Gibbsian hydrodynamics, or Quantum gravity for poor people

G.Torrieri

2307.07021 , 2309.05154 2007.09224 (JHEP),
2109.06389 (Annals of Physics, With T.Dore,M.Shokri,L.Gavassino,D.Montenegro)
Answers somewhat speculative... but I think I am asking good questions!

What is ideal hydrodynamics?

Hydrodynamics is an "effective theory", built around coarse-graining and "fast thermalization". Fast w.r.t. Gradients of coarse-grained variables
If thermalization instantaneus, then isotropy,EoS enough to close evolution

$$T_{\mu\nu} = (e + P(e)) u_\mu u_\nu + P(e) g_{\mu\nu}$$

In rest-frame at rest w.r.t. u^μ

$$T_{\mu\nu} = \mathrm{Diag}\,(e(p), p, p, p)$$

(NB: For simplicity we assume no conserved charges, $\mu_B = 0$)

This makes sysem solvable: $\partial_\mu T^{\mu\nu} = 0, p = p(e)$

A beautiful, rigorous theory with a direct connection to statistical mechanics, i.e. fundamental physics and maths. Exciting that HIC can be described by it!

If thermalization not instantaneous,

$$T_{\mu\nu} = T_{\mu\nu}^{eq} + \Pi_{\mu\nu} \quad , \quad u_\mu \Pi^{\mu\nu} = 0$$

$$\sum_n \tau_{n\Pi} \partial_\tau^n \Pi_{\mu\nu} = -\Pi_{\mu\nu} + \mathcal{O}\left(\partial u\right) + \mathcal{O}\left((\partial u)^2\right) + ...$$

A series whose "small parameter" (Barring phase transitions/critical points/... all of these these same order):

$$K \sim \frac{l_{micro}}{l_{macro}} \sim \frac{\eta}{sT} \nabla u \sim \frac{\mathrm{Det}\Pi_{\mu\nu}}{\mathrm{Det}T_{\mu\nu}} \sim ...$$

and the transport coefficients calculable from asymptotic correlators of microscopic theory Navier-Stokes $\sim K$, Israel-Stewart $\sim K^2$ etc.
Non-relativistic version still considered beautiful and profound, but with relativity...

What's wrong with this?

u_μ **ambiguus** many definitions (Landau, Eckart,...)
We think flow is "clear", so this is a bit strange

$\Pi_{\mu\nu}$ **ambiguus** can even be eliminated by carefully choosing u_μ (BDNK)

Fluctuations ...

- Defined linearly
- No clear fluctuation-dissipation relation
- Does "everything" fluctuate? What if fluctuation of $u_\mu, T, \Pi_{\mu\nu}$ leave $T_{\mu\nu}$ invariant?

Entropy current not clearly connected to energy-momentum current, need microscopic theory to "select good EFT" (2nd law)

More concretely

A theorist will say that fluctuations of e.g. $\delta\Pi_{\mu\nu}, \delta f(x,p)$ produce "non-hydrodynamic modes", sensitive to underlying theries, and hydrodynamics is easy to break down

An experimentalist measures neither $\Pi_{\mu\nu}$ nor f but rather, e.g.

$$\frac{dN}{dyp_T dp_T d\phi} \equiv \frac{dN}{dyp_T dp_T}\left[1+2v_n(p_T,y)\cos\left(n\left(\phi-\phi_{0n}\right)\right)\right]$$

i.e. gradients of $T_{\mu\nu}$,entropy and finds hydro everywhere they look!
in a fluctuating medium are "non-hydrodynamic modes" detectable in principle? Can your non-hydro mode be my fluctuating sound-wave?

The two are in a very complicated correspondence which is not $1 \leftrightarrow 1$

Hydrodynamics from microscopic theories

QFT transport coefficients plagued by divergences, need truncation (Schwinger-Keldysh separates "fast", "slow", Kadanoff-Baym needs truncation)

Boltzmann equation Sequential scattering and molecular chaos. Weak coupling, Lose microscopic correlations

AdS/CFT strong coupling and large N_c, lose microscopic correlations

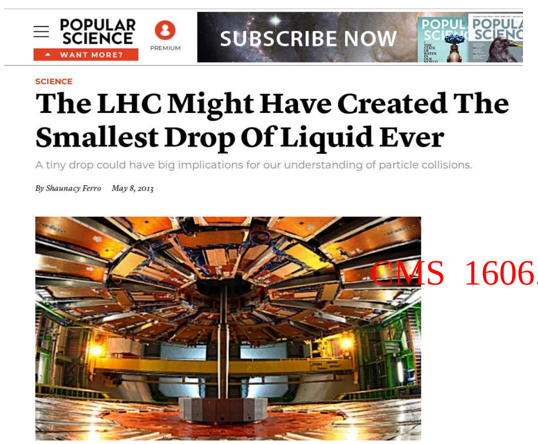
Molecular dynamics keeps microscopic correlations, lose Lorentz invariance (in practice not a problem)

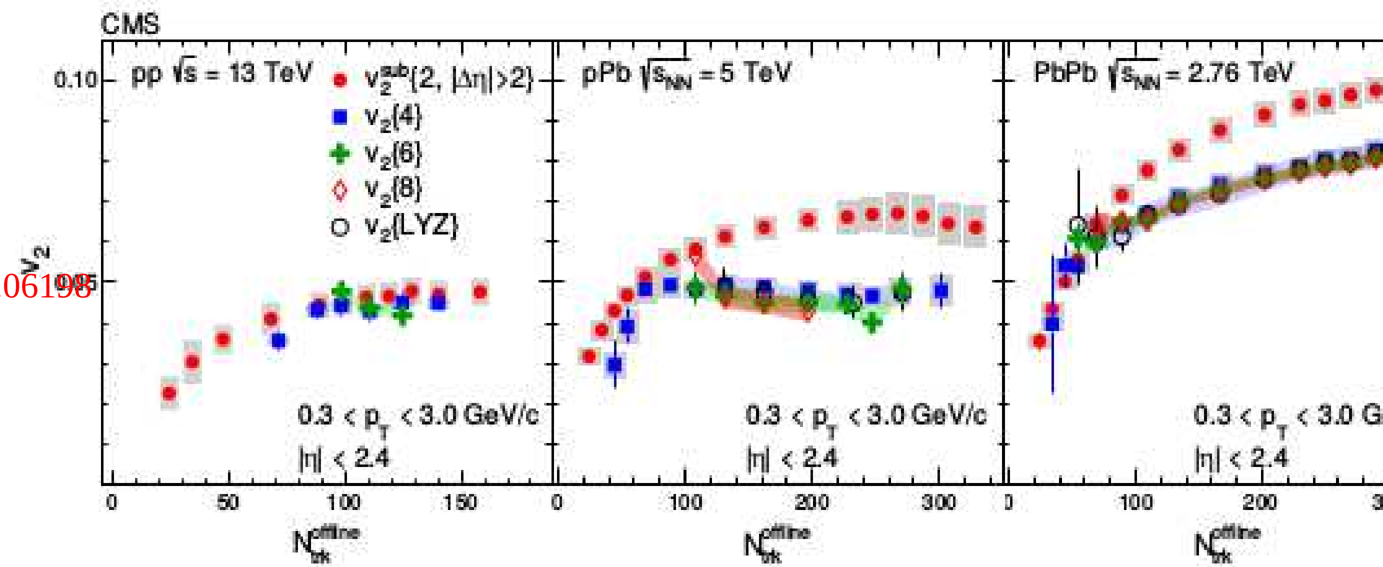
Basic problem with either Lorentz invariance or correlations on scale of gradients! Ambiguity in flow,$\Pi_{\mu\nu}$ comes from here!

In brief most microscopic approaches to EFT hydrodynamics assume that

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

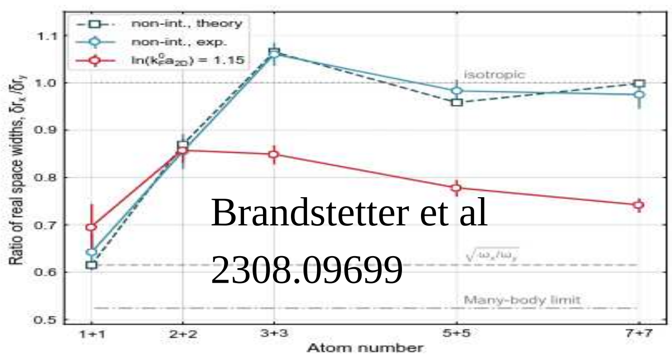
But this seems falsified by hydrodynamics in small systems!


POPULAR SCIENCE
WANT MORE?
PREMIUM
SUBSCRIBE NOW
SCIENCE
The LHC Might Have Created The Smallest Drop Of Liquid Ever
A tiny drop could have big implications for our understanding of particle collisions.
By Shaunacy Ferro May 8, 2013



CMS 1606.06198

Not just in heavy ions



The
Brazil
nut effect

Empirically, strongly coupled systems with enough thermal energy seem to be "fluid" even with a small number of DoFs. EFT does not explain this!

The role of fluctuations in hydrodynamics, and of the exast relation of statistical physics and hydrodynamics, are still ambiguous and this is related to experimental puzzles

Hydrodynamics and statistical mechanics

Equation of state $p(E)$ comes from basic statistical mechanics

$$p = T \ln \mathcal{Z} \qquad , \qquad \frac{dP}{dT} = \frac{dS}{dV} = \frac{p + e - \mu n}{T}$$

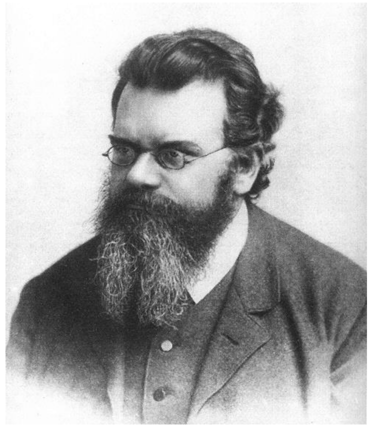
But the same partition function also predicts fluctuations

$$\left\langle (\Delta E)^2 \right\rangle = \frac{\partial \ln \mathcal{Z}}{\partial \beta^2} \sim \frac{1}{(\Delta V) \times s}$$

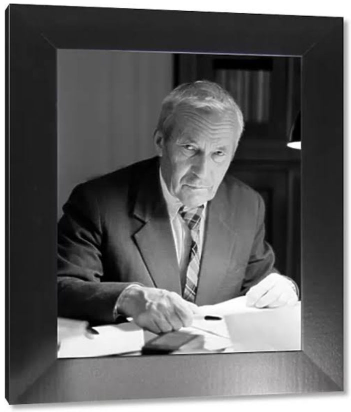
which in a deterministic theory are completely neglected. could this have something to do with the above ambiguity?

the battle

of the entropies



Boltzmann entropy (associated with frequentist probability) a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy (more Bayesian) is the log of the area of phase space, and is justified from coarse-graining and ergodicity . The two are different even in equilibrium, with interactions! (Khinchin,stat.mech.) Note, Von Neumann $\langle ln\hat{\rho}\rangle$ Gibbsian . Gibbs is more general, but...

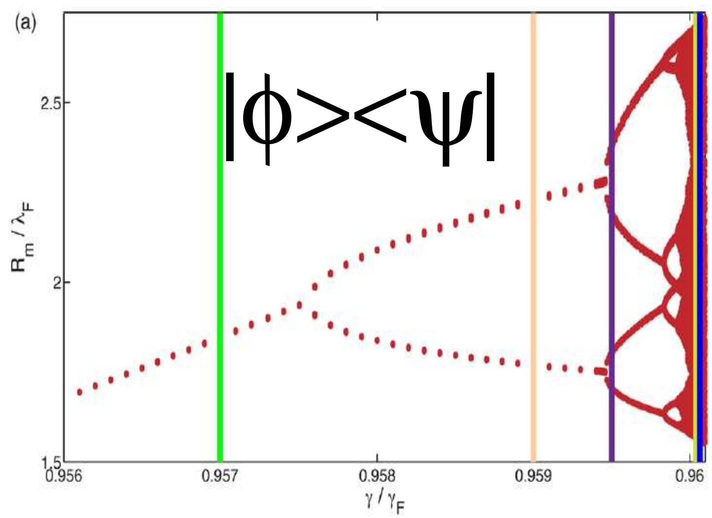


the unreasonable effectiveness of stat mech

Non-ideal hydrodynamics is based around approximate local equilibrium . Boltzmannian global and local equilibrium are defined, but they depend on Boltzmannian physics Only Global equilibrium well defined in Gibbs (what is "approximate maxiumum" Gibbsian entropy?)

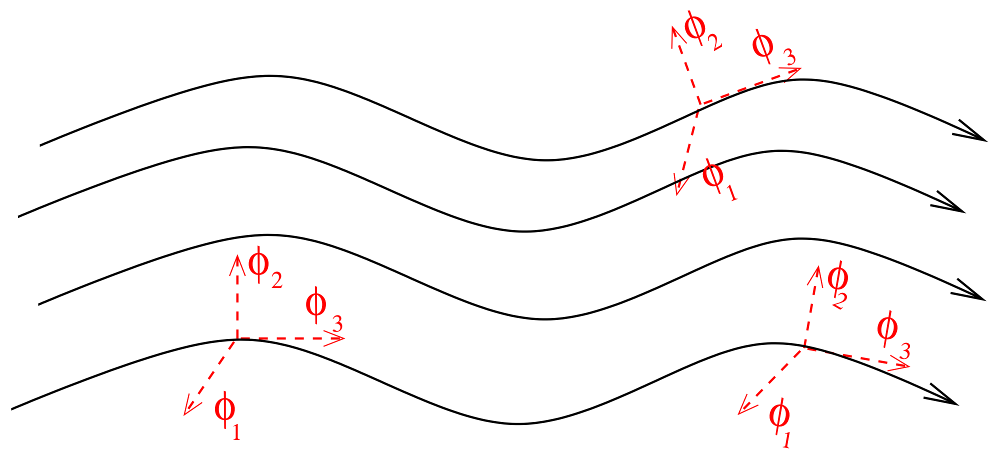
Khinchin's "failed" PhD: Stat Mech just seems wrong but seems to apply everywhere! Just like hydro?

QM to rescue?



Berry/Bohigas/Eigenstate thermalization hypothesys: $\underline{E_{n>>1}}$ of quantum systems whose classical correspondent is chaotic have density matrices that look like pseudo-random. If off-diagonal elements oscillate fast or observables simple, indistinguishable from Micro-canonical ensemble! . Can be valid for arbitrarily small fluctuating systems!

Let's look at this ambiguity a bit deeper: Lagrangian and Eulerian hydrodynamics Hydro as fields: (Nicolis et al,1011.6396 (JHEP))
Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0,x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame) Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons"

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P + \rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK}\, \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K$$

Equation of state chosen by specifying $F(B)$. "Ideal": $\Leftrightarrow F(B) = B^{4/3}$ $\sqrt{B}$ is identified with the entropy and $\sqrt{B}\frac{dF(B)}{dB}$ with the microscopic temperature. u^μ fixed by $u^\mu \partial_\mu \phi^{\forall I} = 0$

Conserved charges (Dubovsky et al, 1107.0731(PRD))

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

obviously can generalize to more complicated groups

This looks a bit like GR and this is not a coincidence!

4D local Lorentz invariance becomes local SO(3) invariance

Vierbein $g_{\mu\nu} = \eta^{\alpha\beta} e_\mu^\alpha e_\nu^\beta$ is

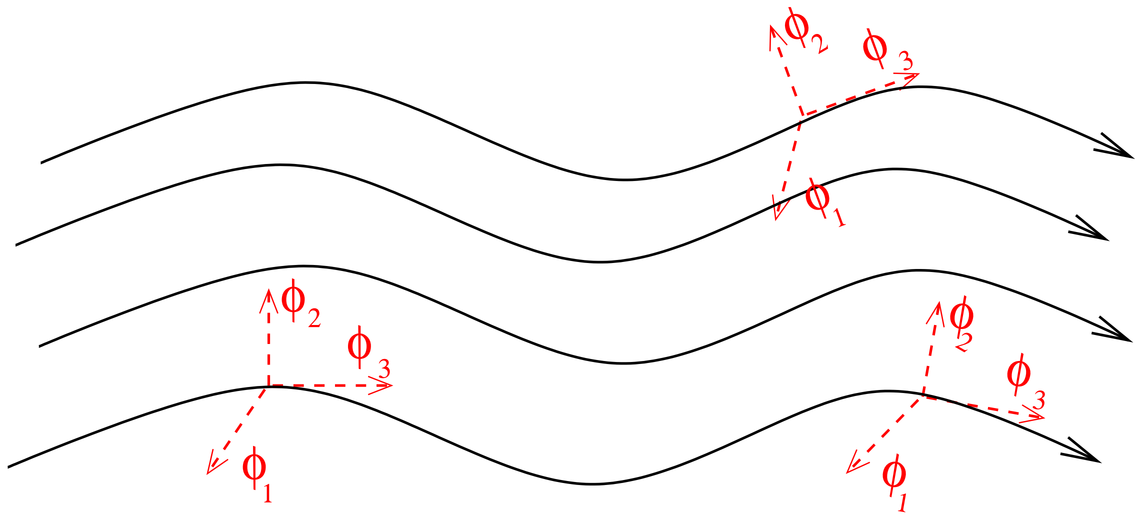
$\frac{\partial x_I^{comoving}}{\partial x_\mu} = \partial_\mu \phi_I$ (with Gauge phase for chemical potential)

Killing vector becomes u_μ

$\mathcal{L} \sim \sqrt{-g}\,(\Lambda + R + ...)$ becomes $\mathcal{L} \sim F(B) \equiv f(\sqrt{-g})$ Just cosmological constant, expanding fluid $\equiv$ dS space

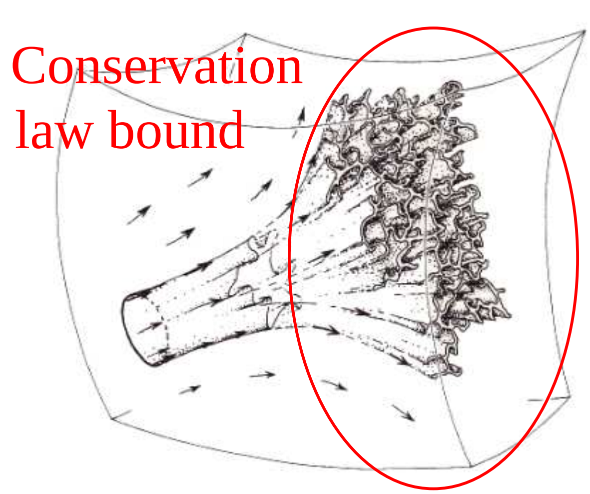
Very nice... but the ambiguities beyond ideal hydro generally break this . Who cares? Should beyond idel hydrodynamics have this general covariance?

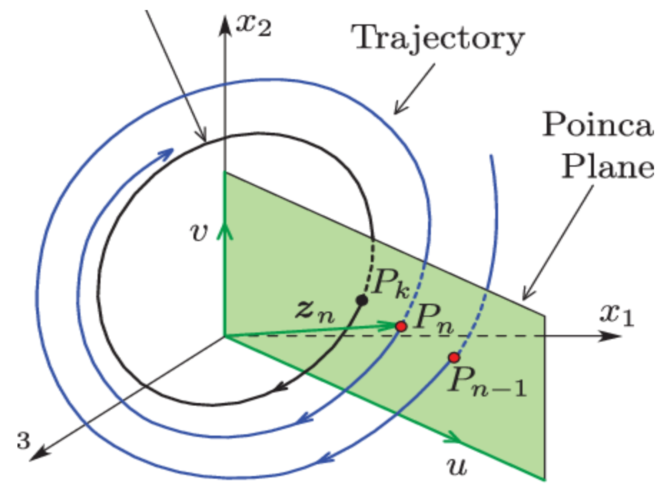
The poor people's quantum gravity: How can fluctuations and dissipation keep hydrodynamic's diffeomorphism invariance?



First step: Lagrangian hydrodynamics very elegant, but where is the connection to local thermalization? Statistical mechanics? Transport? Hint from D.T.Son: it is the largest group of diffeomorphisms <u>where time plays no role!</u>

Where does statistical mechanics come from? Ergodicity





Classical evolution via Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p} \quad , \quad \dot{p} = -\frac{\partial H}{\partial x} \quad , \quad \dot{O} = \{O, H\}$$

"Chaos",conservation laws$\rightarrow$ phase space more "fractal", recurring

"After some time", for <u>any</u> observable <u>ergodic</u> limit applies

$$\int_0^{(large)\quad T} \dot{O}(p,q)dt = \int P(O(p,q))dqdp$$

where $P(...)$ probability independent of time. This probability can only be given by conservation laws

$$P(O) = \frac{\left(\sum_i O_i\right)\delta^4\left(\sum_i P_i^\mu - P^\mu\right)\delta\left(\sum_i Q_i - Q\right)}{N} \quad , \quad N = \int P(O)dO = 1$$

this is the <u>microcanonicanal ensemble</u>. In <u>thermodynamic limit</u>

$$P(O) \rightarrow \delta(O - \langle O \rangle)$$

Hydrodynamics is “thermodynamics in every cell

$$\int_0^{(large)\ T} \dot{O}(p,q)dt \to \frac{\Delta\phi}{\Delta t}$$

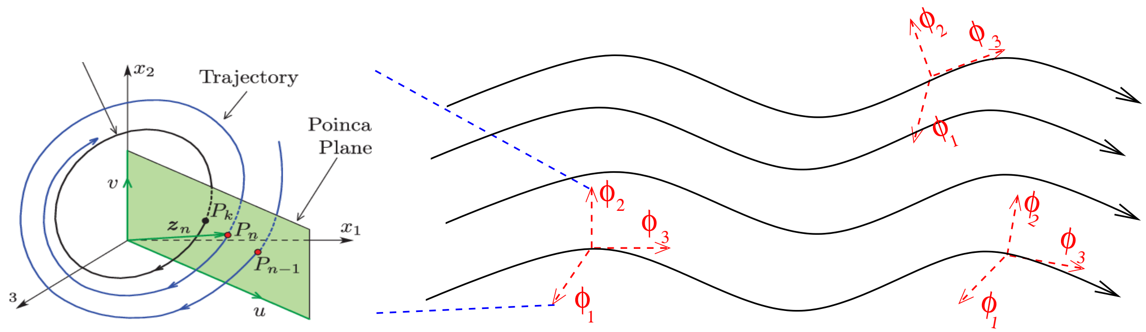
where ϕ is some local observable.

$$\left.\frac{\Delta\phi}{\Delta t}\right|_{t-t'=\Delta} \simeq \frac{1}{d\Omega(Q,E)}\times$$

$$\times \sum \delta^4_{P^\mu,P^\mu_{macro}(t)}\delta_{Q,Q_{macro}(t)}\delta\left(\sum_j^\infty p_j^\mu - P^\mu\right)\delta\left(\sum_j^\infty Q_j - Q\right)$$

Problem: This is not relativistically covariant!

Solution: Foliation!



$$t \to \Sigma_0 \quad , \quad x_\mu \to \Sigma_\mu \quad , \quad \Delta \to \text{``}smooth\text{''} \; \frac{\partial \Sigma_\mu}{\partial \Sigma_\nu}$$

Smooth: $R_{curvature}$ of metric change smaller than "cell size" (New f_{mfp})

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad \Sigma_\mu \to \Sigma'_\mu \quad , \quad \frac{\Delta\phi}{\Delta\Sigma'_0} = \frac{\Delta\phi}{\Delta\Sigma_0}$$

What kind of effective lagrangian would enforce

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi,\Sigma_\mu) d\Sigma_i \quad , \quad \frac{\Delta\phi}{\Delta\Sigma_0'} = \frac{\Delta\phi}{\Delta\Sigma_0}$$

with

$$P(...) \sim \delta(\sum_i P_i^\mu - P)\delta(\sum_i Q_i - Q)$$

Now Remember Noether's theorem!

$$p_\mu = \int d^3\Sigma^\nu T_{\mu\nu} \quad , \quad T_{\mu\nu} = \frac{\partial L}{\partial\partial^\mu\phi}\Delta_\nu\phi - g_{\mu\nu}L \quad , \quad \Delta_\nu\phi(x_\mu) = \phi(x_\mu + dx_\nu)$$

$$Q = \int d^3\Sigma^\nu j_\nu \quad , \quad j_\nu = \frac{\partial L}{\partial\partial^\mu\phi}\Delta_\psi\phi \quad , \quad \Delta_\psi\phi = |\phi(x)| e^{i(\psi(x)+\delta\psi(x))}$$

momentum generates spatial translations, conserved charges generate complex rotations!

Space-like foliations decompose

$$d\Sigma_\mu = \epsilon_{\mu\nu\alpha\beta}\frac{\partial\Sigma^\nu}{\partial\Phi_1}\frac{\partial\Sigma^\alpha}{\partial\Phi_2}\frac{\partial\Sigma^\beta}{\partial\Phi_3}d\Phi_1 d\Phi_2 d\Phi_3$$

where the determinant (needed for integrating out $\delta - functions$ is only in the volume part

$$\frac{\partial\Sigma'_\mu}{\partial\Sigma_\nu} = \Lambda_\mu^\nu \det\frac{d\Phi'_I}{d\Phi_J} \quad , \quad \det\Lambda_\mu^\nu = 1$$

Physically, Λ_μ^ν moves between the frame $d\Sigma^\mu_{rest} = d\Phi_1 d\Phi_2 d\Phi_3 (1,\vec{0})$

so lets try

$$\underbrace{L(\phi)}_{microscopic\ \ DoFs} \simeq L_{eff}(\Phi_{1,2,3})$$

with

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad P(...) = \delta(...)\delta(...)$$

the general covariance requirement of $\frac{\Delta\phi}{\Delta\Sigma_0} = \frac{\Delta\phi}{\Delta\Sigma_0'}$ means the invariance of the RHS

$$\frac{d\Omega(dP_\mu', dQ', \Sigma_0')}{d\Omega(dP_\mu, dQ, \Sigma_0)} =$$

$$= \frac{d\Sigma_0'}{d\Sigma_0} \frac{\int da_\mu d\psi \delta^4 \left(d\Sigma^\nu a_\alpha \partial^\alpha \left(\delta_\nu^\mu L\right) - dP^\mu(\Sigma_0)\right) \delta \left(d\Sigma^\mu \psi \partial_\mu L - dQ(\Sigma_0)\right)}{\int da_\mu' d\psi' \delta^4 \left(d\Sigma_\nu' a_\alpha' \partial^\alpha \left(\delta_\nu^\mu L\right) - dP_\mu'(\Sigma_0')\right) \delta \left(d\Sigma_\mu' \psi' \partial^\mu L - dQ'(\Sigma_0')\right)}$$

It is then easy to see,via

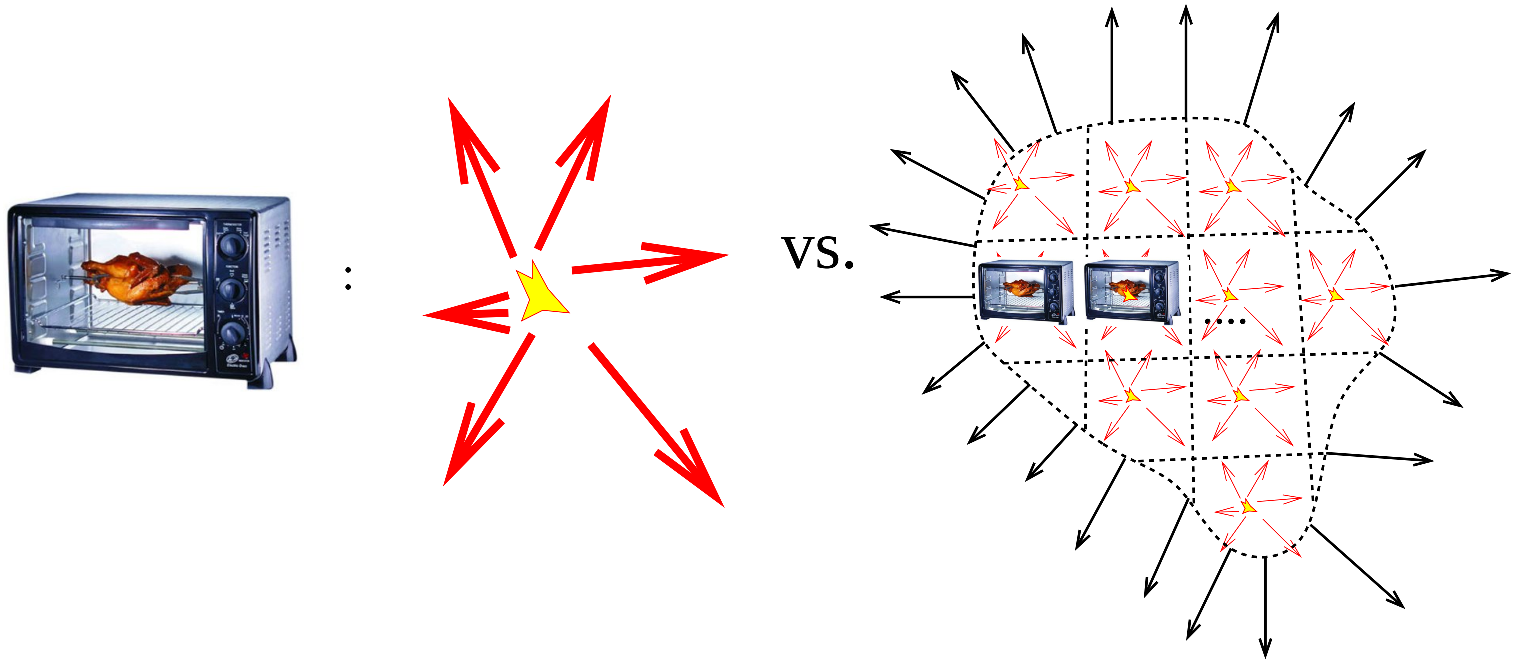
$$\delta((f(x_i))) = \sum_i \frac{\delta(x_i - a_i)}{\underbrace{f'(x_i = a_i)}_{f(a_i)=0}} \quad , \quad \phi'_I = \frac{\partial_\alpha \Sigma'_I}{\partial^\alpha \Sigma^J} \Phi_J \quad , \quad \delta^4(\Sigma_\mu) = \det \left| \frac{\partial \Sigma^\mu}{\partial \Sigma^\nu} \right| \delta^4$$

that for general covariance to hold

$$L(\Phi_I, \psi) = L(\Phi'_I, \psi') \quad , \quad \det \frac{\partial \phi_I}{\partial \phi_J} = 1 \quad , \quad \psi' = \psi + f(\phi_I)$$

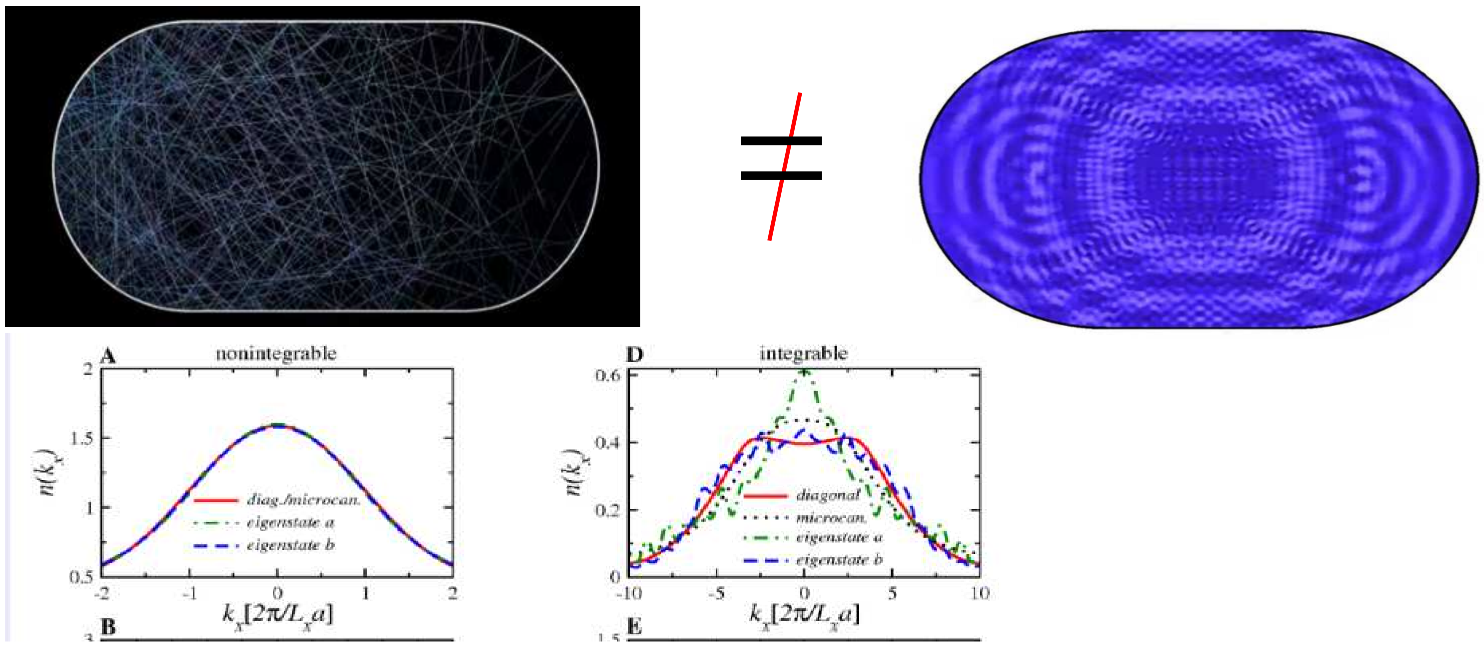
the symmetries of perfect fluid dynamics are equivalent to requiring the ergodic hypothesys to hold for generally covariant causal spacetime foliations!!!!

Classical to quantum F.Becattini, 0901.3643



Berry's conjecture: quantum systems with Chaotic classical counterparts and Above ground state $E_{n\gg 1}$

Density matrix pseudorandom , indistinguishable from microcanonical ensemble. born in equilibrium



M. Rigol, V. Dunjko, and M. Olshanii,Nature 452, 854 (2008)
Quantum billiard balls very different from classical and semi-classical ones! Any "non-integrability" modifies "initial state" which already "looks thermal". All evolution does is randomize phase . Related to divergences in finite temperature QFT? "loop" corrections to transport hard!

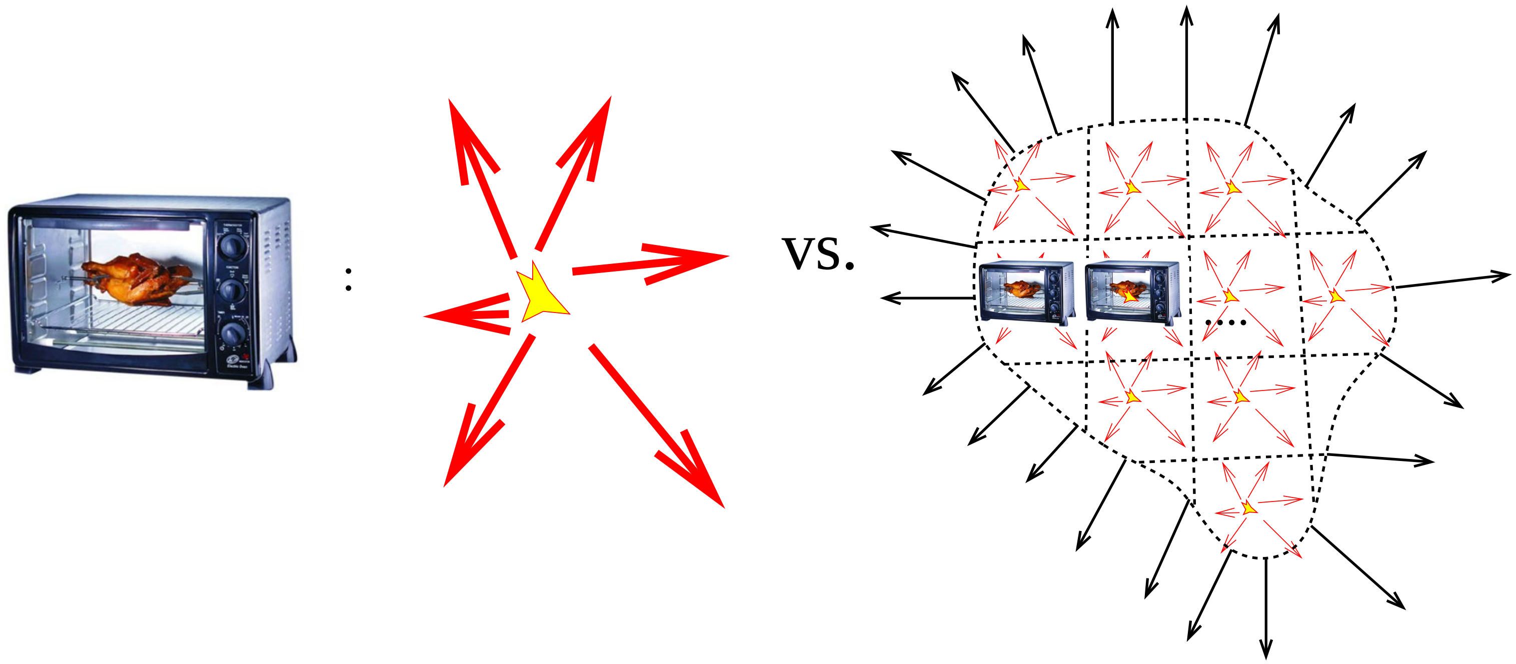
Applying the Eigenstate thermalization hypothesis to every cell in every foliation is equivalent to promoting $J_{\mu\nu}, \theta, P, Q$ to functions of x_μ and imposing foliation independence on the "pseudo-randomness" of $\hat{\rho}$.

$$\left.\frac{d\hat{\rho}}{d\Sigma_0}\right|_{\Sigma_0-\Sigma_0'\simeq\Delta} = 0 \quad , \quad \hat{\rho} \simeq \frac{1}{d\Sigma}\hat{\delta}_{E,E'}\hat{\delta}_{Q,Q'}$$

$$\hat{U}^{-1}(x)\hat{\rho}\hat{U}(x) \simeq \hat{\rho} \quad , \quad \hat{U}(x) = \exp\left[i\hat{T}^{\mu\nu} d^3\Sigma_\mu \delta x_\nu\right] \exp\left[i\partial_\alpha \theta d^3\Sigma^\alpha \delta \hat{Q}\right]$$

for arbitrary $d^3\Sigma_\mu$. Above derivation follows.

So one expects hydro together with statistical hadronization!



So the symmetries of ideal hydrodynamics are equivalent to ideal local ergodicity . So what? turns out one might be able to extend this “close” to equilibrium while retaining these symmetries!

The crucial question: Does this extend to non-ideal hydrodynamics?

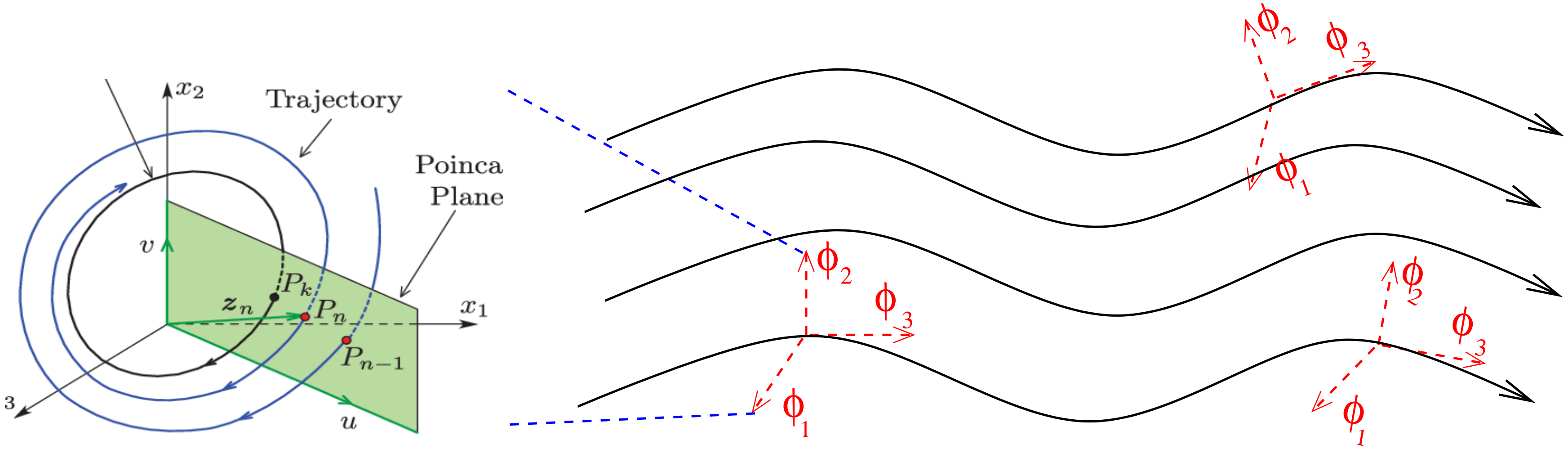
Close to local equilibrium is not on gradient expansion but the approximate applicability of fluctuation-dissipation
These are not automatically the same!

For smaller fluctuating systems many equivalent definitions of $e, u_\mu, J^\mu, \Pi^{\mu\nu}, ...$ leaving $T_{\mu\nu}$ invariant!
Different Boltzmannian entropy but all counted as Gibbsian entropy

If many equivalent choices of $e, u_\mu, J^\mu, \Pi^{\mu\nu}, ...$ likely in one its "small" !
Ideal hydro behavior.

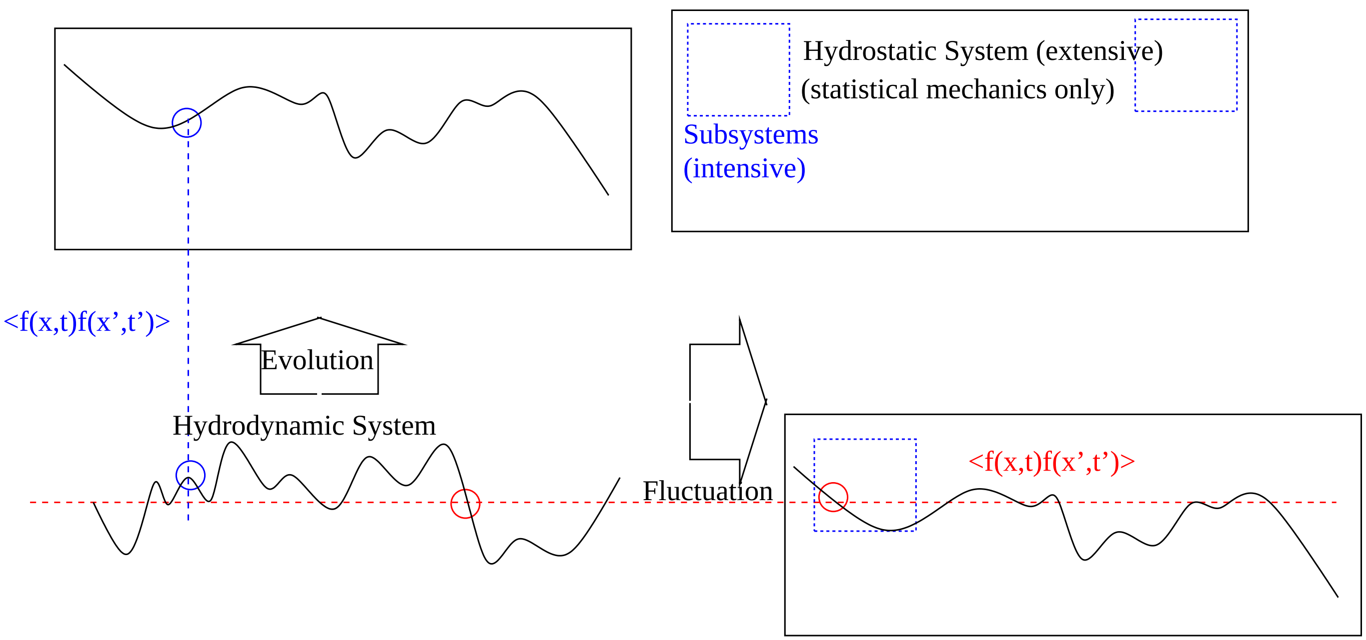
So indeed Ambiguity from fluctuations makes system look like a fluid.

The physical intuition Ergodicity/Poncaire cycles meet relativity slightly away from equilibrium!

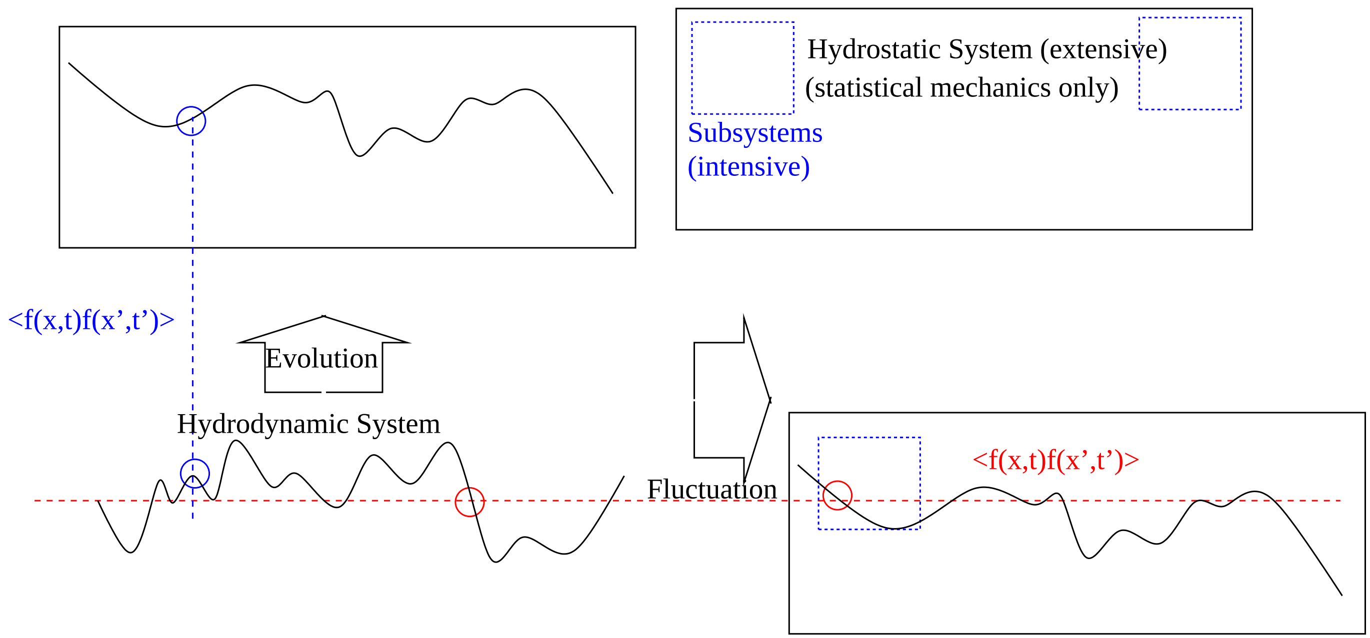


Gibbs entropy level+relativity : Lack of equilibrium is equivalent to "loss of phase" of Poncaire cycles. one can see a slightly out of equilibrium cell <u>either</u> as a "mismatched u_μ" (fluctuation) or as lack of genuine equilibrium (dissipation)

How to make physics fully "gauge"-invariant?



Fluctuation-dissipation at the cell level could do it! We don't know if a "step" is fluctuation ($T_0^{\mu\nu}$ or evolution ($\Pi_{\mu\nu}$)-driven!



But in hydro $T_0^{\mu\nu}, \Pi_{\mu\nu}$ treated very differently! "Sound-wave" $u \sim \exp[ik_\mu x^\mu]$ or "non-hydrodynamic Israel-Stewart mode?"

$D\Pi_{\mu\nu} + \Pi_{\mu\nu} = \partial u$

Only in EFT $1/T \ll l_{mfp}$ they are truly different!

What is a gauge theory,exactly?

$$\mathcal{Z} = \int \mathcal{D}A^\mu \exp\left[S[F_{\mu\nu}\right] \equiv \int \mathcal{D}A_1^\mu \mathcal{D}A_2^\mu \exp\left[S[A_1^\mu\right]$$

$A^\mu_{1,2}$ can be separated since physics sensitive to derivatives of $\ln \mathcal{Z}$

$$\ln \mathcal{Z} = \Lambda + \ln \mathcal{Z}_G \quad , \quad Z_G = \int \mathcal{D}\mathcal{A}^\mu \delta\left(G(A^\mu)\right) \exp\left[S(A_\mu)\right]$$

Ghosts come from expanding $\delta(...)$ term. In KMS condition/Zubarev

$$Z = \int \mathcal{D}\phi \quad , \quad "S" \rightarrow d\Sigma_\nu \beta_\mu T^{\mu\nu}$$

Multiple $T_{\mu\nu}(\phi) \rightarrow$ Gauge-like configuration . Related to Phase space fluctuations of ϕ

In summary,what we need is a hydrodynamics...

Manifestly in terms of observable quantities

Diffeomorphism-invariant at the level of fluctuations

Entropy content a scalar

Zubarev partition function for local equilibrium: think of Eigenstate thermalization...

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T^\mu_\nu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[-\int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiplies β_μ , with microscopic and quantum fluctuations included.

Effective action from $\ln[Z]$. Correction to Lagrangian picture?

All normalizations diverge but hey, it's QFT! (Later we resolve this!)

This is perfect global equilibrium. What about imperfect local?

- Two vectors, $d\Sigma_\mu u_\mu T_0^{\mu\nu}$ $d\Sigma_\mu$ foliation choice not clear (with vorticity it can't be parallel to flow everywhere). Physics should be choice independent. If $d\Sigma_\mu$ close to β_μ , $d\Sigma_\mu$ non-inertial
- Dynamics is not clear. Naively partition function can not depend on time (Adiabatically wrt microscopic scale however it could!) Becattini et al, 1902.01089: Gradient expansion in β_μ . Reproduces Euler and Navier-Stokes, but...
  - 2nd order Gradient expansion (Navier stokes) non-causal perhaps...
  - Use Israel-Stewart, $\Pi_{\mu\nu}$ arbitrary perhaps...
  - Foliation $d\Sigma_\mu$ arbitrary but not clear how to link to Arbitrary $\Pi_{\mu\nu}$
- What about fluctuations? Coarse-graining and fluctuations mix? How does one truncate?

An operator formulation $\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}_{\mu\nu}$

and $\hat{T}_0^{\mu\nu}$ truly in equilibrium! Each microscopic particle "does not know" if it "belongs" to $\hat{T}_0^{\mu\nu}, \hat{\Pi}_{\mu\nu}$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[-\int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

describes all cumulants and probabilities

$$\langle T_0^{\mu\nu}(x_1) T_0^{\mu\nu}(x_2) ... T_0^{\mu\nu}(x_n) \rangle = \prod_i \frac{\delta^n}{\delta \beta_\mu(x_i)} \ln Z$$

Equilibrium at "probabilistic" level and KMS Condition obeyed by "part of density matrix" in equilibrium, "expand" around that! An operator constrained by KMS condition is still an operator! $\equiv$ time dependence in interaction picture

Does this make sense? Nishioka, 1801.10352 $\langle x|\,\rho\,|x'\rangle =$

$$= \frac{1}{Z}\int_{\tau=-\infty}^{\tau=\infty}\int [\mathcal{D}\phi, \mathcal{D}y(\tau)\,\mathcal{D}y'(\tau)]\; e^{-iS(\phi y, y')} \cdot \underbrace{\delta\left[y(0^+) - x'\right]\,\delta\left[y'(0^-) - x\right]}_{\frac{\delta J_i(y(0^+))}{\delta J_i(x')}\frac{\delta J_j(y(0^-))}{\delta J_j(x)}}$$

$$\Rightarrow \frac{\delta^2}{\delta J_i(x)\delta J_j(x')}\ln\left[Z_{T_0}(T^{\mu\nu}, J) \times Z_\Pi(J)\right]_{J=J_1(x)+J_2(x')}$$

$J_1(x) + J_2(x')$ chosen to respect Matsubara conditions!

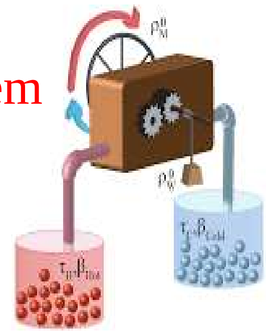
Any ρ can be separated like this for any β_μ . The question is, is this a good approximation? "Close enough to equilibrium"

The source J related to the smearing in "weak solutions". Pure maths angle?

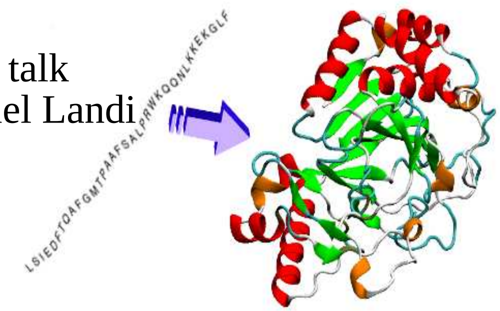
How to go forward... Crooks fluctuation theorem

Crooks fluctuation theorem

$P(W)/P(-W)=e^{\Delta s}$



From talk
Gabriel Landi



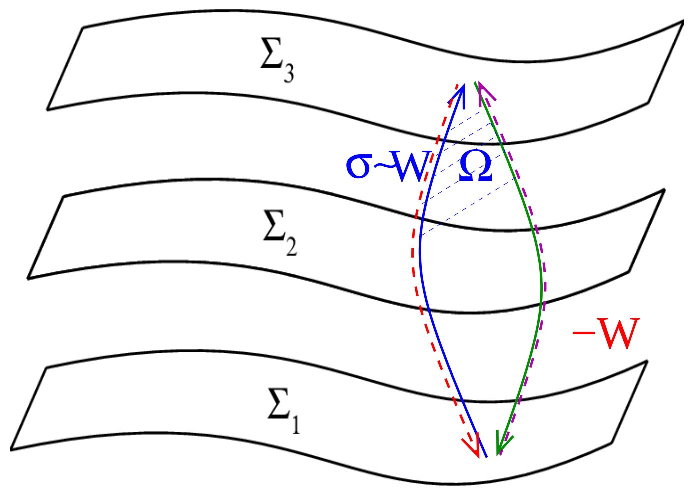
Relates fluctuations, entropy in <u>small</u> <u>fluctuating</u> systems (Nano,proteins)

P(W) Probability system doing work in its usual thermal evolution

P(-W) Probability of the same system "running in reverse" and decreasing entropy due to a <u>thermal fluctuation</u>

ΔS Entropy produced by $P(W)$

How to go forward... Crooks fluctuation theorem redtextApplying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) = -\int_{\Sigma(\tau')} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu\right),$$

In a past work (2007.09224,JHEP) I have shown Zubarev+Crooks fluctuation theorem has right limits and symmetries But highly non-local and non-linear, "lattice" .

A simpler EFT: A Gaussian approximation

General covariance via the Gravitational Ward identity

Gaussian approximation from Zubarev hydrodynamics

Kramers-Konig to enforce fluctuation-dissipation

The gravitational ward identity $\nabla \mathcal{W} = 0$

$$\mathcal{W} = G^{\mu\nu,\alpha\beta}\left(\Sigma_\mu, \Sigma'_\nu\right) - \frac{1}{\sqrt{g}}\delta\left(\Sigma' - \Sigma\right) \times$$

$$\times \left(g^{\beta\mu} \left\langle \hat{T}^{\alpha\nu}\left(x'\right) \right\rangle_\Sigma + g^{\beta\nu} \left\langle \hat{T}^{\alpha\mu}\left(x'\right) \right\rangle_\Sigma - g^{\beta\alpha} \left\langle \hat{T}^{\mu\nu}\left(x'\right) \right\rangle_\Sigma \right)$$

Fancy name and complicated but consequence of elementary properties of the metric and energy conservation

$$\partial_\mu T^{\mu\nu} + \Gamma_{\nu\alpha\beta} T^{\alpha\beta} = 0 \quad , \quad \left\langle T^n_{\mu\nu} \right\rangle = \frac{\delta^n}{\sqrt{-g}\delta g^{\mu\nu(n)}} \ln \mathcal{Z}$$

Cumulant expansion: An possibly diffeomorphism invariant alternative to gradient expansion which isn't!

$$\ln \mathcal{Z} \simeq \ln \mathcal{Z}|_0 - \left.\frac{\partial^2 \ln \mathcal{Z}}{\partial \beta_\mu \partial \beta_\nu}\right|_0 [\, d\Sigma_\alpha d\Sigma'_\tau \left(T^{\mu\alpha}(\Sigma) - \langle T^{\mu\alpha}(\Sigma')\rangle\right)\left(T^{\nu\tau}(\Sigma) - \langle T^{\nu\tau}(\Sigma')\rangle\right)]$$

A covariantization of

$$\left\langle E^2 \right\rangle - \left\langle E \right\rangle^2 \equiv C_V T \Rightarrow\Rightarrow C_{\alpha\beta\mu\nu} \sim \left.\frac{\partial \ln \mathcal{Z}}{\partial \beta_\mu \partial \beta_\nu}\right|_0 F(\Sigma)_{\alpha\beta}$$

This way metric tensor propagator can be modelled as a Gaussian

$$f(...) \sim \prod_{\Sigma(x),\Sigma(x')} \exp\left[-\frac{1}{2}\left(T_{\mu\nu}(\Sigma(x')) - \langle T_{\mu\nu}(\Sigma(x'))\rangle\right) C^{\mu\nu\alpha\beta}(\Sigma(x),\Sigma(x'))\,(T_{\alpha\beta}(\Sigma\right.$$

and Ward identity imposed on width $C_{\alpha\beta\gamma\nu}$.

fluctuation-dissipation relation From Kramers-Konig relations

$$\mathrm{Im}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Re}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right]}{\omega'-\omega}d\omega'$$

$$\mathrm{Re}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right] = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right]}{\omega'-\omega}d\omega'$$

Direct consequence of causality, relate the real and imaginary part of the response function in momentum space But non-local in frequency, generally invalidates gradient expansion! (inherently breaks fluctuation-dissipation)

Apply on the linear response function of energy-momentum tensor

$$T_{\mu\nu}(\Sigma) = \int e^{\epsilon\Sigma_0} G^{\mu\nu,\alpha\beta}(\Sigma_0' - \Sigma_0)\delta g_{\alpha\beta}(\Sigma_0')d\Sigma_0$$

$$\tilde{\mathcal{G}}^{\mu\nu\alpha\beta} = \frac{1}{2i}\left(\frac{\tilde{G}^{\alpha\beta\mu\nu}(\Sigma_0,k)}{\tilde{G}^{\alpha\beta\mu\nu}(-i\epsilon\Sigma_0,k)} - 1\right)$$

These equations together should do it!

Only in terms of $T_{\mu\nu}, J_\mu, \Sigma_\mu$ "observables" and a "gauge" reditemSecond law imposed via fluctuation dissipation (redundances, fluctuations of observables)

Conclusions

Fluctuations in non-ideal hydrodynamics not well understood

Intimately related to entropy current, double counting of DoFs
Could alter fluctuation-dissipation expectation, "fluctuations help dissipate", in analogy to Gauge theory

Approximate local equilibrium not understood in Gibbsian picture
My proposal: applicability of fluctuation-dissipation

Need a covariant description purely in terms of observable quantities
Ergodicity works in ideal hydro, Crooks theorem/K-K beyond it?

Could be relevant for hydro in small systems

SPARE SLIDES

PS: transfer of micro to macro DoFs experimentally proven!

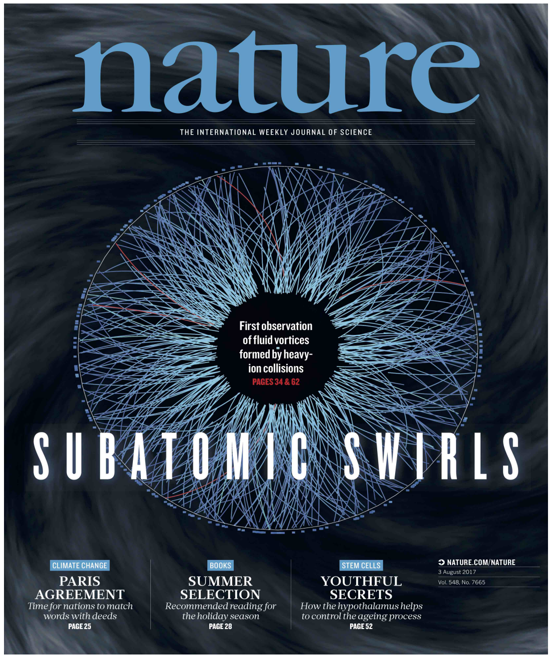
STAR collaboration

1701.06657

NATURE

August 2017

Polarization by vorticity in heavy ion collisions



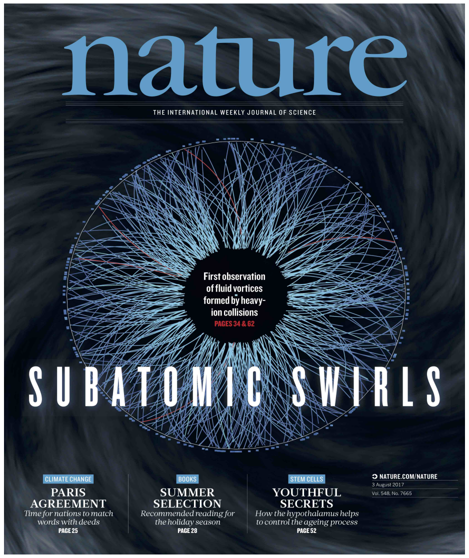
Could give new talk about this, but will mention hydro with spin not developed and a lot of conceptual debates Pseudo-gauge dependence if both spin and angular momentum present in fluid? Gauge symmetry "ghosts"? GT,1810.12468 (EPJA) . redundances?

STAR
collaboration

1701.06657

NATURE
August 2017

Polarization by vorticity
in heavy ion collisions



Pseudo-gauge symmetries physical interpretation: T.Brauner, 1910.12224

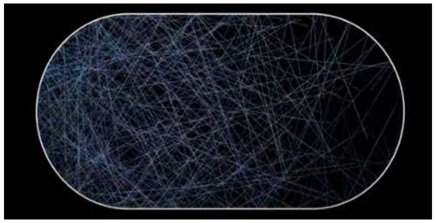
$$x^\mu \to x^\mu + \epsilon \zeta^\mu(x) \quad , \quad \psi_a \to \psi_a + \epsilon \psi'_a \to \mathcal{L} \to \mathcal{L}$$

$\ln \mathcal{Z}$ Invariant, but $\langle O \rangle$ generally is not. Spin $\leftrightarrow$ fluctuation, need equivalent of DSE equations! $D \langle O \rangle = 0 \to D \langle O \rangle = \langle O_I O_J \rangle$

Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T, V, \mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations. many issues connecting to Stat.Mech. Wild weak solutions, millenium problem!

The problem with general "transport thinking"

 $=$  ?

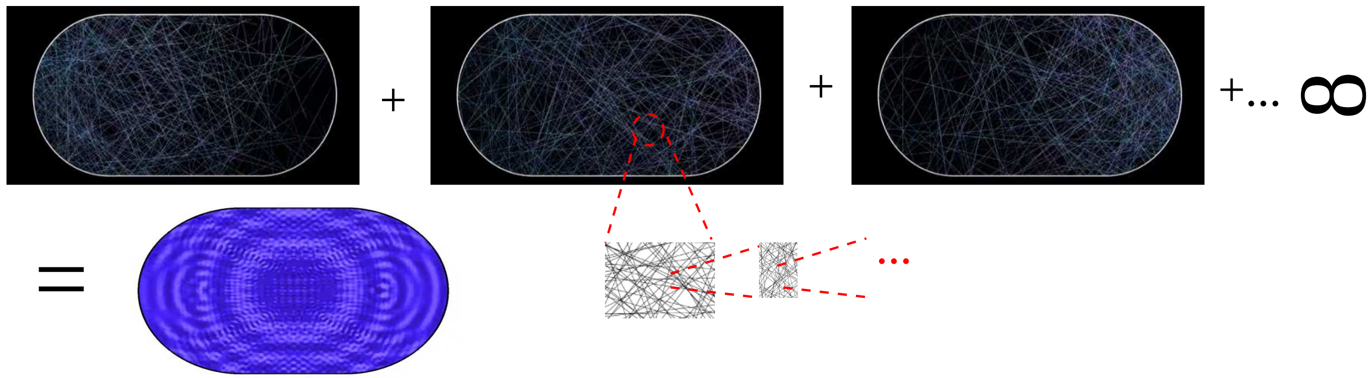
Let's solve the simplest transport equation possible: Free particles

$$\frac{p^\mu}{m}\partial_\mu f(x,p) = 0 \to f(x,p) = f\left(x_0 + \frac{p}{m}t, p\right)$$

obvious solution is just to propagate
What is weird is that "hydro-like" solution possible too (eg vortices)!

$$f(x,p) \sim \exp\left[-\beta_\mu p^\mu\right] \qquad , \qquad \partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

But obviously unphysical, no force! What's up?



This paradox is resolved by remembering that $f(x,p)$ is defined in an ensemble average limit where the number of particles is not just “large” but uncountable . curvature from continuity!

BUt this suggests Boltzmann equation <u>disconnected</u> from <u>any</u> finite number of particles!

What if $e^{-\beta_\mu p^\mu}$ used to <u>sample</u> strongly coupled particles in ”many finite events”? Thermal fluctuations,Vlasov correlations and Boltzmann scattering ”mix these words”. Many ways to mix,some <u>wrong</u>! What is appropriate?

How "different events" correlated is crucial
Villani , https://www.youtube.com/watch?v=ZRPT1Hzze44

Vlasov equation contains all classical correlations. Relativistically numer of particles varies in each event but "evolves" deterministically. but instability-ridden, "filaments", cascade in scales.
$N_{DOF} \rightarrow \infty$ invalidates KAM theorem stability

Boltzmann equation "Semi-Classical UV-completion" ov Vlasov equation, first term in BBGK hyerarchy, written in terms of Wigner functions.

Infinitely unstable jerks on infinitely small scales Random scattering
Statistical behavior emerges from both instabilities (chaos, Poncaire cycles) and scattering (H-theorem) but interplay non-trivial. Strong coupling away from molecular chaos not understood!

There is more to hydro than the Knudsen number

Power counting:

B.Betz,D.Henkel,D.Risc 0812.1440

3 length scales: 2 microscopic, 1 macroscopic

- thermal wavelength $\lambda_{\rm th} \sim \beta \equiv 1/T$
- mean free path $\ell_{\rm mfp} \sim (\langle\sigma\rangle n)^{-1}$

What if these are ~?

$\langle\sigma\rangle$ averaged cross section, $n \sim T^3 = \beta^{-3} \sim \lambda_{\rm th}^{-3}$

- length scale over which macroscopic fluid fields vary $L_{\rm hydro}$, $\partial_\mu \sim L_{\rm hydro}^{-1}$

Note: since $\eta \sim (\langle\sigma\rangle\lambda_{\rm th})^{-1}$ $\Longrightarrow$ $\frac{\ell_{\rm mfp}}{\lambda_{\rm th}} \sim \frac{1}{\langle\sigma\rangle n}\frac{1}{\lambda_{\rm th}} \sim \frac{\lambda_{\rm th}^3}{\langle\sigma\rangle\lambda_{\rm th}} \sim \frac{\lambda_{\rm th}^3}{\langle\sigma\rangle\lambda_{\rm th}} \sim \frac{\eta}{s}$

s entropy density, $s \sim n \sim T^3 = \beta^{-3} \sim \lambda_{\rm th}^{-3}$

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

Second inequality was developed so far, but first is suspect! experimentally

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

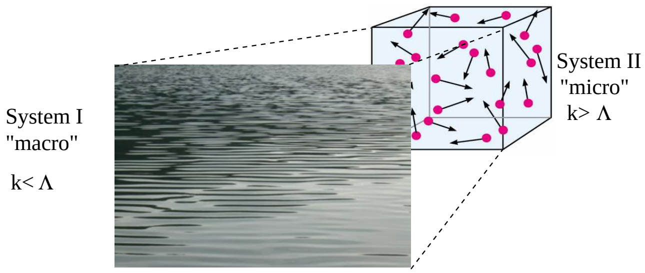
Weakly coupled: Ensemble averaging in Boltzmann equation good up to $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right)$

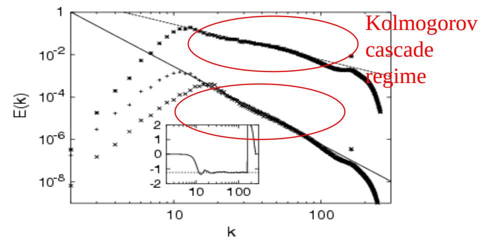
Strongly coupled: classical supergravity requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1$ so

$$\frac{1}{TN_c^{2/3}} \ll \frac{\eta}{sT} \quad \left(\quad or \quad \frac{1}{\sqrt{\lambda} T} \right) \ll L_{macro}$$

QGP: $N_c = 3 \ll \infty$,so $l_{micro} \sim \frac{\eta}{sT}$. Cold atoms: $l_{micro} \sim n^{-1/3} > \frac{\eta}{sT}$?

Why is $l_{micro} \ll l_{mfp}$ necessary? microscopic fluctuations (which have nothing to do with viscosity) will drive fluid evolution. $\Delta\rho/\rho \sim C_V^{-1} \sim N_c^{-2}$



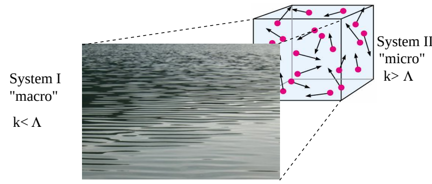


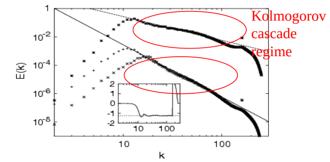
A classical low-viscosity fluid is turbulent. Typically, low-k modes cascade into higher and higher k modes In a non-relativistic incompressible fluid

$$\eta/(sT) \ll L_{eddy} \ll L_{boundary} \quad , \quad E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$$

For a classical ideal fluid, no limit! since $\lim_{\delta\rho\to 0, k\to\infty} \delta E(k) \sim \delta\rho k c_s \to 0$ but quantum $E \geq k$ so energy conservation has to cap cascade.

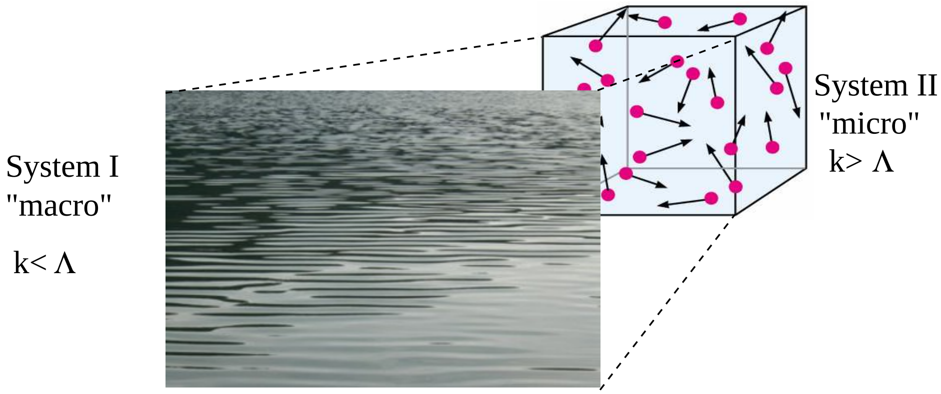
More fundamentally: take stationary slab of fluid at local equilibrium.

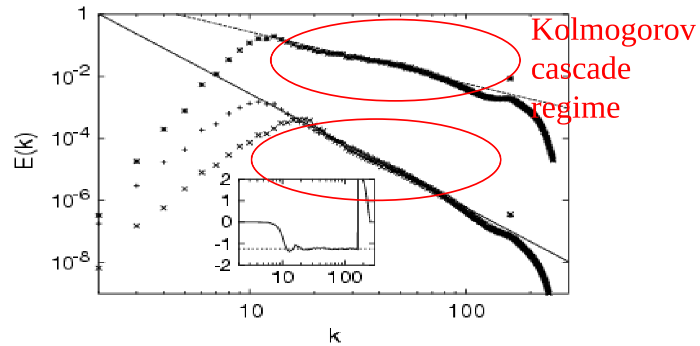




Statistical mechanics: This is a <u>system</u> in <u>global</u> equilibrium, described by a partition function $Z(T,V,\mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\left\langle (\Delta E)^2 \right\rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a <u>field</u> in <u>local</u> equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the <u>Euler/N-S equations</u>. Smaller η/s , the closer to <u>local</u> equilibrium (SM applies to <u>cell</u>) but the longer the timescale to <u>global</u> equilibrium (SM applies to <u>system</u>).





- Provided state is localized, local equilibrium is ”global equilibrium in every cell”, global equilibrium with spin, forces ”non-local” A.Palermo et al,2007.08249,2106.08340 ”global” equilibrium not necessarily stable against hydro perturbations I think ”real” global equilibrium built up from local equilibria

- Dissipation scale in local equilibrium $\eta/(Ts)$, global equilibration timescale $(Ts)/\eta$ .turbulence drastically changes this ,but ”when does a small perturbation become a microstate?”

Some insight from maths

Millenium problem: existence and smoothness of the Navier-Stokes equations

Important tool are "weak solutions" , similar to what we call "coarse-graining".

$$F\left(\frac{d}{dx}, f(x)\right) = 0 \Rightarrow F\left(\int \frac{d}{dx}\phi(x)..., f(x)\right) = 0$$

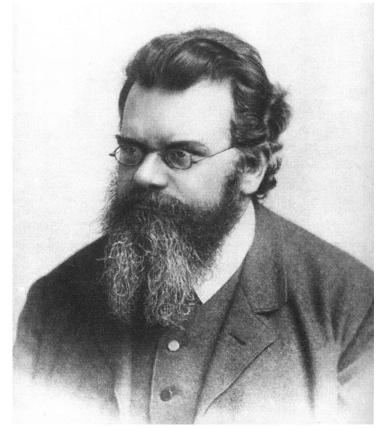
$\phi(x)$ "test function", similar to coarse-graining!

Existance of Wild/Nightmare solutions and non-uniqueness of weak solutions shows this tension is non-trivial, coarse-graining “dangerous”

I am a physicist so I care little about the ”existence of ethernal solutions” to an approximate equation, Turbulent regime and microscopic local equilibria need to be consistent

Thermal fluctuations could both ”stabilize” hydrodynamics and ”accellerate” local thermalization
But where do microstates,”local” microstates fit here?

the battle of the entropies



Boltzmann entropy is usually a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy is the log of the area of phase space, and is justified from coarse-graining and ergodicity , but hard to define it in non-equilibrium . The two are different even in equilibrium, with interactions! Note, Von Neumann $\langle ln\hat{\rho}\rangle$ Gibbsian

Gauge theory and local thermalization

The formalism we introduced earlier is ok for quark polarization but problematic for gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for transversely polarized spin states. So vorticity vs polarization is ambiguus

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \to T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda} \right)$$

where Φ is fully antisymmetric. $\delta S / \delta g_{\mu\nu}$ and canonical tensors are lmits of choice of Φ . But vorticity global (and gauge invariant), $y_{\mu\nu}$ local (and gauge dependent). Affects EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators)

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \qquad , \qquad \exp(i\alpha) \to \exp\left(i\sum_i \alpha_i \hat{T}_i\right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J_a^\mu \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U^{-1}_{ac}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i \right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U^{-1}_{ac}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} U^{-1}_{fg} \partial_\mu \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} \partial_\mu \left(U^{-1}_{fg} \alpha_d U_{bd}(x) \right) - J^{\mu}_{a} \left(U \partial_\mu U \right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J^{\mu}_{j} \partial_\mu \alpha_i\right) \to F\left(b, J^{\mu}_{j} \left(\partial_\mu - U(x) \partial_\mu U(x)\right) \alpha_i\right)$$

Which is totally unexpected, profound and crazy

The swimming ghost!

$$F\left(b, J_j^\mu \partial_\mu \alpha_i\right) \to F\left(b, J_j^\mu \left(\partial_\mu - U(x)\partial_\mu U(x)\right)\alpha_i\right)$$

Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \to 1 \quad , \quad y_{ab} \to \mu_Q \quad , \quad u_\mu \partial^\mu \alpha_i \to A_\tau$$

So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...

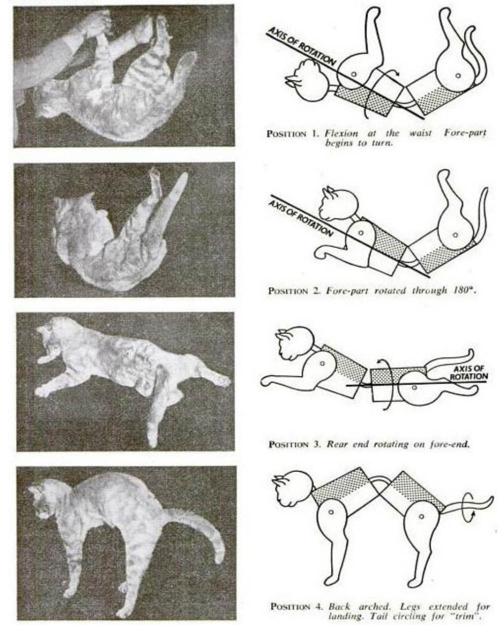
The swirling ghost

Since $u^\mu \partial_\mu$ is in the Lagrangian,let us compare vorticity and Wilson loops!

$$\mathrm{Vorticity}: \oint J_\mu dx^\mu \neq 0 \quad , \quad \mathrm{Wilsonloop}: \oint dx_\mu \partial^\mu U_{ab} \equiv \int_\Sigma d\Sigma_{\mu\nu} F_{ab}^{\mu\nu}$$

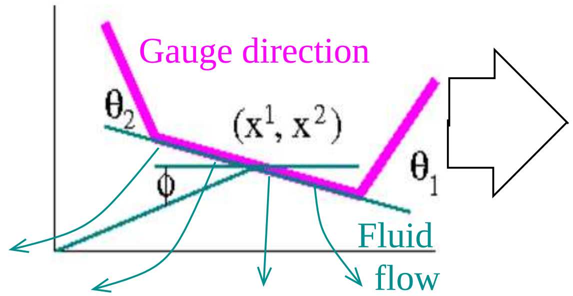
Lagrangian will in general have gauge-invariant terms proportional to $Tr_a \omega_{\mu\nu a} F_a^{\mu\nu}$ Unlike in Jackiw et al , $F_{\mu\nu}$ is not field strength but just a polarization tensor, whose value is set by entropy maximization.

But circular modes correlating angle around vortex of u_μ and direction a of $F_{\mu\nu}^a$ non-dissipative (unlike in polarization hydro described earlier)



S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a "set of gauges" a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a “swimmer”, with arms and legs outstretched in “gauge” directions...

Hydrostatic vacuum unstable against purcell swimmers in Gauge space!

A statistical mechanics/Gauge explanation

Hydrodynamic limit: $\partial^\mu s_\mu \equiv \partial^\mu \left(u_\mu \ln N_{microstates} \right) = 0$

In thermal Gauge theory microstates contain gauge redundancies,

$N_{microstates} \rightarrow N_{microstates} - N_{gauge}$ But s_μ^{real} not parallel to s_μ^{gauge}

so no local equilibrium!. recall hydrostatic limit perturbation

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential". Let us vary the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

"color susceptibility" typically negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes "work" from "heat" is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Development of EoMs, linearization, etc. of this theory in progress!

A crazy guess, speculation Remember that all flow dependence through μ_{ab} color chemical potentials. What if local equilibrium happens when they go to zero, i.e. color density is neutral.

Could colored-swimming ghosts quickly be produced, and then locally thermalize and color-neutralize the QGP?

Similar to Positivity violation picture of confinement (Alkofer)

What about gauge-gravity duality?

Large N non-hydrodynamic modes go away in the planar limit
There are N ghost modes and N^2 degrees of freedom

Conformal fixed point most likely means ghosts non-dynamical
Not yet sure of this, but conformal invariance reduces pseudo-Gauge transformations to

$$\Phi_{\lambda,\mu\nu} \underbrace{\rightarrow}_{conformal} g_{\sigma\mu}\partial_\nu\phi - g_{\sigma\mu}\partial_\mu\phi$$

where ϕ is a scalar function. Irrelevant for dynamics.
As shown in Capri et al (1404.7163) Gribov copies for a Yang-Mills theory non-dynamical there. It would be a huge job to do this for hydrodynamics.