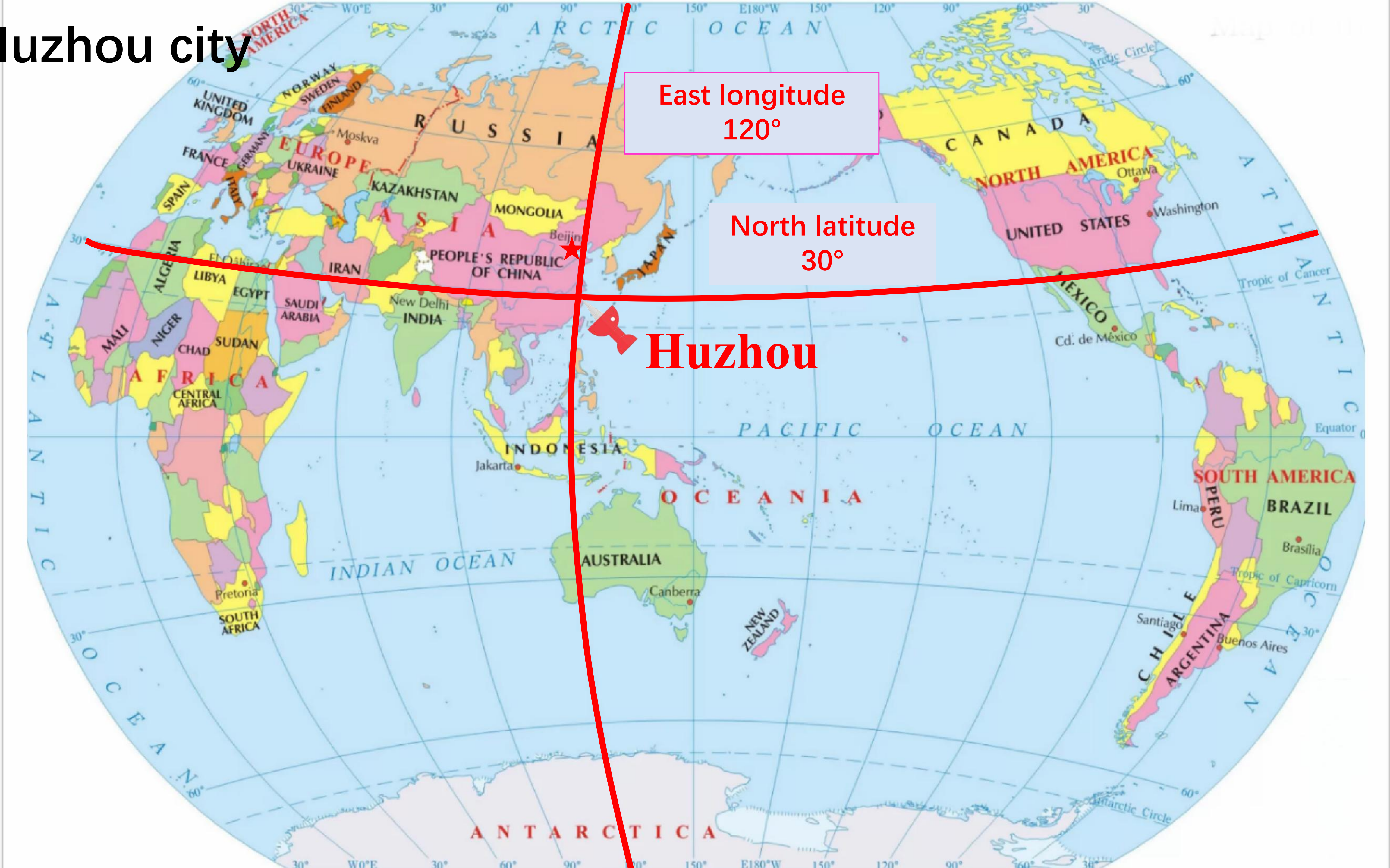
The 10th International Symposium on Non-equilibrium Dynamics (NeD-2024), Nov. 25-29, 2024

Elliptic flow in heavy-ion collisions at intermediate energies

Yongjia Wang (Huzhou University, China)

Huzhou city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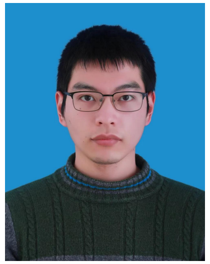
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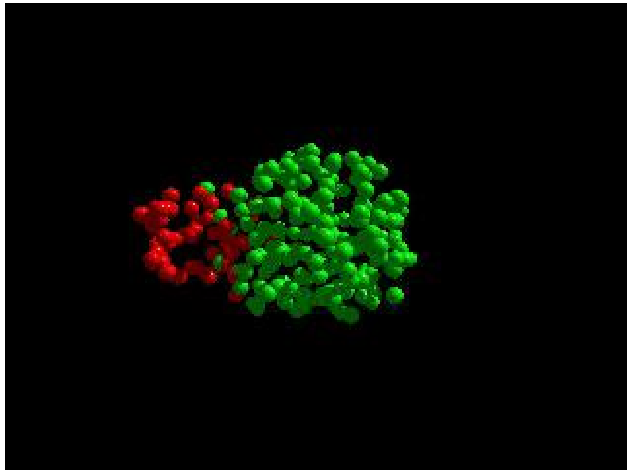
Outline

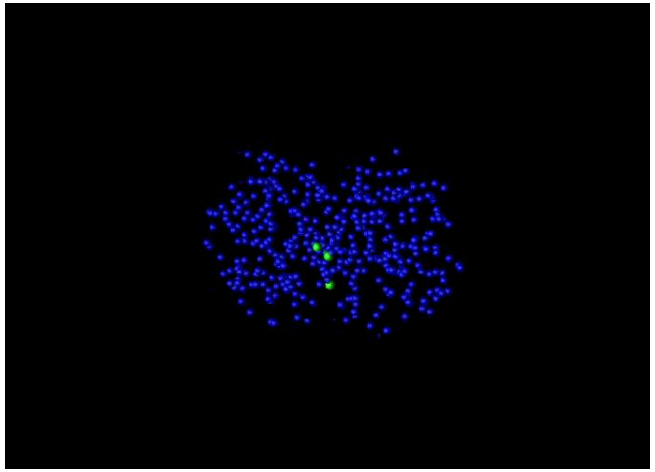
- 1. Background
- 2. Elliptic flow
  - ✓ 2.1 The role of nuclear potential and collision term
  - ✓ 2.2 The density interval at which the nuclear equation of state is probed by the elliptic flow
  - ✓ 2.3 Effects of rapidity and transverse momentum cuts
- 3. Bayesian analysis with FOPI data
- 4. Summary

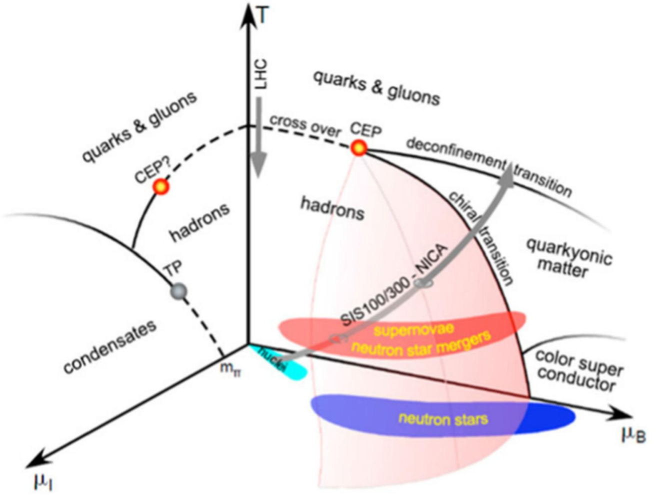
01 Overview of HIC

◆ intermediate energies HIC



Nuclear equation of state





◆ Low energy fusion reaction

~100 MeV- a few GeV/nucleon

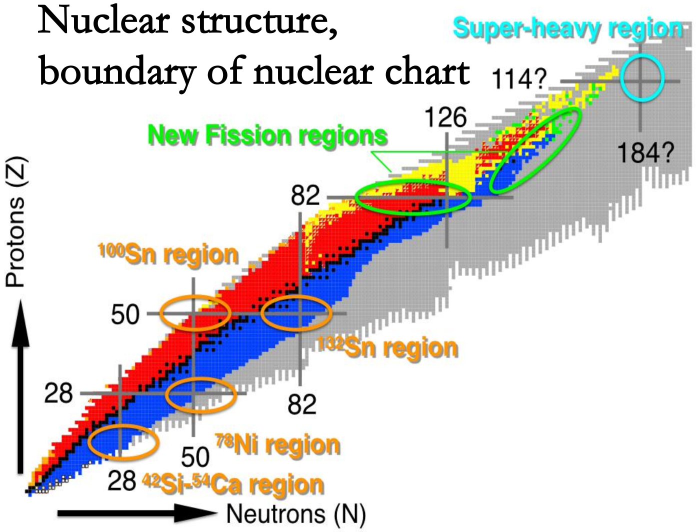
QCD phase diagram, QGP properties

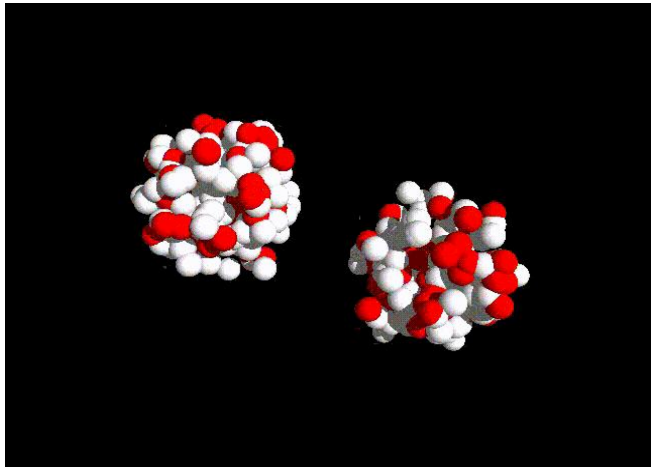
~100 MeV/nucleon

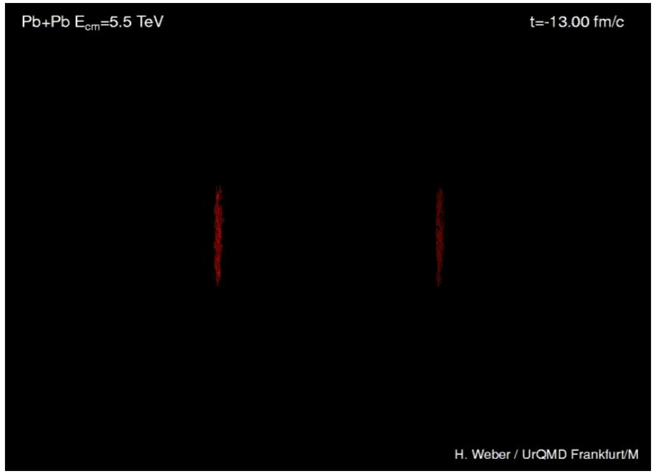
> hundreds of GeV/nucleon

beam energy

Nuclear structure, boundary of nuclear chart







◆ Relativistic HIC

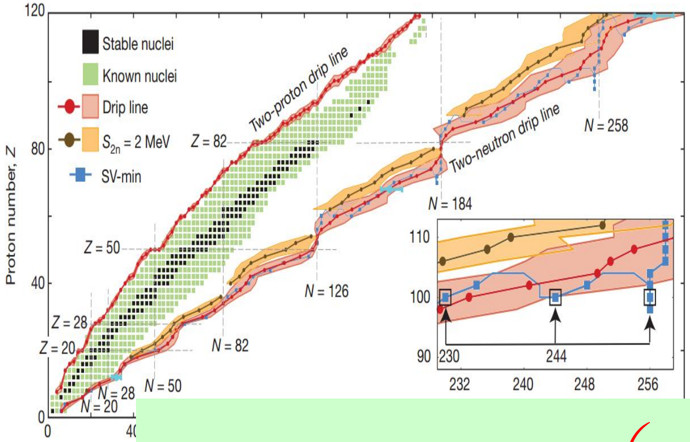
01

Nuclear equation of state (EOS)

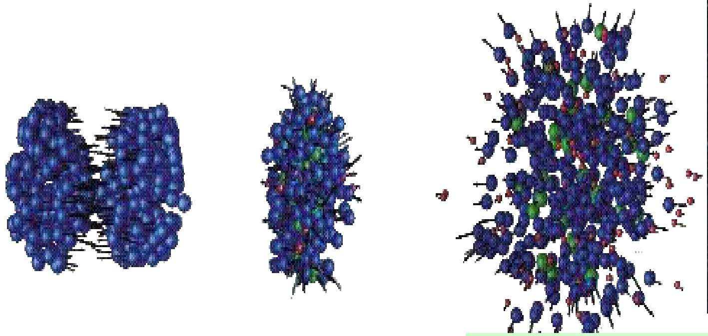
The thermodynamic relationship between the binding energy *E* (or pressure *P*) and density ρ, as well as the isospin asymmetry δ.

$$E(\rho,\delta)=E(\rho,0)+E_{sym}(\rho)\delta^2+\cdots,\qquad \delta=\frac{\rho_n-\rho_p}{\rho_n+\rho_p}$$

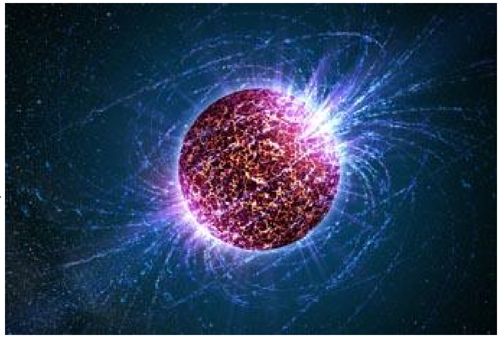
Nuclear landscape



Heavy ion collision



Neutron star



$$E(\rho,0)=E_0+\frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots,$$

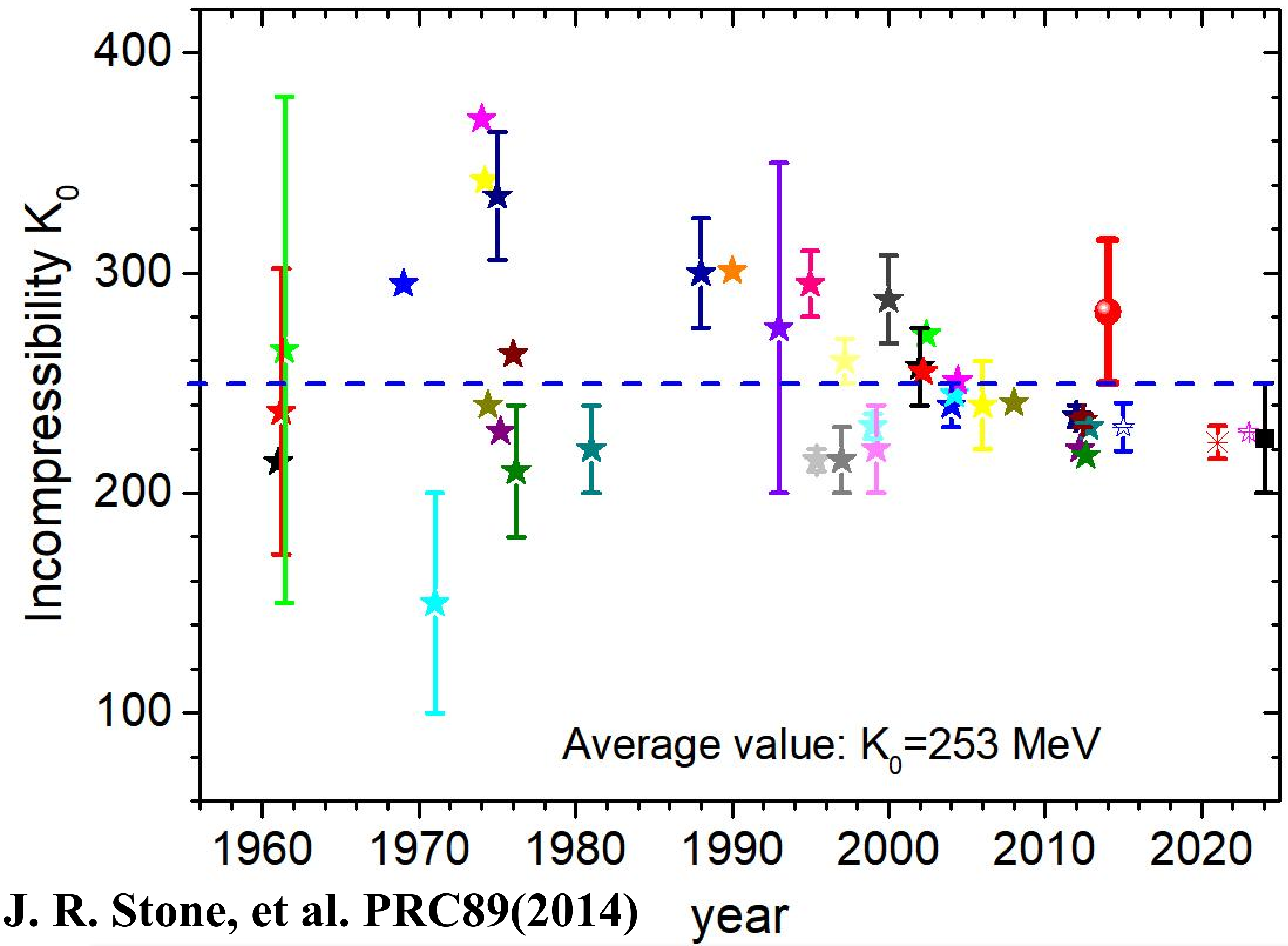
$$K_0=9\rho^2\left(\frac{\partial^2 E}{\partial\rho^2}\right)\Bigg|_{\rho=\rho_0}$$

$$E_{sym}(\rho)=S_0+L\left(\frac{\rho-\rho_0}{3\rho_0}\right)+\frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots$$

$$L=3\rho\frac{dE_{sym}(\rho)}{d\rho}\Bigg|_{\rho=\rho_0}$$

K_0 and L determine the EOS in the vicinity of the saturation density.

The incompressibility K_0 from properties of nuclei



J. R. Stone, et al. PRC89(2014)

Data:isoscalar giant monopole resonances (GMR), nuclear masses, nuclear decay...

Model:
Skyrme-Hartree-Fock model, Skyrme-Hartree-Fock-Bogoliubov, ...

Ning Wang, et al. PLB(2015)

Jun Xu, et al. PRC(2021)

Zhenzhen Li, et al. PRL(2023)

Xiao Liu, et al. PRC(2024)

The incompressibility K_0 from heavy-ion physics

VOLUME 55, NUMBER 24 PHYSICAL REVIEW LETTERS 9 DECEMBER 1985

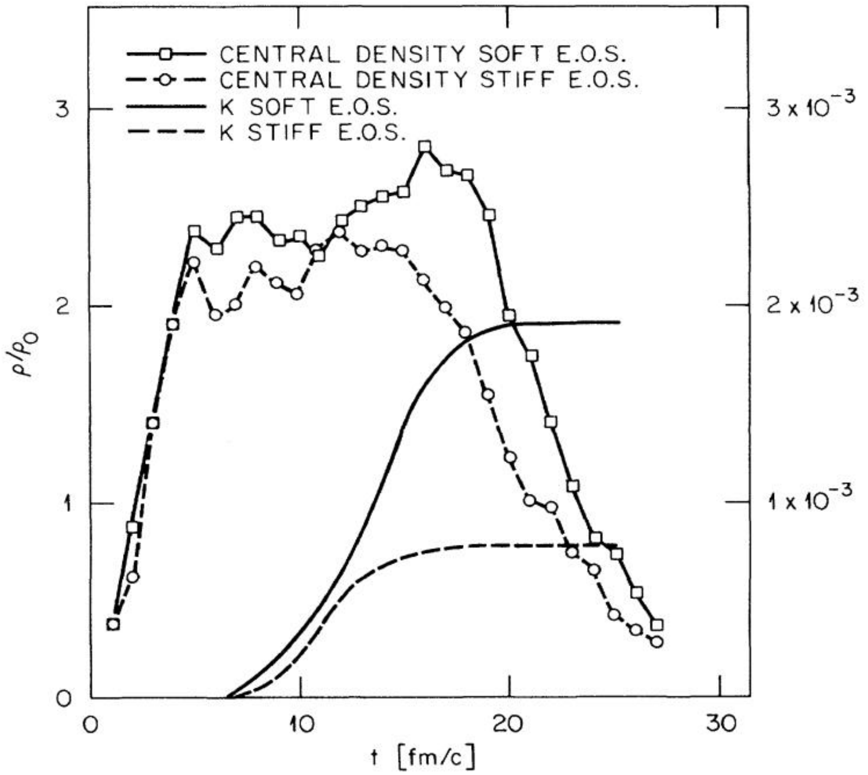
Subthreshold Kaon Production as a Probe of the Nuclear Equation of State

J. Aichelin and Che Ming Ko[(a)]

Joint Institute for Heavy Ion Research, Holifield Heavy Ion Research Facility, Oak Ridge, Tennessee 37831

(Received 11 June 1985; revised manuscript received 23 September 1985)

The production of kaons at subthreshold energies from heavy-ion collisions is sensitive to the nuclear equation of state. In the Boltzmann-Uehling-Uhlenbeck model, the number of produced kaons from central collisions between heavy nuclei at incident energies around 700 MeV/nucleon can vary by a factor of ~3, depending on the equation of state.



Data: pion and kaon production, collective flow

Model: QMD and BUU

Further evidence for a stiff nuclear equation of state from a transverse-momentum analysis of Ar(1800 MeV/nucleon) + KCl

Joseph J. Molitoris and Horst Stöcker

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Phys. Rev. C **32**, 346(R) – **Published 1 July, 1985**

Evidence for a Soft Nuclear Equation-of-State from Kaon Production in Heavy-Ion Collisions

C. Sturm[1], I. Böttcher[4], M. Dębowski[5], A. Förster[1], E. Grosse[6,7], P. Koczoń[2], B. Kohlmeyer[4], F. Laue[2], and M. Mang[2] *et al.*

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Phys. Rev. Lett. **86**, 39 – **Published 1 January, 2001**

Hadronic Matter Is Soft

Ch. Hartnack[1], H. Oeschler[2], and Jörg Aichelin[1]

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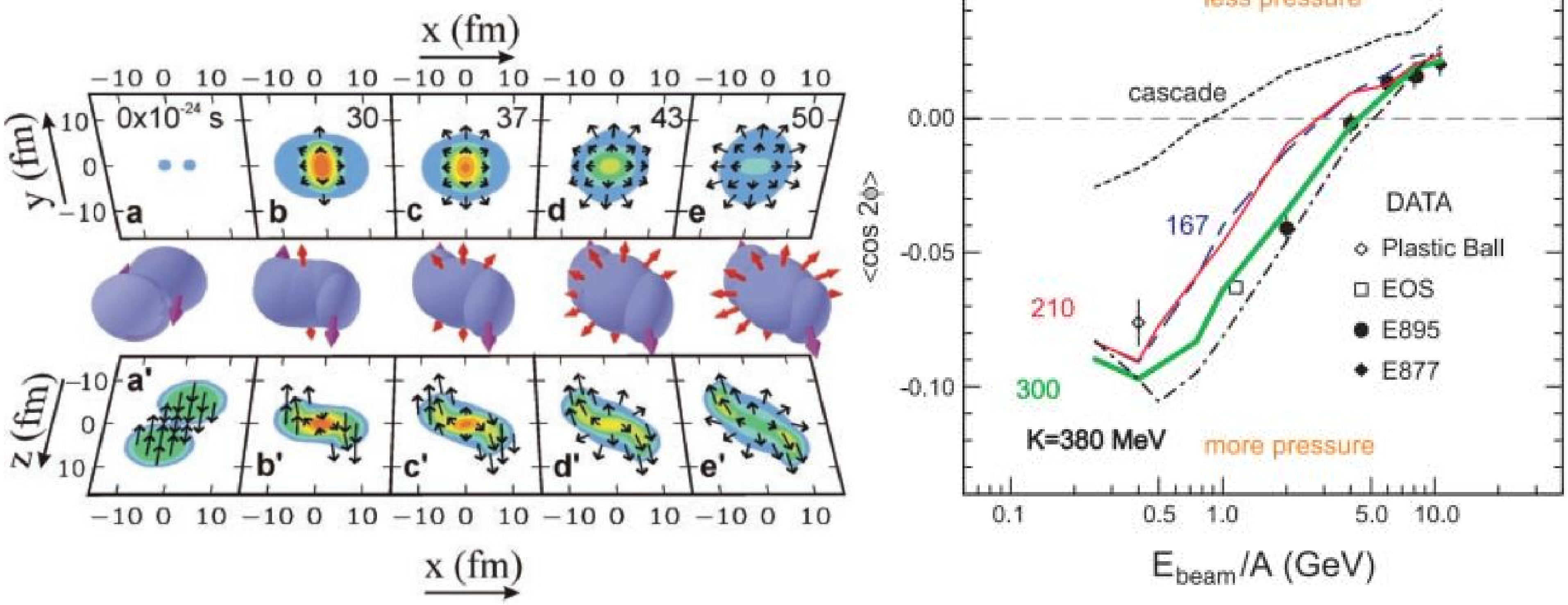
Phys. Rev. Lett. **96**, 012302 – **Published 9 January, 2006**

Stiff: K_0>380 MeV

Soft: K_0<200 MeV

HIC offers a unique way to create nuclear matter with high density and isospin asymmetry in laboratory.

Determination of the Equation of State of Dense Matter
Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
DOI: 10.1126/science.1078070



EOS can be deduced from the comparision bewteen experimental observables and transport model calculations.

01 Nuclear equation of state

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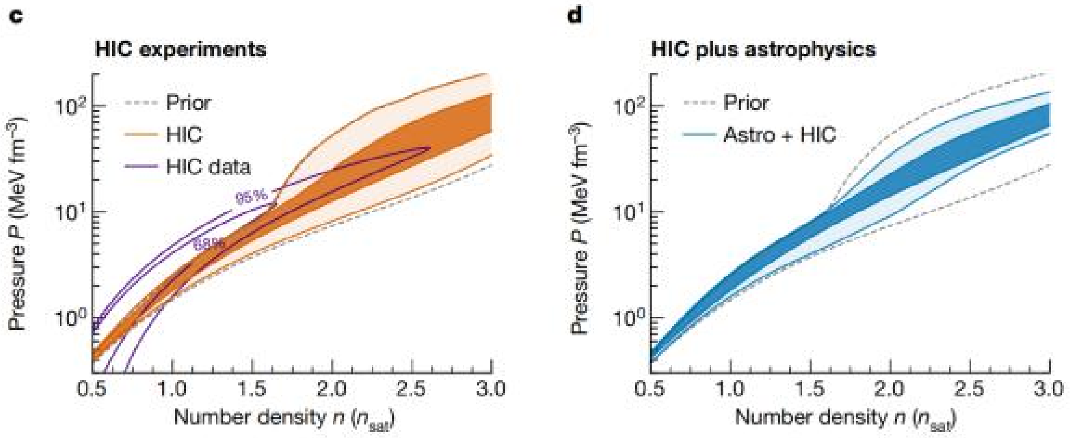
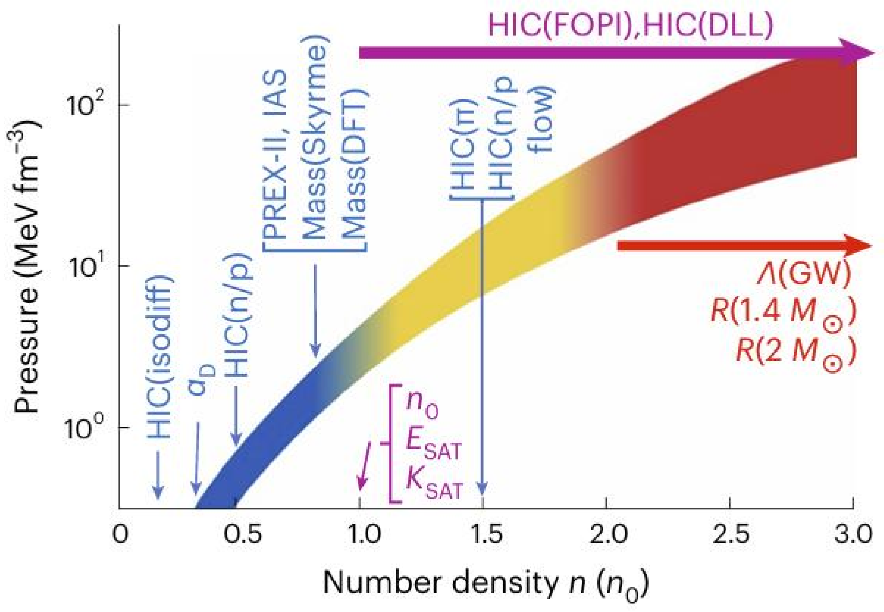
Constraining neutron-star matter with microscopic and macroscopic collisions

Sabrina Huth, Peter T. H. Pang, Ingo Tews, Tim Dietrich, Arnaud Le Fèvre, Achim Schwenk, Wo
Trautmann, Kshitij Agarwal, Mattia Bulla, Michael W. Coughlin & Chris Van Den Broeck





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Determination of the equation of state from nuclear experiments and neutron star observations

Chun Yuen Tsang, ManYee Betty Tsang, William G. Lynch, Rohit Kumar & Charles J. Horowitz

Nature Astronomy **8**, 328–336 (2024) | Cite this article

01 Nuclear equation of state

Two White papers

Progress in Particle and Nuclear Physics
Available online 19 September 2023, 104080
In Press, Journal Pre-proof   What's this?

Review

Dense nuclear matter equation of state from heavy-ion collisions

Agnieszka Sorensen [1], Kshitij Agarwal [2], Kyle W. Brown [3 4], Zbigniew Chajecki [5], Paweł Danielewicz [3 6], Christian Drischler [7], Stefano Gandolfi [8], Jeremy W. Holt [9 10], Matthias Kaminski [11], Che-Ming Ko [9 10], Rohit Kumar [3], Bao-An Li [12], William G. Lynch [3 6], Alan B. McIntosh [10], William G. Newton [12], Scott Pratt [3 6], Oleh Savchuk [3 13], Maria Stefaniak [14 15], Ingo Tews [8], ManYee Betty Tsang [3 6]...Yi Yin [94]

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https://doi.org/10.1016/j.ppnp.2023.104080   Get rights and content

Abstract

This White Paper highlights the essential role of hadronic transport simulations of heavy-ion collisions in studies involving the equation of state of nuclear matter. It also elucidates many connections between inferences of the equation of state from heavy-ion collision data and other efforts aiming to understand the properties of nuclear matter.

arXiv > nucl-th > arXiv:2211.02224

Nuclear Theory

[Submitted on 4 Nov 2022 (v1), last revised 8 Nov 2022 (this version, v2)]

Long Range Plan: Dense matter theory for heavy-ion collisions and neutron stars

Alessandro Lovato, Travis Dore, Robert D. Pisarski, Bjoern Schenke, Katerina Chatziioannou, Jocelyn S. Read, Philippe Landry, Paw
Hannah Elfner, Veronica Dexheimer, Rajesh Kumar, Michael Strickland, Johannes Jahan, Claudia Ratti, Volodymyr Vovchenko, Mikh
Hippert, Jacquelyn Noronha-Hostler, Jorge Noronha, Enrico Speranza, Nicolas Yunes, Chuck J. Horowitz, Steven P. Harris, Larry Mc
Stefano Gandolfi, Ingo Tews, M. Coleman Miller, Cecilia Chirenti, Zohreh Davoudi, Jamie M. Karthein, Krishna Rajagopal, Salvatore
Vladimir Skokov, Ulrich Heinz, Christian Drischler, Daniel R. Phillips, Madappa Prakash, Zoltan Fodor, David Radice, Christopher Plu
Fraga, Aleksi Kurkela, James M. Lattimer, Andrew W. Steiner, Jeremy W. Holt, Bao-An Li, Chun Shen, Mark Alford, Alexander Haber,

Since the release of the 2015 Long Range Plan in Nuclear Physics, major events have occurred that reshaped our understanding of quantum chromodyr
of equilibrium. The US nuclear community has an opportunity to capitalize on advances in astrophysical observations and nuclear experiments and enga
matter that connects low- and high-energy nuclear physics, astrophysics, gravitational waves physics, and data science

Comments: 70 pages, 3 figures, White Paper for the Long Range Plan for Nuclear Science
Subjects: Nuclear Theory (nucl-th); High Energy Astrophysical Phenomena (astro-ph.HE); High Energy Physics - Phenomenology (hep-ph)
Report number: LA-UR-22-31648
Cite as: arXiv:2211.02224 [nucl-th]
(or arXiv:2211.02224v2 [nucl-th] for this version)
https://doi.org/10.48550/arXiv.2211.02224

Submission history

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[v2] Tue, 8 Nov 2022 01:52:13 UTC (2,373 KB)

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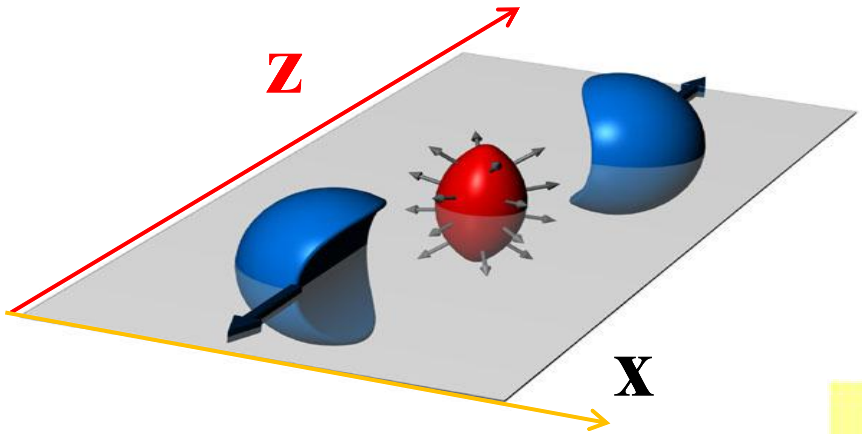
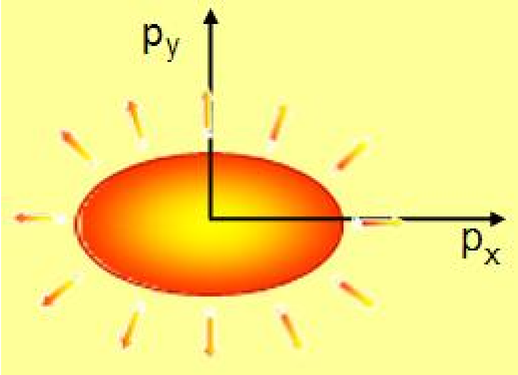
01 Elliptic flow (v_2)

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \qquad p_t = \sqrt{p_x^2 + p_y^2}\,;\; y = \frac{1}{2}\ln\left[\frac{E + p_z}{E - p_z}\right]$$

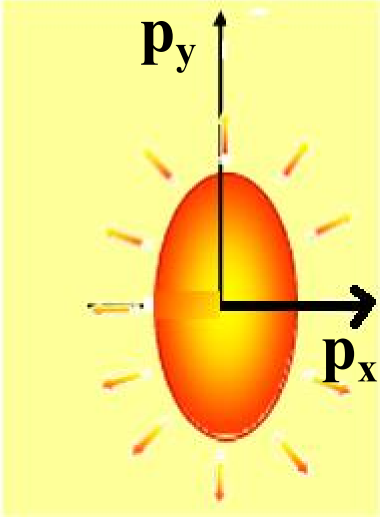
Coordinate-space anisotropy

Momentum-space anisotropy

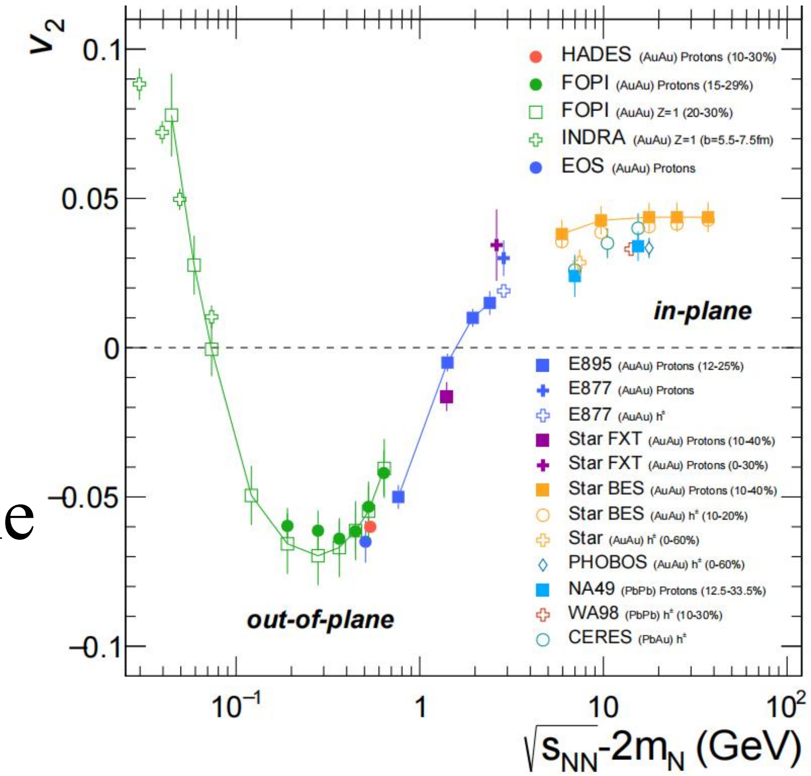




In-plane
$v_2 > 0$



Out-of-plane
Squeeze out
$v_2 < 0$

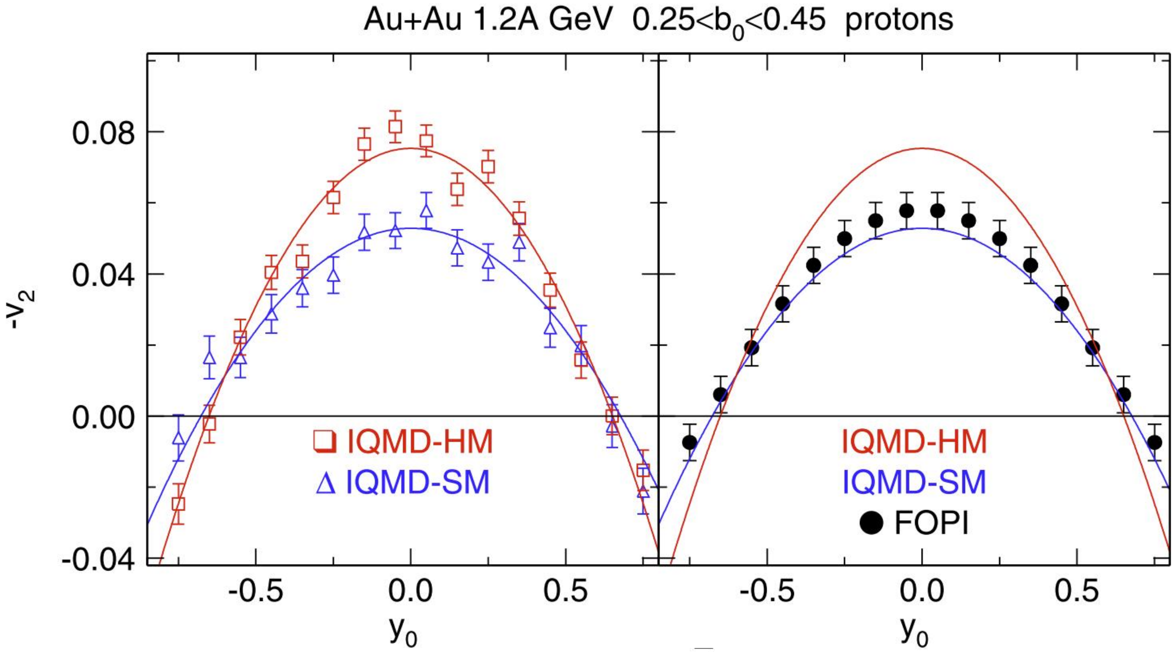


v_2 is a multivariate function of rapidity (y_z) and transverse momentum (p_t)

v_2 has been widely used to probe the nuclear EOS, the nucleon effective mass,...

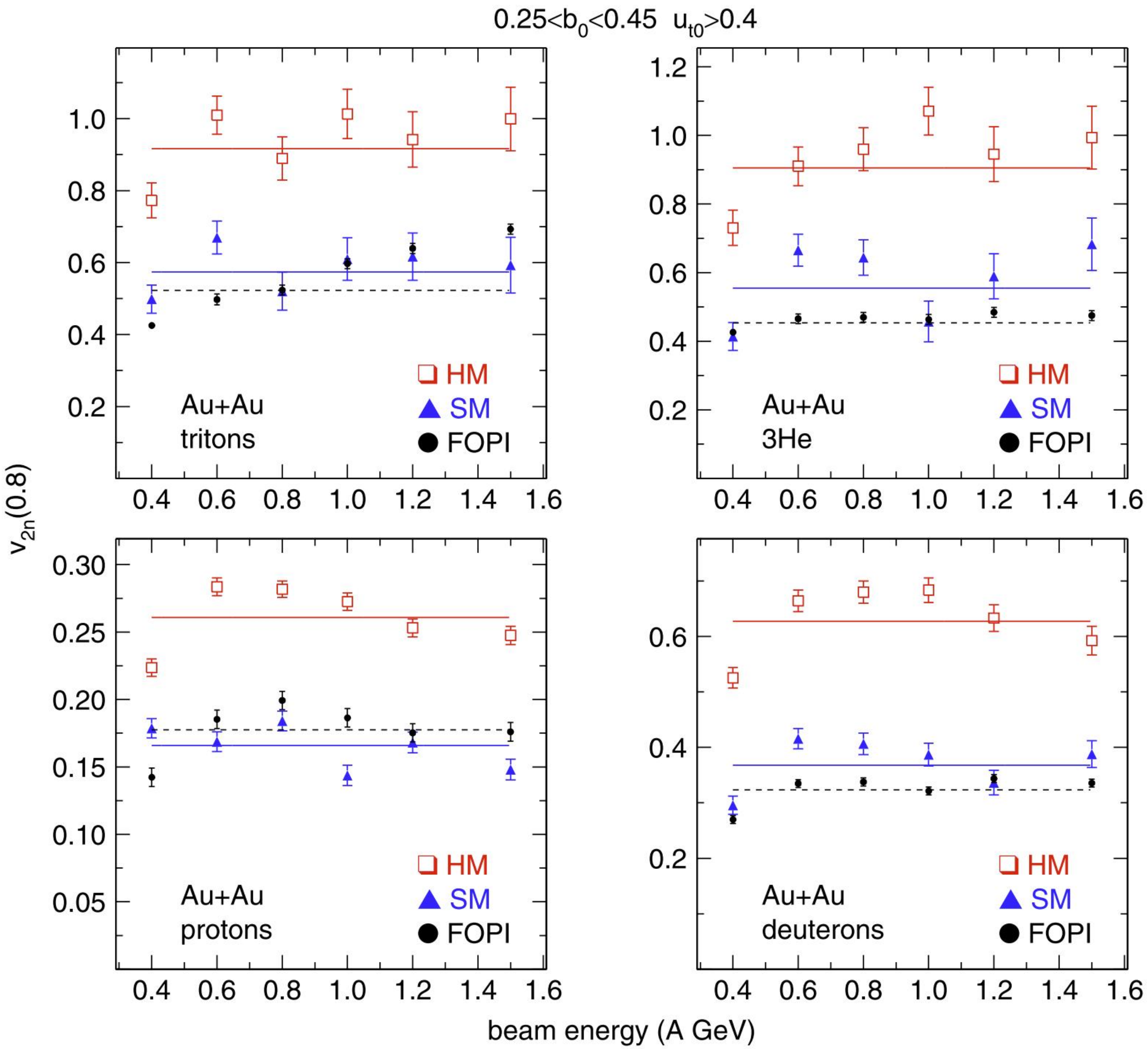
01 Elliptic flow (v_2)

Le Fevre, A., Leifels, Y., Reisdorf, W., Aichelin, J., & Hartnack, C. (2016). Constraining the nuclear matter equation of state around twice saturation density. *Nuclear Physics A*, *945*, 112-133.



$$v_2 = v_{20} + v_{22} \cdot y_0^2.$$

$$v_{2n} = |v_{20}| + |v_{22}|$$



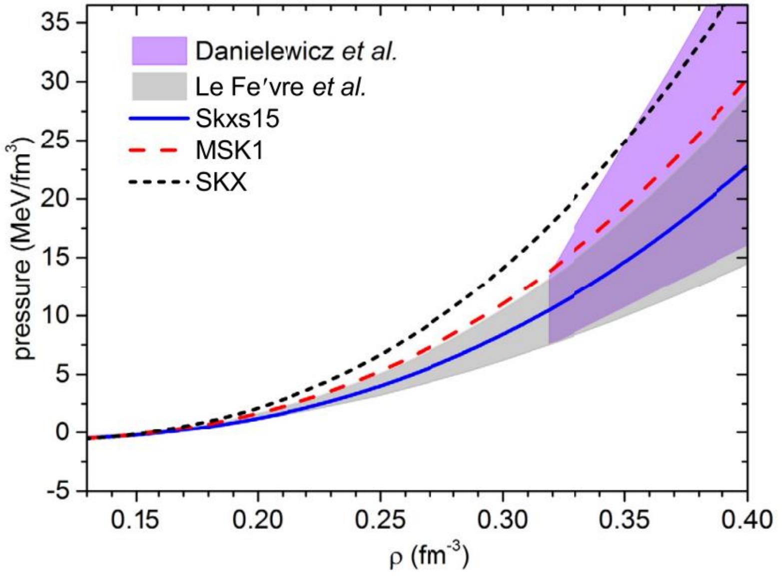
01 Elliptic flow (v_2)

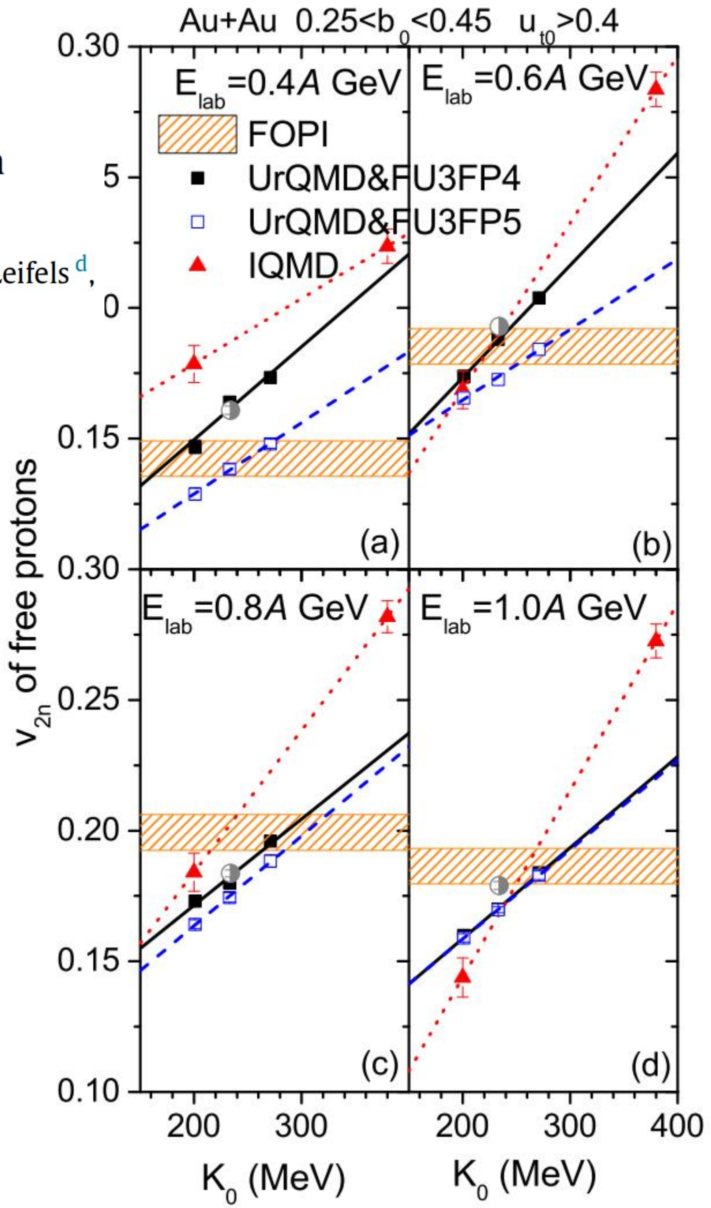
Determination of the nuclear incompressibility from the rapidity-dependent elliptic flow in heavy-ion collisions at beam energies 0.4*A*–1.0*A* GeV

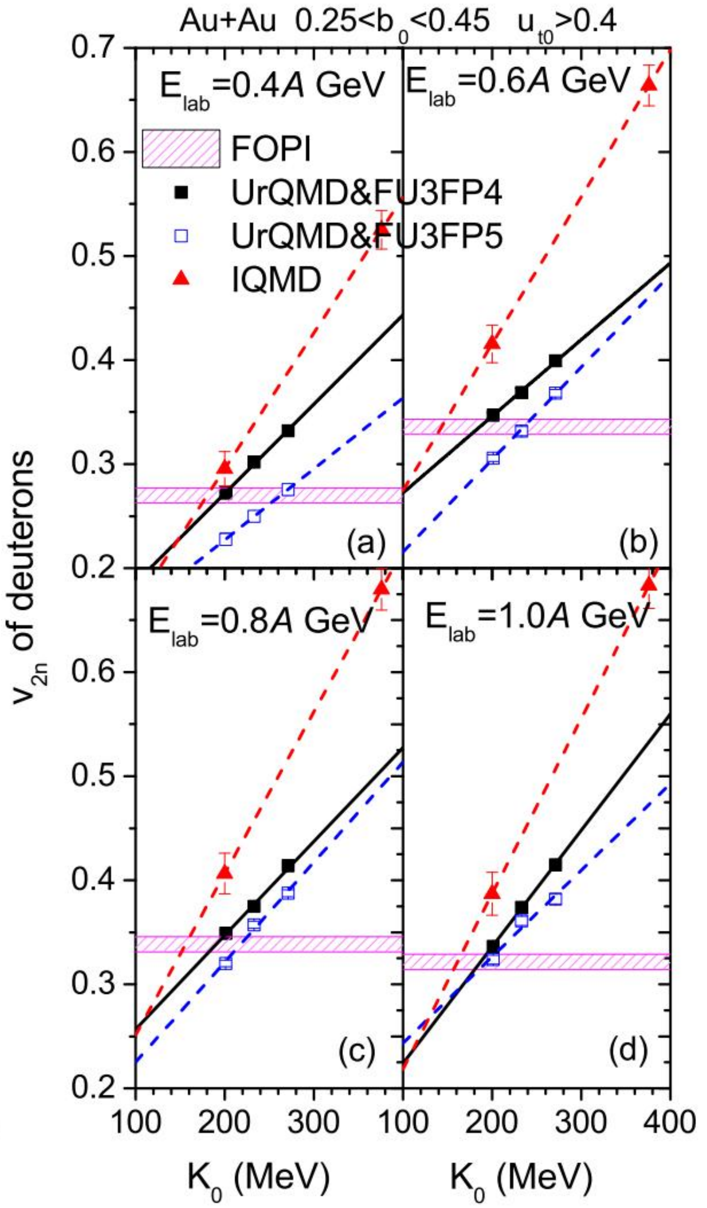
Yongjia Wang [a,*], Chenchen Guo [b], Qingfeng Li [a,c,**], Arnaud Le Fèvre [d], Yvonne Leifels [d], Wolfgang Trautmann [d]

Physics Letters B 778 (2018) 207–212.

	K_0 (MeV)	S_0 (MeV)	L (MeV)
Skxs15	201	31.88	34.79
MSK1	234	30.00	33.92
SKX	271	31.10	33.18
SV-sym34	234	34.00	80.95





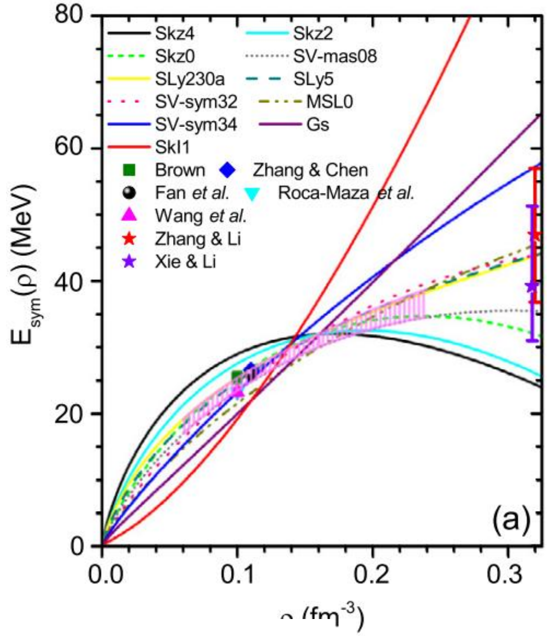


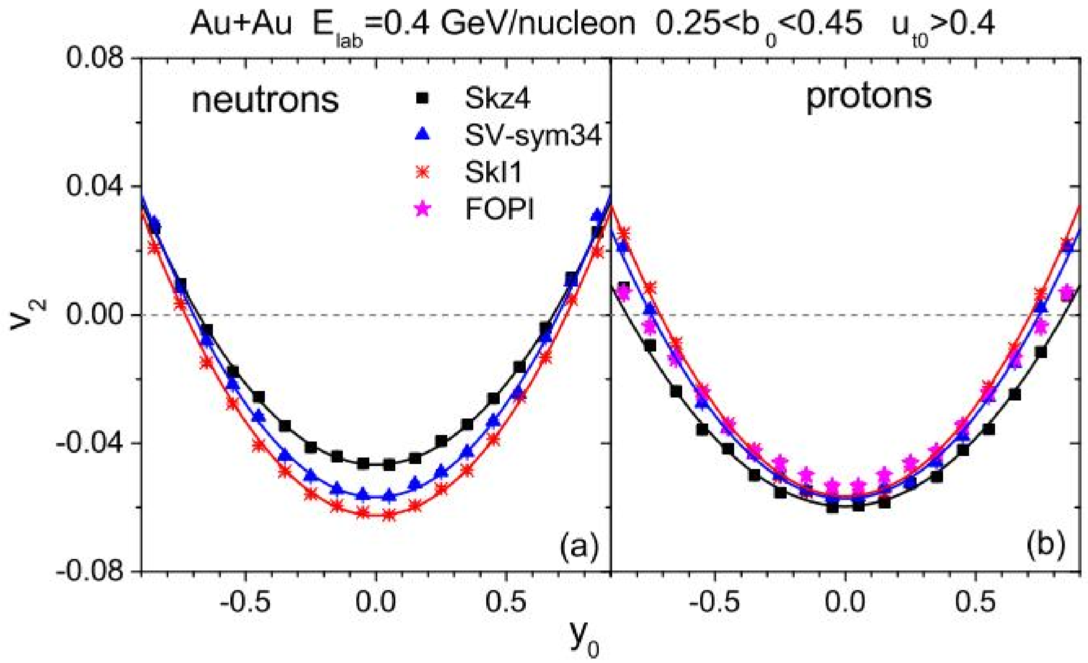
01 Elliptic flow (v_2)

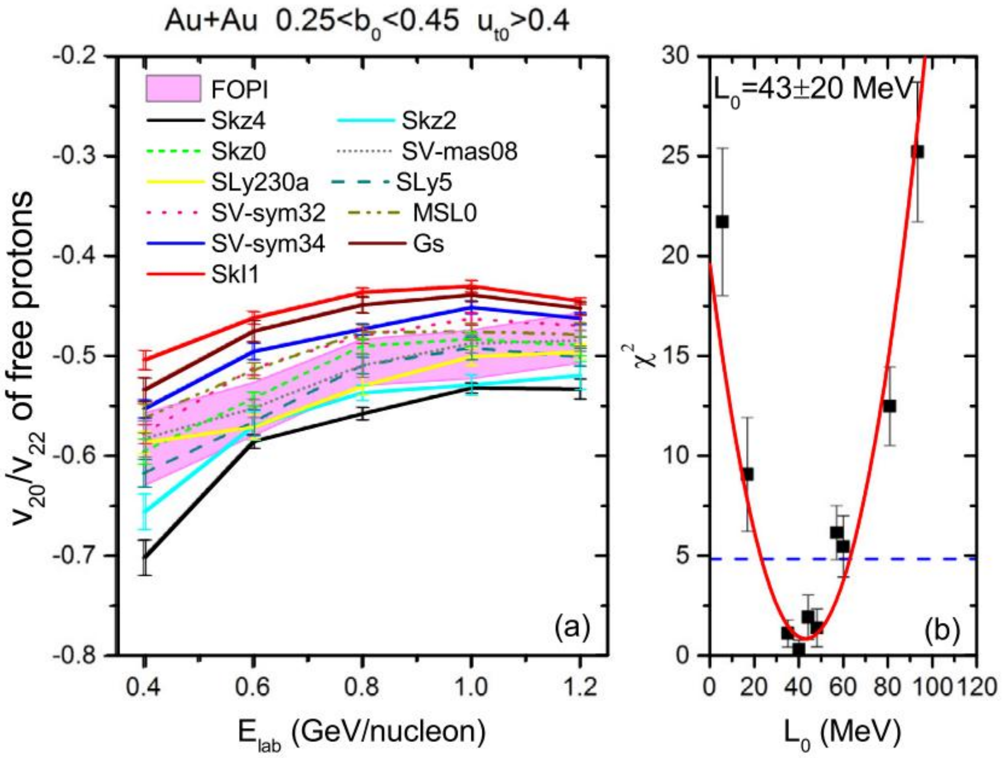
Study of the nuclear symmetry energy from the rapidity-dependent elliptic flow in heavy-ion collisions around 1 GeV/nucleon regime

Yongjia Wang [a], Qingfeng Li [a,b,*], Yvonne Leifels [c], Arnaud Le Fèvre [c]

Physics Letters B 802 (2020) 135249





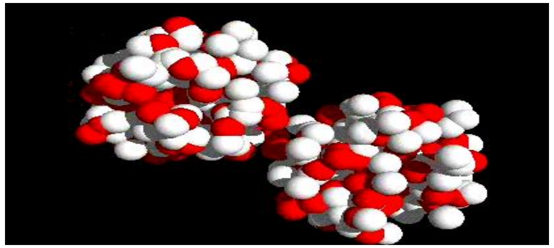


Ultrarelativistic Quantum Molecular Dynamics (UrQMD) Model

1).Initialization

Get the coordinate **r**

and the momentum **p**



$$\phi_i(\vec{r}_i;t) = \frac{1}{(2\pi)^{3/4}(\Delta x)^{3/2}} \exp\left\{-\frac{[\vec{r}_i - \vec{R}_i(t)]^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t)\right\}.$$

2).Propagation

Nucleon moves in nuclear potential.

Density, momentum, isospin-dependent.

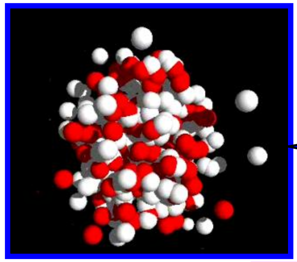
$$\dot{\boldsymbol{p}}_i = -\frac{\partial \langle H \rangle}{\partial \boldsymbol{r}_i} \quad \text{and} \quad \dot{\boldsymbol{r}}_i = \frac{\partial \langle H \rangle}{\partial \boldsymbol{p}_i},$$

Input nuclear potential

EOS, Symmetry energy

3).Collision term

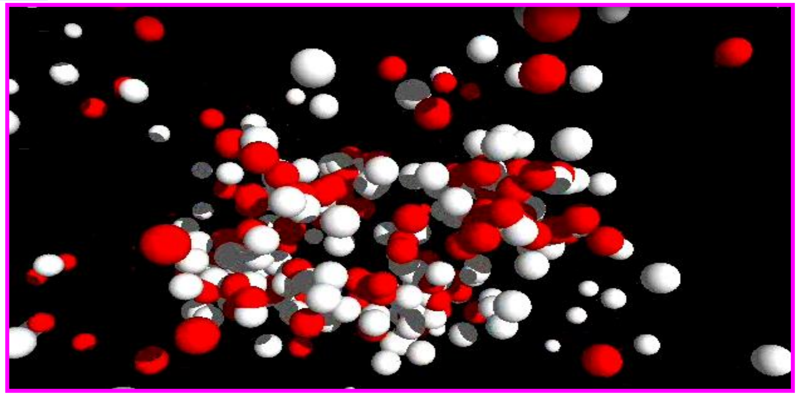
Medium modified cross section. Density, momentum, isospin-dependent. Pauli blocking.



Input: $F * \sigma_{NN}^{free}$

4). Cluster recognition

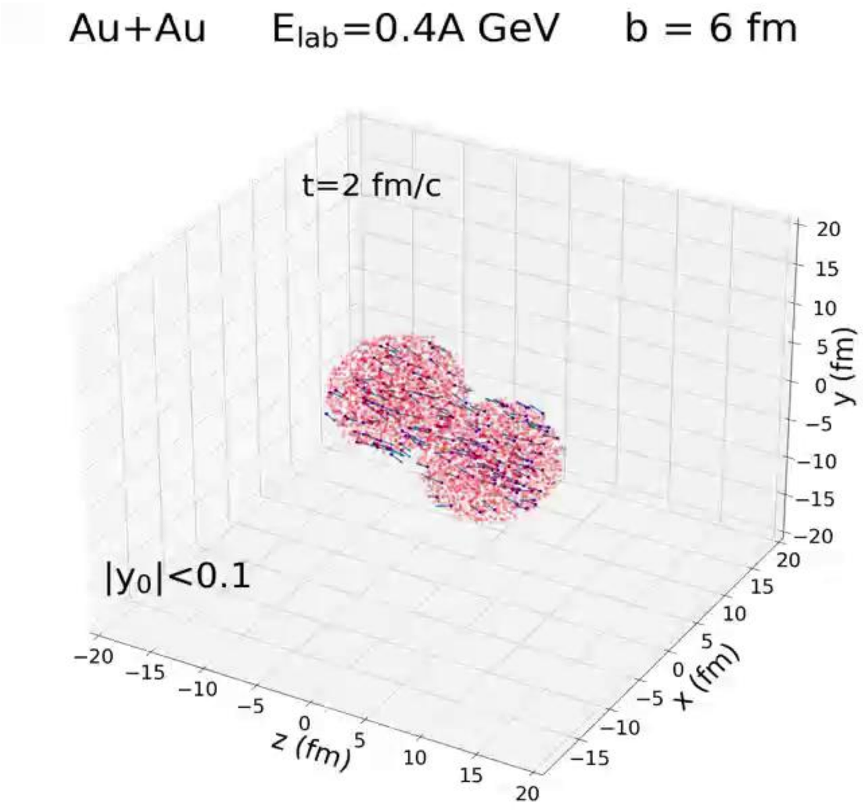
an isospin-dependent Minimum Spanning Tree

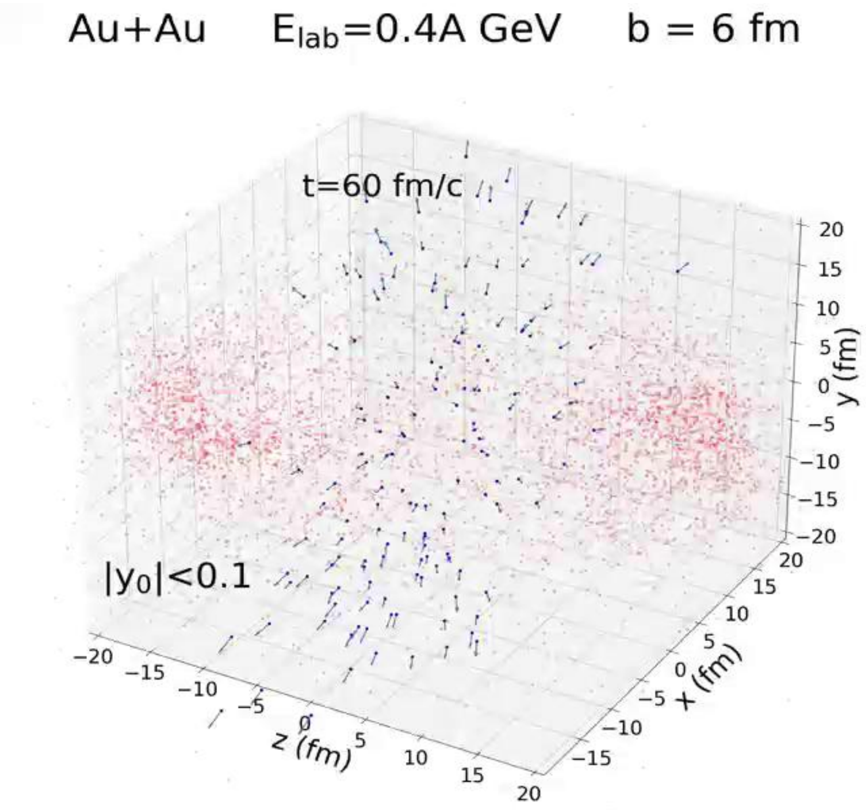


YJW, et al. PRC 83.044617; 89.034606.

02 Elliptic flow

The momentum of each nucleon can be changed either by the force caused by the nuclear mean field (MF) potential or by nucleon-nucleon (NN) scattering.





$$v_2 = \langle V_2 \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle.$$

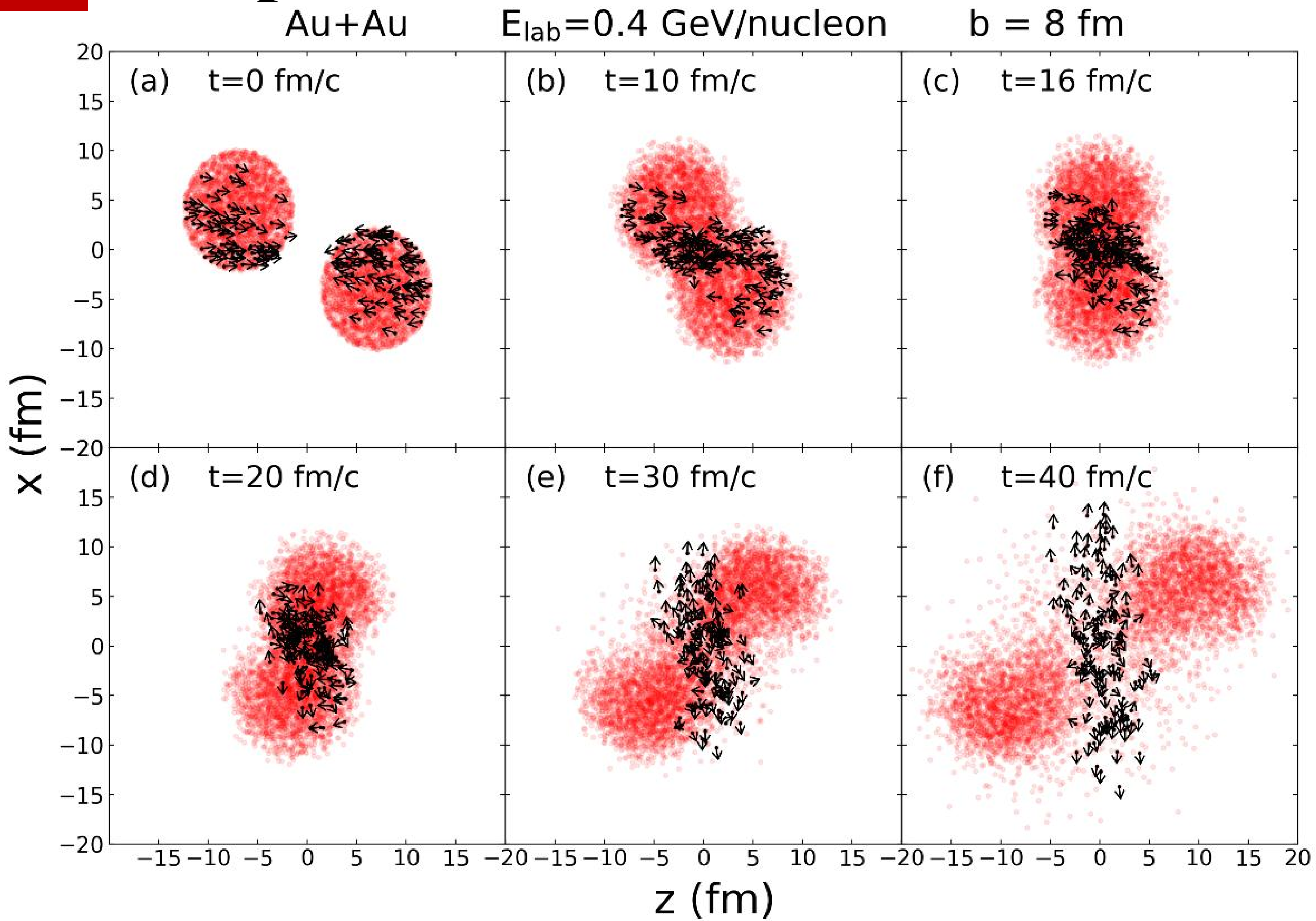
$$\Delta v_2^{\mathrm{coll}}(t) = \left\langle V_2^{\mathrm{aft.coll}}(t) - V_2^{\mathrm{bef.coll}}(t) \right\rangle.$$

$$\Delta v_2^{\mathrm{mf}}(t) = \left\langle V_2^{\mathrm{aft.mf}}(t+\Delta t) - V_2^{\mathrm{bef.mf}}(t) \right\rangle.$$

Reversely tracing nucleons that are finally emitted at mid-rapidity ($-0.1<y_0<0.1$).
Recording the momentum changes casued by MF and NN collisions.

Physics Letters B 838 (2023) 137685

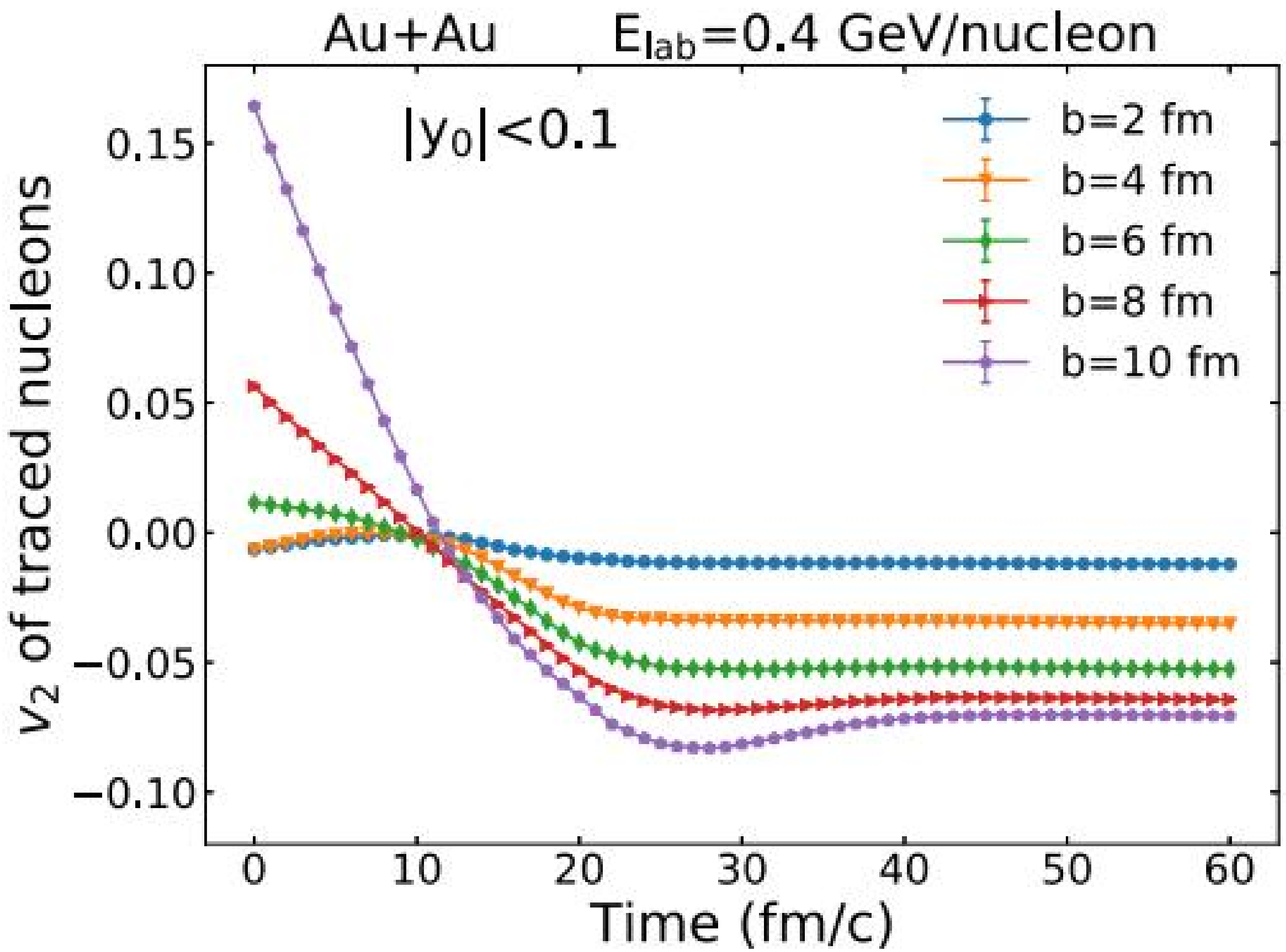
02 Elliptic flow



Solid dots with arrows represent the traced nucleons (those will be appeared at mid-rapidity at the final stage), while circles denote other nucleons.

02 Elliptic flow

Time evolution of v_2 of the **traced nucleons finally emitted at mid-rapidity** ($|y_0| < 0.1$) with different impact parameters.



At the initial time, v_2 is non-zero, indicating that those nucleons **are not randomly "selected"** out of the uniform distribution in the initial state. This is especially so for large b where v_2(t=0 fm/c) is positive because nucleons with larger |px| and smaller |py| tend to have higher probability to experience NN scattering and finally emerge at midrapidity.

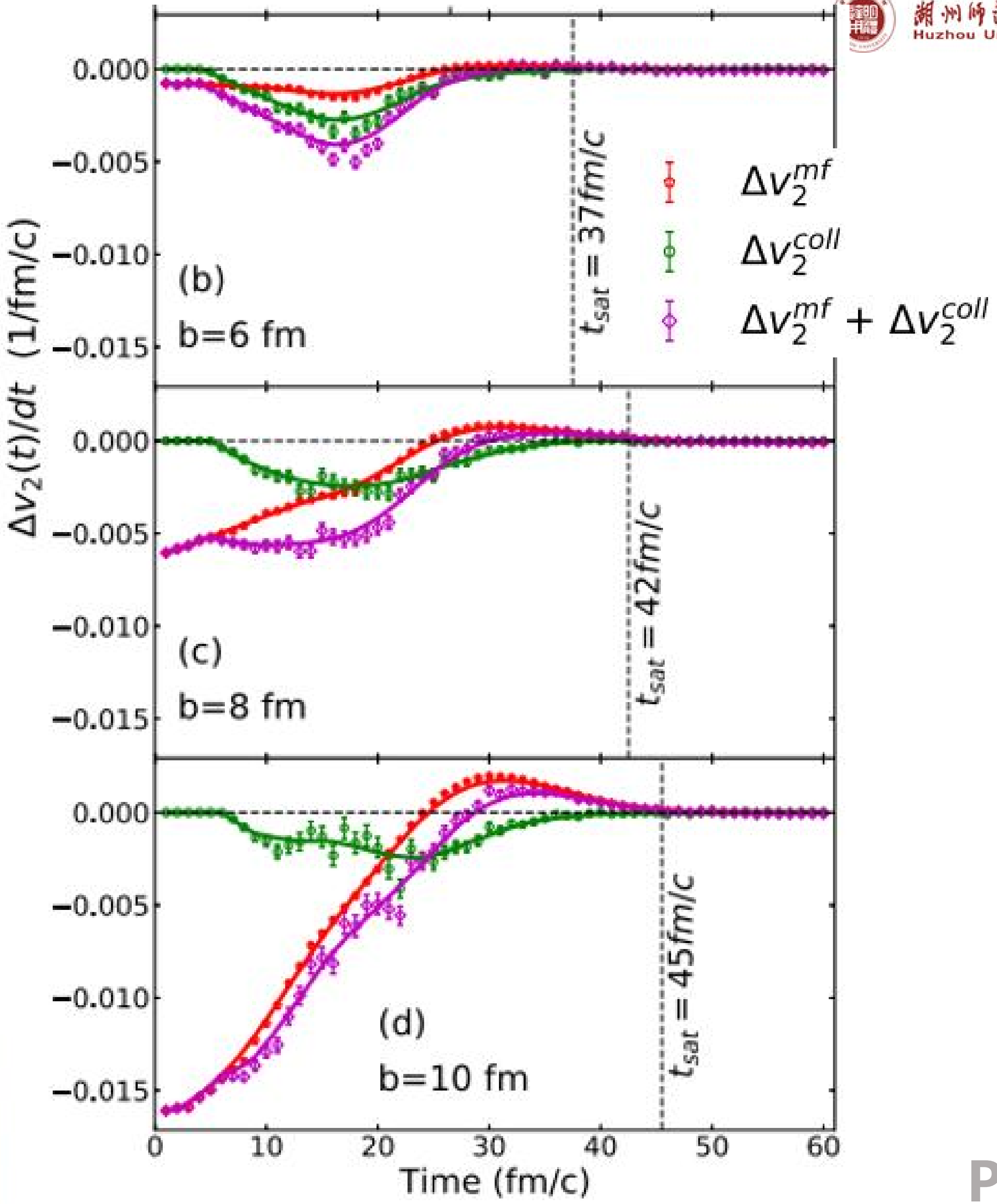
02 Machine learning

$$v_2 = \langle V_2 \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle.$$

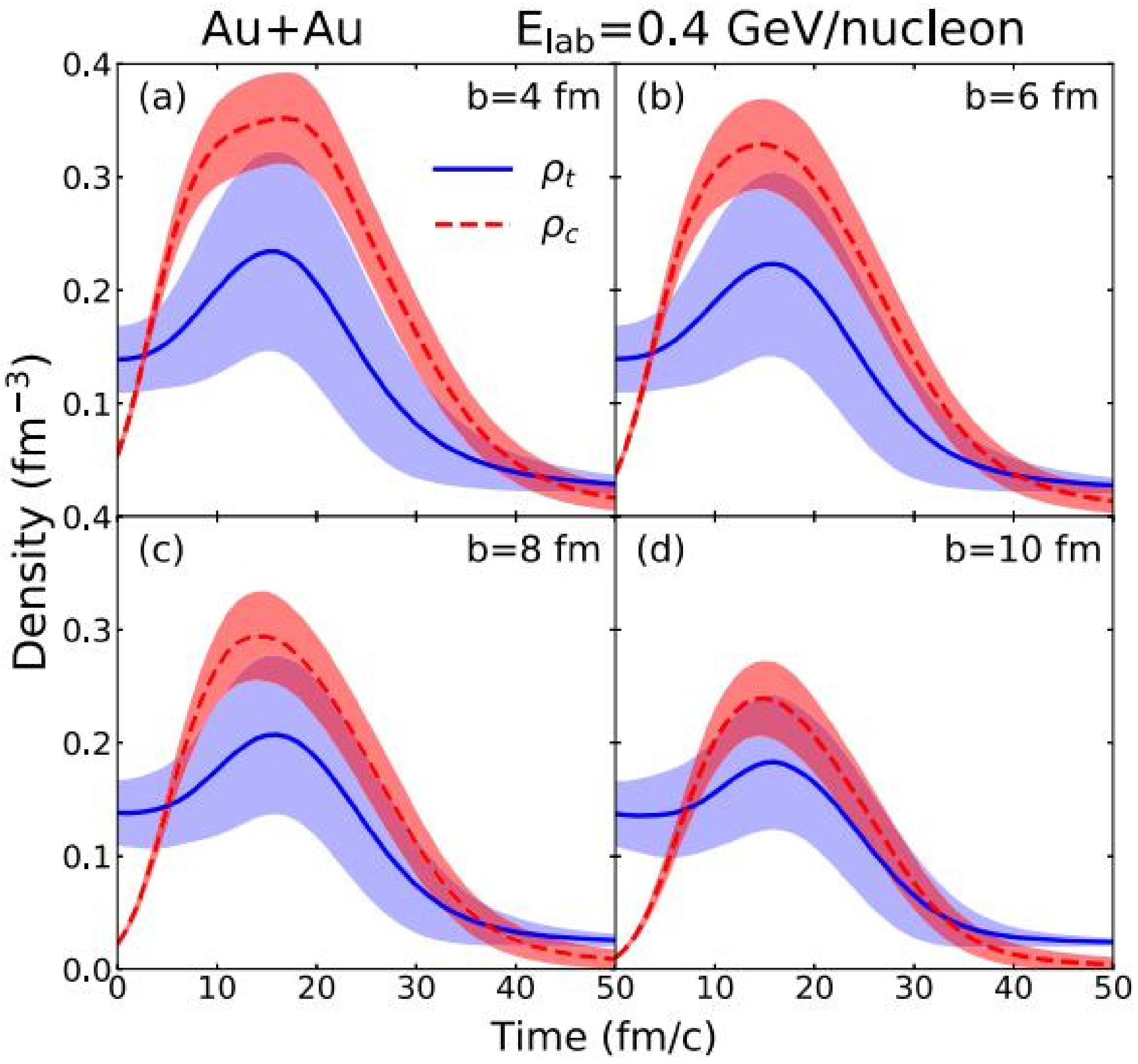
$$\Delta v_2^{\mathrm{coll}}(t) = \left\langle V_2^{\mathrm{aft.coll}}(t) - V_2^{\mathrm{bef.coll}}(t) \right\rangle.$$

$$\Delta v_2^{\mathrm{mf}}(t) = \left\langle V_2^{\mathrm{aft.mf}}(t+\Delta t) - V_2^{\mathrm{bef.mf}}(t) \right\rangle.$$

1. At the initial time, $\Delta v_2^{coll}(t)$ is zero because it takes approximately 5 fm/c for target and projectile nucleons to encounter each other.
2. $\Delta v_2^{coll}(t)$ is always negative which indicates NN collision enhances the signal of out-of-plane elliptic flow (decreases the value of v_2). This is because the outgoing nucleons along the reaction plane will be blocked.
3. The value of $\Delta v_2^{mf}(t)$ can be either negative or positive, depending on the impact parameter and time.



02 The density interval at which the nuclear equation of state is probed by the elliptic flow



ρ_c denotes the density at the central region.
ρ_t denotes the density experienced by the traced nucleons.

$$\langle \rho \rangle_{v_2} = \frac{\int_0^{t_{sat}} |\Delta v_2(t)| \rho_t(t) dt}{\int_0^{t_{sat}} |\Delta v_2(t)| dt}.$$

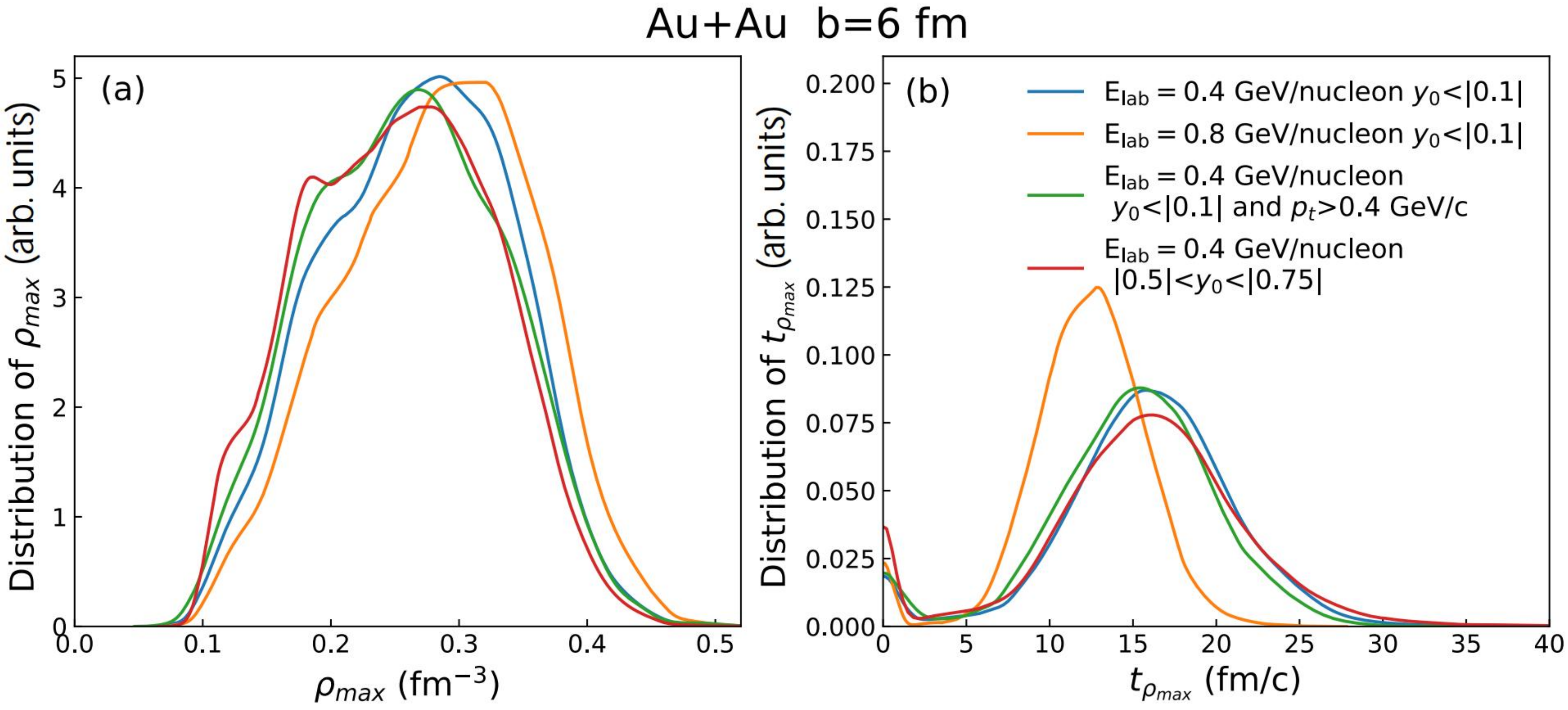
The most relevant density probed by v_2 of nucleons at mid-rapidity lies in **$(0.9\text{-}1.3)\rho_0$** and dependent of the impact parameter.

Their values are found to be **60%** of the maximum density reached during the collisions.

02 The density interval at which the nuclear equation of state is probed by the elliptic flow

Wang, et al. PRC110, 044606 (2024)

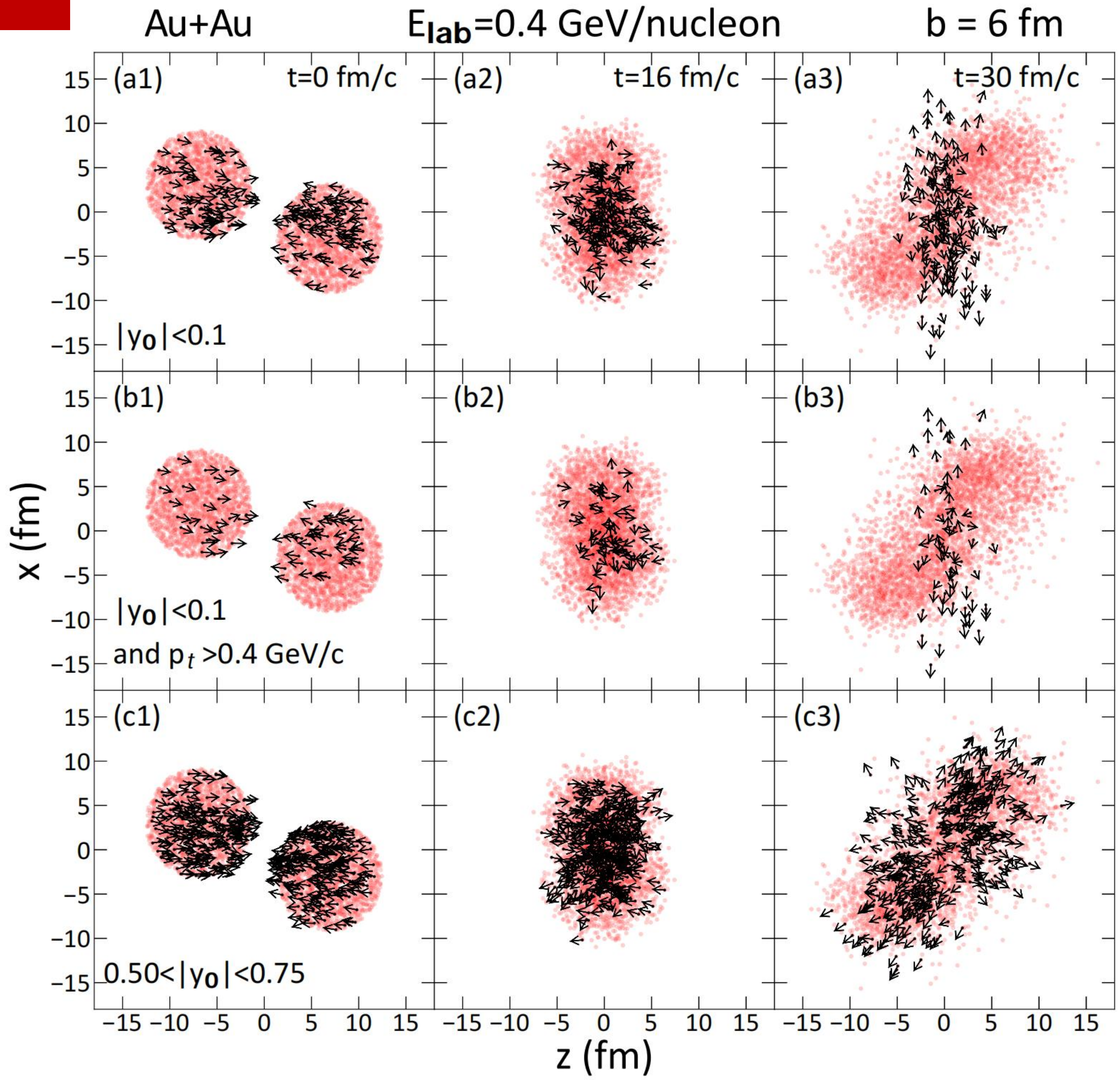
Anonymous referee: different nucleons reach their maximum densities at different times.



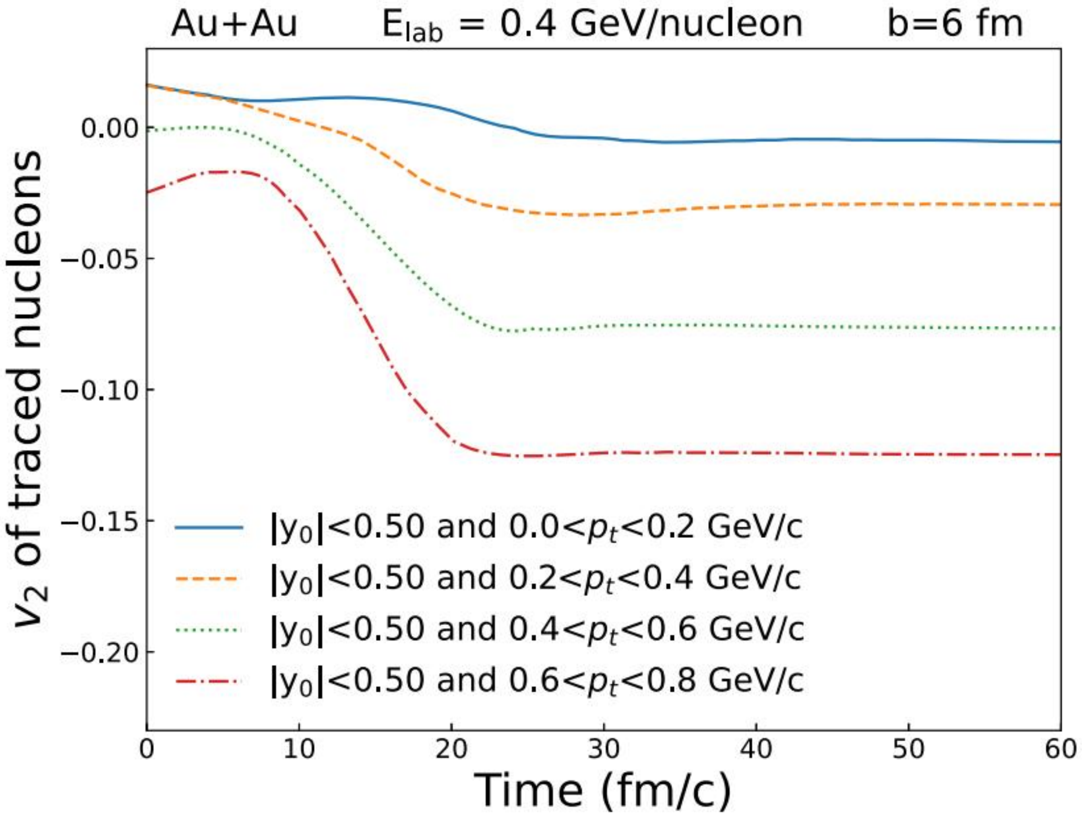
The distributions of the maximum density (ρ_{max}) defined for each traced nucleon.

The average of ρ_{max} is about **80%** of the maximum average density created in the central region.

02 Effects of rapidity and transverse momentum cuts

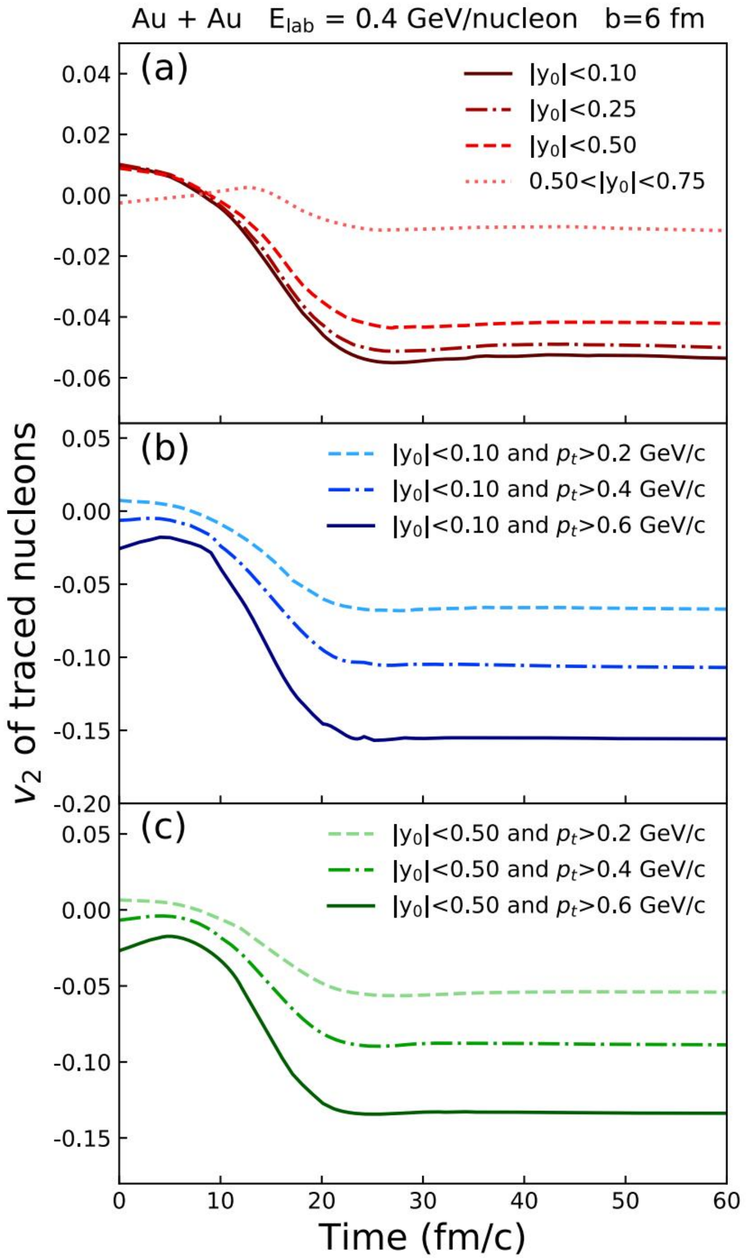


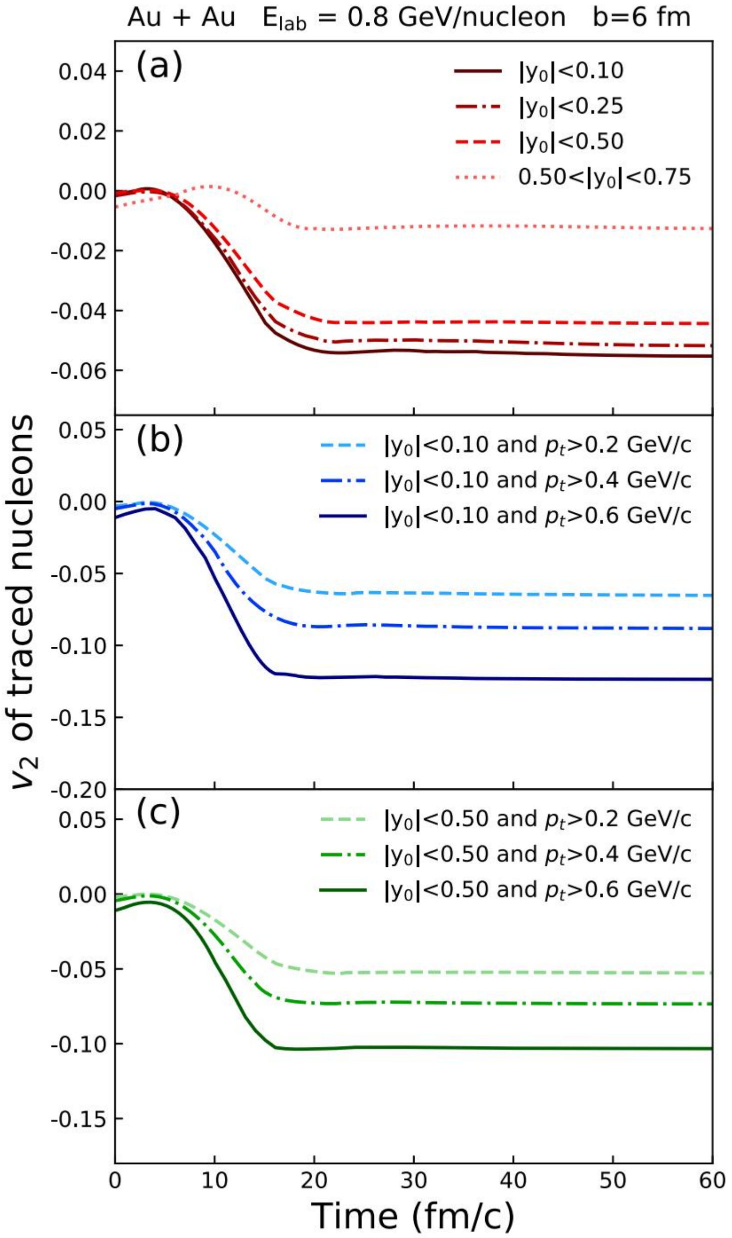
Wang, et al. PRC110, 044606 (2024)



02

Effects of rapidity and transverse momentum cuts





Initial-final state correlation in the momentum space. This correlation becomes rather weak at E_{lab} = 0.8 GeV/nucleon.

03 Bayesian analysis of properties of nuclear matter with FOPI data

150 runs of different parameter sets in UrQMD to train emulator, and 20 runs for validate the trained emulator. 100, 000 events for each run.

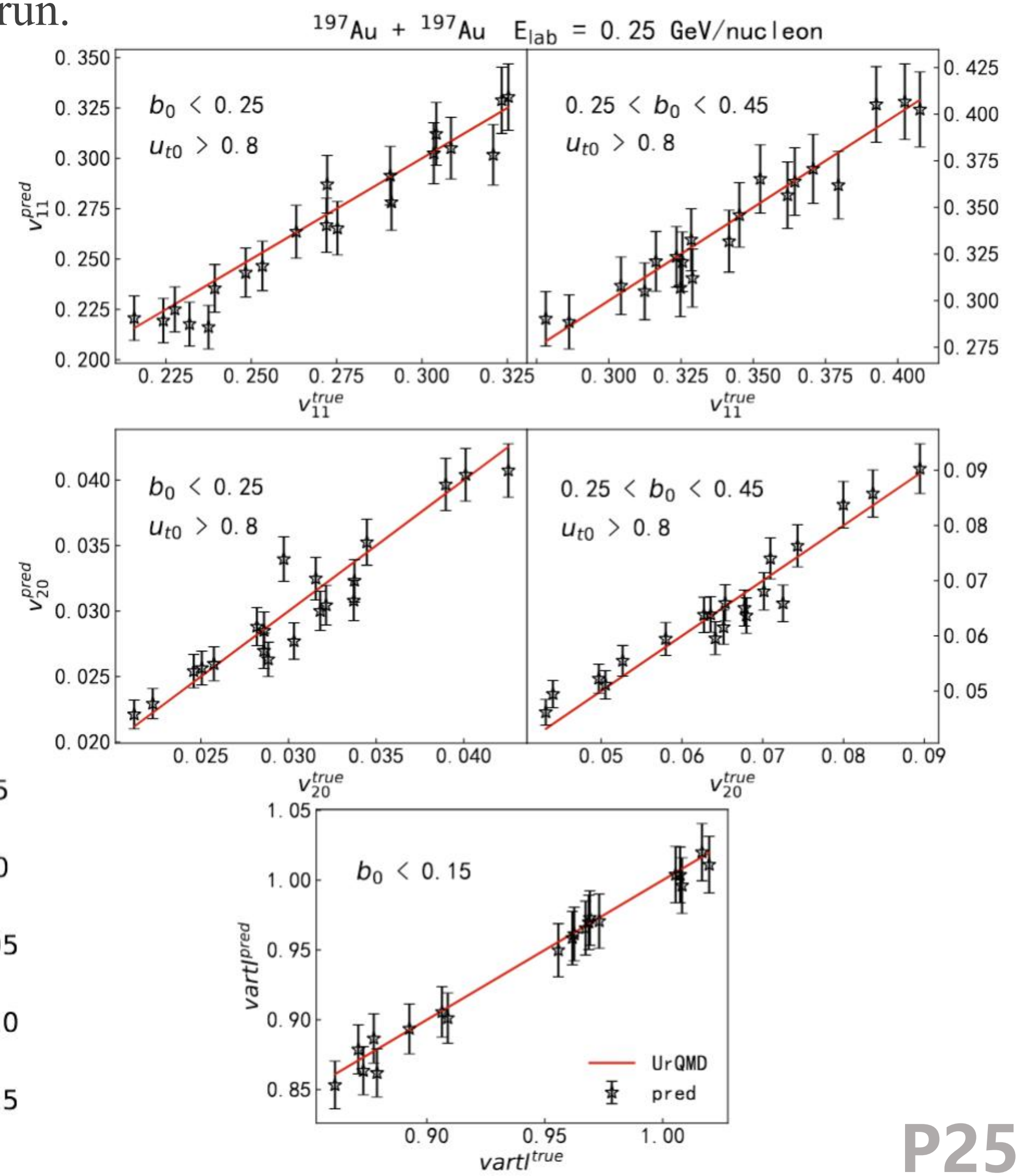
Table I. List of observables used in the analysis in ^{197}Au + ^{197}Au collisions with a beam energy of 0.25 GeV/nucleon

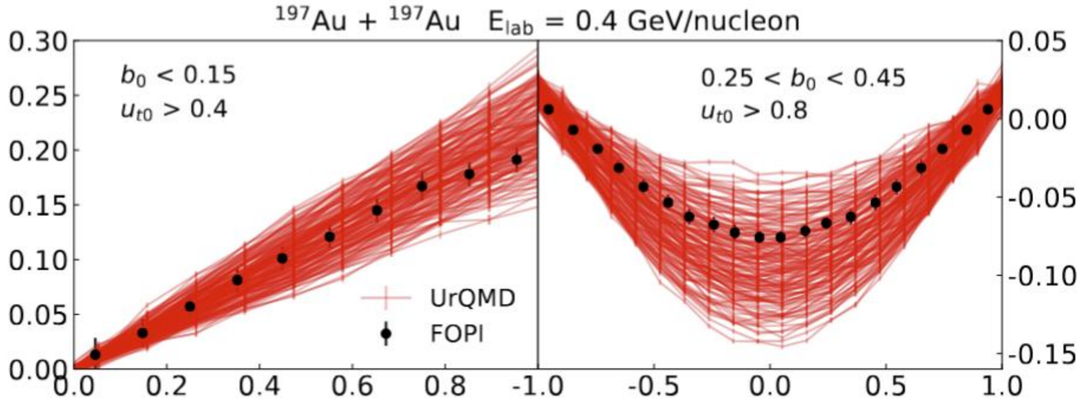
Observable	b_0	u_{t0}	value
v_{11}	$b_0 < 0.25$	$u_{t0} > 0.8$	0.23±0.01
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.37±0.01
$-v_{20}$	$b_0 < 0.25$	$u_{t0} > 0.8$	0.026±0.001
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.046±0.005
$vartl$	$b_0 < 0.15$	None	0.891±0.041

Table III. Parameters used in the present work

Para. Name	Description	Prior ranges
K_0	Incompressibility	[180, 380]
m^*	Isoscalar effective mass	[0.6, 0.95]
F	In-medium correction factor	[0.5, 1.0]

Gaussian process (GP) model is trained as an **emulator** of UrQMD model to interpolate the simulation results in the parameter space.

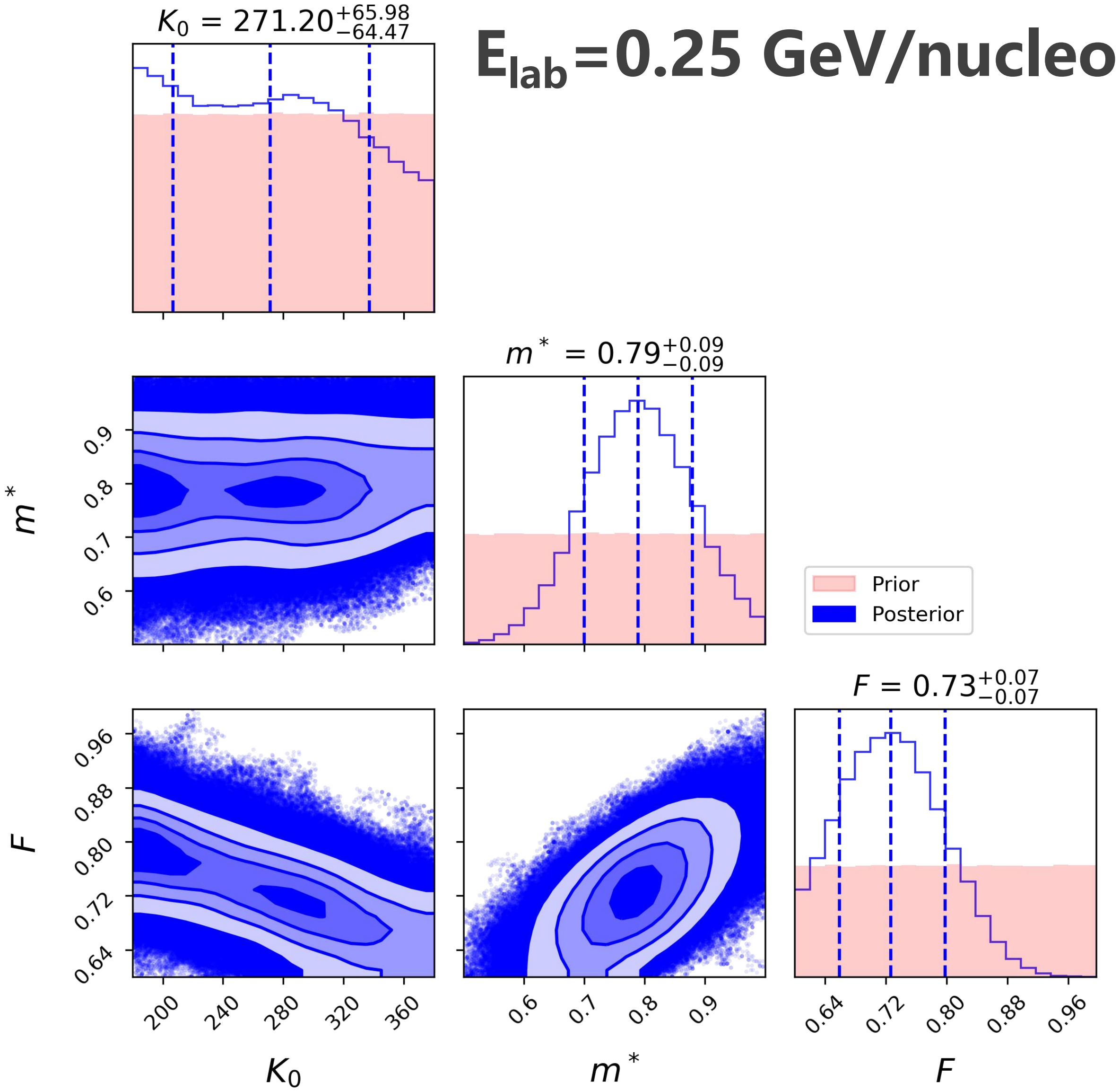


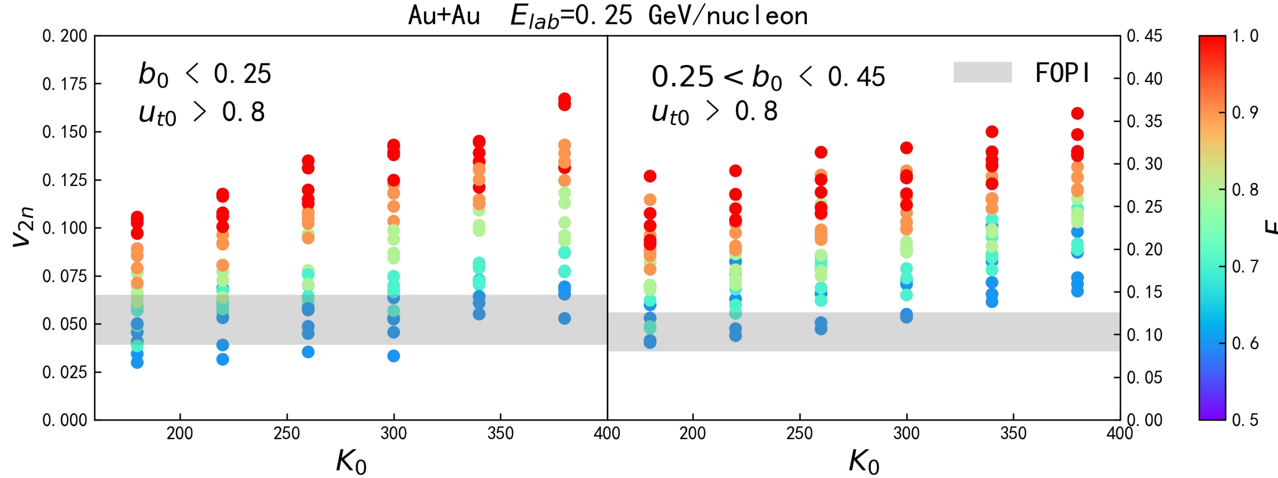


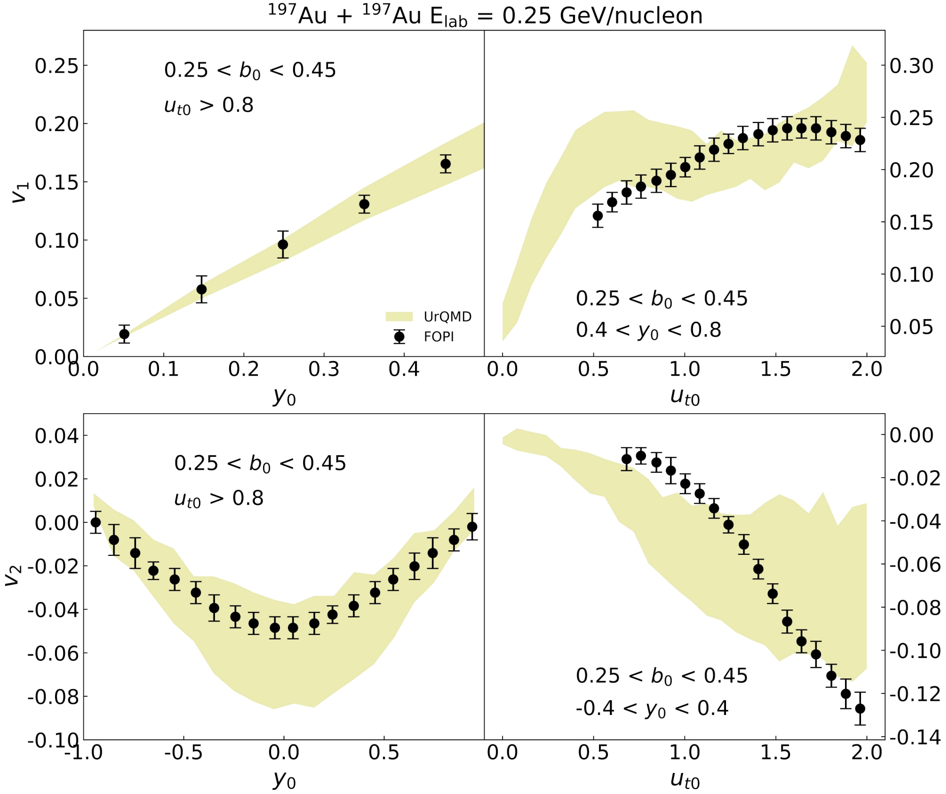
In preparation.

03 Bayesian analysis of properties of nuclear matter with FOPI data

E_{lab}=0.25 GeV/nucleon

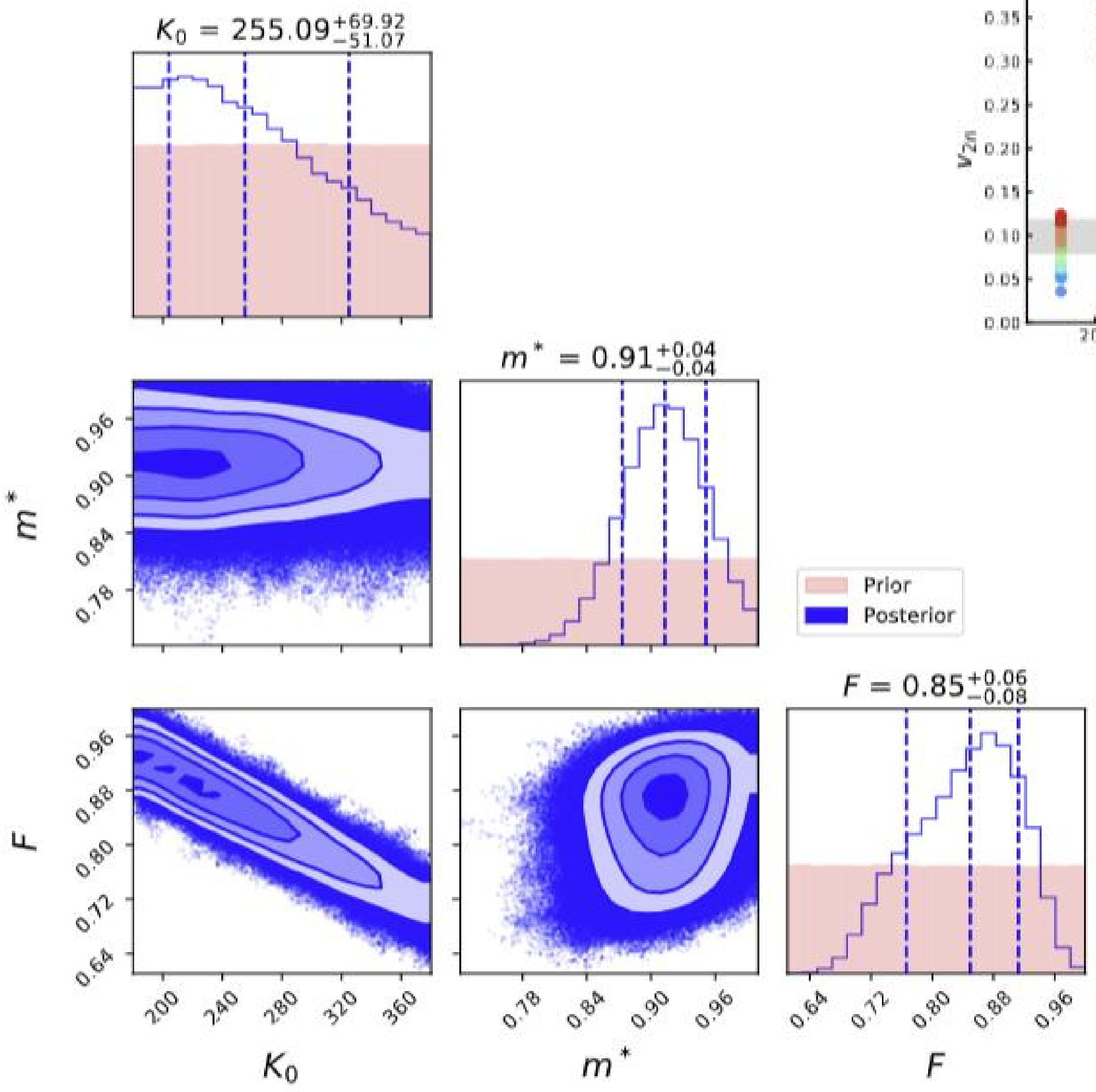


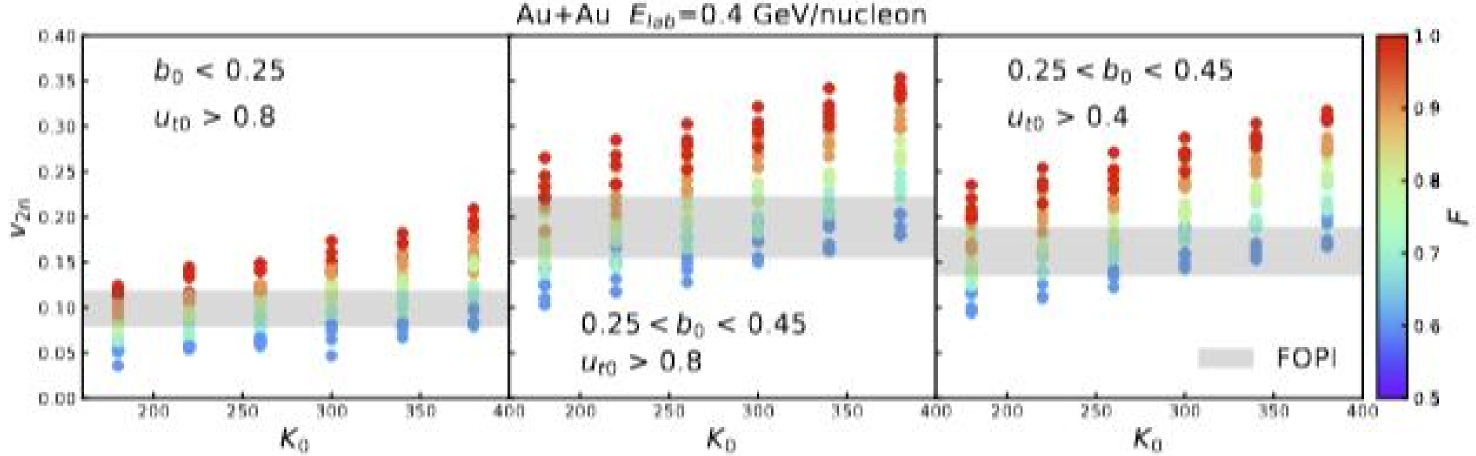


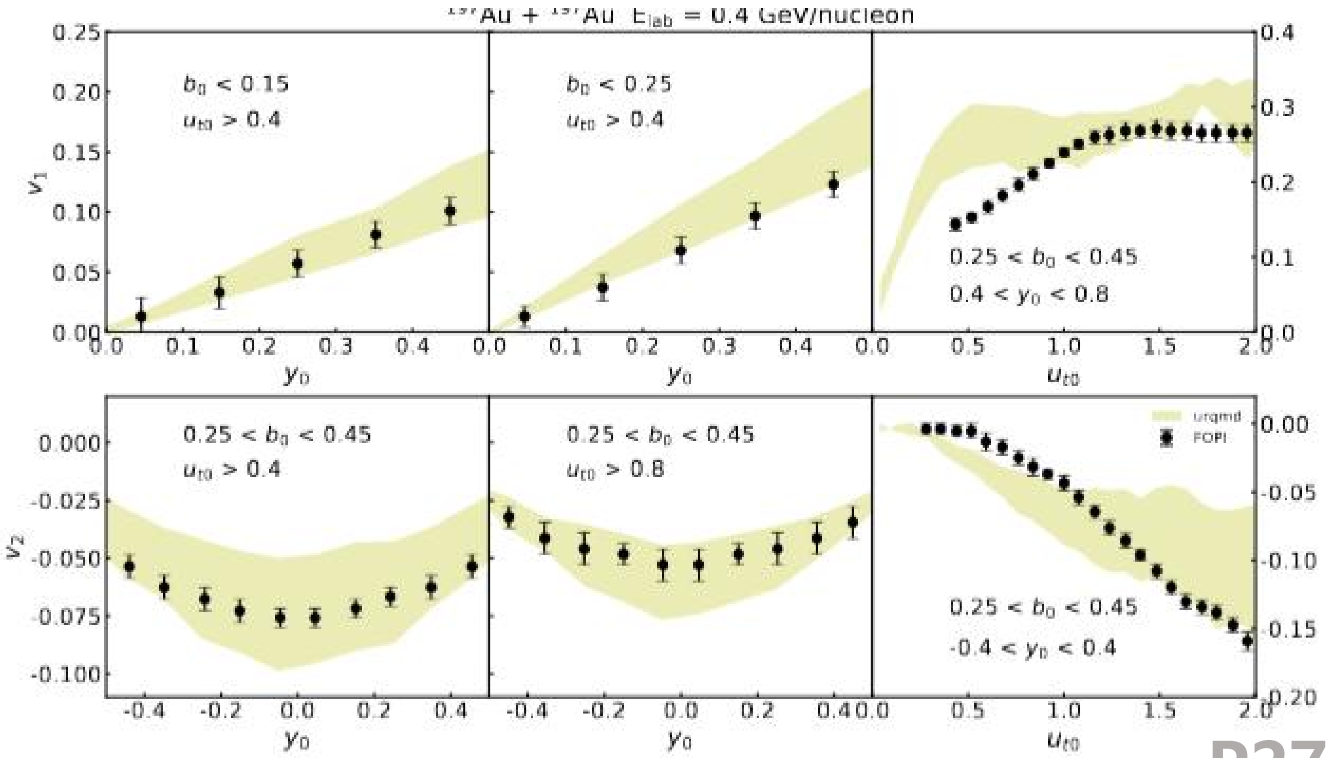


03

Bayesian Inference of properties of nuclear matter with FOPI data







E_{lab}=0.4 GeV/nucleon

04 Summary

1. Nucleon-nucleon collisions always suppress the value of v_2 (enhance the out-of-plane emission), while the nuclear mean field potential effect is more complex, which depends on both impact parameter and reaction time.

2. The density interval at which the nuclear equation of state is probed by the elliptic flow is about 60–80% of the maximum average density created in the central region of heavy-ion collision.

3. $m^*/m=0.79\pm0.09$ and $F=\sigma^{in\text{-}med.}/\sigma^{free}=0.73\pm0.07$ with FOPI data at $E_{lab}=0.25$ GeV/nucleon, and the obtained values increased to $m^*/m=0.91\pm0.04$ and $F=0.85\pm0.07$ at $E_{lab}=0.4$ GeV/nucleon. No tight constraint on K_0!

Thanks!