Impact of momentum-dependent potential on the directed and elliptic flows

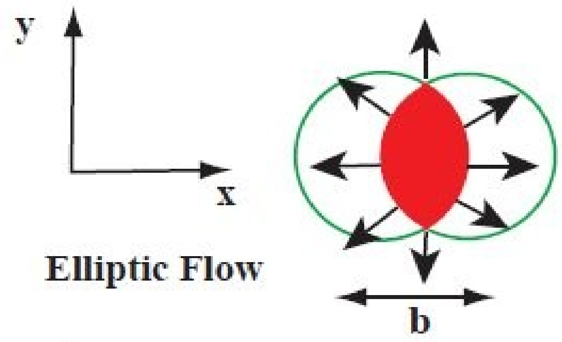
Yasushi Nara (Akita International Univ.)

- Introduction
- Relativistic quantum molecular dynamics (RQMDv)
- RQMDv with the Skyrme + momentum-dependent (MD) potential
- RQMDv with chiral mean-field (CMF) + MD potential (crossover)
- RQMDv with vector density functional (VDF) + MD potential (1st order phase transition)

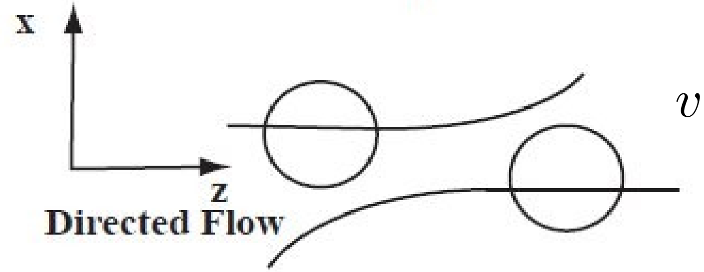
9th international symposium on Non-equilibrium dynamics, NeD-2022 Workshop 2022/11/28

Determination of EoS from flows

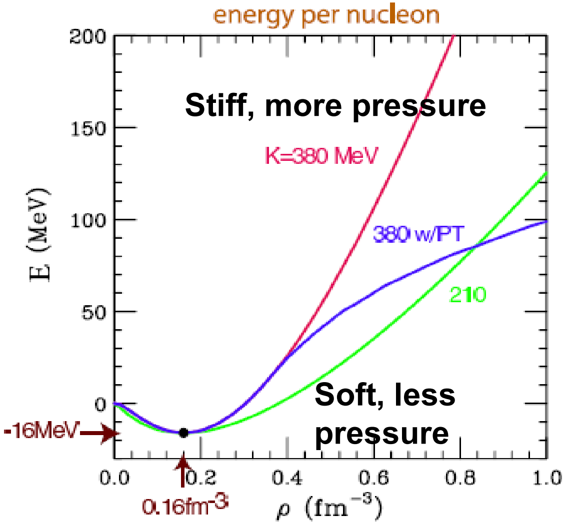
$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Phi_r)]\right)$$



$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$



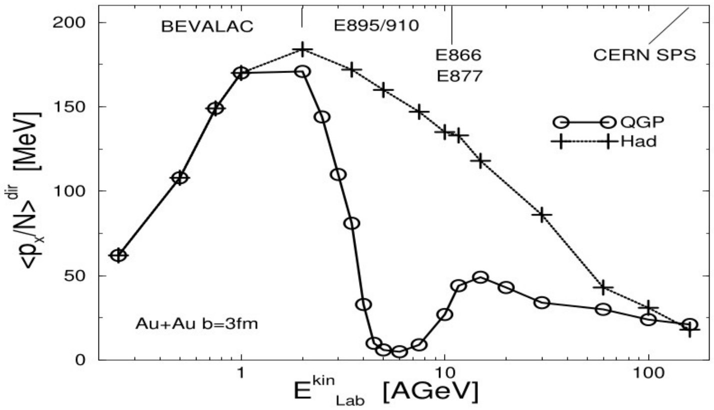
Transport model predicts strong sensitivities of EOS on the directed and elliptic flows.

P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002) 1592

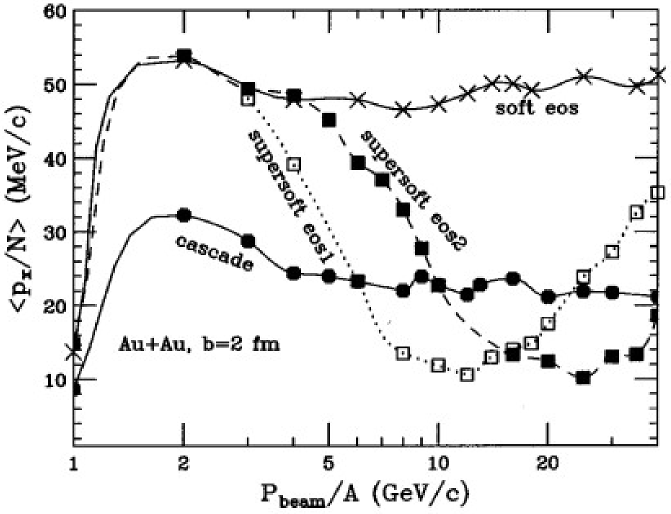
The softest point in the EoS

$$\langle p_x/N\rangle^{dir} = \frac{1}{N}\int_{-y_{CM}}^{y_{CM}} \mathrm{d}y\ \langle p_x/N\rangle(y)\ \frac{\mathrm{d}N}{\mathrm{d}y}\ \mathrm{sgn}(y)$$

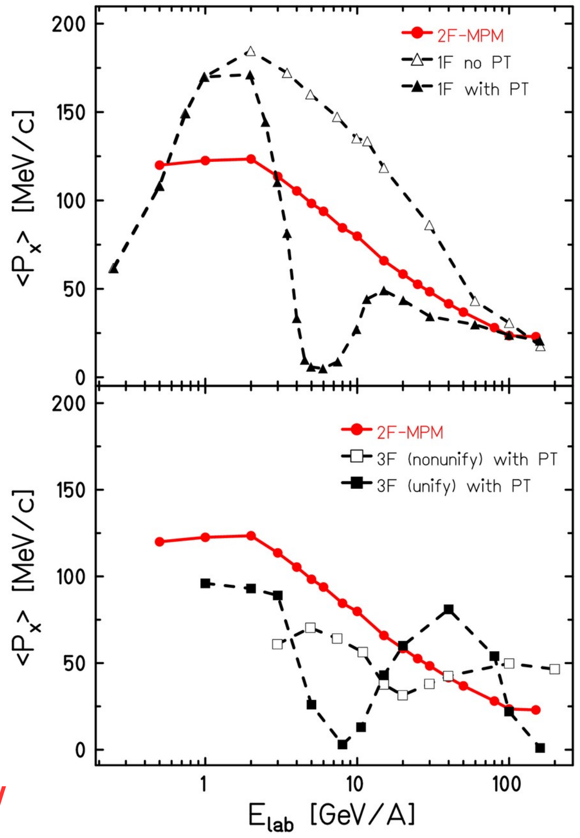
3FD:Ivanov Heavy Ion Phys.15:117-130,2002



1FD: D.H.Rischke, et.al
Heavy Ion Phys.1, 309 (1995)



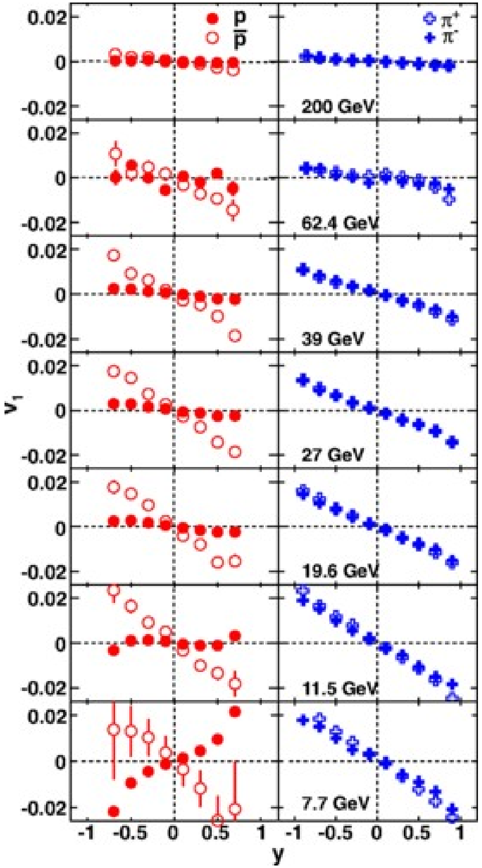
ART:B-A. Li and C.M. Ko,
PRC58(1998)R1382

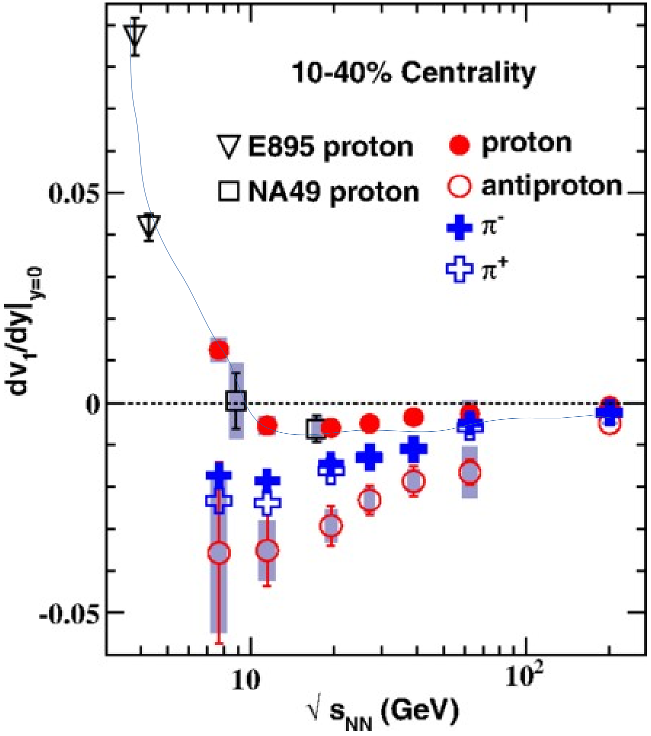


A minimum is predicted in the excitation function of the directed flow

Beam energy dependence of v1: discovery of negative flow

L. Adamczyk et al. (STAR Collaboration) Phys. Rev. Lett. 112, 162301 – 23 April 2014





Proton slope changes sign from positive to negative at 11.5GeV. Signal of a 1st-order phase transition?

But no model with 1st-order phase transition reproduces beam energy dependence of the proton v1 data.

$$v_1(y) = Fy + F_3 y^3, \quad dv_1/dy|_{y=0} = F$$

Quantum molecular dynamics

J. Aichelin, Phys. Rep. 202 (1991)233

Quantum molecular dynamics (QMD) → N-body theory

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\partial\langle H\rangle}{\partial\boldsymbol{p}_i}, \qquad \frac{d\boldsymbol{p}_i}{dt} = -\frac{\partial\langle H\rangle}{\partial\boldsymbol{r}_i} \qquad + \text{Boltzmann type collision term}$$

$$\langle H\rangle = \langle\Phi|H|\Phi\rangle, \qquad \Phi = \text{Gaussian wave packets}$$

One-particle potential V(n) use in QMD is related with the single-particle potential U(n) as

$$V(n) = \frac{1}{n}\int_0^n dn' U(n')$$

Relativistic quantum molecular dynamics

JAM2: RQMD equations of motion: scalar and vector potential

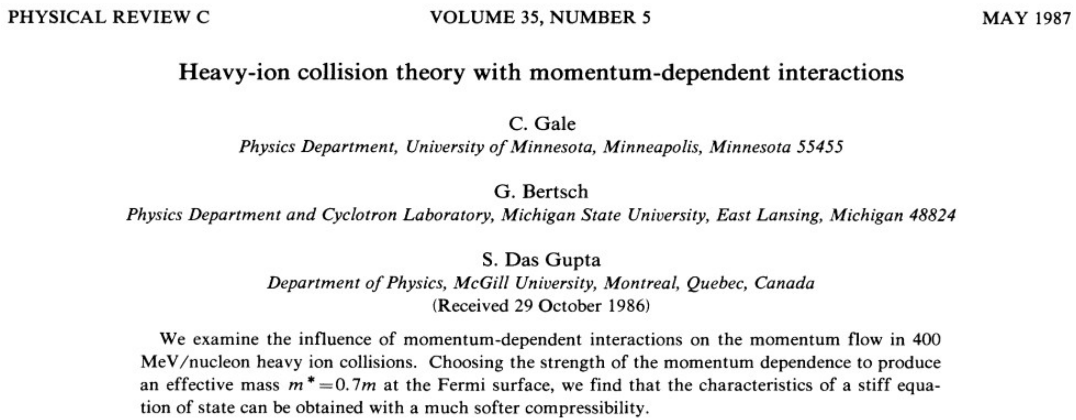
$$\dot{x}_i = \frac{p_i^*}{p_i^{*0}} + \sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial p_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial p_i} \right), \quad \dot{p}_i = -\sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial r_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial r_i} \right)$$

$$m^* = m - S(x,p), \quad p_\mu^* = p_\mu - U_\mu(x,p)$$

- RQMD.RMF: σ-ω model, PRC (2019),(2020)
- RQMDv: Lorentz vector Skyrme potential PRC(2022)

Collision term: hadronic resonances and strings.

Importance of momentum-dependent potential



PHYSICAL REVIEW C VOLUME 35, NUMBER 5 MAY 1987

Heavy-ion collision theory with momentum-dependent interactions

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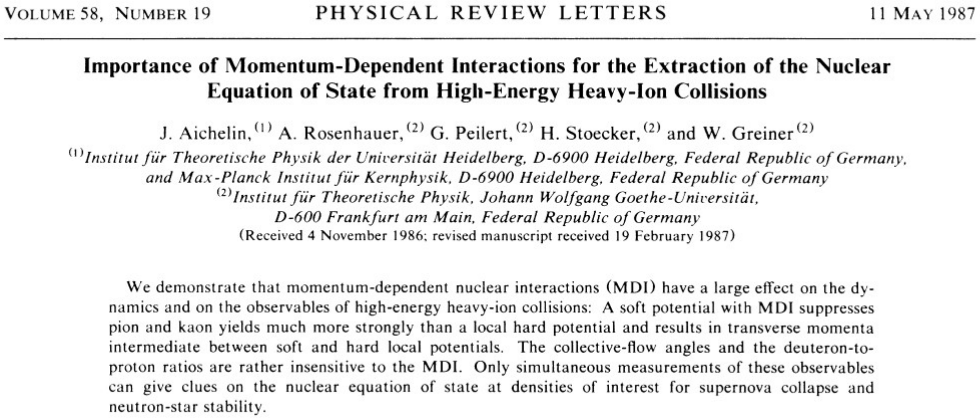
(Received 29 October 1986)

We examine the influence of momentum-dependent interactions on the momentum flow in 400 MeV/nucleon heavy ion collisions. Choosing the strength of the momentum dependence to produce an effective mass $m^* = 0.7m$ at the Fermi surface, we find that the characteristics of a stiff equation of state can be obtained with a much softer compressibility.



VOLUME 58, NUMBER 19 PHYSICAL REVIEW LETTERS 11 MAY 1987

Importance of Momentum-Dependent Interactions for the Extraction of the Nuclear Equation of State from High-Energy Heavy-Ion Collisions

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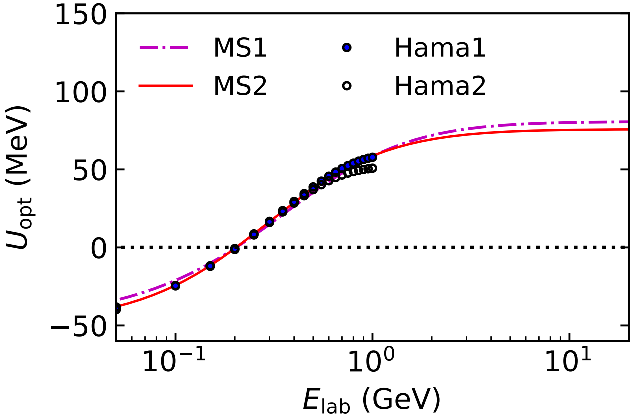
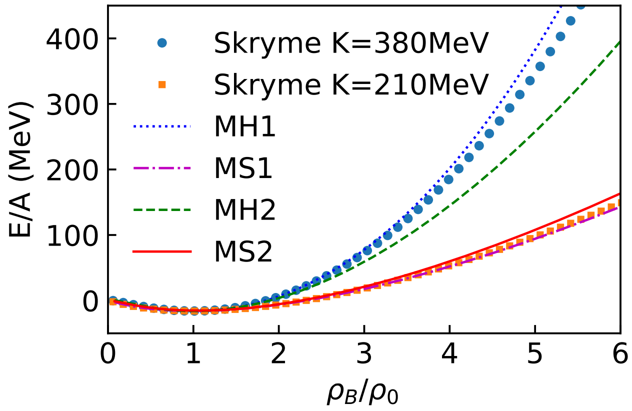
(Received 4 November 1986; revised manuscript received 19 February 1987)

We demonstrate that momentum-dependent nuclear interactions (MDI) have a large effect on the dynamics and on the observables of high-energy heavy-ion collisions: A soft potential with MDI suppresses pion and kaon yields much more strongly than a local hard potential and results in transverse momenta intermediate between soft and hard local potentials. The collective-flow angles and the deuteron-to-proton ratios are rather insensitive to the MDI. Only simultaneous measurements of these observables can give clues on the nuclear equation of state at densities of interest for supernova collapse and neutron-star stability.

EoS in the JAM2/RQMDv mode

Experimentally extracted nuclear potential from pA collisions





Skyrme type Loretnz vector potential:

$$\mathrm{p}^{*\mu} = p^\mu - U^\mu_{\rm sk}(\rho) - U^\mu_m(p).$$

$$U_{\rm sk}(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^\gamma,$$

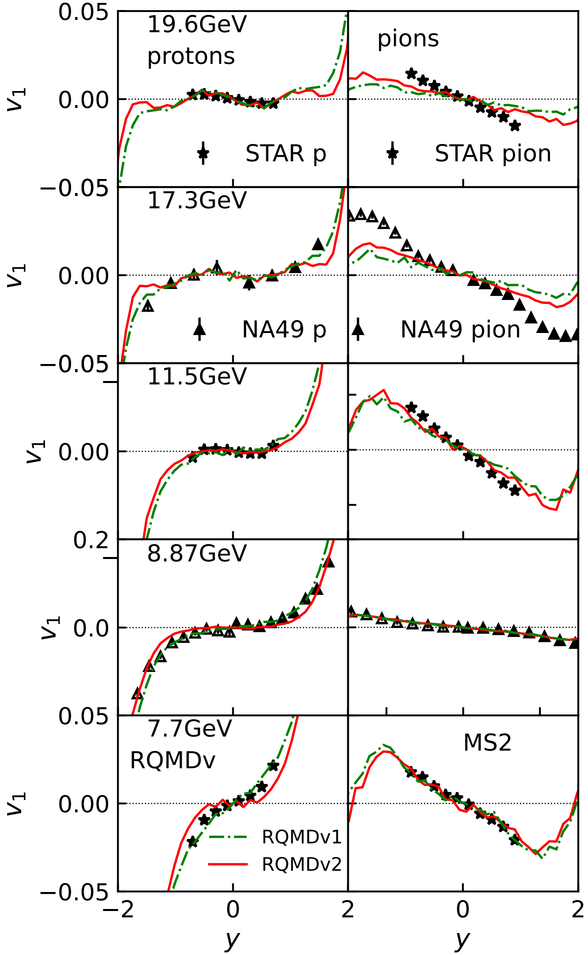
$$U^\mu_m(p) = \frac{C}{\rho_0}\int d^3p' \frac{p^{*\prime\mu}}{e^*}\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$

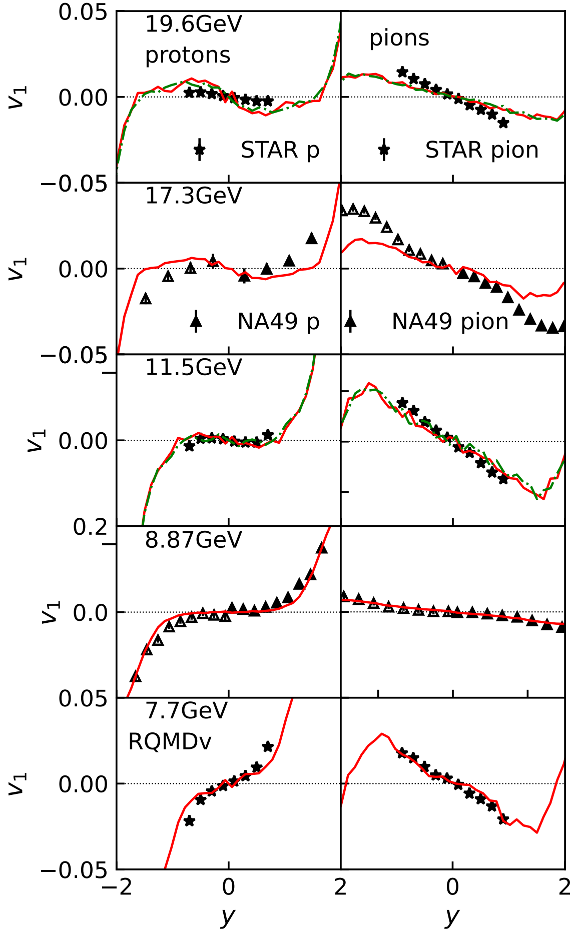
Energy density:

$$e = \int d^3p\left(e^* + U^0_m - \frac{1}{2}\frac{p^*_\mu}{e^*}U^\mu_m(p)\right)f(p) + \int_0^\rho U^0_{\rm sk}(\rho')d\rho'$$

Y.N. and A. Ohnishi, PRC(2022)

Beam energy dependence of v1 from RQMD


RQMDv

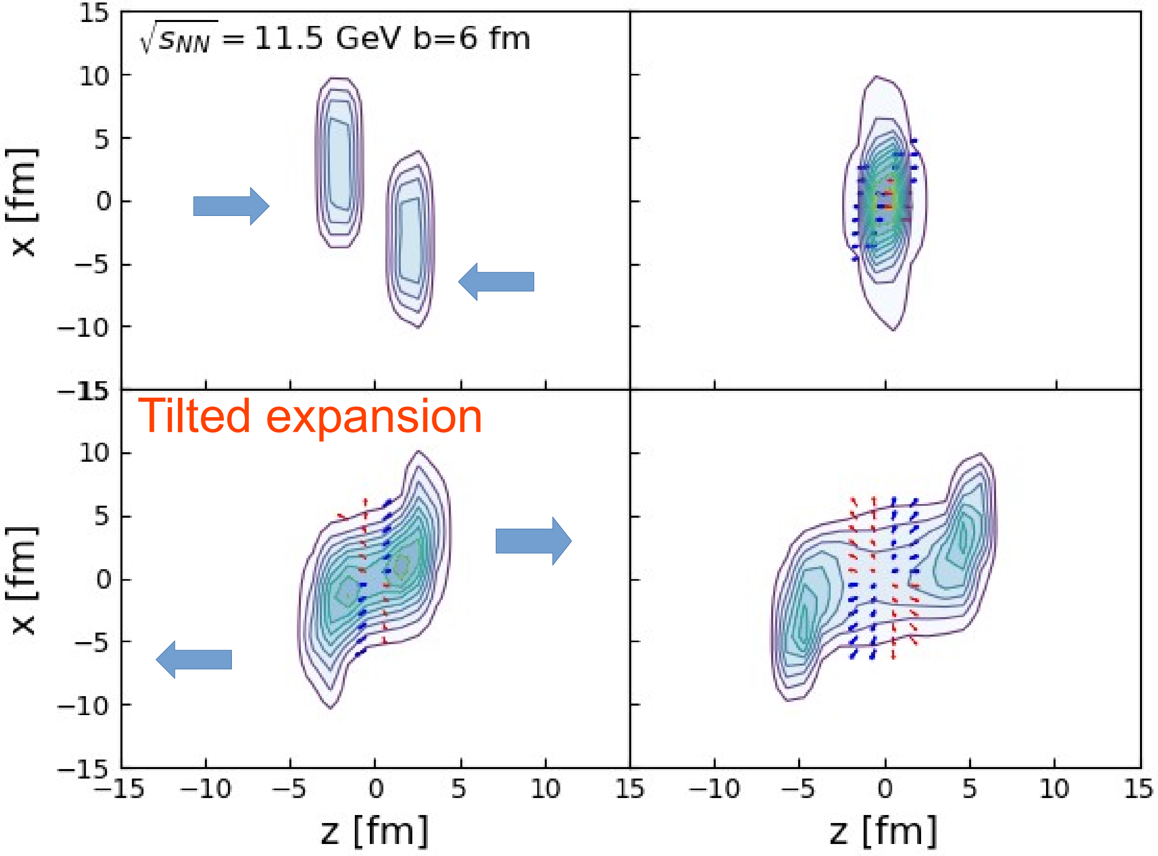

RQMD.RMF

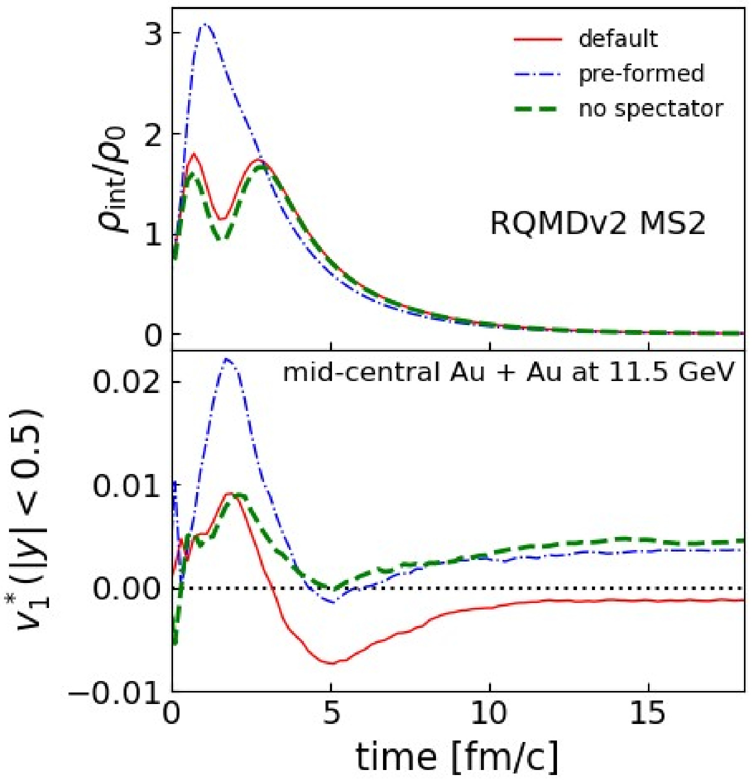
Beam energy dependence of v1 is explained by a mean-field both Skyrme type and sigma-omega.

Y.N, A. Ohnishi, PRC (2022)

Time evolution of v1 at 11.5GeV

Time evolution of the baryon density in Au + Au mid-central collision (b=6fm)



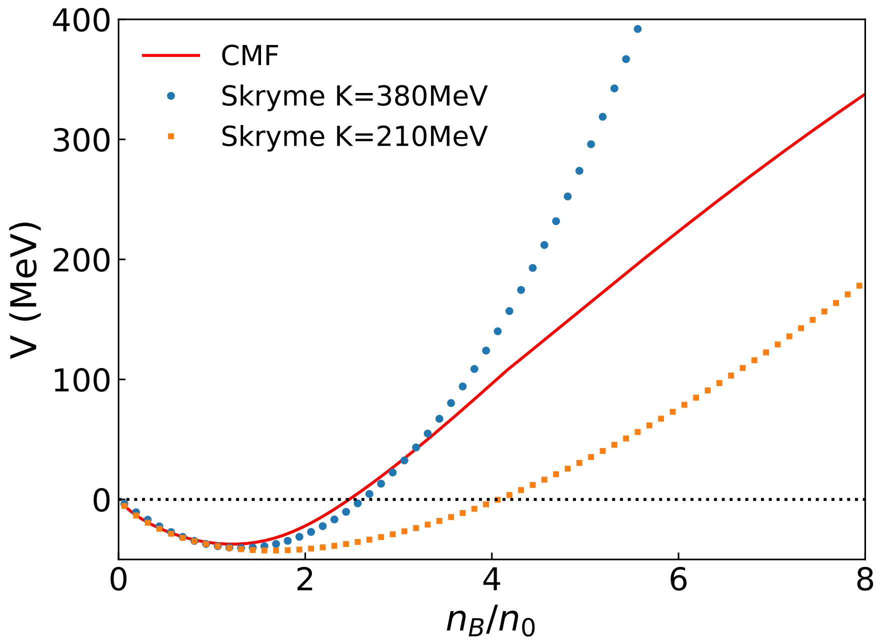


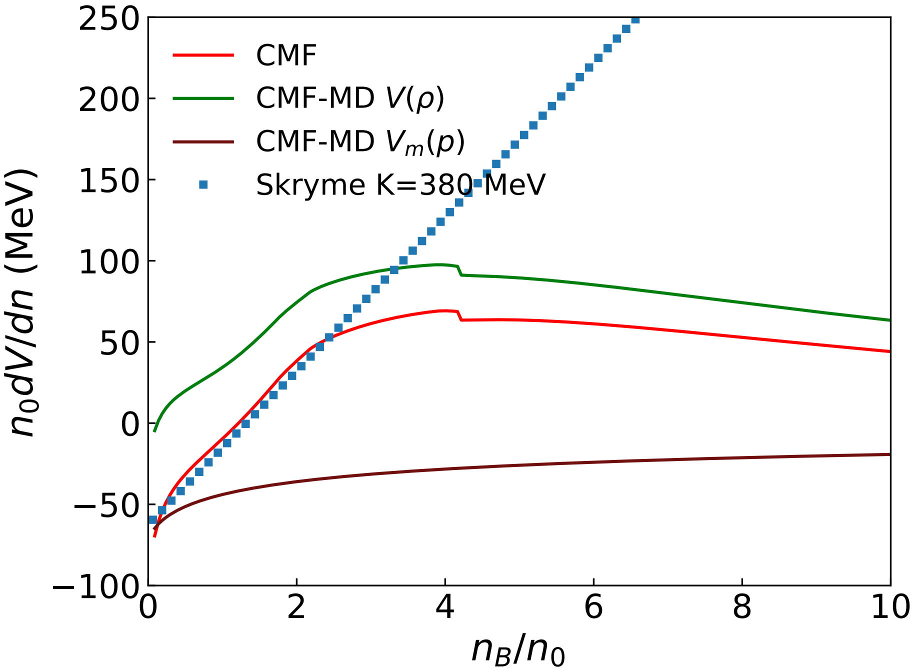
Positive v1 at compression, while negative v1 at expansion.

$$v_1^* = \int_{-0.5}^{0.5} dy v_1(y)\,\mathrm{sgn}(y)$$

QMD potential from the Chiral Mean Field model (CMF)

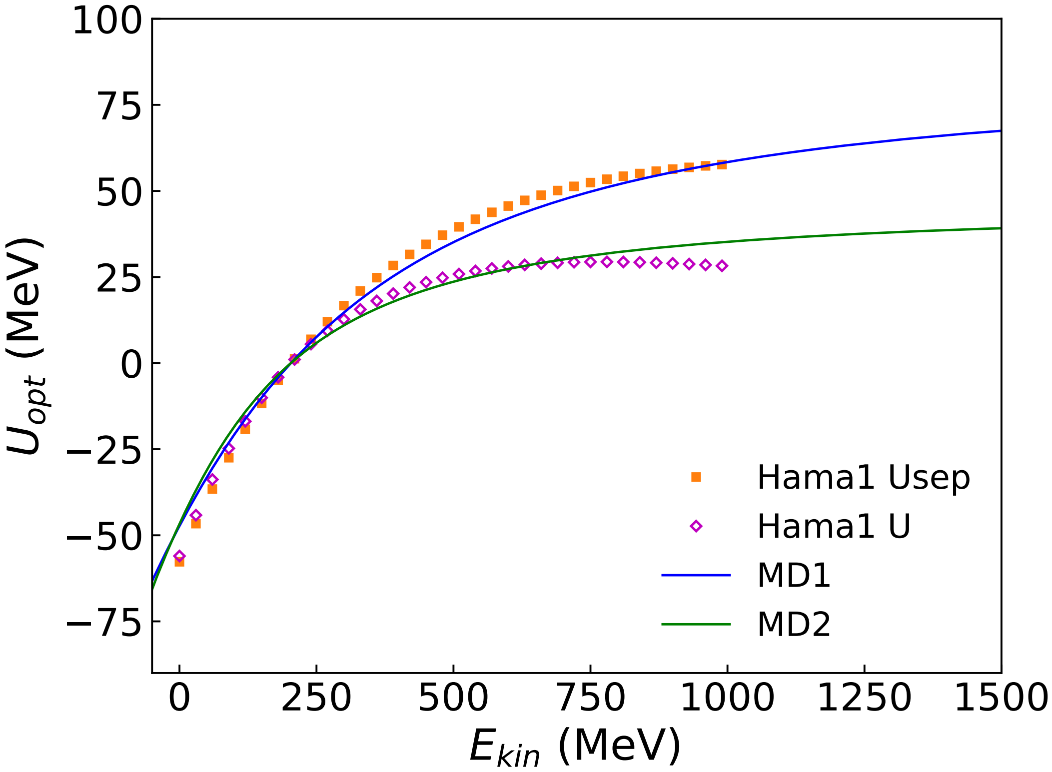
A. Motornenko,et.al PRC103,054908(2021), J.Steinheimer,et.al, EPJC82,911(2022)





$$V = \frac{1}{n}\left[\epsilon - \epsilon_{\mathrm{free}}\right] \qquad n\frac{dV}{dn} = U - V \qquad U = \text{single particle energy} - \sqrt{m_N^2 + k_F^2}$$

Momentum-dependent potential



Schrëdinger-equivalent potential : U_{sep}

$$V^0(n) = V'^0(n) + V^0_m(n,p)$$

$$\frac{dV^0(n)}{dn} = \frac{dV'^0(n)}{dn} + \frac{dV^0_m(n,p)}{dn}$$

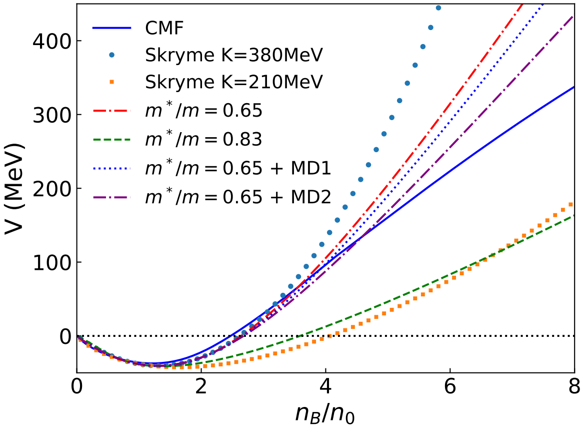
$$V^0_m = \frac{1}{2}\frac{g}{(2\pi)^3}\int d^3p\; U^0_m(p)$$

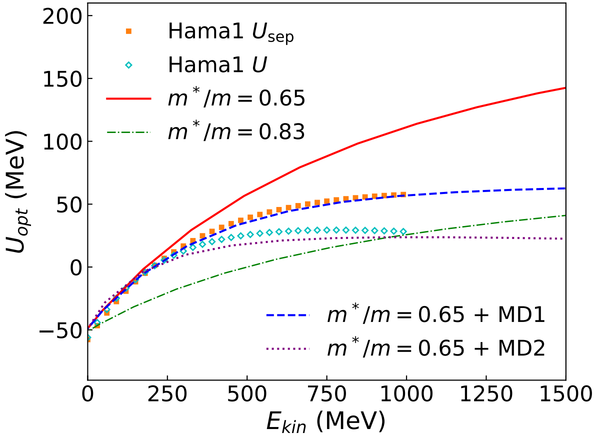
$$U^\mu_m(p) = \frac{C}{n_0}\int d^3p'\,\frac{p^{*'\mu}}{p'^{*0}}\,\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2},$$

$$U = e - \sqrt{m_N^2 + p^2}$$

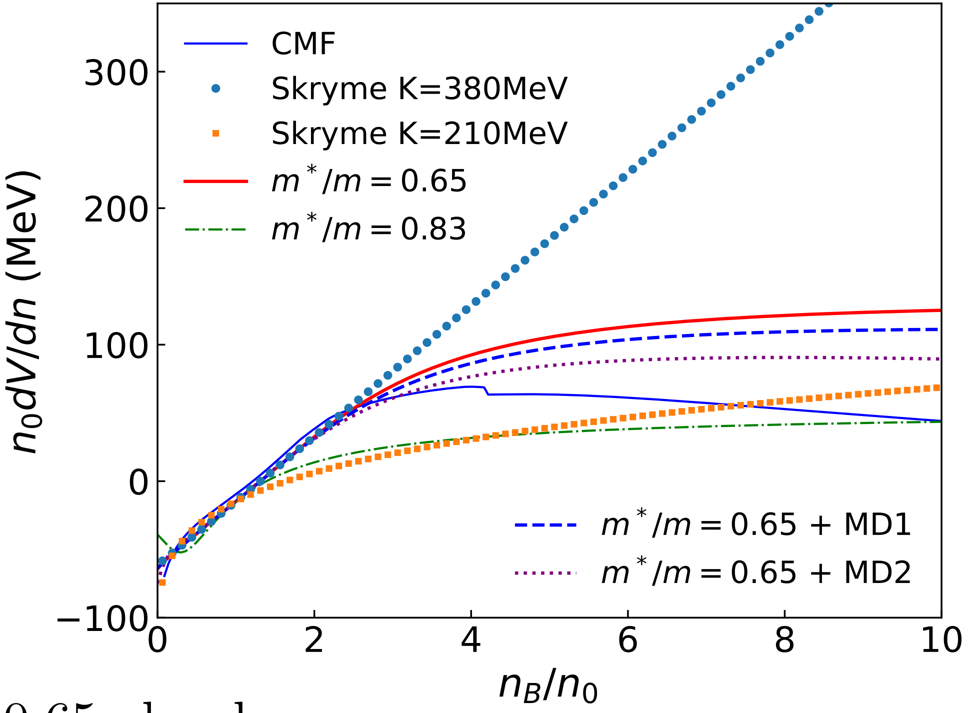
Single potential reduction of RMF

Sigma + omega type Relativistic mean field





derivatives of the QMD potential

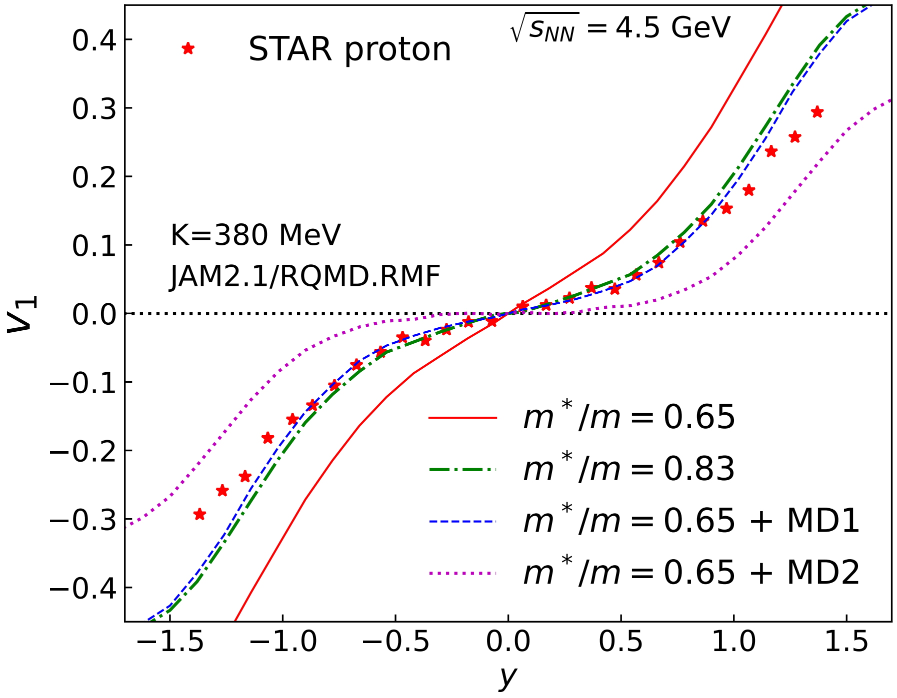


$$m^*/m = 0.65 : \mathrm{hard}$$

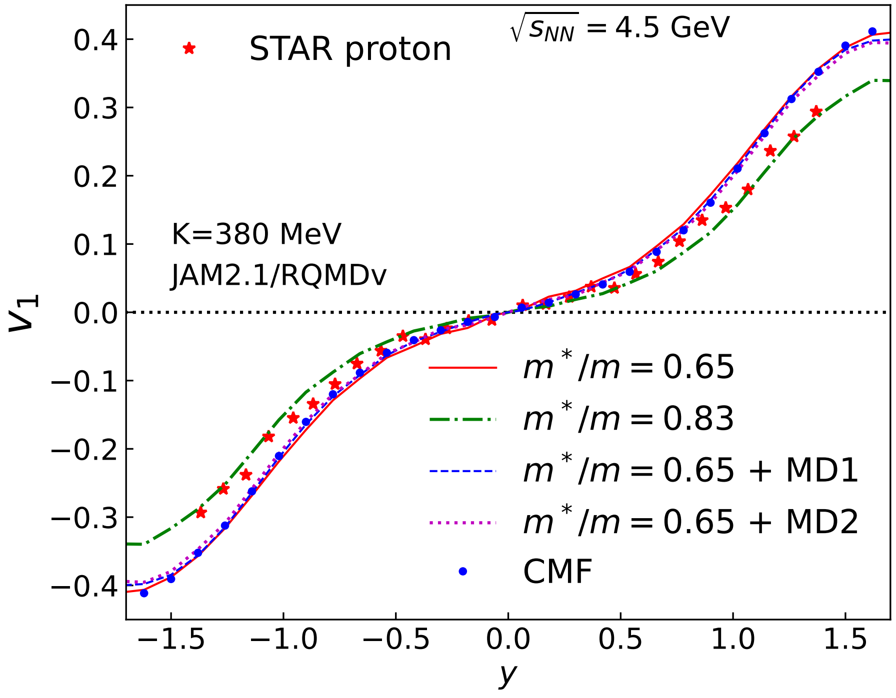
$$m^*/m = 0.83 : \mathrm{soft}$$

Single potential reduction of RMF

Relativistic mean field

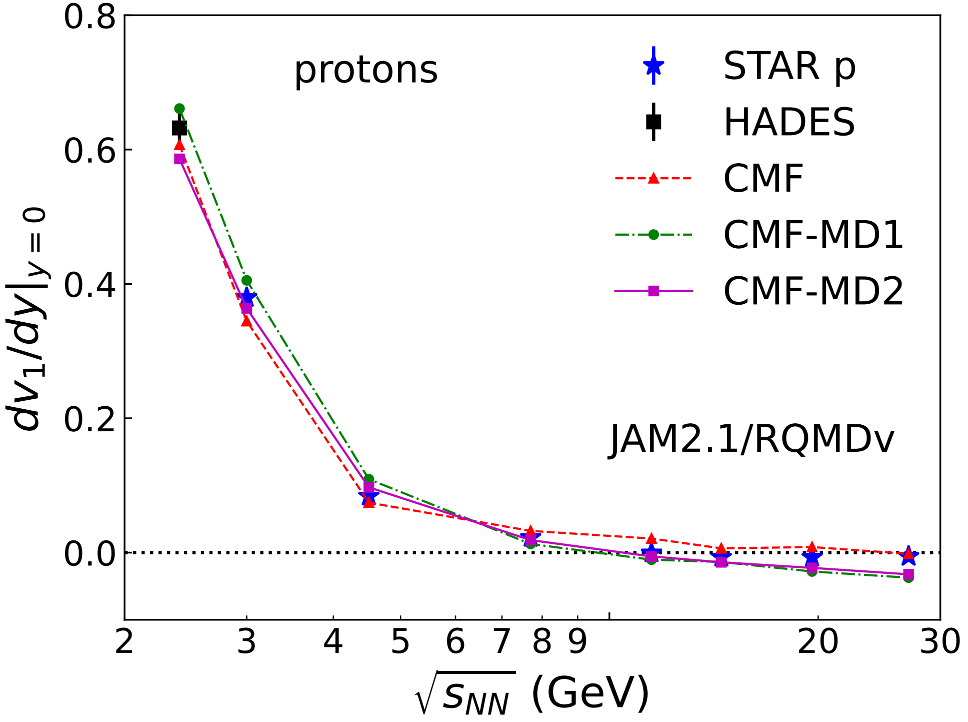


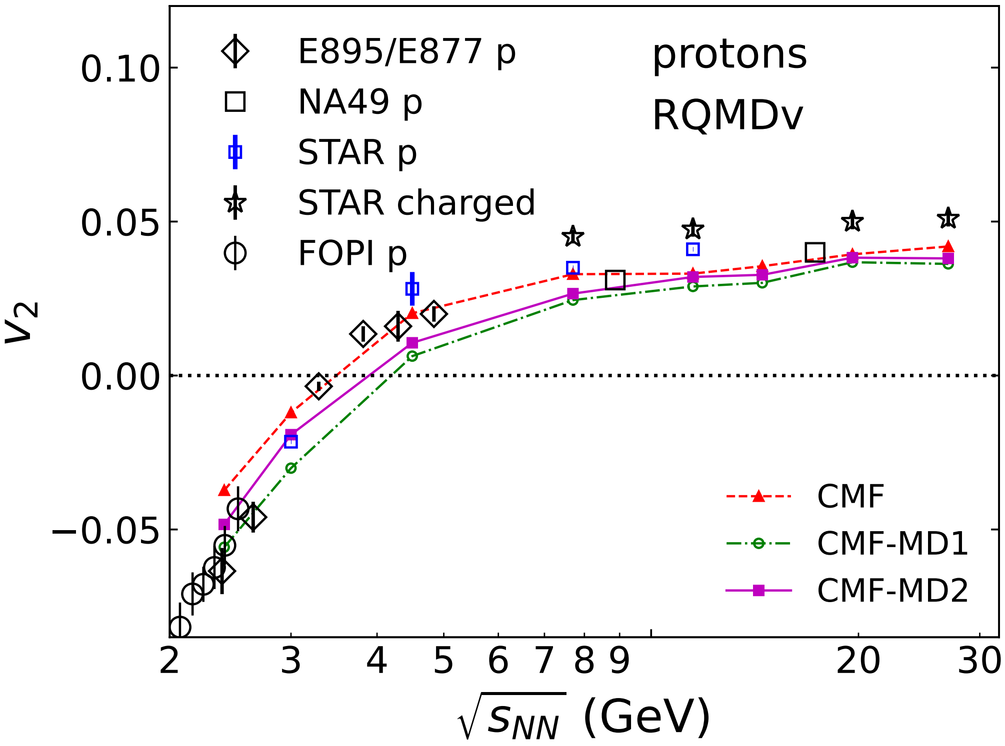
Vector potential



All RMF EoS with m*/m=0.65 predict the same v1 in the RQMDv model.

v1 and v2 from RQMD mode

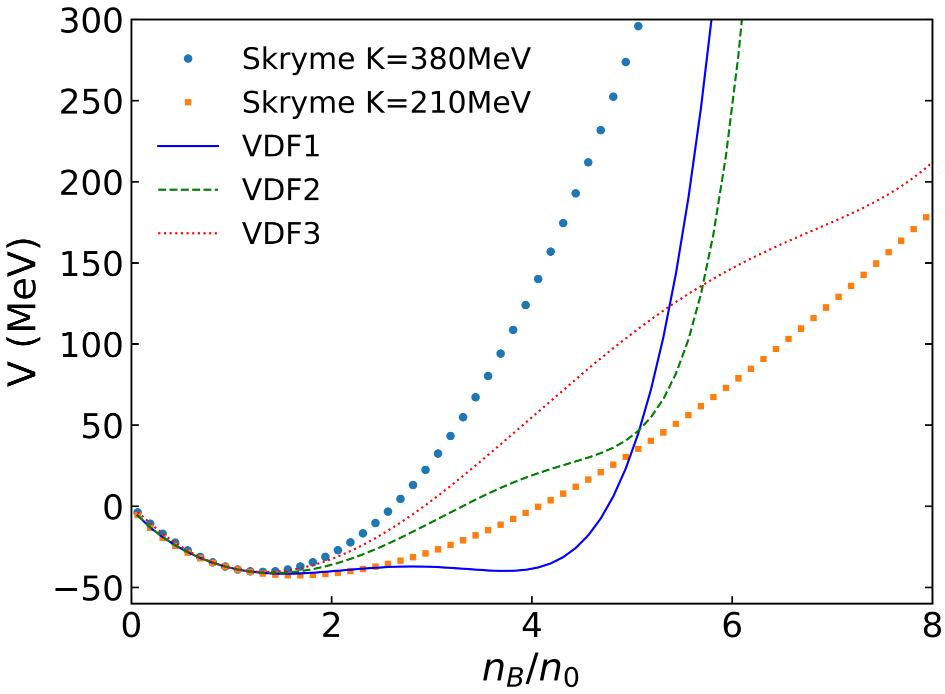


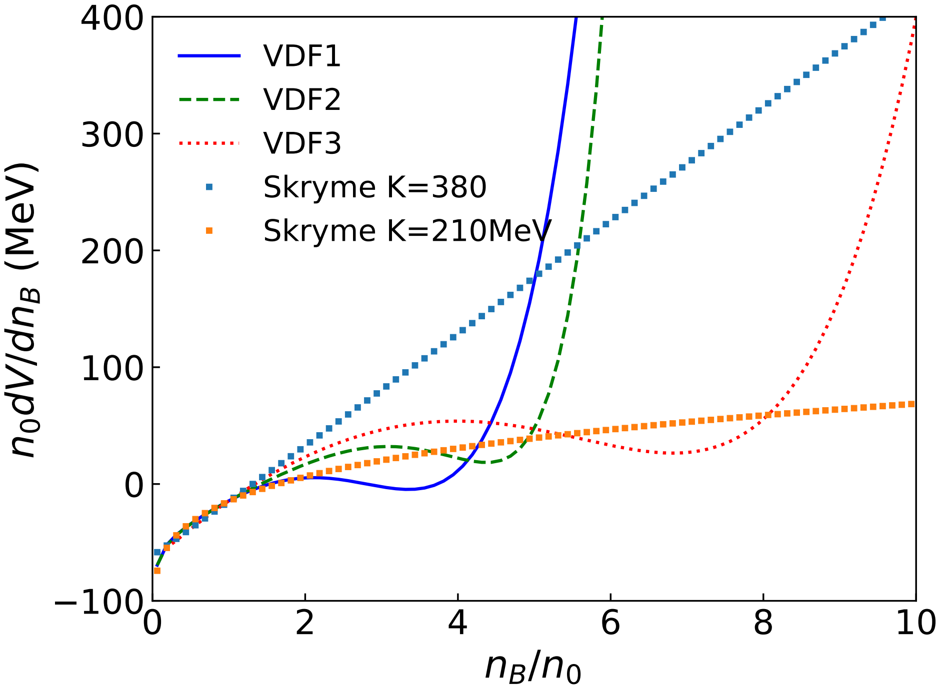


$$V^\mu = V(n_B)u^\mu, \quad u^\mu = \frac{J_B^\mu}{n_B}, \quad n_B = \sqrt{J_B^2}$$

The vector density functional model (VDF)

A. Sorensen, V. Koch, Phys. Rev. C104,034904(2021)

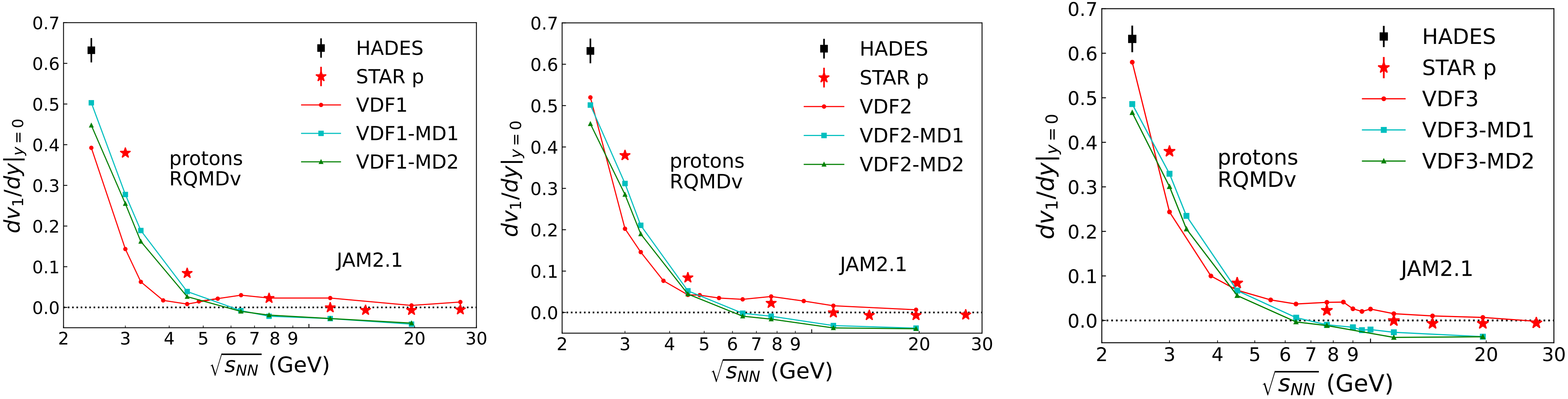




$$V^{\mu}_{\mathrm{VDF}} = \sum_{i=1}^{4} \frac{C_i}{b_i} \left(\frac{n_B}{n_0} \right)^{b_i - 1} \cdot \frac{J^{\mu}}{n_B}$$

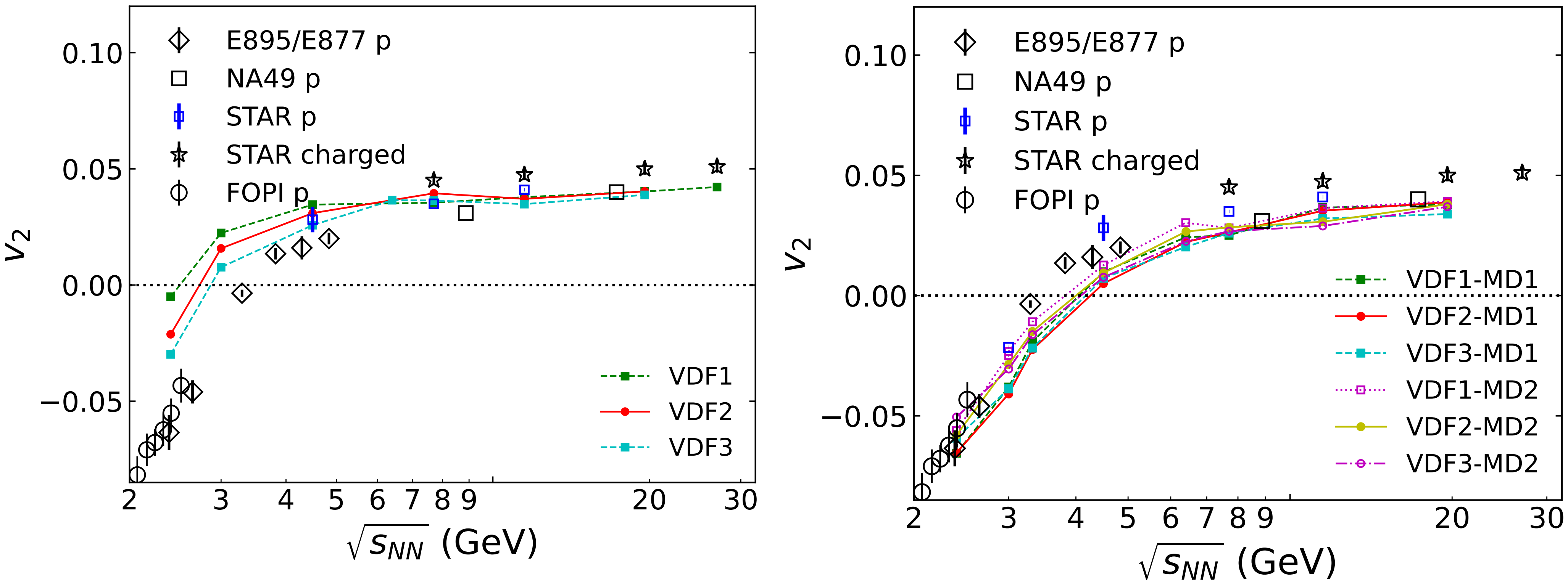
$$P = P_{\mathrm{kin}} + n^2 \frac{dV}{dn}$$

v1 from RQMD + VDF



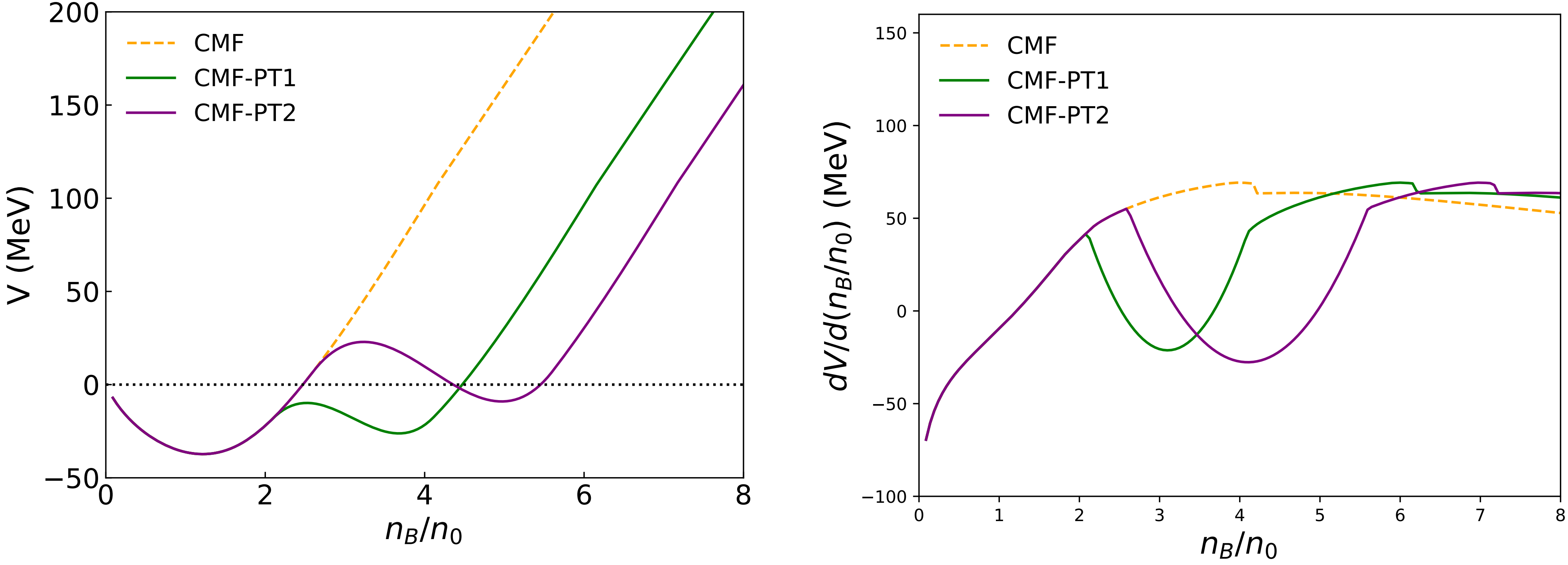
Minium in the slope of the directed flow due to 1st order PT disappears by the MD potential.

v2 from RQMD + VDF



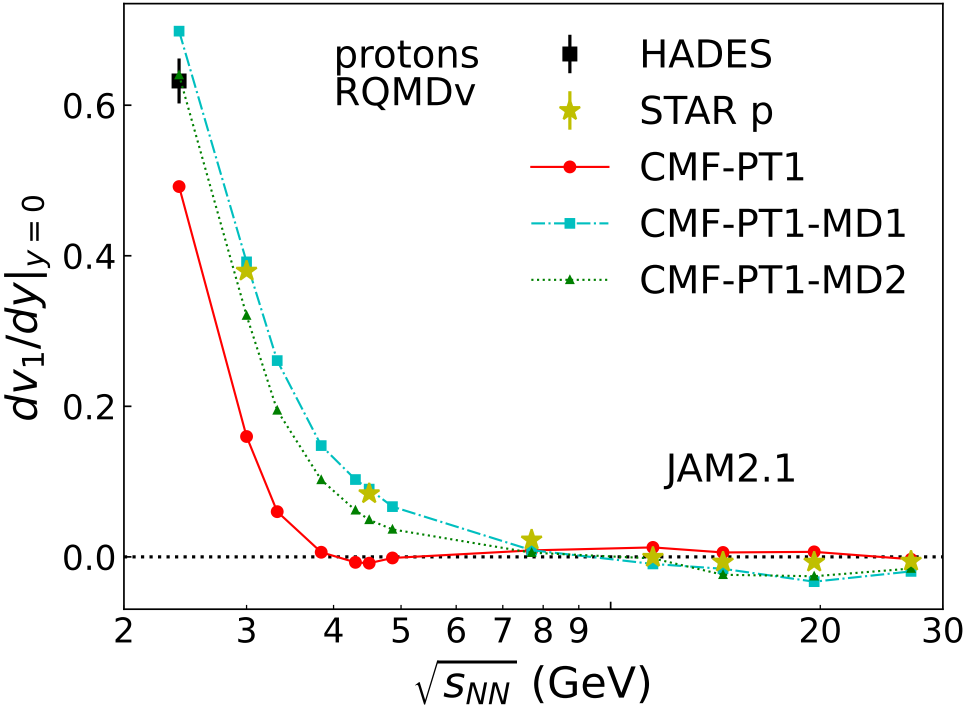
Enhancement of v1 by a first-order phase transition disappears by MD potential.

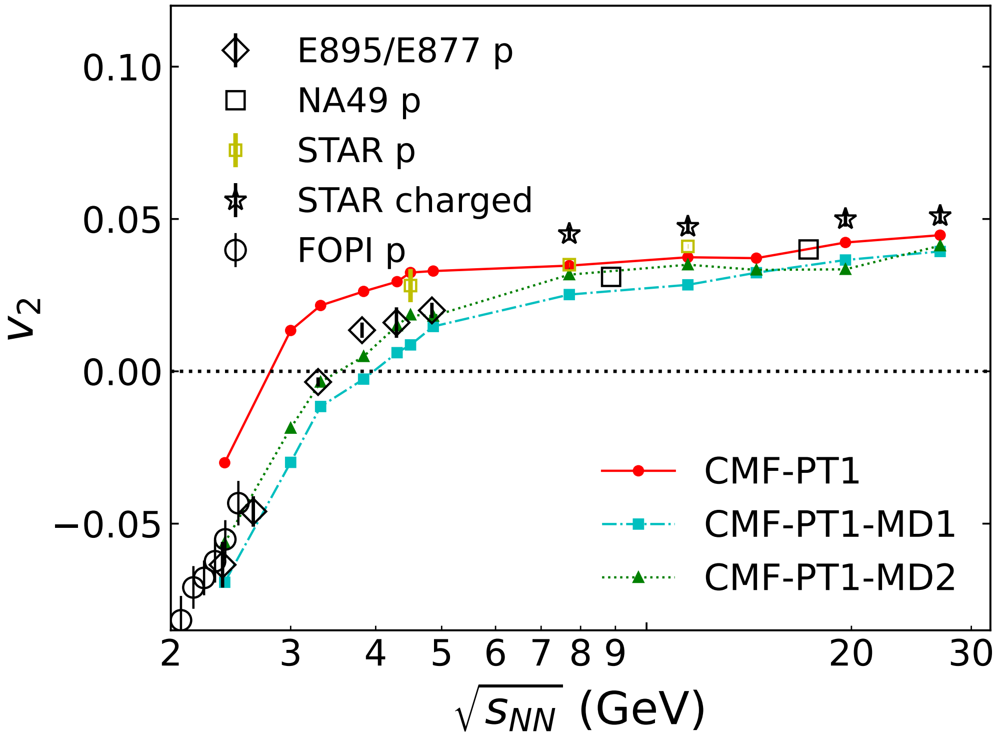
CMF + 1stOPT



Introduce 1stOPT into the CMF model J.Steinheimer, et.al. Eur. Phy.C82(2022)911

V1 and v2 from CMF + 1stOPT





Collapse of v1 slope disappears by the momentum-dependent interaction.

Summary

- The beam energy dependence of the directed flow is explained by the conventional hadronic mean-field.
- Final v1 is determined by the interplay between positive v1 generated in the compression stages and the negative v1 generated during the expansion stage.
- RQMD simulation with CMF + MD potential agrees with the data on v1 and v2.
- RQMD simulation by the potential with phase transition: A. Sorense and V. Koch, PRC104 (2021) 3, 034904 predicts very different results between with and without MD potential.
- Collapse of directed flow for a 1st OPT EoS in the beam energy dependence of v1 disappears, when momentum-dependent interaction is introduced.

Buck up

Relativistic quantum molecular dynamics (RQMD) approach

RQMD was developed based on the constrained Hamiltonian dynamics:
H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192, 266 (1989).

T. Maruyama, et. al. Prog. Theor. Phys. 96, 263 (1996).

Manifestly covariant way: four-vectors $q_i^\mu, \quad p_i^\mu \ (i = 1, N)$

For the description of N-particle system, we have 8N dimension.
In order to reduced the dimension from 8N to 6N, we need 2N constraints.

Hamiltonian is a linear combinations of the constraints, and equations of motion are given by

$$H = \sum_i \lambda_i \phi_i \qquad \frac{dq_i}{d\tau} = \{H, q_i\}, \quad \frac{dp_i}{d\tau} = \{H, p_i\}$$

2N constraints: On-mass shell condition and time fixation.

$$\phi_i = (p_i - V_i)^2 + (m_i - S_i)^2 = p_i^{*2} + m_i^{*2} = 0, \quad (i = 1, \cdots, N)$$

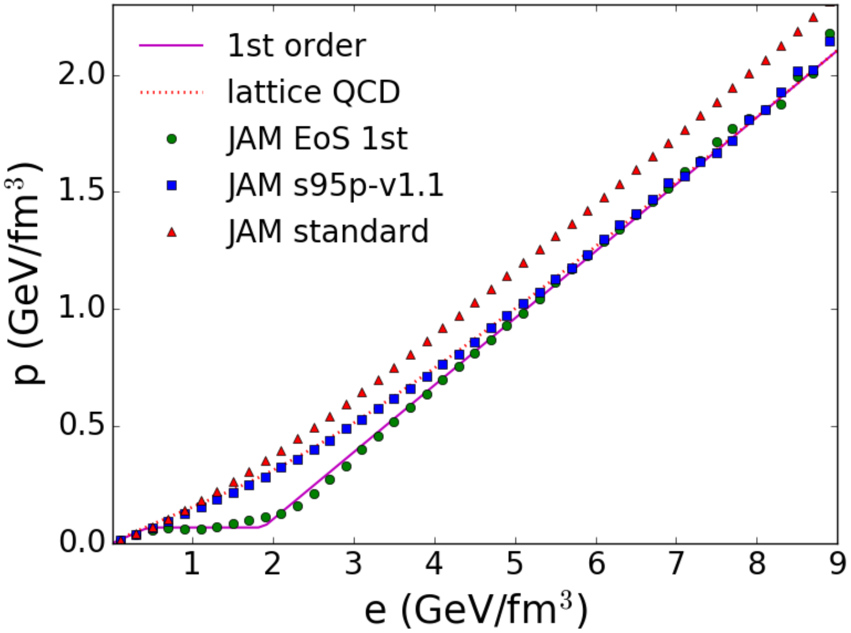
JAM2:micro-macro transport model

- Fortran77 → C++
- Pythia6 → Pythia8
- Update of collision term: include new pp data.
  - ✓ New total hadronic cross section at high energies (PDG2016)
  - ✓ New resonance cross section (Ecm< 4GeV)
  - ✓ New string excitation low (4 < Ecm < 20 GeV)
  - ✓ New multiple-parton scattering (Pythia8) (Ecm > 20GeV)

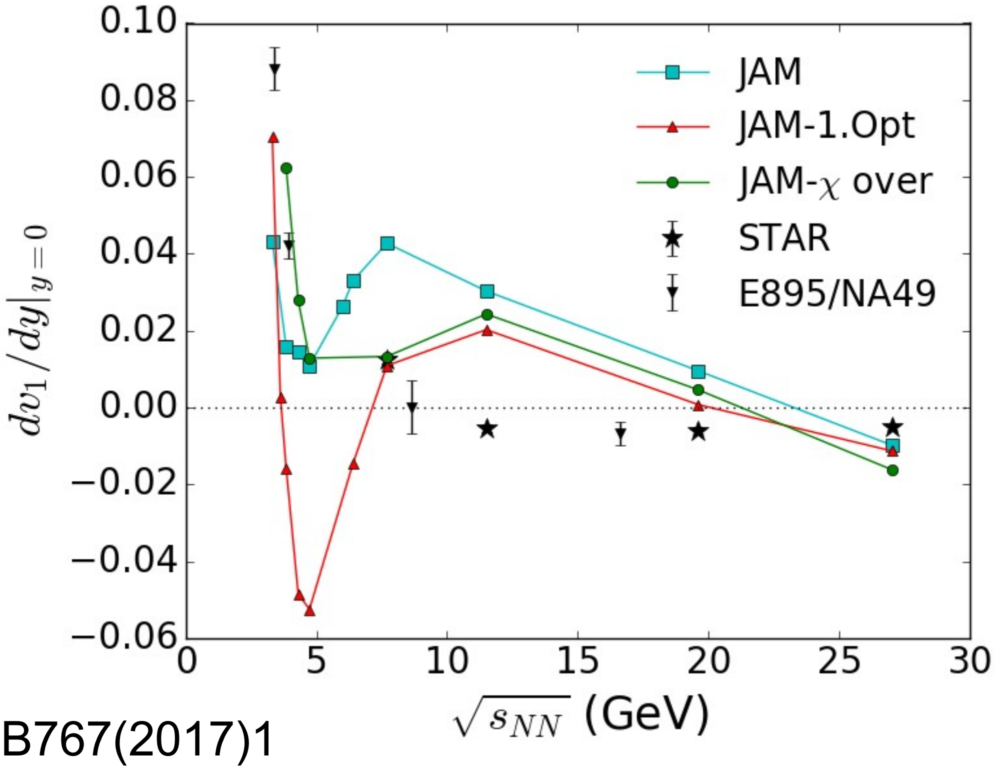
- Quantum Molecular Fluid Dynamics (QMFD): 3D perfect hydro + RQMD model
- RQMD with Skyrme force (Lorentz scalar and vector)
- RQMD.RMF with momentum-dependent potential
- Speeding up computational time by introducing expanding box for both collision term and potential evaluation

v1 from EoS modified collision term

EoS modified collision term
provides efficient method
to control EoS in a
microscopic transport model.



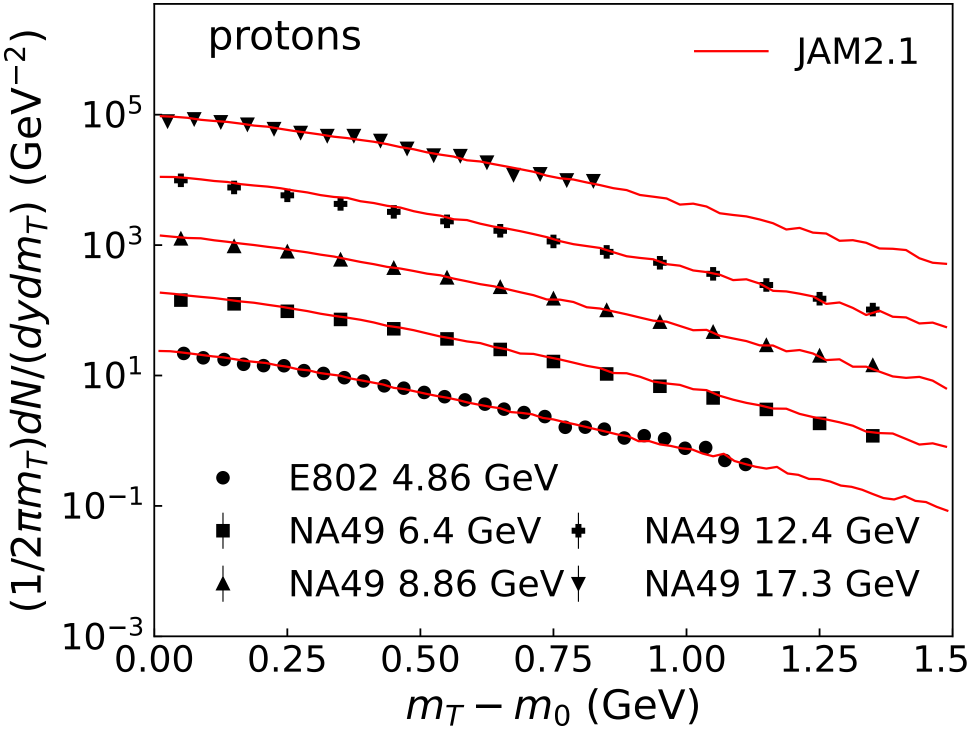
JAM with fully baryon density dependent EoS meets hydro

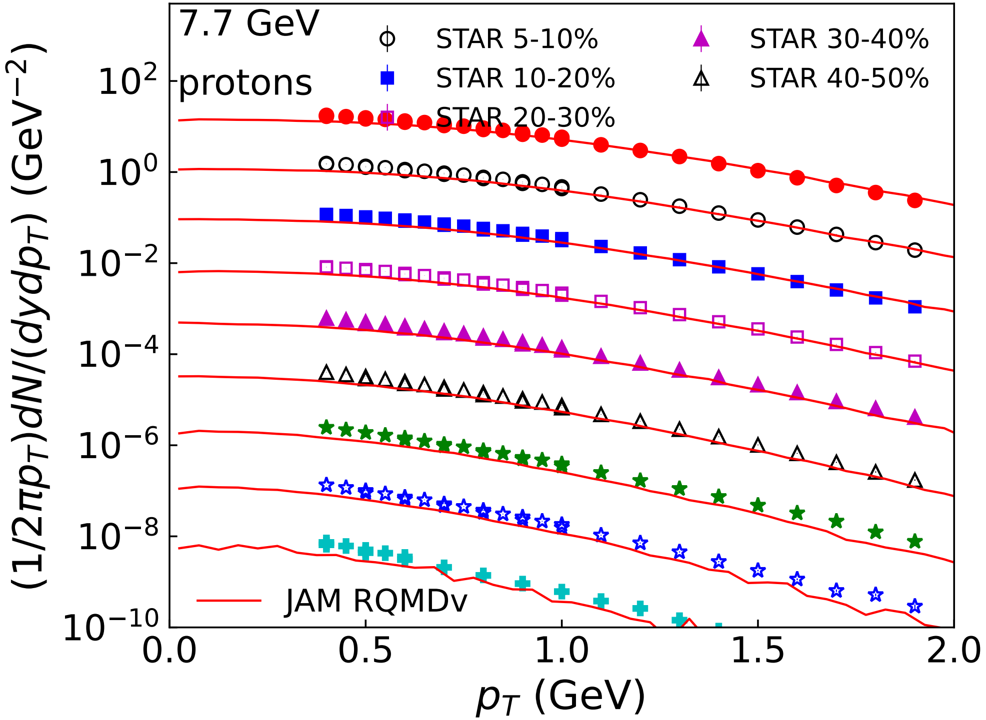


Y.Nara, PLB767(2017)1

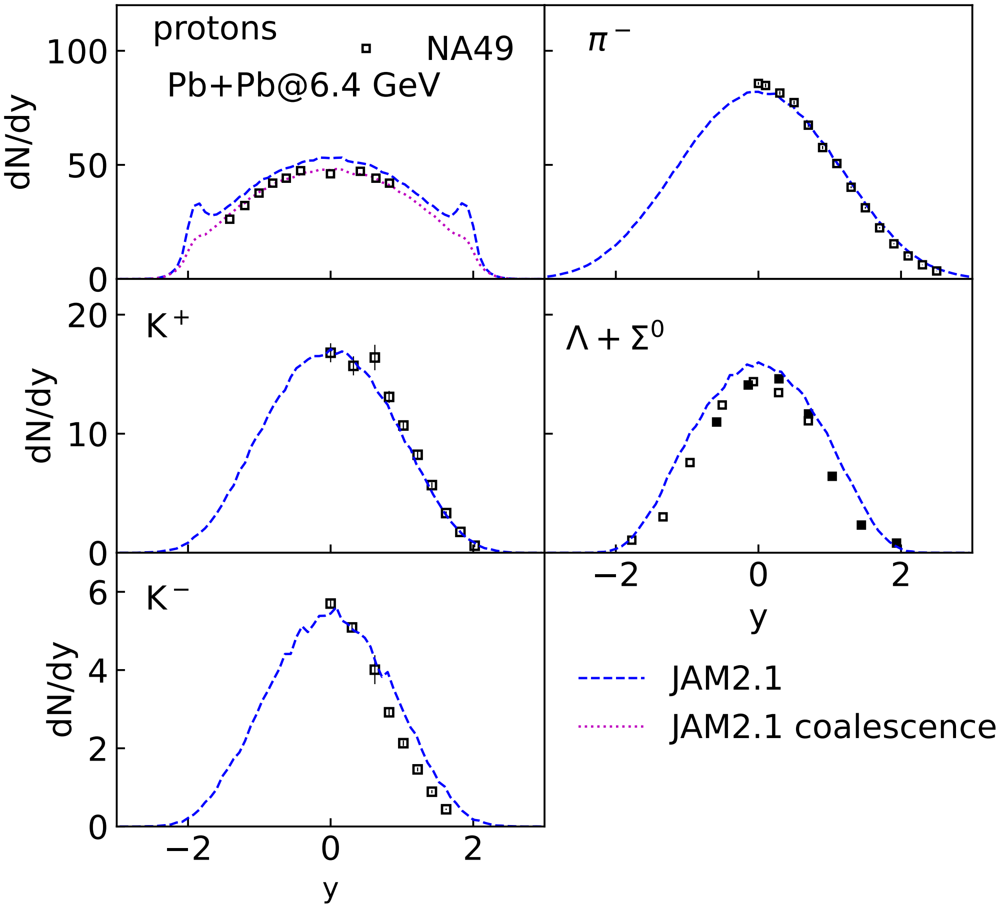
Proton spectra

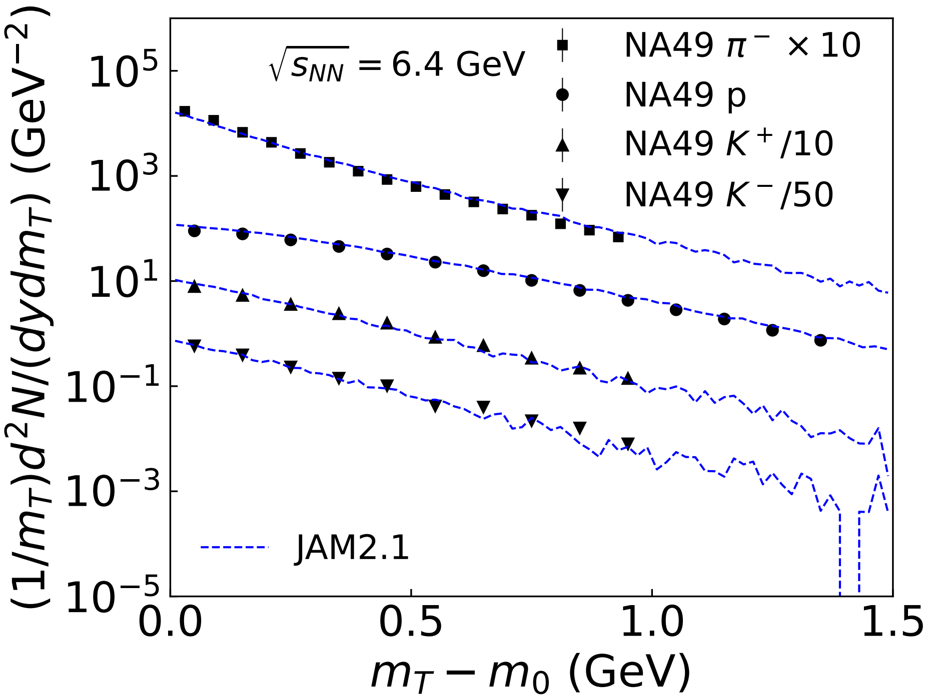
JAM2.108





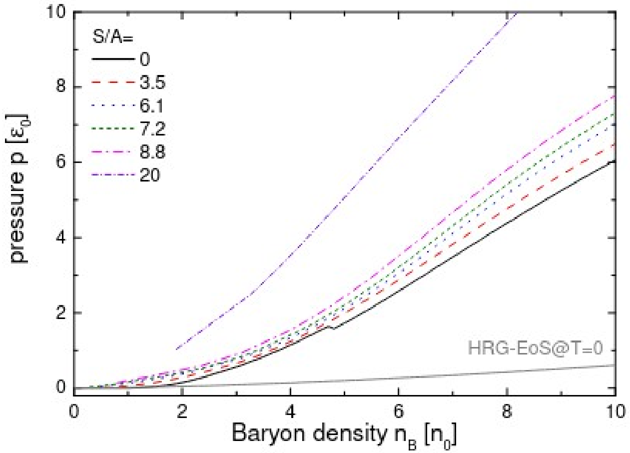
Pb + Pb at Elab=20AGeV



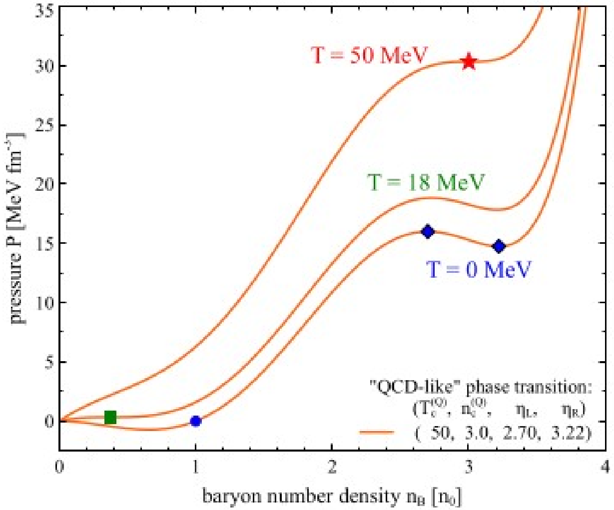


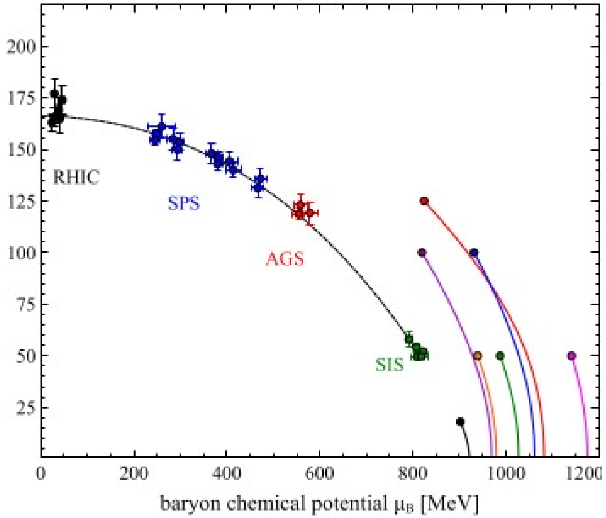
Recent progress in microscopic transport approaches

- SMASH with critical point EoS, A. Sorensen and V. Koch, PRC104(2021)034904
- Chiral mean-field (CMF) EoS in UrQMD, M. Omana Kuttan, et. al. nucl-th2201.01622
- Microscopic transport model with the Parity doublet model
  - ✓ DJBUU: M. Kim, et.al PRC101(2020)064614
  - ✓ GiBUU: A.B.Larionov and L. von Smekal, nucl-th2109.03556
- PHQMD (Parton hadron quantum molecular dynamics) J.Aichelin, PRC101(2020)044905
- JAM RQMD.RMF (2019),(2020), RQMDv (2022)



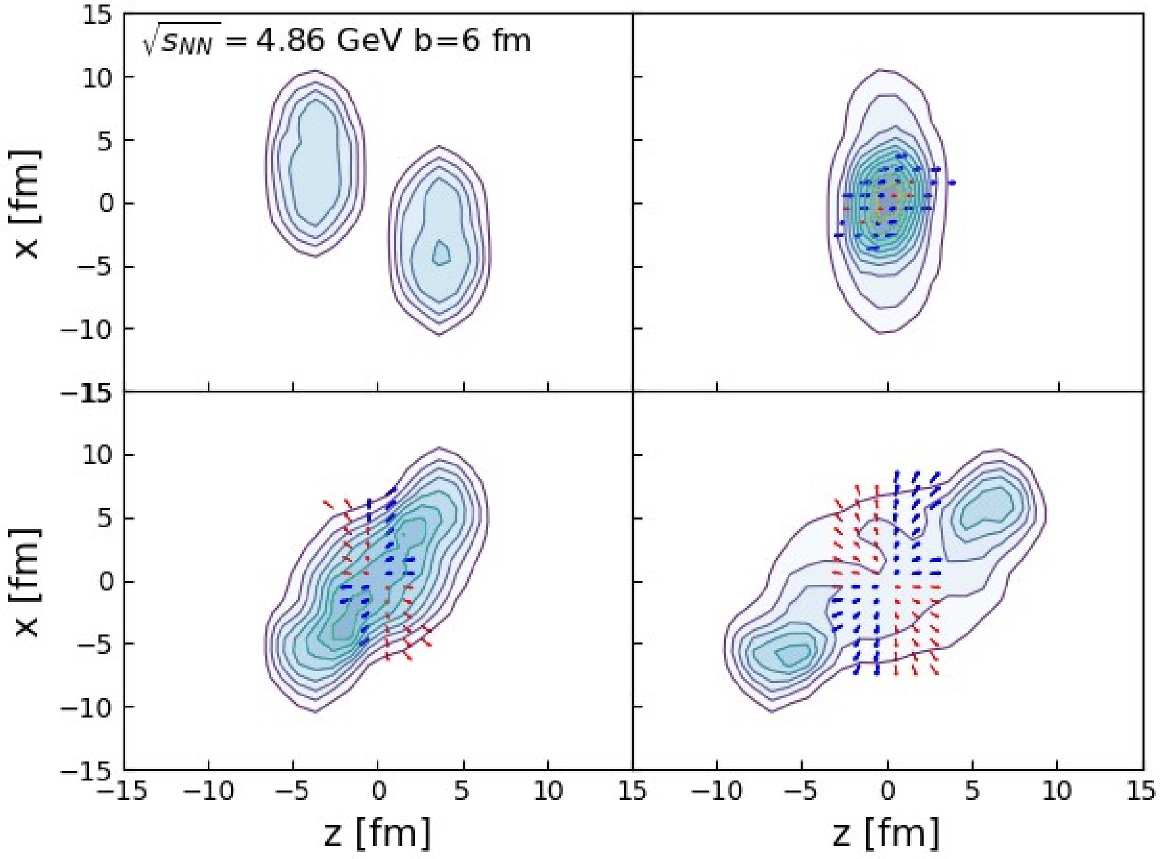
CMF

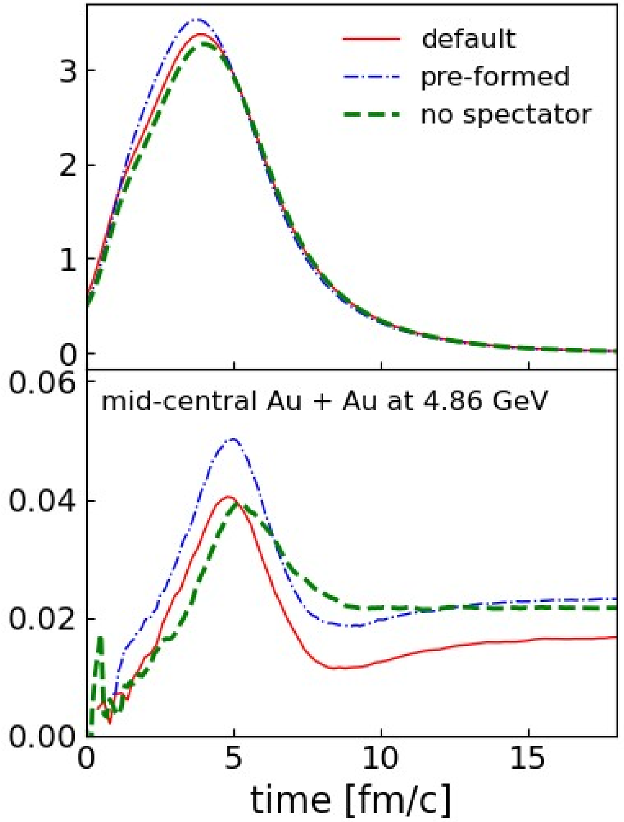




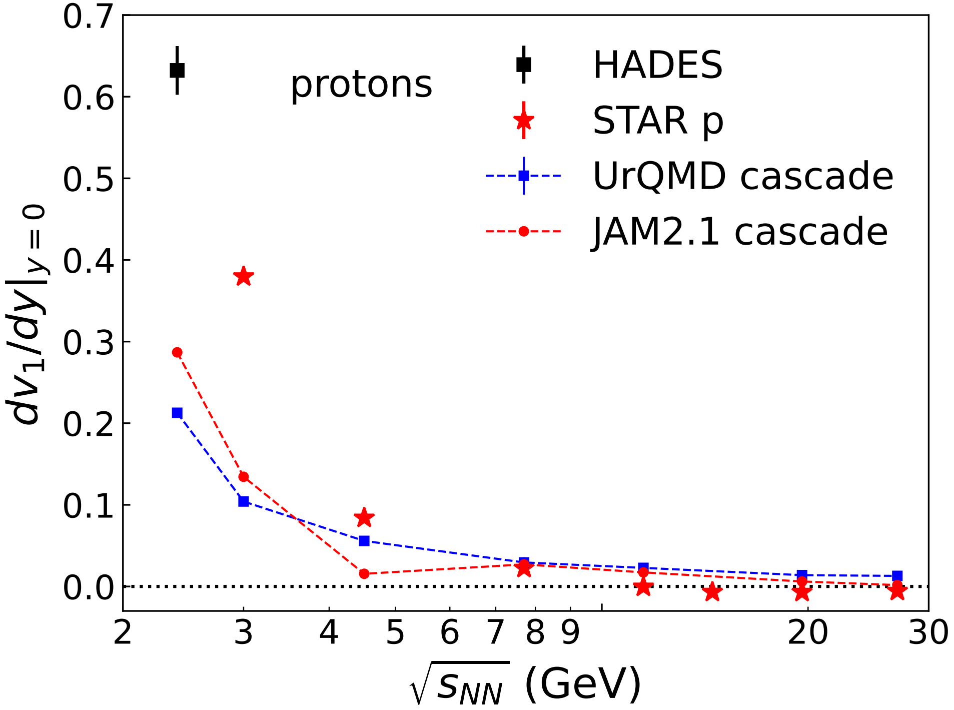
Time evolution of v1 at 4.86 GeV

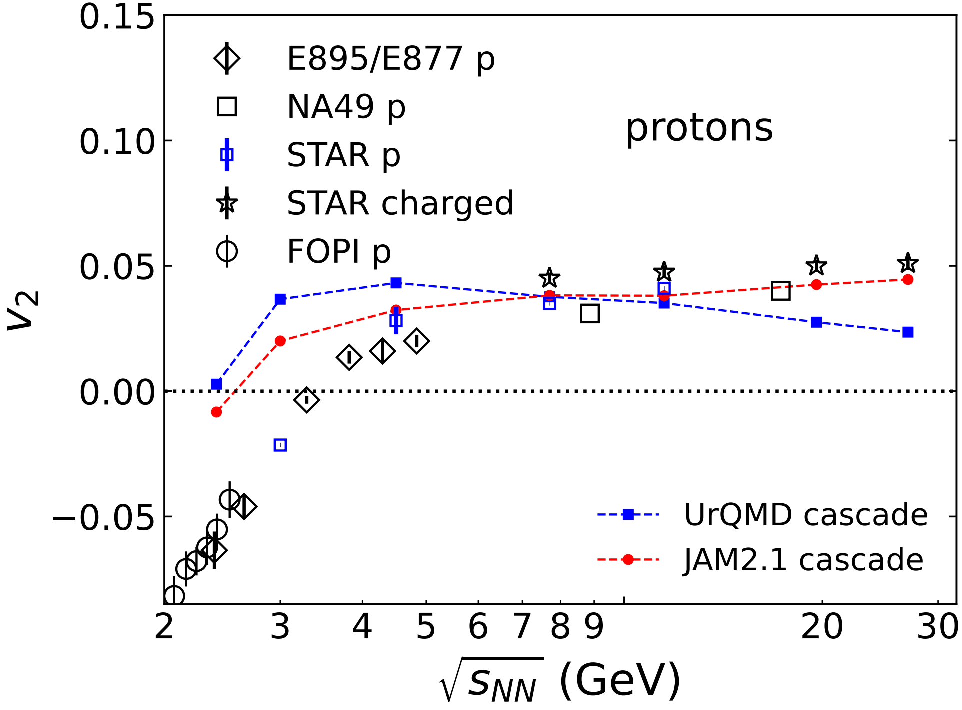
Time evolution of the baryon density in Au + Au mid-central collision (b=6fm)



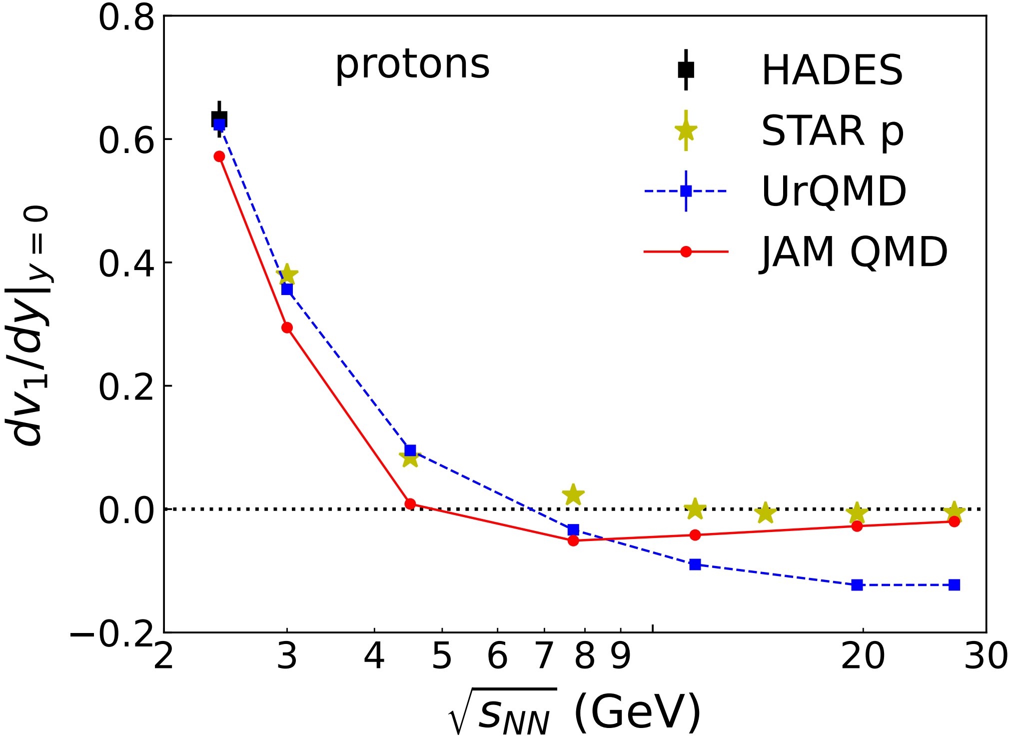


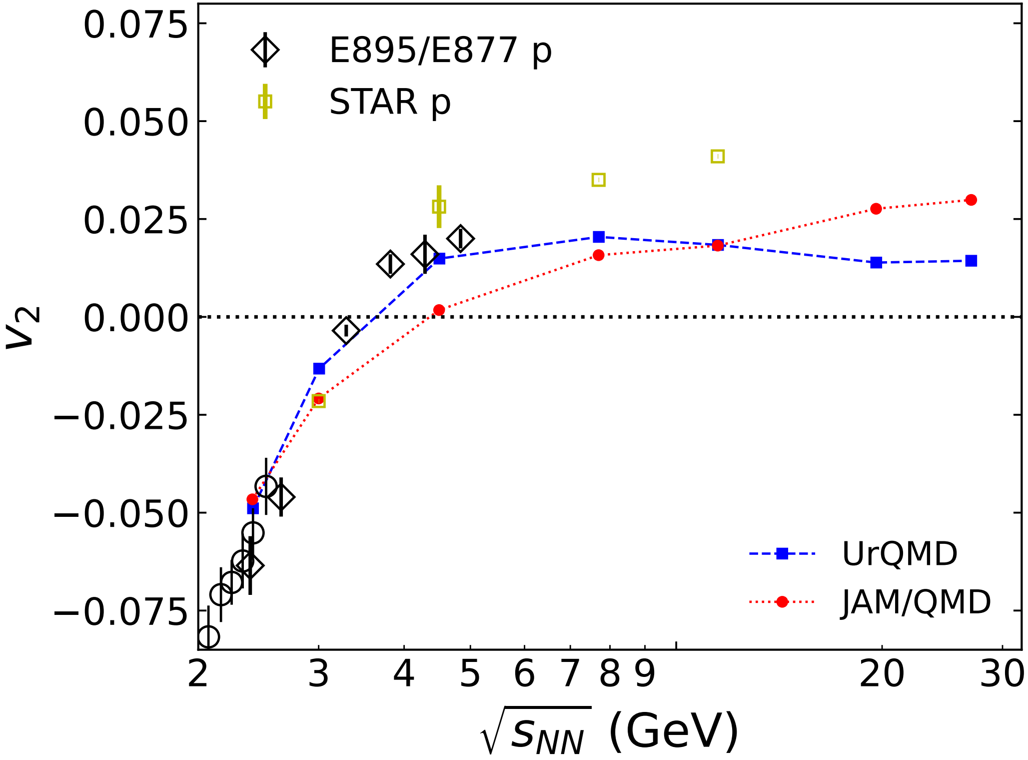
Beam energy dependence of v1 and v2 from cascade mode



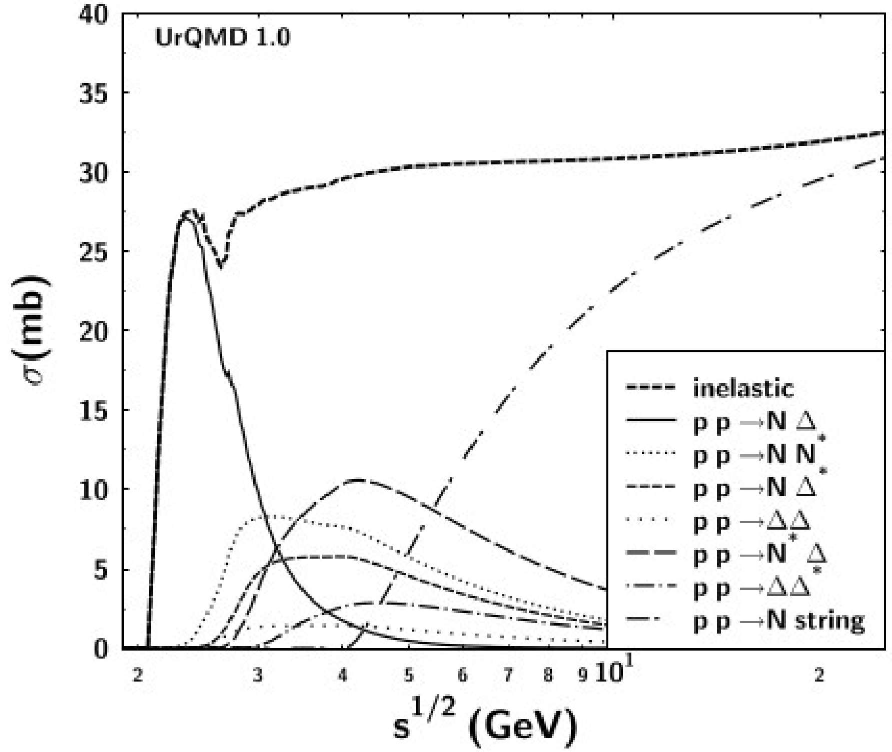


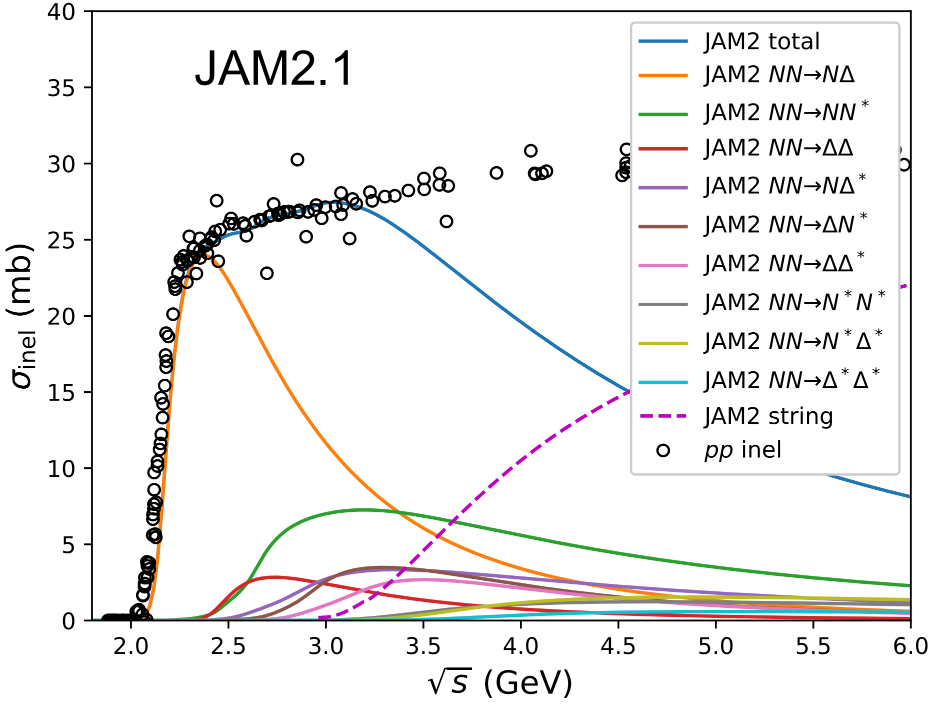
Beam energy dependence of v1 and v2 from QMD mode





UrQMD and JAM cross sections

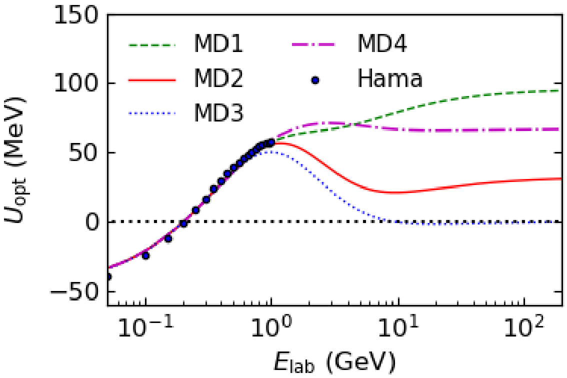


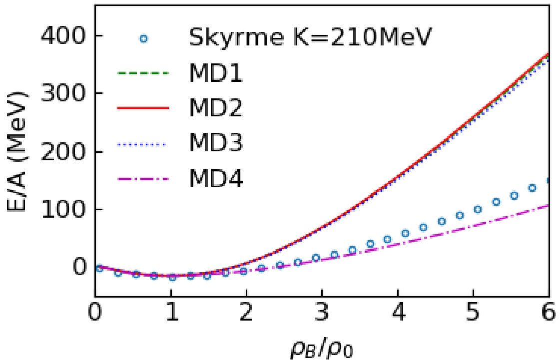


different parametrization

Momentum-dependent potential

K. Weber, B. Blaettel, W. Cassing, H. C. Doenges, V. Koch,
A. Lang and U. Mosel, Nucl. Phys. A 539, 713 (1992).





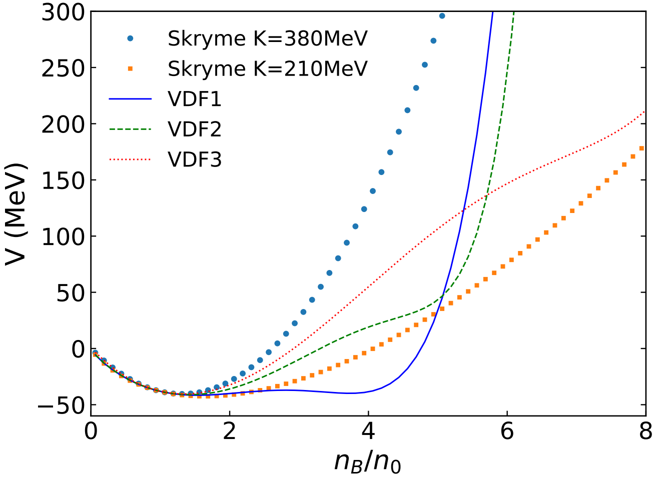
$$V_s^{\rm MD} = \frac{\bar{g}_s^2}{m_s^2} \int d^3p \frac{m^*}{p_0^*} \frac{f(x,p)}{1+(p-p')^2/\Lambda_s^2} \quad V_\mu^{\rm MD} = \frac{\bar{g}_v^2}{m_v^2} \int d^3p \frac{p_\mu^*}{p_0^*} \frac{f(x,p)}{1+(p-p')^2/\Lambda_v^2}$$

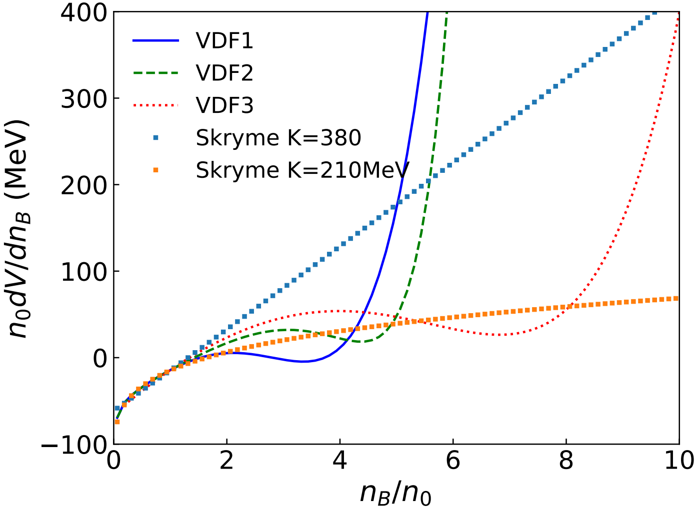
		MD1	MD2	MD3	MD4
K	(MeV)	380	380	380	210
m^*/m		0.65	0.65	0.65	0.83
$U_{\rm opt}(\infty)$	(MeV)	95	30	-0.4	67
g_s		9.030	9.233	5.439	4.059
g_v		6.740	3.888	0.0	5.632
g_2	(1/fm)	4.218	4.012	-15.59	-160.3
g_3		6.667	5.520	391.9	2684
$\bar{g}_s$		3.186	2.502	7.711	5.544
$\bar{g}_v$		8.896	10.43	11.22	3.926
Λ_s	(GeV)	0.641	0.4897	1.702	0.704
Λ_v	(GeV	1.841	2.489	1.898	4.252

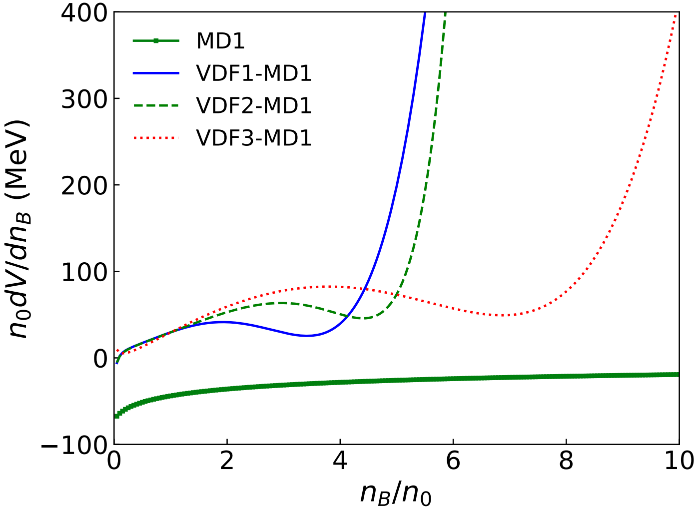
Y.N. T.Maruyama,H.Stoecker,PRC(2020)

The vector density functional model (VDF)

A. Sorensen, V. Koch, Phys. Rev. C104,034904(2021)





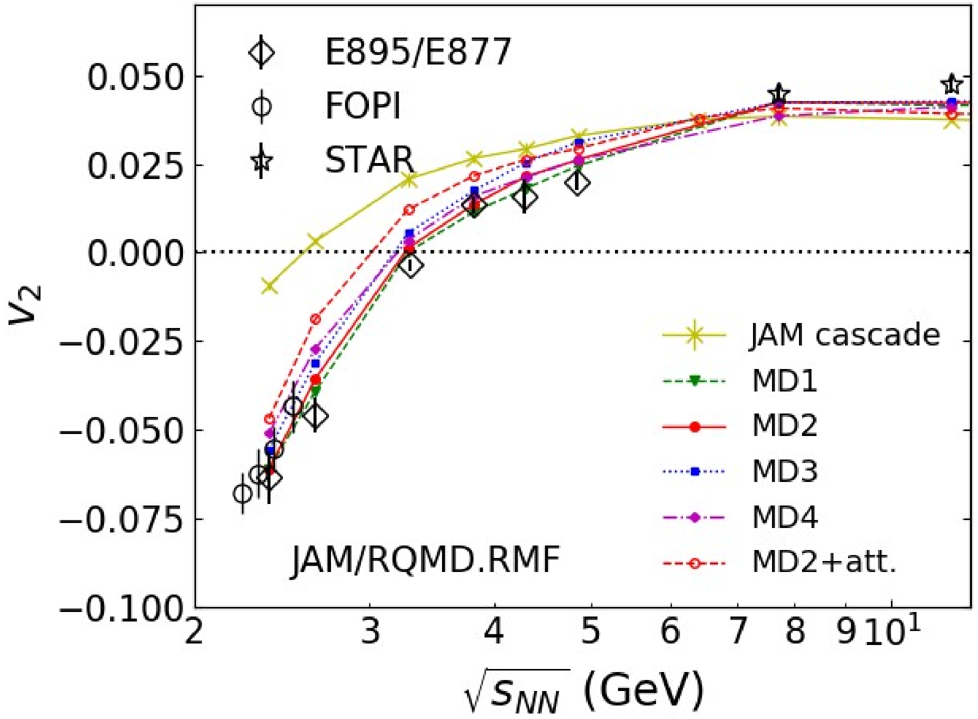


$$V^{\mu}_{\mathrm{VDF}} = \sum_{i=1}^{4} \frac{C_i}{b_i} \left(\frac{n_B}{n_0}\right)^{b_i - 1} \cdot \frac{J^{\mu}}{n_B}$$

$$P = P_{\mathrm{kin}} + n^2 \frac{dV}{dn}$$

v2 from RQMD.RMF

Beam dependence of proton v1 at mid-rapidity



V2 from JAM2/RQMDv