

Chiral symmetry restoration, ϕ , K^* and K_1 in nuclear medium

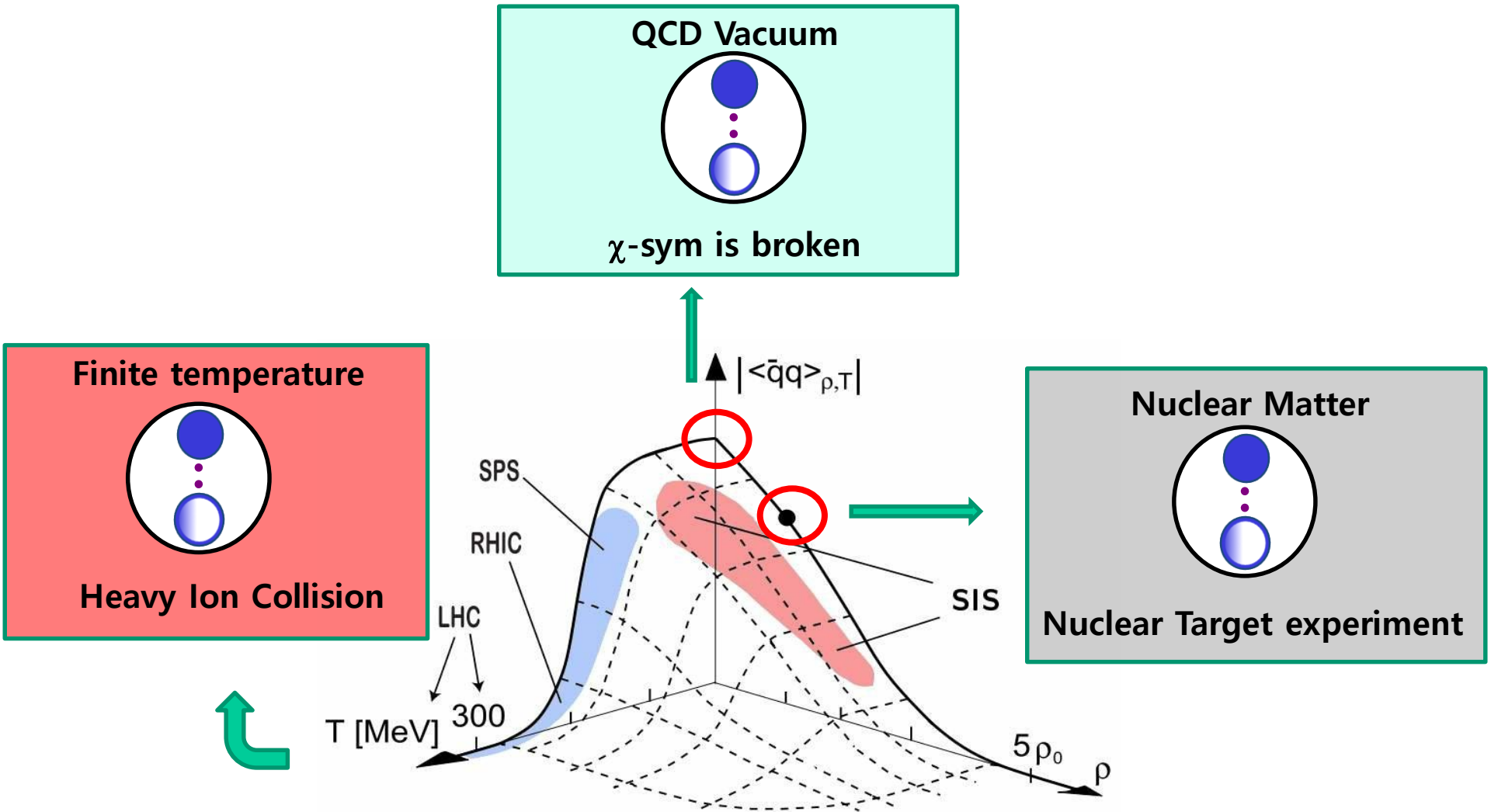
Su Hong Lee



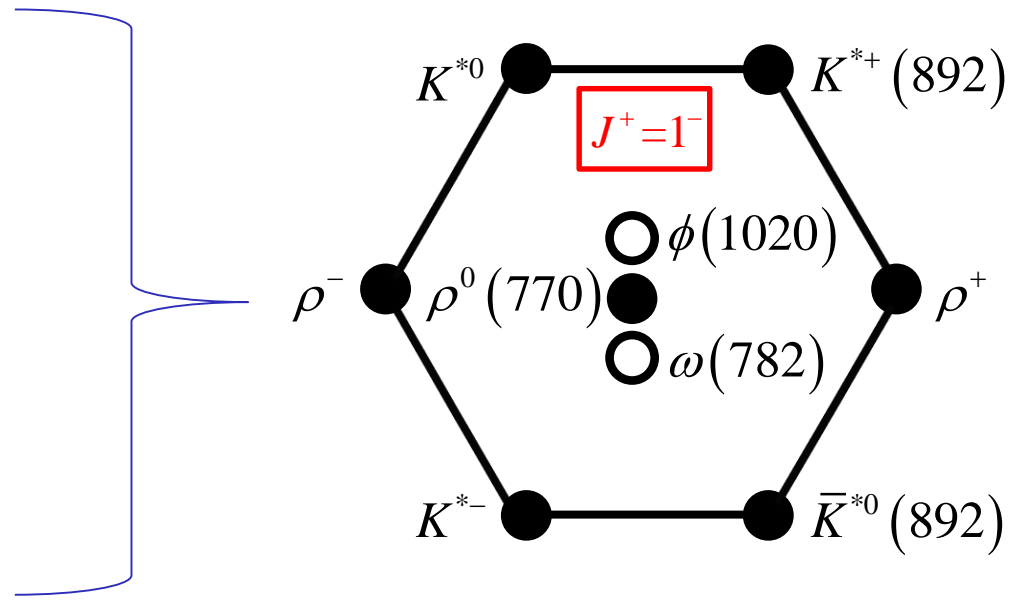
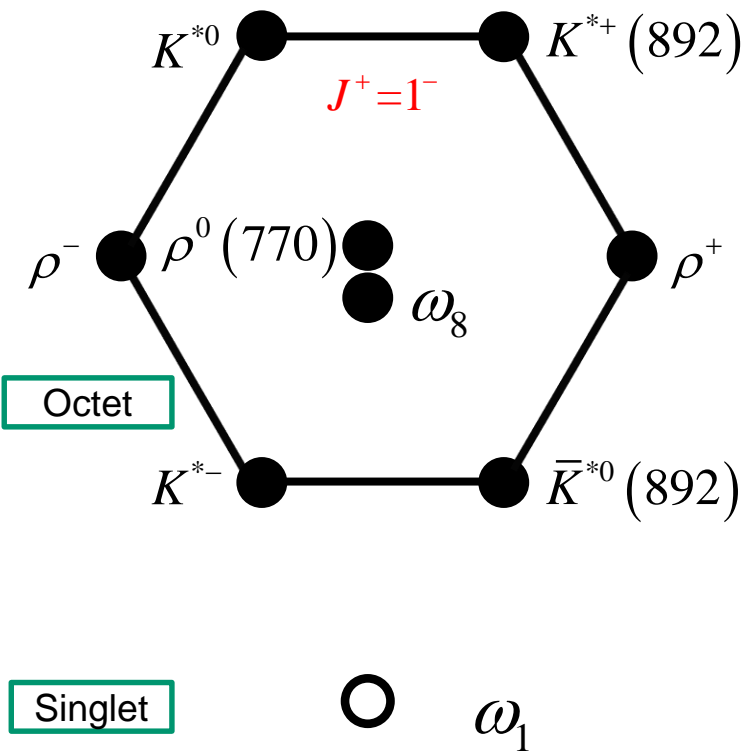
YONSEI
UNIVERSITY

1. Introduction
2. Chiral partners: K_1 , K^*
3. Vector mesons with small width: ϕ meson
4. K_1 and K^* in experiment
5. Outlook

Chiral symmetry restoration and hadron mass



Vector meson in Flavor SU(3) symmetry and broken symmetry



$$\omega_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

$$\omega_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

Mixing due to $m_s \gg m_d \sim m_u$

$$\phi(1020) = \bar{s}s$$

$$\omega(782) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

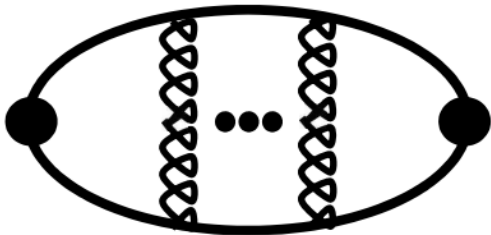
Why does meson representation mix for $Y=0, I=0$?

□ Consider correlation function or Mass matrix

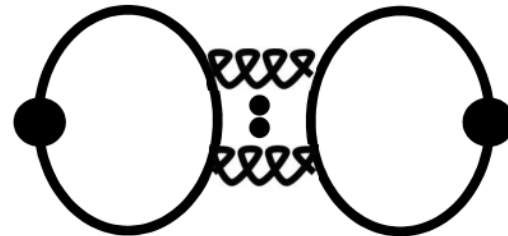
Example in SU(2) case with 1 light 1 strange flavor

$$\begin{aligned} \omega_1 &= \bar{q}q + \bar{s}s \\ \omega_8 &= \bar{q}q - \bar{s}s \end{aligned} \quad \mathbf{M} = \begin{pmatrix} \langle \omega_1 \omega_1 \rangle & \langle \omega_1 \omega_8 \rangle \\ \langle \omega_8 \omega_1 \rangle & \langle \omega_8 \omega_8 \rangle \end{pmatrix} = \begin{pmatrix} a + D & \Delta \\ \Delta & a - D \end{pmatrix}$$

$$\left\{ \begin{aligned} a &= \langle (\bar{q}q)(\bar{q}q) + (\bar{s}s)(\bar{s}s) \rangle \\ \Delta &= \langle (\bar{q}q)(\bar{q}q) - (\bar{s}s)(\bar{s}s) \rangle \\ D &= \langle 2(\bar{q}q)(\bar{s}s) \rangle \end{aligned} \right. \quad \begin{array}{l} \longrightarrow \text{Symmetry breaking} \\ \longrightarrow \text{Disconnected contribution} \end{array}$$



Connected



Disconnected contribution

$$\omega_1 = \bar{q}q + \bar{s}s \quad \omega_3 = \bar{q}q - \bar{s}s$$

$$\square \quad \mathbf{M} = \begin{pmatrix} \langle \omega_1 \omega_1 \rangle & \langle \omega_1 \omega_8 \rangle \\ \langle \omega_8 \omega_1 \rangle & \langle \omega_8 \omega_8 \rangle \end{pmatrix} = \begin{pmatrix} a + D & \Delta \\ \Delta & a - D \end{pmatrix} \quad \left\{ \begin{array}{l} a = \langle (\bar{q}q)(\bar{q}q) + (\bar{s}s)(\bar{s}s) \rangle \\ \Delta = \langle (\bar{q}q)(\bar{q}q) - (\bar{s}s)(\bar{s}s) \rangle \\ D = \langle 2(\bar{q}q)(\bar{s}s) \rangle \end{array} \right.$$

$$\square \quad \text{Eigenvalues} \quad \mathbf{M}_{\pm} = a \pm \sqrt{D^2 + \Delta^2}$$

- If SU(3) symmetric limit: $\Delta = 0$, there is no mixing, and singlet and octet do not mix.

$$\text{If } \Delta = 0 \quad \mathbf{M} = \begin{pmatrix} a + D & 0 \\ 0 & a - D \end{pmatrix} \rightarrow \begin{pmatrix} \langle ((\bar{q}q) + (\bar{s}s))^2 \rangle & 0 \\ 0 & \langle ((\bar{q}q) - (\bar{s}s))^2 \rangle \end{pmatrix}$$

- If SU(3) symmetry is broken: Ideal mixing when $D=0 \ll \Delta$ (Vector meson)

$$\text{If } \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \rightarrow D = 0 \quad \mathbf{M} = \begin{pmatrix} a & \Delta \\ \Delta & a \end{pmatrix} \rightarrow \begin{pmatrix} \langle (\bar{q}q)(\bar{q}q) \rangle & 0 \\ 0 & \langle (\bar{s}s)(\bar{s}s) \rangle \end{pmatrix}$$

Chiral transformation of quark bilinears

- Chiral symmetry $SU(3)_R \times SU(3)_L$

$$q_{R,L} \rightarrow \exp(i\vec{\theta}(1 \pm \gamma^5))q_{R,L} \quad \text{where} \quad \vec{\theta} = \theta^a \tau^a$$

- Spontaneous breaking of Chiral symmetry $SU(3)_R \times SU(3)_L \rightarrow SU(3)_V$

- Broken transformation $q \rightarrow \exp(i\vec{\theta}\gamma^5)q, \quad \bar{q} \rightarrow \bar{q} \exp(i\vec{\theta}\gamma^5)$

- Transformation of Quark bilinear

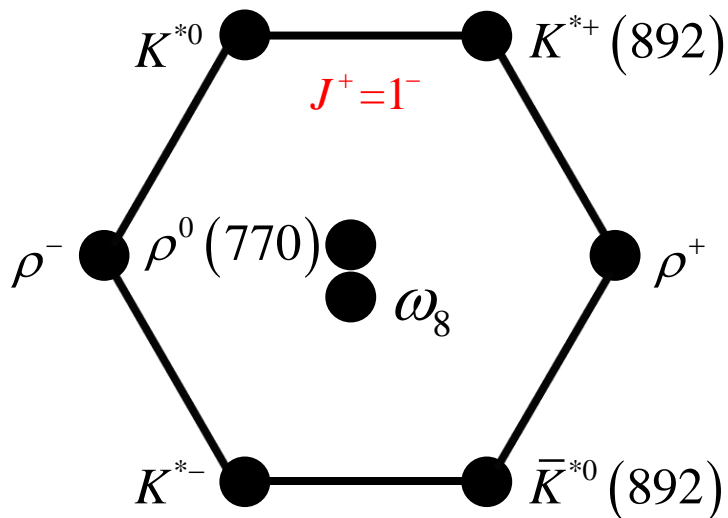
$$\bar{q}\Gamma q \rightarrow \bar{q}(1+i\vec{\theta}\gamma^5)\Gamma(1+i\vec{\theta}\gamma^5)q \rightarrow \bar{q}\Gamma q + \bar{q}(i\vec{\theta}\gamma^5\Gamma + \Gamma i\vec{\theta}\gamma^5)q$$

scalar: $\bar{q}q \rightarrow \bar{q}q + 2\bar{q}(i\vec{\theta}\gamma^5)q$

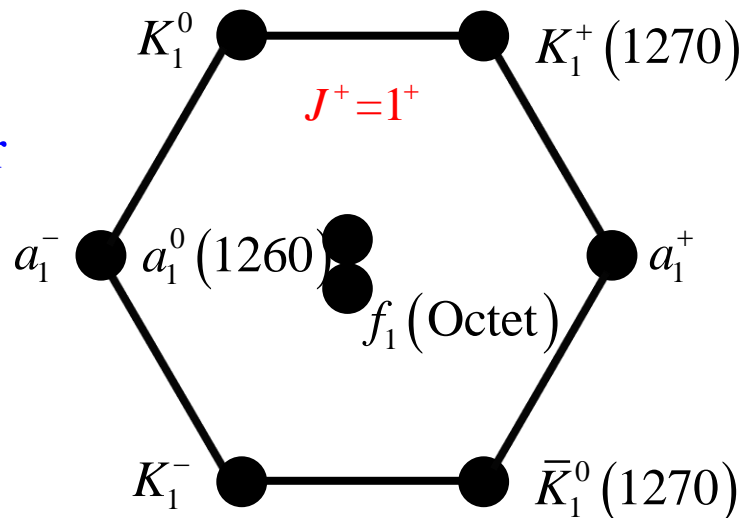
Flavor octet vector meson: $\bar{q}\gamma^\mu \tau^a q \rightarrow \bar{q}\gamma^\mu \tau^a q + \bar{q}i\gamma^5 \gamma^\mu [\vec{\theta}, \tau^a] q$

Flavor singlet vector meson: $\bar{q}\gamma^\mu q \rightarrow \bar{q}\gamma^\mu q + \bar{q}i\gamma^5 \gamma^\mu [\vec{\theta}, 1] q = \bar{q}\gamma^\mu q$

Spin-1 Flavor Octet

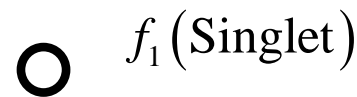


Chiral Partner



Spin-1 Flavor Singlet

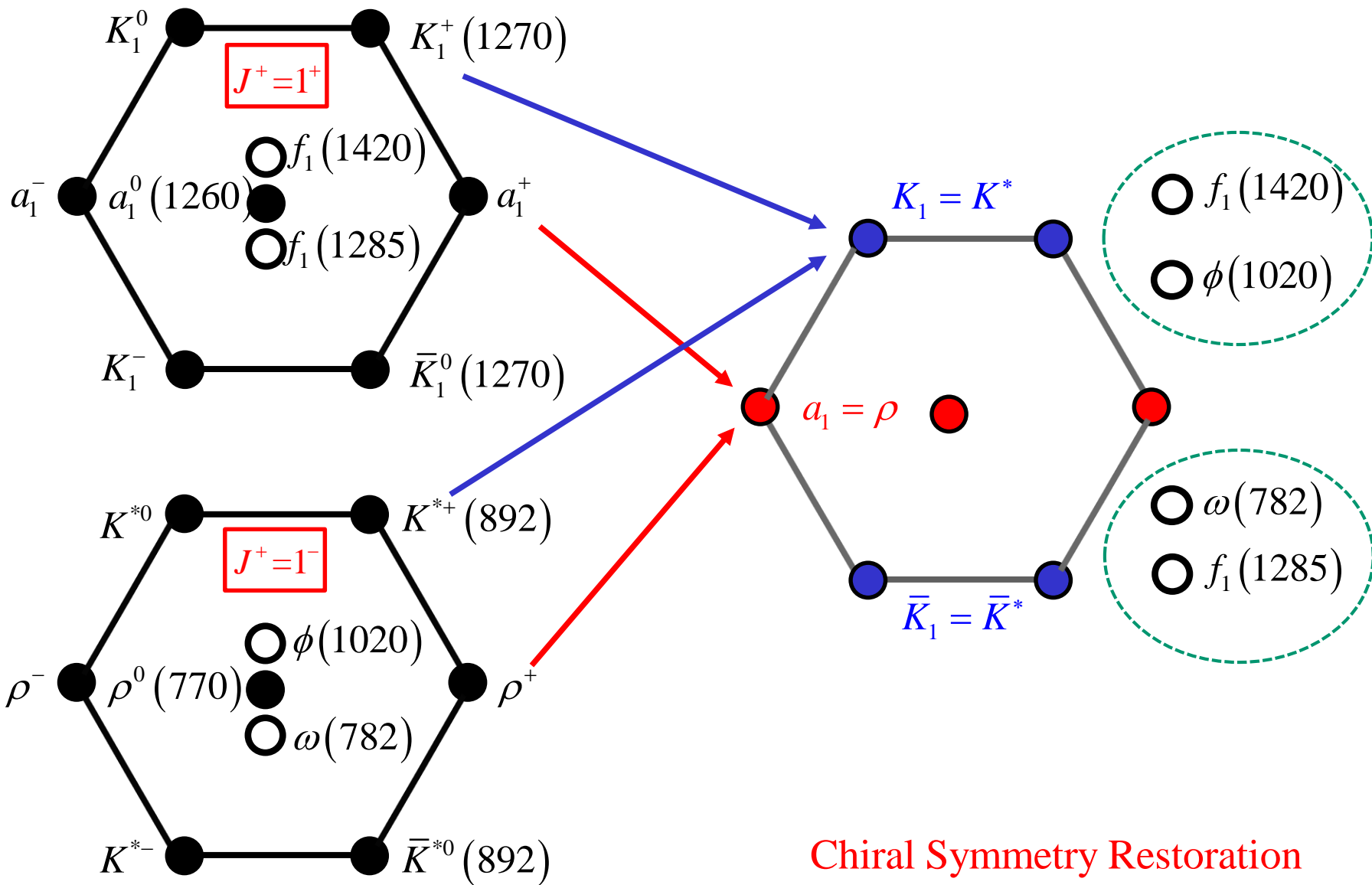
No Chiral Partner



$$\omega_8 = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s)$$

$$\omega_1 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s)$$

$$m_s \gg m_d \sim m_u \left\{ \begin{array}{l} \phi(1020) = \bar{s}s = \frac{1}{\sqrt{3}} (\omega_1 - \sqrt{2}\omega_8) \\ \omega(782) = \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) = \frac{1}{\sqrt{3}} (\omega_8 + \sqrt{2}\omega_1) \end{array} \right.$$



Broken Chiral Symmetry

Chiral Symmetry Restoration

Chiral partners: K^* , K_1

- Operator product expansion for K^* and K_1

$$\square \quad \Pi^{K^*K^*} = \langle (\bar{u}\gamma_\mu s)_x (\bar{s}\gamma_\mu u)_0 \rangle = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{u}\gamma_\mu\gamma^5\lambda^a s)(\bar{s}\gamma_\mu\gamma^5\lambda^a u) \rangle + \frac{1}{9} \left\langle \left(\sum_{su} \bar{q}\gamma_\mu\lambda^a q \right) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \right\rangle \right]$$

$$\Pi^{K_1K_1} = \langle (\bar{u}i\gamma_\mu\gamma^5 s)_x (\bar{s}i\gamma_\mu\gamma^5 u)_0 \rangle = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{u}\gamma_\mu\lambda^a s)(\bar{s}\gamma_\mu\lambda^a u) \rangle + \frac{1}{9} \left\langle \left(\sum_{ud} \bar{q}\gamma_\mu\lambda^a q \right) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \right\rangle \right]$$

- Only connected diagrams contribute in the difference

$$\begin{aligned} \Pi^{K^*K^*} - \Pi^{K_1K_1} &= \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{u}\gamma_\mu\gamma^5\lambda^a s)(\bar{s}\gamma_\mu\gamma^5\lambda^a u) \rangle - \langle (\bar{u}\gamma_\mu\lambda^a s)(\bar{s}\gamma_\mu\lambda^a u) \rangle \right] \\ &= \dots + \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{s}\lambda^a\gamma_\mu [S_u(x,0) - i\gamma^5 S_u(x,0)i\gamma^5] \lambda^a\gamma_\mu s) \rangle - \langle u \leftrightarrow s \rangle \right] \end{aligned}$$

Because they are chiral partners, this part is a chiral order parameter

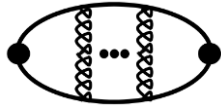
$$\langle \bar{u}u \rangle = -\lim_{x \rightarrow 0} \left\langle \frac{1}{2} [S_u(x,0) - i\gamma^5 S_u(x,0)i\gamma^5] \right\rangle$$

- Operator product expansion for ϕ and $f_1(1420)$

$$\square \quad \Pi^{\phi\phi} = \langle (\bar{s}\gamma_\mu s)_x (\bar{s}\gamma_\mu s)_0 \rangle = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{s}\gamma_\mu\gamma^5\lambda^a s)(\bar{s}\gamma_\mu\gamma^5\lambda^a s) \rangle + \frac{2}{9} \langle (\bar{s}\gamma_\mu\lambda^a s) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \rangle \right]$$

$$\Pi^{f_1 f_1} = \langle (\bar{s}i\gamma_\mu\gamma^5 s)_x (\bar{s}i\gamma_\mu\gamma^5 s)_0 \rangle = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{s}\gamma_\mu\lambda^a s)(\bar{s}\gamma_\mu\lambda^a s) \rangle + \frac{2}{9} \langle (\bar{s}\gamma_\mu\lambda^a s) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \rangle \right]$$

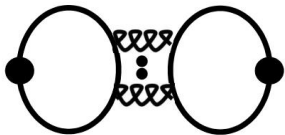
\square The Difference has contribution from disconnect diagram, and is chiral symmetric



$$\lim_{x \rightarrow 0} \langle [S_s(x,0) - i\gamma^5 S_s(x,0) i\gamma^5] \rangle = -\langle \bar{s}s \rangle$$

$$\Pi^{\phi\phi} - \Pi^{f_1 f_1} = \dots + \frac{4\pi\alpha}{Q^6} \left[\langle (\bar{s}\lambda^a\gamma_\mu [S_s(x,0) - i\gamma^5 S_s(x,0) i\gamma^5] \lambda^a\gamma_\mu s) \rangle \right]$$

$$- \frac{2\pi\alpha}{Q^6} \left[\langle \text{Tr}(\lambda^a\gamma_\mu\gamma^5 S_s(x,x)) \text{Tr}(\lambda^a\gamma_\mu\gamma^5 S_s(0,0)) \rangle + \langle \text{Tr}(\lambda^a\gamma_\mu S_s(x,x)) \text{Tr}(\lambda^a\gamma_\mu S_s(0,0)) \rangle \right]$$



$$S_s = \frac{1}{2} [S_s - i\gamma^5 S_s i\gamma^5] + \frac{1}{2} [S_s + i\gamma^5 S_s i\gamma^5] \otimes \langle \bar{s}s \rangle$$

- Disconnect diagram is known to be small in vacuum (ideal mixing)

Which vector meson should one study in medium

- Hadrons with small width: ϕ is the best candidate

J-PARC E16, E88

$J^{PC}=1^{--}$	Mass	Width	$J^{PC}=1^{++}$	Mass	Width
ρ	770 MeV	150 MeV	a_1	1260	250–600
ω	782	8.49	f_1	1285	24.2
ϕ	1020	4.266	f_1	1420	54.9
$K^*(1^-)$	892	50.3	$K_1(1^+)$	1270	90

- Chiral partner: K^* and K_1 is the best candidate

- Decay mode of chiral partners typically has one more pion

$$m_{K^*} - m_{K_1} \propto \langle \bar{q}q \rangle$$

$$K^* \rightarrow K\pi, \quad K_1^- \rightarrow \begin{cases} \rho(\pi\pi) + K \\ \pi + K^*(K\pi) \end{cases}$$

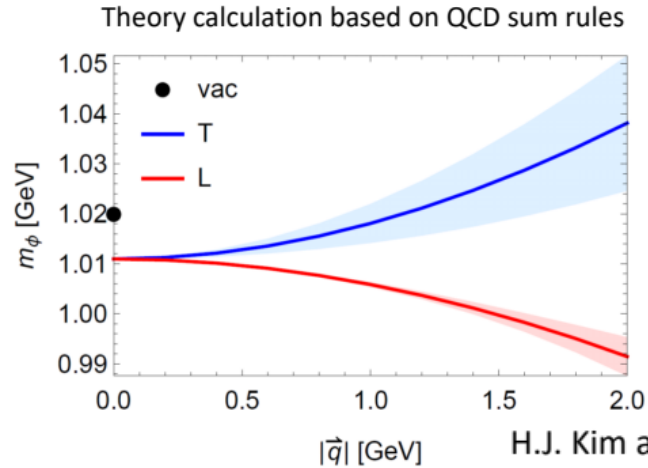
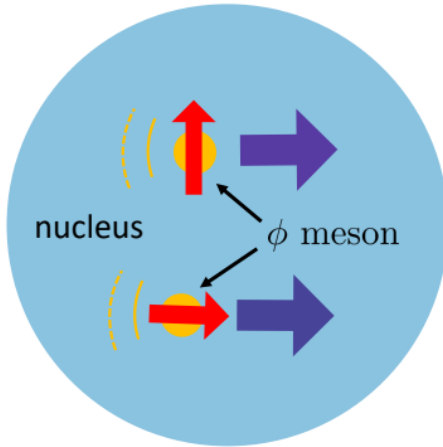
- Measurement of $f_1(1420)$ is still interesting (parity partner of ϕ)

$$m_\phi - m_{f_1(1420)} \propto \langle \bar{s}s \rangle + \text{other}$$

$$\phi \rightarrow K + K \quad f_1(1420) \rightarrow K + K^*(K\pi)$$

ϕ meson with small momentum

□ Transverse and Longitudinal modes at large momentum



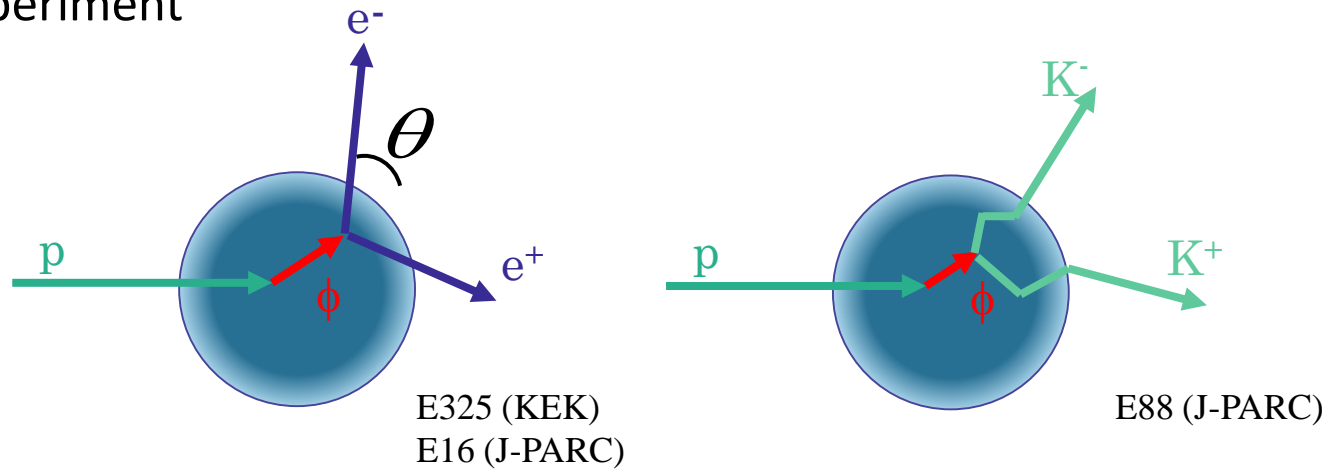
H.J. Kim and P. Gubler,
Phys. Lett. B **805**, 135412 (2020).

□ This momentum dependence is Chiral symmetric contributions

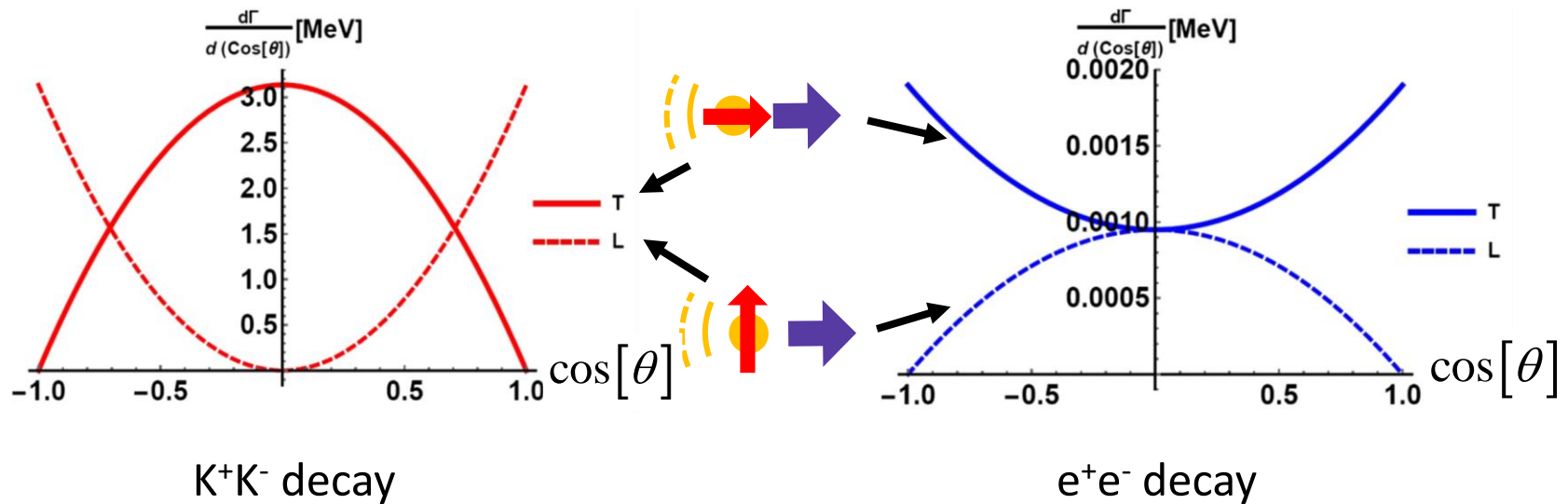
$$\Pi^T(\omega, \vec{q}) - \Pi^L(\omega, \vec{q}) = \frac{mA_2^{u+d} \vec{q}^2}{\omega^6} \rho_n.$$

where $A_2^q \propto \langle N | \bar{q} \gamma_\mu D_\nu q | N \rangle$

❑ Experiment



❑ Angular decay distributions for Transverse and Longitudinal modes



K^+K^- decay

e^+e^- decay

K^* and K_1 mass in the chiral symmetric vacuum

- Sum rules in the chiral symmetric vacuum

$$\begin{aligned}
 \Pi^{\rho\rho} &= \dots - \frac{2\pi\alpha}{Q^6} \left[\frac{28}{9} \langle B_{uu} \rangle + \langle S_{\rho-a_1} \rangle \right] & \Pi^{K^*K^*} &= \dots - \frac{2\pi\alpha}{Q^6} \left[\langle B_{su} \rangle - \frac{1}{9} \langle B_{uu} \rangle + \langle S_{K^*-K_1} \rangle \right] \\
 \Pi^{K_1K_1} &= \dots - \frac{2\pi\alpha}{Q^6} \left[-\frac{44}{9} \langle B_{uu} \rangle + \langle S_{\rho-a_1} \rangle \right] & \Pi^{K_1K_1} &= \dots - \frac{2\pi\alpha}{Q^6} \left[-\langle B_{su} \rangle - \frac{1}{9} \langle B_{uu} \rangle + \langle S_{K^*-K_1} \rangle \right]
 \end{aligned}$$

From phenomenology of ρ , a_1 , K^* , K_1 can determine $\langle B_{su} \rangle, \langle B_{uu} \rangle, \langle S_{\rho-a_1} \rangle, \langle S_{K^*-K_1} \rangle$

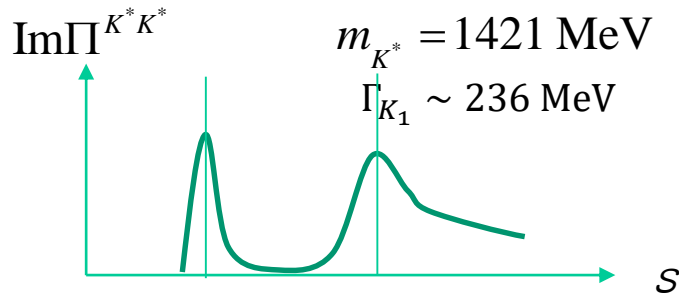
Mass when in the chiral symmetry restored phase can be estimated by

$\langle B_{su} \rangle = \langle B_{uu} \rangle = 0$, but keep vacuum values for $\langle S_{\rho-a_1} \rangle, \langle S_{K^*-K_1} \rangle$

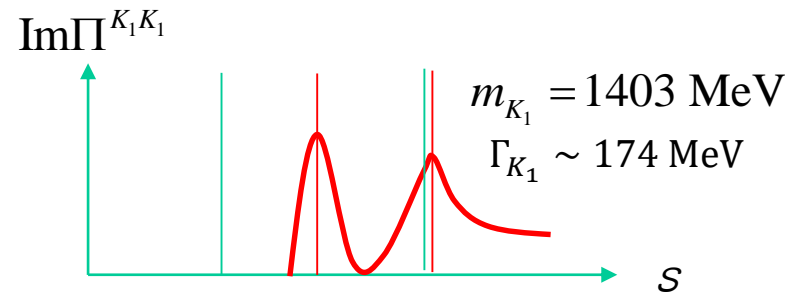
Particle	m_0	m_B	m_{sym}
ρ	775.26 MeV	610 MeV	572.5 MeV
K^*	895.81 MeV	775 MeV	545 MeV
K_1	1272 MeV	1080 MeV	545 MeV

- Weinberg type sum rules

☞
$$\Pi^{K^*} - \Pi^{K_1} = \dots \frac{1}{Q^6} [2 \langle B_{su} \rangle] + \dots = \frac{1}{\pi} \int ds \frac{\text{Im}(\Pi^{K^*} - \Pi^{K_1})}{s - q^2}$$



$m_{K^*} = 892 \text{ MeV}$
 $\Gamma_{K^*} \sim 47 \text{ MeV}$



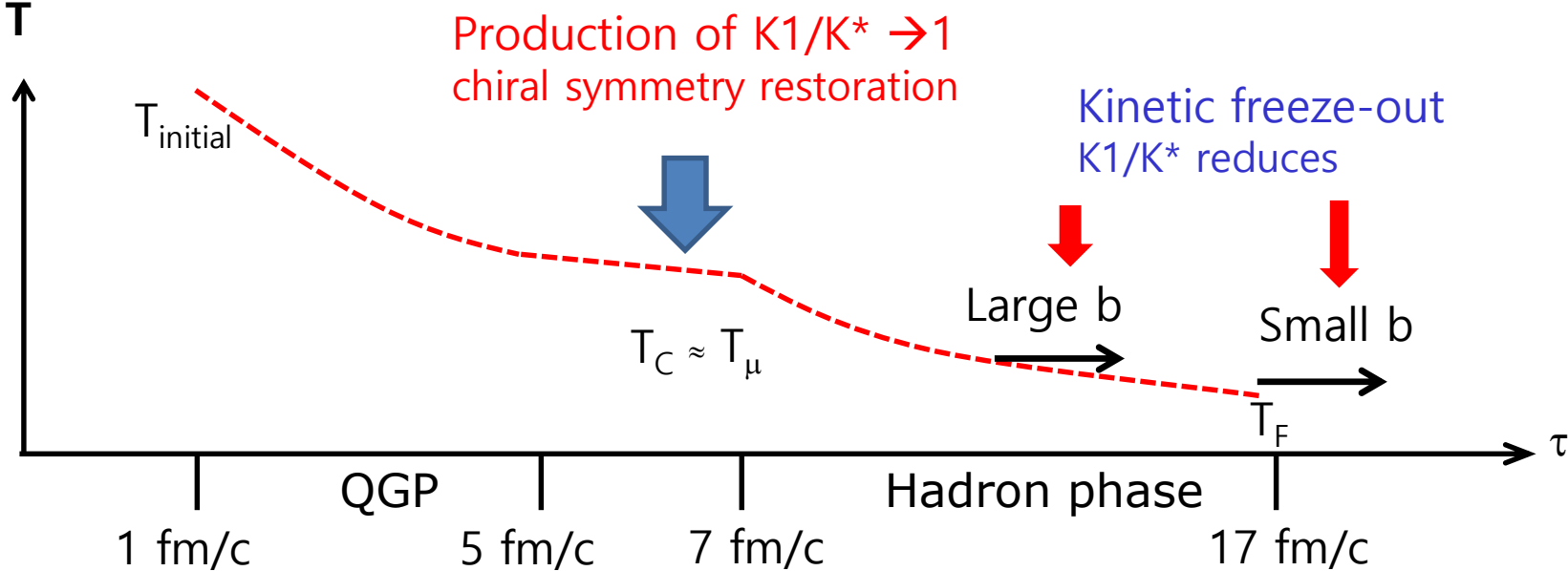
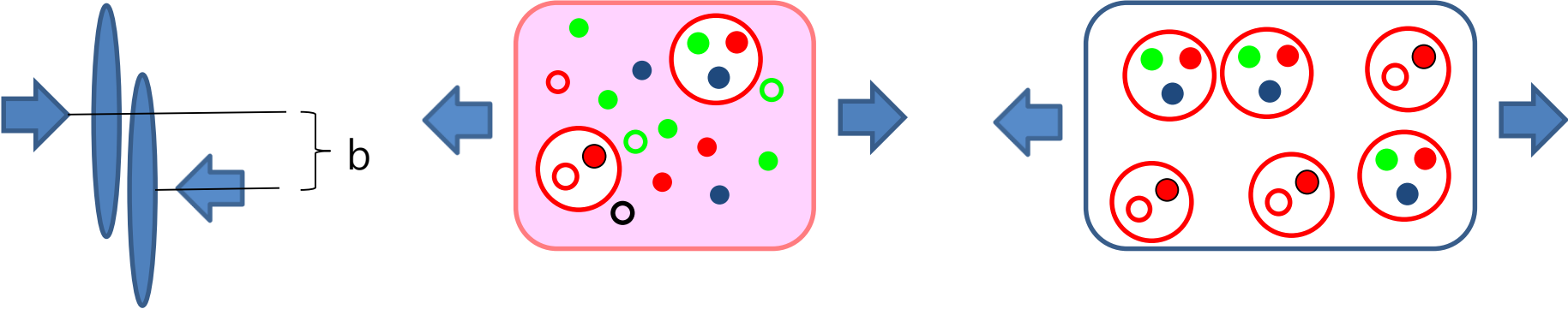
$m_{K_1} = 1272 \text{ MeV}$
 $\Gamma_{K_1} \sim 90 \text{ MeV}$

☞
$$f_{K^*}^2 m_{K^*}^4 - f_{K_1}^2 m_{K_1}^4 = -2m_s \langle \bar{u}u \rangle \quad f_{K^*}^2 m_{K^*}^6 - f_{K_1}^2 m_{K_1}^6 = 2 \langle B_{su} \rangle = -64\pi\alpha_s \langle \bar{u}u \rangle \langle \bar{s}s \rangle$$

➔
$$f_{K^*}^2 m_{K^*}^4 (m_{K_1}^2 - m_{K^*}^2) = 2m_s \langle \bar{u}u \rangle m_{K_1}^2 + 64\pi\alpha_s \langle \bar{u}u \rangle \langle \bar{s}s \rangle$$

➔
$$m_{K_1}^2(T) = m_{K^*}^2 + \frac{\langle \bar{u}u \rangle_T}{\langle \bar{u}u \rangle_0} (m_{K_1}^2 - m_{K^*}^2)$$

K1/K production in heavy ion collision*



- K_1/K^* vs centrality

Haesom Sung, et al. (PLB819(2021) 136388, PRC109 (2024) 4

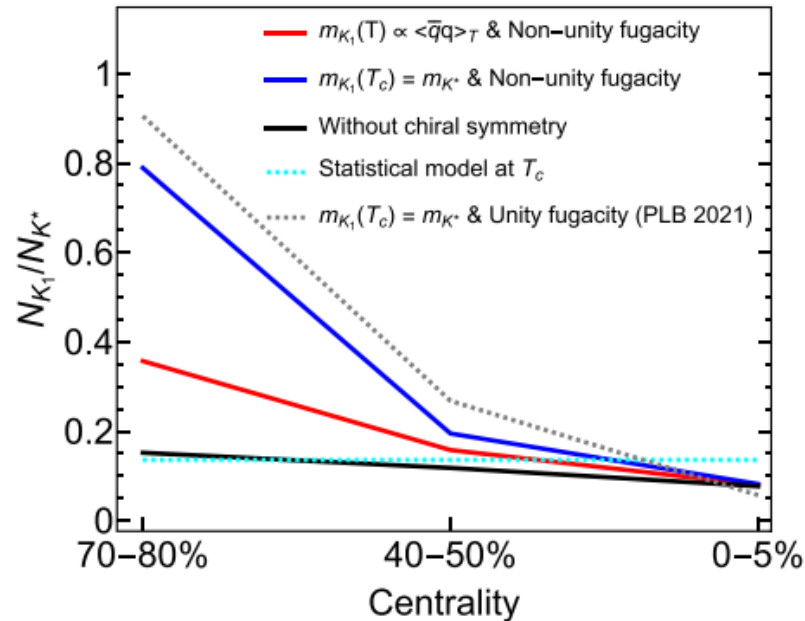


FIG. 5. The yield ratio K_1/K^* in Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV at three centralities of 0–5%, 40–50%, and 70–80% for various scenarios.

K1 measurement in heavy ion collision ?

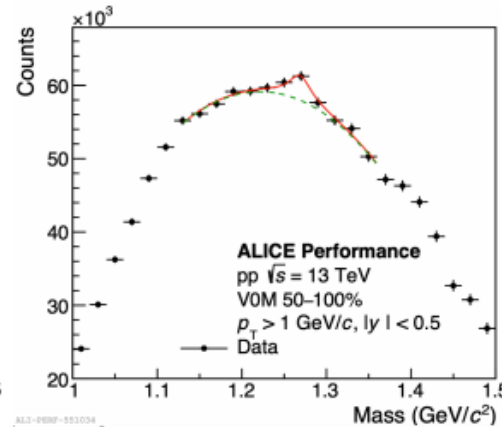
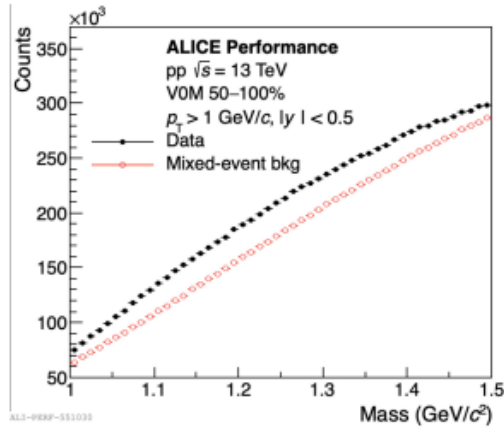


Study of K_1 meson production in pp collisions with ALICE

SU-JEONG JI⁽¹⁾ ON BEHALF OF THE ALICE COLLABORATION | ¹⁾ PUSAN NATIONAL UNIVERSITY



SIGNAL EXTRACTION



Signal: $\pi^\pm \pi^\mp K^\pm$ pairs

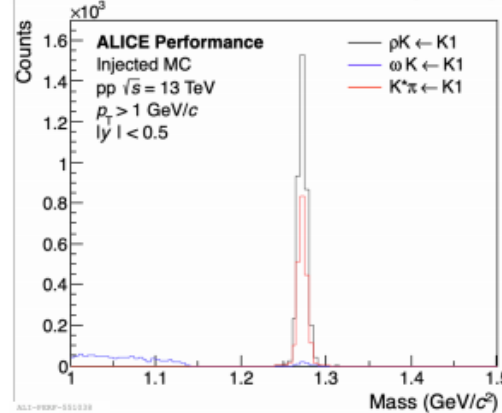
Background:

$$\pi^\pm \pi^\pm K^\mp, \pi^\pm \pi^\pm K^\pm$$

pairs from mixed event

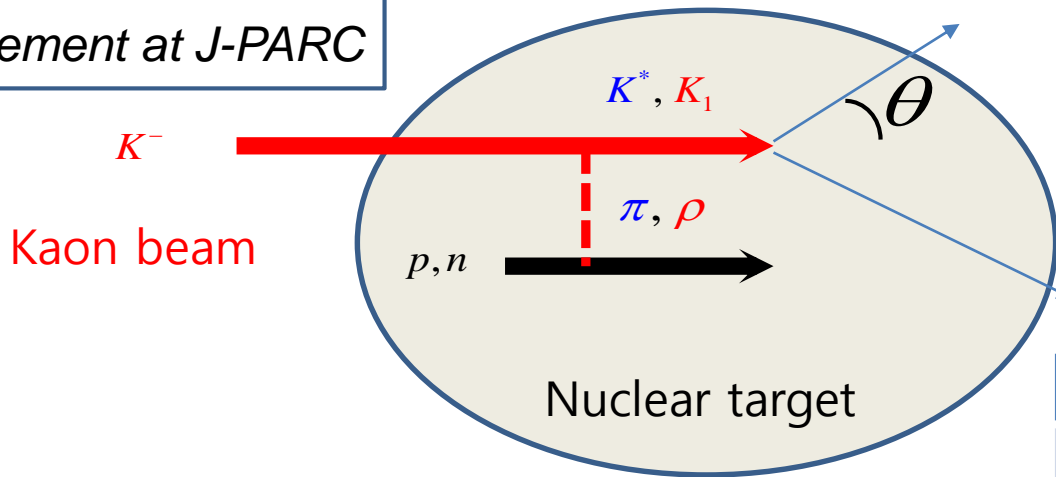
(30 events, $\Delta z_{vtx} \leq 2$ cm)

Fit Function: Breit-Wigner
+ quadratic fn.



The signal peak is seen at 1270 MeV/c² in data.

K1 measurement at J-PARC

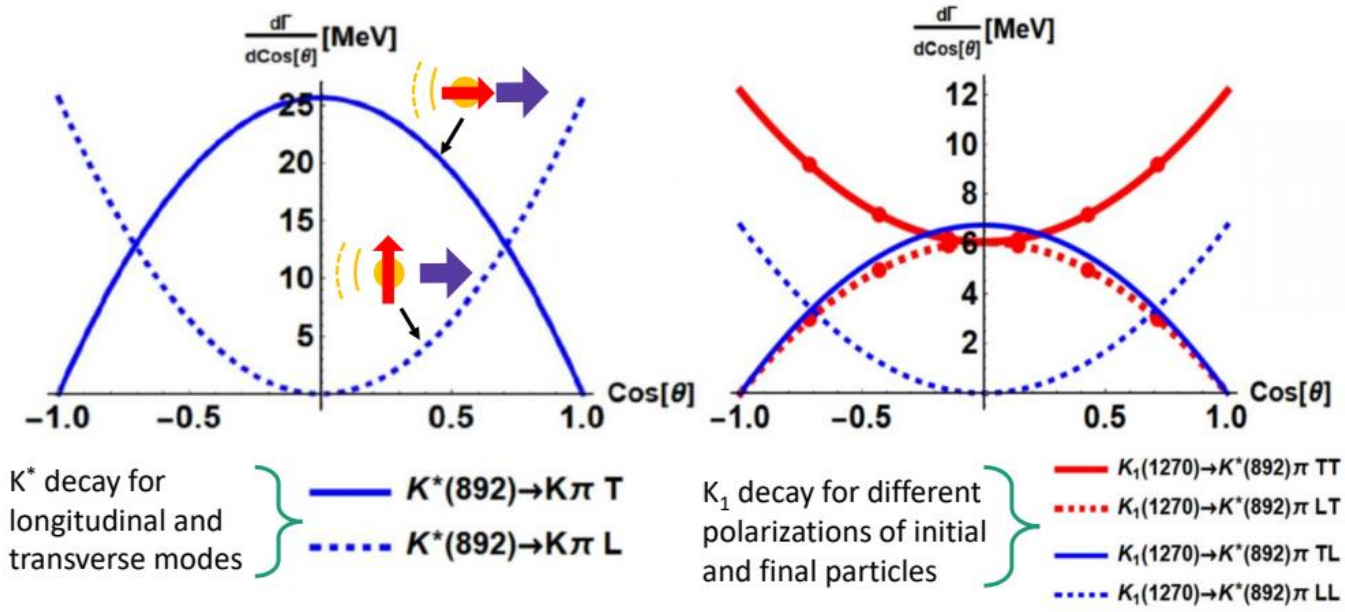


$$K_1^- \rightarrow \begin{cases} \rho^0 K^- \\ \rho^- \bar{K}^0 \end{cases} \quad \begin{cases} \pi^0 K^{*-} \\ \pi^- \bar{K}^{*0} \end{cases}$$

$$\bar{K}_1^0 \rightarrow \begin{cases} \rho^+ K^- \\ \rho^0 \bar{K}^0 \end{cases} \quad \begin{cases} \pi^+ K^{*-} \\ \pi^0 \bar{K}^{*0} \end{cases}$$

Decay mode	Fraction
$K_1(1270) \rightarrow K \rho$	42 %
$K_1(1270) \rightarrow K^* \pi$	16 %

□ Transvers and longitudinal modes from angular dependence of decay particles



I.W. Park, H. Sako, K. Aoki, P. Gubler and S.H. Lee, Phys. Rev. D **109**, 114042 (2024).

1) Hadrons with small width: ϕ is the best candidate

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ρ	770 MeV	150 MeV	a_1	1260	250–600
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J-PARC E16, E88 → ϕ	1020	4.266	f_1	1420	54.9
$K^*(1^-)$	892	50.3	$K_1(1^+)$	1270	90

2) Chiral partner: K^* and K_1 is the best candidate

- Decay mode of chiral partners typically has one more pion

$$m_{K^*} - m_{K_1} \propto \langle \bar{q}q \rangle$$

$$K^* \rightarrow K\pi, \quad K_1^- \rightarrow \begin{cases} \rho(\pi\pi) + K \\ \pi + K^*(K\pi) \end{cases}$$

3) If possible measurement of $f_1(1420)$ is also interesting (parity partner of ϕ)

$$m_\phi - m_{f_1(1420)} \propto \langle \bar{s}s \rangle + \text{other}$$

$$\phi \rightarrow K + K \quad f_1(1420) \rightarrow K + K^*(K\pi)$$

Outlook

1. *K1 and K* are Chiral Partners with $\Gamma < 100$ MeV*
$$m_{K_1}^2(T) = m_{K^*}^2 + \frac{\langle \bar{u}u \rangle_T}{\langle \bar{u}u \rangle_0} (m_{K_1}^2 - m_{K^*}^2)$$
2. *K1/K* production could be measured for different centralities in Heavy Ion Collision*
→ *Signature of chiral symmetry restoration*
3. *K1 K* masses can also be measured in J-PARC*

Some additions

Which vector meson should one study in medium

1) Hadrons with small width

J-PARC E16, E88

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2) Use condensate based Operator Product Expansion (OPE) methods

- Can identify Chiral symmetry breaking effects

$$\langle \bar{q}q \rangle \quad \begin{matrix} \curvearrowright & q \rightarrow \exp(i\gamma_5 \vec{\alpha}) q & \curvearrowleft \end{matrix}$$

$$\Pi^{K^*K^*} - \Pi^{K_1K_1} = \frac{1}{V} \int d^4x e^{iqx} \left[\langle \bar{s}\gamma^\mu q(x), \bar{q}\gamma^\mu s(0) \rangle - \langle \bar{s}i\gamma^5\gamma^\mu q(x), \bar{q}i\gamma^5\gamma^\mu s(0) \rangle \right] = \frac{c_4}{q^4} \langle m_s \bar{q}q \rangle + \frac{c_6}{q^6} \langle \bar{s}s \bar{q}q \rangle \dots$$

- Can relate them to mass

$$\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im}\Pi(s)}{s - q^2}$$

Chiral partners: K^* and K_1 (in a chiral symmetry restored vacuum)

- Operator product expansion

Jisu Kim, SHL: arXiv2012.06463 (PRD21)

Jisu Kim, SHL: arXiv2109.12791 (PRD22)

$$\Pi^{K^*K^*} = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{u}\gamma_\mu\gamma^5\lambda^a s)(\bar{s}\gamma_\mu\gamma^5\lambda^a u) \rangle + \frac{1}{9} \left\langle \left(\sum_{su} \bar{q}\gamma_\mu\lambda^a q \right) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \right\rangle \right]$$

$$\Pi^{K_1K_1} = \dots - \frac{2\pi\alpha}{Q^6} \left[\langle (\bar{u}\gamma_\mu\lambda^a s)(\bar{s}\gamma_\mu\lambda^a u) \rangle + \frac{1}{9} \left\langle \left(\sum_{ud} \bar{q}\gamma_\mu\lambda^a q \right) \left(\sum_{uds} \bar{q}\gamma_\mu\lambda^a q \right) \right\rangle \right]$$

$$\begin{aligned} \Pi^{K^*K^*} &= \dots - \frac{2\pi\alpha}{Q^6} \left[\langle B_{su} \rangle - \frac{1}{9} \langle B_{uu} \rangle + \langle S_{K^*-K_1} \rangle \right] \\ \Pi^{K_1K_1} &= \dots - \frac{2\pi\alpha}{Q^6} \left[-\langle B_{su} \rangle - \frac{1}{9} \langle B_{uu} \rangle + \langle S_{K^*-K_1} \rangle \right] \end{aligned} \quad \left. \vphantom{\begin{aligned} \Pi^{K^*K^*} \\ \Pi^{K_1K_1} \end{aligned}} \right\} \Pi^{K^*K^*} - \Pi^{K_1K_1} = \dots - \frac{2\pi\alpha}{Q^6} [2\langle B_{su} \rangle]$$

$\langle B_{su} \rangle$ Chiral symmetry breaking operator involving u/s quarks

$\langle S_{uds} \rangle$ Chiral symmetric operator involving $u/d/s$ quarks

Chiral symmetry breaking operators

1. Changes in Chiral symmetry breaking operators: $\langle \bar{q}q \rangle_{nm} = \langle \bar{q}q \rangle_0 + \rho \langle P | \bar{q}q | P \rangle$
2. 4-quark operator : $\frac{1}{2} \left[\left\langle \left(\bar{q} \gamma_\mu \gamma^5 \lambda^a \tau^3 q \right)^2 - \left(\bar{q} \gamma_\mu \lambda^a \tau^3 q \right)^2 \right\rangle \right] \propto \langle \bar{q}q \rangle_{nm}^2$
 \Rightarrow Tends to decrease mass

Chiral symmetric operators

1. Appearance of Tensor operators due to density and energy ect. :
 $\langle \bar{q} \gamma_0 q \rangle, \langle \bar{q} \gamma_\mu D_\nu q \rangle, \langle E^2 + B^2 \rangle \dots$
 \Rightarrow Tends to increase mass
 \Rightarrow when $q \neq 0$, Momentum dependent masses; Transverse and Longitudinal Masses