Nonequilibrium effects in Polyakov loop extended chiral fluid dynamics

Christoph Herold

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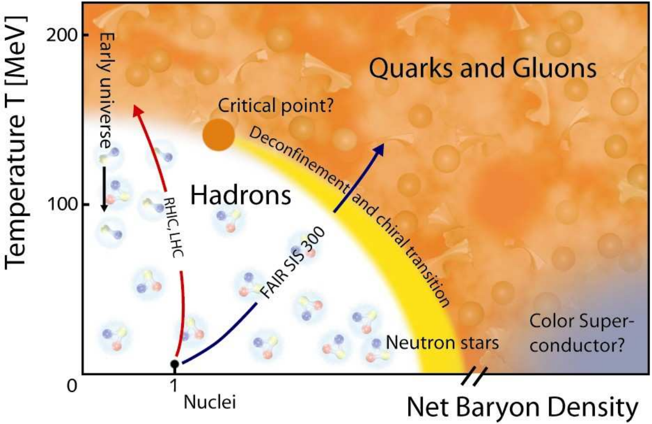
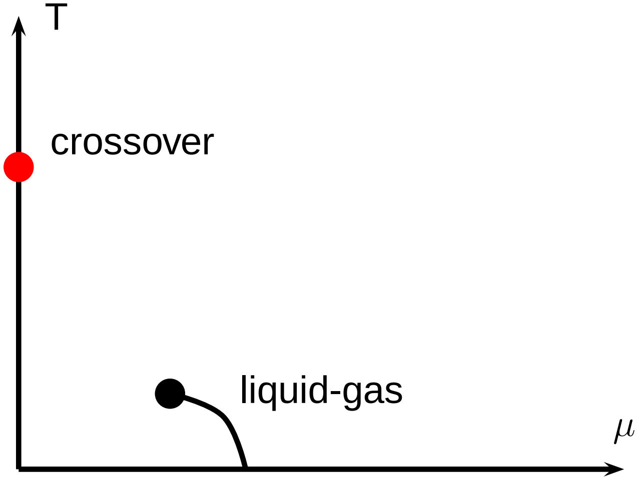
Marcus Bleicher, Carsten Greiner, Igor Mishustin, Marlene Nahrgang

The phase diagram of QCD



The phase diagram of QCD



Symmetries and order parameters in QCD

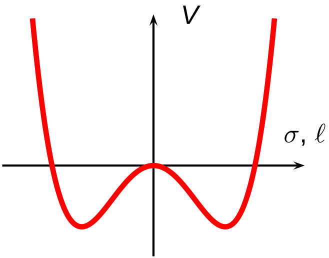
$SU(2)_V \times SU(2)_A$ chiral symmetry

- explicitly broken by m_q
- approximate symmetry for small m_q
- order parameter:
  chiral condensate $\langle \bar{q} q \rangle$, sigma field σ

Z_{N_c} center symmetry of $SU(N_c)$ gauge group

- only exact in pure gauge theory
- approximate symmetry for large m_q
- order parameter for confinement-deconfinement phase transition:
  Polyakov loop $\ell = \frac{1}{N_c} \langle tr_c \mathcal{P} \rangle_\beta$ with $\mathcal{P} = P \exp \left(i g_{QCD} \int_0^\beta \mathrm{d}\tau A_0 \right)$

Phase transitions



first order phase transition

- two degenerate minima at T_c
- phase coexistence

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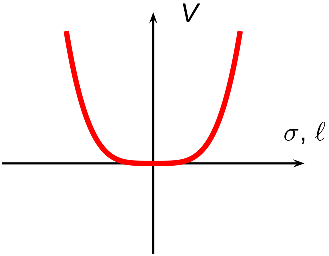
nucleation

- supercooled, $\frac{\partial^2 V}{\partial \sigma^2} > 0$
- large fluctuations
- bubble formation and growth

spinodal decomposition

- unstable, $\frac{\partial^2 V}{\partial \sigma^2} < 0$
- small fluctuations
- phase separation uniformly

Phase transitions



critical point

- $m_\sigma = 0$
- divergent correlation length

critical phenomena

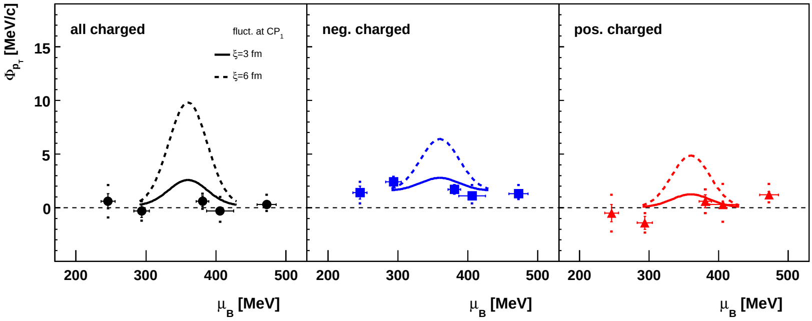
- divergent susceptibilities
- critical slowing down
- long-range fluctuations

The search for the critical point in heavy-ion collisions

- event-by-event fluctuations of multiplicity, mean p_T

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

(Stephanov, Rajagopal and Shuryak, PRD **60** (1999))



(K. Grebieszkow, NA49 collaboration Nucl. Phys. A **830** (2009))

- higher cumulants even more sensitive, e. g. $\kappa_4 \sim \xi^7$

(M. A. Stephanov, Phys. Rev. Lett. **102** (2009))

The search for the critical point in heavy-ion collisions

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$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$

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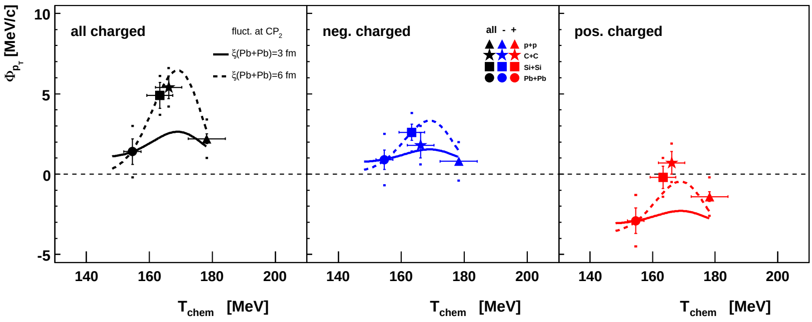
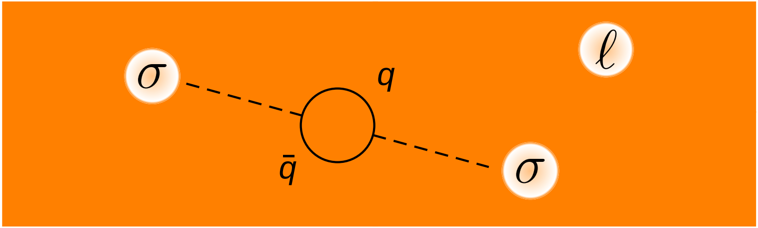
- system size dependence



(K. Grebieszkow, NA49 collaboration Nucl. Phys. A **830** (2009))

Chiral fluid dynamics with a Polyakov loop



- quarks: heat bath in local thermal equilibrium, locally interacting with:
- σ: mesonic field, propagated via Langevin equation
- ℓ: Polyakov loop, coupled to heat bath
- dynamical, self-consistent and energy-conserving
- nonequilibrium effects

(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999),

K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003),

M. Nahrgang, S. Leupold, C. H. and M. Bleicher, Phys. Rev. C **84** (2011))

The Polyakov loop extended linear-σ-model

The Lagrangian

$$\mathcal{L} = \bar{q}\left[\mathrm{i}\left(\gamma^\mu\partial_\mu - \mathrm{i}g_{QCD}\gamma^0 A_0\right) - \mathrm{g}\sigma\right] q + {}^1\!/_2 \left(\partial_\mu\sigma\right)^2 - U\left(\sigma\right) - \mathcal{U}(\ell,\bar{\ell})$$

with the mesonic potential

$$U\left(\sigma\right) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

and the Polyakov loop potential

$$\frac{\mathcal{U}}{T^4}\left(\ell,\bar{\ell}\right) = -\frac{b_2(T)}{4}\left(\left|\ell\right|^2 + \left|\bar{\ell}\right|^2\right) - \frac{b_3}{6}\left(\ell^3 + \bar{\ell}^3\right) + \frac{b_4}{16}\left(\left|\ell\right|^2 + \left|\bar{\ell}\right|^2\right)^2$$

(C. Ratti, M. A. Thaler, W. Weise, Phys. Rev. D **73** (2006), B.-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D **76** (2007))

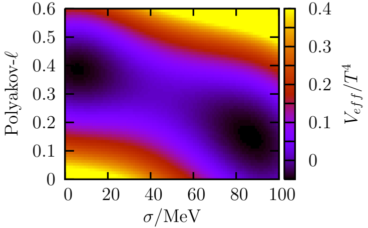
Thermodynamics

grand canonical potential at $\mu_B = 0$, $\ell = \bar{\ell}$, mean-field

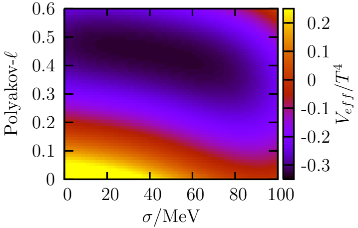
$$\Omega_{\bar{q}q} = -4N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln \left[1 + 3\ell \mathrm{e}^{-\beta E} + 3\ell \mathrm{e}^{-2\beta E} + \mathrm{e}^{-3\beta E}\right]$$

effective potential

$$V_{eff}(\sigma, \ell, T) = U(\sigma) + \mathcal{U}(\ell) + \Omega_{\bar{q}q}(\sigma, \ell, T)$$



first order transition,
$g = 4.7$, $T_c = 172.9$ MeV



critical point,
$g = 3.52$, $T_c = 180.5$ MeV

The equations of motion

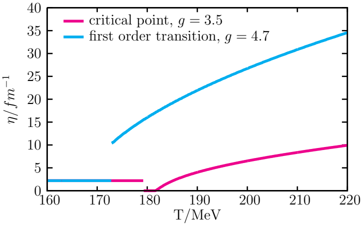
$$\partial_\mu \partial^\mu \sigma + \eta_\sigma \partial_t \sigma + \frac{\partial V_{eff}}{\partial \sigma} = \xi_\sigma$$

with damping coefficient η_σ for $\mathbf{k} = \mathbf{0}$

$$\eta_\sigma = \frac{12g^2}{\pi} \left[1 - 2n_{\mathrm{F}} \left(\frac{m_\sigma}{2}\right)\right] \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

and $\eta_\sigma = 2.2/fm$ for $\sigma \leftrightarrow \pi\pi$ reaction

(T. S. Biro and C. Greiner, Phys. Rev. Lett. **79** (1997))



and the dissipation-fluctuation theorem

$$\langle \xi_\sigma(t) \xi_\sigma(t') \rangle = \frac{1}{V} \delta(t - t') m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right)$$

(M. Nahrgang, S. Leupold, C. H. and M. Bleicher, Phys. Rev. C **84** (2011))

The equations of motion

Allow for dynamical evolution of the Polyakov loop

$$\mathcal{L} \to \mathcal{L} + \frac{N_c}{g_{QCD}^2} |\partial_\mu \ell|^2 T^2$$

and add a phenomenological damping term $\eta_\ell \sim 1/fm$

(A. Dumitru and R. D. Pisarski, Nucl. Phys. A **698** (2002))

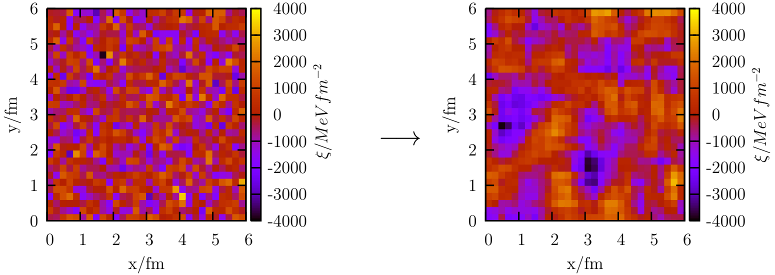
$$\frac{2N_c}{g_{QCD}^2} \partial_\mu \partial^\mu \ell T^2 + \eta_\ell \partial_t \ell + \frac{\partial V_{eff}}{\partial \ell} = \xi_\ell$$

$$\langle \xi_\ell(t) \xi_\ell(t') \rangle = \frac{1}{V} \delta(t - t') 2 \eta_\ell T$$

Thermodynamic consistency currently under investigation ...

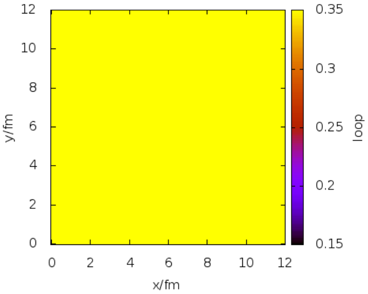
Nucleation and phase coexistence

correlate stochastic noise field over volume of $1fm^3$:



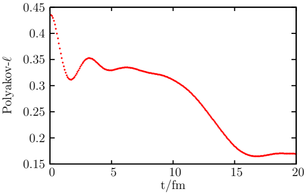
quench first order scenario from
177 MeV to 169 MeV,
phase coexistence

Nucleation and phase coexistence



bubble-

- formation
- growth
- melting



evolution of average value of ℓ

Propagation of the quark fluid

ideal relativistic fluid dynamics

$$\partial_\mu \left(T_q^{\mu\nu} + T_\sigma^{\mu\nu} + T_\ell^{\mu\nu} \right) = 0$$

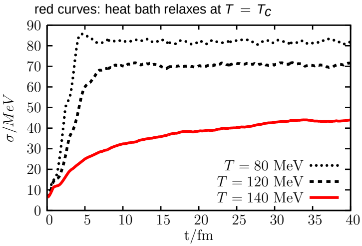
equation of state $e = e(p)$ from

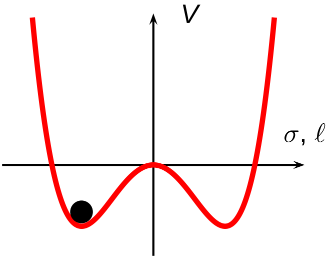
$$\begin{aligned} e(\sigma, \ell, T) &= T \frac{\partial p(\sigma, \ell, T)}{\partial T} - p(\sigma, \ell, T) \\ p(\sigma, \ell, T) &= -\Omega_{\bar{q}q}(\sigma, \ell, T) \end{aligned}$$

investigate two scenarios:

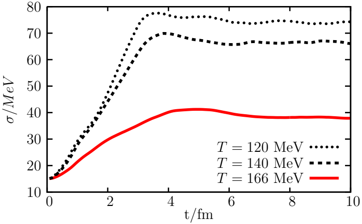
- fluid and fields in a box, temperature quench
- fluid dynamic expansion of a hot and plasma

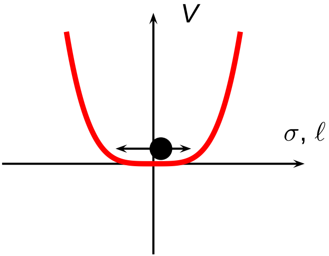
Box: Relaxation to equilibrium





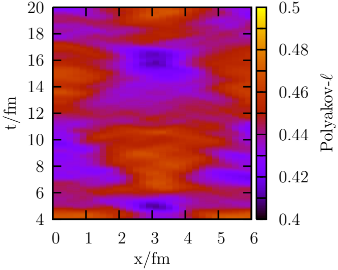
first order phase transition



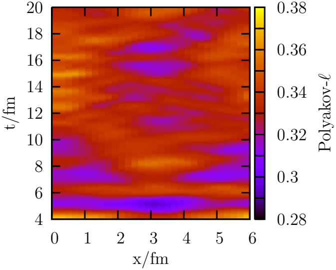


critical point

Box: Fluctuations at the critical point



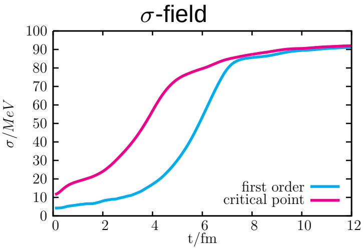
critical point

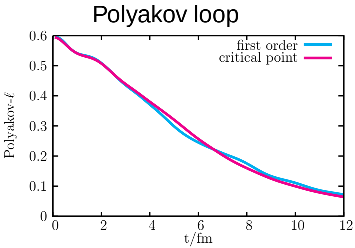


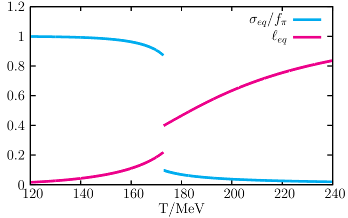
first order phase transition

long-range fluctuations at the CP over space **and** time

The expanding plasma



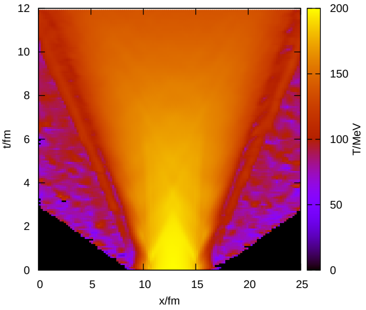




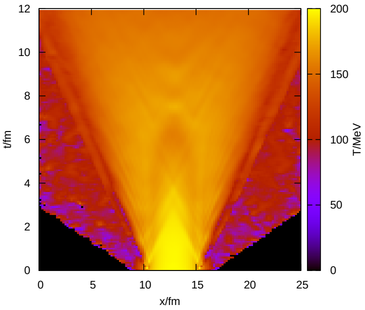
- σ-field: large barrier, different damping
- Polyakov loop: small barrier, equal damping

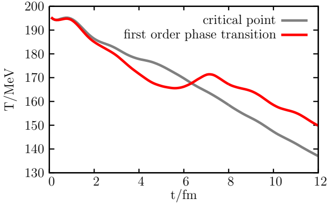
Effects on the temperature: Reheating

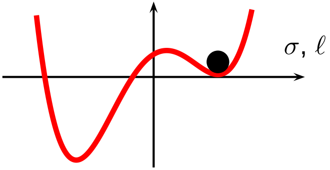
critical point



first order phase transition

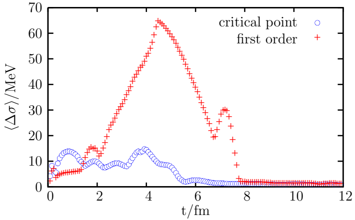


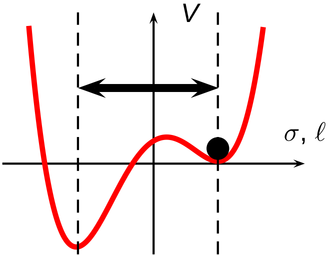


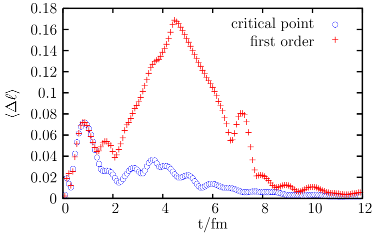


supercooled phase

Nonequilibrium fluctuations







$$\langle \Delta\sigma \rangle = \sqrt{\langle (\sigma - \sigma_{eq})^2 \rangle}$$

$$\langle \Delta\ell \rangle = \sqrt{\langle (\ell - \ell_{eq})^2 \rangle}$$

- enhanced fluctuations at the first order PT
- particle production

Summary and Outlook

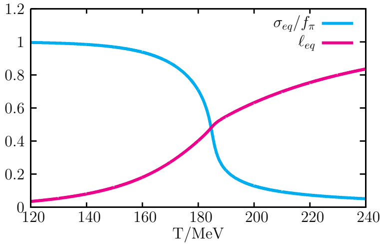
Summary

- Polyakov loop extended chiral fluid dynamics model
- nonequilibrium effects visible:
  - domain formation and growth
  - supercooling and reheating
  - critical slowing down
  - large fluctuations at first order phase transition

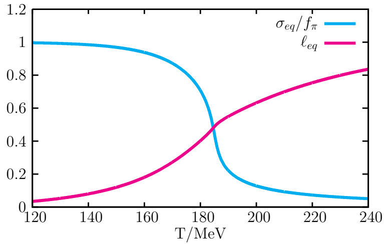
Outlook

- go to finite baryo-chemical potential μ_B
- include pions and study event-by-event fluctuations (Marlene Nahrgang)
- propagate quarks by Vlasov equation (Christian Wesp, Carsten Greiner)

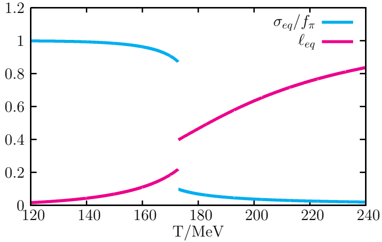
Thermodynamics



crossover



critical point



first order transition

order of the transition at $\mu = 0$
tuned via coupling g
$g = 3.2$ (physical): crossover
$g = 3.5$: critical point
$g = 4.7$: first order

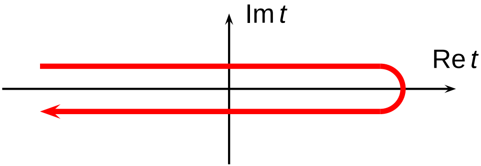
The equations of motion

The influence functional

- split system into relevant (σ) and environmental ($\bar{q}$, q) degrees of freedom
- integrate out the quarks in a path integral over Keldysh contour

$$S_{\mathrm{IF}}\left[\bar{\sigma}, \Delta\sigma\right] = \int \mathrm{d}^4 x D(x) \Delta\sigma(x) + \frac{i}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x) \mathcal{N}(x,y) \Delta\sigma(y)\,,$$

with $\Delta\sigma = \sigma^+ - \sigma^-$ and $\bar{\sigma} = \frac{1}{2}(\sigma^+ + \sigma^-)$ on the CTP contour



The equations of motion

Damping kernel

$$D(x) = ig^2 \int_{y_0}^{x_0} \mathrm{d}^4 y \bar{\sigma}(y) \left[S^<(x-y) S^>(y-x) - S^>(x-y) S^<(y-x) \right]$$

and noise kernel

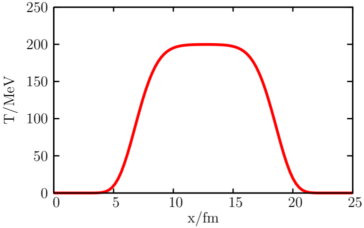
$$\mathcal{N}(x,y) = -\frac{1}{2} g^2 \left[S^<(x-y) S^>(y-x) + S^>(x-y) S^<(y-x) \right]$$

determine equation of motion

$$-\frac{\delta S_{cl}[\bar{\sigma}]}{\delta \bar{\sigma}} - D = \xi \,,$$

$$\langle \xi(x) \xi(y) \rangle = \mathcal{N}(x,y) \,.$$

The expanding plasma: initial conditions



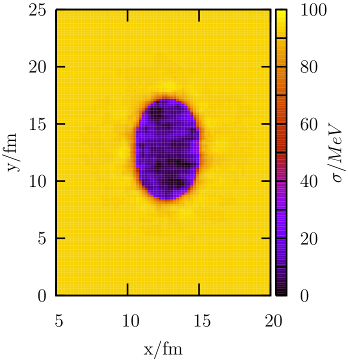
Temperature:
Woods-Saxon distribution

σ-field:

$$\sigma = \sigma_{eq}(T) + \delta\sigma(T)$$

thermal distribution, corr. 1fm

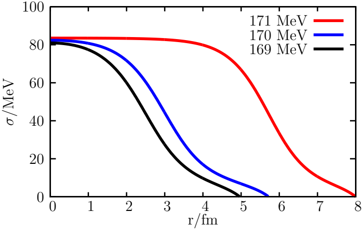
$$\begin{aligned} \ell &= \ell_{eq}(\sigma, T) \\ e &= e(\sigma, \ell, T) \end{aligned}$$

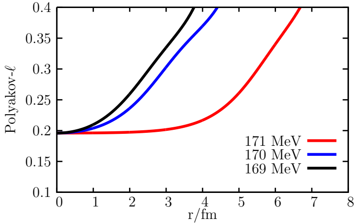


Nucleation: critical bubble profiles

minimize free energy with respect to the fields σ and ℓ

$$F(\sigma, \ell, T) = \int \mathrm{d}^3x \left[\frac{1}{2} (\nabla\sigma)^2 + \frac{N_c}{g_{QCD}^2} (\nabla\ell)^2 T^2 + V_{eff}(\sigma, \ell, T) \right]$$





size of critical bubbles increases significantly near transition temperature

(cf. Scavenius, Dumitru, Fraga, Lenaghan and Jackson, Phys. Rev. D **63** (2001))