Two-Color Quark-Meson-Diquark Model beyond Mean Field

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based on work together with
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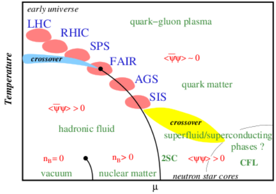
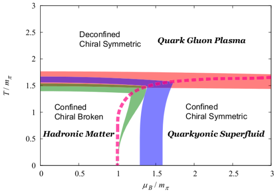
Motivation

Why 2-color QCD?

- sign problem in 3-color QCD
- 3-color QCD with isospin chemical potential
- relativistic analog of models for BEC BCS crossover in ultracold atomic gases
- illustration for importance of baryonic d.o.f.s
- interesting phase structure: BEC-BCS crossover, quarkyonic phase?



[SCHAEFER DELTA MEETING '10]



[BRAUNER, FUKUSHIMA & HIDAKA '09]

Special Features of 2-color QCD

Why is 2-color QCD different from real QCD?

- SU(2) fundamental representation is pseudoreal: $\sigma_a^T = -\sigma_2 \sigma_a \sigma_2$

Consequences:

- antiunitary symmetry of Dirac operator $\mathcal{D}(\mu)$: no sign problem
  $\mathcal{D}(\mu) T_2 C \gamma^5 = T_2 C \gamma^5 \mathcal{D}^*(\mu)$
- enlarged flavor (Pauli Gürsey) symmetry (at $\mu = 0$): SU($2N_f$)
  instead of $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ allowing to rotate ψ_R^C into ψ_L
- color-neutral bound states of two quarks: bosonic baryons
- more complex pattern of symmetry breaking
  Coset S^5: 5 (pseudo) Goldstone bosons (3 pions + 2 diquarks)

Patterns of Symmetry Breaking

2-color 2-flavor QCD: [Kogut et al '00]

$\mu = 0 \quad m = 0$
$SU(4) \simeq SO(6)$

$\mu \neq 0$ →

$\mu \neq 0 \quad m = 0$
$SU(2)_L \times SU(2)_R \times U(1)$

↓ $\langle q\bar{q} \rangle \neq 0$

$\mu = 0 \quad m \neq 0$
$Sp(2) \simeq SO(5)$

↓

$\mu \neq 0 \quad m \neq 0$

$\langle q\bar{q} \rangle \neq 0 \quad \langle qq \rangle = 0$
$SU(2)_V \times U(1)$

$\langle q\bar{q} \rangle \neq 0 \quad \langle qq \rangle \neq 0$
$SU(2)_L \times SU(2)_R$

QMD Model Lagrangian

- 6-plet $\phi = (\sigma, \vec{\pi}, \mathrm{Re}\Delta, \mathrm{Im}\Delta)$ coupled to SO(6) vector of quark bilinears in a linear sigma model Lagrangian

Quark-meson-diquark (QMD) model Lagrangian

$$\mathcal{L}_{\mathrm{QMD}} = \bar{\psi}\left(\not{D} + g(\sigma + \mathrm{i}\gamma^5 \vec{\pi}\vec{\tau})\right)\psi + \frac{g}{2}\left(\Delta^*(\psi^T C\mathrm{i}\gamma^5 \tau_2 T_2 \psi) + \Delta(\psi^\dagger C\mathrm{i}\gamma^5 \tau_2 T_2 \psi^*)\right)$$

$$+ \frac{1}{2}\sum_{i=1}^{4} \partial_\mu \phi_i \partial^\mu \phi_i + \frac{1}{2} D_\mu \Delta D^\mu \Delta^* + V(\phi_i),$$

$$D_\mu \psi = (\partial_\mu - \mu\delta_{\mu 0})\psi, \quad D_\mu \Delta = (\partial_\mu - 2\mu\delta_{\mu 0})\Delta$$

- c.f. NJL Lagrangian after HS transformation [Ratti Weise '04]
- both quarks and (bosonic) diquarks couple to chemical potential
- parameter fixed via N_c scaling e.g. $f_\pi = 76$ MeV, $m_\pi = 138$ MeV

QMD Mean Field

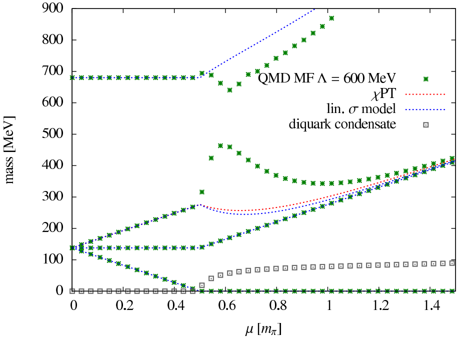
- Grand Potential:

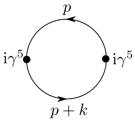
$$\Omega = -4 \sum_{i=\pm} \int \frac{d^3k}{(2\pi)^3} E^i_{\vec{p}} + 2T \log\left(1 + \exp(-E^i_{\vec{p}}/T)\right) + V_{MF},$$

where $$E^{\pm}_{\vec{p}} = \sqrt{g^2|\Delta|^2 + \left(\sqrt{g^2\sigma^2 + \vec{p}^2} \pm \mu\right)^2}$$

- Pole mass vs. screening mass e.g. pion mass:

$$\omega^2 = \frac{\partial^2 V}{\partial \pi_i \partial \pi_i} + M_\pi(\omega^2, T), \quad M_\pi((k^0)^2, T) = \mathrm{Tr}_p \left[i\gamma^5 G_{MF}(p) i\gamma^5 G_{MF}(p+k) \right]$$





- $T = 0$ pole mass spectrum consistent with Silver Blaze property

[Cohen '03]

Functional RG

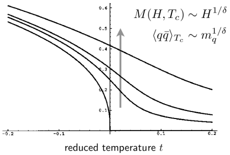
- Effective average action Γ_k interpolates between microscopic bare action $\Gamma_{k=\Lambda}$ and full quantum effective action $\Gamma_{k=0}$
- RG flow equation:[WETTERICH '93]

$$\partial_t \Gamma_k = k\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

- Truncation: local potential approximation (LPA)
  only scale dependence via effective potential U_k parametrized by $\rho^2 = \sigma^2 + \vec{\pi}^2$ and $d^2 = |\Delta|^2$
- Numerical solution of $\partial_t U_k = F(U_r, U_d, U_{rr}, U_{rd}, U_{dd})$
  by discretizing $U_k(\rho^2, d^2)$ on a 2d grid in field space

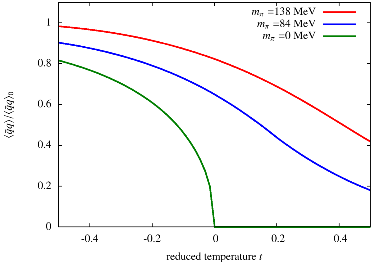
Critical Exponents ($\mu = 0$)

- 3-color 2-flavor QCD:
  - $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1) \to \mathrm{SU}(2)_V \times \mathrm{U}(1)$
  - O(4) universality class [RAJAGOPAL, WILCZEK '93]



$H,\ m_q = 0$:

$M_0(T) \sim \langle \bar{q}q\rangle_T \sim (-t)^\beta$



- 2-color 2-flavor QCD ($\mu = 0$):
  - $\mathrm{SO}(6) \to \mathrm{SO}(5)$
  - O(6) universality class
  - Critical exponents β, δ:

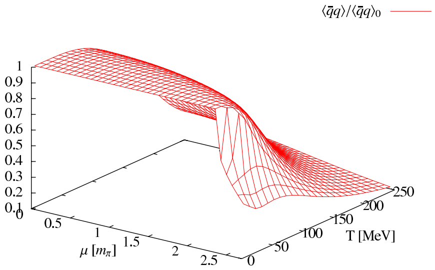
$$\langle q\bar{q}\rangle_T \sim (-t)^\beta \quad \langle q\bar{q}\rangle_{T_c} \sim (m_q)^{1/\delta}$$

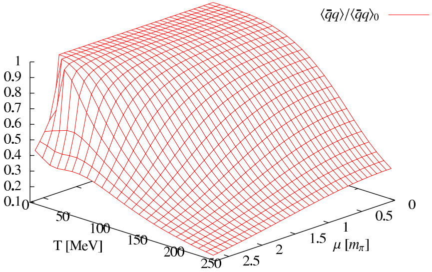
$$\beta = 0.4318(4) \quad \delta = 5.08(8)$$

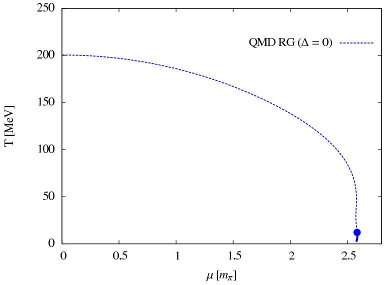
Literature values:
RG, LPA: $\beta = 0.4322 \quad \delta = 5$ [LITIM '02]
Lattice: $\beta = 0.425(2) \quad \delta = 4.77(2)$

[HOLTMANN, SCHULZE '03]

QMD RG, SU(4) symmetric

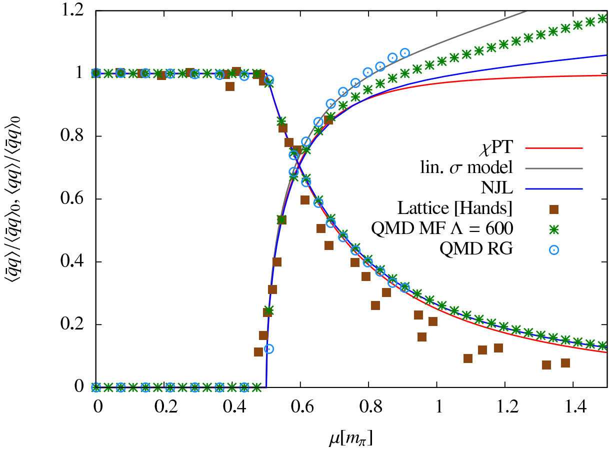






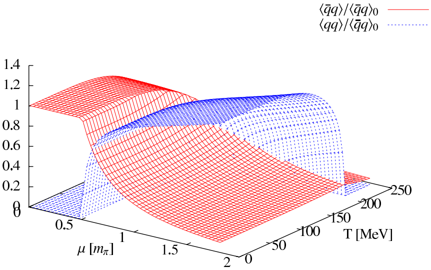
- enforcing $d = 0$ by hand leaves us with 1d flow equation
- typical phase diagram of 3-color QM models
- CEP at $\mu \approx 2.5 m_\pi$?

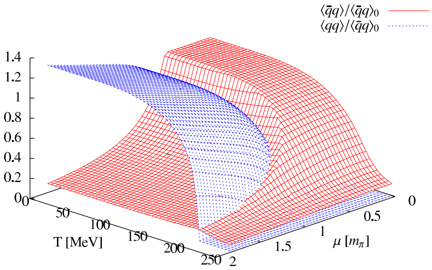
Condensates ($T = 0$)

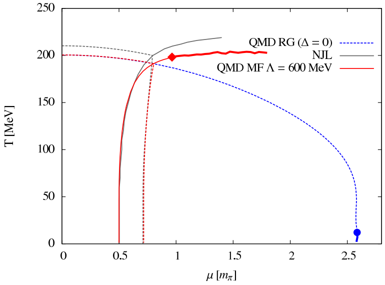


- importance of baryonic degrees of freedom:
  - onset of diquark condensation at $\mu = \frac{m_\pi}{2}$
  - no CEP
- χPT, NJL, Lattice, QMD model agree for small μ

Phase Diagram QMD MF

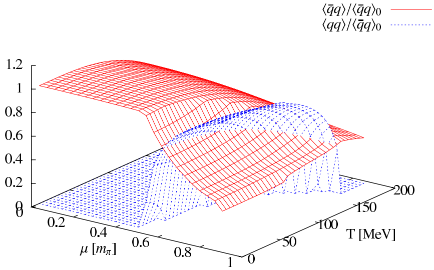


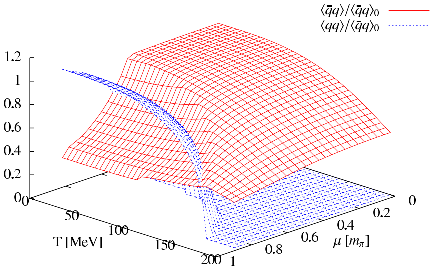


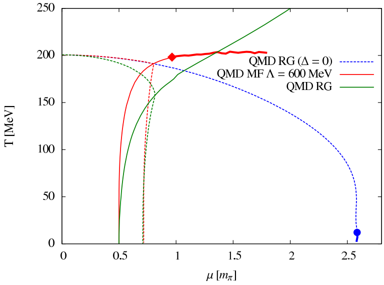


- tricritical point as found in NLO χPT [SPLITTORF ET AL '02] but none in NJL [BRAUNER ET AL '06]
- mean field induced first order transition?

Phase Diagram QMD RG

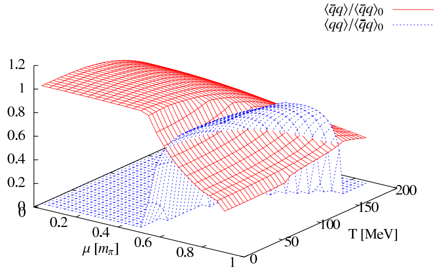


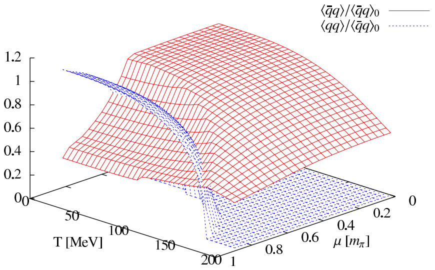


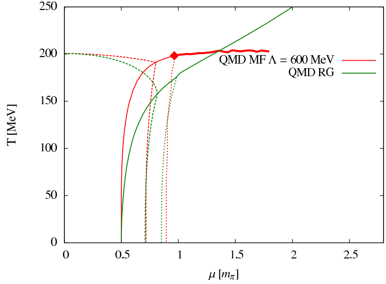


- all transitions now 2nd order or crossover
- fluctuations wash out sharp transitions found in mean field approximation

Phase Diagram QMD RG







- all transitions now 2nd order or crossover
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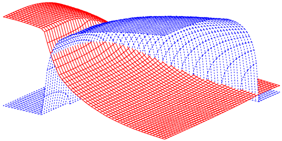
Summary and Outlook

Summary:

- quark-meson-diquark model for 2-color QCD
- MF and FRG flow equations in LPA
- mass definition and the Silver Blaze problem
- numerical solution: phase diagram
- impact of baryonic d.o.f. for the phase diagram



Outlook:

- coupling to gauge sector
- Silver Blaze property in RG setting
- baryonic degrees of freedom in 3-color QCD

Thank you for your attention!