Shear viscosity in an NJL pion gas

TECHNISCHE UNIVERSITÄT DARMSTADT

Michael Buballa

TORIC workshop, Heraklion, Crete, September 5 – 8, 2011

Collaborators

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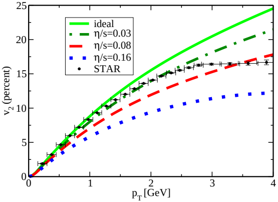
- based on
  - Klaus Heckmann, Ph.D. thesis, TU Darmstadt, 2011
  - K. Heckmann, M.B., J. Wambach, work in progress

Motivation

- flow data at RHIC



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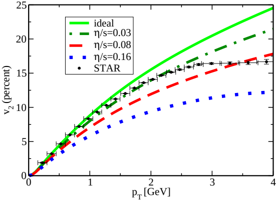

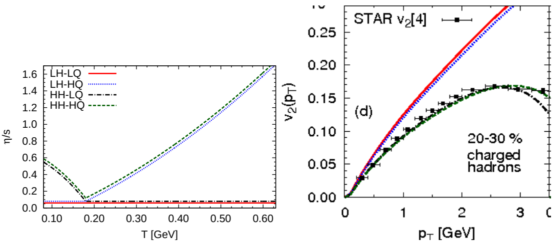

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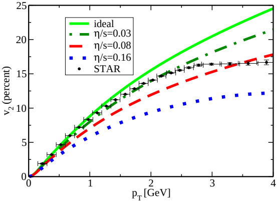

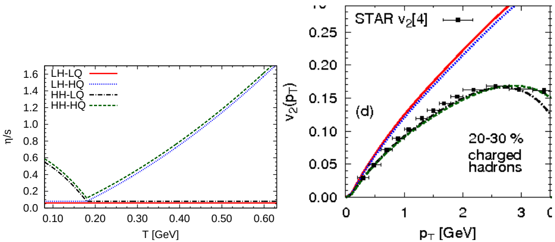

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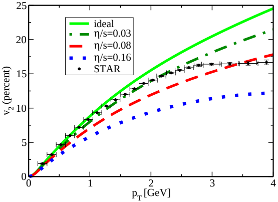

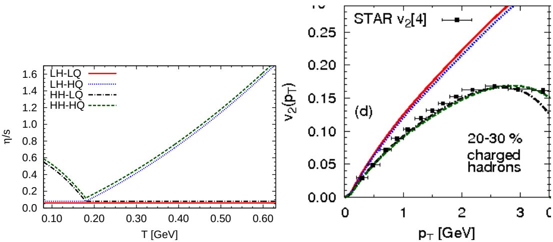

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- more recent: hydrodynamics more sensitive to hadronic phase

➜ aim: microscopic understanding of the shear viscosity in the hadronic phase

- here: BUU approach to $\pi\pi$-scattering in the NJL model
  - correct low-temperature limit
  - imprints of the chiral crossover and the compositeness of the pions

Viscous relativistic hydrodynamics

- basic ingredients and conservation laws:
  - fluid 4-velocity $u^\mu(x)$, $\quad u^\mu(x)u_\mu(x) = 1$
  - energy-momentum tensor $T^{\mu\nu}(x)$, $\quad \partial_\mu T^{\mu\nu}(x) = 0$
  - particle current $J^\mu(x) = n(x)u^\mu(x)$, $\quad \partial_\mu J^\mu(x) = 0$
- additional assumption:

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- gradient expansion:

  $T^{\mu\nu} = T^{(0)\mu\nu} + T^{(1)\mu\nu} + \ldots$ (and similar for J^μ)
  - ideal fluid:

    $$T^{(0)\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$$
  - 1st-order viscous correction:

    $$T^{(1)\mu\nu} = \eta \left(\partial^\mu u^\nu + \partial^\nu u^\mu + u^\mu u^\lambda \partial_\lambda u^\nu + u^\nu u^\lambda \partial_\lambda u^\mu\right) + \left(\zeta - \tfrac{2}{3}\eta\right)\left(g^{\mu\nu} - u^\mu u^\nu\right)\partial_\lambda u^\lambda$$

Quantum relativistic kinetic theory

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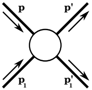

- Boltzmann-Uehling-Uhlenbeck (BUU) equation $(2 \to 2)$

$$\frac{d}{dt} f_a(\vec{x}, \vec{p}, t) = \sum_b \frac{g_b}{1+\delta_{ab}} \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p'_1}{(2\pi)^3} \Big\{ |\mathcal{M}_{ab}|^2 \frac{(2\pi)^4 \delta^4(p+p_1-p'-p'_1)}{16 E E_1 E' E'_1} \times \Big[f'_a f'_{1b}(1+f_a)(1+f_{1b}) - f_a f_{1b}(1+f'_a)(1+f'_{1b}) \Big] \Big\}$$

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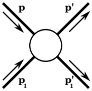

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$$\times \left[f_a' f_{1b}' (1+f_a)(1+f_{1b}) - f_a f_{1b} (1+f_a')(1+f_{1b}') \right] \Big\}$$

- linearization (2nd-order Chapman-Enskog expansion)

$f_a = f_a^{(0)} + f_a^{(1)} + \dots ,$ local equilibrium: $f_a^{(0)}(x,p) = \frac{1}{\exp\left[\left(p^\mu u_\mu(x) - \mu_\pi(x)\right)/T(x) - 1\right]}$

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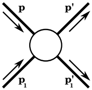

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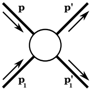

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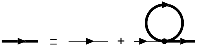
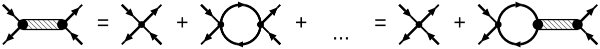
➜ linear integral equation for η

- physics input: scattering matrix element $\mathcal{M}_{ab}$ here: $\pi\pi \to \pi\pi$ in NJL

Mesons in the NJL model

- Lagrangian: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 

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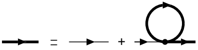
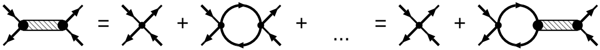
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 
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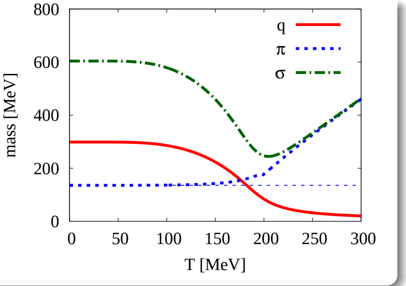

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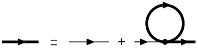
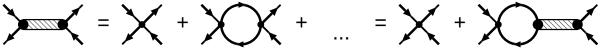
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 
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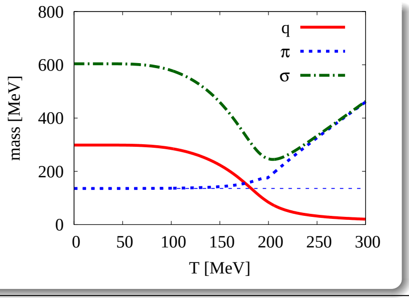
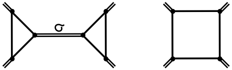


- characteristic temperatures:
  [Quack et al., PLB (1995)]
  - σ-dissociation temperature:
    $m_\sigma(T_{diss}) = 2m_\pi(T_{diss})$
    here: T_{diss} = 180 MeV
  - Mott temperature:
    $m_\pi(T_{Mott}) = 2m_\pi(T_{Mott})$
    here: T_{Mott} = 199 MeV

In-medium $\pi\pi$-scattering

- scattering amplitude

  [Bernard et al., PLB (1991), Schulze, JPG (1995)]



  - leading order $1/N_c$
  - to be taken in *s*-, *t*-, and *u*-channel
  - consistent with chiral low-energy theorems

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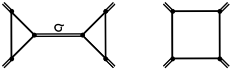

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- scattering length

$$a^I = \frac{1}{32\pi m_\pi} \mathcal{M}^I_{\pi\pi}(s = 4m_\pi^2, t = u = 0)$$

- chiral expansion [Weinberg, PRL (1966)]

$$a^0_W = \frac{7m_\pi}{32\pi f_\pi^2}, \quad a^2_W = -\frac{2m_\pi}{32\pi f_\pi^2}$$

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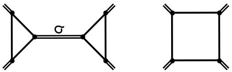

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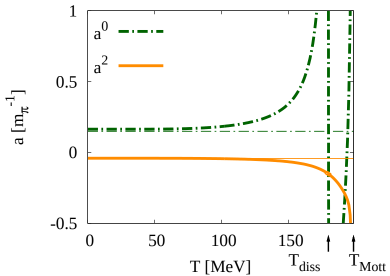
- numerical results

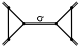
  cf. [Quack et al. PLB (1995)]



  - Weinberg values at low T
  - "Feshbach resonances" at T_{diss} and T_{Mott}

In-medium cross section

- isospin averaged cross section: $\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{9}\sum_{I=0}^{2}(2I+1)\frac{\left|\mathcal{M}^{I}_{\pi\pi}\right|^{2}}{64\pi^{2}s}$

- $i\mathcal{M}_{\pi\pi}$ = 

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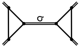
- $i\mathcal{M}_{\pi\pi}$ = 

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- approximations:
  1. Weinberg amplitude

     $\mathcal{M}^{I}_{\pi\pi} = 32\pi\, m_\pi\, a^{I}_{W}$

     (T and momentum independent)
  2. evaluate $\mathcal{M}$ at threshold

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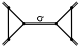
- $i\mathcal{M}_{\pi\pi}$ =  + 

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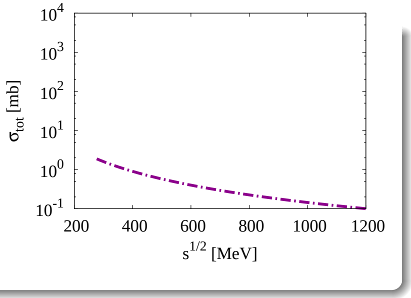
total cross section ($T = 0$)



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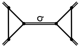
- $i\mathcal{M}_{\pi\pi}$ =  + 

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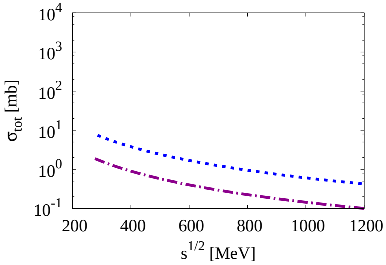
total cross section (T = 150 MeV)



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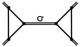
- $i\mathcal{M}_{\pi\pi}$ = 

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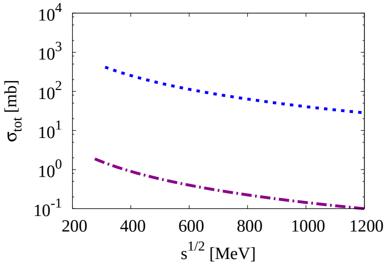
total cross section (T = 177 MeV)



In-medium cross section

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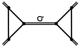
- isospin averaged cross section: $\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{9}\sum_{I=0}^{2}(2I+1)\frac{\left|\mathcal{M}^{I}_{\pi\pi}\right|^2}{64\pi^2 s}$
- $i\mathcal{M}_{\pi\pi}$ =  + 

- approximations:
  1. Weinberg amplitude
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     (T and momentum independent)
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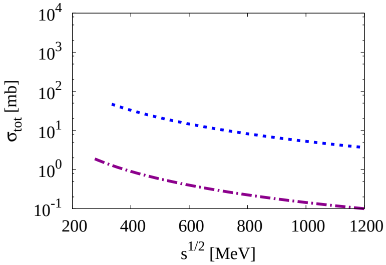
total cross section (T = 188 MeV)



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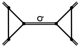
- $i\mathcal{M}_{\pi\pi}$ = 

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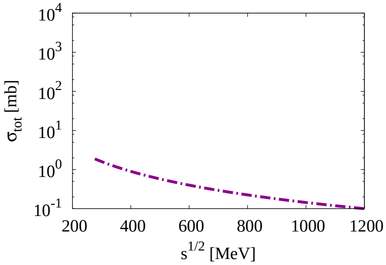

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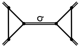
- $i\mathcal{M}_{\pi\pi}$ = 

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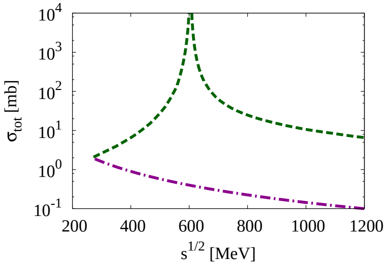

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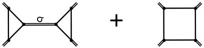

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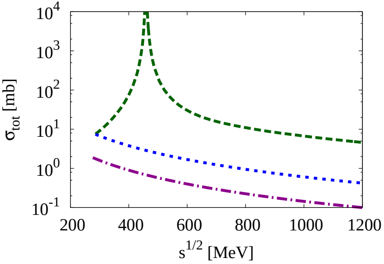

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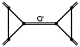
- $i\mathcal{M}_{\pi\pi}$ = 

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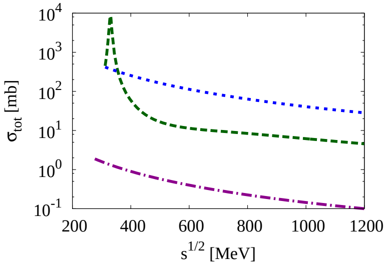
total cross section (T = 177 MeV)



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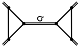
- $i\mathcal{M}_{\pi\pi}$ = 

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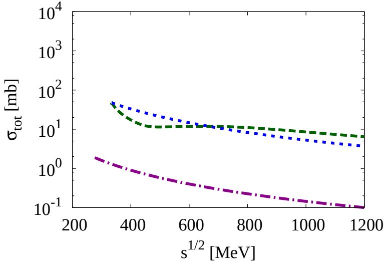
total cross section (T = 188 MeV)



Including the sigma-decay width

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- physical inconsistency:

  $\sigma \leftrightarrow \pi\pi$ considered in scattering,
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  $\Rightarrow$ width strongly underestimated

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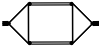
➜ include 

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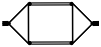
➜ include 

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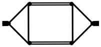
➜ include 

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$$\rho_\sigma(q) = -2\,\mathrm{Im}\,D_\sigma(q)$$

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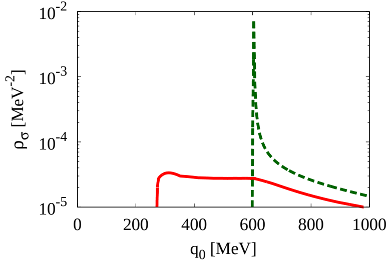

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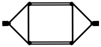
➜ include 

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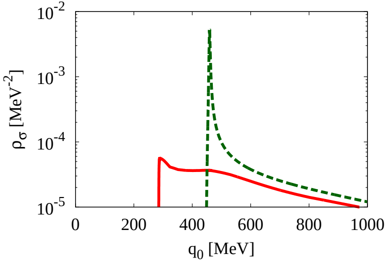

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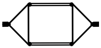
➜ include 

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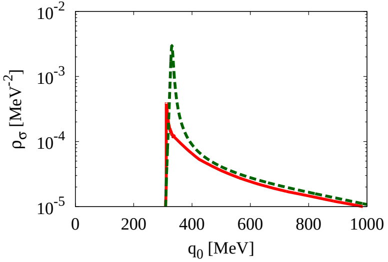

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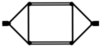
➜ include 

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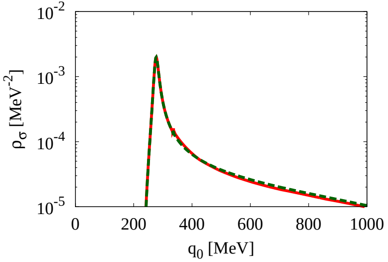

Cross section with sigma-decay width

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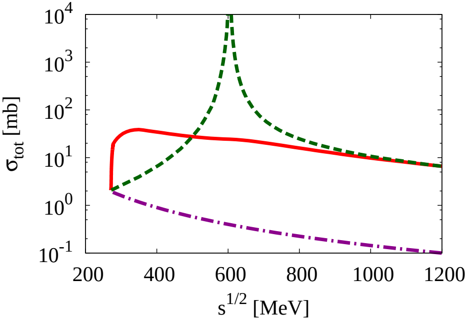
total cross section ($T = 0$)



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Cross section with sigma-decay width

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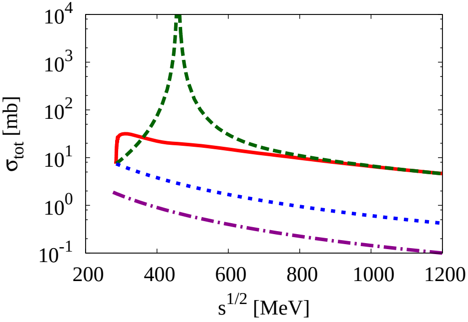
total cross section (T = 150 MeV)



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- small and intermediate T:

  $\sigma \gg \sigma(\mathcal{M})_{thresh} \gg \sigma_{Weinberg}$

Cross section with sigma-decay width

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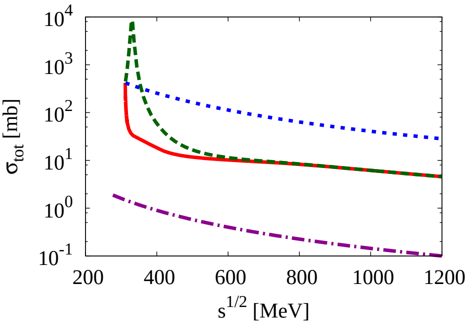

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TECHNISCHE UNIVERSITÄT DARMSTADT

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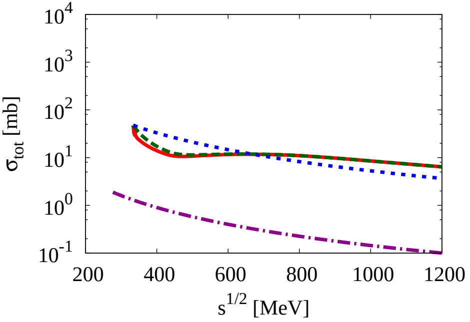

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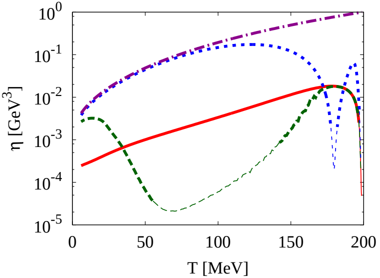
- $T > T_{diss}$:

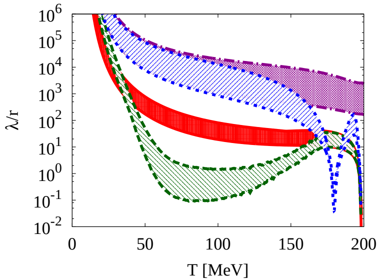
  $\sigma \to \pi\pi$ irrelevant

Shear viscosity: numerical results



- Weinberg
- $\mathcal{M} = \mathcal{M}_{threshold}$
- RPA σ-propagator
- dressed σ-propagator

- validity of the kinetic aproach:
  - criterion: $\frac{\lambda}{r} \gg 1$ (dilute gas)
  - $\lambda = \frac{1}{n\sigma}$ mean free path
  - r = interaction range
    (e.g., $1/m_\sigma$, $1/m_\pi$, hard sphere: $\sqrt{\frac{\sigma}{\pi}}$)



Fluidity

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  [Liao & Koch, PRC (2010)]
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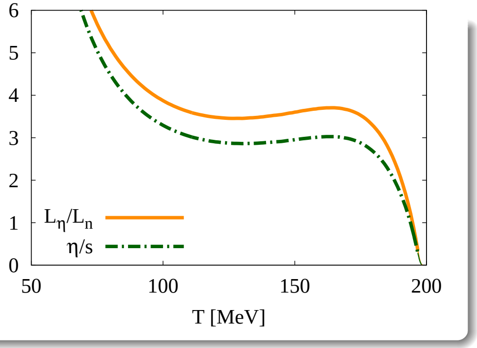

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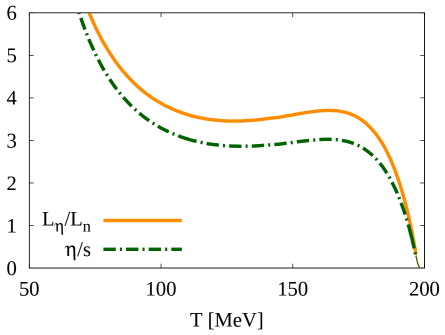

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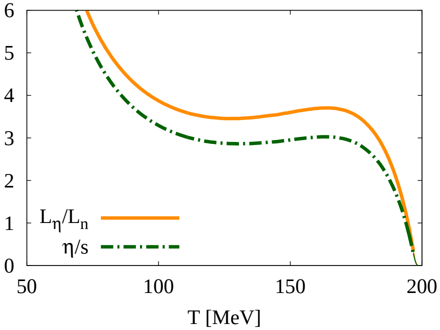

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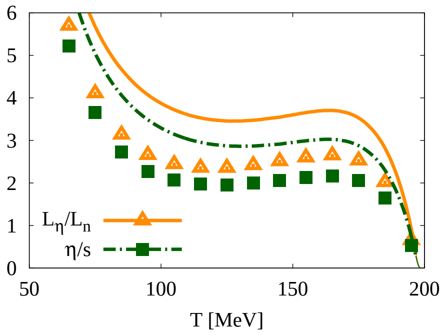

Conclusions

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  - quantitative results very sensitive to details of the model
- outlook:
  - better description of *p*-wave $\pi\pi$ scattering (include ρ-meson)
  - further scattering channels, e.g., kaons
  - ...