

# Non-relativistic bound states across a thermal medium

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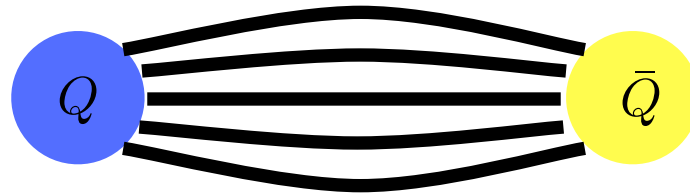
In collaboration with M.A.Escobedo and J. Soto, *Phys.Rev. D84 (2011) 016008*

# Outline

- Heavy quarkonia as a thermometer
- Dissociation processes:  
Debye screening vs. Landau damping
- Moving bound state
- Velocity dependent EFT
- Conclusions

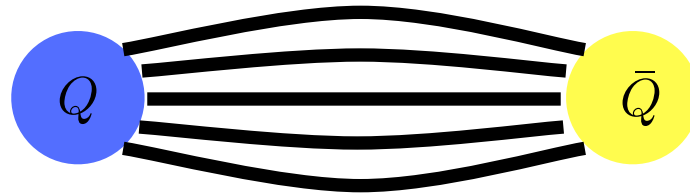
# Dissociation of heavy quarkonia

In vacuum, the static chromo-electric field leads to the formation of a heavy quark bound state. Very similar to (muonic) hydrogen.



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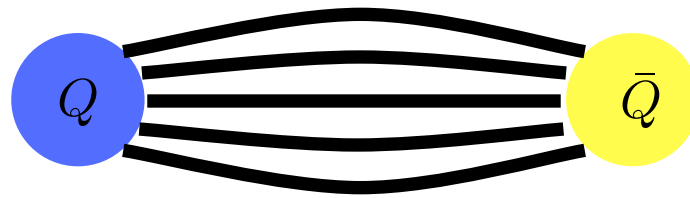
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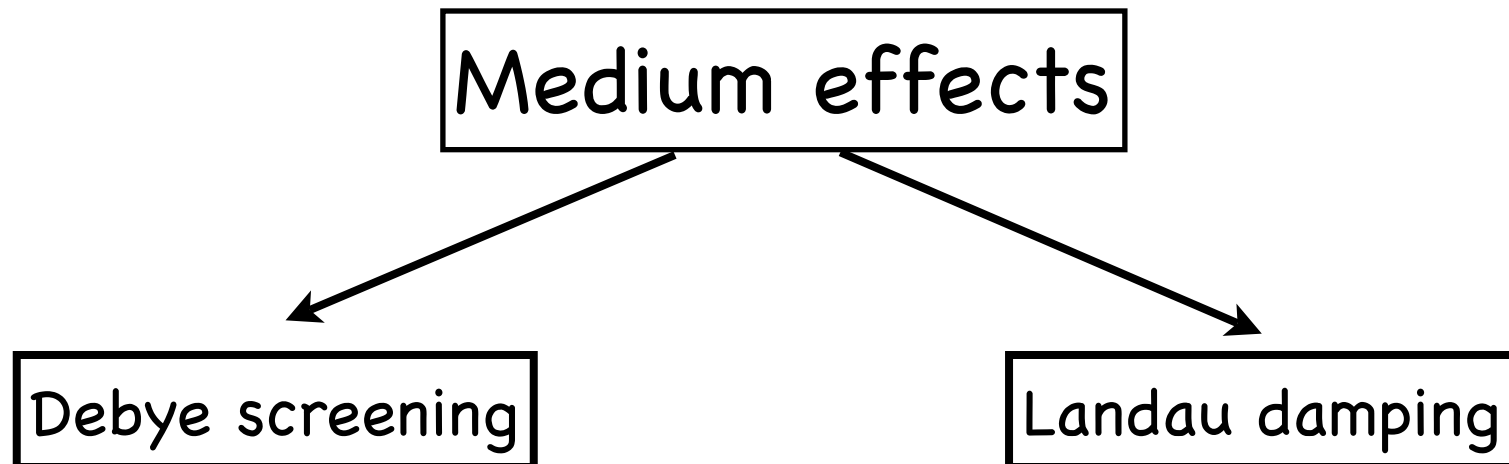
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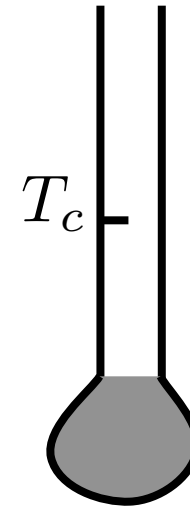
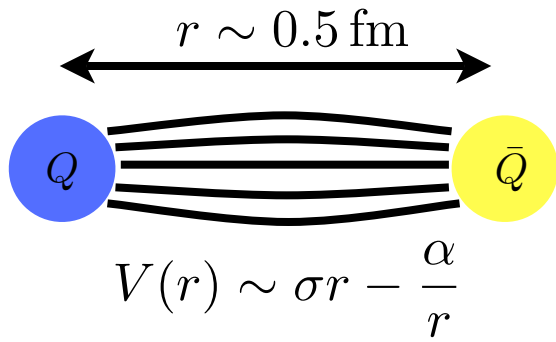
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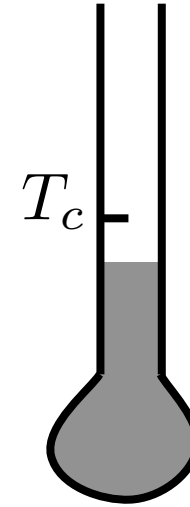
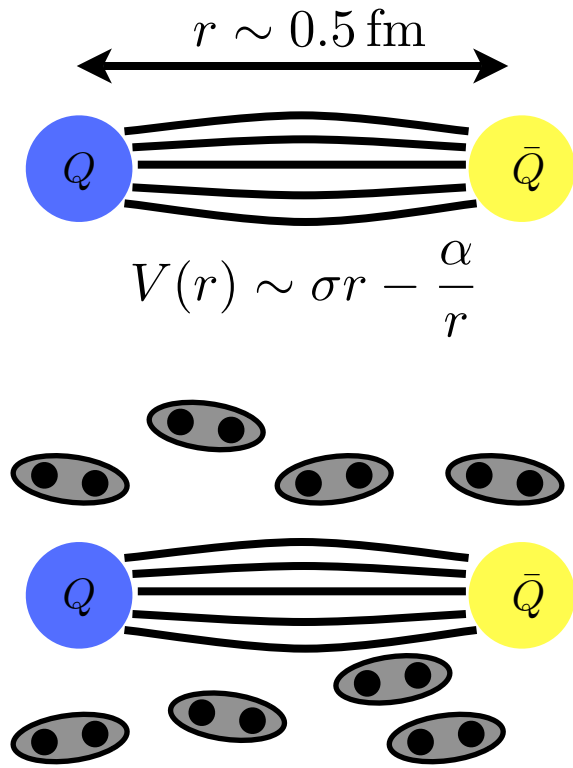
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# Debye Screening

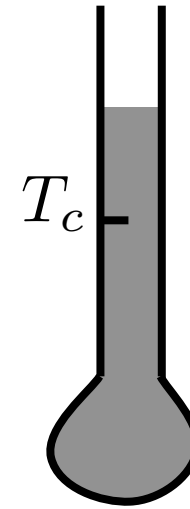
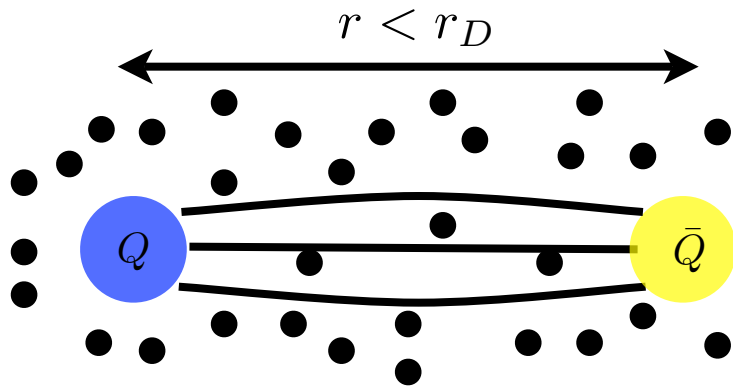
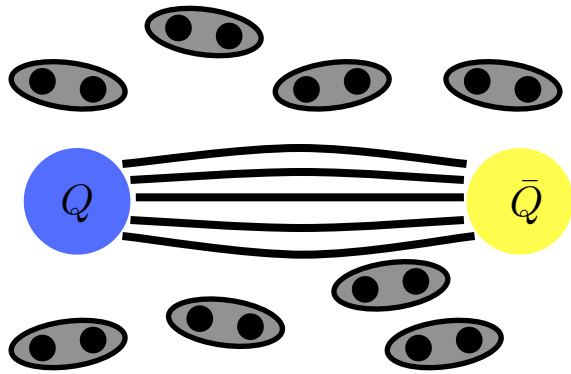
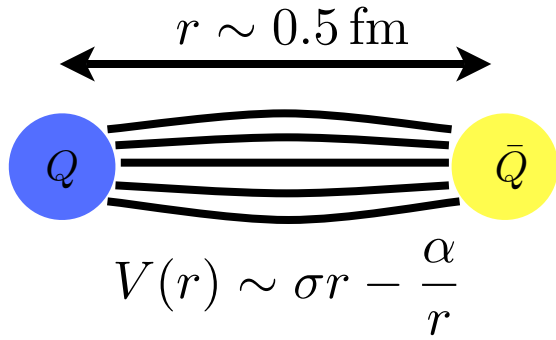


# Debye Screening



$$T < T_c$$

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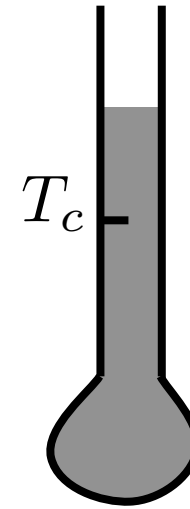
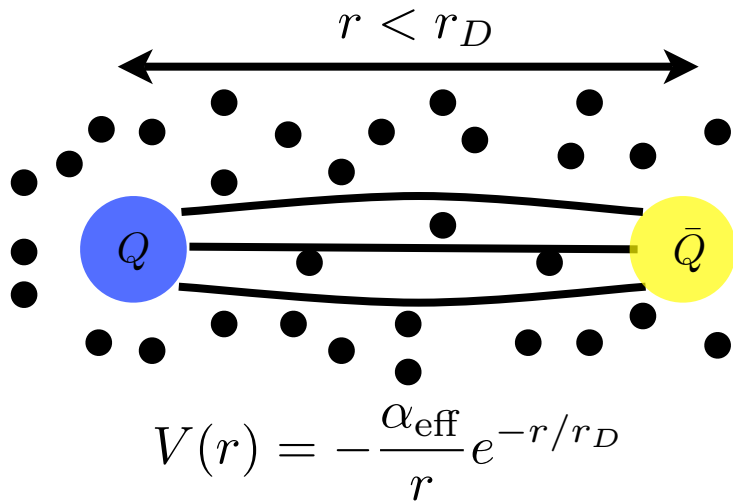
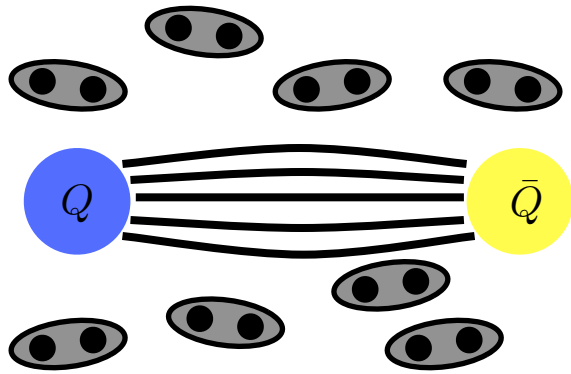
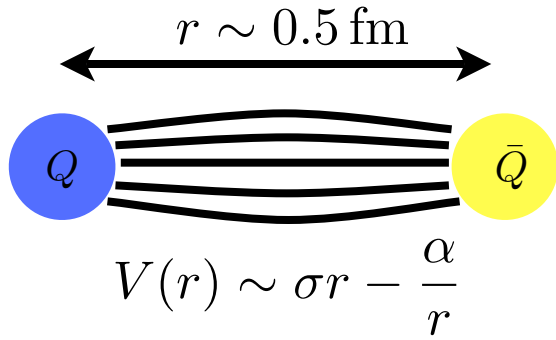


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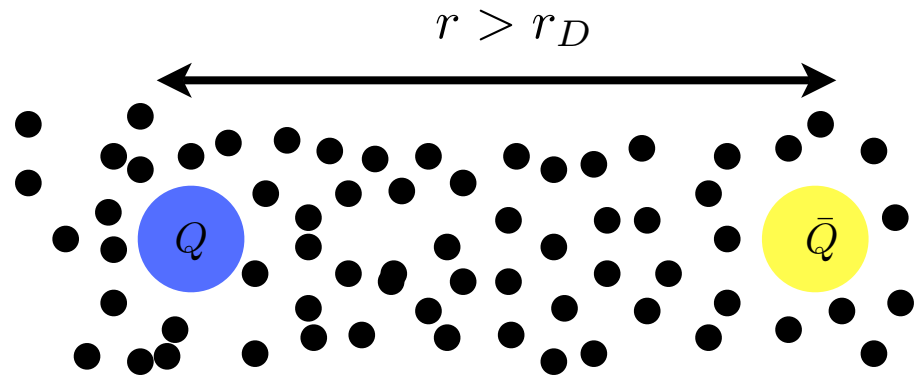
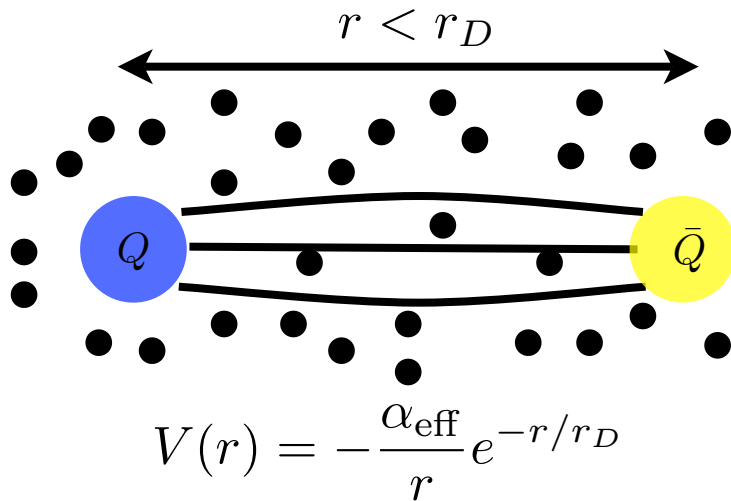
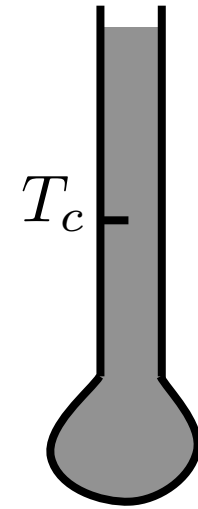
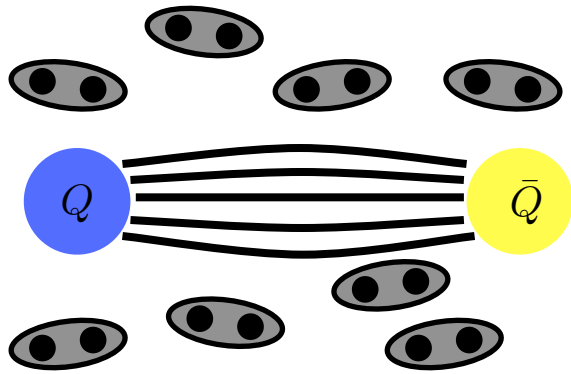
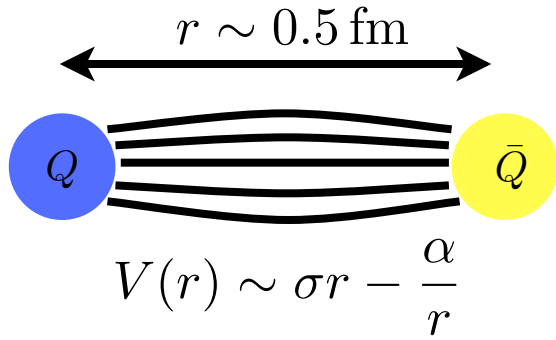
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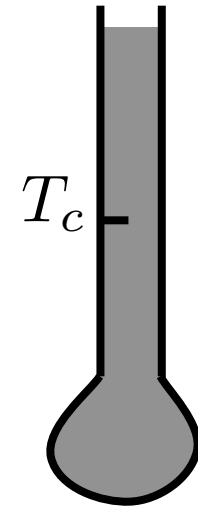
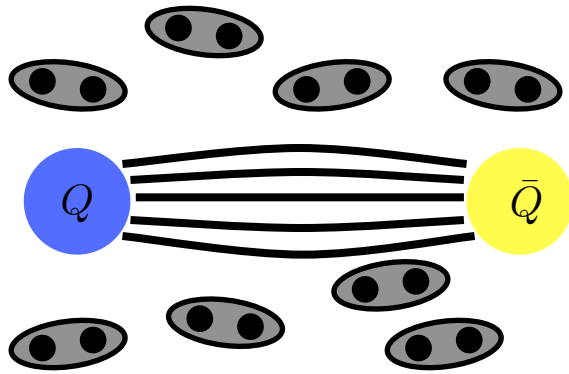
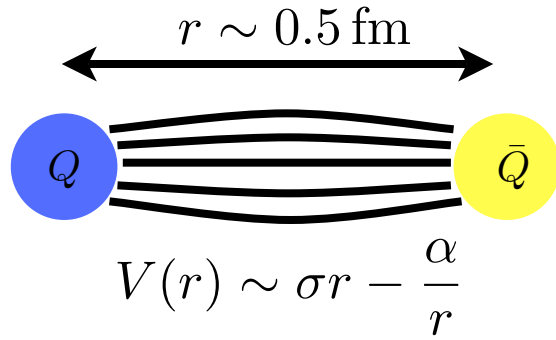
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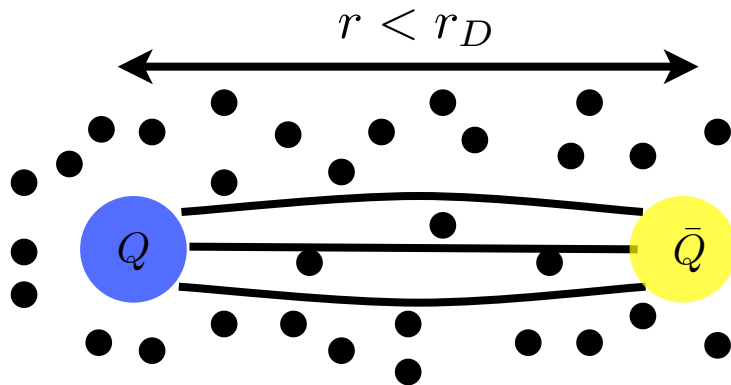
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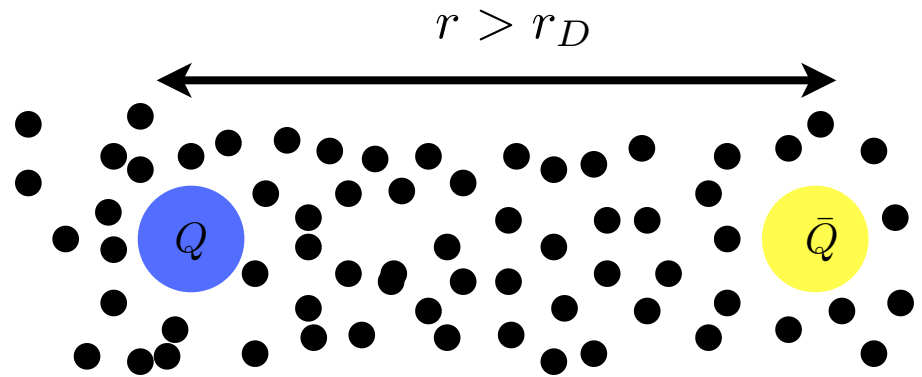
$$T > T_d$$

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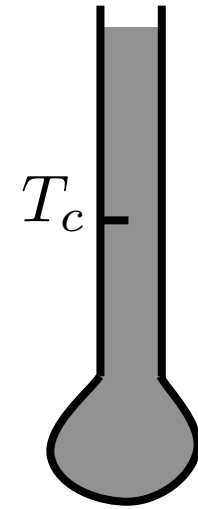
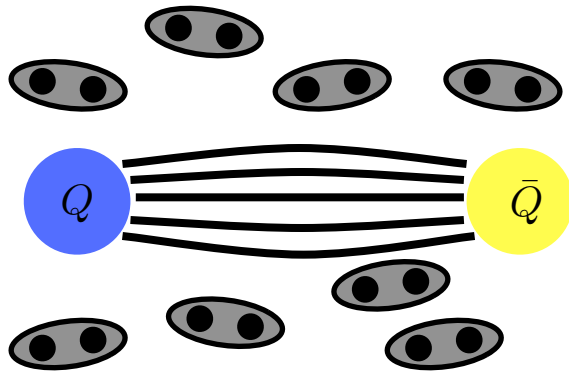
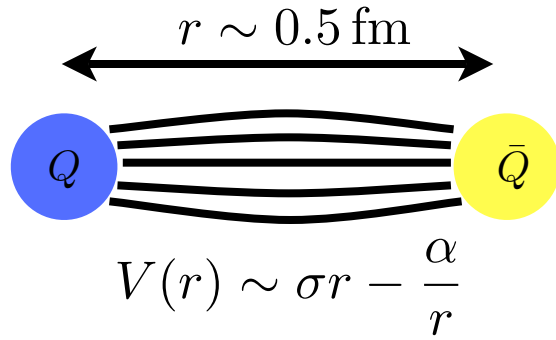


$$V(r) = -\frac{\alpha_{\text{eff}}}{r} e^{-r/r_D}$$



The plasma screens the static chromo-electric fields, leading to unbinding of quarkonium Matsui and Satz (1986).

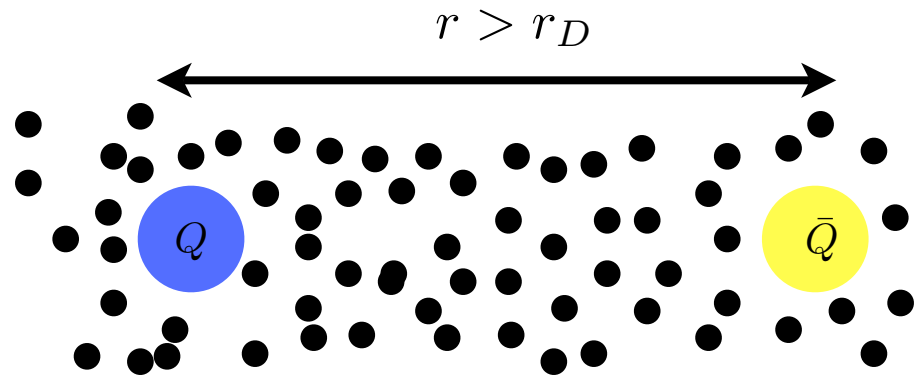
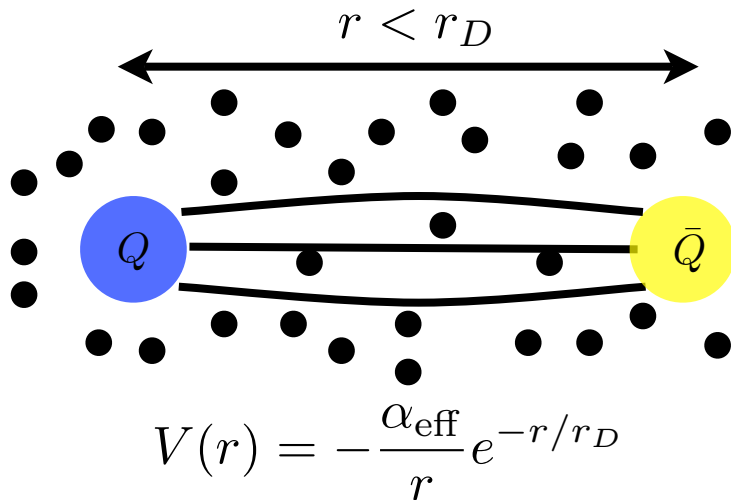
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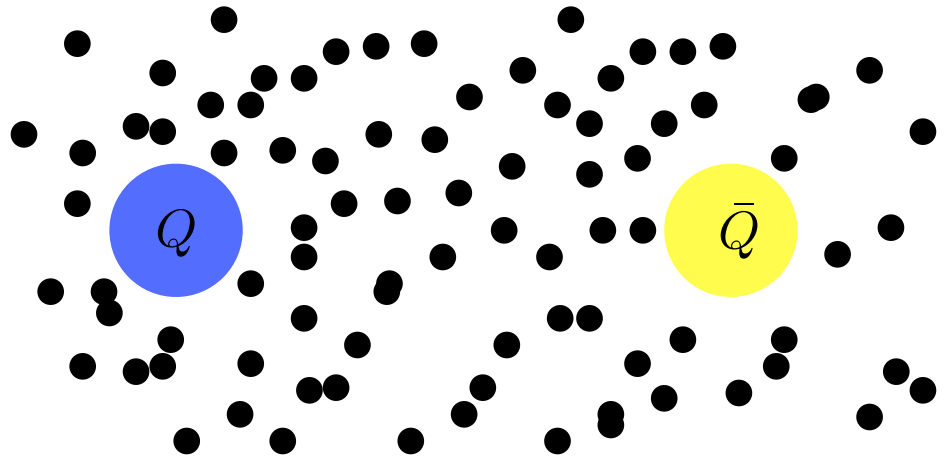
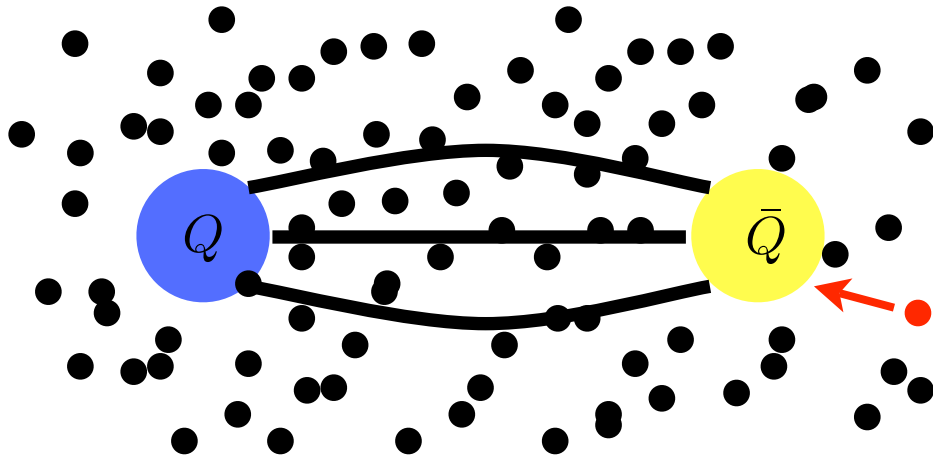


The plasma screens the static chromo-electric fields, leading to unbinding of quarkonium Matsui and Satz (1986). **Dissociation as a thermometer.**

# Landau damping

In a thermal medium, no strictly stationary bound state exists.

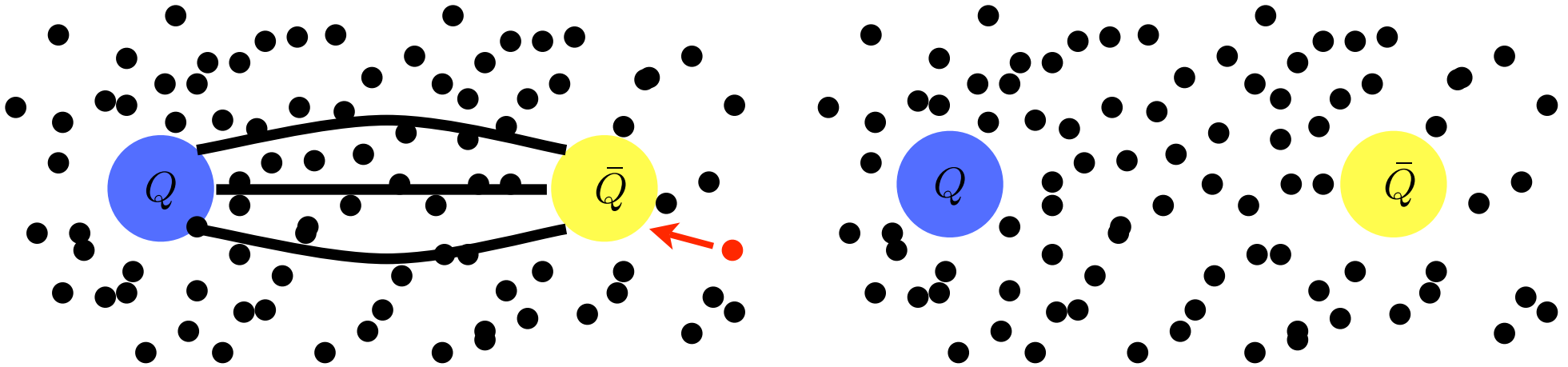
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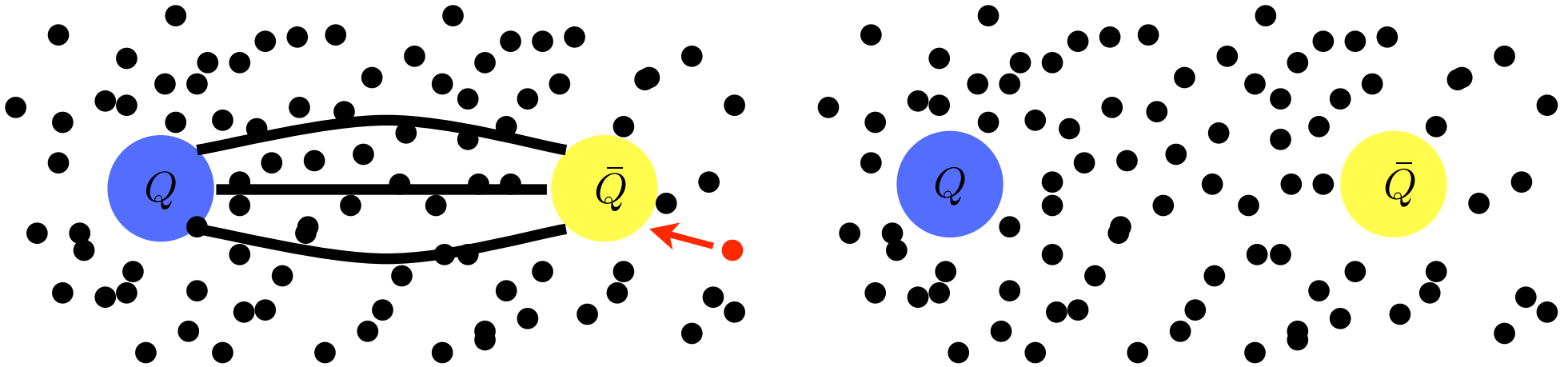


broadening of the energy levels: imaginary part of the energy eigenvalues.  
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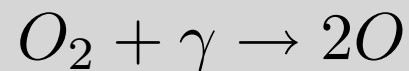
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Analogous to photodissociation of molecules like



in a heat bath.

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Debye screening and Landau damping in principle operate simultaneously.

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## Coulombic bound state

Radius of a bound state

$$r_B \sim \frac{1}{m_Q \alpha_{\text{eff}}}$$

dissociation

$$r_D = 1.2 r_B$$

Debye screening length

$$r_D = \frac{1}{m_D}$$

$$T_d \sim m_Q \frac{\alpha_{\text{eff}}}{g}$$

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Debye screening length	$r_D = \frac{1}{m_D}$	$T_d \sim m_Q \frac{\alpha_{\text{eff}}}{g}$

## Imaginary potential

The real part of the potential is smaller than the imaginary part for

$$1/r \sim k > g^{2/3} T$$

The imaginary part dominates for  $g^{2/3} T > m_D$  Laine et al.  
and leads to dissociation for  $T \gg m_Q C_F g^2 / 16\pi$  hep-ph/0611300

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## Setting:

Plasma (or black-body radiation) in thermal equilibrium at a temperature  $T$  and moving with a velocity  $v$  with respect to the bound state.

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## Previous works:

Dynamic Debye screening considered by M.C. Chu and T. Matsui (1989)

Dynamic Landau damping considered by T. Song et al. (2008) and by F.Dominguez and B.Wu (2009)

# Thermal medium

Distribution function  
of particles in the plasma

$$f(\beta^\mu k_\mu) = \frac{1}{e^{|\beta^\mu k_\mu|} \pm 1}$$

where  $\beta^\mu = \frac{\gamma}{T}(1, \mathbf{v}) = \frac{u^\mu}{T}$

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then

$$f(k, T, \theta, v) = \frac{1}{e^{k/T_{\text{eff}}(\theta, v)} \pm 1}$$

Effective temperature  
(massless particles)

$$T_{\text{eff}}(\theta, v) = \frac{T\sqrt{1-v^2}}{1-v\cos\theta}$$

relative change of temperature as in the Doppler effect

# Light-cone variables

We choose  $v$  in the  $z$  direction and define

$$k_+ = k_0 + k_3 \quad \text{and} \quad k_- = k_0 - k_3$$

Then the distribution function depends on

$$\beta^\mu k_\mu = \frac{1}{2} \left( \frac{k_+}{T_+} + \frac{k_-}{T_-} \right)$$

where

$$T_+ = T \sqrt{\frac{1+v}{1-v}} \quad \text{and} \quad T_- = T \sqrt{\frac{1-v}{1+v}}$$



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In the construction of the effective theory one has to take into account the existence of these scales. In particular for  $v \sim 1$

$$T_- \ll T \ll T_+$$

for more details see Escobedo, MM, Soto **Phys.Rev. D84 (2011) 016008**

# Evaluating the potential

## Temperature range

$$m_Q \gg T \gg \Lambda_{QCD} \gg m_q$$
$$T \gg 1/r$$

HTL approximation, Coulomb gauge.

The potential is obtained by a Fourier transform of the longitudinal gauge boson propagator

$$\Delta_{11}(k) = \frac{1}{2} [\Delta_R(k) + \Delta_A(k) + \Delta_S(k)]$$

where

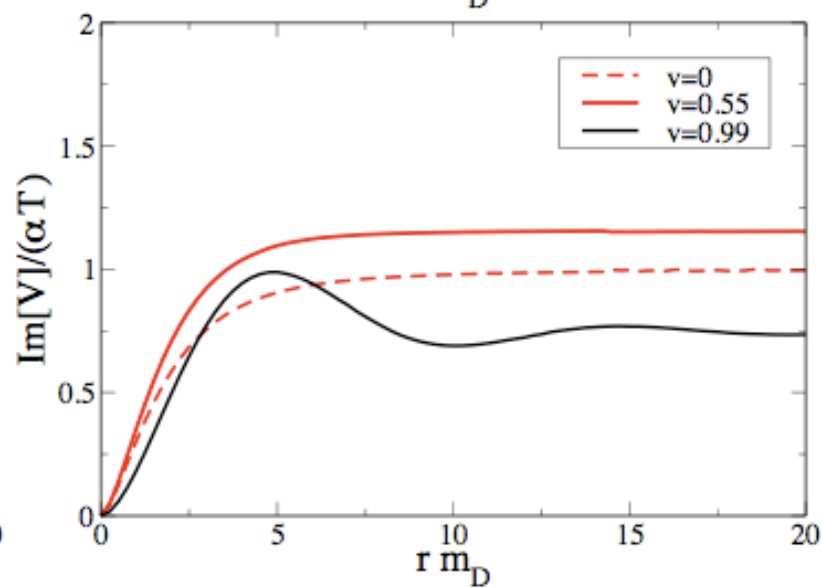
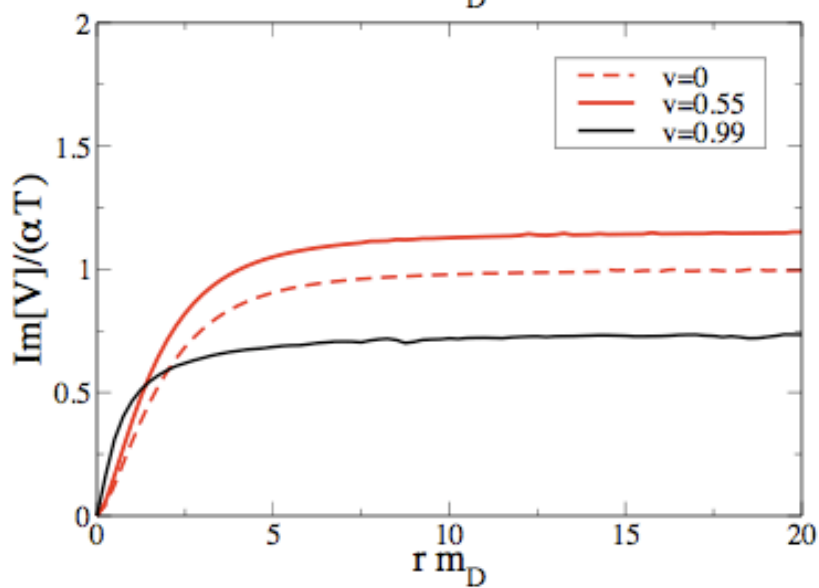
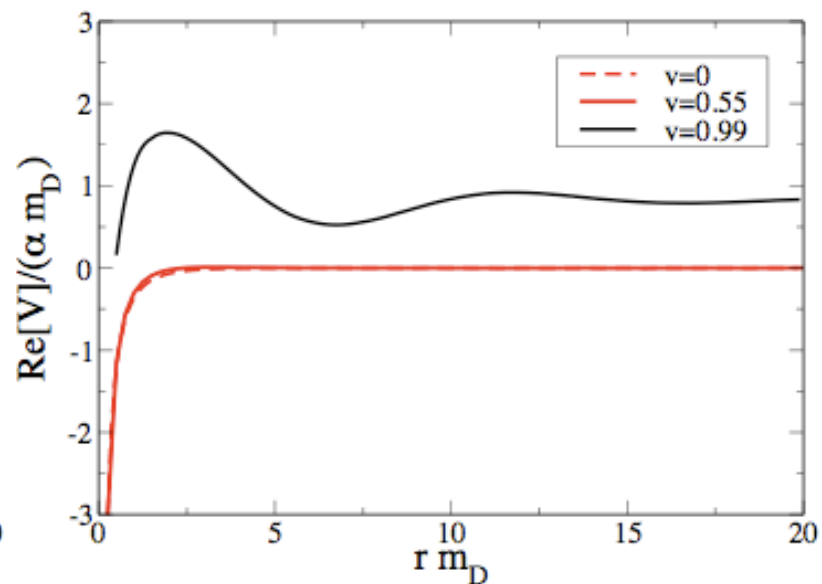
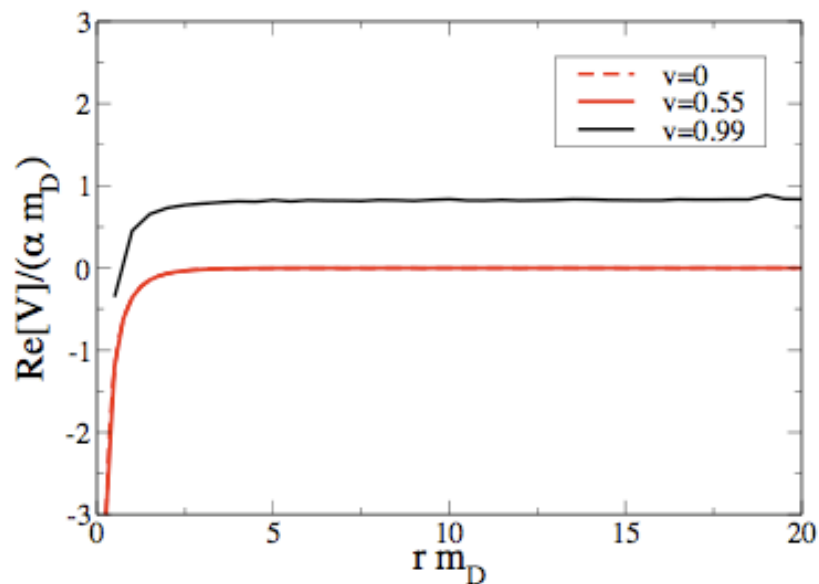
$$\Delta_R^*(k) = \Delta_A(k)$$
$$\Delta_S(k, u) = \frac{\Pi_S(k, u)}{2i\Im\Pi_R(k, u)} (\Delta_R(k, u) - \Delta_A(k, u))$$

# RESULTS

# Potential

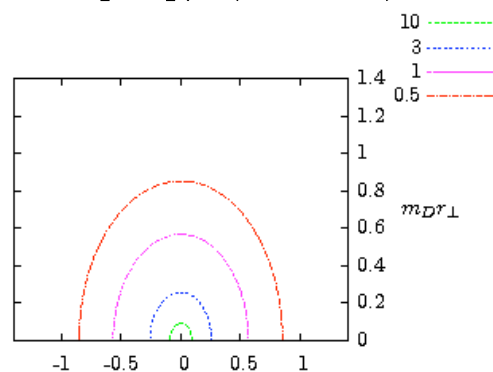
$$\theta = 0$$

$$\theta = \pi/2$$



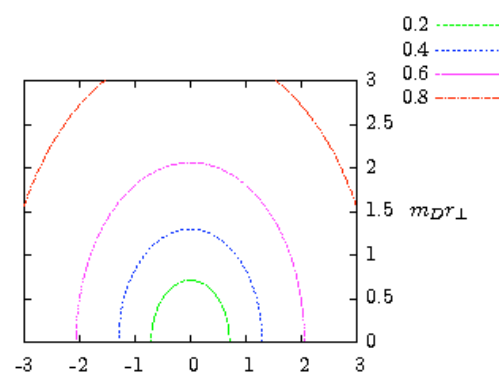
# Contour lines

$\text{Re}[V]/(\alpha m_D)$

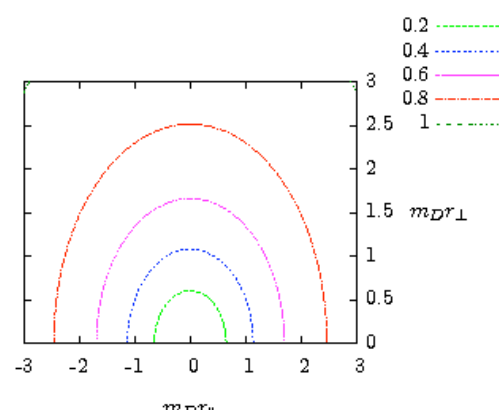
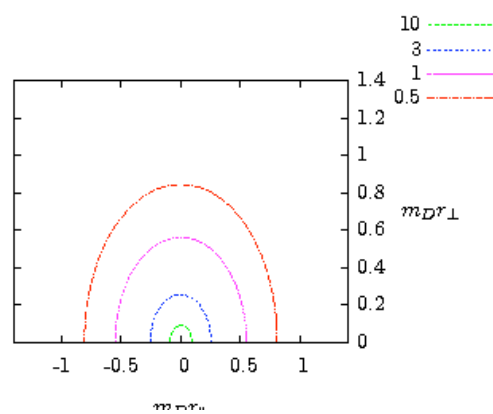


$v = 0$

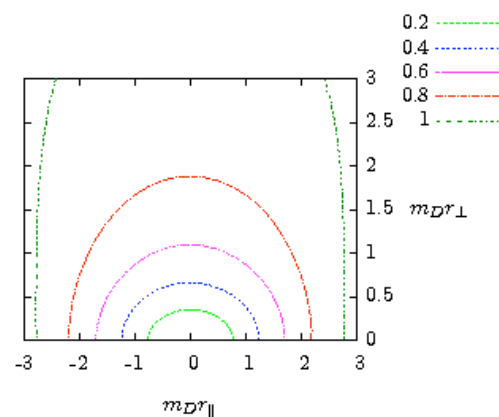
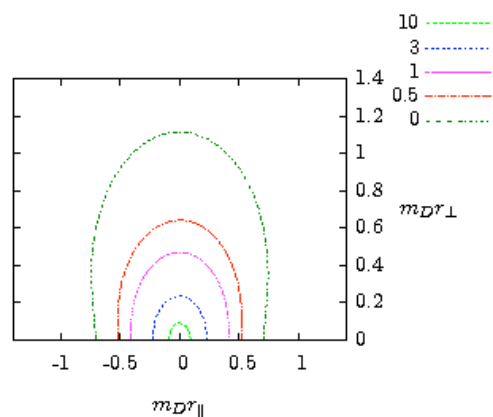
$\text{Im}[V]/(\alpha T)$



$v = 0.5$



$v = 0.95$



# "critical" velocity

	$v = 0$	$v \neq 0$
$m_D$	$gT$	$\sim gT$
$\text{Re}[V] = \text{Im}[V]$	$\bar{k} \simeq g^{2/3}T$	$\bar{k}(v) \simeq g^{2/3}T\sqrt{1-v^2}$

For  $k < \bar{k}(v)$  the real part dominates and if  $\bar{k}(v) < m_D$  the dissociation is due to screening

"critical" velocity:  $v_{cr} = \sqrt{1 - ag^{2/3}}$

If  $v > v_{cr}$  dissociation is due to Debye screening

# Summary

- 🎤 The properties of bound states change in the presence of a thermal medium
- 🎤 In the HTL approximation for a static bound state, Landau damping dominates
- 🎤 We find that there should be a “critical” velocity,  $v_{cr}$ , such that for  $v > v_{cr}$ , Debye screening dominates.
- 🎤 Next step: evaluation of the yield of HQ in a heavy-ion collision