Investigation of $\Delta(1232)$, $N(1520)$, and $N(1535)$ Structures in $p\gamma^* \to N^*$ Reactions

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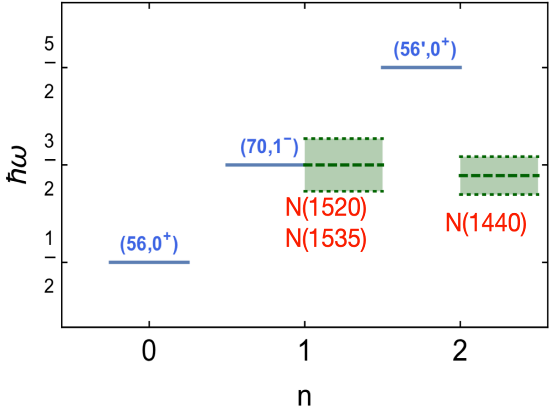
Outline

- Introduction
- $\Delta(1232)$ with $L = 2$ Components
- Pentaquark States in Constituent Quark Model.
- Transition Amplitudes of $N(1520)$ and $N(1535)$ in $q^3 + q^4\bar{q}$ Picture.
- Summary

Nucleon Resonances below 2 GeV

Resonance	Status	J^P	M_{BW}^{exp} MeV	Γ_{BW}^{exp} MeV
$N(1440)$	****	$\frac{1}{2}^+$	1410-1470	250-450
$N(1680)$	****	$\frac{5}{2}^+$	1665-1680	115-130
$N(1710)$	****	$\frac{1}{2}^+$	1680-1740	80-200
$N(1720)$	****	$\frac{3}{2}^+$	1680-1750	150-400
$N(1860)$	**	$\frac{5}{2}^+$	1800-1980	220-410
$N(1880)$	***	$\frac{1}{2}^+$	1830-1930	200-400
$N(1900)$	****	$\frac{3}{2}^+$	1890-1950	100-320
$N(1990)$	**	$\frac{7}{2}^+$	1950-2100	200-400
$N(2000)$	**	$\frac{5}{2}^+$	2030-2090	335-445
$N(1520)$	****	$\frac{3}{2}^-$	1510-1520	100-120
$N(1535)$	****	$\frac{1}{2}^-$	1525-1545	125-175
$N(1650)$	****	$\frac{1}{2}^-$	1645-1670	100-150
$N(1675)$	****	$\frac{5}{2}^-$	1670-1680	130-160
$N(1685)$	*	$\frac{1}{2}^-?$	1665-1675	15-45
$N(1700)$	***	$\frac{3}{2}^-$	1650-1750	100-300
$N(1875)$	***	$\frac{3}{2}^-$	1850-1920	120-250
$N(1895)$	****	$\frac{1}{2}^-$	1870-1920	80-200

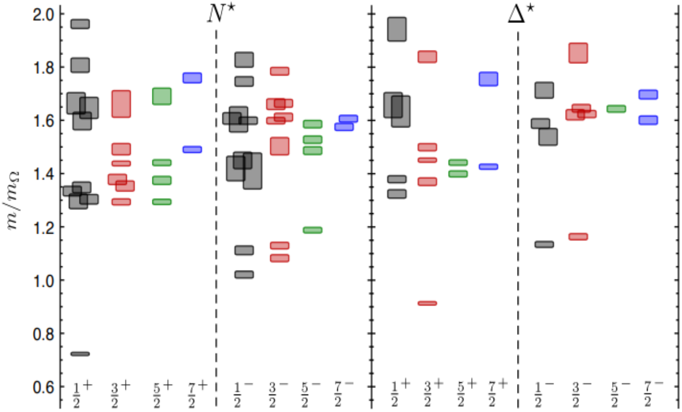
Wrong Mass Ordering



In the traditional q^3 picture, the Roper $N(1440)$ usually gets a mass ~ 100 MeV above $N(1520)$ and $N(1535)$, but not 100 MeV below it.

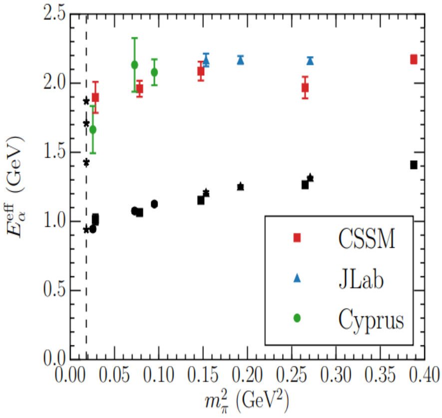
LQCD Data on N^*

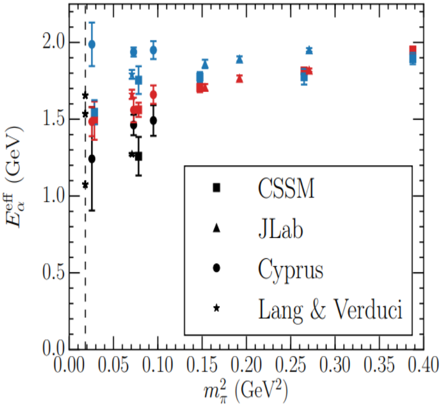
Spectrum of Nucleons and Deltas at $m_\pi = 396$ MeV, in units of Ω mass (HSC: PRD84 (2011) 074508)



LQCD Data on N^*

Review paper: arXiv:1511.09146

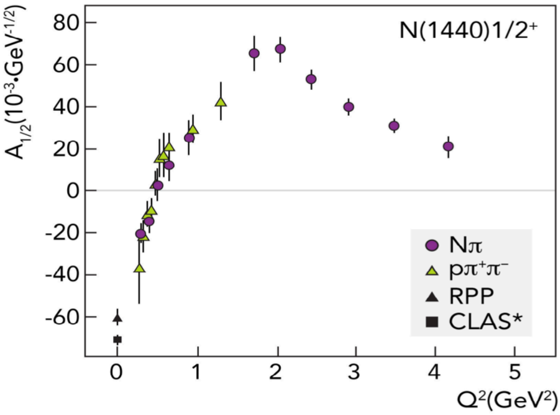




On Roper

- $N(1440)$ is usually blamed sitting at a wrong place or intruding the q^3 spectrum.
- $N(1400)$ has been studied intensively in various pictures like $q^4\bar{q}$ pentaquark, q^3g hybrid, $q^3(q\bar{q})$ resonance, radial excitation of nucleon, ...

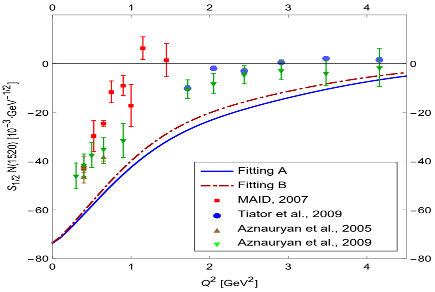
Data of Helicity Transition Amplitude of $N_{1/2+}(1440)$

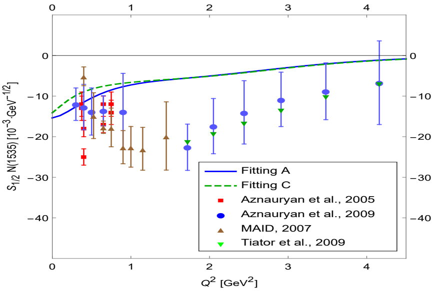


- The sign change in the helicity amplitude may suggest a node in the wave function, and hence $N(1440)$ ia likely the first radial excitation state.

$S_{1/2}$ Amplitudes of $N(1535)/N(1520)$ in q^3 Picture [EPJA **58**, 185 (2022)]

- Fitting A: $N(1520)$ and $N(1535)$ as $L = 1$ excitations, taking the same spatial wave function
- Fitting B (C): $N(1520)$ ($N(1535)$) is fitted alone as $L = 1$ excitations

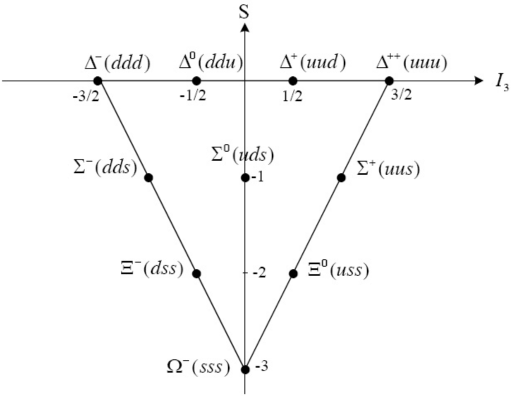




- The results are upper limits of the three-quark contributions
- $N(1520)$ and $N(1535)$ resonances may have considerable other components besides q^3 $L = 1$ three-quark state.

On $\Delta(1232)$

- $\Delta(1232)$ is traditionally interpreted as an $L = 0$ q^3 resonance with $S = 3/2$ and $I = 3/2$,



- For an $L = 0$ $\Delta(1232)$, the helicity transition amplitude $S_{1/2}$ in $p\gamma^* \to \Delta(1232)$ vanishes in quark models.

Ansatz on $\Delta(1232)$, $N(1440)$, $N(1520)$ and $N(1535)$

Considering

- Proton form factors and $N(1440)$ transition amplitudes are well reproduced in q^3 picture [PRD105, 016008 (2022)]
- It is difficult, if not say impossible, to understand $S_{1/2}$ of $N(1520)$ and $N(1535)$ in q^3 picture
- Helicity transition amplitude $S_{1/2}$ of $L = 0$ $\Delta(1232)$ vanishes.

One may assume

- $\Delta(1232)$ may have considerable $L = 2$ components.
- $N(1440)$ is dominantly the first redial excitation of q^3.
- $N(1520)$ and $N(1535)$ may have large ground-state ($L = 0$) pentaquark components.

Wave Function of $\Delta(1232)$

- The spin-spatial part of $\Delta(1232)$ state may take the form in q^3 picture,

$$A\,\psi^S_{l=0}\chi^S + B\,\psi^S_{l=2}\chi^S + C\,\frac{1}{\sqrt{2}}\left(\psi^\lambda_{l=2}\chi^\lambda + \psi^\rho_{l=2}\chi^\rho\right),$$

  with

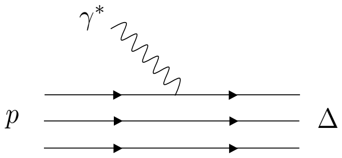
$$\begin{aligned} \psi^S_{2m} &= Y_{2m}(\boldsymbol{\rho}) + Y_{2m}(\boldsymbol{\lambda}), \\ \psi^\rho_{2m} &= \left[Y_{1m_\rho}(\boldsymbol{\rho}) \otimes Y_{1m_\lambda}(\boldsymbol{\lambda})\right]_{2m},\ \psi^\lambda_{2m} = Y_{2m}(\boldsymbol{\rho}) - Y_{2m}(\boldsymbol{\lambda}) \end{aligned}$$

- ψ^α_{lm} may be expanded in the complete basis of H.O. wave functions,

$$\psi^\alpha_{lm} = \sum_n a_n\,\Psi^\alpha_{nlm},$$

$$\Psi^\alpha_{nlm} = \sum_{n_\lambda,n_\rho,l_\lambda,l_\rho} A(n_\lambda,n_\rho,l_\lambda,l_\rho)\cdot\psi_{n_\lambda l_\lambda m_\lambda}(\boldsymbol{\lambda})\cdot\psi_{n_\rho l_\rho m_\rho}(\boldsymbol{\rho}) \cdot C(l_\lambda,m_\lambda;l_\rho,m_\rho;l,m)$$

Quark Core Contribution to $p\gamma \to \Delta(1232)$



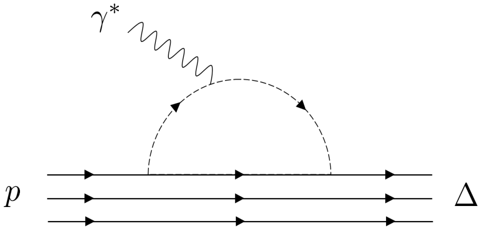
$$P_i = (E_N, 0, 0, -|\boldsymbol{k}|)$$
$$P_f = (M_{N^*}, 0, 0, 0)$$
$$k = (\omega, 0, 0, |\boldsymbol{k}|)$$
$$|\boldsymbol{k}| = \left[Q^2 + \left(\frac{M_{N^*}^2 - M_N^2 - Q^2}{2M_{N^*}}\right)^2\right]^{\frac{1}{2}},$$
$$\omega = \frac{M_{N^*}^2 - M_N^2 - Q^2}{2M_{N^*}}.$$

- Helicity amplitudes of $p\gamma \to \Delta(1232)$

$$A_{1/2}^{q^3} = \frac{1}{\sqrt{2K}} \langle N', S_z' = 1/2 \,|\, q_1' q_2' q_3' \rangle\, T^+(q_1 q_2 q_3 \to q_1' q_2' q_3')\, \langle q_1 q_2 q_3 \,|\, N, S_z = -1/2 \rangle\,,$$
$$A_{3/2}^{q^3} = \frac{1}{\sqrt{2K}} \langle N', S_z' = 3/2 \,|\, q_1' q_2' q_3' \rangle\, T^+(q_1 q_2 q_3 \to q_1' q_2' q_3')\, \langle q_1 q_2 q_3 \,|\, N, S_z = 1/2 \rangle\,,$$
$$S_{1/2}^{q^3} = \frac{1}{\sqrt{2K}} \langle N', S_z' = 1/2 \,|\, q_1' q_2' q_3' \rangle\, T^0(q_1 q_2 q_3 \to q_1' q_2' q_3')\, \langle q_1 q_2 q_3 \,|\, N, S_z = 1/2 \rangle\,.$$

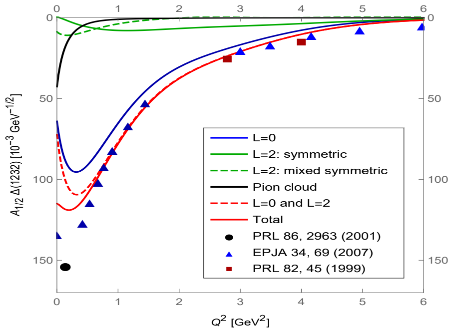
$$T^\lambda(q_1 q_2 q_3 \to q_1' q_2' q_3') = \langle q_1' \,|\, Q_1 \bar{u}_{s'}(p')\gamma^\mu u_s(p) \epsilon_\mu^\lambda(k) \,|\, q_1 \rangle\, \langle q_2' q_3' \,|\, q_2 q_3 \rangle$$
$$\epsilon_\mu^0 = \tfrac{1}{Q}(|\boldsymbol{k}|, 0, 0, \omega),\;\; \epsilon_\mu^+ = -\tfrac{1}{\sqrt{2}}(0, 1, i, 0),\;\; Q^2 = -k^2,\;\; K = \frac{M_{N^*}^2 - M_N^2}{2M_{N^*}}$$

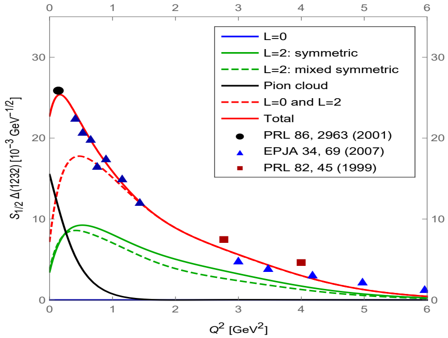
Meson Cloud Contribution to $p\gamma \to \Delta(1232)$



- Interaction Lagrangians:
  $\mathcal{L}_{\pi\pi\gamma} = e\epsilon_{3jk}\pi_j\partial^\nu\pi_k A_\nu,$
  $\mathcal{L}_{qq\pi} = \frac{1}{2F}\partial_\mu\pi_i\bar{u}\gamma^\mu\gamma^5\tau^i u \qquad \Longleftarrow F = 88$ MeV
- Form factor: $\frac{1}{1+k^2/\Lambda_\pi^2}, \qquad \Longleftarrow \Lambda_\pi = 0.732$ GeV
- Pauli-Villars Regularization: $\frac{1}{p^2-m_\pi^2} \to \frac{1}{p^2-m_\pi^2} - \frac{1}{p^2-M^2},$
  $\Longleftarrow M = 0.154$ GeV

Results of Helicity Transition Amplitudes of $p\gamma \to \Delta(1232)$





- For $S_{1/2}$, no contribution from $\Delta(1232)$ with $L = 0$. Only the $L = 2$ component contributes to $S_{1/2}$.
- In this work, $\Delta(1232)$ resonance consists of:
  - $L = 0$: 83% ($A = 0.908$)
  - Symmetric $L = 2$: 8% ($B = 0.29$)
  - Mixed-symmetric $L = 2$: 9% ($C = 0.3$)

Pentaquarks

- Pentaquarks are states of four quarks and one antiquark, $q^4\,\overline{q}$. QCD does not rule out the existence of pentaquarks, but there is no solid experimental evidence of compact pentaquarks.

- Pentaquarks may be analyzed under the fundamental representation of $SU(n)$ for quarks and the conjugate representation of $SU(n)$ for antiquarks, with $n = 2, 3, 3, 6$ for the spin, flavor, color and spin-flavor degree of freedom, respectively.

- The corresponding algebraic structure consists of the usual spin-flavor and color algebras

$$SU_{\mathrm{sf}}(6) \otimes SU_{\mathrm{c}}(3) \tag{1}$$

with

$$SU_{\mathrm{sf}}(6) = SU_{\mathrm{f}}(3) \otimes SU_{\mathrm{s}}(2) \tag{2}$$

$q^4\,\overline{q}$ Systems

- One may construct pentaquark states by considering
  - It is a color singlet;
  - It is antisymmetric under any permutation of q^4 configurations.
- The permutation symmetry of q^4 configurations is characterized by S_4 Young tabloids $[4]$, $[31]$, $[22]$, $[211]$ and $[1111]$.
- That it is a color singlet demands that the color part must be a $[222]_1$ singlet.
- Since the color part of antiquark is a $[11]_3$ antitriplet

$$\psi^c_{[11]}(\overline{q}) = \yng(1,1) \tag{3}$$

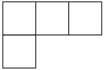
the color wave function of q^4 must be a $[211]_3$ triplet

$$\psi^c_{[211]}(q^4) = \yng(2,1,1) \tag{4}$$

q^4 Wave Functions

- The total state of q^4 is antisymmetric implies that the orbital-spin-flavour part must be a [31] state

$$\psi^{osf}_{[31]}(q^4) = \tag{5}$$



which is the conjugacy of the Young tabloid of the colour part.

- Total wave function of q^4 configuration may be written in the general form

$$\Psi^{(q^4)} = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\psi^{c}_{[211]_i}\psi^{osf}_{[31]_j} \tag{6}$$

By applying the permutations of S_4 to the above equation, one gets,

$$\Psi^{(q^4)}_A = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{7}$$

Spatial-Spin-Flavor Configurations of q^4

All the possible spin-flavor, spin, and flavor configurations may be determined by the representation characters of S_4.

- .

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}, [211]_{SF}$
$[211]_O$	$[31]_{SF}, [211]_{SF}, [22]_{SF}, [1111]_{SF}$
$[31]_O$	$[4]_{SF}, [31]_{SF}, [211]_{SF}, [22]_{SF}$

- For example for the $[31]_{FS}$ configuration,

$[31]_{FS}$			
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	

Constituent Quark Model with Cornell-like Potential

- Realistic Hamiltonian for a N-quark system:

$$\begin{aligned} H &= H_0 + H_{hyp}^{OGE}, \\ H_0 &= \sum_{k=1}^{N}(m_k + \frac{p_k^2}{2m_k}) + \sum_{i<j}^{N}(-\frac{3}{8}\lambda_i^C \cdot \lambda_j^C)(A_{ij}r_{ij} - \frac{B_{ij}}{r_{ij}}), \\ H_{hyp}^{OGE} &= -C_{OGE}\sum_{i<j}\frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j, \end{aligned} \tag{8}$$

- The model parameters are determined by fitting theoretical results to experimental data:

  1. The mass of all the ground state baryons.
  2. The mass of light baryon resonances up to $N \leq 2$, including the first radial excitation state $N(1440)$, and orbital excited $l = 1$ and $l = 2$ baryons.

Ground State Pentaquark Mass Spectra

J^P	$q^4\bar{q}$ configurations	$(S^{q^4}, S^{\bar{q}}, S)$	$M^{EV}(q^4\bar{q})$
$\frac{5}{2}^-$	$\Psi^{sf}_{[31]_F[4]_S}(q^4\bar{q})$	(2,1/2,5/2)	2269
$\frac{3}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	(1,1/2,3/2)	2269
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[4]_S}(q^4\bar{q}) \\ \Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})\end{pmatrix}$	(2,1/2,3/2) (1,1/2,3/2)	$\begin{pmatrix}1805 \\ 2269\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	(1,1/2,3/2)	2049
$\frac{1}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	(1,1/2,1/2)	2562
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q}) \\ \Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})\end{pmatrix}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}1986 \\ 2162\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	(1,1/2,1/2)	1683

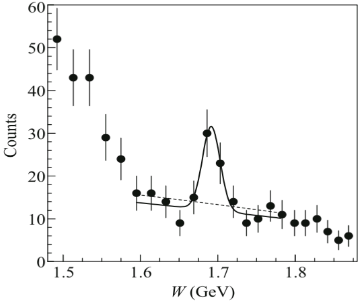
A Surprising Byproduct [PRD. **101**, 076025 (2020)]:

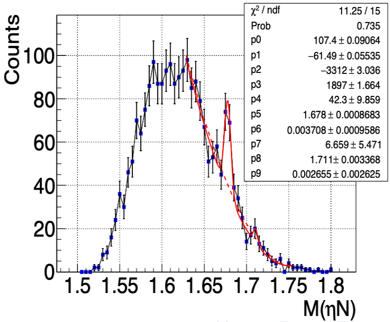
- $q^4\bar{q}$ state with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ in $[31]_{FS}[22]_F[31]_S$ configuration has the lowest mass, 1683 MeV.
- The mass is quite close to the $I = 1/2$ narrow resonance $N(1685)$, which can not be accommodated in the baryon spectrum in q^3 picture.

Experimental Situation of $N(1685)$

- $N(1685)$ was firstly reported in the photoproduction of η meson off the quasi-free neutron. Its width is less than $30\ \mathrm{MeV}$, much less than the width of other low-lying nucleon resonances [PLB 647, 23-29(2007)].

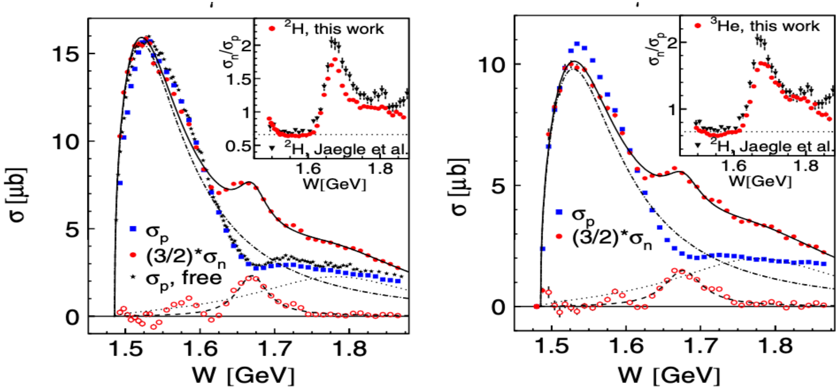
- Quasifree Compton scattering $\gamma N \to \gamma N$ on the neutron [PRC 83, 022201(R) (2011)] and the invariant mass spectra of ηN at GRAAL [JETP Letters 106, 693-699(2017)]



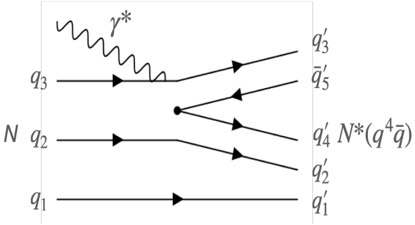


Experimental Situation of $N(1685)$

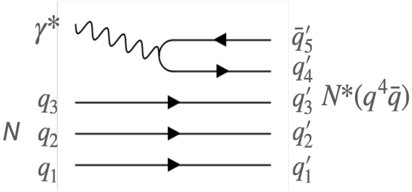
- A2 at Mainz MAMI accelerator, η photoproduction with deuterium and ^3He target also establish this narrow structure [PRL 111, 232001 (2013), PRL 117, 132502 (2016)].



Helicity Transition Amplitudes of N^* in $q^4\bar{q}$ picture



Process with γqq vertex



Process with $\gamma q\bar{q}$ vertex

$$A_{1/2(3/2)} = \frac{1}{\sqrt{2K}} \langle N^*, S_z' = 1/2(3/2) \,|\, q_1' q_2' q_3' q_4' \bar{q}_5' \rangle \, T^{+}_{s's}(q_1 q_2 q_3 \gamma^* \to q_1' q_2' q_3' q_4' \bar{q}_5')$$
$$\cdot \langle q_1 q_2 q_3 \,|\, N, S_z = -1/2(1/2) \rangle \,,$$
$$S_{1/2} = \frac{1}{\sqrt{2K}} \langle N^*, S_z' = 1/2 \,|\, q_1' q_2' q_3' q_4' \bar{q}_5' \rangle \, T^{0}_{s's}(q_1 q_2 q_3 \gamma^* \to q_1' q_2' q_3' q_4' \bar{q}_5')$$
$$\cdot \langle q_1 q_2 q_3 \,|\, N, S_z = 1/2 \rangle \,,$$

γqq vertex: $T^{\lambda}_{s's}(q_1 q_2 q_3 \gamma^* \to q_1' q_2' q_3' q_4' \bar{q}_5') = e_3 \lambda^c \bar{u}_s'(q_3') \gamma_\mu u_s(q_3) \epsilon^{\lambda}_{\mu}(k) \langle q_1' q_2' \,|\, q_1 q_2 \rangle$

$\gamma q\bar{q}$ vertex: $T^{\lambda}_{s's}(q_1 q_2 q_3 \gamma^* \to q_1' q_2' q_3' q_4' \bar{q}_5') = e_4 \bar{u}_{s'}(q_4') \gamma^\mu v_s(q_5') \epsilon^{\lambda}_{\mu}(k) \langle q_1' q_2' q_3' \,|\, q_1 q_2 q_3 \rangle$

$N(1520)$ and $N(1535)$ with q^3 and $q^4\bar{q}$ Components

- These baryon resonances may be expressed as linear combinations of an $l = 1$ q^3 state and ground $q^4\,\bar{q}$ states which have the same quantum numbers as the q^3 state,

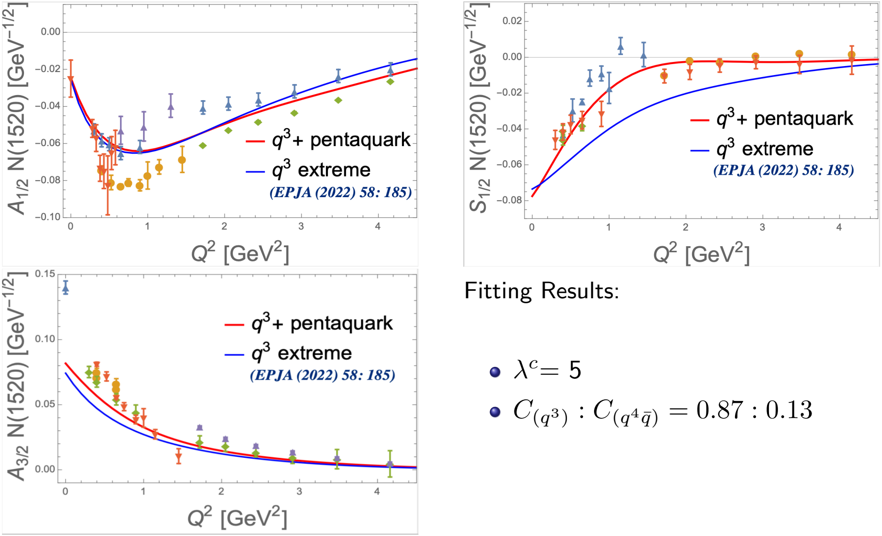
$$a_0|q^3\rangle + \sum_\alpha a_\alpha|q^4\bar{q}\rangle^\alpha \,.$$

- We consider a simple case, where only the $[31]_{SF}[22]_F[31]_S$ pentaquark configuration is included in the present work. That is

$$\begin{aligned} |N(1520)\rangle &= a_0|q^3(l=1, s=3/2)\rangle + a_1|q^4\bar{q}(2049)\rangle, \\ |N(1535)\rangle &= b_0|q^3(l=1, s=1/2)\rangle + b_1|q^4\bar{q}(1683)\rangle \end{aligned}$$

- Spatial wave functions of q^3 and $q^4\bar{q}$ pentaquark are fitted to the helicity amplitudes of $N(1520)$ and $N(1535)$ in the mixing pictures.

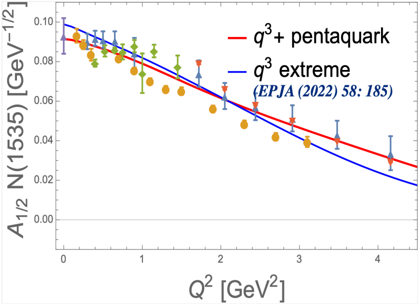
Helicity Transitions Amplitude of $N(1520)$ with q^3 and $q^4\bar{q}$ Components

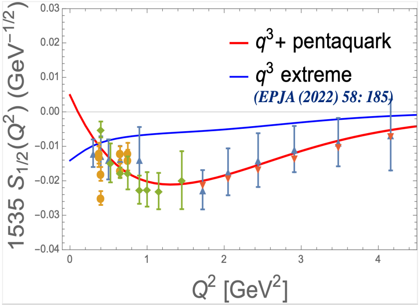


Fitting Results:

- $\lambda^c = 5$
- $C_{(q^3)} : C_{(q^4\bar{q})} = 0.87 : 0.13$

Helicity Transitions Amplitude of $N(1535)$ with q^3 and $q^4\bar{q}$ Components





- $\lambda^c = 5$
- $C_{(q^3)} : C_{(q^4\bar{q})} = 0.66 : 0.34$

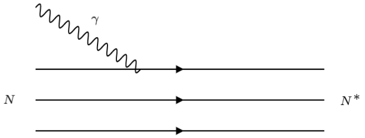
Summary

- Helicity transition amplitude $S_{1/2}$ vanishes for $\Delta(1232)(L=0)$ in q^3 picture.
- $\Delta(1232)$ may comprise large $L=2$ components.
- Inclusion of pentaquark components in $N(1520)$ and $N(1535)$ largely improves the results of helicity transition amplitude $S_{1/2}$.

Thank You Very Much for Your Patience

Form Factor and Helicity Transition Amplitudes in q^3 Picture

- Proton electric form factor G_E in the impulse approximation,



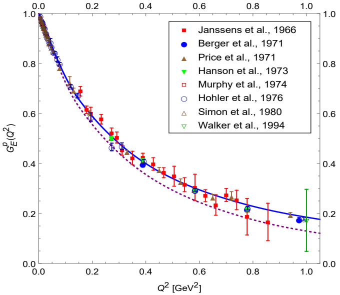
$$G_E = \left\langle N, S_z' = \tfrac{1}{2} \,\middle|\, q_1' q_2' q_3' \right\rangle T_B(q_1 q_2 q_3 \to q_1' q_2' q_3') \times \left\langle q_1 q_2 q_3 \,\middle|\, N, S_z = \tfrac{1}{2} \right\rangle \Big|_{\text{Breit frame}} ,$$

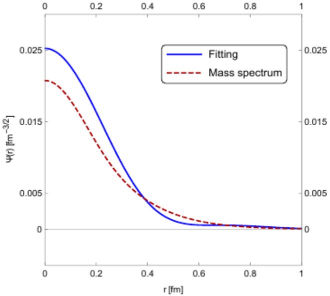
$$T^{\lambda}_{s's}(q_1 q_2 q_3 \to q_1' q_2' q_3') = e_3 \bar{u}_{s'}(q_3') \gamma^{\mu} u_s(q_3) \epsilon^{\lambda}_{\mu}(k) \left\langle q_1' q_2' \,\middle|\, q_1 q_2 \right\rangle \qquad (9)$$

In the Breit frame, the momenta of the initial and finil proton states and the photon are respectively $P_i = (E_N, 0, 0, k)$ and $P_f = (E_N, 0, 0, -k)$ and $k = (0, 0, 0, k)$.

Proton Form Factor and Quark Distribution

- Dashed Curves: Proton spatial wave function is imported from the mass spectrum calculations
- Solid Curves: Proton spatial wave function is fitted to experimental data.





Figures from [PRD **105**, 016008 (2022)]

One may conclude:

- The three-quark core dominantly contribute to the proton electric form factor.

Spatial-Spin-Favor Configurations of q^4

- **Definition:** Let $\Gamma = \{D(g)\}$ be the representation of the group G of order n, the traces of the n $D(g)$ form the characters of the representation Γ.
- The orthogonal theorem in group theory leads to the property for the characters of a group,

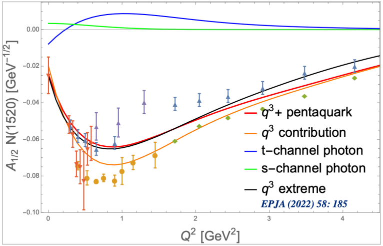
$$\chi(g) = \sum_{\beta=1}^{h} m_\beta \chi^{(\beta)}(g), \quad m_\beta = \frac{1}{n}\sum_g \chi^{(\beta)*}(g)\,\chi(g) \tag{10}$$

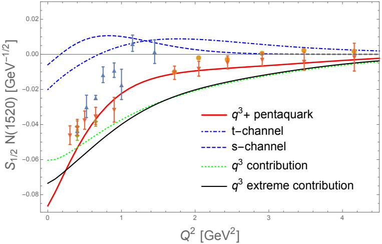
  where g are group elements, $\chi(g)$ are the characters of a product (reducible) representation of the group and $\chi^{(\beta)}(g)$ are the characters of the irreducible representation labeled by β.
- From the above equation and the properties of characters, one gets

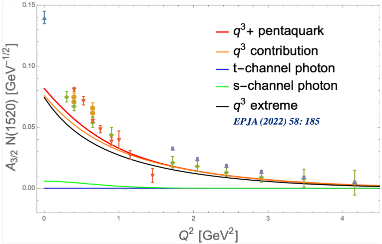
$$m_{[31]_{OSF}} \quad = \quad \frac{1}{n}\sum_g \chi^{[31]_{OSF}*}(g)\,\left(\chi^{[X]_O}(g)\chi^{[Y]_{SF}}(g)\right) \tag{11}$$

- By applying Eq. (11), we get all the spatial-spin-flavor configurations. The spin-flavor configurations of the q^4 cluster of pentaquarks can be derive the same way.

Helicity Transitions Amplitude of $N(1520)$ in mixing picture



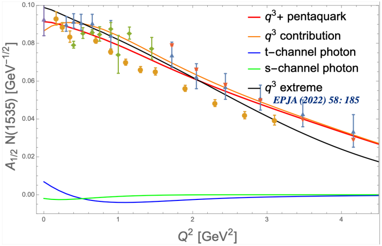


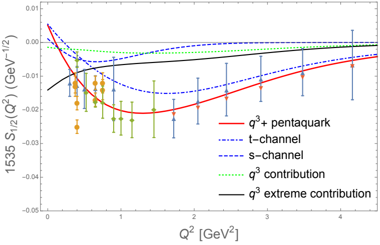


- N(1520):
  $C_{(q^3)} : C_{(q^4\bar{q})} = 0.50 : 0.50$

- Helicity amplitudes of both $A_{3/2}$ and $S_{1/2}$ were better fitted in the mixing q^3 and pentaquark picture.

Helicity Transitions Amplitude of $N(1535)$ in q^3 picture





- $N(1535)$: $C_{(q^3)} : C_{(q^4\bar{q})} = 0.66 : 0.34$

- Pentaquark component largely contributes to $S_{1/2}$, more than the contribution of the q^3 components, but very little to $A_{1/2}$.

- Inclusion of sizable pentaquark contributions in the three-quark state helps to give a better description of helicity amplitudes of N(1520) and N(1535).

Contributions of q^3 and Pentaquarks for varied λ^c (added)

- The ratios of contributions of three-quark and pentaquark components in N(1520) and N(1535) when we get best fit for varied range of λ^c from 1-10.

λ^c	1	2	3	4	5	6	7	8	9	10	20
N(1520): $\frac{C_{(q^3)}}{C_{(q^4\bar{q})}}$	$\frac{0.650}{0.350}$	$\frac{0.590}{0.410}$	$\frac{0.538}{0.462}$	$\frac{0.510}{0.490}$	$\frac{0.498}{0.501}$	$\frac{0.497}{0.503}$	$\frac{0.499}{0,501}$	$\frac{0.502}{0.498}$	$\frac{0.504}{0.496}$	$\frac{0.506}{0.494}$	$\frac{0.509}{0.491}$
N(1535): $\frac{C_{(q^3)}}{C_{(q^4\bar{q})}}$	$\frac{0.624}{0.376}$	$\frac{0.618}{0.382}$	$\frac{0.620}{0.380}$	$\frac{0.630}{0.370}$	$\frac{0.657}{0.343}$	$\frac{0.692}{0.308}$	$\frac{0.720}{0.280}$	$\frac{0.739}{0.261}$	$\frac{0.751}{0.249}$	$\frac{0.758}{0.242}$	$\frac{0.774}{0.226}$

- The pentaquark components contribute to N(1520) at maximum 50.3% when λ=6, to N(1535) at maximum 38.2% when λ=2.

- λ=4 should be the best case for both N(1520) and N(1535),
  $N(1520)$: $C_{(q^3)} : C_{(q^4\bar{q})} = 0.51 : 0.49$,
  $N(1535)$: $C_{(q^3)} : C_{(q^4\bar{q})} = 0.63 : 0.37$.

Model parameters

- The model parameters are determined by fitting the theoretical results to the experimental data:

  1. The mass of all the ground state baryons.
  2. The light baryon resonances in Harmonic oscillation model of energy level $N \leq 2$, including the first radial excitation state $N(1440)$, and orbital excited $l = 1$ and $l = 2$ baryons.

  All these baryons are believed to be mainly $3q$ states whose masses were taken from Particle Data Group.

- The 3 model coupling constants and 4 constituent quark masses are fitted,

$$
\begin{aligned}
m_u = m_d = 327 \text{ MeV}\,, \quad m_s = 498 \text{ MeV}\,, \\
m_c = 1642 \text{ MeV}\,, \quad m_b = 4960 \text{ MeV}\,, \\
C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75
\end{aligned}
\tag{12}
$$

Possible Mixtures of q^3 and $q^4\,\bar{q}$ States

- We expect to derive the right mass for $N(1520)$, $N(1535)$, $\Delta(1620)$ and $\Delta(1700)$ by mixing the q^3 states with $q^4\,\bar{q}$ components.
- All four q^3 states take the same mass, 1380 MeV. As examples, we take the lowest pentaquark states (except the 1683 MeV state) to mix with the four q^3 states.

J^P	ψ_1 State	ψ_2 State	q^3	$q^4\bar{q}$ config.	$q^4\bar{q}$ Mass
$\frac{1}{2}^-$	1530	1882	1380	$q^3 s\bar{s}_{[211]_F[31]_S}$	2032
$\frac{3}{2}^-$	1515	1895	1380	$q^4\bar{q}_{[31]_F[4]_S}$	2025
$\frac{1}{2}^-$	1610	1893	1380	$q^4\bar{q}_{[31]_F[31]_S}$	2123
$\frac{3}{2}^-$	1700	1923	1380	$q^3 s\bar{s}_{[22]_F[31]_S}$	2243

- $N(1520)3/2^-$ and $N(1875)3/2^-$ form a non-strange pair, and $N(1535)1/2^-$ and $N(1895)1/2^-$ form a strange pair.
- $\Delta(1620)1/2^-$ and $\Delta(1900)1/2^-$ form a non-strange pair. $\Delta(1700)3/2^-$ and $\Delta(1940)3/2^-$ form a strange pair.

Normalized q^3 symmetric type spatial wave functions

- $N(1520)$ and $N(1535)$ wave functions are fitted to experimental data of helicity transition amplitudes:

$$
\begin{aligned}
b_n &= \{0.799, -0.143, 0.563, 0.045, 0.136, 0.059\},\\
c_n &= \{0.852, -0.081, 0.515, -0.031, -0.014, 0.009\},\\
d_n &= \{0.613, 0.134, 0.631, -0.261, 0.352, -0.128\}
\end{aligned} \tag{13}
$$

$\Psi_1^{[21]\rho}$	$\phi_{011}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})$
$\Psi_2^{[21]\rho}$	$\frac{\sqrt{10}}{4}\phi_{111}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})+\frac{\sqrt{6}}{4}\phi_{011}(\boldsymbol{\rho})\phi_{100}(\boldsymbol{\lambda})$
$\Psi_3^{[21]\rho}$	$\frac{\sqrt{7}}{4}\phi_{211}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})+\frac{\sqrt{6}}{4}\phi_{111}(\boldsymbol{\rho})\phi_{100}(\boldsymbol{\lambda})+\frac{\sqrt{3}}{4}\phi_{011}(\boldsymbol{\rho})\phi_{200}(\boldsymbol{\lambda})$
$\Psi_4^{[21]\rho}$	$\frac{\sqrt{21}}{8}\phi_{311}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})+\frac{\sqrt{21}}{8}\phi_{211}(\boldsymbol{\rho})\phi_{100}(\boldsymbol{\lambda})+\frac{\sqrt{15}}{8}\phi_{111}(\boldsymbol{\rho})\phi_{200}(\boldsymbol{\lambda})+\frac{\sqrt{7}}{8}\phi_{011}(\boldsymbol{\rho})\phi_{300}(\boldsymbol{\lambda})$
$\Psi_5^{[21]\rho}$	$\frac{\sqrt{66}}{16}\phi_{411}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})+\frac{\sqrt{18}}{8}\phi_{311}(\boldsymbol{\rho})\phi_{100}(\boldsymbol{\lambda})+\frac{\sqrt{15}}{8}\phi_{211}(\boldsymbol{\rho})\phi_{200}(\boldsymbol{\lambda})+\frac{\sqrt{10}}{8}\phi_{111}(\boldsymbol{\rho})\phi_{300}(\boldsymbol{\lambda})$ $+\frac{\sqrt{18}}{16}\phi_{011}(\boldsymbol{\rho})\phi_{400}(\boldsymbol{\lambda})$
$\Psi_6^{[21]\rho}$	$\frac{\sqrt{858}}{64}\phi_{511}(\boldsymbol{\rho})\phi_{000}(\boldsymbol{\lambda})+\frac{3\sqrt{110}}{64}\phi_{411}(\boldsymbol{\rho})\phi_{100}(\boldsymbol{\lambda})+\frac{15}{32}\phi_{311}(\boldsymbol{\rho})\phi_{200}(\boldsymbol{\lambda})+\frac{5\sqrt{7}}{32}\phi_{211}(\boldsymbol{\rho})\phi_{300}(\boldsymbol{\lambda})$ $+\frac{15\sqrt{2}}{64}\phi_{111}(\boldsymbol{\rho})\phi_{400}(\boldsymbol{\lambda})+\frac{3\sqrt{22}}{64}\phi_{011}(\boldsymbol{\rho})\phi_{500}(\boldsymbol{\lambda})$

$\Psi_1^{[21]\lambda}$	$\phi_{000}(\boldsymbol{\rho})\phi_{011}(\boldsymbol{\lambda})$
$\Psi_2^{[21]\lambda}$	$\frac{\sqrt{10}}{4}\phi_{000}(\boldsymbol{\rho})\phi_{111}(\boldsymbol{\lambda})+\frac{\sqrt{6}}{4}\phi_{100}(\boldsymbol{\rho})\phi_{011}(\boldsymbol{\lambda})$
$\Psi_3^{[21]\lambda}$	$\frac{\sqrt{7}}{4}\phi_{000}(\boldsymbol{\rho})\phi_{211}(\boldsymbol{\lambda})+\frac{\sqrt{6}}{4}\phi_{100}(\boldsymbol{\rho})\phi_{111}(\boldsymbol{\lambda})+\frac{\sqrt{3}}{4}\phi_{200}(\boldsymbol{\rho})\phi_{011}(\boldsymbol{\lambda})$
$\Psi_4^{[21]\lambda}$	$\frac{\sqrt{21}}{8}\phi_{000}(\boldsymbol{\rho})\phi_{311}(\boldsymbol{\lambda})+\frac{\sqrt{21}}{8}\phi_{100}(\boldsymbol{\rho})\phi_{211}(\boldsymbol{\lambda})+\frac{\sqrt{15}}{8}\phi_{200}(\boldsymbol{\rho})\phi_{111}(\boldsymbol{\lambda})+\frac{\sqrt{7}}{8}\phi_{300}(\boldsymbol{\rho})\phi_{011}(\boldsymbol{\lambda})$
$\Psi_5^{[21]\lambda}$	$\frac{\sqrt{66}}{16}\phi_{000}(\boldsymbol{\rho})\phi_{411}(\boldsymbol{\lambda})+\frac{\sqrt{18}}{8}\phi_{100}(\boldsymbol{\rho})\phi_{311}(\boldsymbol{\lambda})+\frac{\sqrt{15}}{8}\phi_{200}(\boldsymbol{\rho})\phi_{211}(\boldsymbol{\lambda})+\frac{\sqrt{10}}{8}\phi_{300}(\boldsymbol{\rho})\phi_{111}(\boldsymbol{\lambda})$ $+\frac{\sqrt{18}}{16}\phi_{400}(\boldsymbol{\rho})\phi_{011}(\boldsymbol{\lambda})$
$\Psi^{[21]\lambda}$	$\sqrt{858}\ \phi_{000}(\boldsymbol{\rho})\phi_{511}(\boldsymbol{\lambda})+3\sqrt{110}\ \phi_{100}(\boldsymbol{\rho})\phi_{411}(\boldsymbol{\lambda})+15\ \phi_{200}(\boldsymbol{\rho})\phi_{311}(\boldsymbol{\lambda})+5\sqrt{7}\ \phi_{300}(\boldsymbol{\rho})\phi_{211}(\boldsymbol{\lambda})$

Nucleon resonances of positive parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	M^{exp}(MeV)	M^{cal}(MeV)
$N(56, {}^28, 0, 0^+)$	****	$\frac{1}{2}^+$	939	939
$N(56, {}^28, 2, 0^+)$	****	$\frac{1}{2}^+$	N(1440)	1499
$N(56, {}^28, 2, 2^+)$	****	$\frac{5}{2}^+$	N(1720)	1655
$N(56, {}^28, 2, 2^+)$	****	$\frac{3}{2}^+$	N(1680)	1655
$N(20, {}^21, 2, 1^+)$	***	$\frac{1}{2}^+$	N(1880)	1749
$N(20, {}^41, 2, 1^+)$	-	$\frac{3}{2}^+$	missing	1749
$N(70, {}^210, 2, 0^+)$	****	$\frac{1}{2}^+$	N(1710)	1631
$N(70, {}^410, 2, 0^+)$	****	$\frac{3}{2}^+$	N(1900)	1924
$N(70, {}^210, 2, 2^+)$	-	$\frac{3}{2}^+$	missing	1702
$N(70, {}^210, 2, 2^+)$	**	$\frac{5}{2}^+$	N(1860)	1702
$N(70, {}^410, 2, 2^+)$	***	$\frac{1}{2}^+$	N(2100)	1994
$N(70, {}^410, 2, 2^+)$	*	$\frac{3}{2}^+$	N(2040)	1994
$N(70, {}^410, 2, 2^+)$	**	$\frac{5}{2}^+$	N(2000)	1994
$N(70, {}^410, 2, 2^+)$	**	$\frac{7}{2}^+$	N(1990)	1994

Baryons Masses in the Constituent Quark Model

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$
$N(70, {}^{2}10, 1, 1^-)$	****	$\frac{3}{2}^-$	N(1520)	1380
$N(70, {}^{2}10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1535)	1380
$N(70, {}^{4}10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1650)	1672
$N(70, {}^{4}10, 1, 1^-)$	****	$\frac{5}{2}^-$	N(1675)	1672
$N(70, {}^{4}10, 1, 1^-)$	***	$\frac{3}{2}^-$	N(1700)	1672
$\Delta(70, {}^{2}10, 1, 1^-)$	****	$\frac{1}{2}^-$	Δ(1620)	1380
$\Delta(70, {}^{2}10, 1, 1^-)$	****	$\frac{3}{2}^-$	Δ(1700)	1380

- All the ground state baryons are well reproduced.
- $N(1520)$, $N(1535)$, $\Delta(1620)$ and $\Delta(1700)$ are poorly described.
- No room for $N(1685)$ in the q^3 negative party spectrum.

Baryons masses assumption

- A simple unitary transformation of the baryon wave functions may take the form,

$$\begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |q^3\rangle \\ |q^4\overline{q}\rangle \end{pmatrix}, \tag{14}$$

where ψ_1 and ψ_2 are respectively the lower and higher negative-parity physical states, and the mixing angle θ between the q^3 and $q^4\bar{q}$ states is generally complex.

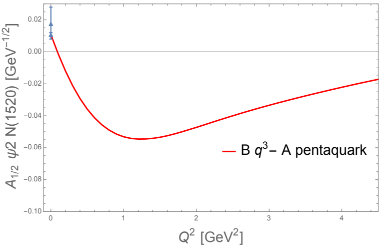
- The masses of the physical states, M_{ψ_1} and M_{ψ_2} are derived as follows:

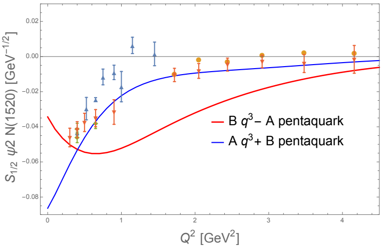
$$\begin{aligned} M_{\psi_1} &= M_{q^3}\cos^2\theta + M_{q^4\bar{q}}\sin^2\theta - m_\delta\,, \\ M_{\psi_2} &= M_{q^3}\sin^2\theta + M_{q^4\bar{q}}\cos^2\theta + m_\delta\,, \\ m_\delta &= \frac{(M_{q^4\bar{q}} - M_{q^3})}{2}\tan 2\theta \sin 2\theta \end{aligned} \tag{15}$$

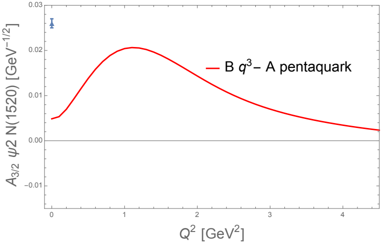
- We always have the mass condition: $M_{\psi_1} + M_{\psi_2} = M_{q^3} + M_{q^4\bar{q}}$.
- General wave functions for lower and higher mixing states,

$$\begin{aligned} \psi_1 &= A|q^3\rangle + B|q^4\bar{q}\rangle\,, \\ \psi_2 &= B|q^3\rangle - A|q^4\bar{q}\rangle\,, \\ & A^2 + B^2 = 1. \end{aligned} \tag{16}$$

Helicity Transitions Amplitude of $N(1520)$ in mixing picture



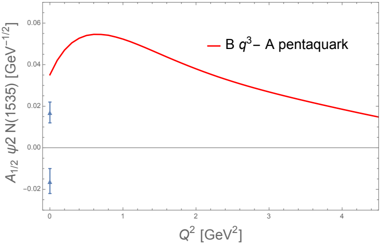


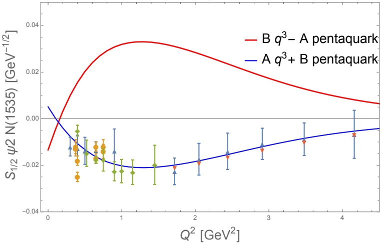


- From the relation of the mixing three-quark and pentaquark picture, we can derive the helicity amplitudes of higher energy state.

- Compare to N(1875)$3/2^-$ and N(1895)$1/2^-$

Helicity Transitions Amplitude of $N(1535)$ in q^3 picture





- Upper limit of both three quark and pentaquark pictures, not same as the mixing states of the mass eigenstates.

- Predictions of possible mixing states of higher energy.

LARGE Model Parameters

- The model parameters are determined by fitting the theoretical results to the experimental data:

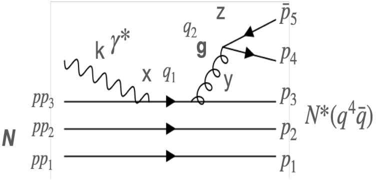
  1. The mass of all the ground state baryons.
  2. The light baryon resonances in Harmonic oscillation model of energy level $N \leq 2$, including the first radial excitation state $N(1440)$, and orbital excited $l = 1$ and $l = 2$ baryons.

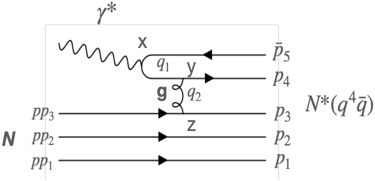
  All these baryons are believed to be mainly $3q$ states whose masses were taken from Particle Data Group.

- The 3 model coupling constants and 4 constituent quark masses are fitted,

$$
\begin{gathered}
m_u = m_d = 327 \text{ MeV}\,, \quad m_s = 498 \text{ MeV}\,, \\
m_c = 1642 \text{ MeV}\,, \quad m_b = 4960 \text{ MeV}\,, \\
C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75
\end{gathered}
\tag{17}
$$

Feynman Diagrams with one gluon exchange





$$
\begin{aligned}
T_1^g =&(-ieg_s^2)\bar{u}_{s'}(p_3)\gamma_S\epsilon_S^\lambda(k)QS_F(p_3+p_4+p_5)\lambda^\alpha u_s(pp_3)\,\bar{u}_{s'}(p_4)D_F(p_4+p_5)\lambda^\alpha v_s(p_5)\\
T_2^g =&(-ieg_s^2)\bar{u}_{s'}(p_3)\lambda^\alpha u_s(pp_3)D_F(k-p_4-p_5)\,\bar{u}_{s'}(p_4)S_F(k-p_5)\gamma_S\epsilon_S^\lambda(k)Q\lambda^\alpha v_s(p_5)
\end{aligned}
\tag{18}
$$

- Effective vertex diagram was supposed to contain all the possible interaction including gluon, meson exchange...

helicity amplitudes in effective scalar vertex and photon creation diagrams

- $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ of ground state pentaquark in possible configurations for t channel.
- helicity amplitudes of isospin $I = 1/2$ for effective scalar vertex diagram,

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$A_{1/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2562	$-\frac{\sqrt{5}}{9\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
	$\frac{3}{2}^-$	2269	$\frac{\sqrt{5}}{18\sqrt{6}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2123	$-\frac{1}{6\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$-\frac{\sqrt{2}}{3\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
$\Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2025	$\frac{1}{2\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	1683	$-\frac{1}{6\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$-\frac{\sqrt{2}}{3\sqrt{3}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$

helicity amplitudes in effective scalar vertex and photon creation diagrams

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$S_{1/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2123	$-\frac{1}{6\sqrt{3}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$-\frac{\sqrt{2}}{3\sqrt{3}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2025	$\frac{1}{2\sqrt{3}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	1683	$-\frac{1}{6\sqrt{3}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$-\frac{\sqrt{2}}{3\sqrt{3}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$A_{3/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	$\frac{3}{2}^-$	2269	$\frac{\sqrt{5}}{6\sqrt{2}}T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}}$

- $A_{1/2}$, $A_{3/2}$ and $S_{1/2}$ of ground state pentaquark in possible configurations for s channel.

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$A_{3/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{3}{2}^-$	2049	$-\frac{1}{3\sqrt{2}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{3}{2}^-$	2049	$\frac{1}{3\sqrt{2}}T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$

$A_{1/2}$ and $S_{1/2}$ of pentaquarks in photon creation diagram

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$A_{1/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2123	$\frac{1}{6\sqrt{3}} T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$-\frac{1}{3\sqrt{6}} T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2025	$-\frac{1}{2\sqrt{3}} T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	1683	$-\frac{1}{3\sqrt{6}} T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$
	$\frac{3}{2}^-$	2049	$\frac{1}{3\sqrt{6}} T_{S_{\frac{1}{2}},S_{\frac{1}{2}}}$

$q^4\bar{q}$ configurations	J^P	$M(q^4\bar{q})$ (MeV)	$S_{1/2}^{q^4\bar{q}}$
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2123	$\frac{1}{6\sqrt{3}} \left(T_{S_{-\frac{1}{2}},S_{\frac{1}{2}}} - 2T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}} \right)$
	$\frac{3}{2}^-$	2049	$-\frac{1}{3\sqrt{6}} \left(T_{S_{-\frac{1}{2}},S_{\frac{1}{2}}} + T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}} \right)$
$\Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	2025	$\frac{1}{2\sqrt{3}} T_{S_{-\frac{1}{2}},S_{\frac{1}{2}}}$
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	1683	$\frac{1}{3\sqrt{6}} \left(T_{S_{-\frac{1}{2}},S_{\frac{1}{2}}} + T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}} \right)$
	$\frac{3}{2}^-$	2049	$\frac{1}{3\sqrt{6}} \left(T_{S_{-\frac{1}{2}},S_{\frac{1}{2}}} + T_{S_{\frac{1}{2}},S_{-\frac{1}{2}}} \right)$

Matrix elements of the single quark transition $\gamma q \to q'$ for the helicity $\lambda = 0, 1$

$$
\begin{aligned}
T^{0}_{\uparrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[1 + \frac{p'_z p_z + 2p'_- p_+}{(E'+m)(E+m)}\right], \\
T^{0}_{\uparrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{\sqrt{2}(p'_z p_- - p'_- p_z)}{(E'+m)(E+m)}\right], \\
T^{0}_{\downarrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{\sqrt{2}(-p'_z p_+ + p'_+ p_z)}{(E'+m)(E+m)}\right], \\
T^{0}_{\downarrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[1 + \frac{p'_z p_z + 2p'_+ p_-}{(E'+m)(E+m)}\right], \\
T^{+}_{\uparrow\uparrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{2p_+}{E+m}\right], \\
T^{+}_{\uparrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[-\frac{\sqrt{2}p_z}{E+m} + \frac{\sqrt{2}p'_z}{E'+m}\right], \\
T^{+}_{\downarrow\uparrow} &= 0, \\
T^{+}_{\downarrow\downarrow} &= \left[\frac{(E'+m)(E+m)}{4E'E}\right]^{\frac{1}{2}} \left[\frac{2p'_+}{E'+m}\right],
\end{aligned} \tag{19}
$$

Matrix elements of quark-antiquark pair creation for the helicity $\lambda = 0, 1$

$$\mathcal{M}^{(0)}_{\frac{1}{2}\frac{1}{2}} = ie\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{p_-}{E+m} + \frac{p'_-}{E'+m}\right] \quad (20)$$

$$\mathcal{M}^{(0)}_{\frac{1}{2}-\frac{1}{2}} = -ie\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{p_z}{E+m} + \frac{p'_z}{E'+m}\right] \quad (21)$$

$$\mathcal{M}^{(0)}_{-\frac{1}{2}\frac{1}{2}} = ie\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{-p_z}{E+m} + \frac{-p'_z}{E'+m}\right] \quad (22)$$

$$\mathcal{M}^{(0)}_{-\frac{1}{2}-\frac{1}{2}} = -ie\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{p_+}{E+m} + \frac{p'_+}{E'+m}\right] \quad (23)$$

$$\mathcal{M}^{(1)}_{\frac{1}{2}\frac{1}{2}} = i\sqrt{2}e\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[1 - \frac{p'_z p_z}{(E'+m)(E+m)}\right] \quad (24)$$

$$\mathcal{M}^{(1)}_{\frac{1}{2}-\frac{1}{2}} = -i\sqrt{2}e\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{\sqrt{2}p'_z p_+}{(E'+m)(E+m)}\right] \quad (25)$$

$$\mathcal{M}^{(1)}_{-\frac{1}{2}\frac{1}{2}} = i\sqrt{2}e\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{-\sqrt{2}p'_+ p_z}{(E'+m)(E+m)}\right] \quad (26)$$

$$\mathcal{M}^{(1)}_{-\frac{1}{2}-\frac{1}{2}} = -i\sqrt{2}e\sqrt{\frac{(E'+m)(E+m)}{4E'E}}\left[\frac{2p'_+ p_+}{(E'+m)(E+m)}\right] \quad (27)$$

$q^4\bar{q}$ spatial wave functions

- The spatial wave functions are in symmetric type for ground state pentaquarks. And we constructed the spatial wave functions to high orders in the harmonic oscillator interaction as complete bases.

$\Psi^{q^4\bar{q}}_{000_{[4]_S}}$	$\psi^{q^4}_{000_{[4]_S}}\psi_{0,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{200_{[4]_S}}$	$\psi^{q^4}_{200_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{000_{[4]_S}}\psi_{1,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{400_{[4]_S}}$	$\psi^{q^4}_{400_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{200_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{000_{[4]_S}}\psi_{2,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{600_{[4]_S}}$	$\psi^{q^4}_{600_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{400_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{200_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{000_{[4]_S}}\psi_{3,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{800_{[4]_S}}$	$\psi^{q^4}_{800_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{600_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{400_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{200_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{000_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{1000_{[4]_S}}$	$\psi^{q^4}_{1000_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{800_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{600_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{400_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{200_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{1200_{[4]_S}}$	$\psi^{q^4}_{1200_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{1000_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{800_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{600_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{400_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{1400_{[4]_S}}$	$\psi^{q^4}_{1400_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{1200_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{1000_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{800_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{600_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{1600_{[4]_S}}$	$\psi^{q^4}_{1600_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{1400_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{1200_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{1000_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{800_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{1800_{[4]_S}}$	$\psi^{q^4}_{1800_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{1600_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{1400_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{1200_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{1000_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{2000_{[4]_S}}$	$\psi^{q^4}_{2000_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{1800_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{1600_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{1400_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{1200_{[4]_S}}\psi_{4,0}(\vec{\xi})$
$\Psi^{q^4\bar{q}}_{2200_{[4]_S}}$	$\psi^{q^4}_{2200_{[4]_S}}\psi_{0,0}(\vec{\xi}),\ \psi^{q^4}_{2000_{[4]_S}}\psi_{1,0}(\vec{\xi}),\ \psi^{q^4}_{1800_{[4]_S}}\psi_{2,0}(\vec{\xi}),\ \psi^{q^4}_{1600_{[4]_S}}\psi_{3,0}(\vec{\xi}),\ \psi^{q^4}_{1400_{[4]_S}}\psi_{4,0}(\vec{\xi})$

q^4 sub-group spatial wave function

NLM	Wave function
0 00	$\Psi^0_{00S} = \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta)$
2 00	$\Psi^2_{00S} = \frac{1}{\sqrt{3}}[\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta)]$
4 00	$\Psi^4_{00S} = \sqrt{\frac{5}{33}}[\Psi_{200}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{200}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{200}(\eta) + \sqrt{\frac{6}{5}}\Psi_{100}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \sqrt{\frac{6}{5}}\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta) + \sqrt{\frac{6}{5}}\Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{100}(\eta)]$
6 00	$\Psi^6_{00S} = \sqrt{\frac{35}{429}}[\Psi_{300}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{300}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{300}(\eta) + \frac{3}{\sqrt{7}}\Psi_{200}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}\Psi_{200}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta) + \frac{3}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{200}(\lambda)\Psi_{100}(\eta) + \frac{3}{\sqrt{7}}\Psi_{100}(\rho)\Psi_{200}(\lambda)\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{200}(\eta) + \frac{3}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{200}(\eta) + 3\sqrt{\frac{6}{35}}\Psi_{100}(\rho)\Psi_{100}(\lambda)\Psi_{100}(\eta)]$
8 00	$\Psi^8_{00S} = \sqrt{\frac{7}{143}}[\Psi_{400}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{400}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{400}(\eta) + \frac{2}{\sqrt{3}}\Psi_{300}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \frac{2}{\sqrt{3}}\Psi_{300}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta) + \frac{2}{\sqrt{3}}\Psi_{000}(\rho)\Psi_{300}(\lambda)\Psi_{100}(\eta) + \frac{2}{\sqrt{3}}\Psi_{100}(\rho)\Psi_{300}(\lambda)\Psi_{000}(\eta) + \frac{2}{\sqrt{3}}\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{300}(\eta) + \frac{2}{\sqrt{3}}\Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{300}(\eta) + \sqrt{\frac{10}{7}}\Psi_{200}(\rho)\Psi_{200}(\lambda)\Psi_{000}(\eta) + \sqrt{\frac{10}{7}}\Psi_{200}(\rho)\Psi_{000}(\lambda)\Psi_{200}(\eta) + \sqrt{\frac{10}{7}}\Psi_{000}(\rho)\Psi_{200}(\lambda)\Psi_{200}(\eta) + 2\sqrt{\frac{3}{7}}\Psi_{200}(\rho)\Psi_{100}(\lambda)\Psi_{100}(\eta) + 2\sqrt{\frac{3}{7}}\Psi_{100}(\rho)\Psi_{200}(\lambda)\Psi_{100}(\eta) + 2\sqrt{\frac{3}{7}}\Psi_{100}(\rho)\Psi_{100}(\lambda)\Psi_{200}(\eta)]$