Mini-Review: T_c from the Wuppertal-Budapest collaboration

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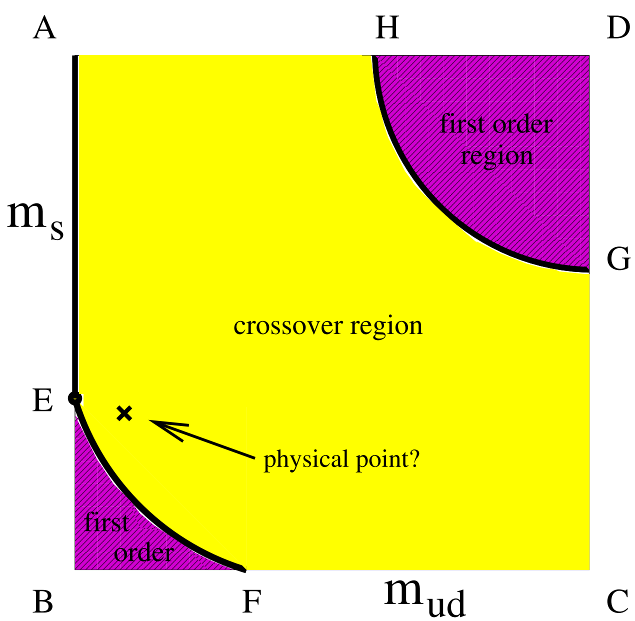
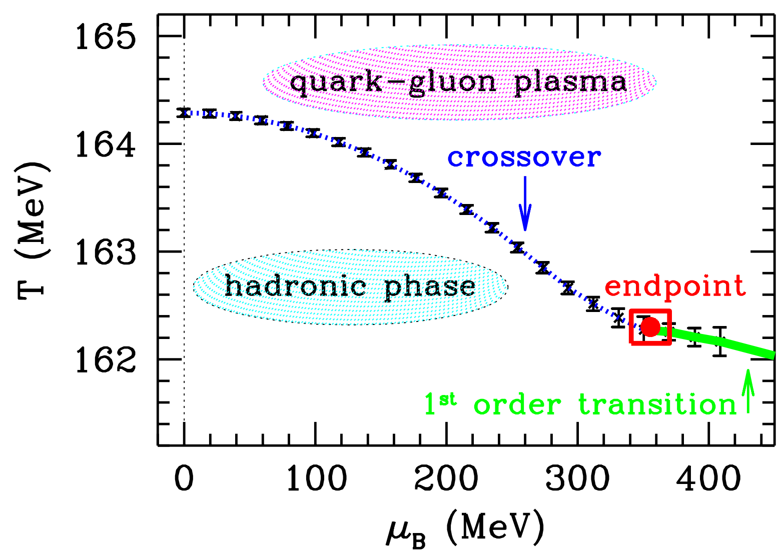
TORIC Network Workshop
5-8 September 2011

Introduction

Basic facts:

- In real-world QCD ($N_f = 2{+}1$ with $m_{ud,s}^{\text{phys}}$) transition (at $\mu = 0$) is a crossover [Y. Aoki et al, Nature 443, 675 (2006)].
- Nevertheless, after specifying the test quantity the resulting crossover temperature "T_c" is unique.





Talk outline

- Lattice QCD: quick consumer guide
- WB T_c paper 1 [Y. Aoki *et al*, PLB 643, 46 (2006)]
- WB T_c paper 2 [Y. Aoki *et al*, JHEP 0906, 088 (2009)]
- WB T_c paper 3 [Sz. Borsanyi *et al*, JHEP 1009, 073 (2010)]
- Explanation 0: taste-splittings
- Explanation 1: HRG
- Explanation 2: XPT

Lattice QCD: quick consumer guide

Points to be considered when using/comparing LQCD results:

(1) Has the continuum limit ($a \to 0$) been taken ?

(2) Are the simulations carried out with $M_\pi = 135\,\mathrm{MeV}$?

(3) Are finite-volume effects (from $L < \infty$) under control ?

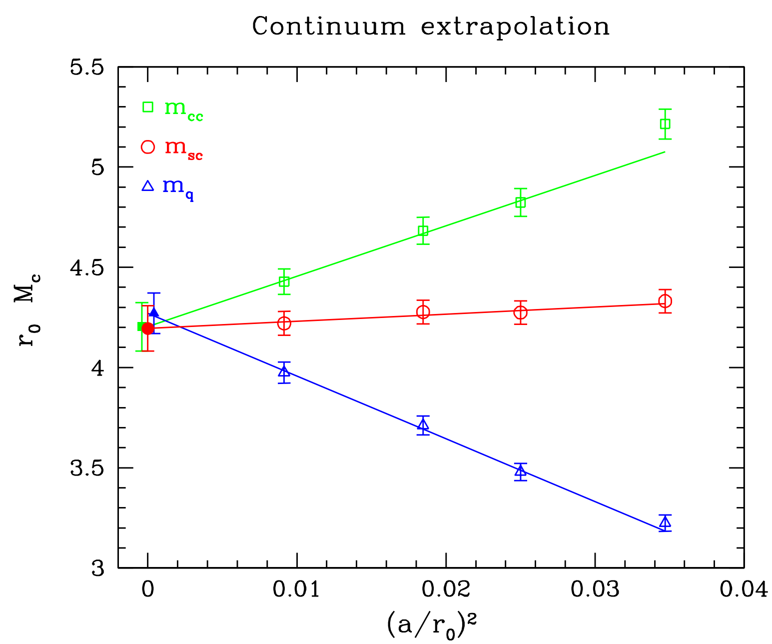
(4) Advanced: are theoretical uncertainties properly assessed/propagated ?

(5) Advanced: in particular if interim/artificial scale (e.g. r_0) is used ?

Example regarding first point:

◇ continuum limit is *universal*

◇ deviation at finite a may be *large*



J.Rolf, S.Sint [ALPHA], JHEP 0212, 007 (2002)

... with an immediate application to the three WB papers:

- **WB T_c paper 1: PLB 643, 46 (2006)**

$M_\pi = 135\,\mathrm{MeV}$ for $T > 0$ investigation
$M_\pi > 135\,\mathrm{MeV}$ for $T = 0$ scale-setting
Continuum limit from $N_t = 4,6,8,10$ lattices

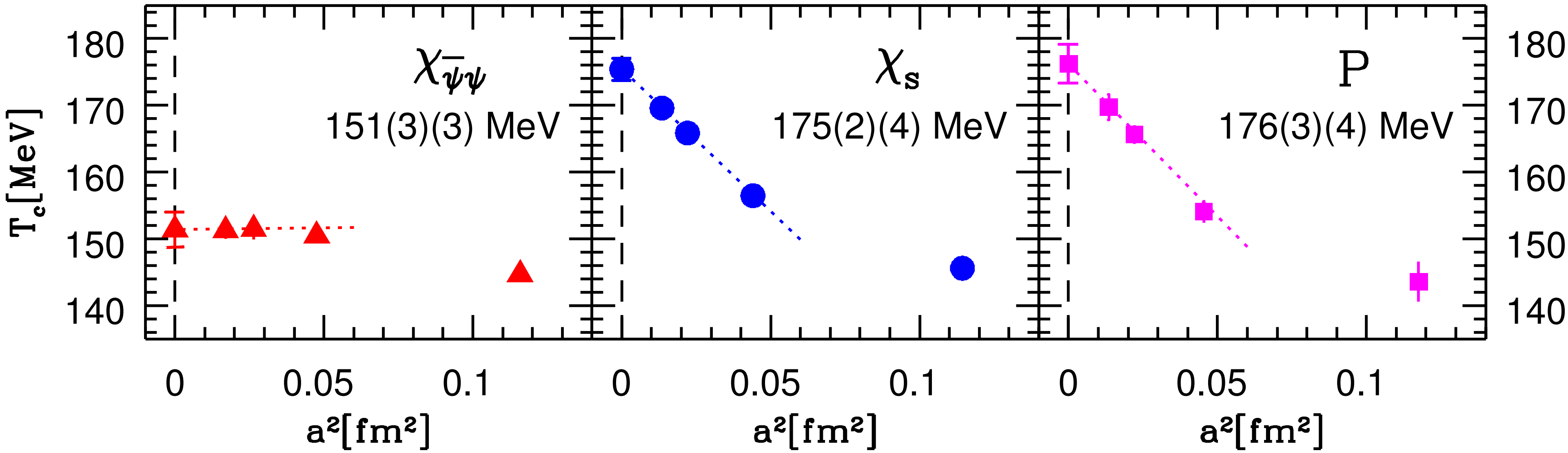
- **WB T_c paper 2: JHEP 0906, 088 (2009)**

$M_\pi = 135\,\mathrm{MeV}$ for $T > 0$ investigation
$M_\pi = 135\,\mathrm{MeV}$ for $T = 0$ scale-setting
Continuum limit from $N_t = 6,8,10,12$ lattices

- **WB T_c paper 3: JHEP 1009, 073 (2010)**

$M_\pi = 135\,\mathrm{MeV}$ for $T > 0$ investigation
$M_\pi = 135\,\mathrm{MeV}$ for $T = 0$ scale-setting
Continuum limit from $N_t = 8,10,12,16$ lattices

WB T_c paper 1: PLB 643, 46 (2006)



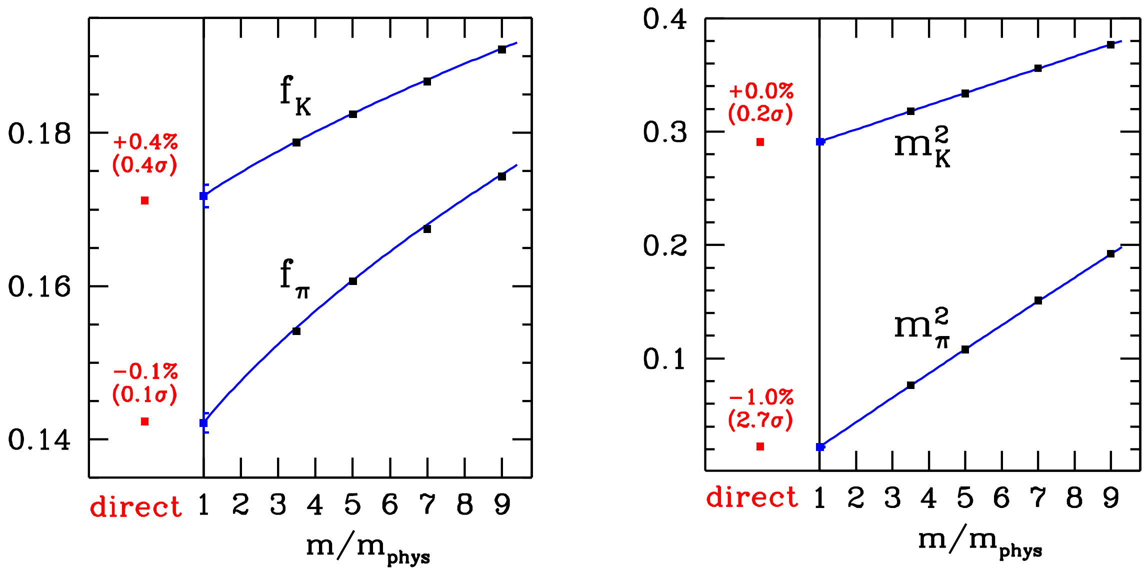
⟶ Renormalized condensate, strange quark number susceptibility and renormalized Polyakov loop give not necessarily the same T_c.

⟶ $N_t = 4$ is not in the Symanzik scaling regime, while $N_t = 6{,}8{,}10$ seem to be (for stout action).

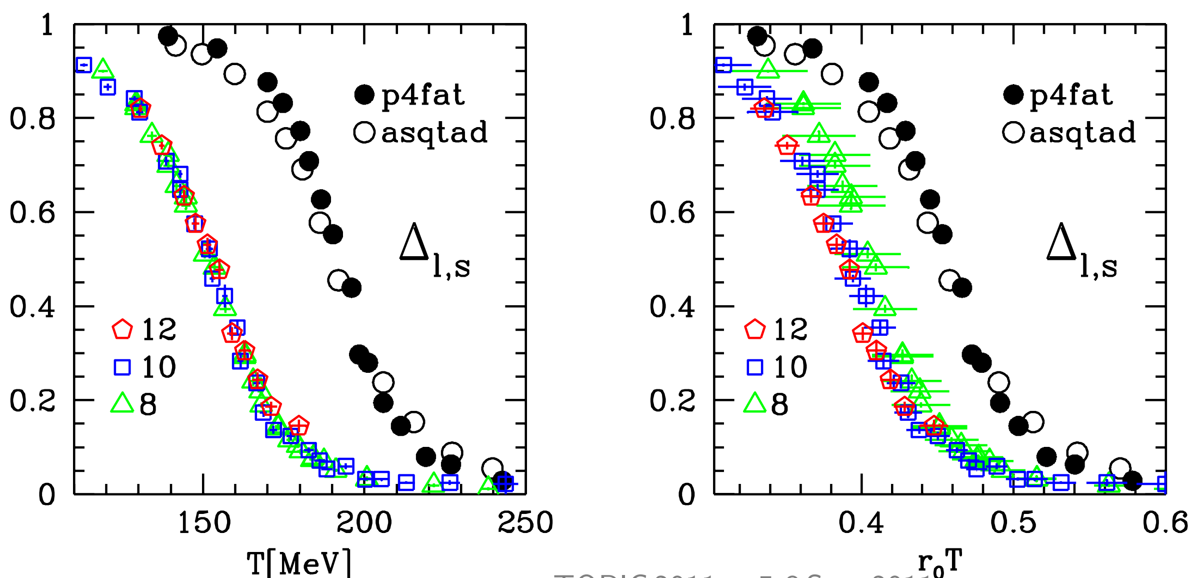
⟶ There is an overall scale setting uncertainty of $\sim 4\%$ (shift in f_K in 2008).

WB T_c paper 2: JHEP 0906, 088 (2009)

- $T=0$ scale-setting with physical pion/kaon mass

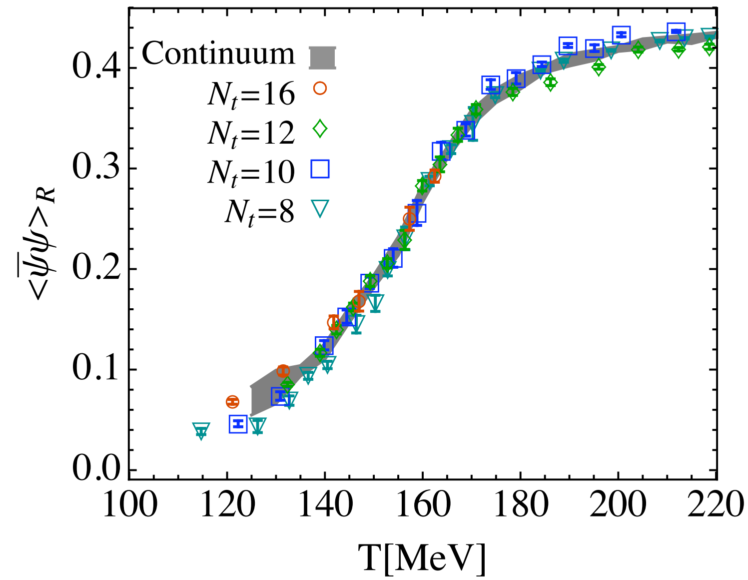


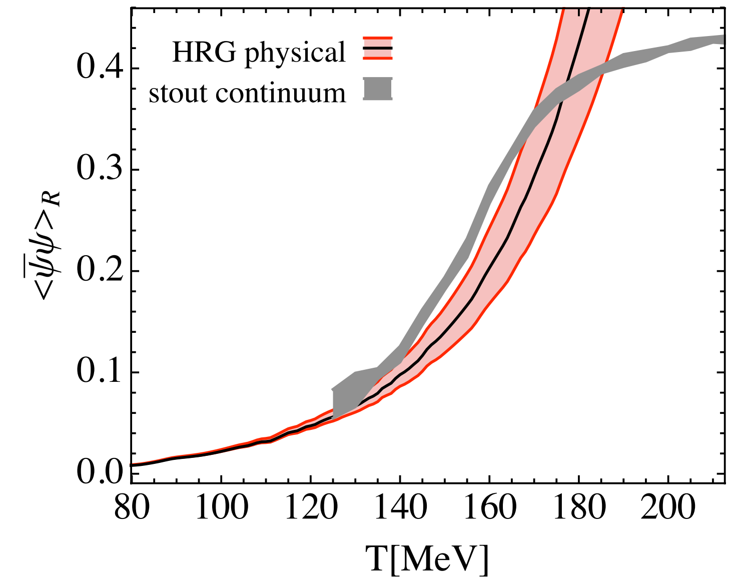
- $T>0$ investigation shows good scaling in transition region



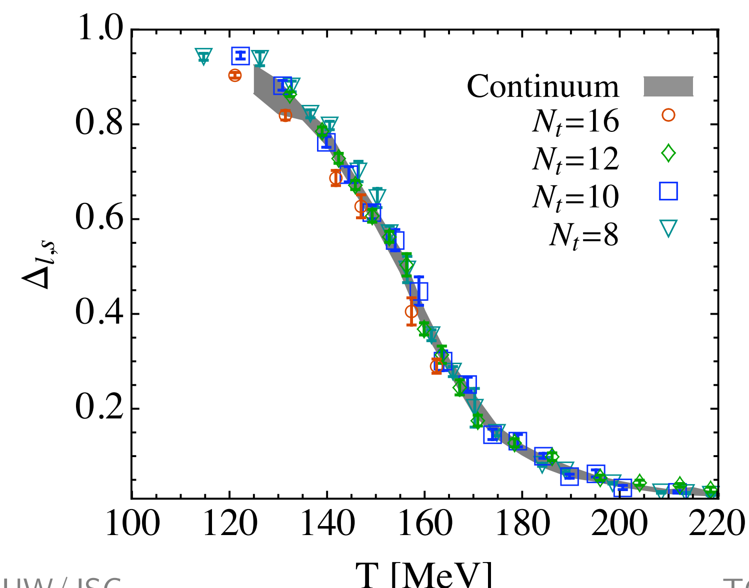
WB T_c paper 3: JHEP 1009, 073 (2010)

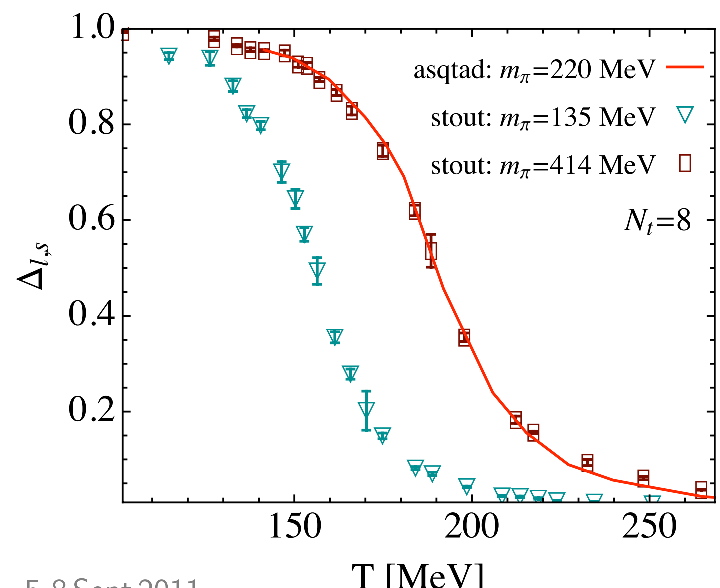
- **Renormalized chiral condensate**



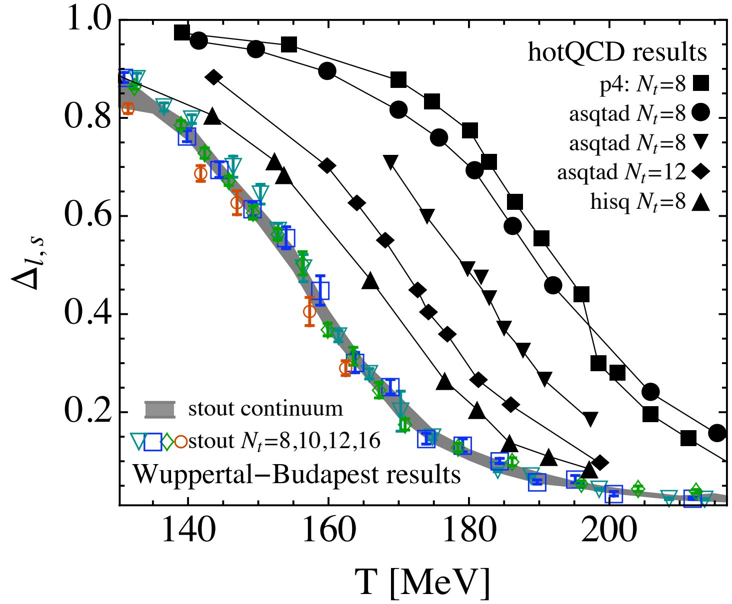


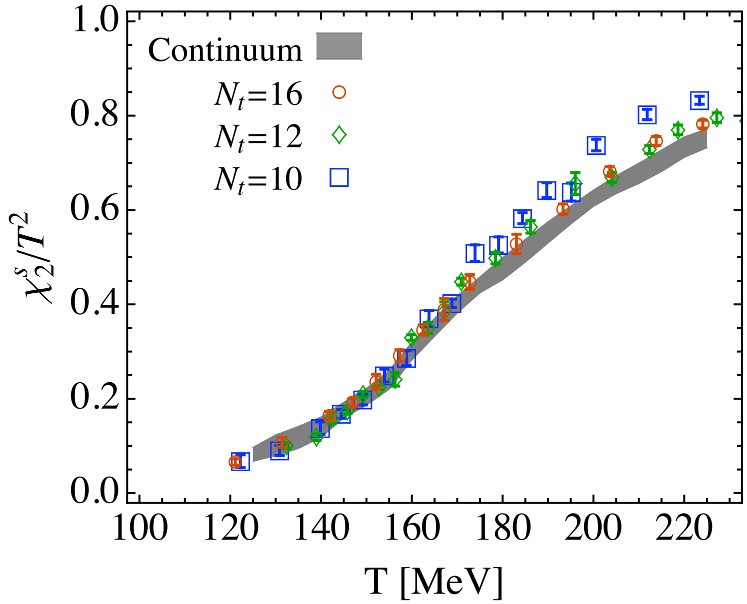
- **Subtracted light and strange condensates**



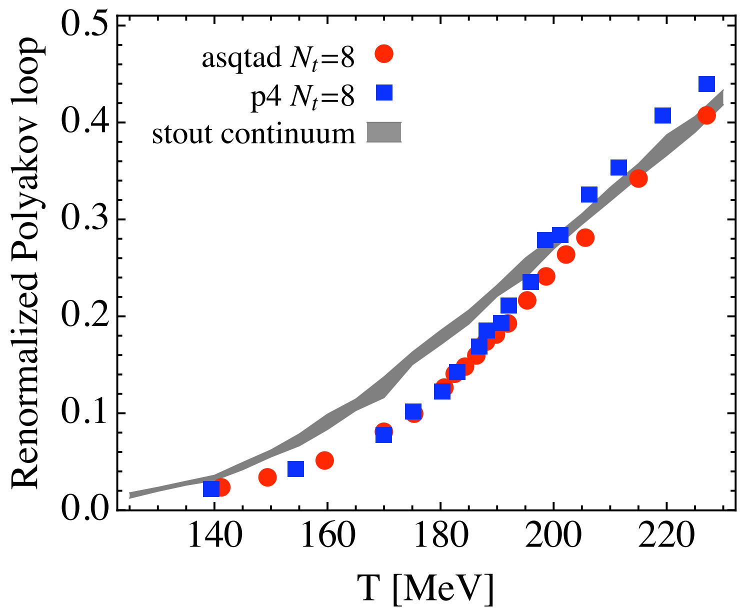


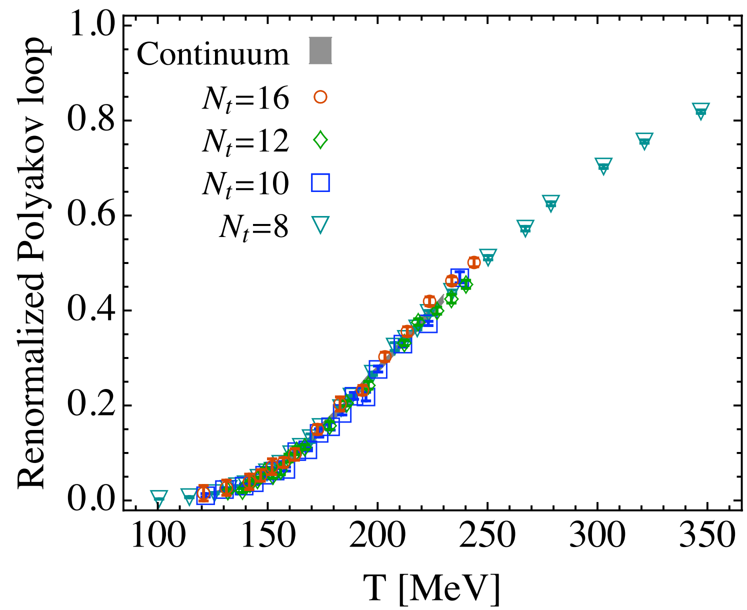
- **Subtracted condensate and strange quark number susceptibility**





- **Renormalized Polyakov loop**



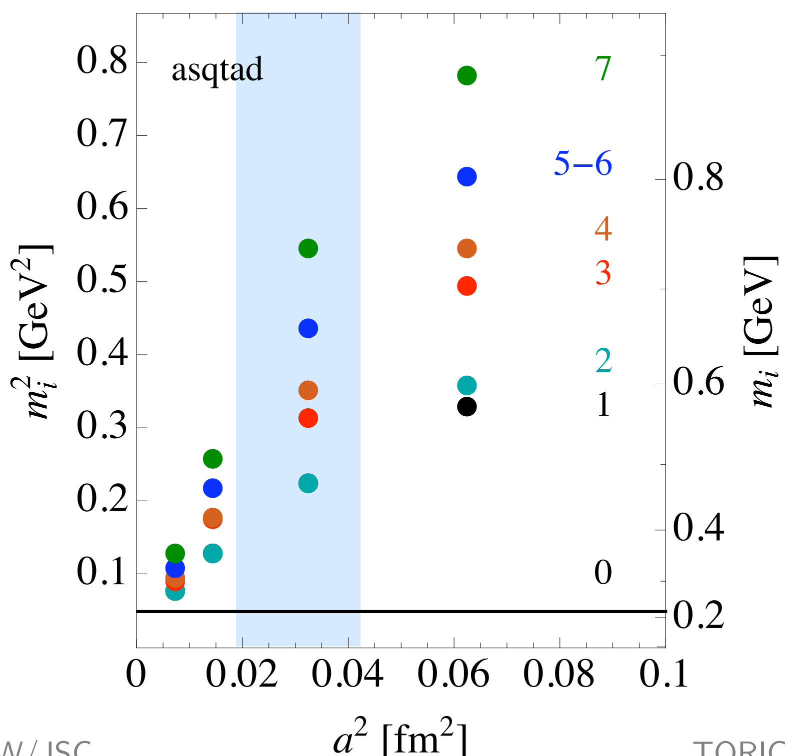


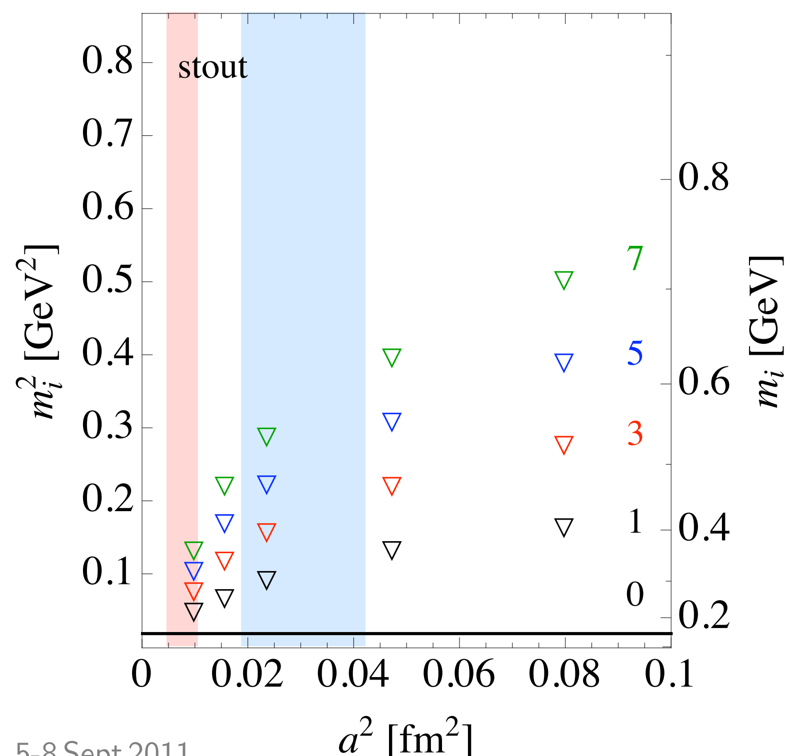
Explanation 0: taste splittings

Staggered action "doubled", i.e. 1 field corresponds to 4 quarks in the continuum.
Accordingly, QCD with 2 staggered fields has rich spectrum of pseudoscalar bosons:

- 3 Goldstone pions, i.e. $M=0$ at $m=0$
- 60 non-Goldstone pions, i.e. $M \propto a^2$ at $m=0$
- 1 η'-type state, i.e. $M>0$ even after $a \to 0$ at $m=0$

For physics (i.e. in the "valence sector") one projects onto the Goldstone bosons.
For MC sampling (i.e. in the "sea sector") the fourth root takes an average over all.
Evidence is mounting that these recipes work out in concert *in the continuum* limit !

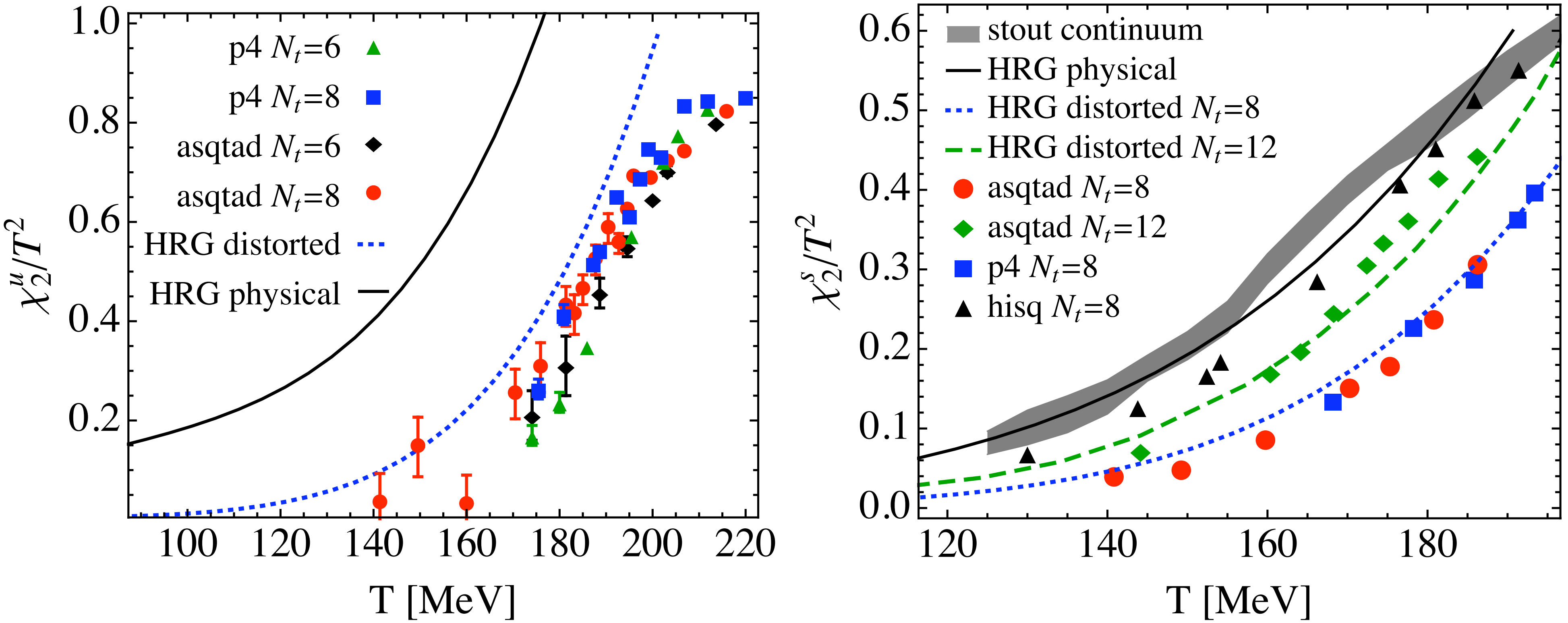




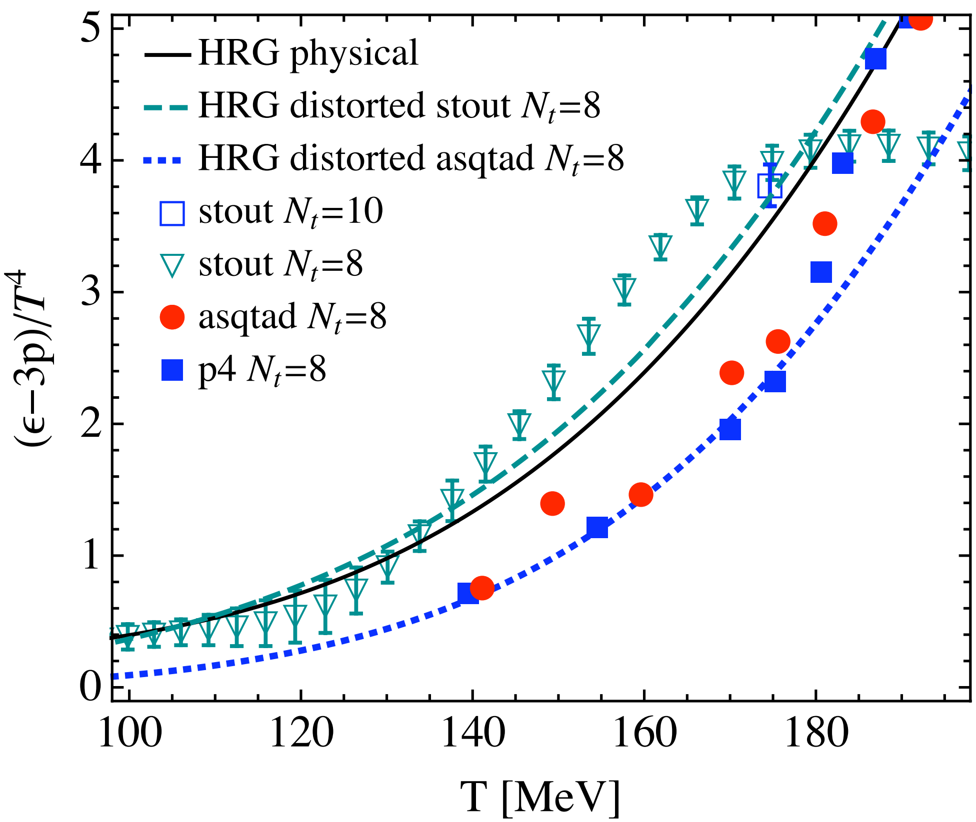
Explanation 1: HRG

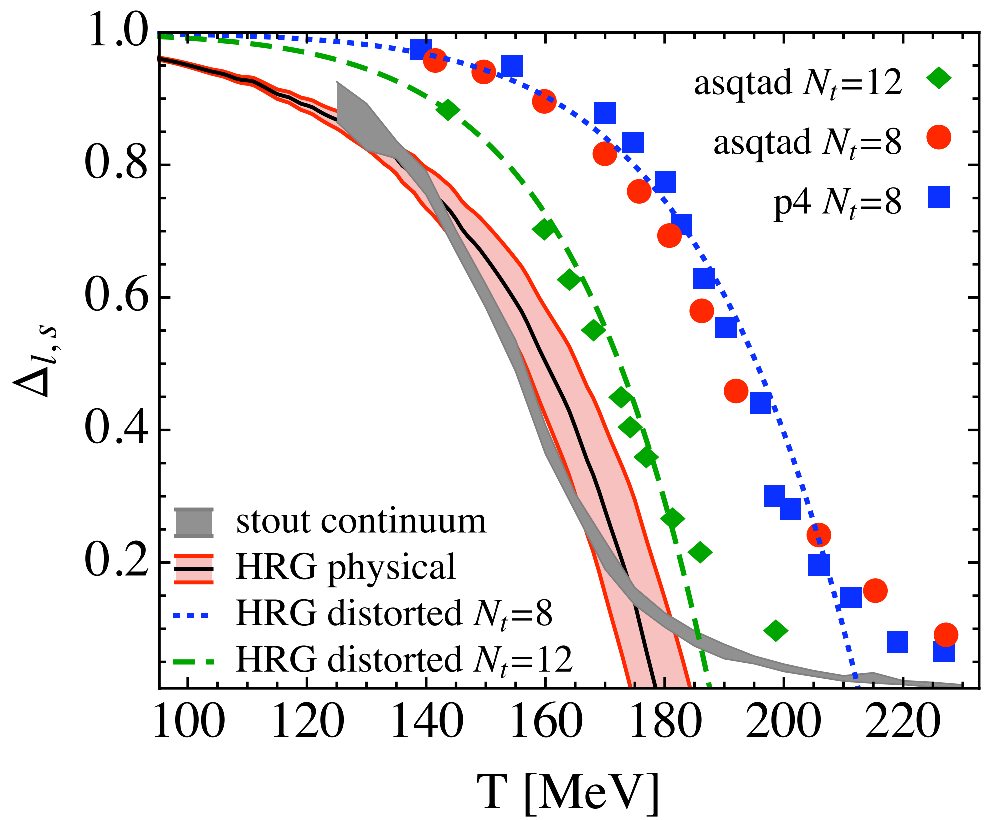
The hadron resonance gas model (HRG) can be extended to account for cut-off effects and pion mass dependence [Huovinen and Petreczky, NPA 837, 26 (2010)].

- **light and strange quark number susceptibility**



• Trace anomaly and subtracted chiral condensate





• Tentative summary

(i) Cut-off induced taste splittings increase the *average* pion mass in the sea.

(ii) Larger-than-physical average sea pion masses tend to *artificially increase* T_c.

(iii) In the *continuum limit* these effects disappear (more easily achieved for stout).

Explanation 2: XPT

In 1986 Gasser and Leutwyler presented a compact formula for the combined M_π, T, L dependence of the chiral condensate to 1-loop order in XPT

$$\langle\bar{\psi}\psi\rangle_{T=0} = -F^2 B\left\{1 - \frac{N_f^2-1}{N_f}\frac{M_\pi^2}{(4\pi F)^2}\left[\log(M_\pi^2/\Lambda^2)+\frac{1}{2}\right]+O(p^6)\right\}$$

$$\langle\bar{\psi}\psi\rangle_{T} = \langle\bar{\psi}\psi\rangle_{T=0}\left\{1 - \frac{N_f^2-1}{N_f}\frac{M_\pi^2}{(4\pi F)^2}\tilde{g}_1(M_\pi^2,L,T)+O(p^6)\right\}$$

where ψ represents any of the N_f light (degenerate) flavors. They also suggest using it to estimate the chiral transition temperature. In other words, they suggest solving $\langle\bar{\psi}\psi\rangle_T = 0$ for T at given values of M_π and L. The dimensionless function

$$\tilde{g}_k(M_\pi^2,L,T) = \int_0^\infty t^{k-3}\sum_{n\neq 0} e^{-t-[(n_1 M_\pi L)^2+(n_2 M_\pi L)^2+(n_3 M_\pi L)^2+(n_4 M_\pi/T)^2]/[4t]}\,dt$$

can be represented [with $a=(n_1M_\pi L)^2+(n_2M_\pi L)^2+(n_3M_\pi L)^2+(n_4M_\pi/T)^2$] as

$$\tilde{g}_k(M_\pi^2,L,T) = 2^{3-k}\sum_{n\neq 0} a^{k/2-1}K_{k-2}(a^{1/2})$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$m(n)$	1	6	12	8	6	24	24	0	12	30	24	24	8	24	48	0	6	48	36	24	24

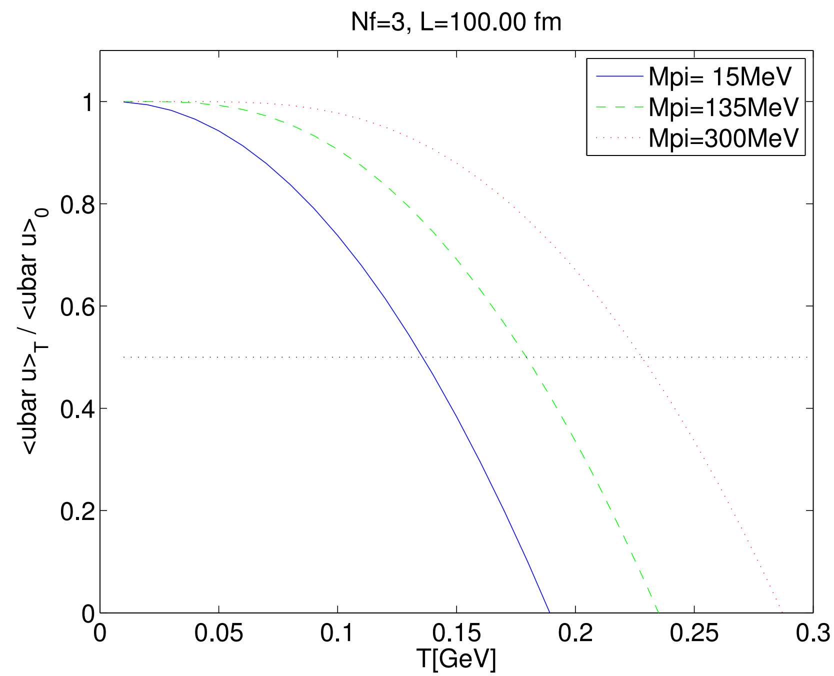
where K_{k-2} denotes a modified Bessel function of the second kind. Then

$$\tilde{g}_k(M_\pi^2, L, T) = 2^{3-k} \sum_{n\geq 1} m(n)(\ldots)^{k-2} K_{k-2}(\ldots) + 2^{4-k} \sum_{\ell\geq 1} \sum_{n\geq 1} m(n)(\ldots)^{k-2} K_{k-2}(\ldots)$$

depends only on $M_\pi L$ and M_π/T, and the replacement, relative to the previous formula, was $n_1^2 + n_2^2 + n_3^2 \to n$ and $n_4 \to \ell$. With the multiplicities $m(n)$ given in the table, this is a convenient way to evaluate the 4-fold infinite sum. Consider

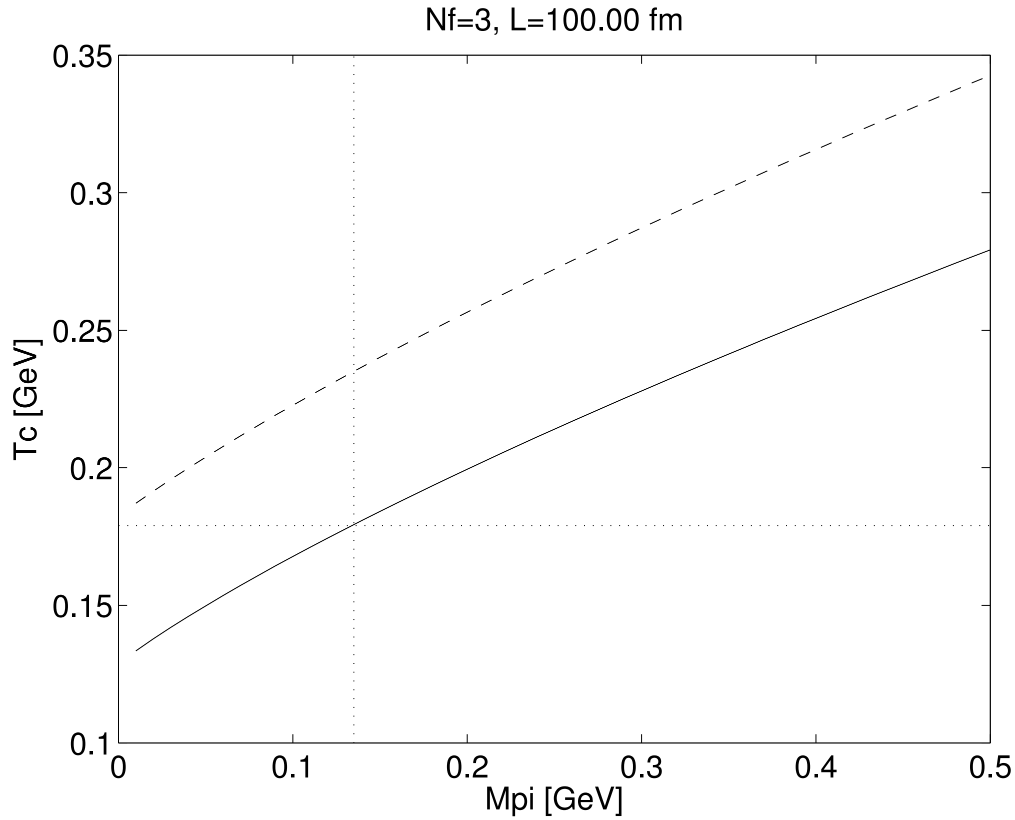
$$1 - \frac{N_f^2 - 1}{N_f} \frac{M_\pi^2}{(4\pi F)^2} \tilde{g}_1(M_\pi^2, L, T)$$

as a function of T, for given values of M_π and L. To determine T_c it is sufficient to solve for this expression being equal to 0 or $1/2$, for given values of M_π and L.



Regardless of the precise T_c definition, one finds that

$$T_c(M_\pi=300\,\mathrm{MeV})-T_c(M_\pi=135\,\mathrm{MeV}) \simeq 50\,\mathrm{MeV}\ .$$



$\Longrightarrow$ Higher-oder contributions $[O(p^6)]$ may decrease each $T_c(M_\pi)$, but the shift from $M_\pi \sim 300\,\mathrm{MeV}$ to $M_\pi \sim 135\,\mathrm{MeV}$ is likely robust.

$\Longrightarrow$ XPT alone can explain the origin of the (meanwhile resolved) discrepancy between the Budapest-Wuppertal results and the old hotQCD results.

Summary

- Crucial issues in LQCD calculations: $a \to 0$ and $M_\pi \sim M_\pi^{\mathrm{phys}}$
- Previously clear disagreement for pseudocritical temperatures T_c
- Meanwhile emerging consensus for pseudocritical temperatures T_c
- Explanation 1: HRG says that T_c increases with increasing (average) M_π
- Explanation 2: XPT says that T_c increases with increasing (average) M_π