Dyson-Schwinger Studies of Color Superconductivity

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HGS-HIRe for FAIR
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Outline

Motivation

Quark Dyson-Schwinger equation

Medium modifications and color superconductivity

Results

Summary and outlook

Motivation

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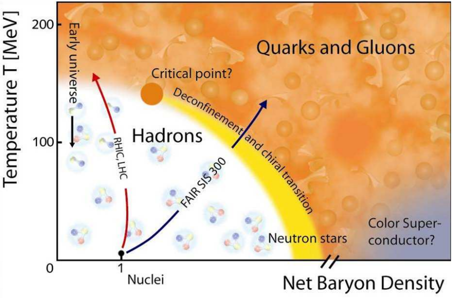


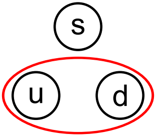
GSI Darmstadt

- high densities: color superconducting regime
- studies with NJL-models (Alford, Rajagopal, Wilczek (1998), Rapp, Schäfer, Shuryak, Velkovsky (1998)) and weak coupling methods (Schäfer, Wilczek (1999), Pisarski, Rischke (1999))
- Dyson-Schwinger equations at $T = 0$ (Nickel, Wambach, Alkofer (2006))
- our aim: DSE studies at finite T and μ, phase diagram

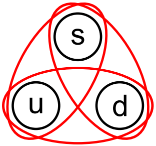
Motivation 2: Color Superconductivity

Cooper instability

- fermionic system + attractive force $\rightarrow$ Cooper pairs
- in QCD: diquarks $\langle q^T C \mathcal{O} q \rangle$
- most important phases:



2sc phase
high m_s / low μ



cfl (-like) phase
low m_s / high μ

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Dyson-Schwinger equations (DSEs)

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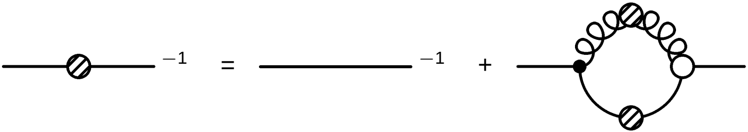
Quark DSE

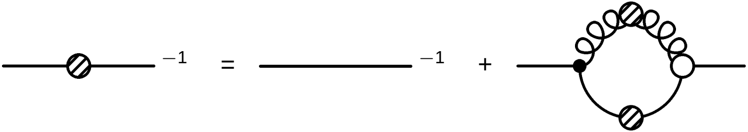


$$S^{-1}(p) = Z_2 \left(S_0^{-1}(p) + \Sigma(p) \right)$$

$$Z_2 \Sigma(p) = Z_{1F} g^2 \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu \frac{\lambda_a}{2} S(q) \Gamma_\nu^b(p,q) D_{\mu\nu}^{ab}(k = p - q)$$

Dyson-Schwinger equations (DSEs)

Quark DSE



$$S^{-1}(p) \quad = \quad Z_2\left(S_0^{-1}(p) \quad + \quad \Sigma(p)\right)$$

$$Z_2\Sigma(p) \;=\; Z_{1F}g^2\int\frac{d^4q}{(2\pi)^4}\gamma_\mu\frac{\lambda_a}{2}S(q)\Gamma^b_\nu(p,q)D^{ab}_{\mu\nu}(k=p-q)$$

- need: gluon propagator and full quark gluon vertex $\rightarrow$ gluon DSE, vertex DSE
- infinite tower of equations $\rightarrow$ truncation

Gluon Truncation

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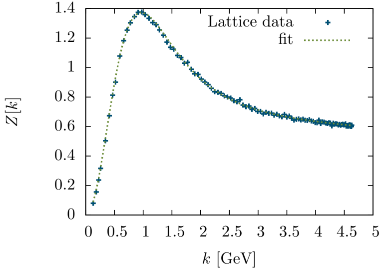
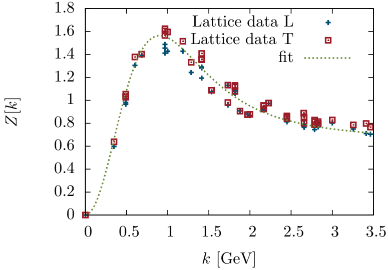
Gluon propagator (Landau gauge):

$$D_{\mu\nu}^{ab}(k) = \delta^{ab} \left(\frac{Z_T(k^2)}{k^2} P_{\mu\nu}^T(k) + \frac{Z_L(k^2)}{k^2} P_{\mu\nu}^L(k) \right)$$



vacuum Lattice data

(Sternbeck et al (2006))



medium Lattice data (shown at $T = 0$)

(Fischer, Maas, Müller (2010))

Vertex Truncation

- abelian vertex construction:

$$\Gamma^a_\mu(p,q;k) \to g\Gamma(k^2)\gamma_\mu \frac{\lambda^a}{2}$$

- Ansatz for the dressing function $\Gamma(k)$
- correct UV behaviour + some infrared strength
- strong running coupling:

$$\alpha_s(k^2) = \frac{Z_{1F}}{Z_2^2} g^2 \Gamma(k^2) Z(k^2)$$

Vacuum results

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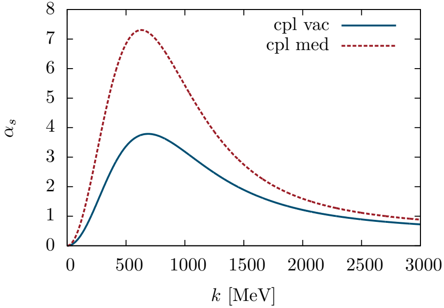
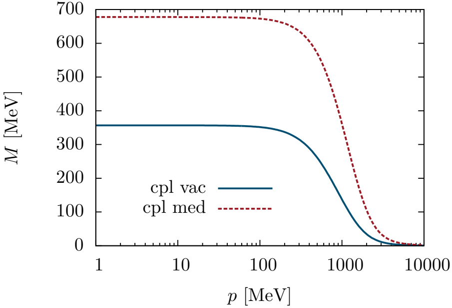
Dirac structure: $S^{-1}(p) = -i\not{p}A(p) + B(p), \quad M(p) = B(p)/A(p)$



strong running coupling



quark mass function

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Gluon Polarization 1

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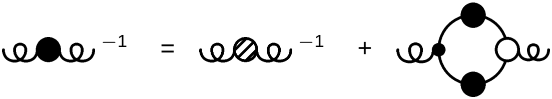
Gluon DSE (truncated)

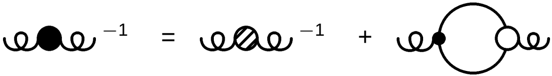


$$D^{-1,ab}_{\mu\nu}(k) = D^{-1,ab}_{\mu\nu,qu}(k) + \Pi^{ab}_{\mu\nu}(k)$$

$$\Pi^{ab}_{\mu\nu}(k) = -Z_{1F} 2\pi\alpha_s(\nu) \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr} \left(\Gamma^{(0)a}_{NG,\mu} \mathcal{S}(p = k + q) \Gamma^{b}_{NG,\nu}(p, q) \mathcal{S}(q) \right)$$

$$D_{\mu\nu}(k) = \frac{Z_T(k^2)}{k^2 + G(k)} P^T_{\mu\nu}(k) + \frac{Z_L(k^2)}{k^2 + F(k)} P^L_{\mu\nu}(k)$$

Gluon Polarization 2



HDL / HTL - like approximation:

- bare quark propagators
- large Temperatures or chemical potentials $T, \mu \gg k$
- (constant) vacuum parts absorbed in renormalization

$$F(k_4, |\vec{k}|) = 2m_g^2(k) \cdot ..., \quad G(k_4, |\vec{k}|) = \frac{\pi}{2} m_g^2(k) \frac{k_4}{|\vec{k}|} \cdot ...$$

$$m_g^2 = \alpha_s(k^2) \left(\frac{N_f \mu^2}{\pi} + \frac{N_f T^2 \pi}{3} \right)$$

Color superconductivity in the Dyson-Schwinger framework

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Nambu Gor'kov formalism

- define bispinors $\Psi = \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix}$, $\bar{\Psi} = \begin{pmatrix} \bar{\psi} & \psi^T C \end{pmatrix}$
- $\mathcal{S}_0 = \begin{pmatrix} S_0^+ & 0 \\ 0 & S_0^- \end{pmatrix}$, $\mathcal{S} = \begin{pmatrix} S^+ & T^- \\ T^+ & S^- \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma^+ & \Phi^- \\ \Phi^+ & \Sigma^- \end{pmatrix}$
- T, Φ: anomalous propagators / self energies, representing color superconducting phases
- new set of quark DSE's

$$(Z_2 S^+)^{-1} = \left((S_0^+)^{-1} + \Sigma^+ - \Phi^- \left((S_0^-)^{-1} + \Sigma^-\right)^{-1} \Phi^+\right)$$

$$T^+ = -\left((S_0^-)^{-1} + \Sigma^-\right)^{-1} \Phi^+ S^+$$

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Results in the chiral limit

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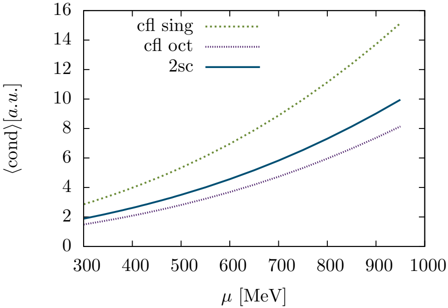
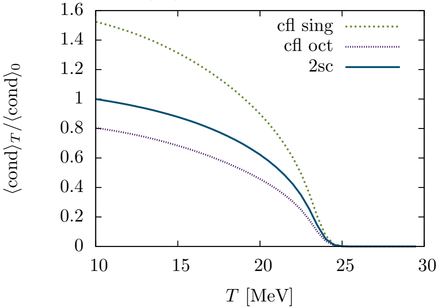
CSC condensates: $\langle q^T C \mathcal{O} q \rangle = -Z_{1F} Z_2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\, [\mathcal{O} T^+]$



dependence of csc condensates on the chemical potential ($T = 0$)



dependence of csc condensates on the temperature ($\mu = 500$)

Results at finite m_s

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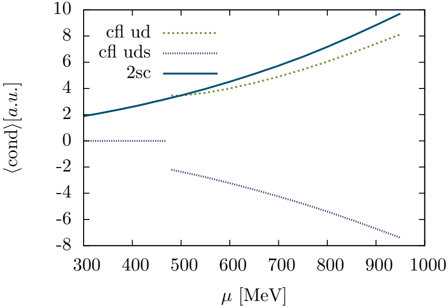
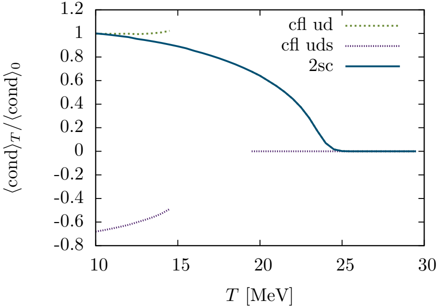
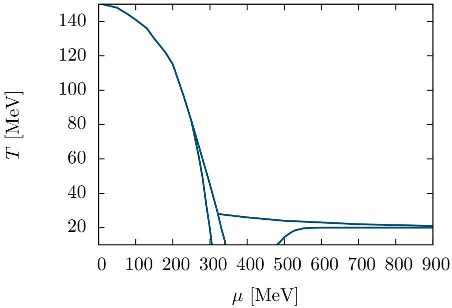


dependence of csc condensates on the chemical potential ($T = 0$)

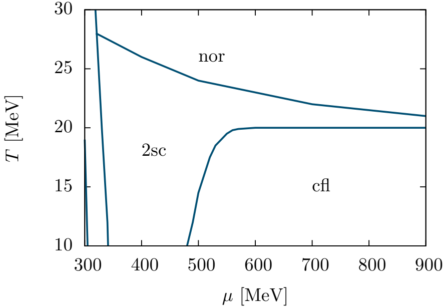


dependence of csc condensates on the temperature ($\mu = 500$)

Phase diagram

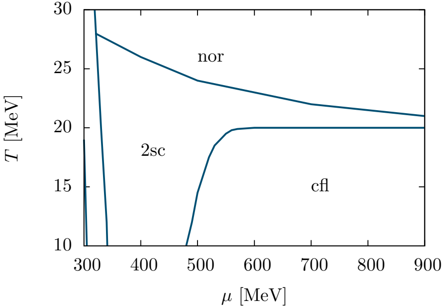


full diagram

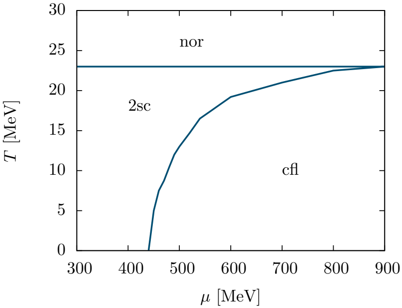


zoom to csc phases

Phase diagram



Coupling: medium fit



Coupling: vacuum fit

Vacuum results with full gluon

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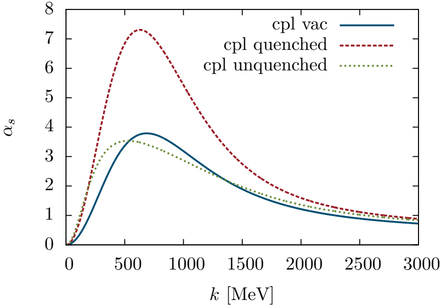
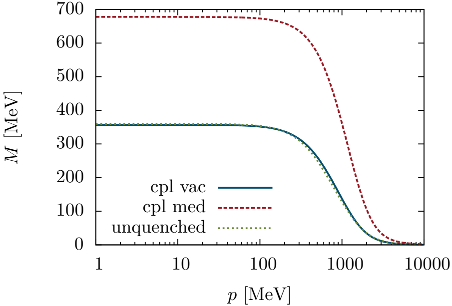
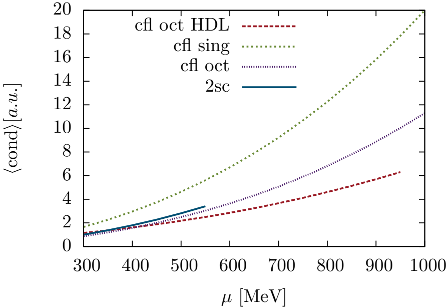


"effective" running coupling $k^2 \frac{\alpha_s}{k^2+\Pi(k)}$

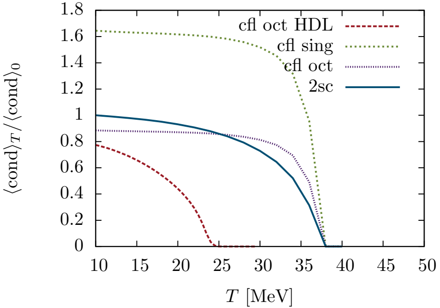


quark mass function

chiral limit

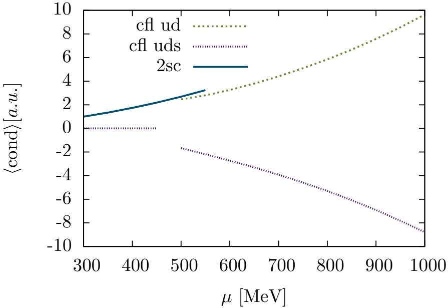


dependence of csc condensates on the chemical potential (T = 0)

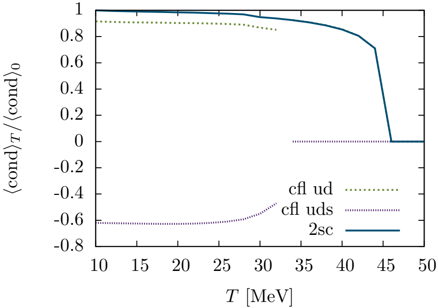


dependence of csc condensates on the temperature (μ = 500)

finite strange quark mass



dependence of csc condensates on the chemical potential ($T = 0$)



dependence of csc condensates on the temperature ($\mu = 500$)

Summary and Outlook

Summary

- Color superconductivity with Dyson-Schwinger equations
- large cfl-phase for $\mu > 500$ MeV
- 2sc phase at finite T
- beyond HDL: larger condensates and critical temperatures

Outlook

- calculate phase diagrams with improved gluon (and other things)
- improvement of the vertex?
- inhomogenous phases?
- ...

THANK YOU