Quarkonium dynamics: From quantum to semi-classical description

Pol B Gossiaux, SUBATECH (NANTES)

Xth NED

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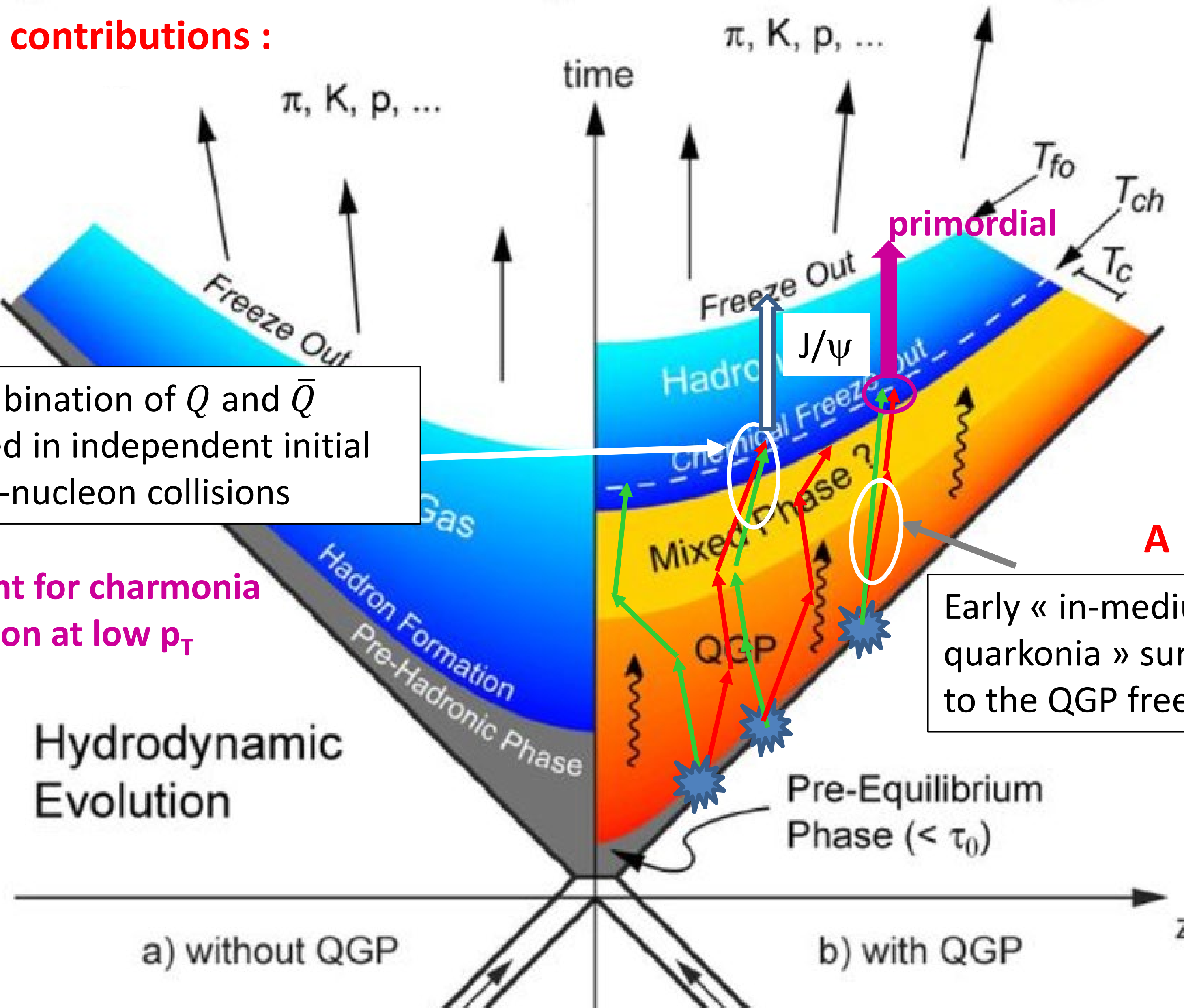
Krabi (Thailand)

and Pays de la Loire

Probing URHIC with quarkonia production

2 typical contributions :



B

(Re)combination of Q and $\bar{Q}$ produced in independent initial nucleon-nucleon collisions

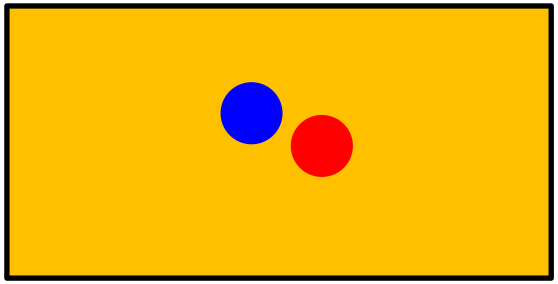
Dominant for charmonia production at low p_T

A

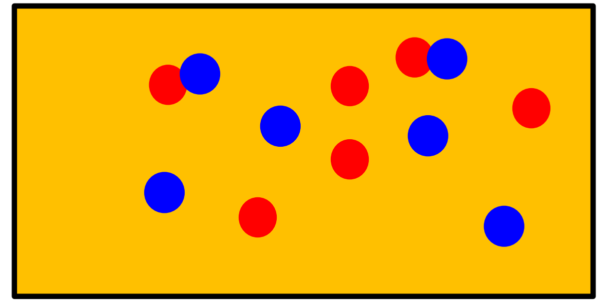
Early « in-medium quarkonia » surviving up to the QGP freeze out

Regeneration: Dilute vs Dense

Bottomia (single pair)



Charmonia (many pairs)

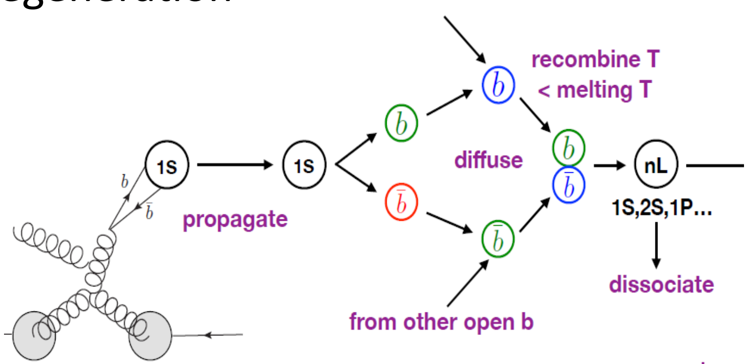


No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states.

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

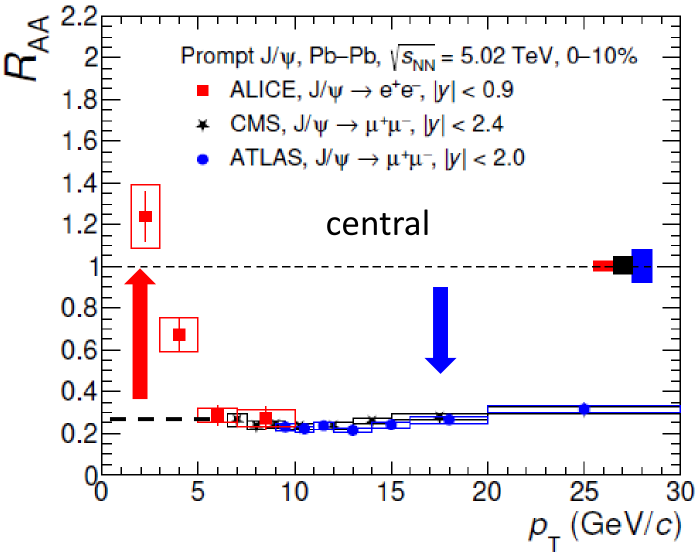
Full quantum treatment affordable

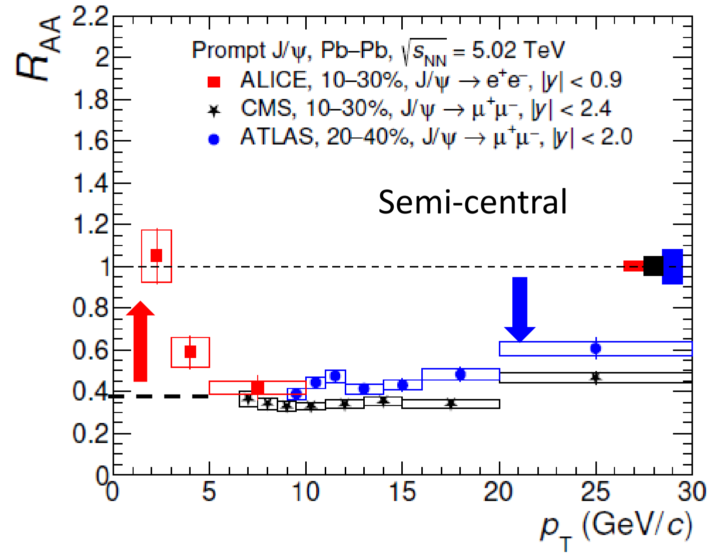
N.B.: In some SC formalisms : intermediate regeneration

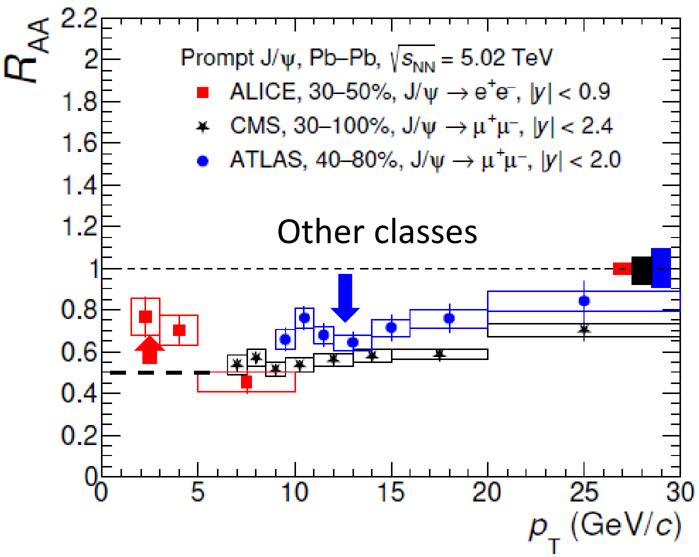


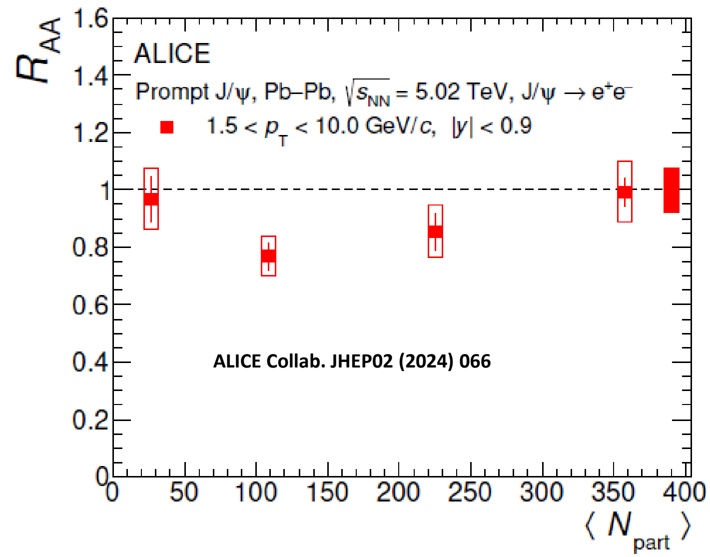
Yao, Mehen, Müller (2019)

What experiment tells us





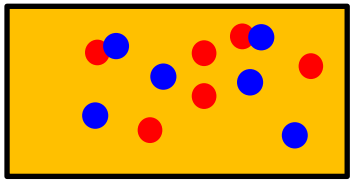




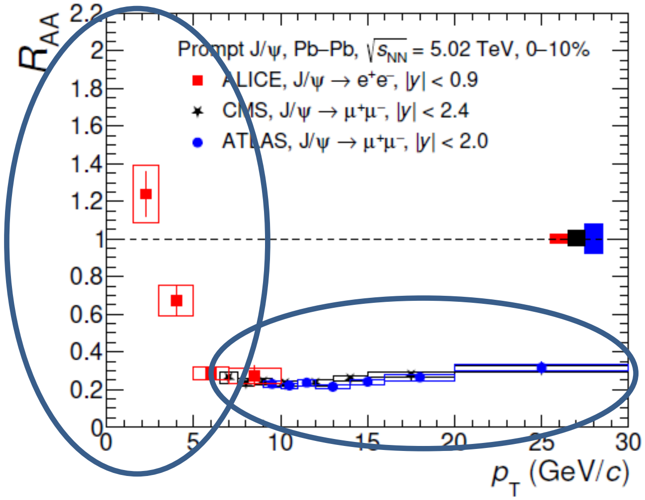
- Quarkonia production in AA strongly affected by the presence of the QGP => good probe of the QGP properties on small scales ($1/M_Q$)
- Increasing suppression with centrality at intermediate and high p_T
- Increasing yield with centrality at low p_T
- Increasing experimental precision => need for the models to gain in accuracy

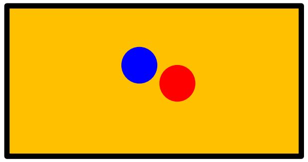
What experiment tells us

ALICE Collab. JHEP02 (2024)



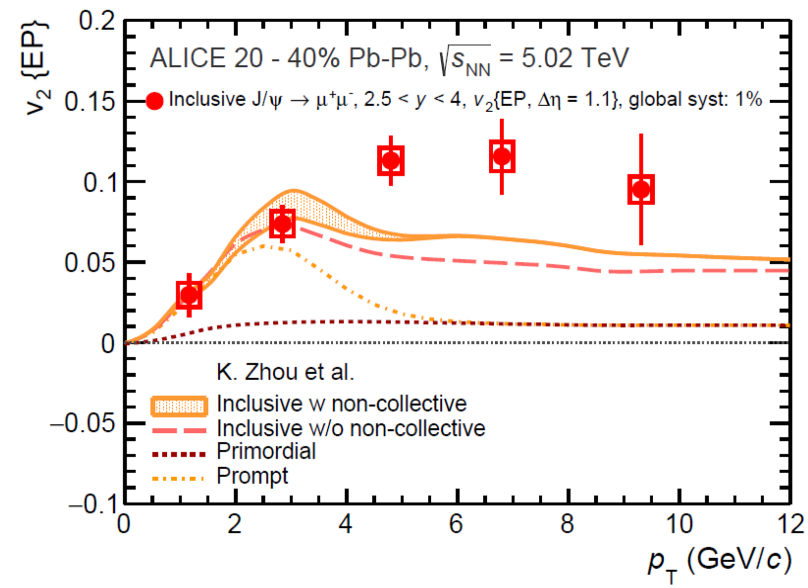
Dense (in phase space) => recombination





Dilute (in phase space)

Alternate possible explanation : p_T-dependent absorption cross section : not excluded, but not favored by the finite v_2 observed for J/ψ by ALICE



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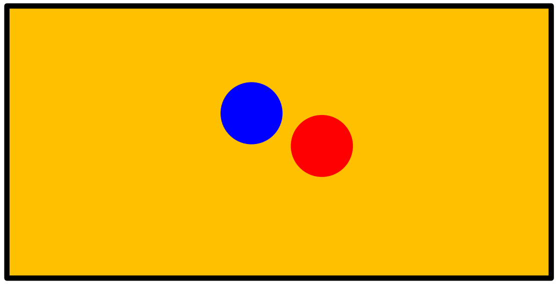

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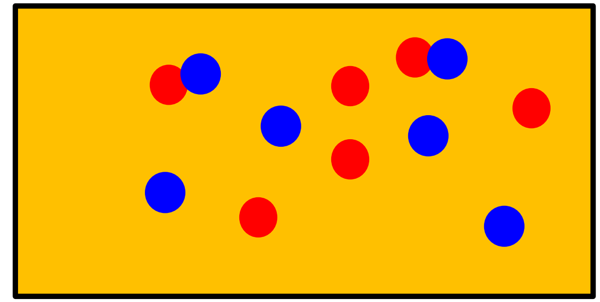

No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states.

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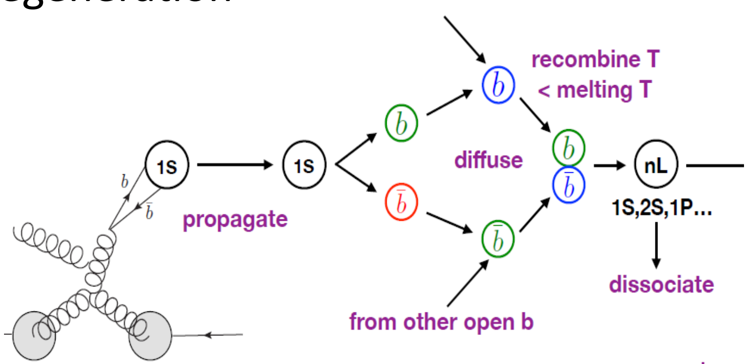
Full quantum treatment affordable

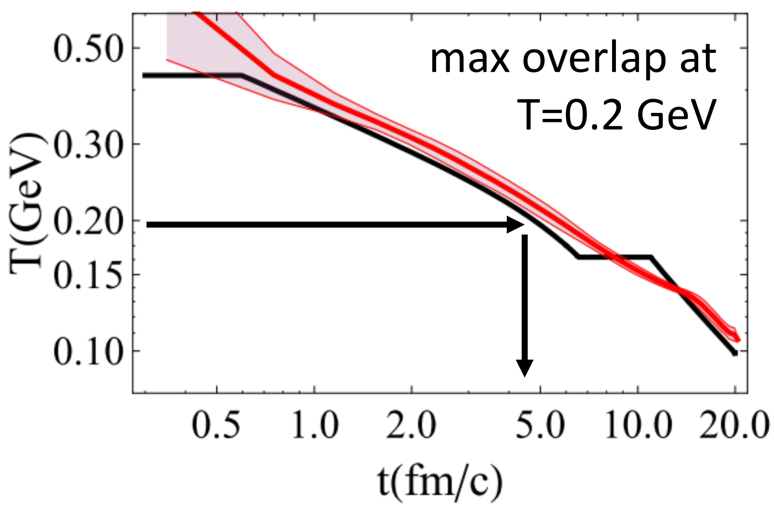
No full quantum treatment possible … but some inspiration from simpler situations…

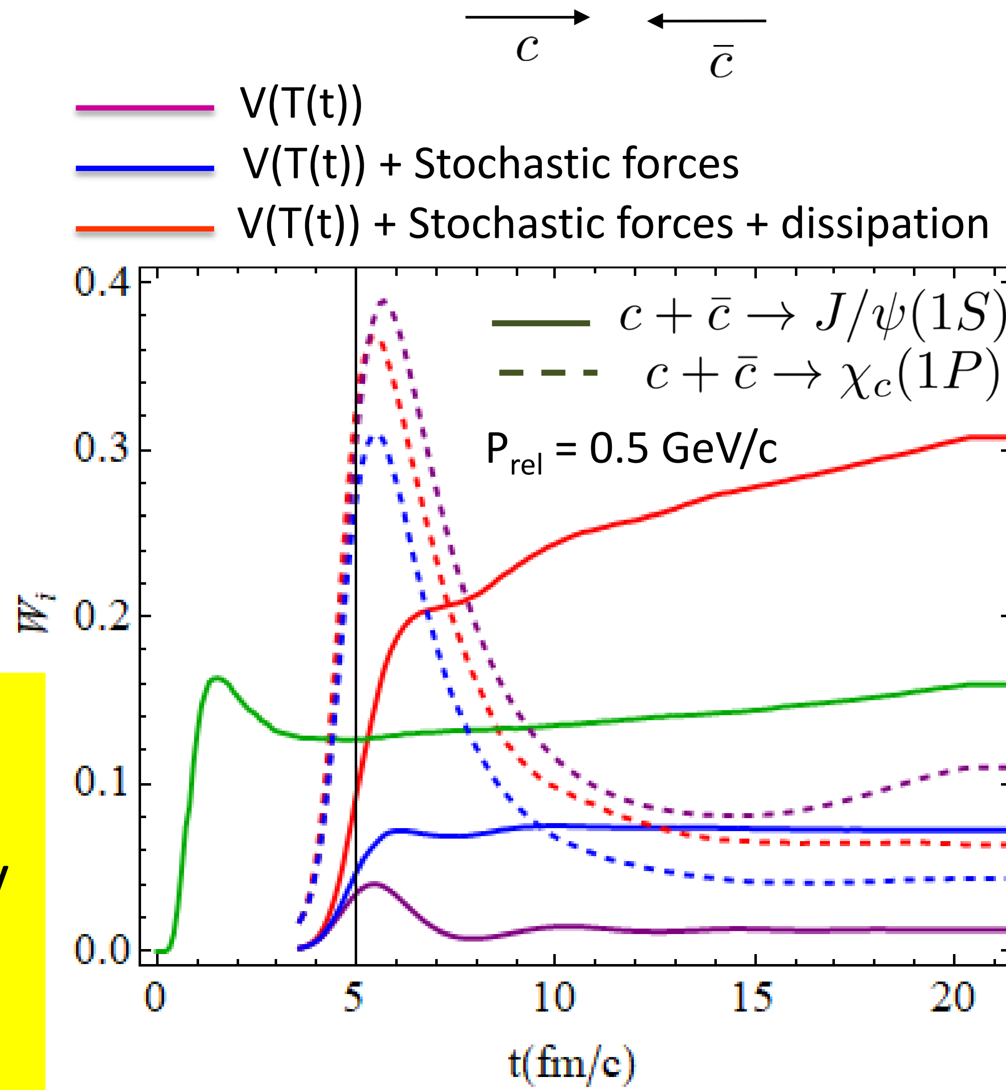
N.B.: In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller (2019)

Stochastic Langevin Equation in *evolving QGP*





✓ Need the full combination (reconfining V(t) + equilibration with environment) to substantially produce lowest state…

✓ Possible cross talk with fragment production at lower energies

Semi-classical treatment through HQ "trajectories" ?

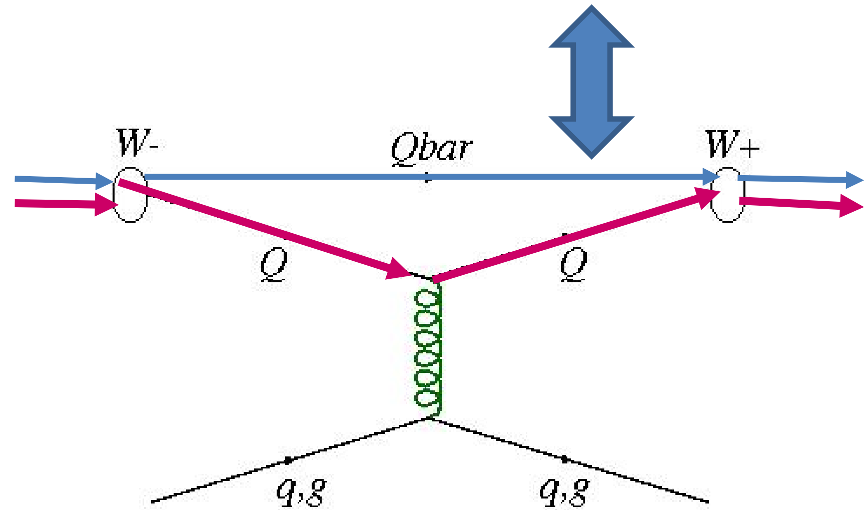
Remler formalism at w

Frankfurt-Nantes Approach
Gossiaux, NED 2022

Combining the expression of the Wigner's functions and substituting in the **effecti rate equation** :

$$\Gamma^\Psi(t) = \sum_{i=1,2} \sum_{j\geq 3} \delta(t-t_{ij}) \int \frac{d^3p_i d^3x_i}{h^3} W^{\Psi}_{Q\bar{Q}}(p_1,x_1;p_2,x_2) \left[W_N(t+\epsilon) - W_N(t-\epsilon)\right]$$

- The quarkonia production in this model is a three body process; the HQs interact only by collisions with the QGP !!!
- The "details" of H_{int} between HQ and bulk partons are incorporated into the evolution of W_N after each collision / time step (nice feature for the MC simulations)
- $W_N(t+\varepsilon)$ and $W_N(t-\varepsilon)$ are NOT the equivalent of gain and loss terms in usual rate equations
- Dissociation and recombination treated in the same scheme



Interaction of HQ with the QGP are carried out by EPOS2+MC@HQ (good results for D and B mesons production)

Then: $P^\Psi(t) = P^\Psi(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$

NB: Also possible to generate similar relations for differential rates

Gossiaux, NED 2022

The 3 layers of the numerical modelling

	Initial state		evolution
Background	EPOS2 initial condition +transverse positions of the NN collisions	→	Fluid dynamics
	↓		↓ {T(x),v(x)}
HQ level	Initialisation of c-cbar pairs according to FONLL[2] ⊗ EPOS2 positions	→	Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**
	↓		↓ $W(p_1,x_1,p_2,x_2)$ ⇔ {Correlated SC trajectories}
Quarkonia level	Initial J/ψ(T) P (t_0) when $T(x) < T_{diss}$	→	Evaluation of Γ according to improved Remler + Γ_{loc}.

We do not have J/ψ quasi particles in our approach, just correlated c-cbar trajectories

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DY Arrebato Villar et al., Phys.Rev.*C* 107 (2023) 5, 054913

Gossiaux, NED 2022

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Obvious possible improvement : revisit with EPOS4 and its c-cbar initial correlations

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DY Arrebato Villar et al., Phys.Rev.*C* 107 (2023) 5, 054913

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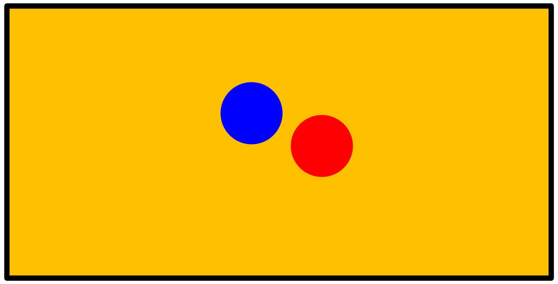

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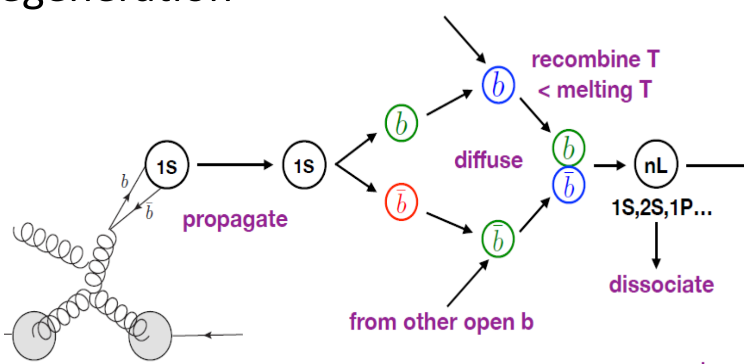

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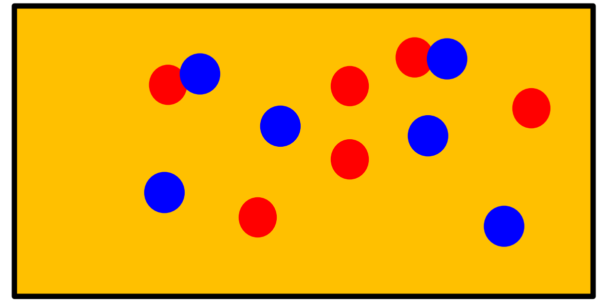

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No full quantum treatment possible => semi-classical approximation (to be specified later)

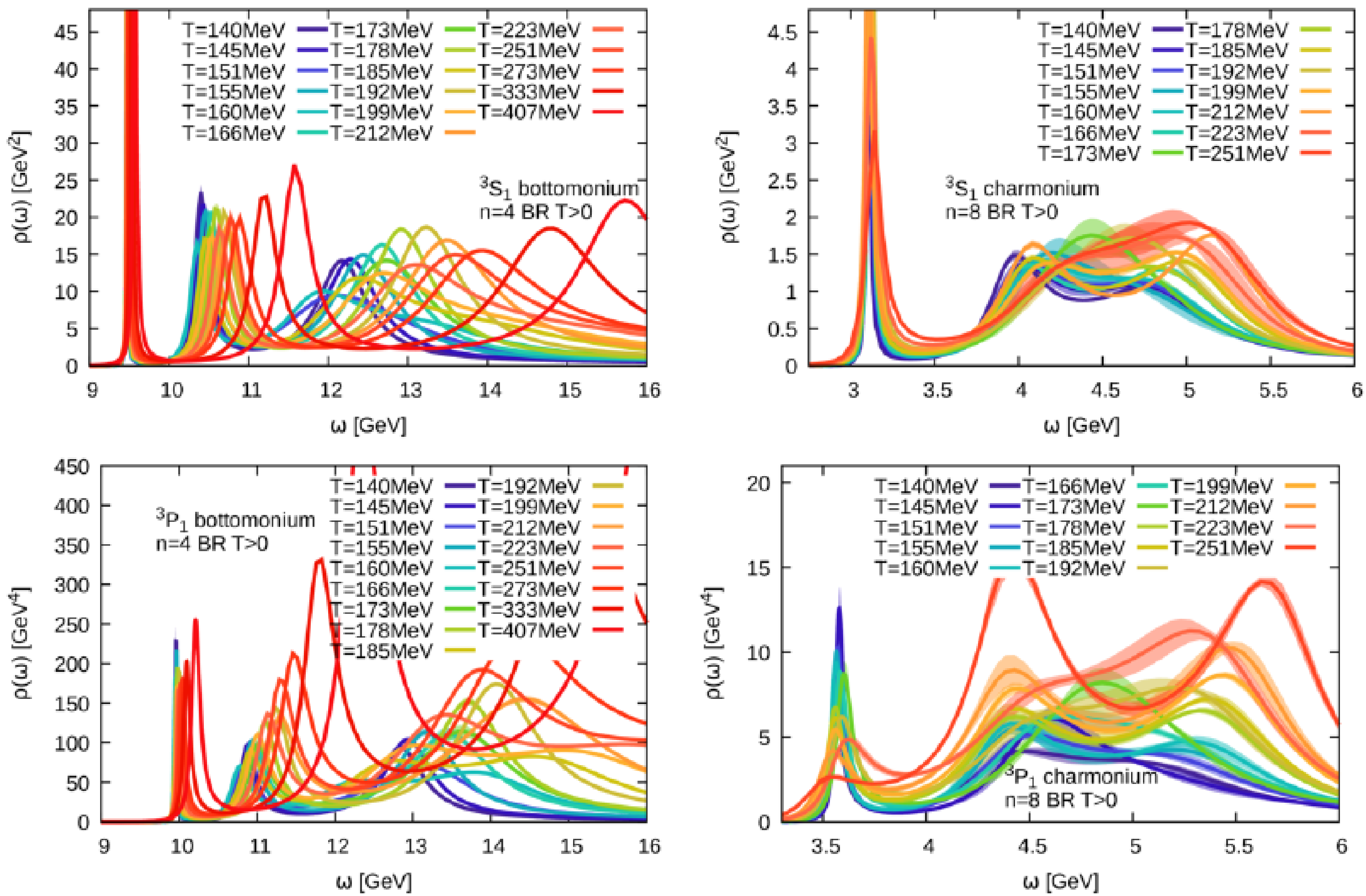
Level of accuracy ?

Structure of the talk

1. Solve the quantum problem in a "simple" situation (single pair)
2. Use this solution to benchmark the semi-classical approximation

What is a quarkonia at finite T ?

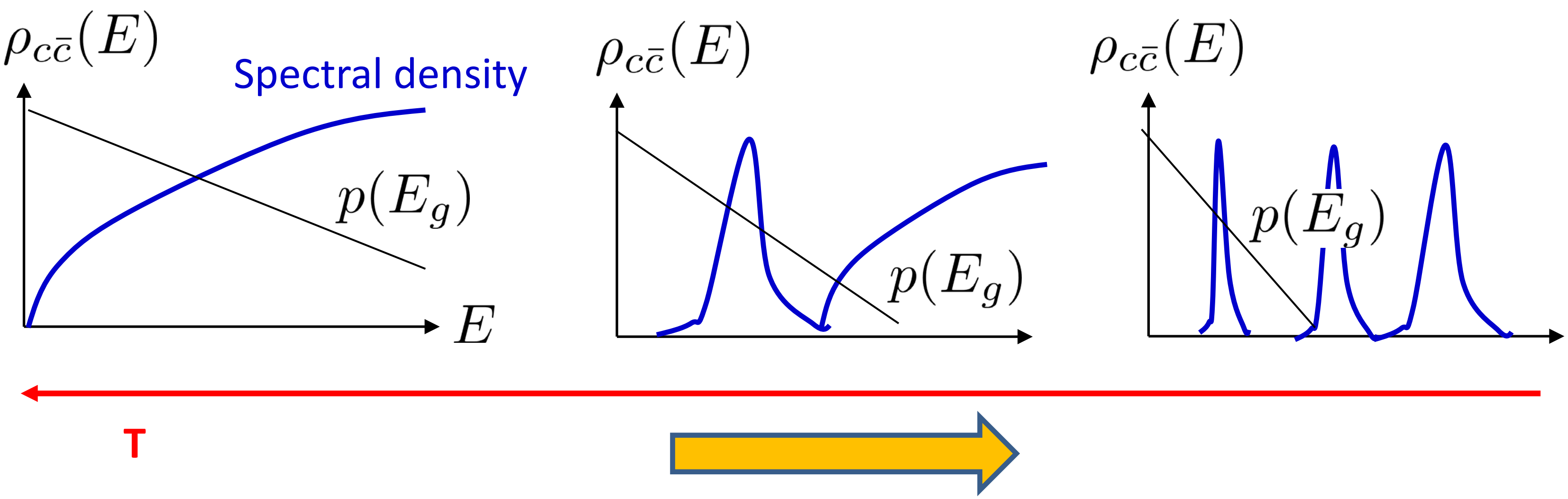
Kim et al, JHEP11(2018)088



Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

a) Screened real potential and b) inelastic interactions with the QGP

Two clear regimes



Time

High Temperature

Multiple scattering on quasi free states

Best d.o.f. : individual heavy quarks

Quantum Brownian Regime

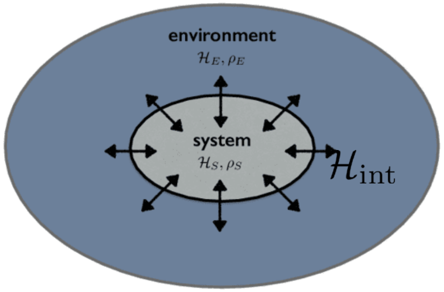
Low Temperature

Dissociation of well identified bound states by scarce “high-energy” gluons (dilute medium => cross section ok)

Best d.o.f. : quarkonia bound states

Quantum Optical Regime

Quantum Master Equations



Quite generally, the system builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}_S^{(0)} + \hat{H}_E + \hat{H}_{\text{int}}$

Von Neumann equation for the total density operator $\hat{\rho}$

System + environment

$$\hat{\rho}(t=0) = \hat{\rho}_S(t=0) \otimes \hat{\rho}_E$$

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}]$$

Evolution of the total system

$$\hat{\rho}(t) = \hat{U}(t,0)\left[\hat{\rho}_S(t=0) \otimes \hat{\rho}_E\right]\hat{U}^\dagger(t,0)$$

Trace out environment degrees of freedom =>
Reduced density operator $\hat{\rho}_S^{\text{red}}$

System

$$\hat{\rho}_S^{\text{red}}(t=0) = \hat{\rho}_S(t=0)$$

Evolution of the system

$$\hat{\rho}_S^{\text{red}}(t) = \text{tr}_E\left[\hat{U}(t,0)\left[\hat{\rho}_S(t=0) \otimes \hat{\rho}_E\right]\hat{U}^\dagger(t,0)\right]$$

Evol. eq. on the red. Density: $\frac{\mathrm{d}\hat{\rho}_S^{\text{red}}}{\mathrm{d}t} = \mathcal{L}[\hat{\rho}_S^{\text{red}}]$ (linear mapping)

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time

Non abelian Quantum Master Equation for a $Q\bar{Q}$ pair

$$\hat{\rho}_S = \mathcal{D}_{\rm s}|1\rangle\langle 1| + \mathcal{D}_{\rm o}\sum_a |o_a\rangle\langle o_a|$$

2 coupled color representations (singlet octet)

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

The Linblad Operator contains various terms representing several aspects of HQ physics

Unitary dynamics
- $\mathcal{L}_0$: kinetic term
- $\mathcal{L}_1$: (screened) real potential term

Imaginary potential W → Non-Unitary dynamics
- $\mathcal{L}_2$: fluctuations => heating and decoherence
- $\mathcal{L}_3$: dissipation
- $\mathcal{L}_4$: mandatory to preserve positivity (but sub-dominant)

J-P.Blaizot & M.A. Escobedo, JHEP 2018.6 (2018)

Sketch of the appearance of an imaginary part to V

$$\frac{d}{dt}\hat{\rho}_S^{\mathrm{red}}(t) = -i[\hat{H}_S^{(0)}, \hat{\rho}_S^{\mathrm{red}}] + \sum_i \gamma_i \left[\hat{L}_i \hat{\rho}_S^{\mathrm{red}} \hat{L}_i^\dagger - \frac{1}{2}\left(\hat{L}_i^\dagger \hat{L}_i \hat{\rho}_S^{\mathrm{red}} + \hat{\rho}_S^{\mathrm{red}} \hat{L}_i^\dagger \hat{L}_i\right)\right]$$

$$\hat{H} = \hat{H}_S + \hat{H}_{\mathrm{sto}}$$

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] \;\Rightarrow\; \hat{\rho}(t+dt) = \hat{\rho}(t) + dt\,\cancel{\frac{d\hat{\rho}}{dt}} + \frac{dt^2}{2}\,\frac{d^2\hat{\rho}}{dt^2}$$

0 stochastic average

At 2nd order : $$\frac{d^2\hat{\rho}}{dt^2} = \frac{1}{\hbar^2}\left(2\hat{H}_{\mathrm{sto}}\hat{\rho}\hat{H}_{\mathrm{sto}} - \hat{H}_{\mathrm{sto}}^2\hat{\rho} - \hat{\rho}\hat{H}_{\mathrm{sto}}^2\right)$$

$$\frac{d^2\langle x|\hat{\rho}|x'\rangle}{dt^2} = \frac{1}{\hbar^2}\left(2\underbrace{\hat{H}_{\mathrm{sto}}(x)\langle x|\hat{\rho}|x'\rangle\hat{H}_{\mathrm{sto}}(x')}_{W(x-x')/dt} - \underbrace{\hat{H}_{\mathrm{sto}}^2(x)}_{W(x-x)/dt}\langle x|\hat{\rho}|x'\rangle - \langle x|\hat{\rho}|x'\rangle\underbrace{\hat{H}_{\mathrm{sto}}^2(x')}_{W(x'-x')/dt}\right)$$

$$\frac{d\rho(x,x')}{dt} = \cdots - 2\underbrace{\left(W(x-x') - W(0)\right)}_{\Gamma(x-x')}\rho(x,x')$$

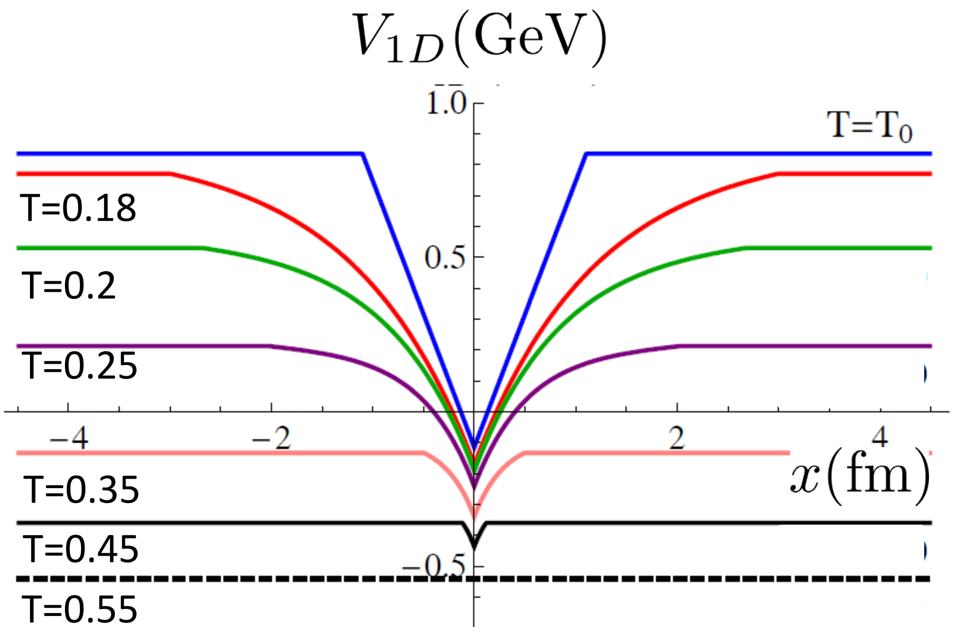
Further implementation features

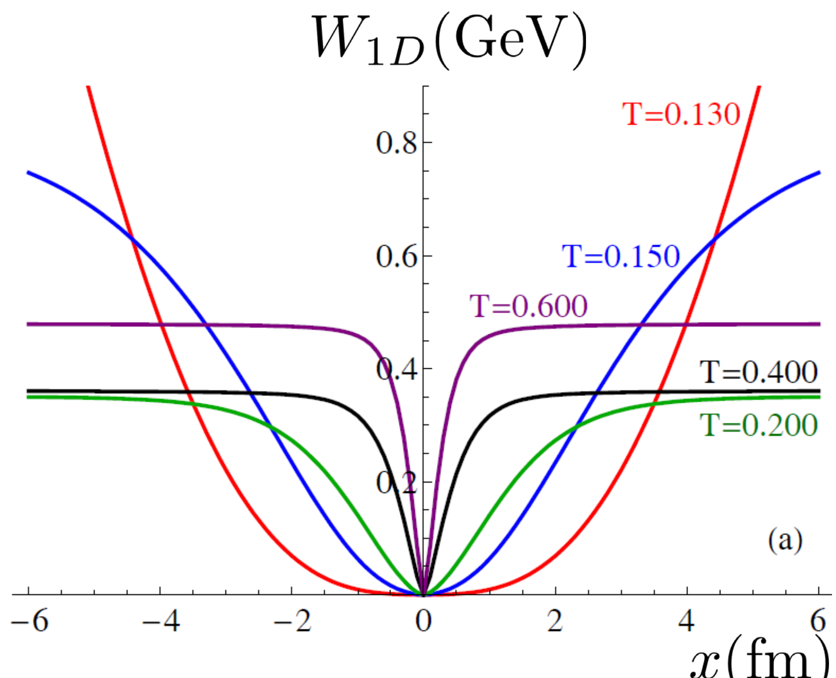
➢ 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths)



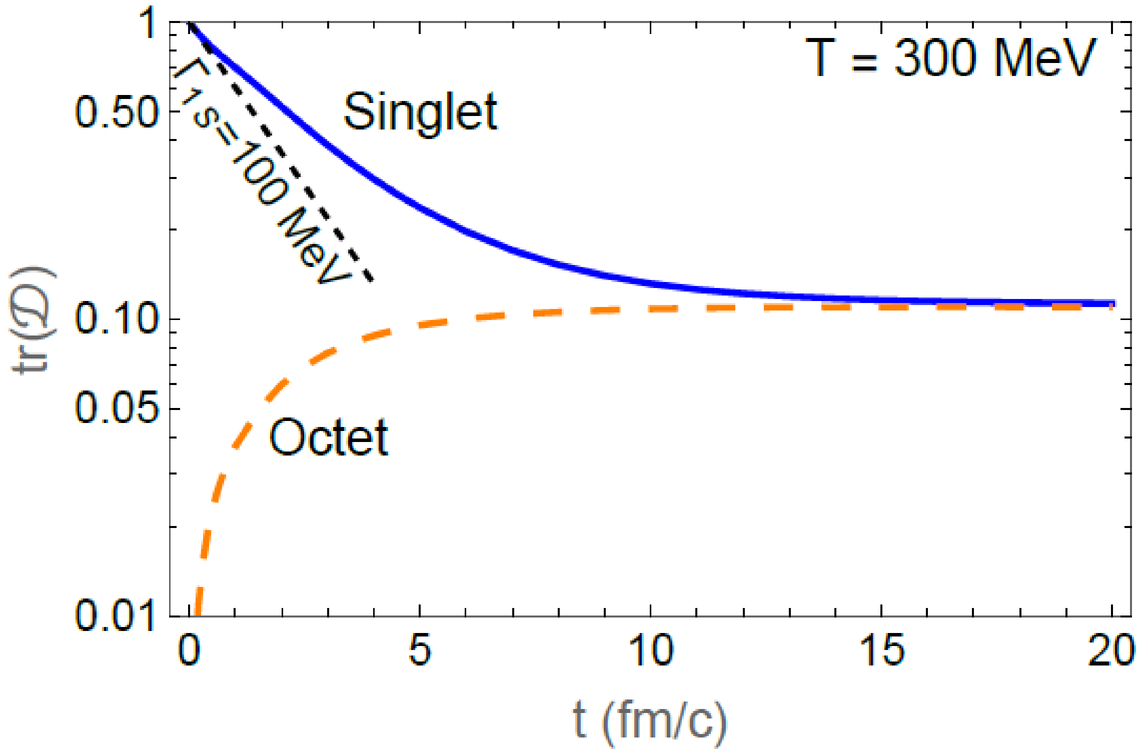


1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Some selected results for 1 $c\bar{c}$ pair

Color Dynamics : Singlet – octet probabilities:

- Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{eq} = D_o^{eq} = \frac{1}{9}$ $(1+8) \times \frac{1}{9}$

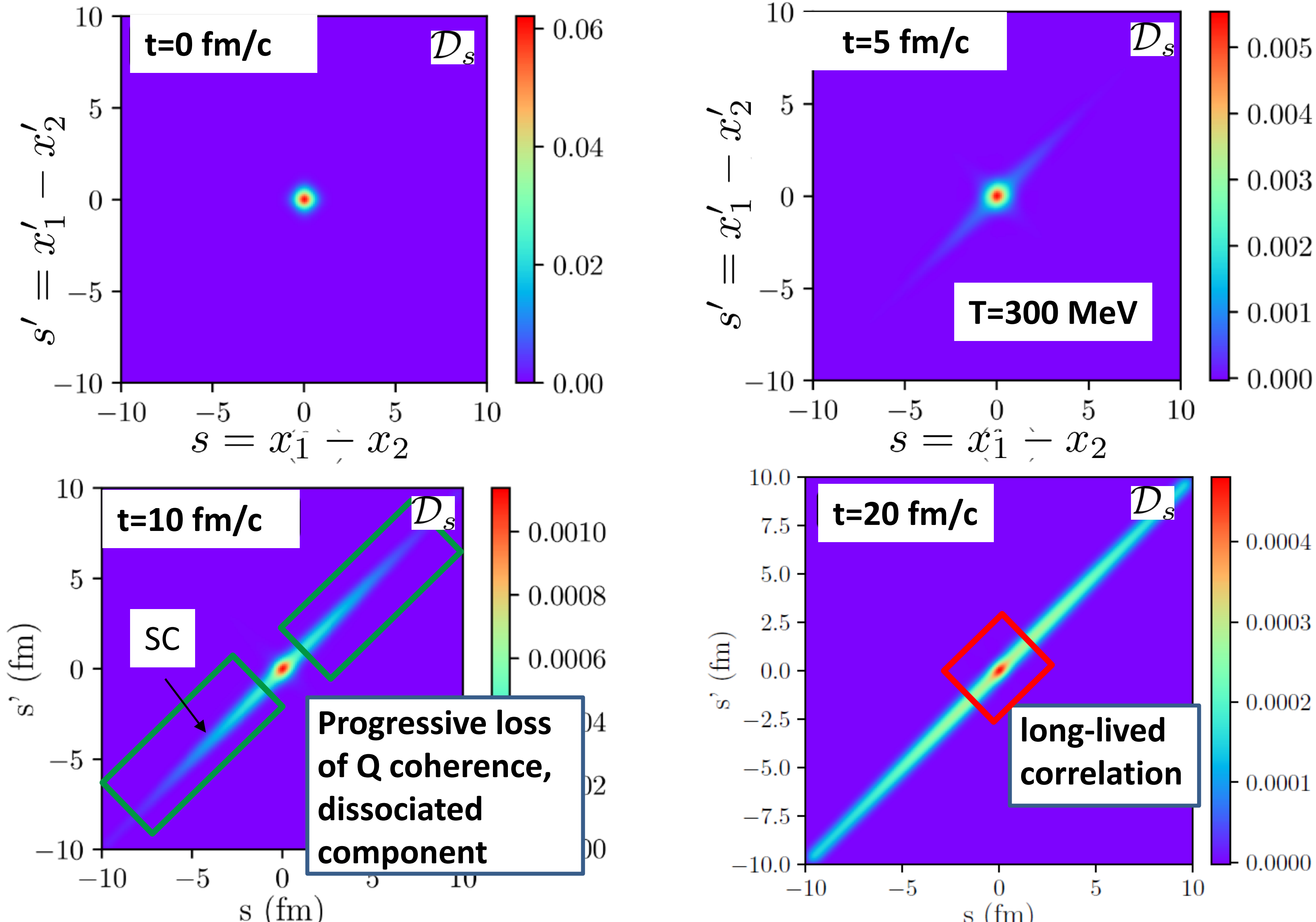


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx$ 2 fm/c
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.
- C-cbar can interact with the surrounding QGP as an octet => energy loss

S.Delorme et al. JHEP 06 (2024) 060

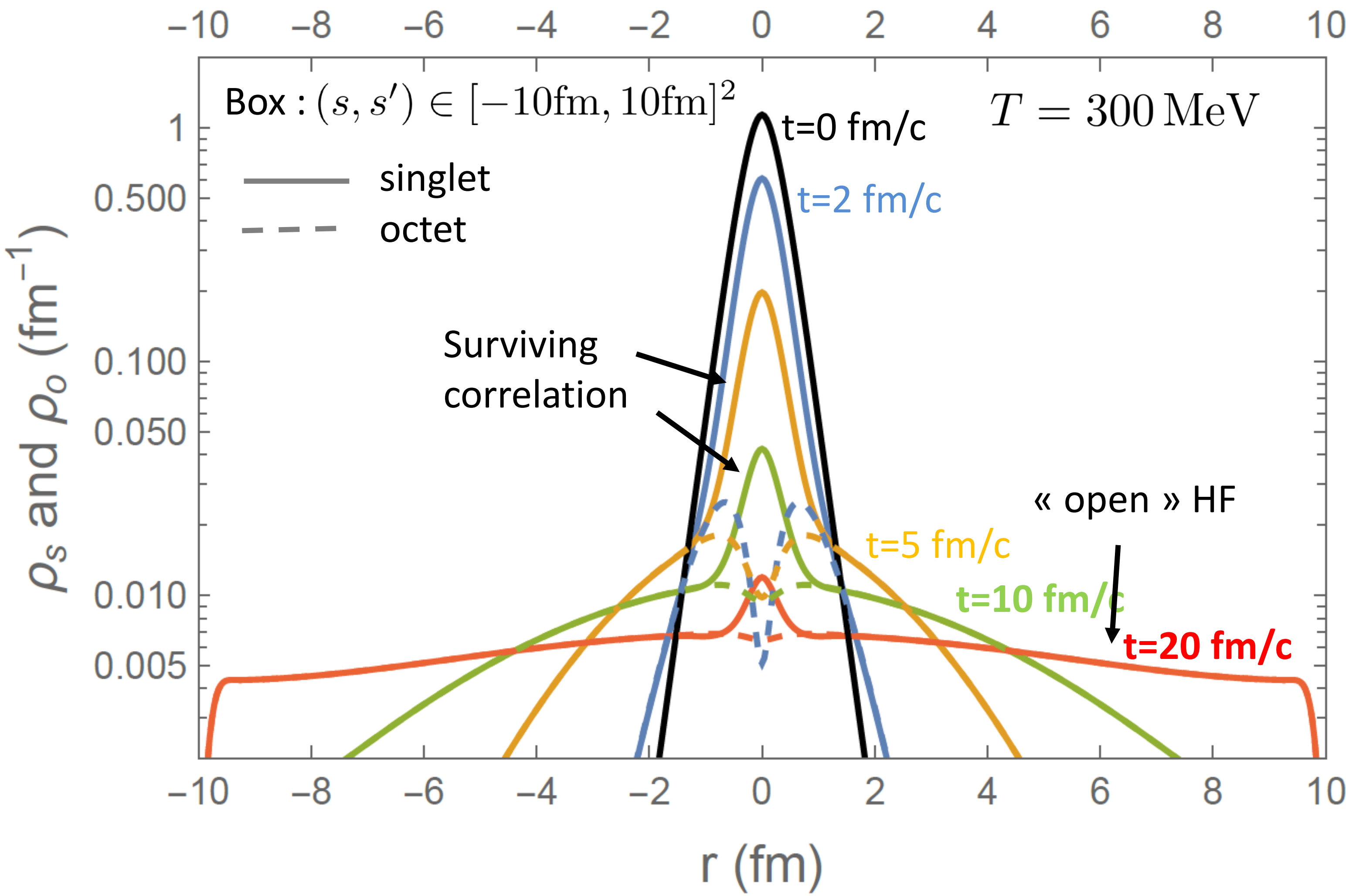
Evolution of the Density matrix

1S singlet initial state:



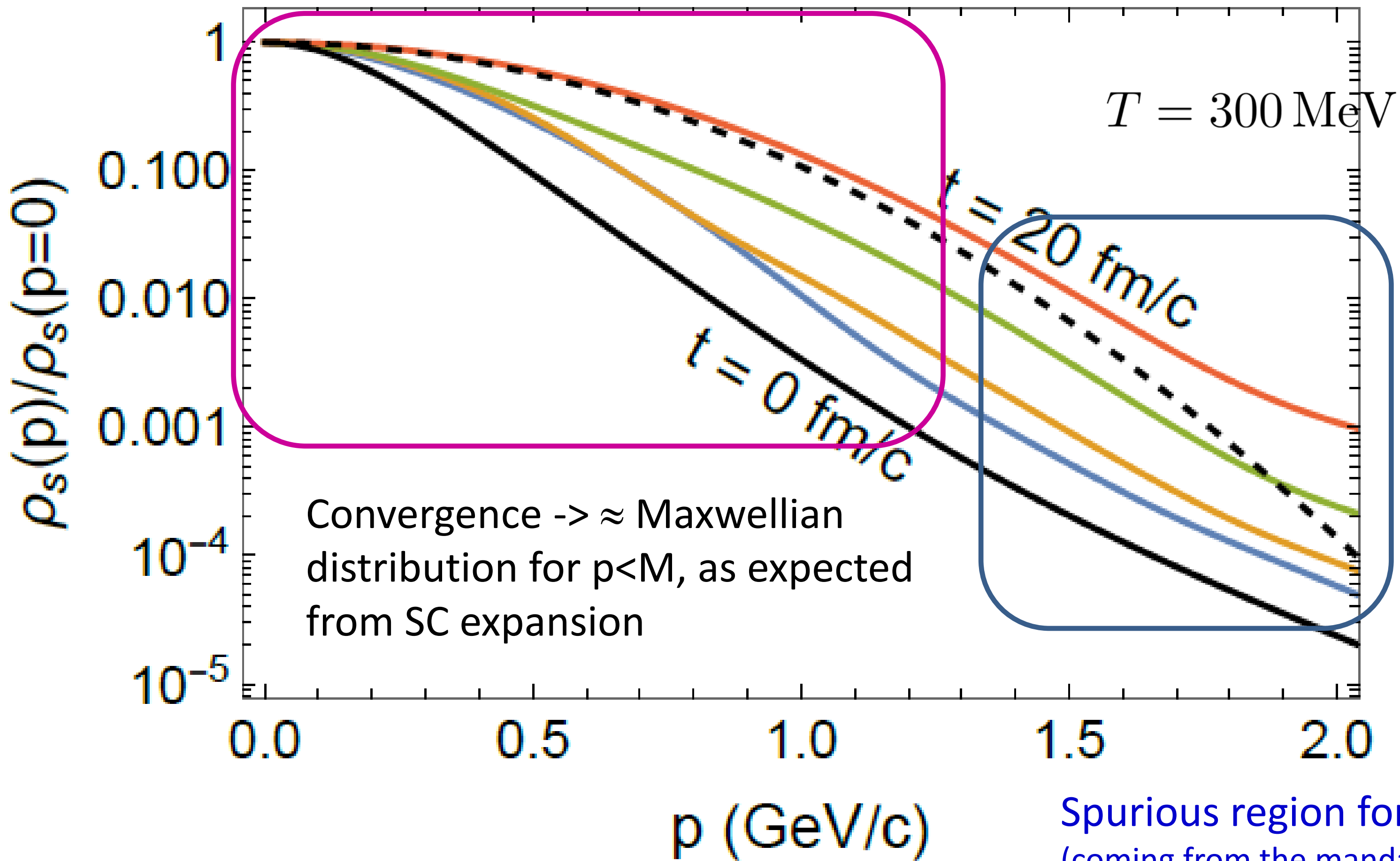
Evolution of the spatial density

1S singlet initial state:



Some c-cbar stay at intermediate distance (“recombination”) … remaining peak in the asymptotic distribution

Evolution of the momentum density

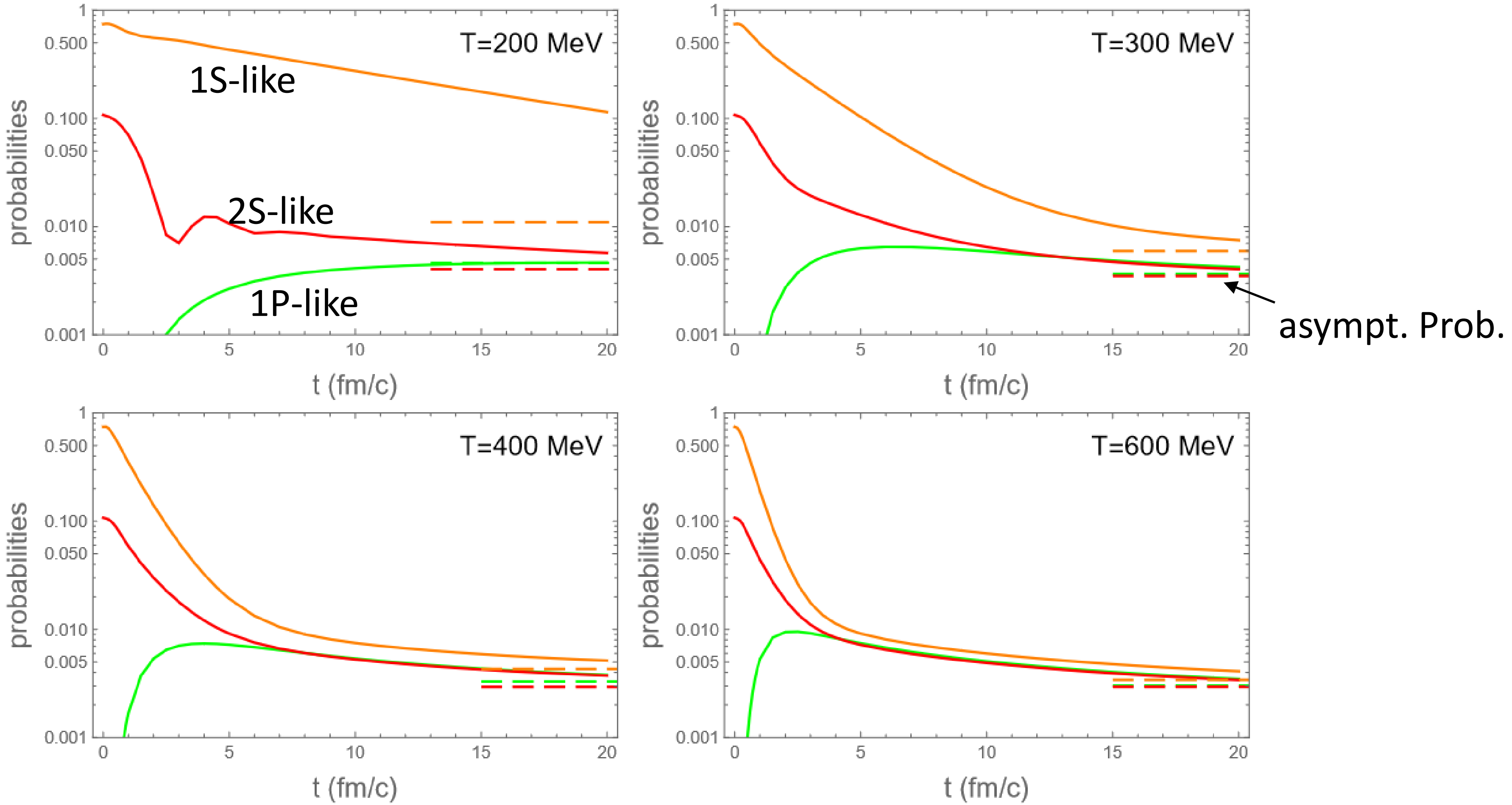


Spurious region for p>M
(coming from the mandatory regularization of the imaginary potential…room for improvement)

Mostly sensitive to the distribution at large relative distance (individual c quarks)

Results for projection on vacuum states

Starting from a compact S-like state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.165\,\mathrm{fm}$ $p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$

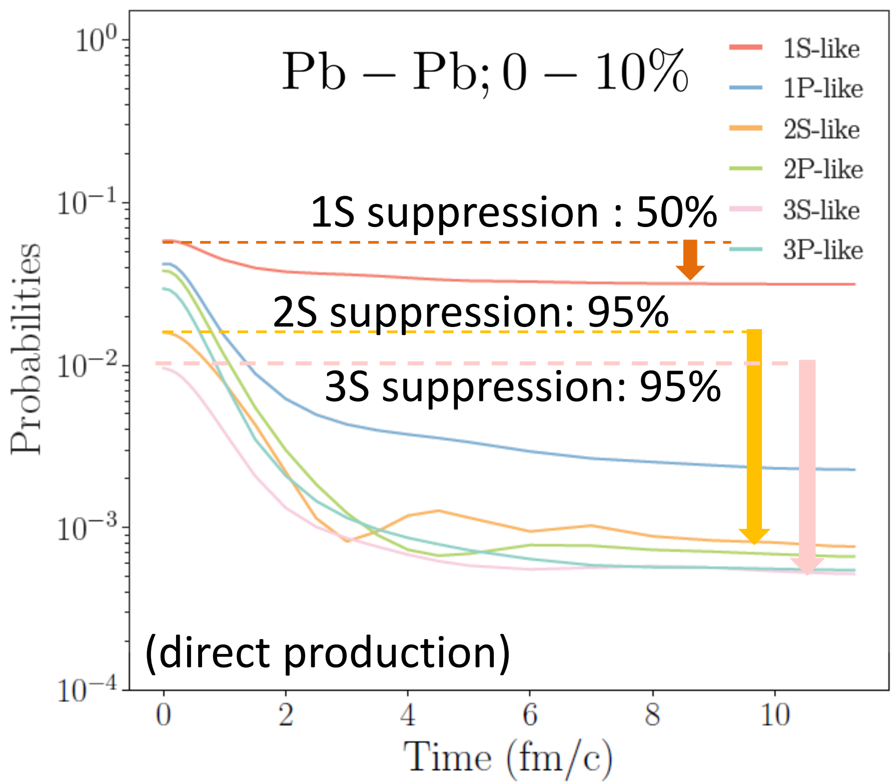


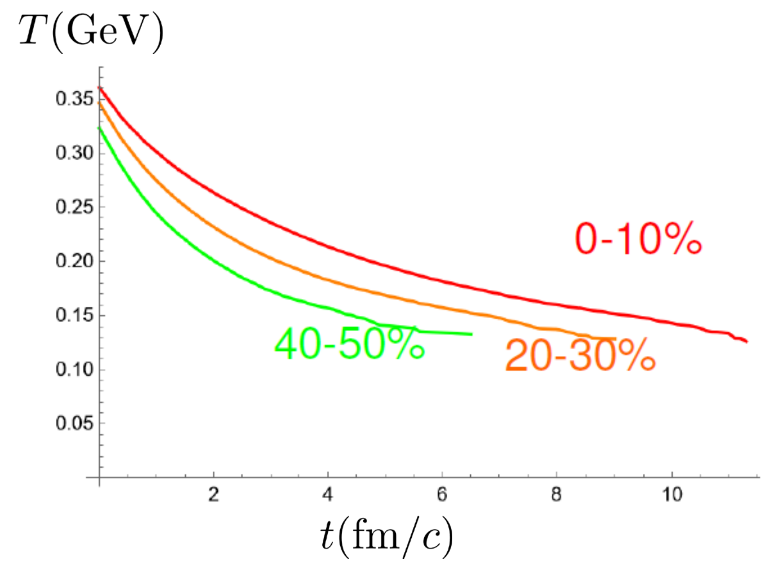
- Natural evolution for 1S-like suppression, from low to high T
- 2S state do not decay $\alpha e^{-\Gamma_{2S} t}$ at early time… partly driven by the ground state at later time.

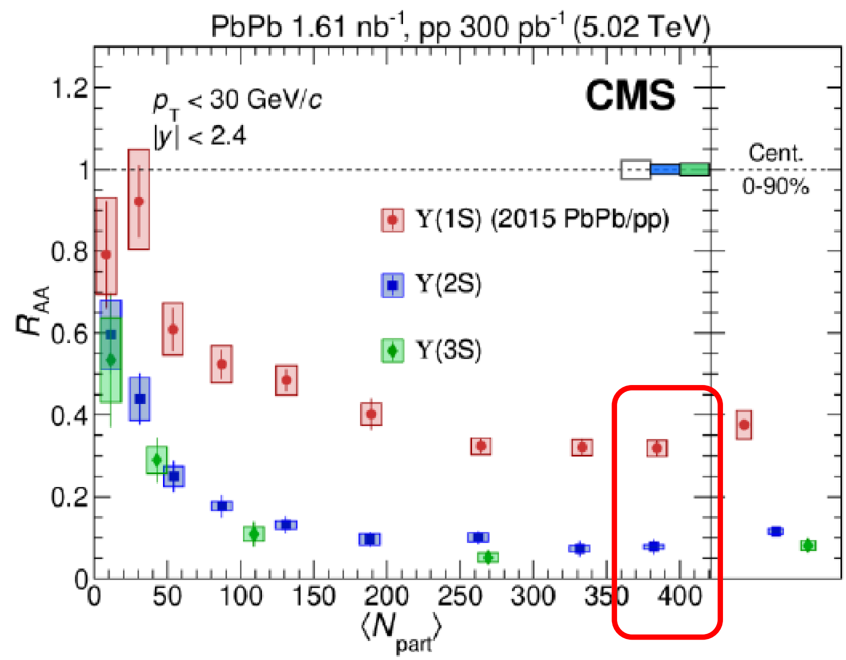
Contact with experiment ($b\bar{b}$)

- Bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.

$$\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}\left(1 + a_{\text{odd}}\frac{s}{\sigma}\right) \qquad \sigma = 0.045\,\text{fm}\; a_{\text{odd}} = 3.5$$







- See Stephane Delorme's talk at SQM24 for more details.

Semiclassical approximation

… and now, we will consider the semi-classical evolution of the lowest $Q\bar{Q}$ bound state

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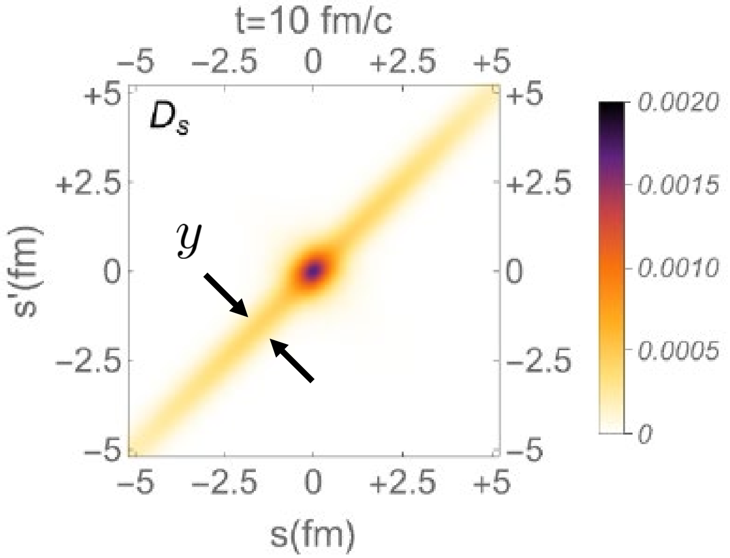
SC = limit of small $\hbar$ ⬄ large action of the system… ok for ground state ?

Semiclassical approximation

- For the relative motion (2 body): $\left.\begin{array}{l}\vec{s}=\vec{x}_1-\vec{x}_2\\ \vec{s}\,'=\vec{x}_1'-\vec{x}_2'\end{array}\right\}\ \vec{r}=\frac{\vec{s}+\vec{s}\,'}{2}$ and $\vec{y}=\vec{s}-\vec{s}\,'$



$$\frac{d}{dt}\mathcal{D}(r,y)=\mathcal{L}\mathcal{D}(r,y)$$

Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** of the Linblad equation: power series in *y* up to 2nd order)

- Wigner transform : $\mathcal{D}(\vec{r},\vec{y})\rightarrow W(\vec{r},\vec{p})$ and $\{\vec{y},\nabla_y\}\rightarrow\{\nabla_p,\vec{p}\}$
- => Usual Fokker Planck equation :

fluctuations dissipation

$$\frac{\partial W}{\partial t}=\left[-\frac{2\vec{p}\cdot\nabla_r}{M}-\nabla_r V\cdot\nabla_p+\frac{\eta(r)}{2}\nabla_p^2+\frac{\gamma(r)}{M}\nabla_p\cdot\vec{p}\right]W$$

- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

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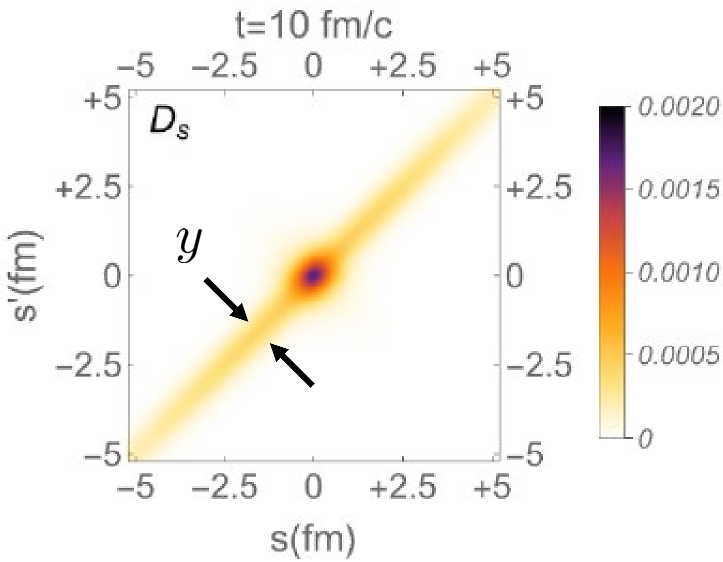


$$\frac{d}{dt}\mathcal{D}(r,y)=\mathcal{L}\mathcal{D}(r,y)$$

Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** of the Linblad equation: power series in *y* up to 2nd order)

- When / why does it work ?
  - The unitary term : $\mathcal{L}_1[\rho]=[V,\rho]=\rho(s,s')(V(s)-V(s'))=V'(r)y+\mathcal{O}(y^3)$

    Wigner-Moyal expansion, valid when y << variation scale of the real potential

  - The interaction with the environment : L_2

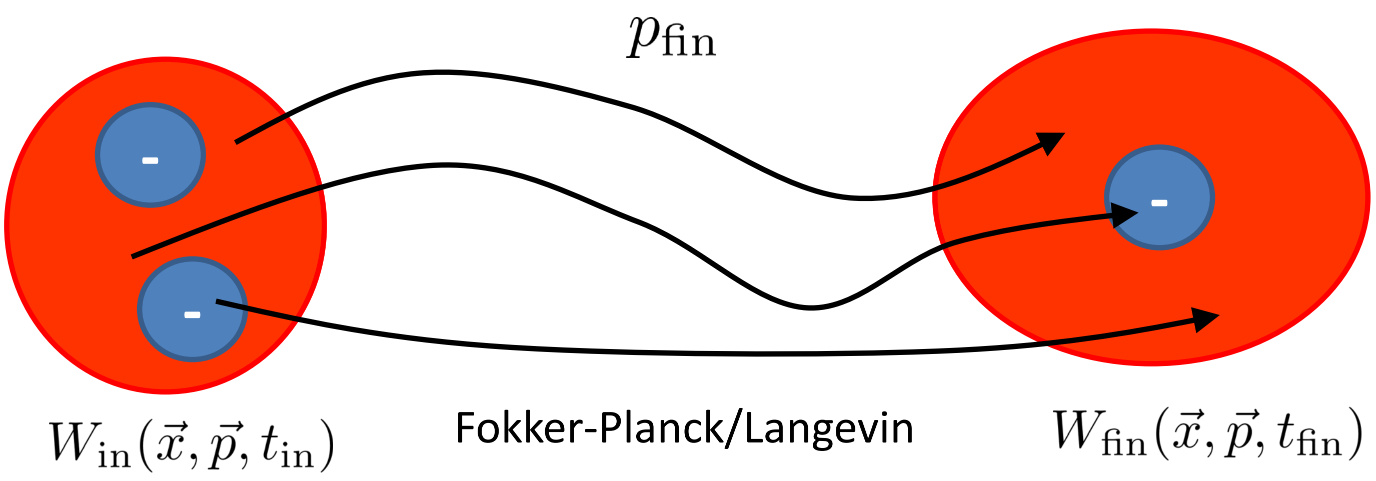
$$\Gamma(y)\rho(s,s')\approx\Gamma''(0)y^2\times\left(1+\mathcal{O}(y^2m_D^2)\right)\rho(s,s')$$

$$\Gamma''(0)\partial_p^2W(r,p) \qquad \approx\frac{T}{m_Q}\ll 1$$

Classical noise

Quantum vs SC dynamics

- SCA : linear mapping



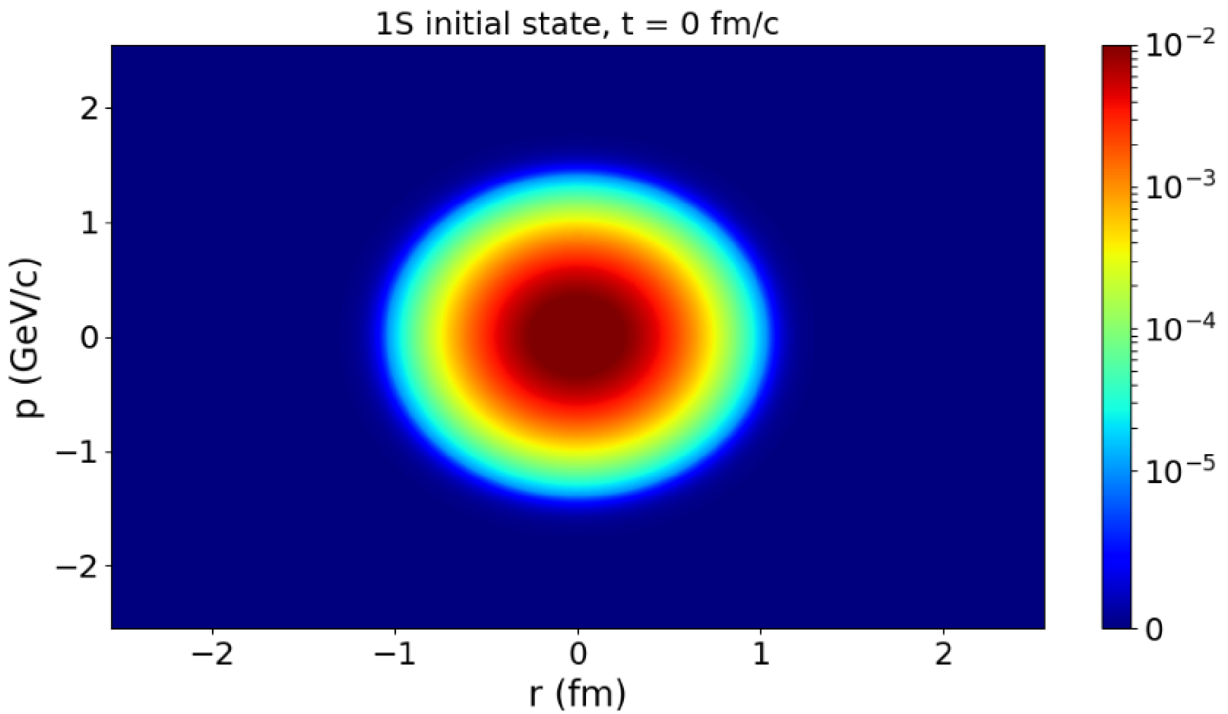
Always positive defined even if W_{in} and W_{fin} are not positive defined

- Several aspects :
  - Temperature
  - Initial state
  - Property considered

In the following : only a limited set of results; manuscript to come soon

Concrete implementation

- 1D (same as for the QME), **1 $c\bar{c}$ pair**
- Same real potential, W in the QME ⬄ η and γ in the FP
- **Abelian case** (for the time, not clear how to deal with the singlet <-> octet transition in a semiclassical approach)
- Yet, not trivial…
- Initial state vacuum 1S state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.38\,\mathrm{fm}$

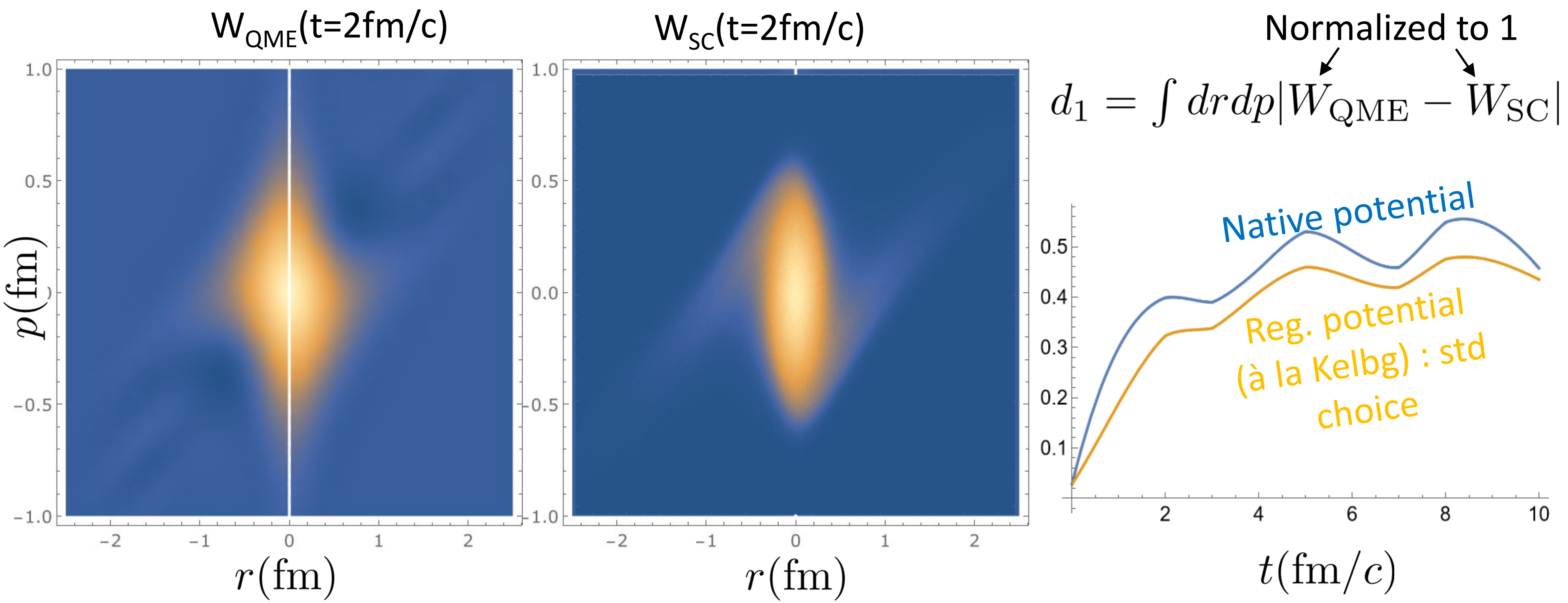


Unitary evolution

Unitary ⇔ no fluctuation and dissipation by coupling with the QGP

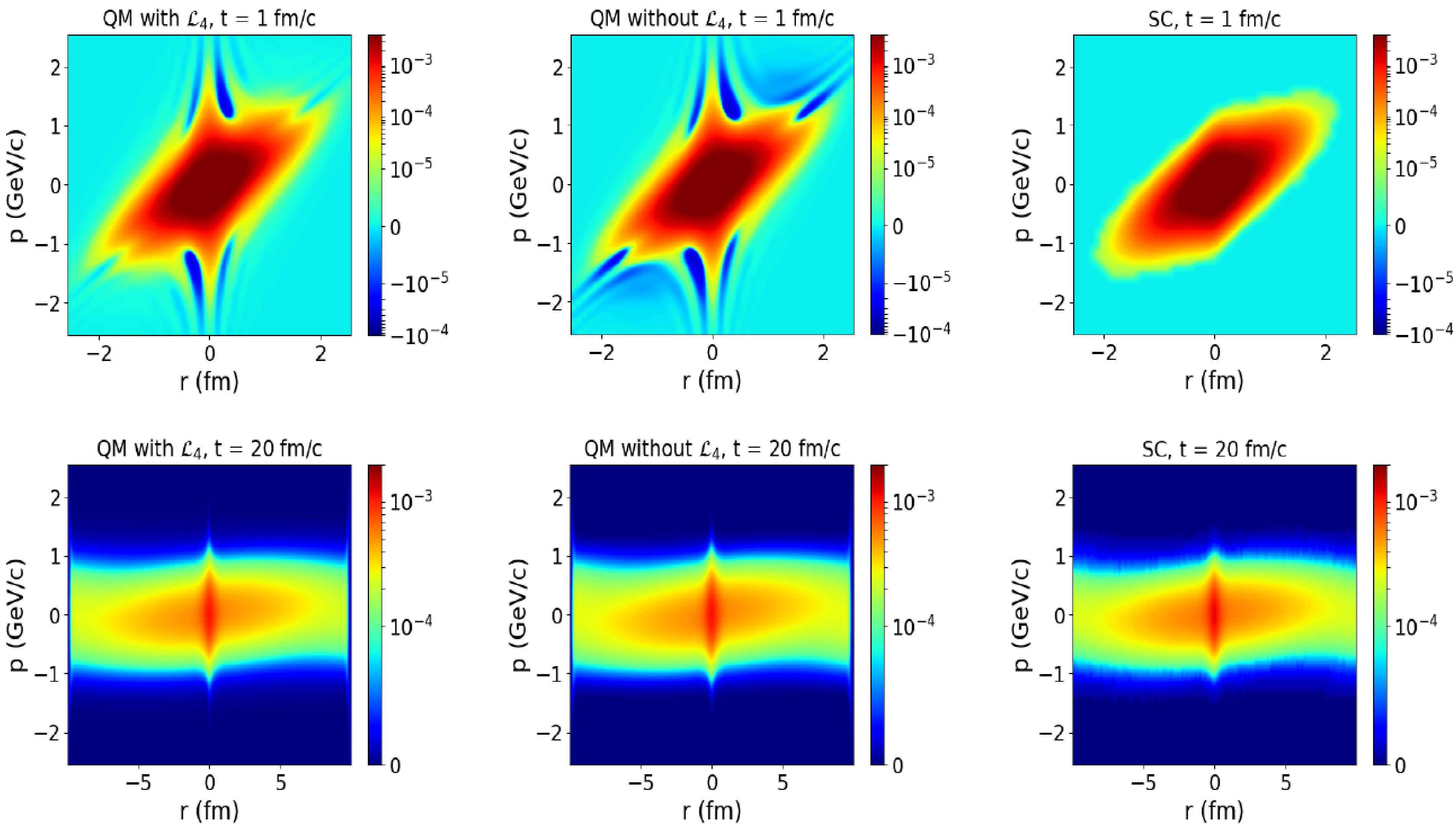
evolution of a vacuum 1S state in a screened V (T=200MeV) … still some evolution

$$H(T)|\psi\rangle_0 \neq E|\psi_0\rangle$$



Non-unitary evolution

Full coupling with the QGP

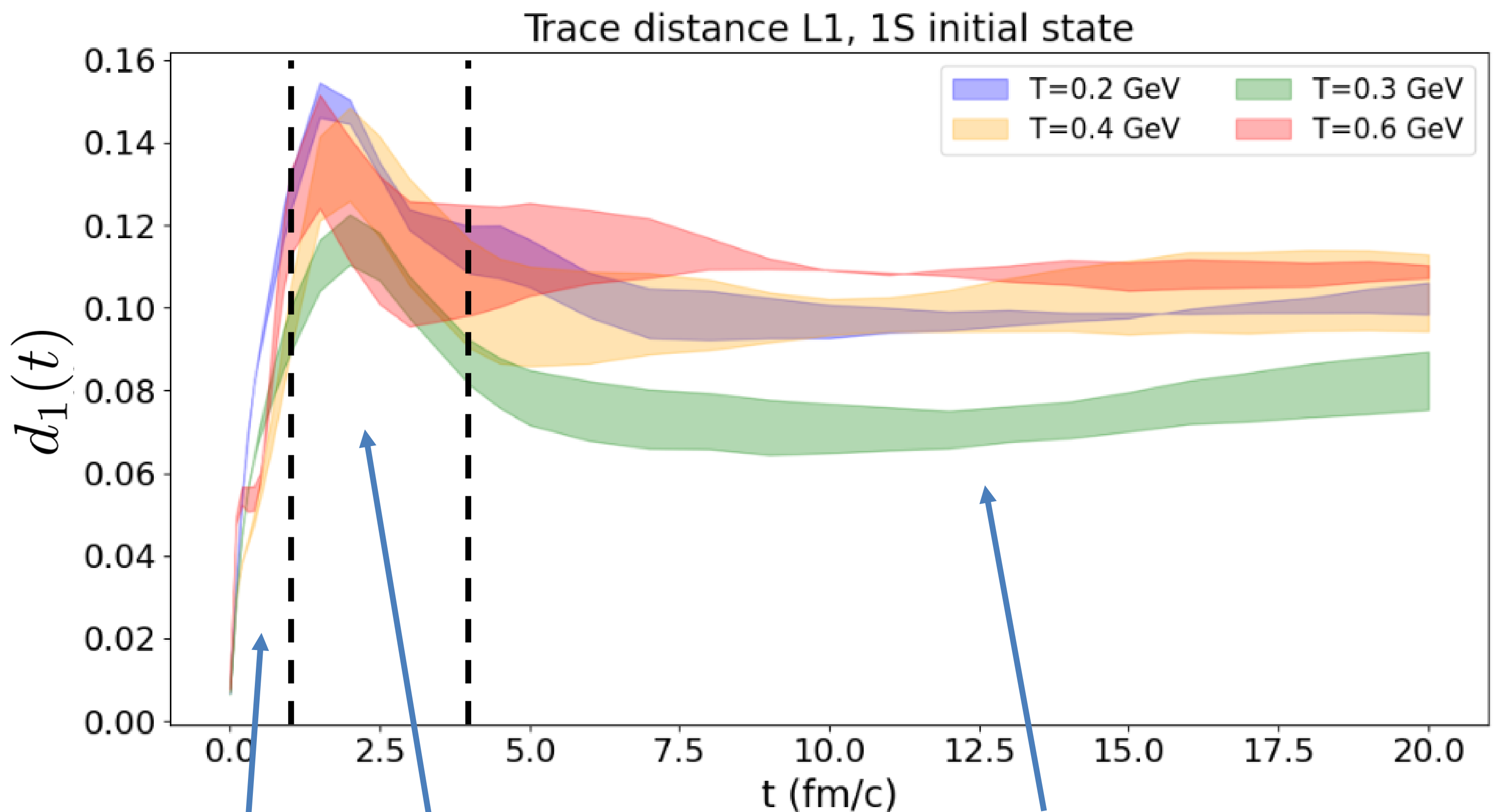


- Some specific signs of genuine QM evolution at small time
- Better agreement between the two descriptions at late times.

Non-unitary evolution

Full coupling with the QGP

$$d_1 = \int dr dp |W_{\mathrm{QME}} - W_{\mathrm{SC}}|$$



I. Growth of the ≠ between QME and SC evolution : genuine QM features

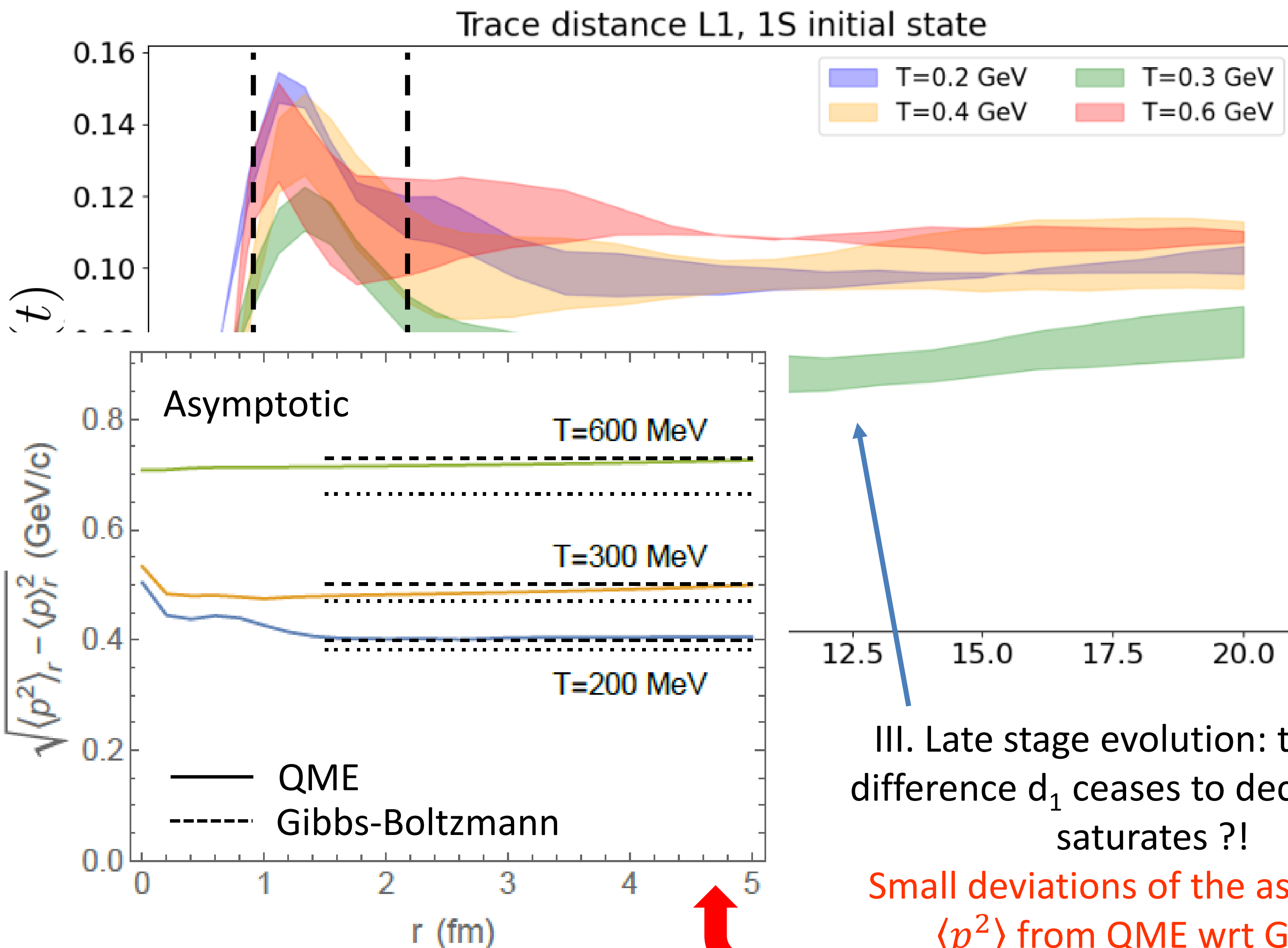
II. Saturation and decrease of the ≠ between QME and SC evolution : **classicalization**

III. Late stage evolution: the norm difference d_1 ceases to decrease and saturates ?!

Non-unitary evolution

Full coupling with the QGP

$$d_1 = \int dr dp |W_{\text{QME}} - W_{\text{SC}}|$$



I. (
betv
evc

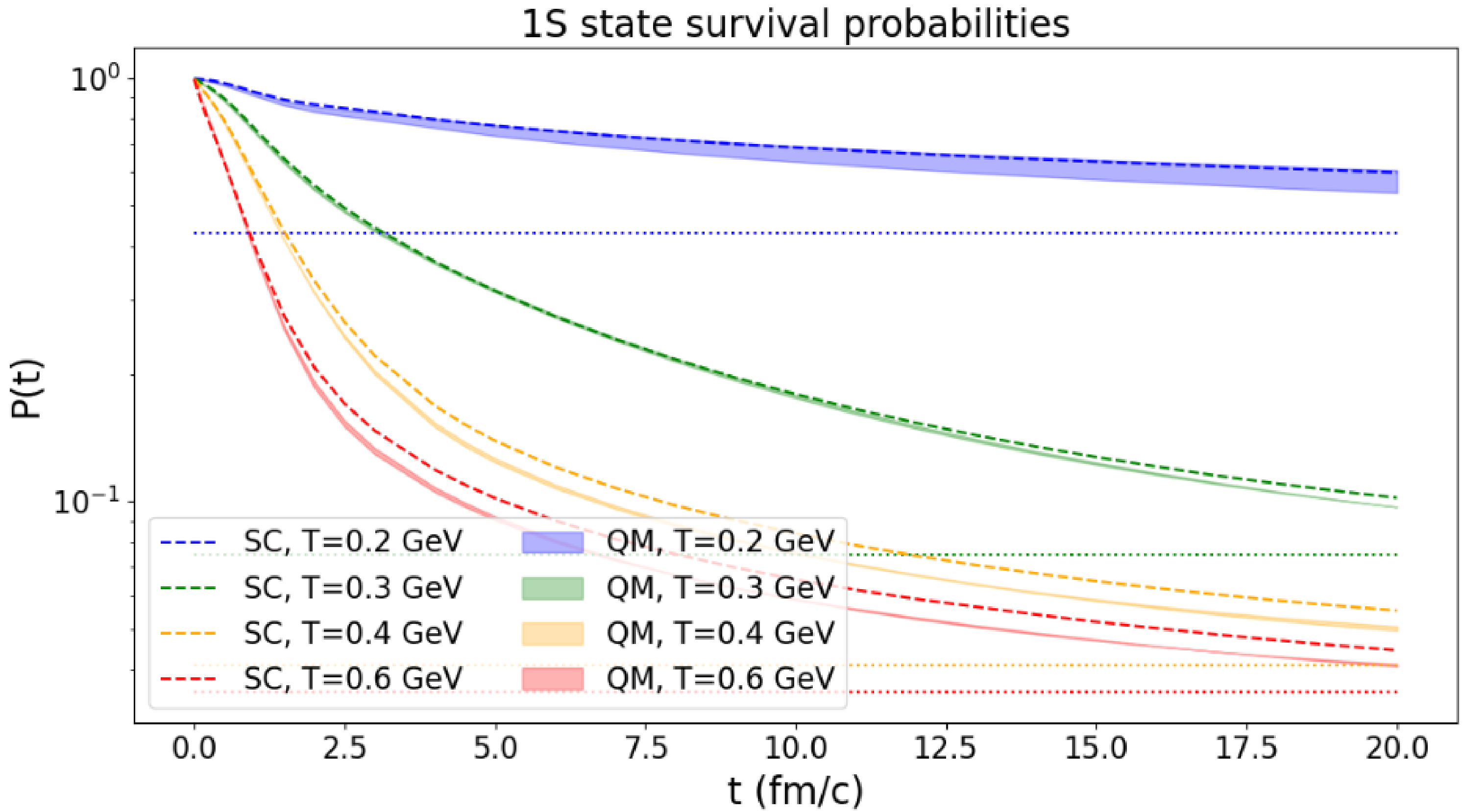
III. Late stage evolution: the norm difference d_1 ceases to decrease and saturates ?!

Small deviations of the asymptotic $\langle p^2 \rangle$ from QME wrt Gibbs-Boltzmann

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **Survival probability of the 1S initial state**:

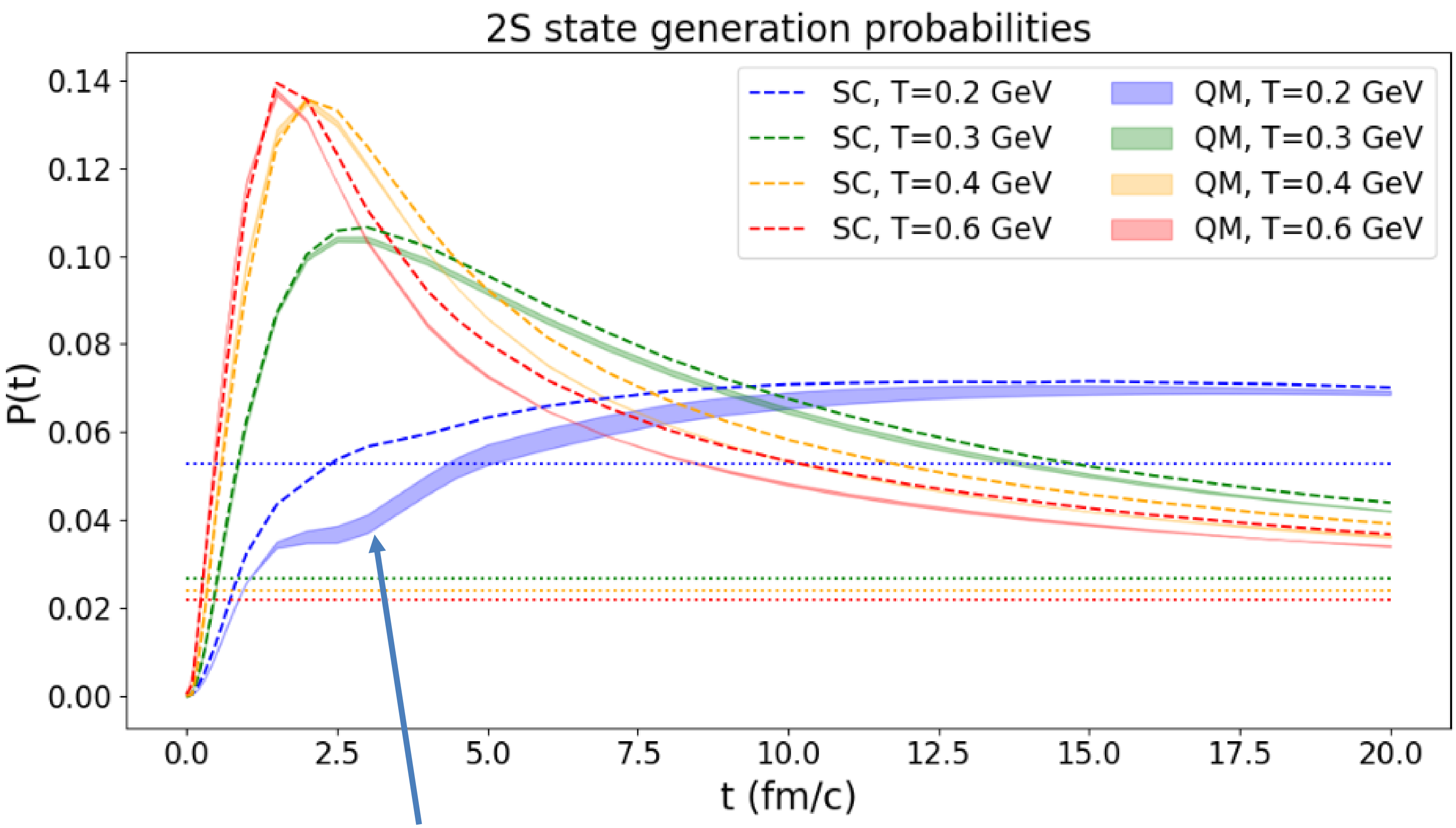


- Good agreement of the SC calculation with the QME benchmark
- Slight over suppression for the QME (overheating)

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T

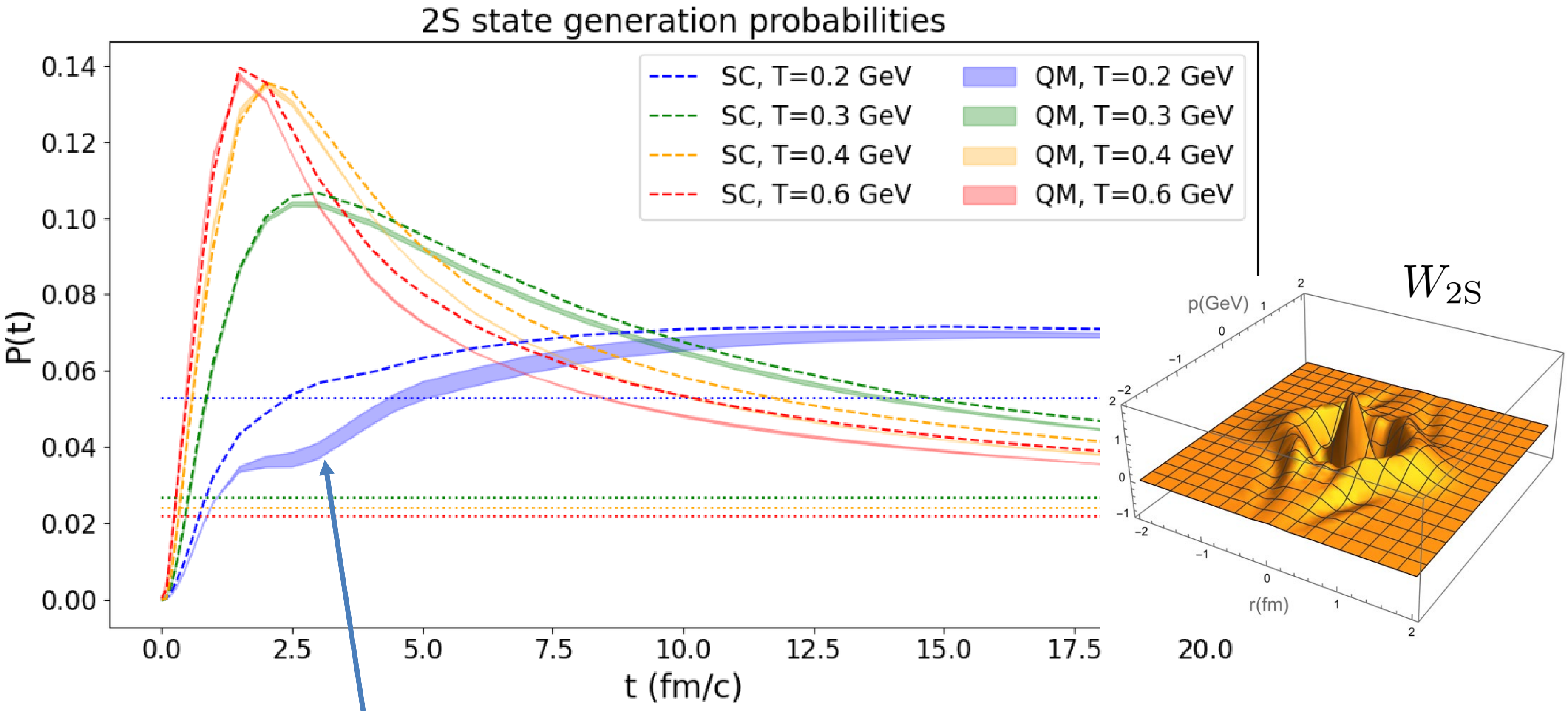


- Most significant disagreement for low T, around t = 2.5 fm/c (beginning of the classicalization)

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T

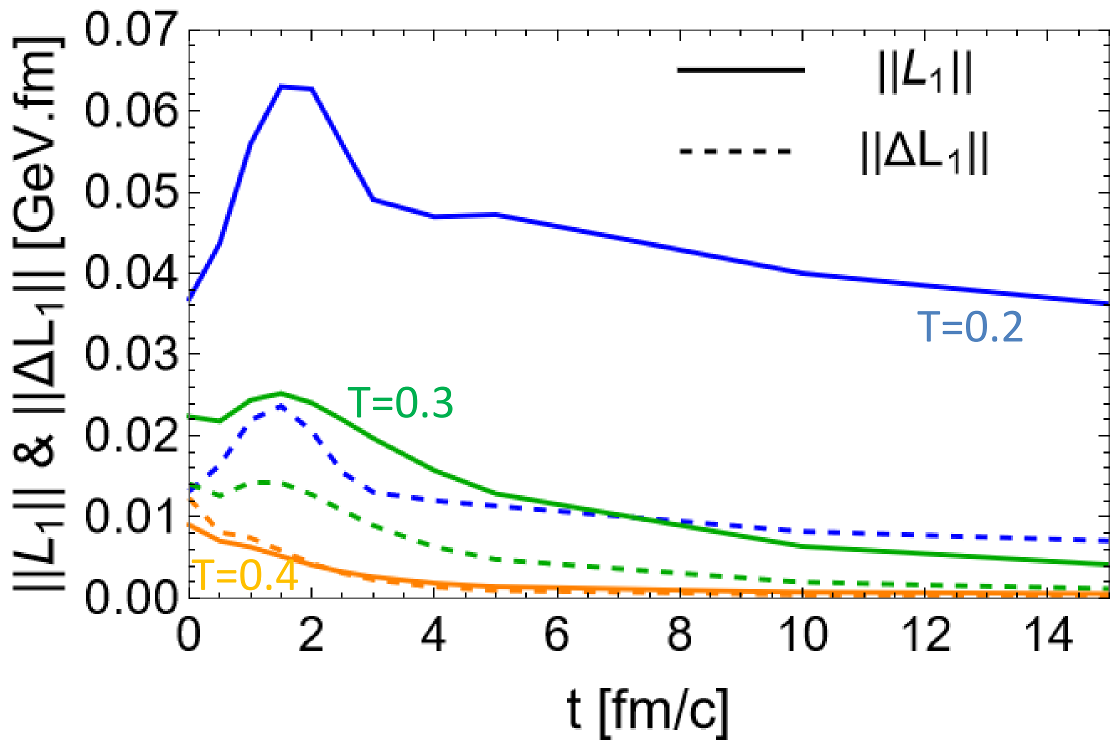


- Higher effect of the genuine interference quantum effects due to the mixture of positive and negative regions in W_{2S}.

Why does it work ?

- When / why does it work ?
  - The unitary term : $\mathcal{L}_1[\rho] = [V, \rho] = \rho(s, s')(V(s) - V(s')) = V'(r)y + \mathcal{O}(y^3)$

    Wigner-Moyal expansion, valid when y << variation scale of the real potential



➢ With increasing time, <y^2> decreases -> the de Broglie thermal length $\propto \sqrt{\frac{1}{T m_Q}}$ and the Wigner-Moyal expansion works better and better.

Conclusions

- The Lindblad equation succeeds in producing the bottomonia sequential suppression observed in R_{AA}. As next step, we will compute this observable and have direct comparison with experimental data.
- After some “decoherence time”, The semiclassical description reproduces very well the results of the exact quantum description, especially, at high temperatures
- The late time discrepancies are, mainly, due to the relaxation into different steady states. The steady sate of an open quantum system is still an active research topic !
- To come : generalization of the comparison for the non-abelian case.