP.

$$l_{\mathrm{correl}} \sim \frac{1}{m_D}$$ (soft modes)

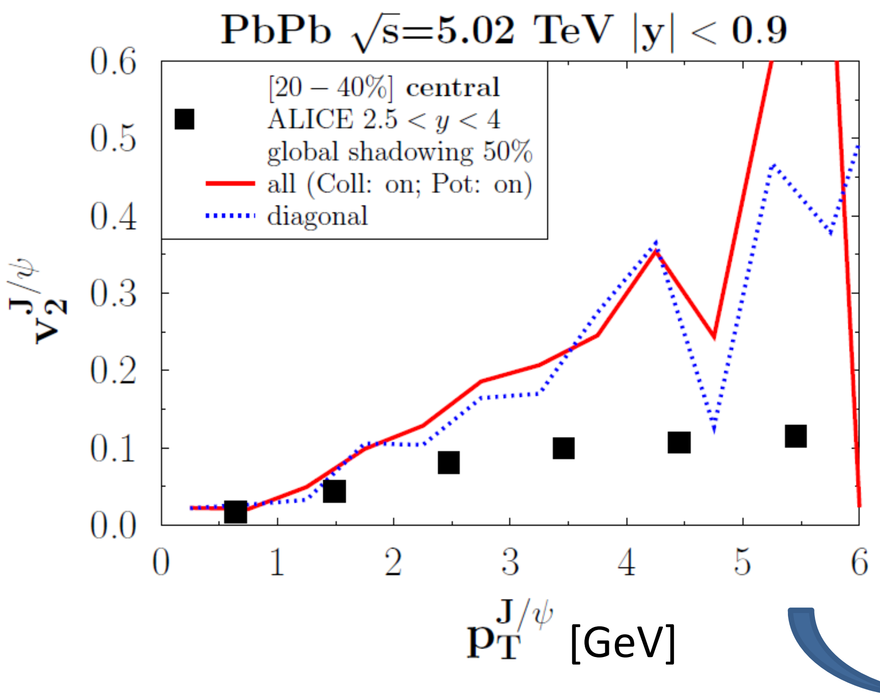
Small T : $r_{\mathrm{rel}} \approx \frac{1}{\alpha_s m_c}$

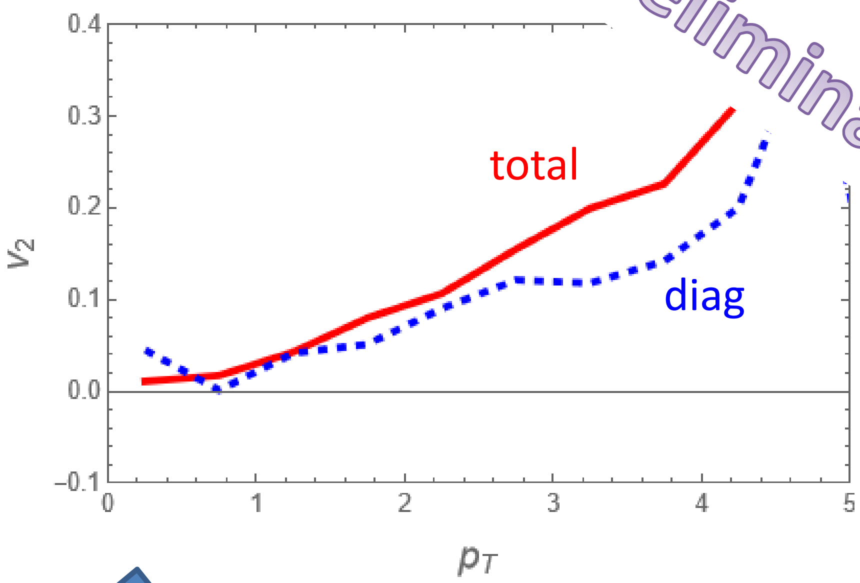
Large T : $r_{\mathrm{rel}} \gtrsim \frac{1}{m_D} \approx \frac{1}{gT}$



- Most of the transport models have considered up to now that primordial charmonia can just be destroyed (with a small probability), but not deflected.
- In our approach, we have investigated the consequences of considering the opposite limit… with too large v_2 resulting from this prescription…

Results : J/ψ v_2





Preliminary

Dipole-like penalty factor for HQ-QGP scatterings

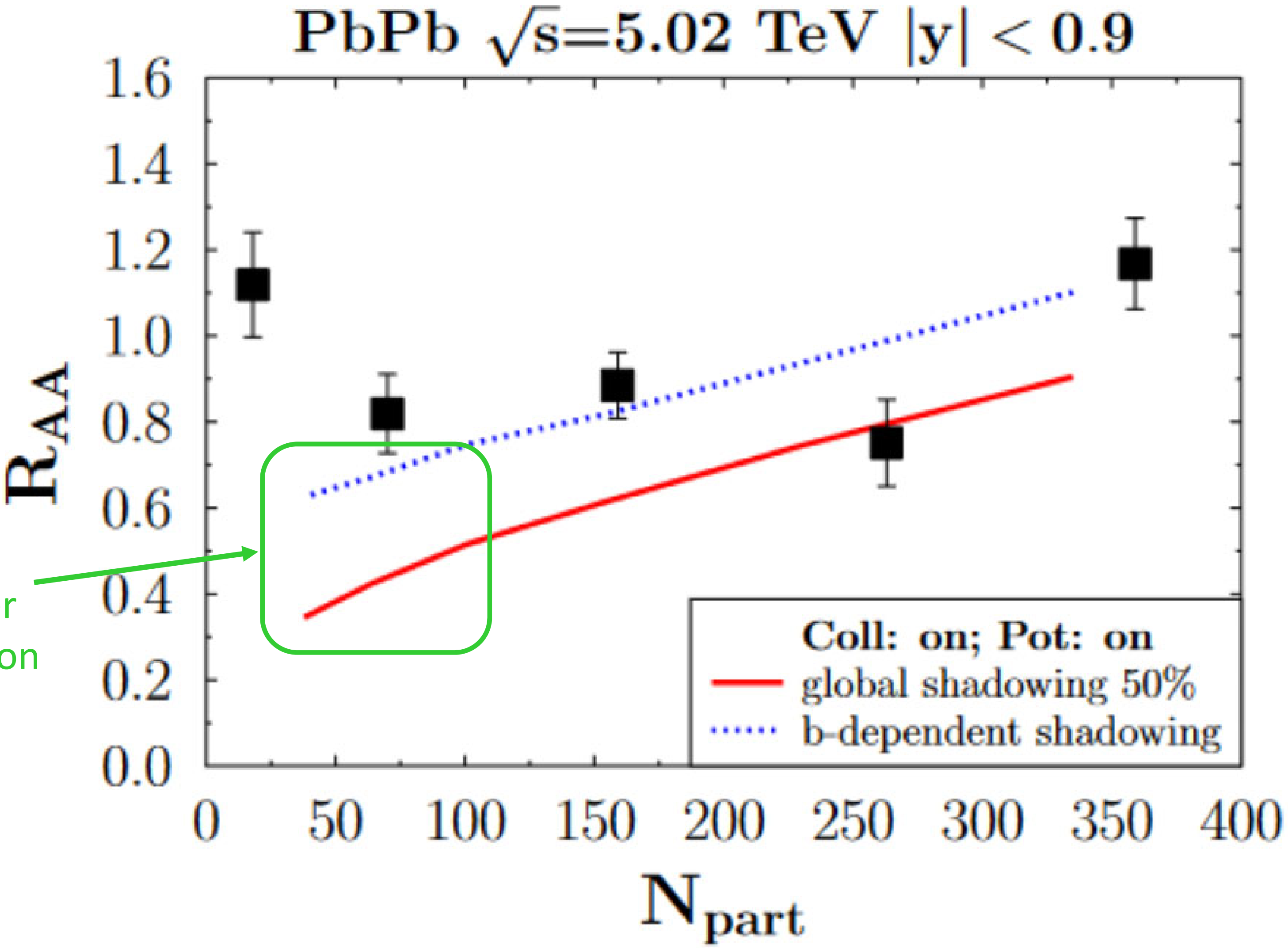
- v_2 of the diagonal component seems to be reduced wrt the v_2 of the total production
- To be further explored

Conclusions and Perspectives

- First move towards a microscopic approach based on individual c and cbar dynamics implementing some of the open quantum systems features for charmonia production in realistic HI conditions : **dynamical coalescence**
- Encouraging results, but still many features to be refined
- Rooms for improvement :
  - Include excited states decay
  - Including color transparency and more generally color dynamics
  - More reliable treatment of the fully relativistic evolution of a N-body coupled system (under construction)
  - More realistic « initial state » for the c-cbar pairs, including correlations at intermediate p_T.
  - Late interactions with hadronic phase
  - … (suggestions welcome)

Back up

R_{AA} vs N_{part}

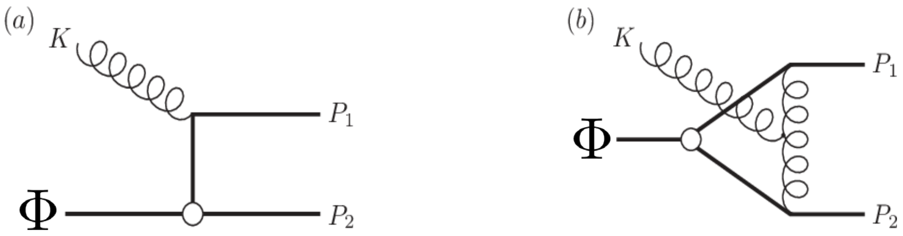


Corona part missing, under implementation in EPOSHQ

A central quantity: the decay rate Γ

Several approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t, \vec{r}) \approx \langle \psi(t, \tfrac{\vec{r}}{2})\bar{\psi}(t, -\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

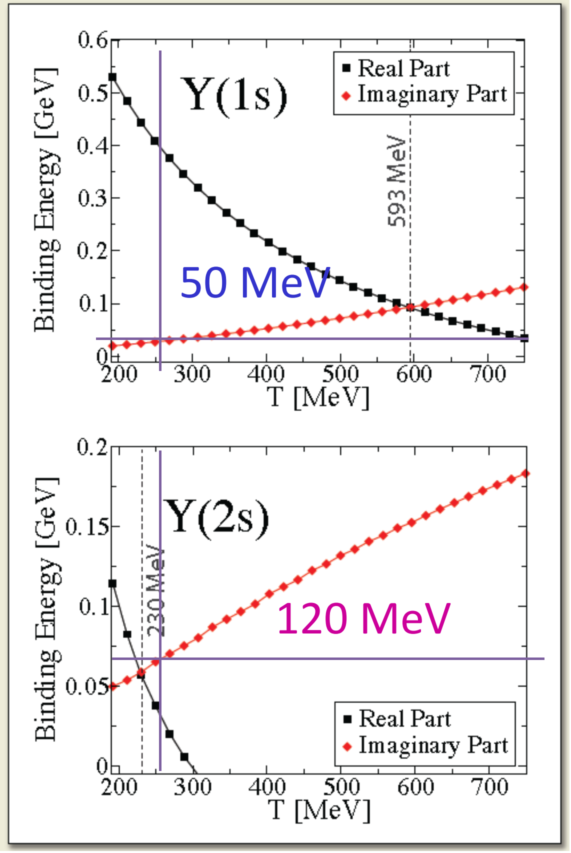
Satisfies Schroedinger equation with imaginary potential iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

Suppression of the bottomonium « candle »

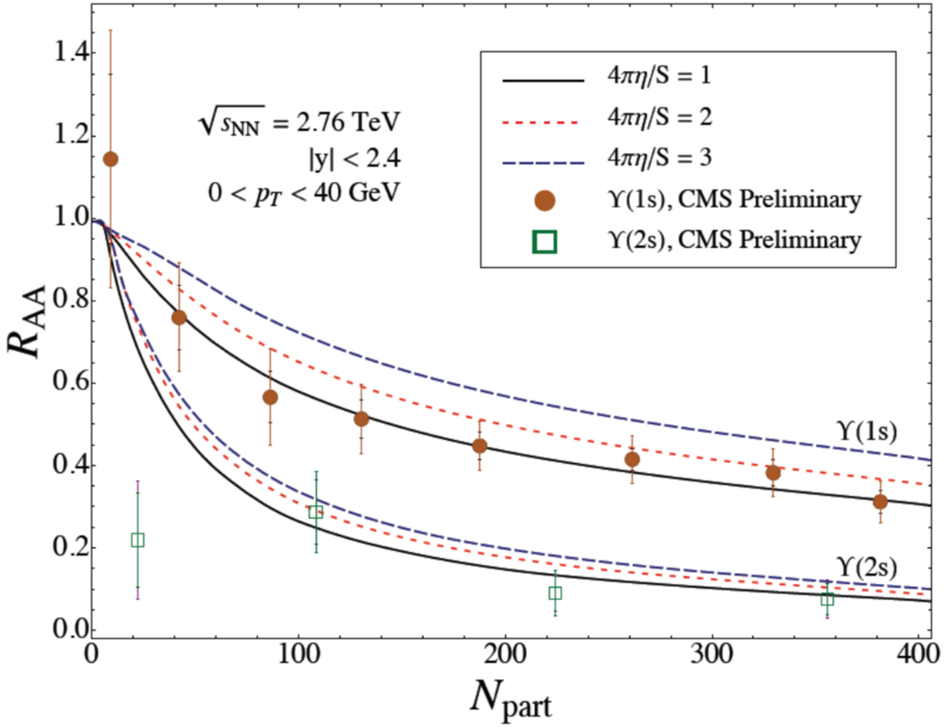
Strickland et al.



Resulting decay rate $\Gamma_T \equiv -2\,\mathrm{Im}[E_{bind}]$

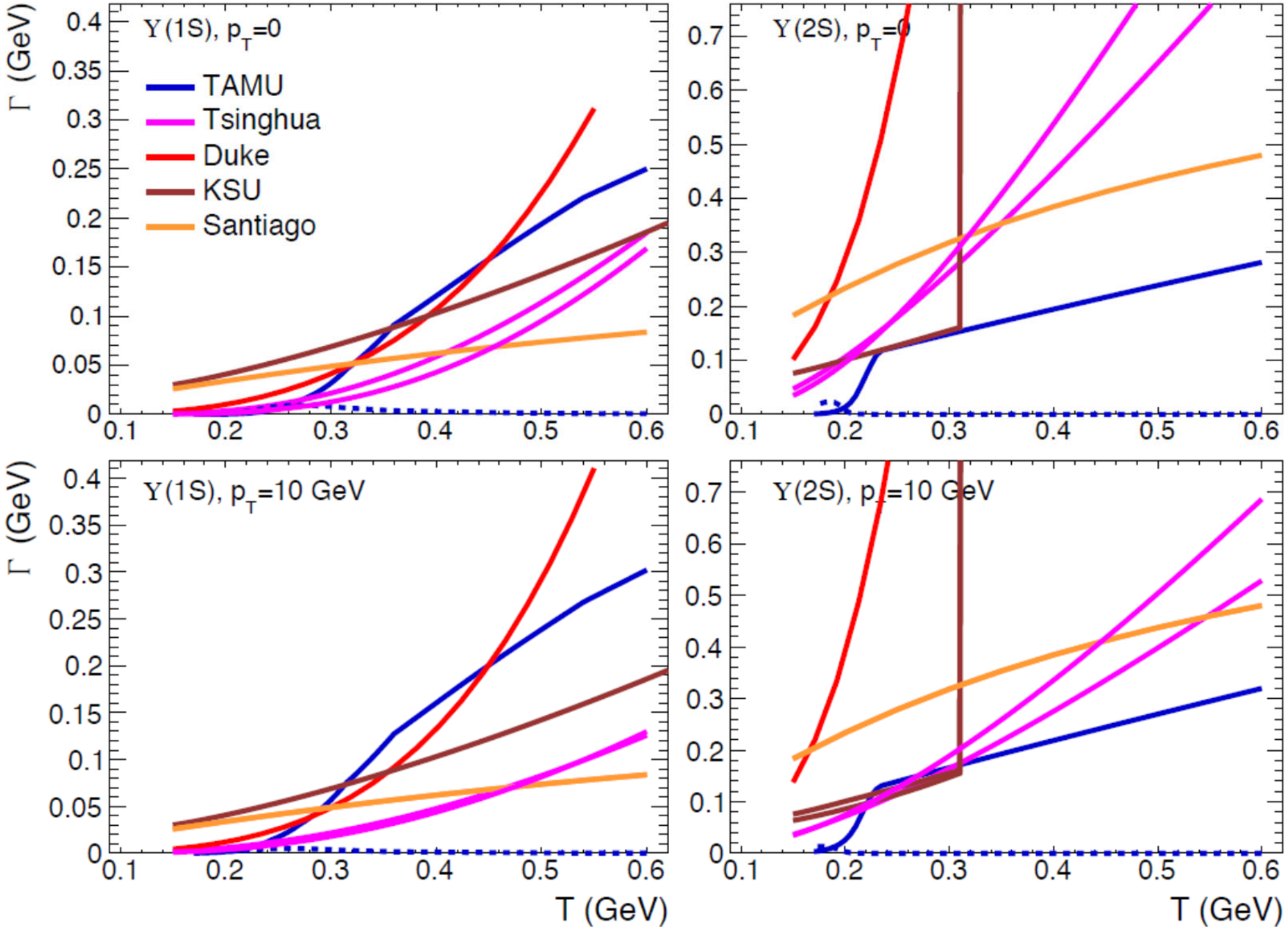
$b\bar{b} \Rightarrow$
- Less shadowing
- Less final state effect

B. Krouppa, R. Ryblewski, and M Stickland, Phys. Rev. C 92, 061901(R) (2015).



… as well as QGP viscosity

Diversity in the dissociation rates



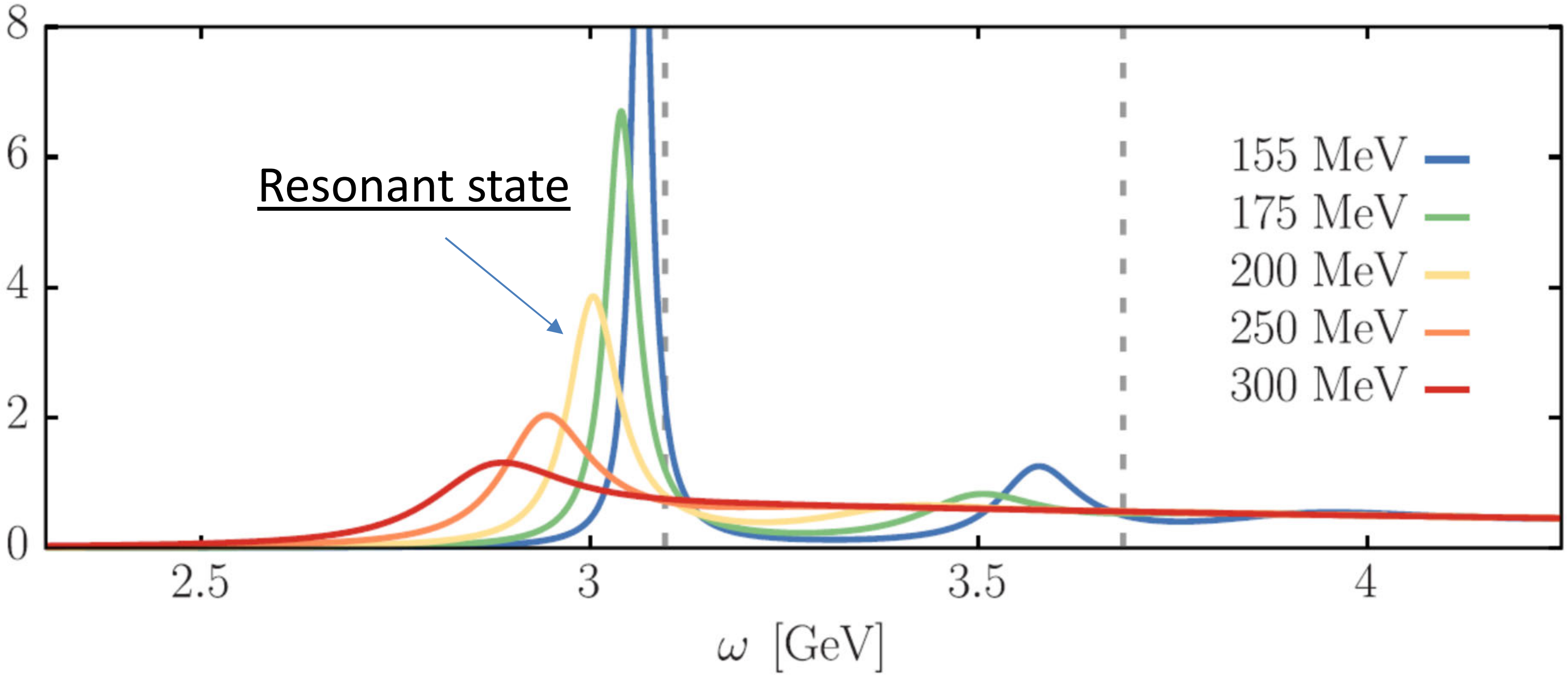
EMMI RRTF on QUARKONIA (Dec 2019)

See https://indico.gsi.de/event/9314/overview and manuscript in preparation

Extra Motivations

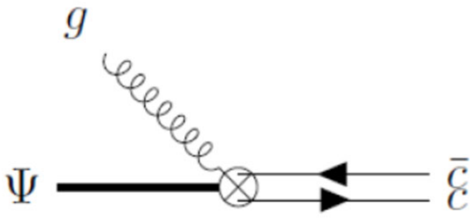
Spectral density

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)

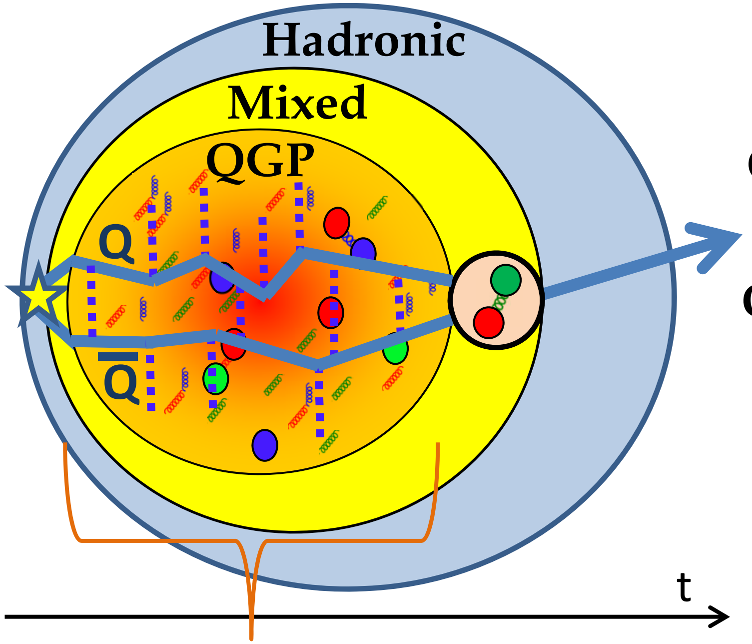


Resonant state

How justified is it to deal with the quantum evolution of such state with cross sections, meant to describe the reactions of asymptotic states ?



Quantum coherence



Quarkonium
?
Open mesons

only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

~~Very complicated QFT problem at finite T(t) !!!~~

How to proceed ?

J/ψ are *quantum* bound states => need for a formalism that preserves *quantum* properties… and continuous transitions between bound and unbound states

Dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

τ_E: environment correlation time

$$\tau_E \sim \frac{1}{T}$$

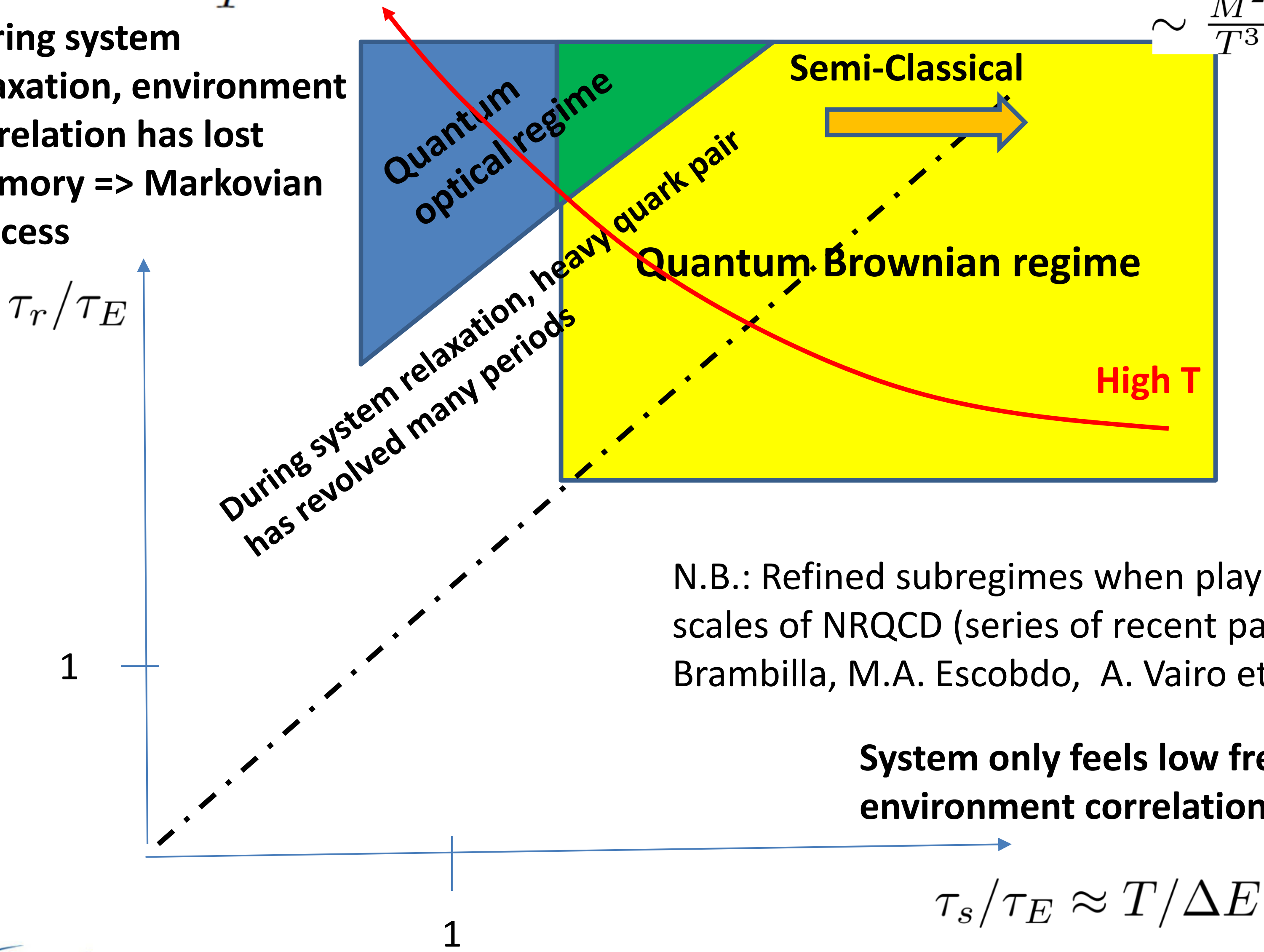
τ_S: system intrinsic time scale

$$\tau_S \approx \frac{1}{Mv^2} \approx \frac{1}{\Delta E}$$

τ_R: system relax time

$$\tau_r \sim \frac{1}{\Gamma} \approx \frac{1}{\alpha T(m_D^2\langle r^2\rangle)} \sim \frac{M^2}{T^3} \gg \tau_E$$

During system relaxation, environment correlation has lost memory => Markovian process



$$\frac{\tau_r}{\tau_E} \sim \frac{M^2}{T^2}$$

N.B.: Refined subregimes when playing with the scales of NRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo et al)

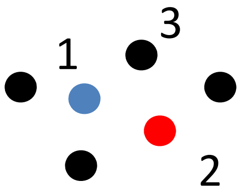
System only feels low frequency part of environment correlation

Not clear all states goes from one regime to the other at the same T

Remler formalism at work

Combining the rate definition + VN equation : $\Gamma^\Psi(t) = -iTr[\rho^\Psi[H_N,\rho_N(t)]]$

Generic case where $H_N = \Sigma_i K_i + \Sigma_{i>j} V_{ij}$



1 & 2 : $c \,\&\, \bar{c}$

3, 4, …… : light quarks

Strictly speaking, not QCD. Important process partly missing : gluo-dissociation

$$H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$

$c\bar{c}$ Internal Hamiltonian

Light quarks

$\Sigma_{i>2} V_{i1}$ $\Sigma_{i>2} V_{i2}$

Heavy – light interaction

$$\Gamma^\Psi(t) = -iTr[\rho^\Psi[H_N,\rho_N(t)]] = -iTr[\rho_N(t)\boxed{[\rho^\Psi, H_N]}]$$

Only U_1 and U_2 => ≠0 (as $[\rho^\Psi, H_{1,2}] = 0$)

$$\Gamma^\Psi(t) = -iTr[\hat{\rho}^\Psi\boxed{[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]}]$$

Sub-part of the VN equation, still impossible to deal with exactly at the quantum n-body level

Remler formalism at work

Passing to the Wigner representation:

$$W_N(\{r\},\{p\}) = \int \Pi d^3y e^{ipy} \langle r - \frac{y}{2} | \hat{\rho}_N | r + \frac{y}{2} \rangle$$

Direct space

$$\begin{aligned} \partial\rho_N(t)/\partial t \quad &= \quad -i\Sigma_j[K_j,\rho_N(t)] \\ &\quad\;\; -i\Sigma_{j>k}[V_{jk},\rho_N(t)] \end{aligned}$$

Wigner space….

$$\begin{aligned} \partial W_N(t)/\partial t \quad = \quad & \langle \Sigma_i v_i \cdot \partial_r W_N(\mathbf{r},\mathbf{p},t) \rangle + \\ & \langle \Sigma_{i\geq j} \Sigma_n \delta(t - t_{ij}(n)) \times \\ & (W_N(\mathbf{r},\mathbf{p},t+\epsilon) - W_N(\mathbf{r},\mathbf{p},t-\epsilon)) \rangle \end{aligned}$$

One to one correspondance

… treated at the semi-classical level :

Wigner distribution ⬄ {trajectories in phase space}

$[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]$ can be modelized from the trajectories evolution in Wigner space

Remler formalism at work

The effective rate for quarkonia state creation (dissociation) in the medium is

$$\Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Working in the phase space through Wigner distribution

$$W^{\Psi_i}_{Q\bar{Q}} = \int d^3y e^{ipy}\langle r - \tfrac{y}{2}|\Psi^i\rangle\langle\Psi^i|r + \tfrac{y}{2}\rangle$$

Quarkonia: Double Gaussian approximation

$$W^{\Psi}_{Q\bar{Q}}(r_{\rm rel}, p_{\rm rel}) = C e^{r_{\rm rel}^2\sigma^2} \times e^{\frac{p_{\rm rel}^2}{\sigma^2}}$$

Parameter: The Gaussian width $\sigma \approx 0.35$ fm

$$[\frac{\hbar^2}{2\mu}\nabla^2 + V(r)]\Psi_{QQ}(r) = E_{QQ}\Psi_{QQ} \longrightarrow \langle r^2\rangle \longrightarrow W^{\Psi}$$

W_N : Semi-classical approach

$$W_N = \Pi_i \hbar^3 \delta(x_i - x_{i0}(t))\delta(p_i - p_{i0}(t))$$

… but no explicit description of W_N required (as it appears in the trace)

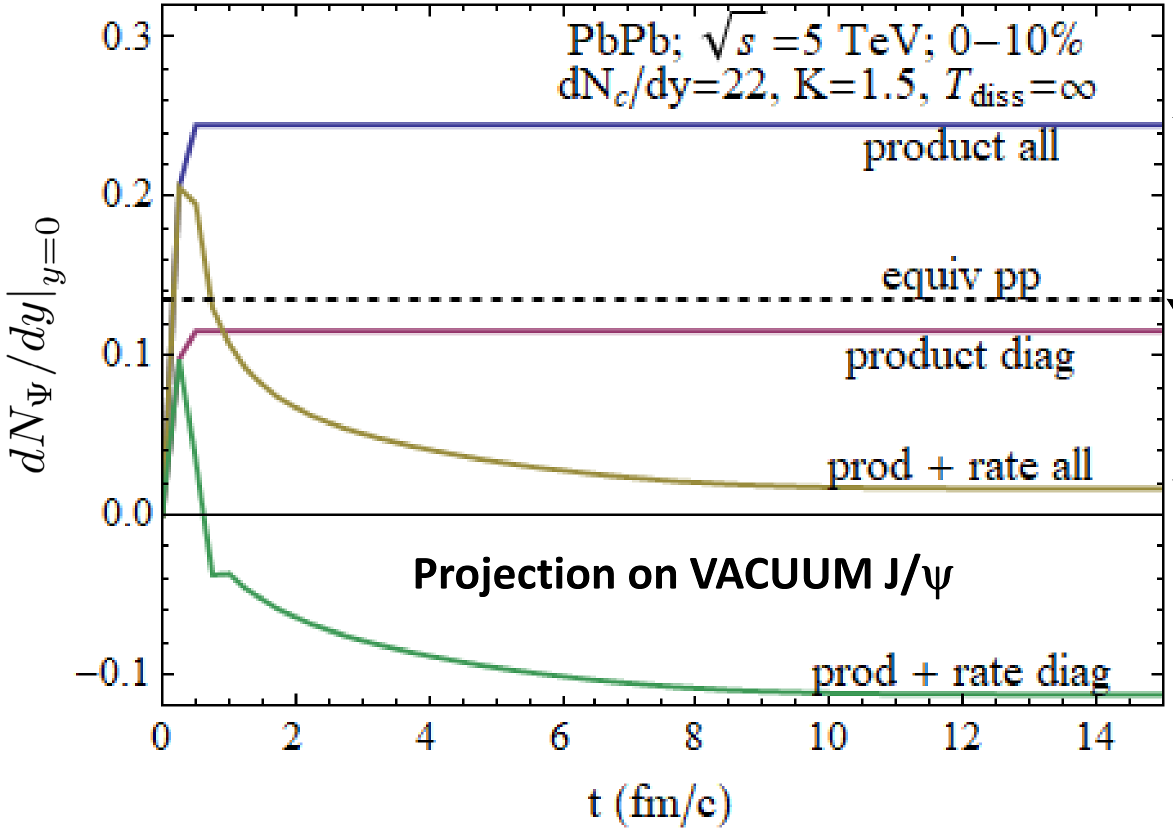
and (less trivial) : generalisation at finite 4-velocity u; fully relativistic… to warrant orthogonality of states

$$\mathrm{Tr}[W_u^{J/\psi} W_u^{\psi'}] = 0$$

Preliminary results for J/ψ production in Pb-Pb

Word of caution: Exploratory phase => not meant to have an exact comparison with exp. data

$$P^{\Psi}(t) = \boxed{\boxed{P^{\Psi}(t_0) +} \int_{t_0}^{t} \Gamma(t')dt'}$$



Projection on VACUUM J/ψ

Cumulated « production » (if no rate equation), indeed oveshoots pp due to off-diagonal contributions

The denominator in the R_{AA}

The full production (i.e. the numenator in the R_{AA})

First answer to puzzle found in Song et al: the primordial production is killed rather fast by the « loss » rate.

Remler formalism for the QGP : last ingredient

Combining the rate definition + VN equation : $\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]]$

$$\Rightarrow \quad H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$

$c\bar{c}$ Internal Hamiltonian … … …

In QGP, 2 body T-dependent effective potential => ….

$$\Gamma^{\Psi}(t) = -iTr[\rho^{\Psi}[H_N, \rho_N(t)]] = -iTr[\rho_N(t)\boxed{[\rho^{\Psi}, H_N]}]$$

$$\Downarrow \quad [\rho^{\Psi}, H_{1,2}(T)] = 0$$

$$= -iTr[\hat{\rho}^{\Psi}(T)[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

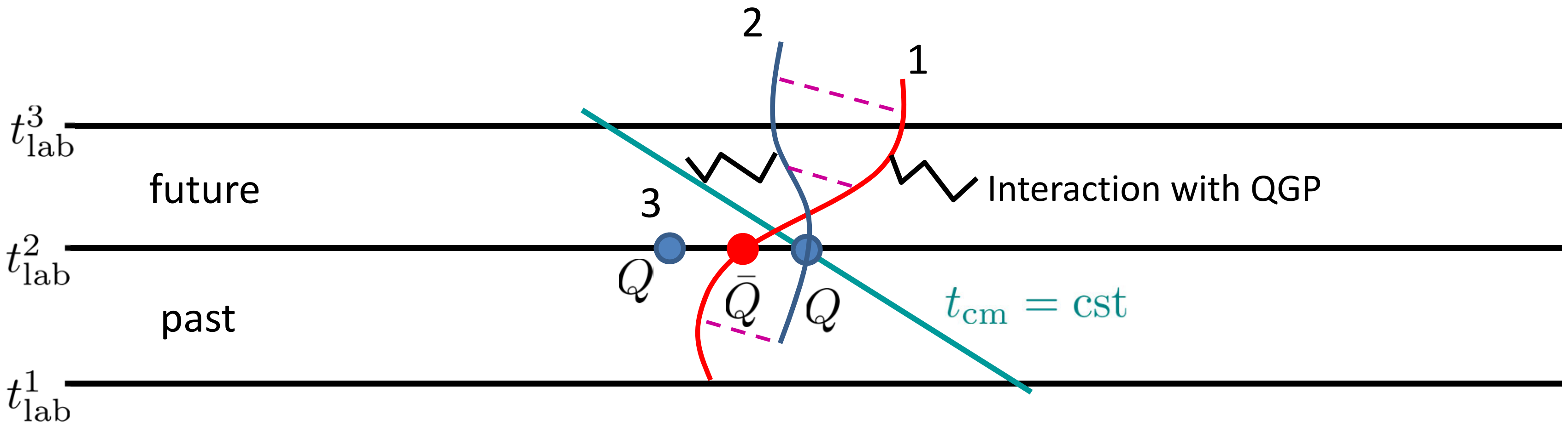
One only preserves the structure of the Remler « collisional rate » if one works in the « local » basis ρ^{Ψ} (T) !!!
Accessible for T > T_{dissoc}^{Ψ} (=0.4 GeV for J/ψ)

Back to the rate : $\Gamma^{\Psi}(t) = \frac{dP^{\Psi}(t)}{dt} = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}} \frac{d\hat{\rho}_N(t)}{dt}\right]$

$$\Rightarrow \quad \Gamma^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}(T(t)) \frac{d\hat{\rho}_N(t)}{dt}\right] + \underbrace{\frac{dT}{dt}\mathrm{Tr}\left[\frac{\partial \hat{\rho}^{\Psi}_{Q\bar{Q}}(T)}{\partial T}\hat{\rho}_N(t)\right]}_{\text{New contribution to the rate (so-called « local rate »)}}$$

The Q-Qbar dynamics… the CM strategy

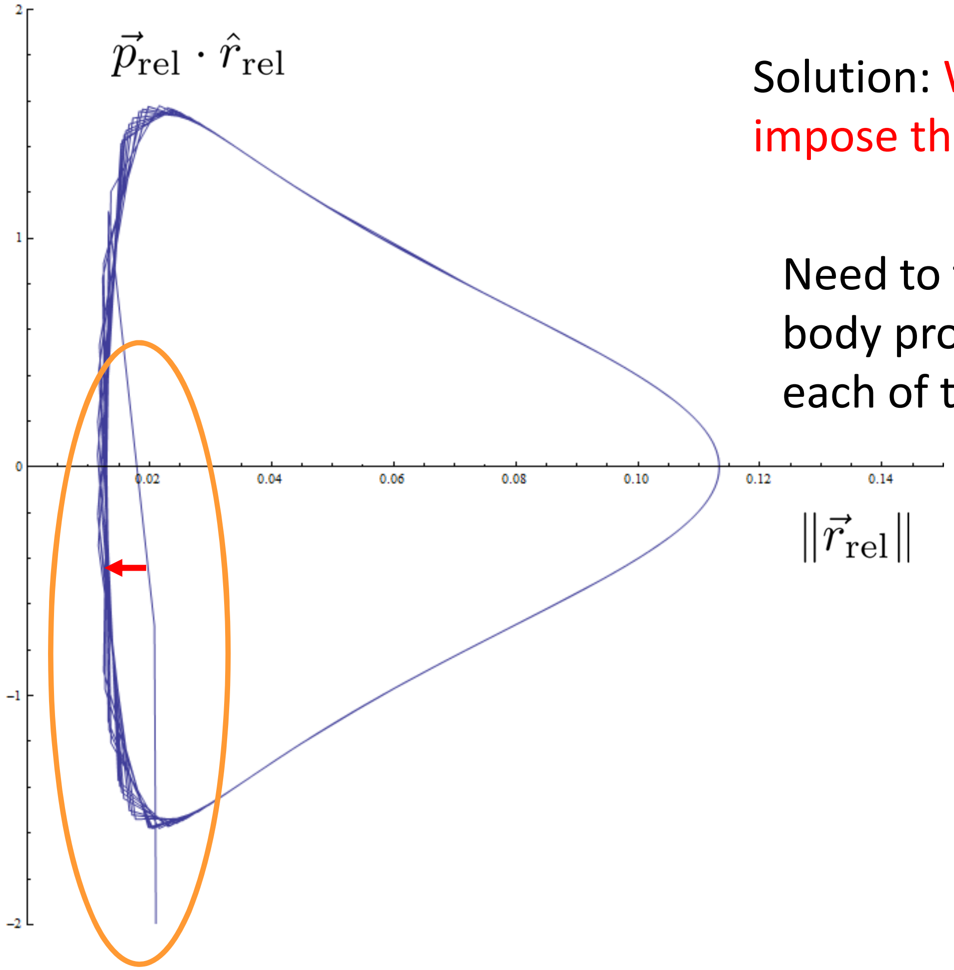
- Main objective : evaluate the propagation towards future of N Q-Qbar interacting pairs
- Strategy (adopted presently) : For each time step in the laboratory frame, pass to the cm frame and perform the evolution in the cm frame (where the potential is well defined)



- “Issue” : slicing the global time evolution (usual strategy in MC) is not 100% compatible with passing to c.m. frame as 2 particles are usually not at the same relative time in both frames (residual glitches ⇔ numerical noise)

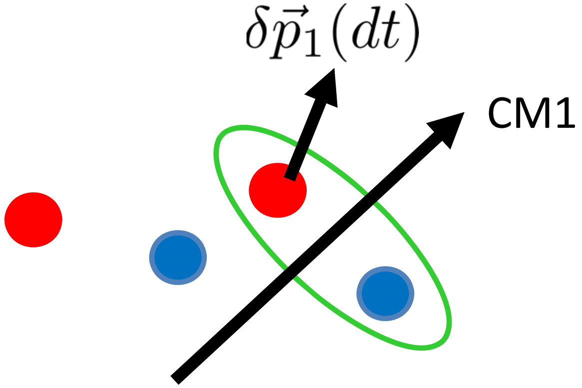
The Q-Qbar dynamics… the CM strategy

- “Minor problem” #1: Classical equations of motion are **unstable** (in the CM):



Solution: Work in Hamilton – Jacobi coordinates or impose the conserved quantities (L and Etot)

Need to factorize the N-body problem as an {} of 2-body problems for some evolution over time step dt, each of them to be solved in the CM



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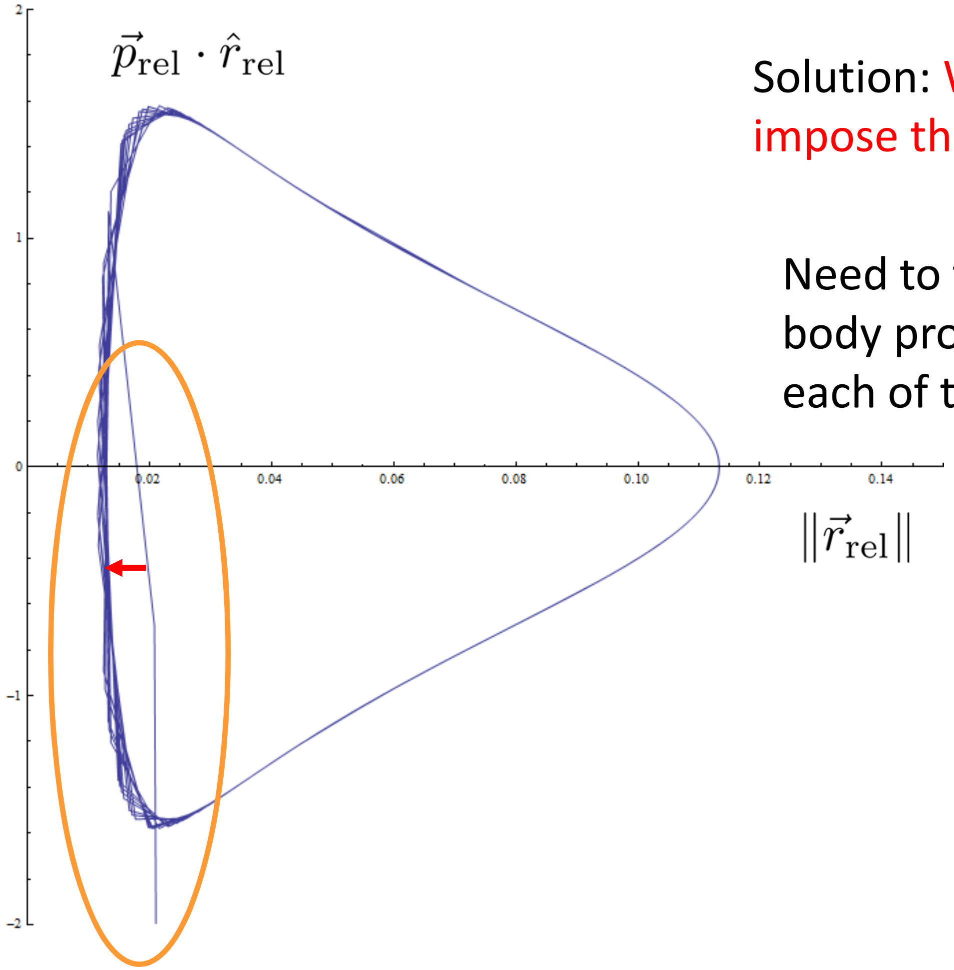

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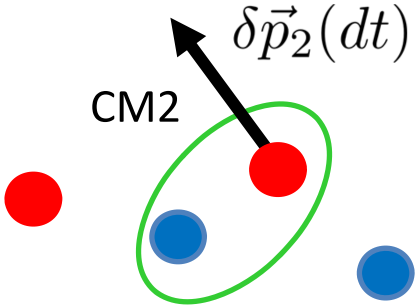

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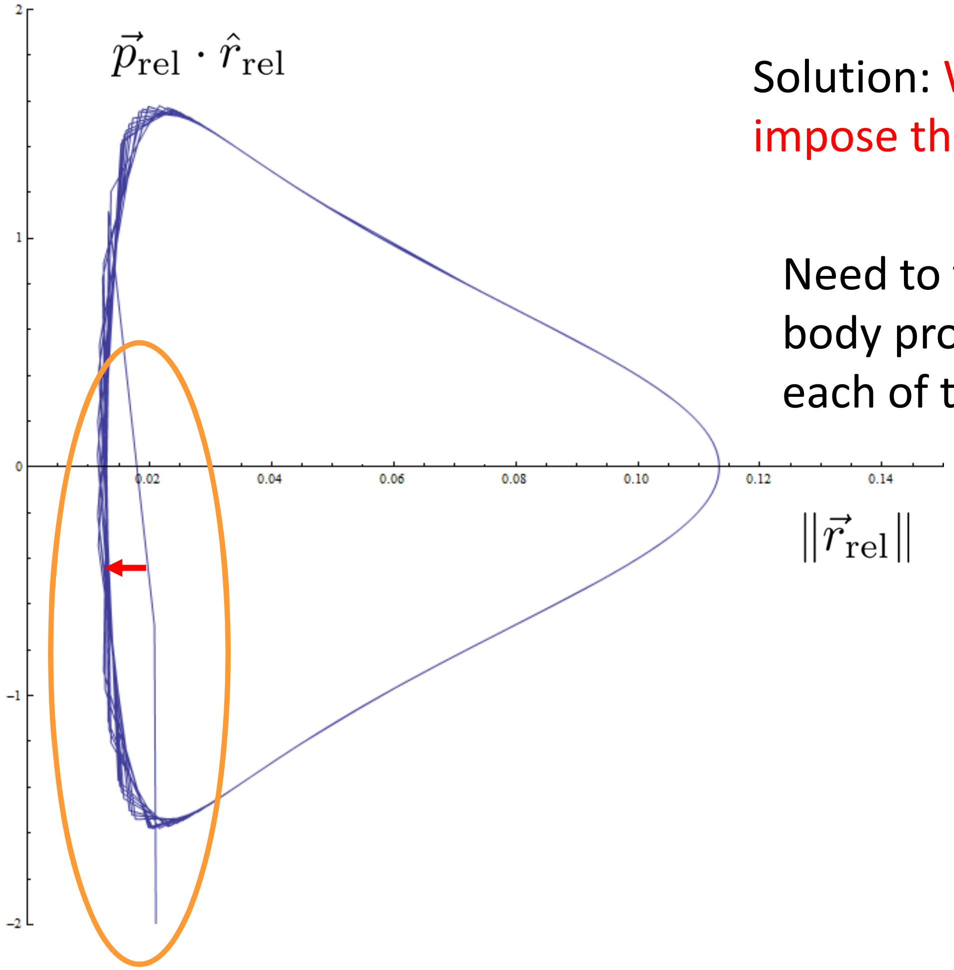

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